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Forecasting the capacity of LTE mobile networks

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A dissertation presented in partial fulfillment of the requirements for the degree of Master in Computer Science and Business Management

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Abstract

The ever increasing usage of networks around the world made the telecommunication companies to start planning ahead out of necessity.

The present work is focused on analysing and understanding which of the tested predictive models best suits Long Term Evolution (LTE) behaviour regarding its capacity, by forecasting several Key Performance Indicators (KPI) originated from network daily cells and dedicated to the same subject.

Many were the tested models, ranging from the benchmark models (which comprise naïve, seasonal naïve and drift), to Exponential Smoothing (ES), AutoRegressive Integrated Moving Average (ARIMA), Theta and Linear Regression and also including models used in the latest M4 competition.

The inherent purpose was not to find a model that was definitely better than the remaining, but instead to understand which model can best serve the KPI under analysis and the predicted forecasted horizon.

The present study forecasts and analyses several different models in order to achieve better predictive results so that telecommunication companies can make more informed decisions regarding network planning.

Keywords: Forecasting, analytics, predictive models, time series, data science

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Resumo

O contínuo aumento global da utilização das redes de telecomunicação fez com que para os operadores de telecomunicações o planeamento deste tipo de infraestrutura fosse uma necessidade a considerar atempadamente.

O presente estudo é focado na análise e compreensão sobre qual o modelo preditivo que melhor se adapta ao comportamento da rede Long Term Evolution (LTE) no que respeita à previsão da sua capacidade, ao calcular os valores futuros de vários indicadores de performance (KPI) originados por células, com frequência diária.

Foram testados vários modelos, que incluem não apenas modelos de referência como naïve, seasonal naïve e drift, mas também Exponential Smoothing (ES), AutoRegressive Integrated Moving Average (ARIMA), Theta e o modelo Regressão Linear. Contudo, este estudo contou também com a utilização de outros modelos provenientes da competição M4.

O propósito deste trabalho não é o de encontrar um modelo que se destaque de todos os outros nas várias previsões feitas, mas em vez disso compreender qual o modelo que melhor pode prever os futuros valores de um determinado KPI.

Este trabalho analisa as várias previsões feitas pelos modelos estudados de forma a poder obter valores mais fidedignos para que dessa forma as operadores do mercado, possam tomar decisões mais bem informadas.

Palavras Chave: Previsões, analytics, modelos preditivos, series temporais, data science

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Acknowledgements

Throughout the development of this dissertation I received a great deal of support and assistance.

Firstly, I would like to thank my supervisors, Dr. João Pedro Oliveira and Dr. João Bastos, whose expertise was invaluable in the present research.

Secondly, and because many times the work of one is not done without the effort of many, I must thank my family who always helped me supported me unconditionally and who suffered the most by my unavailability and absence.

I also would like to thank my colleagues who allowed me time to focus and develop the present study.

And to all who helped, making this work possible, thank you.

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Glossary

COMB On the M4 competition there was an improvement of the benchmark by using an arithmetic average combination of ten standard methods (both statistical and ML) for benchmarking the accuracy of the methods submitted to the competition [1] 8

ES-ATASD Representation of the model Exponential Smoothing with damped additive trend and additive seasonality 53, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72–74, 76, 101

ES-ATASND Representation of the model Exponential Smoothing with additive trend, additive seasonality and not damped 53, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76

ES-ATMSND Representation of the model Exponential Smoothing with additive trend, multiplicative seasonality and not damped 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76

ES-ATNSD Exponential Smoothing model with additive damped trend and no seasonality 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 101

ES-ATNSND Exponential Smoothing model with additive trend and no seasonality 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100

ES-MTMSD Representation of the model Exponential Smoothing with damped multiplicative trend and multiplicative seasonality 53, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72–74, 76, 101, 104

ES-MTNSD Exponential Smoothing model with damped multiplicative trend and no seasonality 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 101

ES-NTASND Representation of the model Exponential Smoothing with no trend, hence not damped and additive seasonality 53–56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 101, 104

ES-NTMSND Representation of the model Exponential Smoothing with no trend, hence not damped and multiplicative seasonality 53–56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 101

Exogenous Regressor An exogenous regressor is variable which cannot be determined by other variables in the system, it is an external varible. An example in a time series that is measured using a year of data, would be Holiday seasons. 28

Gatech The used name for identifying the M4 model created by Fiorucci, J. A. and Louzada, F. 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100

KaterinaKou One of the two statistical models from M4 used in this study, Katerinakou is the name that identifies the model created by Legaki, N. Z. and Koutsouri, K 54, 56, 58, 60, 62, 64, 66–68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 101, 103, 104

Pmontman The name that identifies the M4 model created by Montero-Manso, O., Talagala, T., Hyndman R. J. and Athanasopoulos, G 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100

Prologistica The name of the M4 model created by Pawlikowski, M., Chorowska, A. and Yanchuk, O. 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100

SNAIVE Short form for the model Seasonal Naïve 33, 46, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76

Vangspiliot The used name to identify the M4 model created by Spiliotis, E. and Asimakopoulus, V 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 101, 103, 104

Acronyms

1G 1^{st} Generation 1, 12

2G 2^{nd} generation 1, 12

3G 3^{rd} generation 1, 12

3GPP 3G Partmership Project 12

ACF Auto-Correlation Function 47, 49

AIC Akaike's Information Criterion 10, 11

AIC$_c$ Corrected Akaike's Information Criterion 11

AR Auto Regression IX, 26, 33

ARIMA AutoRegressive Integrated Moving Average III, V, IX, 6, 8, 13, 14, 19, 25, 27, 28, 32, 34, 46, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76

ARMA Auto Regression Moving Average 32

BIC Bayesian Information Criterion 11

CAGR Compound Annual Growth Rate 3

CDMA Code Division Multiple Access 12

DM Diebold Mariano Test 9, 10

DOTM Dynamic Optimized Theta Model 35, 36

DSTM Dynamic Standard Theta Model 35

ENN Elman Neural Network 13

ES Exponential Smoothing III, V, IX, 6, 14, 19–21, 31–34, 38, 46, 47

ES - WA Exponential Smoothing - Weighted Average 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100

FDD Frequency Division Duplexing 13

GMRAE Geometric Mean Relative Absolute Error 42

IFF International Journal Of Forecasting 31

KPI Key Performance Indicators III, V, X–XIII, XV, XVI, 3, 15–18, 45–50, 52, 53, 55–60, 62–68, 70–76, 78, 80, 82–84, 86–92, 94–101, 103–105

LTE Long Term Evolution III, V, 1–3, 5, 12, 13, 15, 45

MA Moving Average IX, 26

MAE Mean Absolute Error 38–41

MAPE Mean Absolute Percentage Error 13, 41, 51, 72

MASE Mean Absolute Scaled Error XI–XIII, XV, XVI, 9, 13, 35, 42, 51–77, 79–100

MdAE Median Absolute Error 40, 41

MdAPE Median Absolute Percentage Error 41

MdRAE Median Relative Absolute Error 42

MIMO Multiple-Input Multiple-Output 12

ML Machine Learning XVII, 6, 8

MLP Multilayer Perceptron 13

MRAE Mean Relative Absolute Error 42

MSE Mean Squared Error 13, 40, 41

NN Neural Networks 6, 31

ODFM Orthogonal Frequency Division Multiplexing 12

OFDMA Orthogonal Frequency Division Multiple Access 13

OTM Optimized Theta Model 35

OWA Overall Weighted Average 9, 33

PI Performance Indicators 9

RACH Random Access Channel 77

RMdsPE Root Median Percentage Error 41

RMSD Root Mean Squared Deviation 13

RMSE Root Mean Squared Error 13, 40, 41

RMSPE Root Mean Square Percentage Error 41

RNN Recurrent Neural Network 31, 32

SARIMA Seasonal AutoRegressive Integrated Moving Average 28

SARIMAX Seasonal AutoRegressive Integrated Moving Average with eXogenous regressors model 28, 46, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76

SES Simple Exponential Smoothing 25, 35, 36, 38, 78–82, 84, 86–88, 90, 92, 94–96, 98, 100, 101, 104

sMAPE Symmetric Mean Absolute Percentage Error XI–XIII, XV, XVI, 9, 34, 35, 41, 42, 51–77, 79–100

STheta Standard Theta Method 35

STM Standard Theta Model 35

TDD Time Division Duplexing 13

UE User Equipment XI, XII, XV, XVI, 16, 17, 48, 49, 52, 53, 55–59, 61–84, 90, 98, 104

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Chapter 1

Introduction

In the past decades the use of cellphones, namely smartphones, has evolved from only using them to make phone calls to a constant more diverse use. They have also become instruments of communication between communities rather than singular people, a necessity rather than an utility. And so it justifies the fast-paced growing of the cellphone networks.

Mobile devices and connections are not only getting smarter in their computing capabilities but are also evolving from lower-generation network connectivity (2G) to higher-generation network connectivity (3G, 3.5G and 4G or LTE) [2].

Since the launch of the first-generation (1G) in 1979, the evolution has been continuous - with great improvements in the following generations. As a result, networks became more and more complex.

The first-generation has fulfilled the basic mobile voice, while the second-generation dealt with capacity and coverage. The third-generation focused on higher data rate, multimedia support and spread spectrum, and it was followed by fourth-generation which provided access to a wide range of telecommunication services including advanced mobile services along with a support from low to higher mobility application. The use of mobile/cellular phones has been increasing in the last 8 years [3].

Networks have been growing to support more and more simultaneous users that have a diverse variety of utilization and purposes. This explosion in growth results in a network that also grows in complexity, consequently becoming an increasing management challenge to the operators.

In 2017 Cisco published a study on worldwide telecommunication analysis. The paper approaches the network growth and predicts the next five years. This study mentions that global mobile data traffic grew 63% in 2016. Global mobile data traffic reached 7.2 exabytes per month at the end of 2016, up from 4.4 exabytes per month at the end of 2015 [2].

Mobile data traffic has grown 18-fold over the past 5 years. Mobile networks carried 400 petabytes per month in 2011 [2].

Regarding LTE, the same study refers fourth-generation (4G) traffic accounted for 69% of mobile traffic in 2016. Although 4G connections represented only 26% of mobile connections in 2016, they already accounted for 69% of mobile data traffic while 3G connections represented 33% of mobile connections and 24% of the traffic. In 2016 a 4G connection generated four times more traffic on average than a 3G connection. Almost half a billion (429 million) mobile devices and connections that were added in 2016, smartphones accounted for most of that growth, followed by M2M mobiles. Global mobile devices and connection in 2016 grew to 8 billion, up from 7.6 billion in 2015 [2].

This study is corroborated by Ericsson mobility report published in June 2018. In retrospect, two decades led to fundamental changes in our society. When mobile data traffic surpassed mobile voice traffic in 2009, it was difficult to know what today's use of mobile technology would look like [4].

Worldwide total monthly mobile data traffic will increase to 107 exabytes by 2023 from 15 exabytes in 2017. By the end of 2023 close to 95% of all subscriptions will be from mobile broadband. Mobile data traffic grew 54% between the first quarter of 2017 and the first quarter of 2018. Monthly mobile data traffic per smartphone continues to increase in all regions. Total mobile data traffic is forecast to rise at a Compound Annual Growth Rate (CAGR) of 43%, reaching close to 107 exabytes per month by the end of 2023. Today mobile networks cover around 95% of the world population and this figure continues to grow [4].

Currently, a 4G connection generates nearly four times more traffic than a 3G connection. There are two reasons for the higher usage per device on 4G. The first is that many 4G connections today are for high-end devices, which have a higher average usage. The second is that higher speed encourages the adoption and usage of high-bandwidth applications. By 2021 a 4G connection will still generate two times more traffic than a 3G connection [2].

Every time we turn on our cellphones there is data being generated. In order for telecommunication operators (telco) to do an optimized and accurate management of the network, they collect every data generated by the cells, in the context of this work LTE cells. The collected data is raw; since it has not been processed yet, it does not contain useful information. Afterwards raw data is processed and aggregated to generate key performance indicators (KPIs). Each KPI represents a specific behaviour of the network, giving telecommunication operators a perspective of the networks whole behaviour, for network management this is core. Knowing the future behavior of networks is vital for its management.

This work will use LTE capacity KPIs, each one will be represented by a set of time series. The prediction of future values for those time series will be made by several different models, whose results will then be compared in order to determine the best forecasting model for a given KPI.

Concerning its structure, the second chapter addresses the review of published related work regarding predictive models, accuracy measures and specific articles where forecast models have been applied to networks, namely LTE.

Chapter 3, describes tools used to elaborate this study. It characterizes the data, predictive models, and the measures used to determine the accuracy of the forecasts.

Chapter 4 is dedicated to the practical part of the present study, how the study was conducted, the necessary decision making and the obtained results, as well as its analysis.

The last chapter, chapter 5 will showcase the conclusions taken from this study and present the future work.

Chapter 2

Related Work

2.1 Introduction

Analytics is an area of research that aims to discover, analyse and interpret meaningful patterns present in data. This field is still growing and this is the reason why there are so many studies from so many different areas related to forecasting. Data forecasts are made by applying predictive models to the data under study and these models will in turn try to find patterns namely seasonal, trend or even both in order to achieve better results.

This chapter will describe related work on predictive models, accuracy measures and selection, and studies regarding network forecasting, specifically Long Term Evolution (LTE).

2.2 Studies on predictive models

Predictive models have been the subject of much research in the second half of the last century, and its interest continues to grow nowadays.

Forecasts are divided into two groups, they can either be point forecasts or prediction intervals. Point forecasts are made by extrapolating the future values of a time series through a line, whereas prediction intervals forecasts result in intervals of trust for future values, which can be achieved by the use of state space models. The present work only regards point forecasts for n-steps ahead.

Time series is the name given to the collected data for forecasting. Time Series prediction represent an important area of forecasting, in which past observation of the same variable are collected and analyzed to develop a model describing the underlying relationship. The model is then used to extrapolate the time series into the future [5].

The interest of the scientific community in the prediction area has resulted in the creation of many different forecast models, that vary in their intricate complexity. The simplest methods comprise average, naïve, seasonal naïve and drift which will be later described in this document.

Despite their simplicity these methods are many times used as a form of benchmark for other more evolved models, seasonal naïve was even used in the M-competitions [6], but these models may not be considered to develop a forecast mechanism.

This study will rely on the analysis of more complex models which will also be described later in this document. These models encompass a set comprised of Exponential Smoothing (ES), AutoRegressive Integrated Moving Average (ARIMA), Theta, Linear Regression and also the models that resulted from the latest M4-Competition.

The set of used models in the present study stands for the successful results obtained by each of them. Predictive models are still evolving and their complexity continues to increase, examples of this are Neural Networks (NN) and native Machine Learning (ML), these types of models were not considered in the present study.

2.3 Studies on accuracy measures and model selection

There are many different ways to select a model given a time series, it will always depends on the time series at hand and on the purpose of the forecast. Forecasting accuracy is of obvious importance to users of forecasts because forecasts used to guide decisions [7].

Next will be presented some of the most used and well-known ways of selecting a given prediction model.

2.3.1 Accuracy Measures

Forecasts accuracy is of vital importance whether to confirm a models adequacy to a certain time series or to rule it out, declaring it as not fit for the time series under study. This field of study has started not long after the start of usage of predicting models, in 1969 by the work of Reid and Newbold and Granger in 1974 [6]. Though Makridakis and Hibon (1979) made the first effort to compare a large number of major time series methods across multiple series [6].

These tests evolved into M-Competition, M3-Competition, and later M4 Competition, which tested many different models, using several different time series, and compared the results applying different accuracy measures. However they have reached some criticized conclusions when addressing the models accuracy, the four most important conclusions are as follows:

- For previous M-Competitions statistically sophisticated or complex methods did not necessarily provide more accurate forecasts than simpler ones, although M4 brings new light on this particular matter by M4 Competition this conclusion became half truth since the best model was one of the most complex presented there. The forecasting accuracies of simple methods, such as the eight statistical benchmarks included in M4, will not be too far from those of the most accurate methods. This is only partially true. On the one hand, the hybrid method and several of the combination methods all outperformed COMB, the statistical benchmark, by a percentage ranging between 9.4% for the first and 5% for the ninth. These percentages are higher than the 4% improvement of the best method in the M3 relative to the Comb benchmark. On the other hand, the improvements below the tenth method were lower, and one can question whether the level of improvement was exceptional, given the advanced algorithms utilized and the computational resources used [1].

- The relative ranking of the performance of the various methods varies according to the accuracy measure being used [6];
- The accuracy when various methods are being combined outperforms on average the individual methods by themselves [6];
- The accuracy of the various methods depends on the length of the forecasting horizon involved [6].

These conclusions are valid for all of M-Competitions.

In 2017 the M3-Competition had an improvement on the extent of their results, comparing ARIMA with ML models, showed that ARIMA still outperforms ML models [8]. M4 competition also regards this issue and yet Machine Learning methods produce worse results than combination models.

One innovation of the M4 was the introduction of ten standard methods (both statistical and Machine Learning) for benchmarking the accuracy of the methods submitted to the competition [1].

One of the major findings of the competition, was the demonstration that none of the pure ML used managed to outperform Comb and that only one of them was more accurate than the Naive2 [1].

An example of critique of M-Competitions might be found in the work of Hyndman and Koehler [9]. Here the authors present many of the measures used in such competitions, pointing why despite the usefulness of the M-competition, they should have been done differently, namely using other accuracy measures, since most of them have problems regarding infinite results and scale dependency error calculation.

Moreover, the M4-Competition used Symmetric Mean Absolute Percentage Error (sMAPE) and Overall Weighted Average (OWA) of the relative sMAPE and the relative Mean Absolute Scaled Error (MASE) for accuracy comparison.

The combination of methods was the king of the M4. Of the 17 most accurate methods, 12 were "combinations" of mostly statistical approaches [1].

The biggest surprise was a "hybrid" approach that utilized both statistical and ML features. This method produced both the most accurate forecasts and the most precise PIs, and was submitted by Slawek Smyl, a Data Scientist at Uber Technologies[1].

2.3.2 Diebold Mariano Test

There is also the work of Diebold and Mariano (1995) - the Diebold Mariano Test (DM) [7]. The essence of the DM approach is to take forecast errors as primitives, intentionally, and to make assumptions directly on those forecast errors [10]. This test consists of comparing the predictive accuracy of two forecasts. Forecast models predict future values of the same time series, associated with the forecast values there is a loss function $g(e_{it})$ which is zero when no errors are made, its value is always equal or greater then zero and it increases in size as errors became large in magnitude. Loss differential between two forecasts is defined by:

$$d_t = g(e_{1t}) - g(e_{2t}) \tag{2.1}$$

The test consists of a null hypothesis where the two forecasts have the equal accuracy only if the loss differential equals zero:

$$H_0 : E(d_t) = 0 H_1 : E(d_t) \neq 0 \tag{2.2}$$

This method can be applied to a very wide range of class of loss functions. Comparison of forecast accuracy is but one of many diagnostics that should be examined when comparing models. The superiority of a particular model in terms of forecast accuracy does not necessarily imply that forecasts from other models contain no additional information [7]. Diebold in the revision of 2015 states that the DM test was intended for comparing forecasts; it has been, and remains, useful in that regard. The DM test was not intended for comparing models [10].

2.3.3 Akaike's Information Criterion (AIC)

$$AIC = T log\left(\frac{SSE}{T}\right) + 2(k+2) \tag{2.3}$$

where T is the number of observations used for estimation, k is the number of predictors in the model and SSE is the minimum sum of squared errors, $SSE = \sum_{t=1}^{T} e_t^2$

The model with the minimum value of the AIC is often the best model for forecasting [11]. In the AIC, as more parameters are added to the model, the first term becomes smaller, representing an increased fit, whereas the second component, or penalty term, becomes larger [12]. Indeed, when the sample is large, the number of adjustable parameters makes a negligible difference, and more complex models will be favored (Forster and Sober, 1994) [12].

2.3.4 Corrected Akaike's Information Criterion (AIC$_c$)

For small values of T, the AIC tends to select too many predictors, and so a bias-corrected version of the AIC has been developed [11],

$$AIC_c = AIC + \frac{2(k+2)(k+3)}{T-k-3} \tag{2.4}$$

Note that in this case the inclusion of branch lengths as estimated parameters can change the order of the AIC$_c$ values, and therefore, the selected model [12].

2.3.5 Bayesian Information Criterion (BIC)

Schwarz's Bayesian Information Criterion (BIC) dates from 1978 and was developed as an approximation of the log marginal likelihood of a model. A related measure is Schwarz's Bayesian Information Criterion (usually abbreviated to BIC):

$$BIC = Tlog\left(\frac{SSE}{T}\right) + (k+2)log(T) \tag{2.5}$$

As with AIC, minimizing the BIC is intended to give the best model. The model chosen by the BIC is either the same as those chosen by the AIC, or one with fewer terms [11].

In this study only accuracy measures were considered for model accuracy. Nonetheless it is important to mention that there are other methods.

2.4 Studies on Long Term Evolution (LTE) and capacity forecasting

The last of the reviewed literature is the core of the future presented work. In the last decades the wireless telecommunications have had a continuous gigantic growth.

There has been a phenomenal growth in the wireless industry-widespread technologies, increasing variety of user-friendly and multimedia-enabled terminals and wider availability of open source tools for content generation that encouraged user-centric networks resulting in a need for efficient network design [3].

In 1979 the first (1G) commercial cellular network was launched in Japan based on the Nordisk Mobil Telefoni (NMT) standard which had been developed in the north European countries. A digital cellular standard known as Global System for Mobile Communications (GSM; originally Groupe Special Mobile) was founded in 1987 and became the standard for 2^{nd} generation (2G) cellular system, improving modulation, voice and security service as compared to 1G analog system. The 3^{rd} generation Partnership Project (3GPP) and 3GPP2 were founded in 1998 from groups of telecommunication associations to develop the 3^{rd} generation (3G) cellular standard based on CDMA technology [13]. Code Division Multiple Access (CDMA) is a channel access method that supports multiple access.

As the CDMA based network reached its limit in accommodating rapidly increasing demand for wireless data traffic, 3GPP decided to develop a standard based on a new access technology, which is called Long Term Evolution (LTE). LTE adopted Orthogonal Frequency Division Multiplexing (ODFM) instead of CDMA as multiple access technology, in order to efficiently support wide band transmission. In addition the use of Multiple-Input Multiple-Output (MIMO) techniques, which plays an important role in improving the spectral efficiency [13].

The main advantages of LTE are high throughput, low latency, plug and play, FDD and TDD. LTE downlink transmission scheme is based on Orthogonal Frequency Division Multiple Access (OFDMA) – which converts the wide-band frequency selective channel into a set of many at fading subchannels [14].

As already presented LTE is the most used mobile network in the world [3], predicting its capacity is crucial for any organization present in the telecommunication market, since forecasts are used to make decisions, and network management is a critical process.

The research did not find many papers on network forecasting subject. Though there is a study on data analytics for forecasting cell congestion on LTE Networks [15], that applies the Auto Regressive Integrated Moving Average to a set of four weeks of LTE data and compares its performance to naïve model only through the result of Mean Squared Error. However, this might be a good measure, it is insufficient to test and measure the accuracy of models, besides the obvious lack of data.

From 2012 there is a paper regarding a modified ARIMA model for Channel Quality Indicator prediction in LTE-based Mobile satellite communications [16]. However, it does not address forecasting LTE networks issue, and although it applies ARIMA model for forecasting, does not regard any other predictive model neither it validates its accuracy.

From 2018 there is a paper regarding a comparative analysis of telecommunication network traffic forecasting using a three model approach [17]. However this paper was the closest and related to what will be done in this study, the paper regards 3G uplink instead of LTE and lacks a correct comparison between models.

Whilst ARIMA accuracy was measured using Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), Mean Absolute Scaled Error (MASE), the other model Elman Neural Network (ENN) and Multilayer Perceptron (MLP) used Mean Squared Error (MSE) and Root Mean Squared Deviation (RMSD), and these were never compared to ARIMA at least in the same accuracy measure.

In 2019 a study using ARIMA, Exponential Smoothing, Theta, Linear Trend and Random walk, shows that despite no method dominates the others across all time series of prediction horizons, exponential smoothing and ARIMA models are good alternatives to forecast this type of data. Combining forecasts generated by these models is also a sensible option. The forecasting models are also rather robust to abrupt shifts in network traffic due to changes in the network configuration [18].

Chapter 3

Methodology

3.1 Data Characterization

The data sets used in this study are from LTE capacity Key Performance Indicators (KPI) and were collected from a telecommunication company located in the north of Europe. This data presents a daily frequency and was collected from LTE cells during a whole year.

In order to carry out this study six capacity KPIs were considered. There is a total of 468 series, divided among the KPIs. Each Key Performance Indicators contains seventy eight cells, or in other words seventy eight series, and since the collected data corresponds to daily data, and was collected for one year, each series should be comprised of 365 days.

Presented below is a short description for each of the selected KPIs with a plot of a cell, that represents most of the cells that make up the entire KPI behaviour.

- Amount of User Equipment (UE) connected to the cell. This KPI is an average;

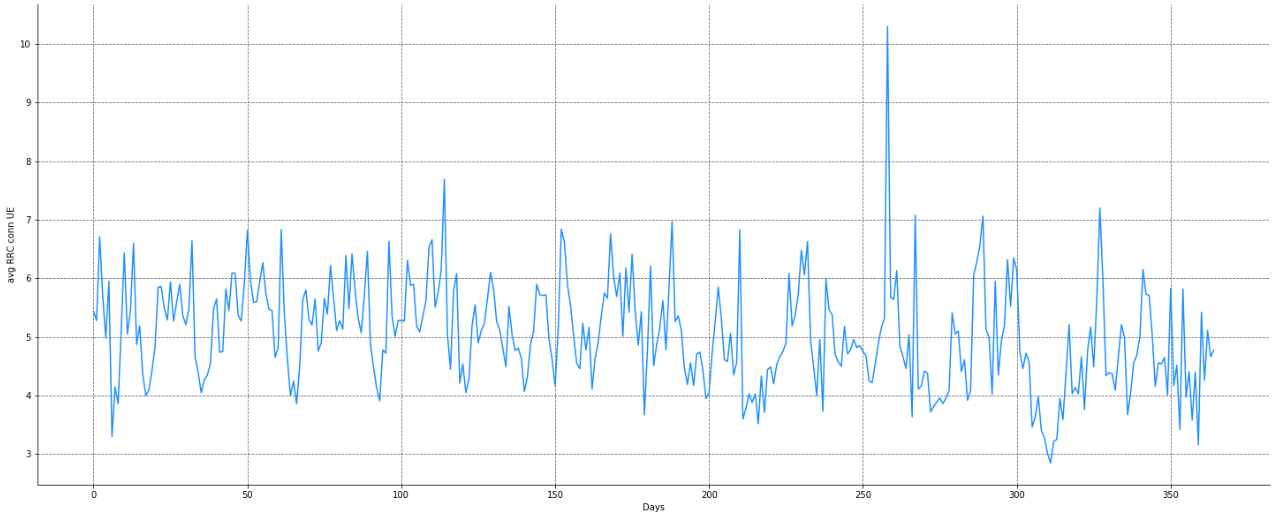

FIGURE 3.1: A series from Average UE's connected KPI

- Requests to network access made by user equipments during random access procedure. This KPI is a ratio hence the values are presented in percentages;

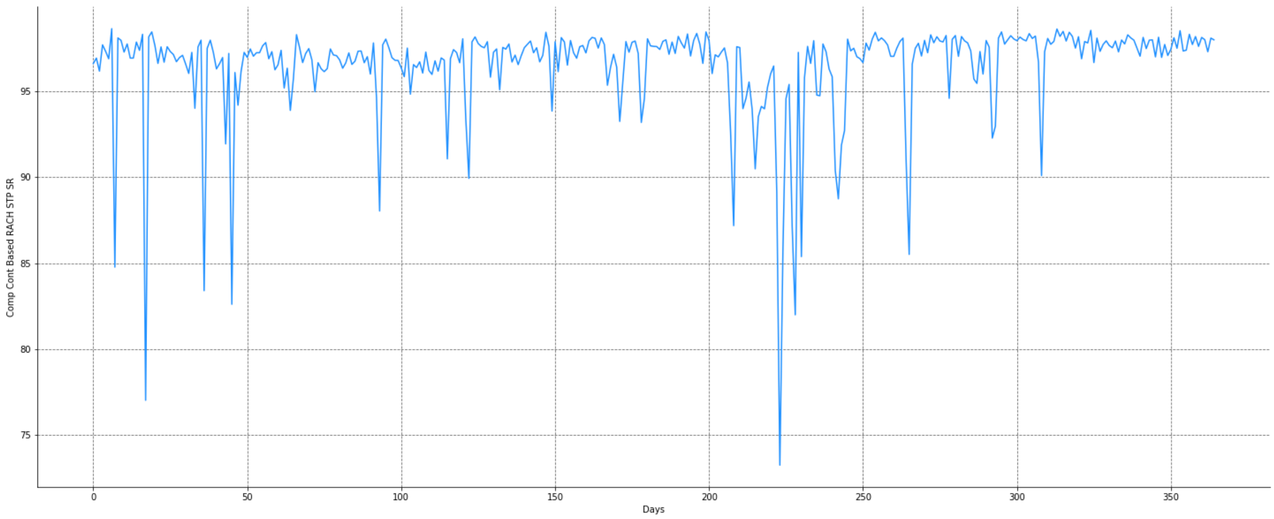

FIGURE 3.2: A series from Request to network access by UE's KPI

- The maximum number of active user equipment's per cell;

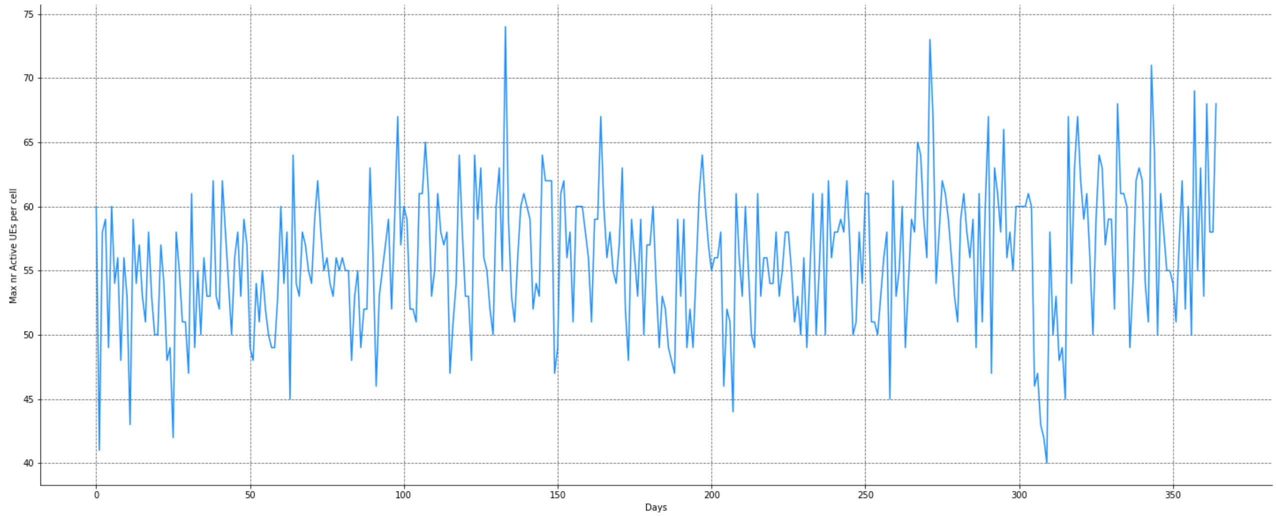

Figure 3.3: A series from maximum number of active UE's per cell KPI

- The maximum number of connected user equipment's per cell;

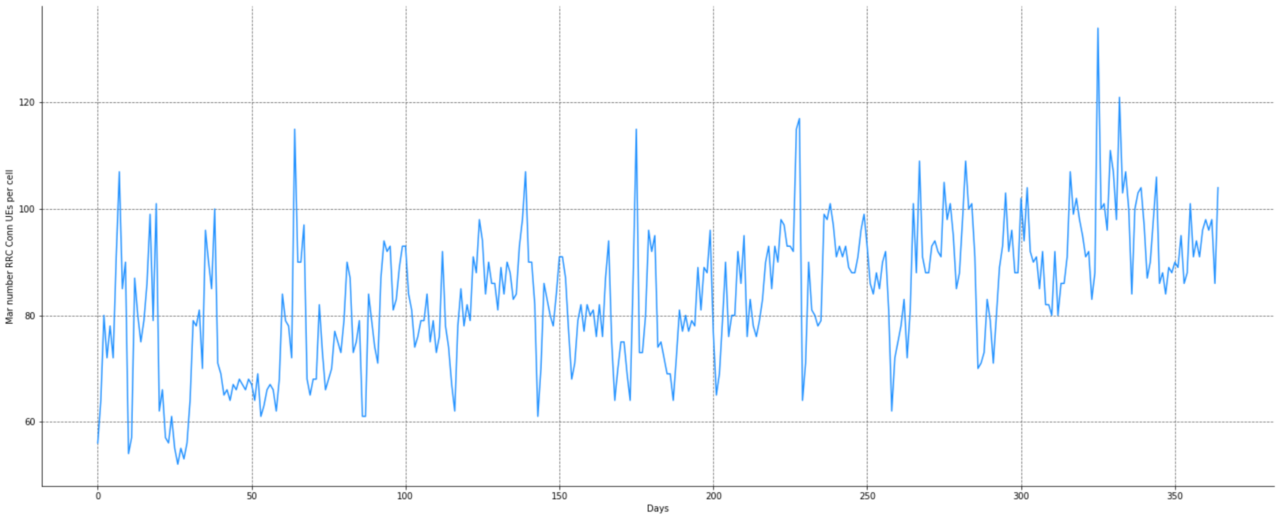

Figure 3.4: A series from maximum number of connected user equipment's per cell KPI

- Allocation of the transport channel for user data downlink. This KPI is a ratio, consequently its values are presented by a percentage;

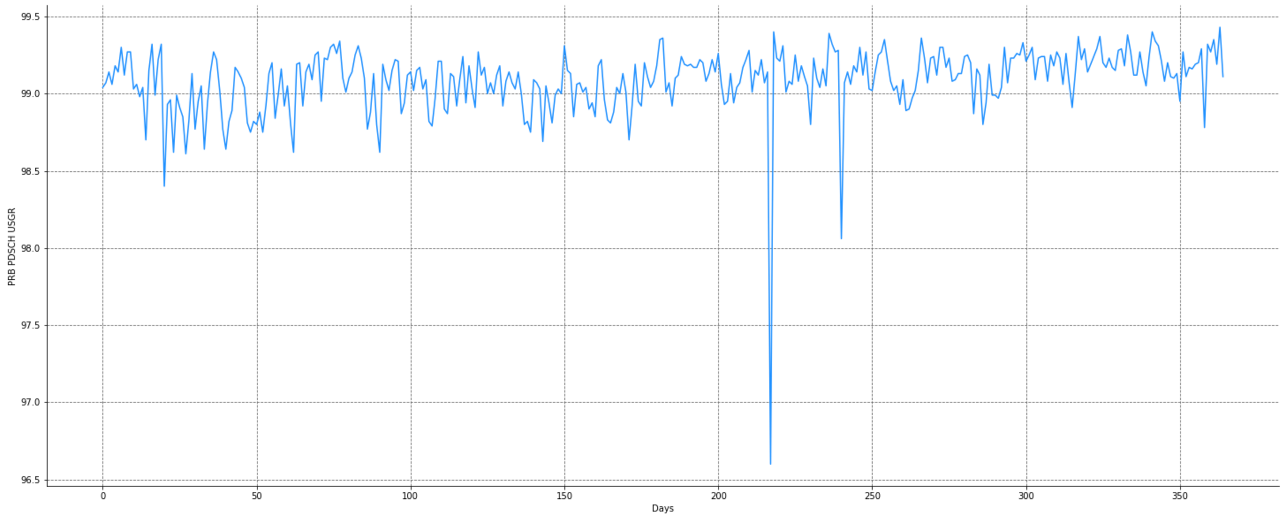

FIGURE 3.5: A series from allocation of the transport channel for user data downlink KPI

- Allocation of transport channel for data uplink. This is also a ratio.

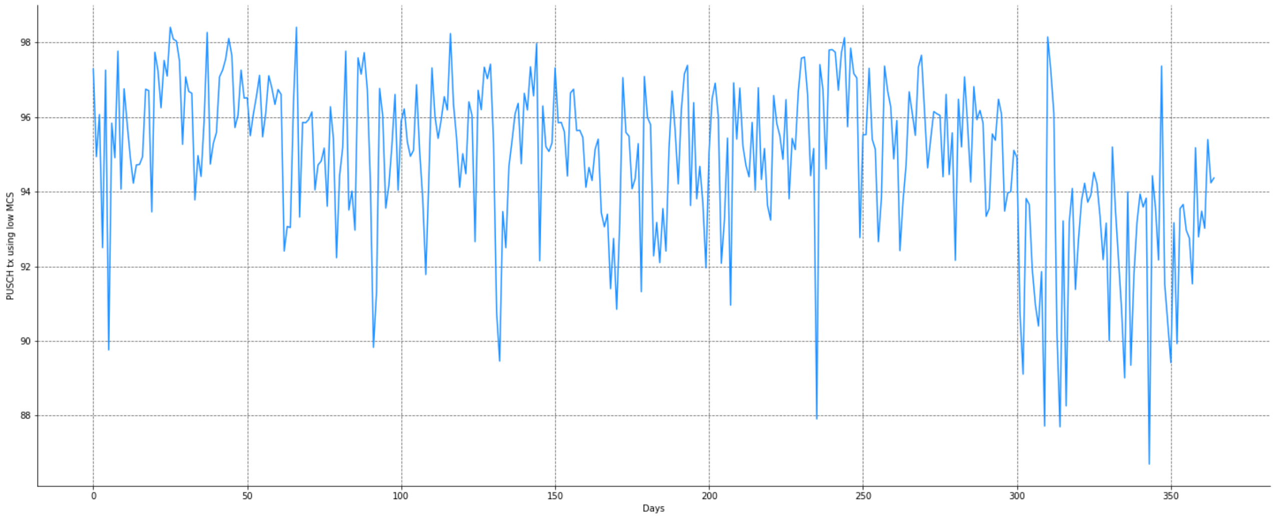

FIGURE 3.6: A series from allocation of the over the air transport channel for user data uplink KPI

3.2 Forecasting Models

In this work several forecasting models were used in order to validate which model or which combination of models could better adapt to a given data set.

The used models can be grouped as follows:

- Benchmark, which will comprise Naïve, Seasonal Naïve and Drift methods;
- Exponential Smoothing which will regard the different aspects of this family of models;
- ARIMA;
- Theta;
- Linear Trend;
- Finally the M4 models section which will reprise all M4 models considered in this study.

3.2.1 Benchmark

These are the simplest models applied to the collected data. They comprise three models which will be presented.

3.2.1.1 Naïve

The naïve method predicts the future values of a time series y as being equal to the last recorded value on it [11].

$$\hat{y}_{T+h|T} = y_t \tag{3.1}$$

3.2.1.2 Seasonal Naïve

Naïve method has a derivation, for seasonal naïve, where instead of the last value, it is considered the last season to be forecast.

$$\hat{y}_{T+h|T} = y_t + h - m(k+1) \tag{3.2}$$

where m represents the seasonal period and k is the integer part of $\frac{h-1}{m}$, h being the number of steps ahead [11].

3.2.1.3 Drift

The last of these models is the drift model. This is a variation on the naïve method that allows forecasts to increase or decrease over time. The amount of change over time is called drift.

$$\hat{y}_{T+h|T} = y_t + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1}) = y_t + h\frac{(y_t - y_1)}{T-1} \tag{3.3}$$

3.2.2 Exponential Smoothing

Exponential Smoothing was proposed in the late 1950s (Brown 1959; Holt 1957; Winters 1960), and has motivated some of the most successful forecasting methods [19, 20].

Holt-Winters exponential smoothing is a popular approach to forecasting seasonal time series. The robustness and accuracy of exponential smoothing methods have led to their widespread use in applications where many series necessitate an automated procedure, such as inventory control [21]. Exponential smoothing is a simple and pragmatic approach to forecasting, whereby the forecast is constructed from an exponentially weighted average of past observations [22].

3.2.2.1 Simple Exponential Smoothing

The simplest of the exponentially smoothing methods was developed by R. G. Brown during World War II (1944), to model the trajectories of bombs fired at submarines, though its work was only published in 1959 [19, 23]. This method is suitable for forecasting data with no clear trend or seasonal pattern.

$$\hat{y}_{T+1|T} = \alpha y_T + \alpha(1-\alpha)y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + ... \tag{3.4}$$

consider y_T an observed value at time T, α is the smoothing parameter as follows $0 \leqslant \alpha \leqslant 1$, and $\hat{y}_{T+1|T}$ represent the one-step-ahead forecast value.

The one-step-ahead forecast for time $T+1$ is a weighted average of all of the observations in the series $y_1 \dots y_t$. The rate at which the weights decrease is controlled by the parameter α [11].

When $\alpha = 1$ the method behaves like the naïve method, forecast of future values will be equal to the last value recorded in the time series.

3.2.2.2 Trend methods

Extended the simple exponential smoothing to allow the forecast of data with a trend. This was developed in 1957 by Holt [9]. This method involves a forecast equation and two smoothing equations (one for the level and one for the trend) [11].

$$ForecastEquation : \hat{y}_{t+h|t} = l_t + hb_t \tag{3.5}$$

$$LevelEquation : l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1}) \tag{3.6}$$

$$TrendEquation : b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1} \tag{3.7}$$

consider l_t to be the estimate of the level of the series at time t, b_t is an estimate of the trend (slope) of the series, α is the smoothing parameter, $0 \leqslant \alpha \leqslant 1$ and β^* is the smoothing parameter for the trend, $0 \leqslant \beta^* \leqslant 1$.

l_t is a weighted average of observation y_t and the one step ahead training forecast for time t.

b_t is a weighted average of the estimated trend at time t base on $l_t - l_{t-1}$ and b_{t-1}, the previous estimate of the trend. The forecast function is trendy.

The h-step-ahead forecast is equal to the last estimated level plus h time the last estimated trend value hence the forecasts are a linear function of h.

Damped Trend methods

Forecasts generated by Holt's linear trend method display a constant trend (increasing or decreasing) indefinitely into the future. This methods tend to over-forecast, especially for longer forecast horizons [11]. Motivated by this observation, Gardner and McKenzie (1985) introduced a parameter that "dampens" the trend to a flat line some time in the future [11].

$$ForecastEquation : \hat{y}_{t+h|t} = l_t + (\theta + \theta^2 + ... + \theta^h)b_t \tag{3.8}$$

$$LevelEquation : l_t = \alpha y_t + (1 - \alpha)(l_{t-1} + \theta b_{t-1}) \tag{3.9}$$

$$TrendEquation : b_t = \beta^*(l_t - l_{t-1}) + (1 - \beta^*)\theta b_{t-1} \tag{3.10}$$

θ is the damped parameter if $\theta = 1$ the method is identical to Holt's Linear Method.

For $0 \leqslant \theta =< 1$, θ dampens the trend so that if approaches a constant some time in the future.

3.2.2.3 Seasonal method

The Holt Winters seasonal method (1960) is an extent to Holt's method, that captures seasonality. This method comprises the forecast equation and three smoothing equations (one for level l_t, one for the trend b_t, and one for seasonal component s_t).

There are two variations on this method that differ in the nature of the seasonal component - additive or multiplicative.

The additive method is preferred when seasonal variations are roughly constant through the series. With the additive method, the seasonal component is expressed in absolute terms in the scale of observed series and in the level equation, the series is seasonally adjusted by subtracting the seasonal component [11]. Within each year, the seasonal component will add up to approximately zero [11].

The multiplicative method is preferred when seasonal variations are changing proportionally to the level of the series [11].

Additive Method

$$ForecastEquation : \hat{y}_{t+h|t} = l_t + hb_t + s_{t+h-m(k+1)} \tag{3.11}$$

$$LevelEquation : l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + bt - 1) \tag{3.12}$$

$$TrendEquation : b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1} \tag{3.13}$$

$$SeasonalEquation : s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m} \tag{3.14}$$

where k is the integer part of $\frac{(h-r)}{m}$, which ensures that the estimates of seasonal indexes are used for forecasting from the final year of the sample.

The level equation shows a weighted average between the seasonally adjusted observation $y_t - s_t - m$ and the non-seasonal forecast $l_{t-1} + b_{t-1}$ for time t.

The trend equation is equal to the Holt's linear method. The seasonal equation shows a weighted average between the current seasonal index $y_t - l_{t-1} - b_{t-1}$, and the seasonal index of the same season last year (m periods ago).

A small value of β^* for the additive model means the slope component hardly changes over time [11].

Multiplicative Method

$$ForecastEquation : \hat{y}_{t+h|t} = (l_t + hb_t)s_{t+h-m(k+1)} \tag{3.15}$$

$$LevelEquation : l_t = \alpha\frac{y_t}{s\ _{t-m}} + (1-\alpha)(l_{t-1} + bt - 1) \tag{3.16}$$

$$TrendEquation : b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1} \tag{3.17}$$

$$SeasonalEquation : s_t = \gamma\frac{y_t}{(l_{t-1} + b_{t-1})} + (1-\gamma)s_{t-m} \tag{3.18}$$

where s refers to the seasonality, k is the integer part of $\frac{(h-r)}{m}$, m defines the frequency of seasonality, for example, the number of seasons in a year - quarterly data $m = 4$, monthly data $m = 12$.
γ is the restriction parameter which is defined like $0 \leqslant \gamma \leqslant 1 - \alpha$

A small value of γ for multiplicative model means that the seasonal component hardly changes over time [11].

3.2.2.4 Holt-Winters' Damped Method

Damping is possible in both multiplicative and additive Holt Winters' methods. A method that often provides accurate and robust forecasts for seasonal data is the Holt Winters' method with damped trend and multiplicative seasonality [11]:

$$ForecastEquation : \hat{y}_{t+h|t} = [l_t + (\theta + \theta^2 + ... + \theta^h)b_t]s_{t+h-m(k+1)} \tag{3.19}$$

$$LevelEquation : l_t = \alpha(\frac{y_t}{s_{t-m}}) + (1-\alpha)(l_{t-1} + \theta bt - 1) \tag{3.20}$$

$$TrendEquation : b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)\theta b_{t-1} \tag{3.21}$$

$$SeasonalEquation : s_t = \gamma\frac{y_t}{(l_{t-1} + \theta b_{t-1})} + (1-\gamma)s_{t-m} \tag{3.22}$$

The Hyndman, Koehler, Snyder, and Grose (2002) taxonomy (extended by Taylor, 2003) provides a helpful categorization for describing the various methods.

Each method consists of one of five types of trend (none, additive, multiplicative, and damped multiplicative) and one of three types of seasonality (none, additive, and multiplicative). Thus, there are 15 different methods, the best known of which are Simple Exponential Smoothing with no trend and no seasonality, Holt's linear method (additive trend, no seasonality), Holt-Winters' additive method (additive trend, additive seasonality), and Holt Winters' multiplicative method (additive trend, multiplicative seasonality) [20].

3.2.3 AutoRegressive Integrated Moving Average (ARIMA)

AutoRegressive Integrated Moving Average models provide another approach to time series forecasting. Early attempts to study time series particularly in the 19^{th} century were generally characterized by the idea of a deterministic world. It was the major contribution of Yule (1927) which launched the notion of stochasticity in time series by postulating that every time series can be regarded as the realization of a stochastic process [20].

This model had a great contribution by Box and Jenkins (1970) [20].

There are two important concepts regarding ARIMA models: stationarity and differencing.

Stationary Time Series

A stationary time series is time series where properties do not depend on the time at which the series is observed - time series with seasonality or trend are not stationary. A time series with cyclic behaviour can be stationary - if it has no trend or seasonality [11].

Typically these time series will have no predictive patterns in the long term.

Time plots can show a series to be roughly horizontal. White noise series is stationary [11].

Differencing

It is the computation of the differences between consecutive observations. Differencing can help stabilize the mean of a time series by removing changes in the level of a time series, therefore reducing the trend or seasonality of the series [11].

When the differenced series is white noise, the model for the original series can be written as $y_t - y_{t-1} = \epsilon_t$, where ϵ_t denotes white noise.

A differenced time series can be easily plotted by its Auto Correlation Function. If it drops to zero relatively quick the time series is differenced [11].

3.2.3.1 Auto Regression (AR) Models

Autoregressive model forecasts the variable of interest using a linear combination of past values of the variable. Auto regression indicates that it is a regression of the variable against itself.

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \epsilon_t \tag{3.23}$$

where c represents the average of changes between observations, ϕ the trend damped factor and ϵ_t denotes white noise. This is an **AR(p) model**, an autoregressive model in order of p and is like a multiplicative regression but with lagged values of y_t as predictors.

Autoregressive models are remarkably flexible at handling a wide range of different time series patterns.

3.2.3.2 Moving Average (MA) Models

Rather than using past values of the forecast variable in a regression, a moving average model uses past forecast errors in a regression like model:

$$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_q \epsilon_{t-q} \tag{3.24}$$

this is called a moving average of order q, MA(q), where ϵ_t denotes white noise, c is the average of changes between observations as in AR and θ is the number of lags.

Each value of y_t can be thought as a weighted moving average of the past few forecast errors.

3.2.3.3 Non Seasonal Models

Non-seasonal ARIMA is the combination of differencing, autoregression and moving average.

$$y'_t = c + \phi_1 y'_{t-1} + ... + \phi_p y'_{t-p} + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q} + \epsilon_t \tag{3.25}$$

where y'_t is the differenced series, p the order of the autoregressive part, d is the degree of first differencing part and q is the order of the moving average part.

3.2.3.4 Seasonal Models

ARIMA models are capable of modelling a wide range of seasonal data. Seasonal ARIMA model is formed by including additional seasonal terms in the ARIMA models.

$$Non - Seasonal : ARIMA(p, d, q) \tag{3.26}$$

$$Seasonal : ARIMA(p, d, q)(P, D, Q)_m \tag{3.27}$$

where m stands for the number of observations per year. Uppercase denotes the seasonal part, while lower case is used for non-seasonal part of the model. The additional seasonal terms are just multiplied by the non-seasonal terms [11].

The popularity of the ARIMA models is due to its statistical properties as well as the well-known Box- Jenkins methodology. In addition, various exponential smoothing models can be implemented by ARIMA models. Although ARIMA models are quite flexible in that, they can represent several different types of time series, their major limitations is the pre-assumed linear form of the model. That is, a linear correlation structure is assumed among the time series values and therefore, no nonlinear patterns can be captured by the ARIMA model. The approximation of the linear models to complex real-world problem is not always satisfactory [5]. SARIMAX is a particular case of Seasonal AutoRegressive Integrated Moving Average (SARIMA), which uses external variables - Exogenous Regressors.

The present study calculated the forecasts of ARIMA model by using a value o 2 for both the AR term and MA term, and using one differencing required to make the time series stationary, hence:

$$ARIMA(2,1,2). \tag{3.28}$$

3.2.4 Linear Trend

Regression analysis is primarily used for predicting values of the response variable at interesting values of the predictor variables, discovering the predictors that are associated with the response variable, and estimating how changes in the predictor variables affects the response variable [24]. A linear regression is an approach to modelling a relationship between a dependent variable and one or more independent variables, depending on it being one or more the regression is called simple or multiple linear regression. In this study a simple linear regression was used. To better adapt the obtained forecasts the used method also applies the least squares method applied to the slope. The least-squares method can be used to compute reliable realizations of the minimum-variance unbiased estimates for linear regression models, and the conjugate gradient method is a very effective iterative algorithm for solving the related normal equations system [25].

3.2.5 Theta

The Theta-model proposes a different approach to decomposition: a decomposition of the seasonally adjusted series into short and long term components [25].

Theta challenge was to increase the degree of exploitation of the embedded useful information in the data, before the application of a forecasting method. Viewed intuitively, such information has long and short term components. These components are identified using the Theta-model and are then extrapolated separately. The Theta-model is analogous to the operation of a magnifying glass though which the time series fluctuations are minimized or maximized accordingly [25].

The model is based on the concept of modifying the local curvatures of the time series. This change is obtained form a coefficient, called Theta coefficient, which is applied directly to the second differences of the time series [25].

If the local curvatures are gradually reduced then the time series is deflated. The smaller the value of the Theta-coefficient, the larger the degree of deflation. In the extreme case where $\Theta = 0$ the time series is transformed to a linear regression line [25].

Conversely if the local curvature is increased ($\Theta > 1$), then the time series is dilated. The larger the degree of dilation, the larger the magnification of the short-term behaviour [25].

The initial time series is decomposed into two or mode Theta-lines. Each of the Theta-lines is extrapolated separately and the forecasts are simply combined. Any forecasting method can be used for the extrapolation of a Theta-line. A different combination of Theta-lines can be employed for each forecasting horizon [25].

The strong point of the method lies in the decomposition of the initial data. The two components include information, which is useful for the forecasting procedure but is lost of cannot completely be taken into account by the existing methods when they are directly applied to the initial data [25].

3.2.6 M4 Models

M4 was the latest edition of M competition, which had some surprising results involving the participating models.

M4 contestants were advised to publish the code which had been used for forecasting, some of the models became available on GitHub, and thanks to this fact, some of the models of in the M4 competition were used in this study. Table 3.1 shows the selection that was made.

In the M4 competition all models performed both point forecasts and intervals of confidence, however in the present study only point forecasts are considered, consequently these models were only used and compared based on such forms of forecast.

Overall Standings	**Type of Model**	**Author**	**Organization**	**ID**
1^{st}	Hybrid	Smyl, S.	Uber Technologies	Slaweks
2^{nd}	Combination	Montero-Manso, O., Talagala, T., Hyndman R. J. & Athanasopoulos, G.	University of Coruña & Monash University	Pmontman
3^{rd}	Combination	Pawlikowski, M., Chorowska, A. & Yanchuk, O.	Prologistica Soft	Prologistica
5^{th}	Combination	Fiorucci, J. A. & Louzada, F.	University of Brasilia & University of São Paulo	Gatech
8^{th}	Statistical	Legaki, N. Z. & Koutsouri, K.	National Technical University of Athens	KaterinaKou
11^{th}	Statistical	Spiliotis, E. & Asimakopoulus, V.	National Technical University of Athens	Vangspiliot

Table 3.1: Models selected from M4 competition

The first four selected models reflect their standings in the competition. Despite being advised, not all participants provided their code - the 4^{th} best qualified model is an example of such behaviour. The last two models present in the table were selected because of their different nature, being statistical instead of a combination.

In the M4 competition all models provided point forecasts and prediction intervals, though as already stated before in this study, only point forecasts are to be considered. Moreover, all models provided several different horizons, namely hourly, daily, monthly, quarterly and yearly, yet the models used here were only forecasting daily data.

The best modules of the M4 competition will have a detailed description in a future release of the International Journal Of Forecasting (IFF), however the next section will contain the description for each of the used modules.

3.2.6.1 Slaweks[1]

This model was tried and tested in the current study. Nonetheless, due to its heavy time consumption to generate forecasts, it was discarded.

This hybrid model combines Exponential Smoothing (ES) formulas, with a black-box Recurrent Neural Network (RNN) forecasting engine. The models do not constitute an ensemble of Exponential Smoothing (ES) and Neural Networks (NN), instead, they are truly hybrid algorithms in which all parameters, like the initial ES seasonality and smoothing coefficients, are fitted concurrently with the NN weights by the same Gradient Descent method [26].

Generally, NN do not have a great track record in time series forecasting.The models described here try to solve this problem by being hierarchical – partly time series specific and partly global [26]. Seasonal models use algorithms inspired by Holt-Winters multiplicative seasonality, either single, or, in case of the hourly series, double seasonal (24 and 168). In the former case, the update formulas are:

$$l_t = \frac{\alpha * y_t}{st} + (1 - \alpha) * l_{t-1} \tag{3.29}$$

$$s_{t+K} = \frac{\beta * y_t}{l_t} + (1 - \beta) * s_t \tag{3.30}$$

where l_t is level, y_t is value of the series, and s_t is the seasonality coefficient at the time t. K is the seasonality size, e.g. 12 for monthly series.α and β

[1] This model was tried and tested in the current study. Nonetheless, due to its heavy time consumption to generate forecasts, it was discarded.

are smoothing coefficients, between 0 and 1, and $s_t > 0$. For completeness, here it is the forecasting formula:

$$\hat{y}_{t+1..t+h} = RNN(x_t) * l_t * s_{t+1..t+h} \tag{3.31}$$

where x_t is the preprocessed (deseasonalized and normalized) input vector. The first K seasonal coefficients are also parameters of the model, fitted by the Gradient Descent. Multiplication in formula 1c is element-wise, bold letters denote vectors. The RNN forecasts whole horizon at once.

The output size was always equal to the forecasting horizon [26]. The model does multiple runs which then merges the results to generate an output.

3.2.6.2 Pmontman

A set of 9 forecasting methods is used. These methods are fitted on each individual series and produce the forecasts at the asked horizon. These forecasts are linearly combined to produce the final point forecast. The rationale behind the pool of methods is to use easily available forecasting methods that can be fitted automatically [26].

The models used in this combination are as follows:

1. The ARIMA model. Parameters like the order of differencing, p, q are obtained through an exhaustive search;
2. Exponential Smoothing state space model with parameters fitted automatically;
3. A feed-forward neural network with a single hidden layer is fitted to the lags. The number of lags is automatically selected;
4. The Exponential smoothing state space Trigonometric, Box-Cox transformation, ARMA errors, Trend and Seasonal components model. Parameters

like the application of a Box-Cox transformation, to include trend, among others, are automatically fitted;

5. Seasonal and Trend decomposition using Loess with AR modeling of the seasonally adjusted series;

6. Random Walk with Drift;

7. The theta method of Assimakopoulos and Nikolopoulos (2000);

8. The naïve method, using the last observation of the series as the forecasts;

9. The forecast are the last observed value of the same season;

In the case of an error when fitting the series (e.g. a series is constant), the forecast method is used instead [26].

In essence, it is a feature-based gradient tree boosting approach where the loss or error function to minimize is tailored to the OWA error used in the M4 competition. The implementation of gradient tree boosting is xgboost, a tool that is computationally efficient and allows a high degree of customization [26].

The final forecasts are calculated by combining the individual forecasts of each model.

3.2.6.3 Prologistica

The final predictions are weighted averages of forecasts produced by statistical methods. The following models were used [26]:

1. Naïve;

2. Naïve2 (M4 benchmark);

3. Simple Exponential Smoothing;

4. Exponential Smoothing (with automatic parameters choice);

5. Damped Exponential Smoothing (automatic);
6. ARIMA (automatic);
7. Simple Theta (modified to never crash on very short series);
8. Optimised Theta;
9. econometric models (linear regression on different types of trend) (custom);

For a given series y the model will take the following steps:

1. Choose a set of models to combine. The selection is based upon y's characteristics: period (Yearly, Daily, etc.), existence of seasonality and/or trend [26];
2. Using the chosen set of models, it generates one-step-ahead forecasts for the last N prefixes of y and measure their accuracy according to sMAPE. The number N depends on y's period and it is typically slightly smaller than forecast horizon [26];
3. For each model, summarize errors of different prefixes. The summarization method is a weighted mean with either equal or exponential weights. This gives scores that measure performance of chosen models on y's history [26];
4. From the scores, create a set of weights, that will be used in calculation of final forecasts. The method of translating scores to weights depends on y's period. Usually the weights are squares of inverted scores [26];
5. Calculate future forecasts for y for all models. The final prediction is defined as a weighted mean of those forecasts, using weights described on previous step [26].

As stated the model utilizes several common statistical models, which are weighted according to their performance on historical data.

3.2.6.4 Gatech

The Gatech model is based on an improvement of Theta model.

The Theta method attracted the attention of researchers by its simplicity and surprisingly good performance [27].

Theta model was already explained in the present study, so the description of Gatech will be more dedicated to the improvements made in this version. The Gatech is based in the findings made in Models for optimising the theta method and their relationship to state space models [27], here the methods and models of Theta such as Dynamic Optimized Theta Model (DOTM), Dynamic Standard Theta Model (DSTM), Optimized Theta Model (OTM), Standard Theta Model (STM) and Standard Theta Method (STheta) were tested using a subset of data from M3 Competition, using as benchmark the models present in 3.2.

Method	Reference	Description
Naïve		ARIMA(0,1,0)
SES	Brown (1956)	ETS(A,N,N)
Damped	Gardner and McKenzie (1985)	ETS(A,Ad,N)
ETS	Hyndman and Khandakar (2008)	ETS automatic algorithm based on AICc
ARIMA	Hyndman and Khandakar (2008)	Automatic ARIMA based on AICc

TABLE 3.2: Benchmark methods and models used

Models forecasted original data and seasonally adjusted data, the results obtained were then processed using sMAPE and MASE. The results varied according in conformity with the horizon calculated (yearly, quarterly, monthly, other and all), although DOTM outperformed the other methods in most cases. The results showed that the proposed improvement of DOTM was more efficient than the existing methods and models of Theta.

The DOTM selects the theta line to be used for the extrapolation of the short-term component of the series optimally [27].

In terms of empirical forecasting performances, DOTM demonstrated improvements over the Theta method to all frequencies and error measures considered [27].

An important property of DOTM is that when $\theta = 1$, the model behaves like SES method. When $\theta > 1$ acts as an extension of SES, by adding a long-term component.

3.2.6.5 KaterinaKou

This methodology is composed of multiple steps in order to produce the final forecasts:

1. Import the data set;
2. Decompose the time series (if needed) to estimate the seasonal component: An auto-correlation test is applied for all the series. In case of a positive result (90% confidence), the series is stripped off their seasonal component using the classical multiplicative decomposition, and obtain the seasonally adjusted data.
3. Apply the Box-Cox transformation: On its bases, the Theta method (that will be used for extrapolating the series) generates linear forecasts. In order this simple forecasting model to produce reasonable results, even in cases of nonlinear trend, historical data is adjusted (without the seasonal component) using the Box-Cox transformation. The transformation depends on the parameter λ with values between 0 and 1, defined as follows [26]:

$$w_t = \begin{cases} \log(y_t) & if \lambda = 0; \\ \frac{(y_t^t - 1)}{\lambda} & otherwise \end{cases} \tag{3.32}$$

4. Generate forecasts based on the theta method: The forecasts of each seasonally adjusted and normalized series are produced using the Theta method;

5. Reverse Box-Cox transformation: Having generated our forecast, the data is rescaled using the reverse Box-Cox transformation, which is defined as:

$$y_t = \begin{cases} \exp(w_t) & if \lambda = 0; \\ (\lambda w_t + 1)^{\frac{1}{\lambda}} & otherwise \end{cases} \tag{3.33}$$

   For the reverse transformation and the calculations, the functions: "InvBox-Cox()" is used with the respective parameters that have been calculated in step 3;

6. Re-seasonalization: The scaled forecasts of step 5 are seasonally adjusted using the respective seasonal indices calculated in step 2, if applicable Final results are obtained in a single data set, as proposed [26].

3.2.6.6 Vangspiliot

This model uses the Theta Framework (4Theta).

Given the limitations of Theta classic, some modifications were implemented to generalize its use and enhance its performance for more complex types of data. This includes mechanisms for considering both linear and non-linear patterns of trend, adjusting trend intensity, recognizing either additive or multiplicative seasonality, as well as additive and multiplicative connections between the components of trend and level [26].

The extensions proposed transform Theta classic into an integrated forecasting framework, 4 Theta, which generates various forecasting models, each one of different properties. In this respect, using simplified selection criteria, it becomes possible to identify the best model per time series examined and exploit it for extrapolation [26].

The Theta method is somewhat limited in its original form as it is forced to generate forecasts which are linearly related. Additionally, the trend of the forecasts is half that of the fitted trend line ($a/2$). Thus, if the long-term components of the data persist in the future, the method will be unable to properly follow their pattern and result to rather pessimistic or optimistic forecasts, depending on the sign of the trend. The same stands for damped trend series, where slope a could be further shrunk to enable a better fit, as well as stationary series for which classic Theta will assume a pseudo-trend through Y^0, possibly leading to poor results. In such cases, an optimal θ coefficient could be used in stead of $\theta = 2$ to properly adjust trend intensity [26].

3.2.6.7 Dealing with trend

There are many ways for selecting the optimal coefficient of θ, to do so efficiently, without significantly increasing the computational cost of the method, the parameter estimation in this study is based on minimizing the Mean Absolute Error (MAE) [26]. This enables Theta to properly fit the data without exaggerating [26].

Due to the changes in place the Theta method can presently mime the properties of more flexible models, namely Damped, Holt (linear trend) and Simple (insignificant trend) Exponential Smoothing (Gardner, 2006). Moreover,it can further improve its forecastability by considering non-linearities and exponential growth rates, which for the case of exponential smoothing are limited captured [26].

3.2.6.8 Additive and multiplicative expression of the Theta method

Many classical forecasting models, are expressed both in a multiplicative and additive form, meaning that the components of the series (level, seasonality and trend) may be combined in two different ways and relatively interact [26]. The Theta method averages the forecasts of SES and regression in time to extrapolate the data. Thus, the level and trend component are independent from each other and additively connected.The same stands for seasonal component which, given that the data are externally seasonally adjusted (if needed), is not directly handled by the method itself [26]. The multiplicative expression of the Theta method can be exploited to handle data displaying multiplicative relations of the level and trend. In total, after the expansion suggested, the Theta method can generate up to six models of different properties but similar principles [26]:

1. Negligible seasonality - Additive relations
2. Negligible seasonality - Multiplicative relations
3. Additive seasonality - Additive relations
4. Additive seasonality - Multiplicative relations
5. Multiplicative seasonality - Additive relations
6. Multiplicative seasonality - Multiplicative relations

3.2.6.9 Model Selection

The present model became more flexible and can capture more easily complex patterns present in the data, special if compared with classical theta. Moreover, it can also select the best performing model available in Theta framework. Therefore, for each time series examined, all possible models are estimated through 4 Theta and MAE is applied to determine which one should be selected for extrapolation [26].

3.3 Accuracy Measures

This section is dedicated do describe each of the considered accuracy measures used during the development of the present study.

3.3.1 Scale-dependent measures

- Mean Squared Error (MSE)

$$MSE = mean(e_t^2) \tag{3.34}$$

- Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{MSE} \tag{3.35}$$

- Mean Absolute Error (MAE)

$$MAE = mean(|e_t|) \tag{3.36}$$

- Median Absolute Error (MdAE)

$$MdAE = median(e_t^2) \tag{3.37}$$

The error at time t is given by:

$$e_t = y - \hat{y}_t \tag{3.38}$$

where y corresponds to the actual value recorded and $\hat{y}_t$ is the forecasted value, accordingly the error corresponds to the difference between the recorded value and the forecasted value.

Scale dependent measures have as the category mentions, the problem of being scale dependent, thereafter errors cannot be compared amongst different scales.

They are interpreted by value, the smaller the error is closer the forecast is to best fit. Often RMSE is preferred to the MSE as it is on the same scale as the data. RMSE and MSE are more sensitive to outliers than MAE or MdAE [9].

3.3.2 Measures based on percentage errors

- Mean Absolute Percentage Error (MAPE)

$$MAPE = mean(|p_t|) \tag{3.39}$$

- Median Absolute Percentage Error (MdAPE)

$$MdAPE = median(|p_t|) \tag{3.40}$$

- Root Mean Square Percentage Error (RMSPE)

$$RMSPE = \sqrt{mean(p_t^2)} \tag{3.41}$$

- Root Median Percentage Error (RMdsPE)

$$RMdsPE = \sqrt{median(p_t^2)} \tag{3.42}$$

- Symmetric Mean Absolute Percentage Error (sMAPE)

$$sMAPE = \frac{100\%}{n} * \sum_{t-1}^{n} \frac{|\hat{y}_t - y_t|}{|\hat{y}_t| + |y_t|} \tag{3.43}$$

where

$$p_t = 100\frac{e_t}{Y_t} \tag{3.44}$$

Percentage errors have the advantage of being scale independent and so are frequently used to compare forecast performance across different data sets. Still these measures have the disadvantage of being infinite if $Y_t = 0$ for any t in the period of interest. It is impossible to use these measures as zero values of Y_t occur frequently [9]. Exception made in the percentage errors to the infinite problem is sMAPE which has a lower and upper boundary.

3.3.3 Measures based on relative errors

- Mean Relative Absolute Error (MRAE)

$$MRAE = mean(|r_t|) \tag{3.45}$$

- Median Relative Absolute Error (MdRAE)

$$MdRAE = median(|r_t|) \tag{3.46}$$

- Geometric Mean Relative Absolute Error (GMRAE)

$$GMRAE = gmean(|r_t|) \tag{3.47}$$

being

$$r_t = \left| \frac{y_t - \hat{y}_t}{y_t - \hat{y}_t^*} \right| \tag{3.48}$$

$$\hat{y}_t^* = \begin{cases} y_{t-1} & \text{Non seasonal} \\ y_{t-M} & \text{Seasonal} \end{cases} \tag{3.49}$$

where M corresponds to the seasonal period.

A serious deficiency of relative error measures is that forecast error can be small, resulting in relative error with infinite variance [9].

- Mean Absolute Scaled Error (MASE)

$$MASE = mean(|q_t|) \tag{3.50}$$

where

$$q_t = \frac{e_t}{\frac{1}{n-1}\sum_{i=2}^{n}|Y_i - Y_{i-1}|} \tag{3.51}$$

When MASE<1, the proposed method, on average gives smaller errors than the one-step errors from the naïve method. If multi-step forecasts are being computed, it is possible to scale by the in-sample MAE computed from multi-step naïve forecasts [9]:

$$MAE = \frac{\sum_{i=1}^{n}|e_i|}{n} = mean(|e_t|) \tag{3.52}$$

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Chapter 4

Experimental Study

4.1 Handling Data

4.1.1 Preparing data

The forecasting process does not start by applying a model to a given time series or set of time series, it starts much earlier with an initial analysis of the nature, purpose, consistency and characteristics of the given data set. In the present study the set is of daily collected LTE data, that contains the values of an entire year, as was previously mentioned.

Half of the KPIs regards ratios and the other half measure network volumes, this is relevant because data that addresses the measurement of volumes is typically characterized by having seasonal behaviour whereas ratios are usually non-seasonal.

Knowing this, the next step is to analyse the characteristics of each of the series presented in all of the used KPIs.

Each series must contain a set of 365 samples, without any blanks or null values. The used data set did not contain much of null values, rather it had some missing values instead. When a given day didn't have an associated value the row did not exist entirely - both the date and its value were not present. The solution implemented for this case, being it a missing value, a blank or null was substituting or adding (in the case it was missing) the last recorded value, or the next recorded value in cases where the last was not available.

It is important to mention that some of the used models, namely the models from M4 competition, have intricate methods to handle missing data, although the data set provided to each model was the same, in other words, the whole data set was primarily treated and only then provided to the used models.

4.2 Forecasting

Models were applied to the KPIs according with the data seasonality, thereafter they have been divided into two groups, seasonal models and non-seasonal models. The considered seasonal models used were:

- Seasonal Naïve (SNAIVE)
- Exponential Smoothing
  - Seasonal Additive without Trend
  - Seasonal Multiplicative without Trend
  - Seasonal Additive with Additive Trend
  - Seasonal Multiplicative with Multiplicative Trend
  - Seasonal Multiplicative with Additive Trend

  - ARIMA
  - SARIMAX
- Theta
- Linear Trend

The considered non-seasonal models used were:

- Naïve
- Drift
- Exponential Smoothing
  - Simple Exponential Smoothing
  - Non-Seasonal without Additive Trend
  - Non-Seasonal without Multiplicative Trend

M4 models are not part of this preselection since they were always applied regardless of data seasonality.

4.2.1 Autocorrelation Function

As previously stated forecasts had some constraints regarding data seasonality, only if a majority of the series on each of the KPI was seasonal or non-seasonal a specific type of forecast model was applied.

Just as correlation measures the extent of a linear relationship between two variables, autocorrelation measures the linear relationship between lagged values of a time series [11].

To determine the seasonality of each series the Auto-Correlation Function (ACF) was applied to each series and the result plotted.

When data are seasonal, the auto-correlations will be larger for the seasonal lags (at multiples of the seasonal frequency) than for other lags [11].

Below are the plots of ACF, that show seasonal behaviour on the series.

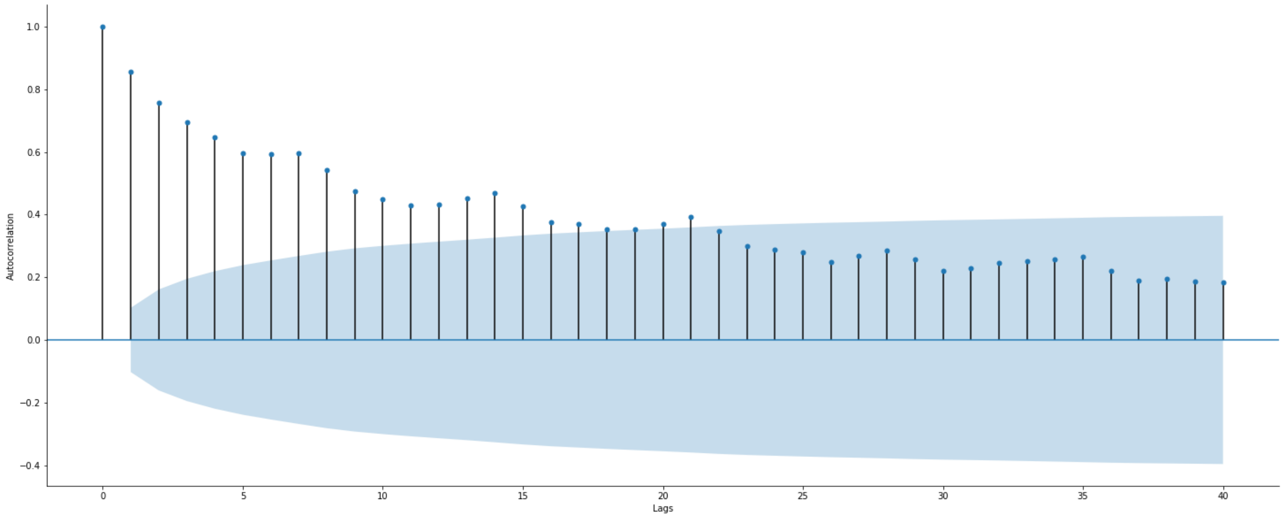

FIGURE 4.1: Plot of the autocorrelation function of a time series from the Average UE's connected KPI

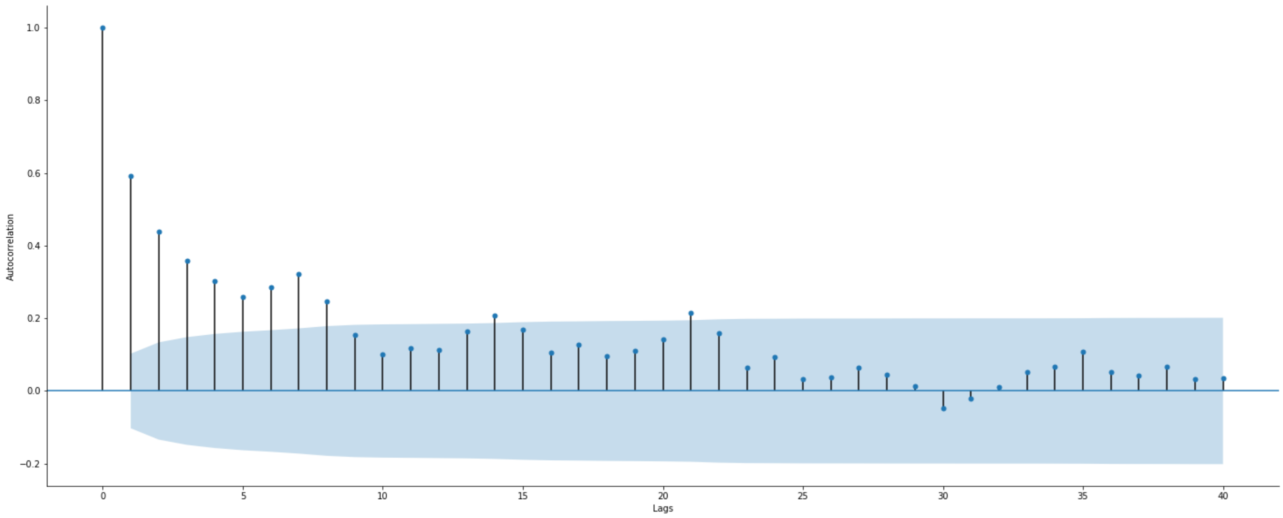

FIGURE 4.2: Plot of a seasonal autocorrelation function of a time series from the maximum number of active UE's per cell KPI

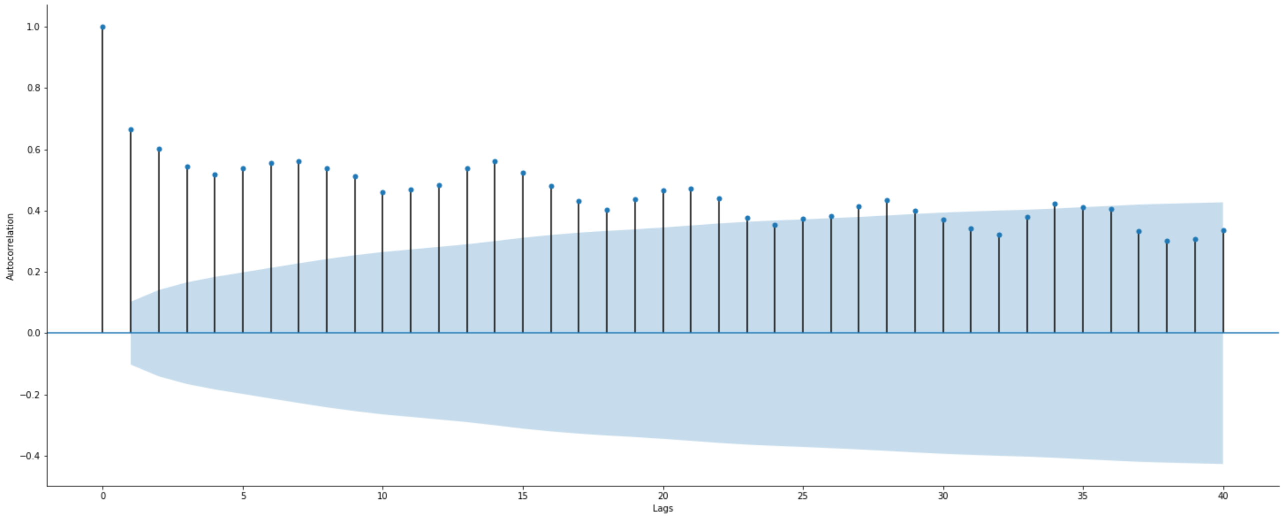

FIGURE 4.3: Plot of a seasonal autocorrelation function of a time series from the maximum number of connected UE's per cell KPI

All the autocorrelations presented represent the ACF function of the large majority of the time series of each of the KPIs analysed. The seasonal behaviour confers the plot with a scalloped look, with peaks according to the seasonality representation - in all of three plots is lagged seven spaces, which indicates a weekly seasonality of the series.

4.2.2 Expanding window

Once the seasonality of the KPIs was defined the models could be applied. The forecast process was incremental and iterative, since some conditions evolved over time, namely the number of analysed models.

For some time, forecasts were calculated using a sample of 350 days per series. Each model would only forecast fifteen days for each series - this was a fast process, although the results were not much consistent.

In order to achieve reliable results as to the performance of each model, the expanding window method was applied. At the time the decision for the future forecasts was to be between Rolling Window or Expanding Window.

In the Rolling or Sliding window methodology, the models would always have a train set with the same number of samples (length), that would move as forecasts were made - for instance in the current study forecasts would always be made using a 250 days window.

As in the Expanding Window the train set expands as forecasts were to be made, consequently models would start with a 250 days window, and would end up with a 350 days window. Since in this method models do not lose any data, and in the current study only a year of data is available for each cell, this methodology seemed to be better hence it was chosen for train the models.

Models were trained using an initial set containing 250 samples, and forecasts were calculated until the training set contained 350 samples. For each new iteration a day was added to the training set and forecasts were calculated. Each model performed this loop 100 times, creating each time forecasts with an horizon of 15 days for each cell.

4.3 Results and Analysis

Once the forecasts were produced a mandatory working of organising the results was necessary, since models produced forecasts in different formats.

It is worth mentioning, before continuing, that each Key Performance Indicators was treated individually.

It was necessary that for each model and KPI the results were aggregated by cell, and since each cell was forecasted 100 times, ordered from the first forecasted, which has the smaller expanding window to the largest, the same is saying that the first forecast of any given cell would be the one which used the 250 sample historic, and the last the one which used the 350 samples of historic values.

4.3.1 Criteria

As already presented (chapter 3), many were the accuracy measures considered during the development of this study. As the study was evolving, they were all calculated and analysed. However the continuous improvement of the present work, was important to reduce their number for analysis purposes, considering that having a large number of measures would not only be costly in terms of calculation time but also the would not help in the subsequent analysis. Having that in mind, the list was incrementally shortened, until only two measures were remaining - Mean Absolute Scaled Error and Symmetric Mean Absolute Percentage Error.

The choice of these two accuracy measures rely on their independence of data scale and the fact that in the case of sMAPE it was already used in previous M-Competitions, and the fact that it is more reliable than MAPE.

MASE in the other hand was recommended on *Other look at measures of forecast accuracy* (R. J. Hyndman and A. B. Koehler, 2006), and later was also used in the M4 Competition.

These accuracy measures offer the possibility of comparing the results of the forecasts in an easy, simple and most of all reliable form.

It is important to take into special consideration that due to the analysed results of both measures, but with an emphasis regarding MASE, being themselves the result of several iterations through the original calculated accuracy, hence the values of MASE tend to usually be over 1, which would indicate a less performing model - hence the value of MASE is considered without giving much importance of its scale.

Models will be compared by the results they achieved in both measures, though sMAPE will be used as a validator of MASE.

Each cell was forecasted using an horizon of 15 days, once the results were obtained and arranged, each forecast was divided then into four different horizons, one day, five days, ten days and 15 days horizons.

For each of those horizons the respectively sMAPE and MASE were calculated - this process allows the identification of a better model for each of the used horizons, hence it was possible for multiple models to perform better in the same KPI, according to the analysed horizon.

4.3.2 Results

This section is dedicated to present the results obtained from the accuracy measures. They will be separated according with the seasonality. For each KPI the forecasting horizons will be analysed separately and each horizon will comprise two graphs (regarding the accuracy measures) and a table that stage both measures.

4.3.2.1 Seasonal KPIs

- Average of UE's connected
  One Day Horizon

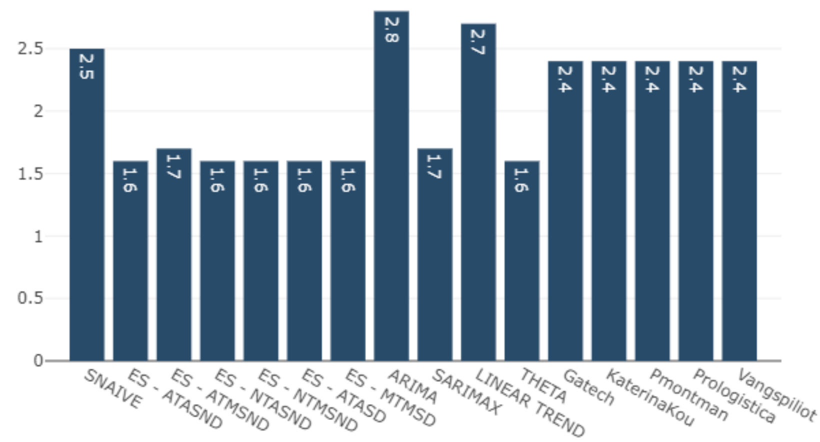

Figure 4.4: Accuracy measured using MASE for the average UE's connected KPI, with one day horizon forecast

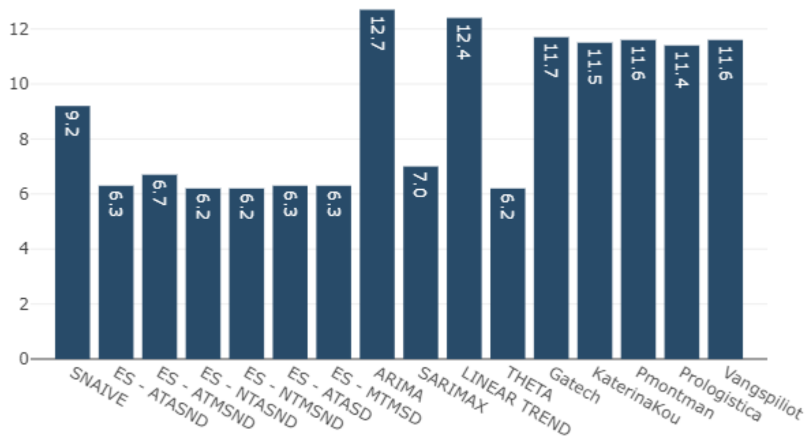

Figure 4.5: Accuracy measured using sMAPE for the average UE's connected KPI, with one day horizon forecast

The obtained differences are not relevant for either of the accuracy measures, this can be better observed on table 4.1 (below).

The models ES-NTASND, ES-NTMSND and Theta have the overall best MASE and sMAPE, however the differences are not sufficiently relevant when comparing these models with ES-ATASND, ES-ATASD or ES-MTMSD.

Model	sMAPE	MASE
SNAIVE	9.2	2.5
ES-ATASND	6.3	1.6
ES-ATMSND	6.7	1.7
ES-NTASND	**6.2**	**1.6**
ES-NTMSND	**6.2**	**1.6**
ES-ATASD	6.3	1.6
ES-MTMSD	6.3	1.6
ARIMA	12.7	2.8
LINEAR TREND	12.4	2.7
SARIMAX	7.0	1.7
THETA	**6.2**	**1.6**
Gatech	11.7	2.4
KaterinaKou	11.5	2.4
Pmontman	11.6	2.4
Prologistica	11.4	2.4
Vangspiliot	11.6	2.4

Table 4.1: Seasonal results of sMAPE and MASE, with horizon equal to one day

. The overall best results are emphasized.

Five Days Horizon

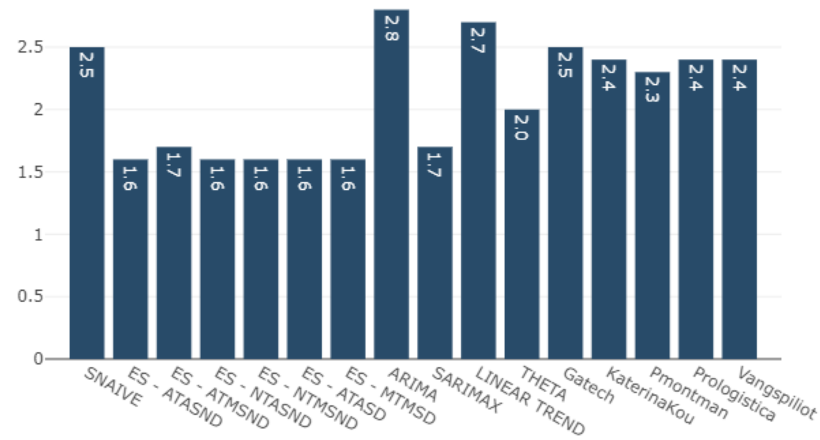

FIGURE 4.6: Accuracy measured using MASE for the average UE's connected KPI, with five day horizon forecast

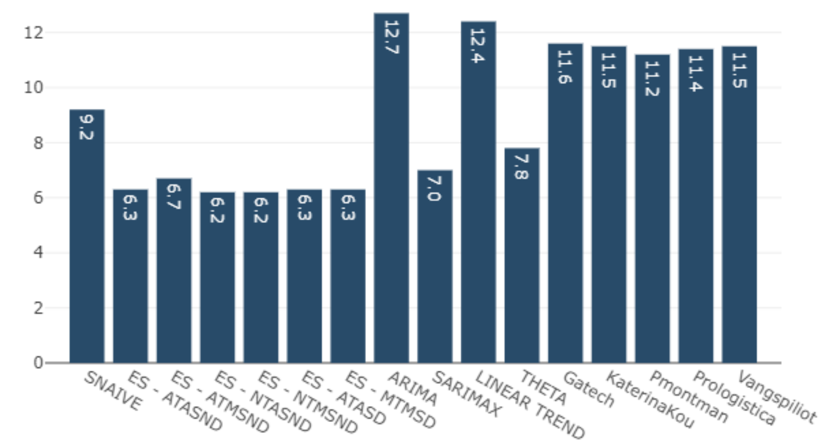

FIGURE 4.7: Accuracy measured using sMAPE for the average UE's connected KPI, with five day horizon forecast

As presented on table 4.2 there is not much difference between some of the studied models. Both ES-NTASND and ES-NTMSND have the same results, therefore either could be chosen as best performing model for an horizon of five days, in spite of the differences to ES-ATASND, ES-ATASD or ES-MTMSD are so little, that can be disregarded.

Model	sMAPE	MASE
SNAIVE	9.2	2.5
ES-ATASND	6.3	1.6
ES-ATMSND	6.7	1.7
ES-NTASND	**6.2**	**1.6**
ES-NTMSND	**6.2**	**1.6**
ES-ATASD	6.3	1.6
ES-MTMSD	6.3	1.6
ARIMA	12.7	2.8
LINEAR TREND	12.4	2.7
SARIMAX	7	1.7
THETA	7.8	2
Gatech	11.6	2.4
KaterinaKou	11.5	2.4
Pmontman	11.2	2.4
Prologistica	11.4	2.4
Vangspiliot	11.5	2.4

TABLE 4.2: Seasonal results of sMAPE and MASE, with an horizon of five days for the average UE's connected KPI

Ten Days Horizon

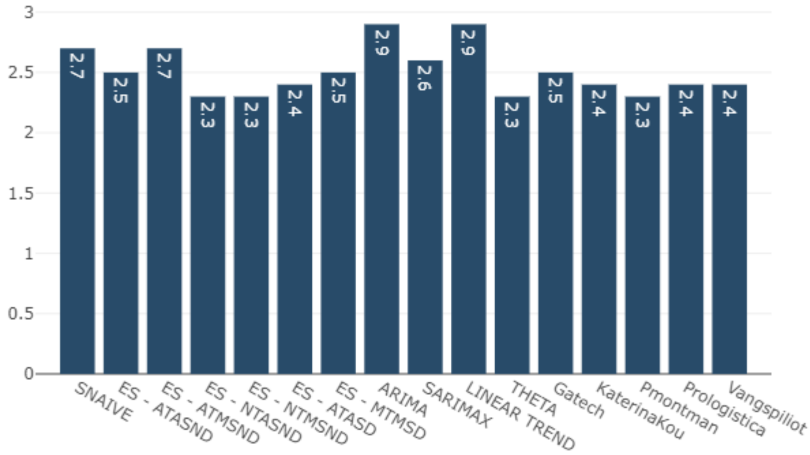

FIGURE 4.8: Accuracy measured using MASE for the average UE's connected KPI, with ten day horizon forecast

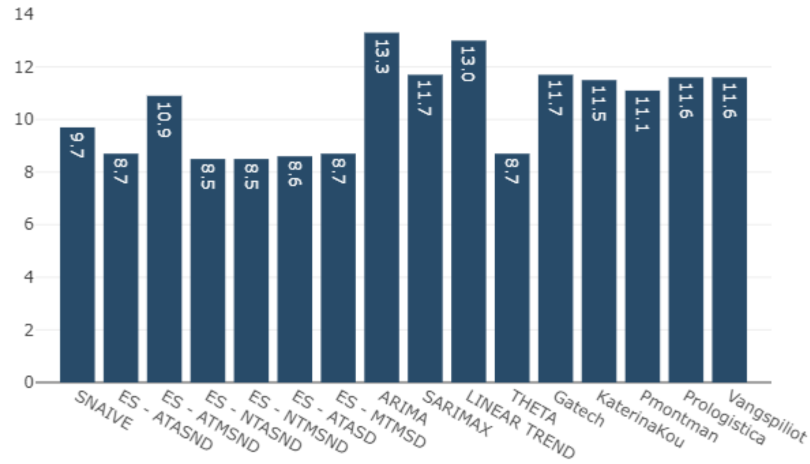

FIGURE 4.9: Accuracy measured using sMAPE for the average UE's connected KPI, with ten day horizon forecast

As when analysing the five day horizon, both ES-NTASND and ES-NTMSND have similar values, consequently either is the best model for this Key Performance Indicators (KPI) with an horizon of ten days. However once again there are other models with really close results, where the difference is not clear, ES-ATASD and Theta can therefore be considered as good as the best.

Model	sMAPE	MASE
SNAIVE	9.7	2.7
ES-ATASND	8.7	2.5
ES-ATMSND	10.9	2.7
ES-NTASND	**8.5**	**2.3**
ES-NTMSND	**8.5**	**2.3**
ES-ATASD	8.6	2.4
ES-MTMSD	8.7	2.5
ARIMA	13.3	2.9
LINEAR TREND	13	2.9
SARIMAX	11.7	2.6
THETA	8.7	2.3
Gatech	11.7	2.5
KaterinaKou	11.5	2.4
Pmontman	11.1	2.3
Prologistica	11.6	2.4
Vangspiliot	11.6	2.4

Table 4.3: Seasonal results of sMAPE and MASE, with horizon ten days for for the average UE's connected Key Performance Indicators (KPI)

Fourteen Days Horizon

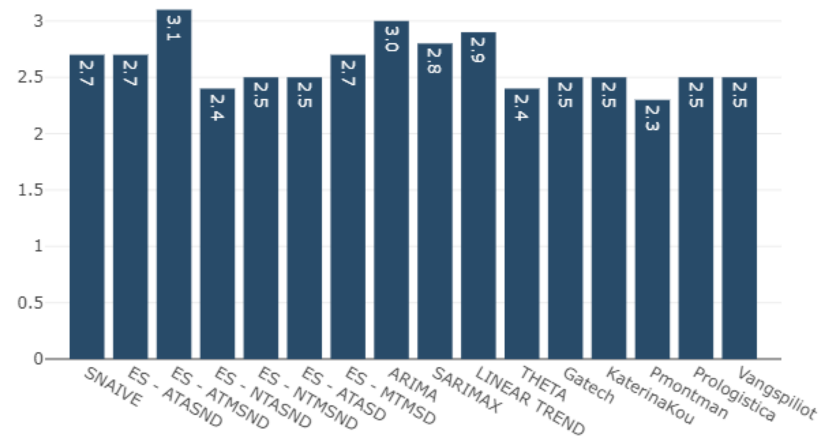

FIGURE 4.10: Accuracy measured using MASE for the average UE's connected KPI, with fourteen day horizon forecast

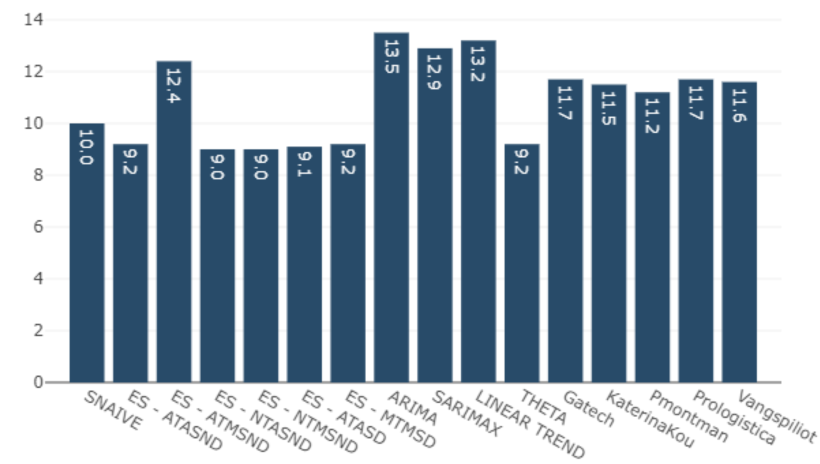

FIGURE 4.11: Accuracy measured using sMAPE for the average UE's connected KPI, with fourteen day horizon forecast

On table 4.4 are represented the results for a fourteen-day horizon of the KPI average UE's connected. The model with the best performance for the horizon and KPI under study in this case is ES-NTASND, despite a really small difference to ES-NTMSND, ES-ATASD and Theta.

Model	sMAPE	MASE
SNAIVE	10	2.7
ES-ATASND	9.2	2.7
ES-ATMSND	12.4	3.1
ES-NTASND	**9**	**2.4**
ES-NTMSND	9	2.5
ES-ATASD	9.1	2.5
ES-MTMSD	9.2	2.7
ARIMA	13.5	3
LINEAR TREND	13.2	2.9
SARIMAX	12.9	2.8
THETA	9.2	2.4
Gatech	11.7	2.5
KaterinaKou	11.5	2.5
Pmontman	11.2	2.3
Prologistica	11.7	2.5
Vangspiliot	11.6	2.5

Table 4.4: Seasonal results of sMAPE and MASE, with horizon fourteen days

- Maximum number of active UE's per cell

  One Day Horizon

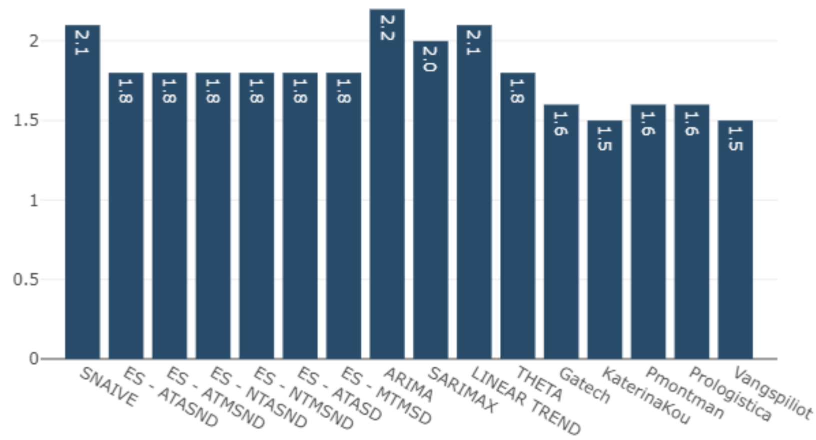

Figure 4.12: Results for MASE having one day horizon

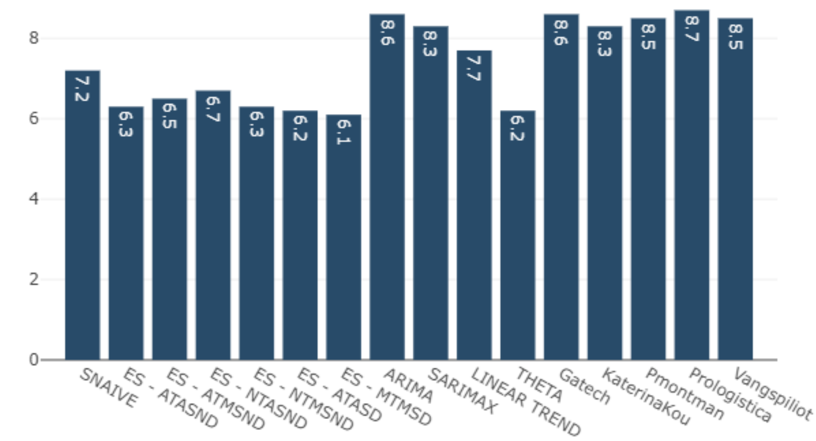

Figure 4.13: Results for sMAPE having one day horizon

Table 4.5 presents the results of sMAPE and MASE with horizon equal to one day.

The model with the best results is ES-MTMSD with a minor difference for Theta, ES-ATASD and ES-ATMSND - although all models from M4 have better performance in MASE than the rest of the competition.

Model	sMAPE	MASE
SNAIVE	7.2	2.1
ES-ATASND	6.3	1.8
ES-ATMSND	6.5	1.8
ES-NTASND	6.7	1.8
ES-NTMSND	6.3	1.8
ES-ATASD	6.2	1.8
ES-MTMSD	**6.1**	**1.8**
ARIMA	8.6	2.2
LINEAR TREND	7.7	2.1
SARIMAX	8.3	2
THETA	6.2	1.8
Gatech	8.6	1.6
KaterinaKou	8.3	1.5
Pmontman	8.5	1.6
Prologistica	8.7	1.6
Vangspiliot	8.5	1.5

TABLE 4.5: Seasonal results of sMAPE and MASE, with horizon one day, for KPI Maximum number of active UE's per cell

Five Days Horizon

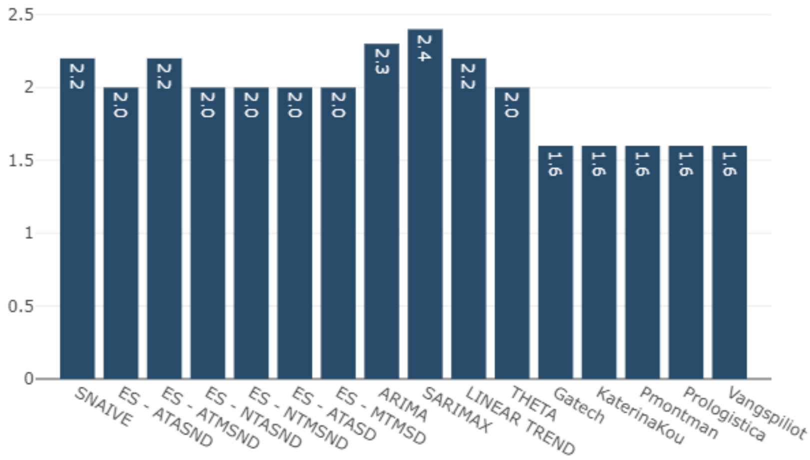

Figure 4.14: Accuracy measured using MASE for the Maximum number of active UE's per cell KPI, with five day horizon forecast

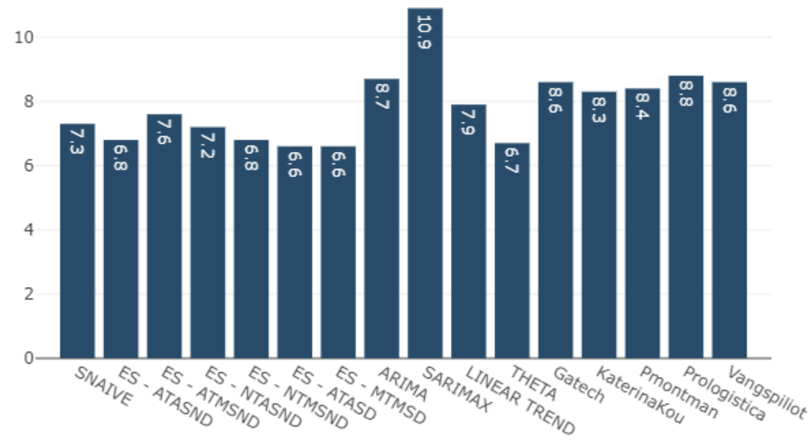

Figure 4.15: Accuracy measured using sMAPE for the Maximum number of active UE's per cell KPI, with five day horizon forecast

As it can be observed (in the table 4.6), the values recorded using MASE are indistinct between the models from M4, and almost indistinct between the remaining models, but there is a distinctive difference between these two groups.

By considering the values of sMAPE, the best models would be either ES-NTMSND or ES-MTMSD, on the other hand if the value considered would only be MASE, the models from M4 would be selected. Considering both measures, even considering MASE more relevant than sMAPE, the best performing model are ES-ATASD, and ES-MTMSD since the difference in sMAPE is greater than the one registered by MASE.

Model	sMAPE	MASE
SNAIVE	7.3	2.2
ES-ATASND	6.8	2
ES-ATMSND	7.6	2.2
ES-NTASND	7.2	2
ES-NTMSND	6.8	2
ES-ATASD	**6.6**	**2**
ES-MTMSD	**6.6**	**2**
ARIMA	8.7	2.3
LINEAR TREND	7.9	2.2
SARIMAX	10.9	2.4
THETA	6.7	2
Gatech	8.6	1.6
KaterinaKou	8.3	1.6
Pmontman	8.4	1.6
Prologistica	8.8	1.6
Vangspiliot	8.6	1.6

Table 4.6: Seasonal results of sMAPE and MASE, with horizon of five days, for KPI Maximum number of active UE's per cell

Ten Days Horizon

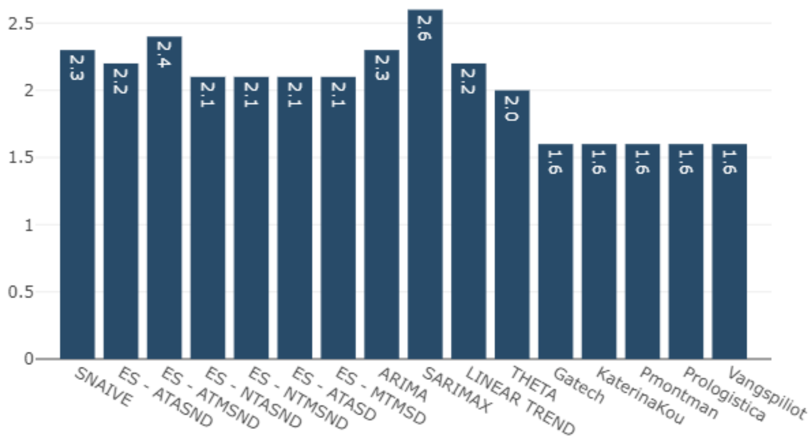

FIGURE 4.16: Accuracy measured using MASE for the Maximum number of active UE's per cell KPI, with ten days horizon forecast

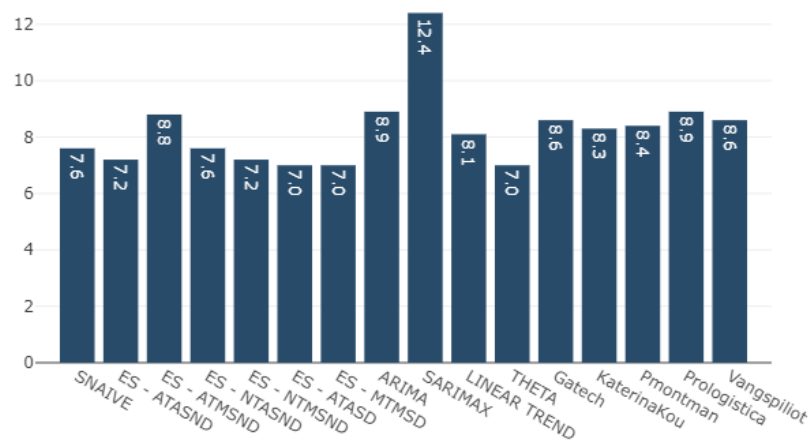

FIGURE 4.17: Accuracy measured using sMAPE for the Maximum number of active UE's per cell KPI, with an horizon of ten days

The table 4.7 shows the results of both sMAPE and MASE with horizon equal to ten days.

Similar to what happened in the previous analysis of the Maximum number of active UE's per cell KPI, M4 models outperform the remaining models. The best models are ES-ATASD and ES-MTMSD, despite the small difference to other models.

Model	sMAPE	MASE
SNAIVE	7.6	2.3
ES-ATASND	7.2	2.2
ES-ATMSND	8.8	2.4
ES-NTASND	7.6	2.1
ES-NTMSND	7.2	2.1
ES-ATASD	**7**	**2.1**
ES-MTMSD	**7**	**2.1**
ARIMA	8.9	2.3
LINEAR TREND	8.1	2.2
SARIMAX	12.4	2.6
THETA	7	2
Gatech	8.6	1.6
KaterinaKou	8.3	1.6
Pmontman	8.4	1.6
Prologistica	8.9	1.6
Vangspiliot	8.6	1.6

Table 4.7: Seasonal results of sMAPE and MASE, with horizon ten days, for KPI Maximum number of active UE's per cell

Fourteen Days Horizon

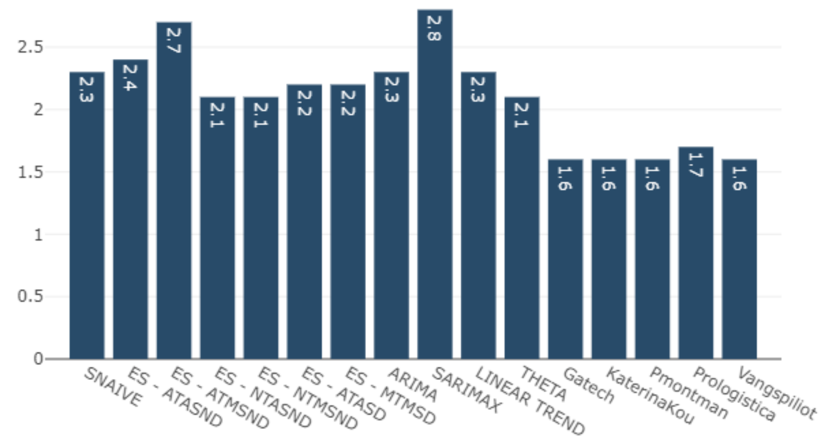

FIGURE 4.18: Accuracy measured using MASE for the Maximum number of active UE's per cell KPI, with fourteen days horizon forecast

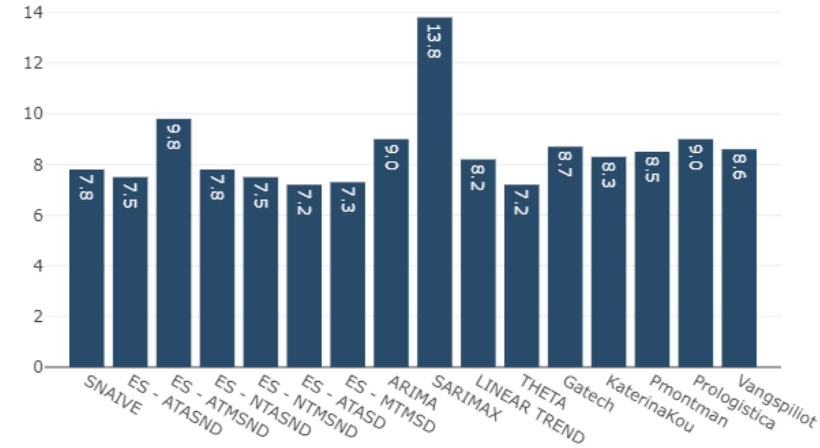

FIGURE 4.19: Accuracy measured using sMAPE for the Maximum number of active UE's per cell KPI, with an horizon of fourteen days

If the result of the best model was to be decided only by MASE, the best model would undoubtedly be one of the M4 models (KaterinaKou), although by observing the values obtained on sMAPE, non M4 models usually performed better.

The decision once again relies on the interpretation of the complete set of results, consequently the best performing model is KaterinaKou.

Model	sMAPE	MASE
SNAIVE	7.8	2.3
ES-ATASND	7.5	2.4
ES-ATMSND	9.8	2.7
ES-NTASND	7.8	2.1
ES-NTMSND	7.5	2.1
ES-ATASD	7.2	2.2
ES-MTMSD	7.3	2.2
ARIMA	9	2.3
LINEAR TREND	8.2	2.3
SARIMAX	13.8	2.8
THETA	7.2	2.1
Gatech	8.7	1.6
KaterinaKou	**8.3**	**1.6**
Pmontman	8.5	1.6
Prologistica	9	1.7
Vangspiliot	8.6	1.6

Table 4.8: Seasonal results of sMAPE and MASE, with horizon fourteen days, for KPI Maximum number of active UE's per cell

- Maximum number of connected UE's per cell

  One Day Horizon

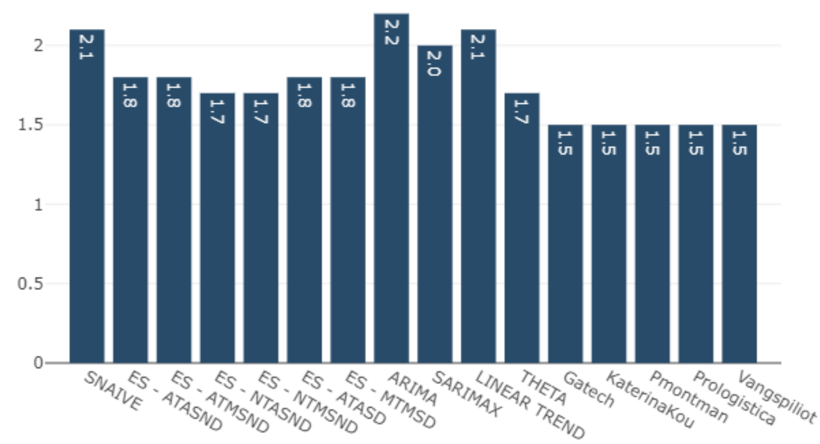

FIGURE 4.20: MASE results, obtained with one day horizon forecast

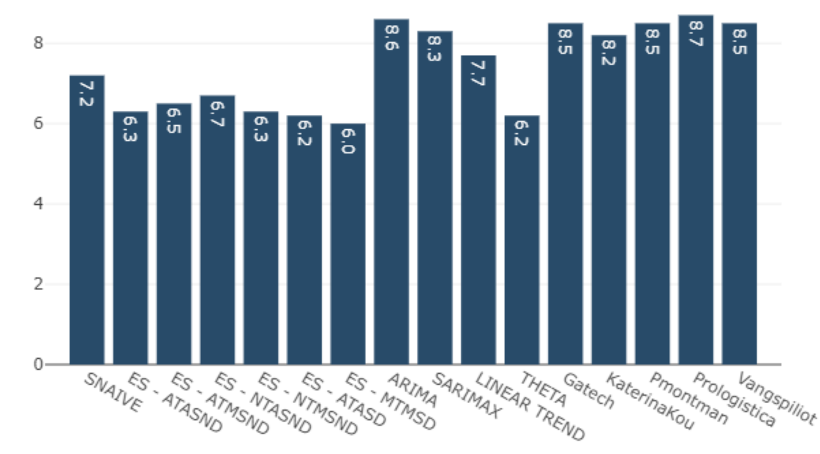

FIGURE 4.21: sMAPE results with one day horizon forecast

The selection of the best model in this case is not obvious, since the values of MASE and sMAPE are not compliant. By considering only MASE, the best models would be either any of the M4 models, however if considering only the sMAPE, the best models are ES-MTMSD. In this case the best model could be either one of the six, since the decision will always depend on the importance given to each of the used accuracy measures, even so the values are not that apart. The case study as already stated before relies on giving a bigger emphasis on MASE, but the difference between KaterinaKou and ES-MTMSD is smaller in the MASE and grew a in sMAPE - the best model is ES-MTMSD.

Model	sMAPE	MASE
SNAIVE	7.2	2.1
ES-ATASND	6.3	1.8
ES-ATMSND	6.5	1.8
ES-NTASND	6.7	1.7
ES-NTMSND	6.3	1.7
ES-ATASD	6.2	1.8
ES-MTMSD	**6**	**1.8**
ARIMA	8.6	2.2
LINEAR TREND	7.7	2.1
SARIMAX	8.3	2
THETA	6.2	1.7
Gatech	8.5	1.5
KaterinaKou	8.2	1.5
Pmontman	8.5	1.5
Prologistica	8.7	1.5
Vangspiliot	8.5	1.5

TABLE 4.9: Seasonal results of sMAPE and MASE, with horizon one day, for KPI Maximum Number of Connected UE's

Five Days Horizon

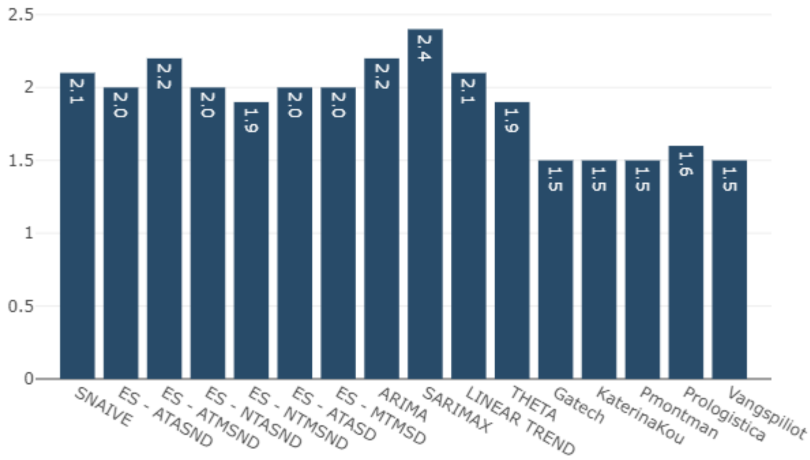

FIGURE 4.22: Accuracy measured using MASE for the Maximum Number of Connected UE's KPI, with five days horizon forecast

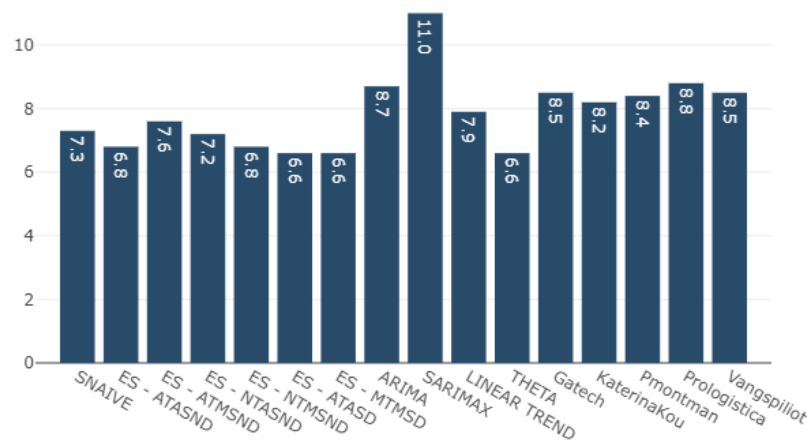

FIGURE 4.23: Accuracy measured using sMAPE for the Maximum Number of Connected UE's KPI, with five days horizon forecast

Table 4.10 contains the results of sMAPE and MASE with horizon equal to five days.

This is another case where MASE and MAPE are not compliant, once again the differences on sMAPE are not too great, in the possible best models. The overall best model here is Theta because of the smaller value of sMAPE and a small difference in MASE for the M4 models.

Model	sMAPE	MASE
SNAIVE	7.3	2.1
ES-ATASND	6.8	2
ES-ATMSND	7.6	2.2
ES-NTASND	7.2	2
ES-NTMSND	6.8	1.9
ES-ATASD	6.6	2
ES-MTMSD	6.6	2
ARIMA	8.7	2.2
LINEAR TREND	7.9	2.1
SARIMAX	11	2.4
THETA	**6.6**	**1.9**
Gatech	8.5	1.5
KaterinaKou	8.2	1.5
Pmontman	8.4	1.5
Prologistica	8.8	1.6
Vangspiliot	8.5	1.5

TABLE 4.10: Seasonal results of sMAPE and MASE, with an horizon of five days, for KPI Maximum Number of Connected UE's

Ten Days Horizon

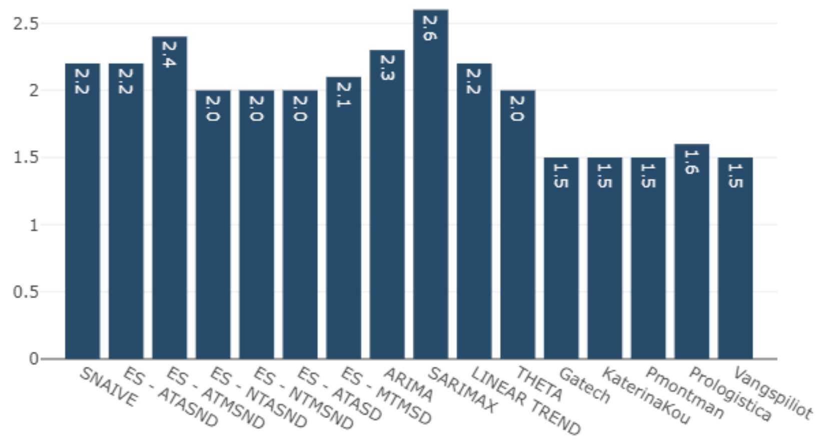

Figure 4.24: Accuracy measured using MASE for the Maximum Number of Connected UE's KPI, with ten days horizon forecast

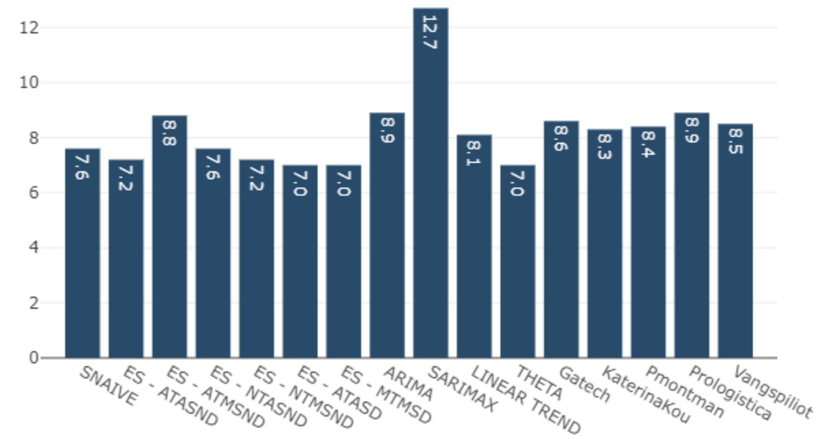

Figure 4.25: Accuracy measured using sMAPE for the Maximum Number of Connected UE's KPI, with ten days horizon forecast

The results in the perspective of sMAPE reveal the best model as being ES-ATASD, ES-MTMSD and Theta, though MASE is once again better on the

M4 models. The issue recorded in this case is the same previously analysed situation. The overall best method is chosen considering a combination of both values (with bigger emphasis on the MASE), hence here the best performing model is Theta.

Model	sMAPE	MASE
SNAIVE	7.6	2.2
ES-ATASND	7.2	2.2
ES-ATMSND	8.8	2.4
ES-NTASND	7.6	2
ES-NTMSND	7.2	2
ES-ATASD	7	2
ES-MTMSD	7	2.1
ARIMA	8.9	2.3
LINEAR TREND	8.1	2.2
SARIMAX	12.7	2.6
THETA	**7**	**2**
Gatech	8.6	1.5
KaterinaKou	8.3	1.5
Pmontman	8.4	1.5
Prologistica	8.9	1.6
Vangspiliot	8.5	1.5

Table 4.11: Seasonal results of sMAPE and MASE, with a ten days horizon, for KPI Maximum Number of Connected UE's

Fourteen Days Horizon

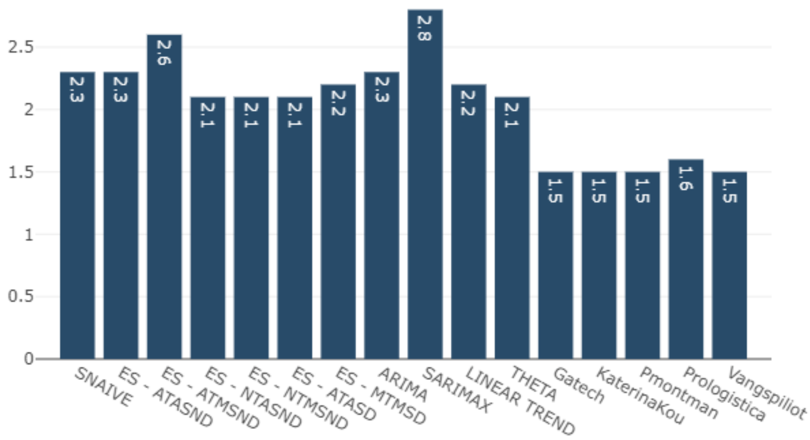

FIGURE 4.26: Accuracy measured using MASE for the Maximum Number of Connected UE's KPI, with fourteen days horizon forecast

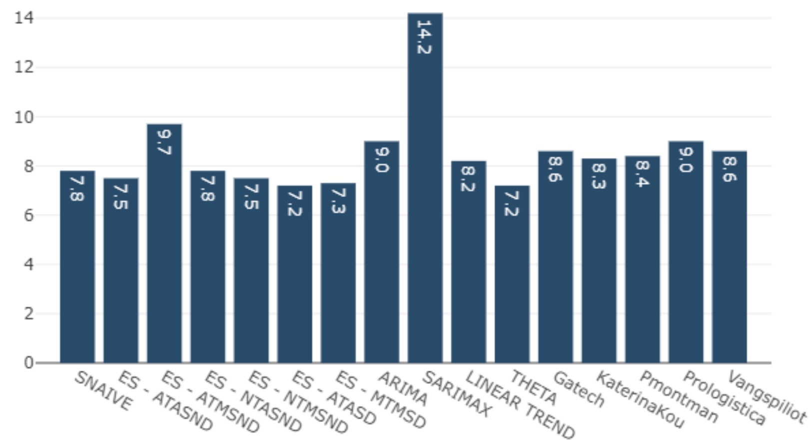

FIGURE 4.27: Accuracy measured using sMAPE for the Maximum Number of Connected UE's KPI, with fourteen days horizon forecast

Table 4.12 contains the results of MASE and sMAPE for a fourteen horizon of forecasting.

The previously analysed situation is repeated for the present horizon of fourteen days.

By the same principle that was applied before, the best model is Theta.

Model	sMAPE	MASE
SNAIVE	7.8	2.3
ES-ATASND	7.5	2.3
ES-ATMSND	9.7	2.6
ES-NTASND	7.8	2.1
ES-NTMSND	7.5	2.1
ES-ATASD	7.2	2.1
ES-MTMSD	7.3	2.2
ARIMA	9	2.3
LINEAR TREND	8.2	2.2
SARIMAX	14.2	2.8
THETA	**7.2**	**2.1**
Gatech	8.6	1.5
KaterinaKou	8.3	1.5
Pmontman	8.4	1.5
Prologistica	9	1.6
Vangspiliot	8.6	1.5

Table 4.12: Seasonal results of sMAPE and MASE, with horizon fourteen day, for KPI Maximum Number of Connected UE's

4.3.2.2 Non-Seasonal

- Requests to network access made by UE's during RACH

  One Day Horizon

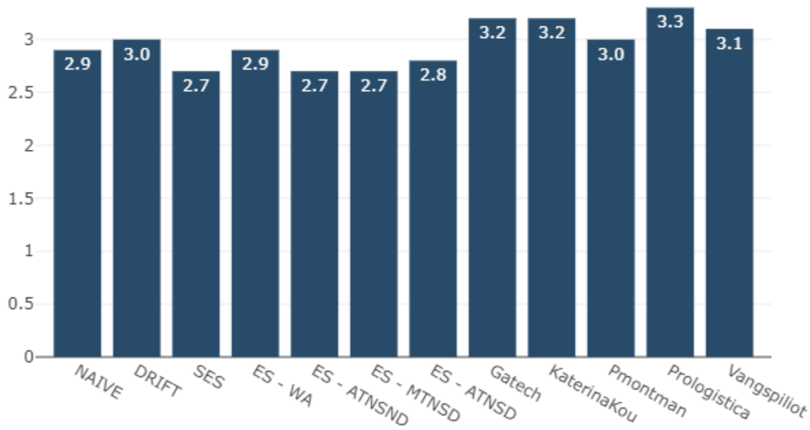

FIGURE 4.28: Accuracy measured using MASE network access request by UE's

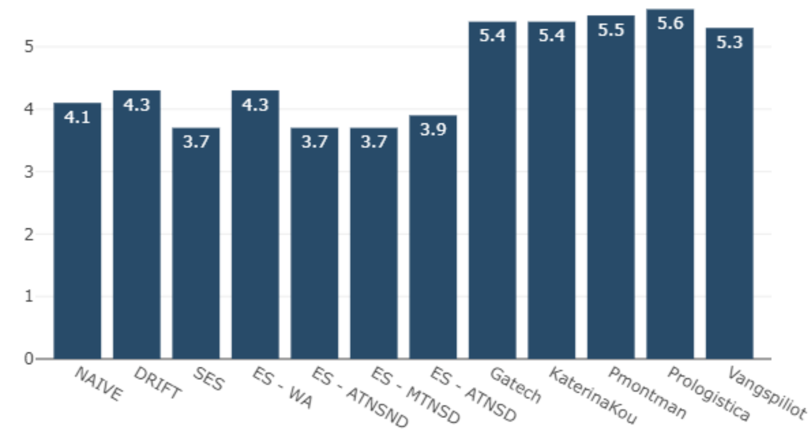

FIGURE 4.29: Accuracy measured using sMAPE network access request to by UE's

The results for the present Key Performance Indicators despite not having a great difference between all the used models, show that SES, ES-MTNSD and ES-ATNSD have better performance than the remaining models.

Model	sMAPE	MASE
NAIVE	4.1	2.9
DRIFT	4.3	3
SES	3.7	2.7
ES - WA	4.3	2.9
ES-MTNSD	**3.7**	**2.7**
ES-ATNSD	**3.7**	**2.7**
ES-ATNSND	3.9	2.8
Gatech	5.4	3.2
KaterinaKou	5.4	3.2
Pmontman	5.5	3
Prologistica	5.6	3.3
Vangspiliot	5.3	3.1

TABLE 4.13: Seasonal results of sMAPE and MASE, with an horizon of one day, for KPI network access request to by UE's

Five Days Horizon

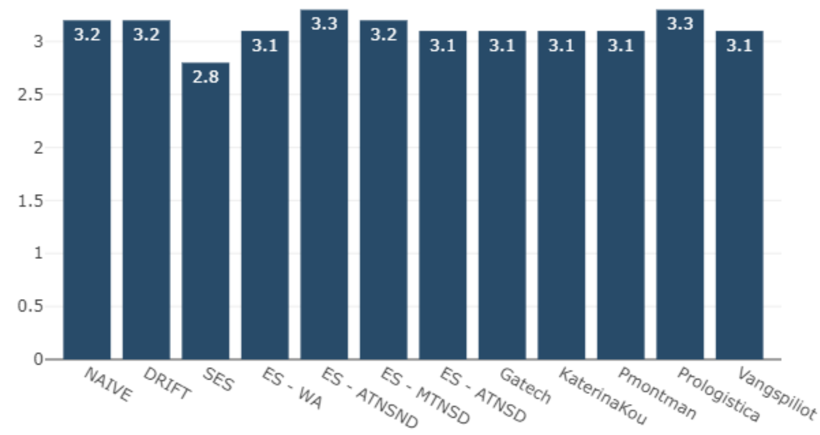

FIGURE 4.30: Accuracy measured using MASE network access request by UE's

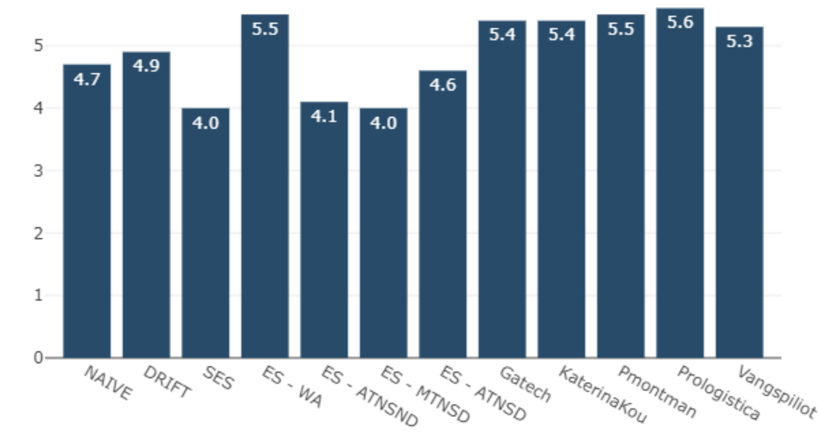

FIGURE 4.31: Accuracy measured using sMAPE network access request to by UE's

For an horizon of five days the results show once again little to no difference between them - though here SES has a better performance than the rest.

Model	sMAPE	MASE
NAIVE	4.7	3.2
DRIFT	4.9	3.2
SES	**4**	**2.8**
ES - WA	5.5	3.1
ES-MTNSD	4.1	3.3
ES-ATNSD	4	3.2
ES-ATNSND	4.6	3.1
Gatech	5.4	3.1
KaterinaKou	5.4	3.1
Pmontman	5.5	3.1
Prologistica	5.6	3.3
Vangspiliot	5.3	3.1

Table 4.14: Seasonal results of sMAPE and MASE, with an horizon equal to five days, for KPI network access request to by UE's

Ten Days Horizon

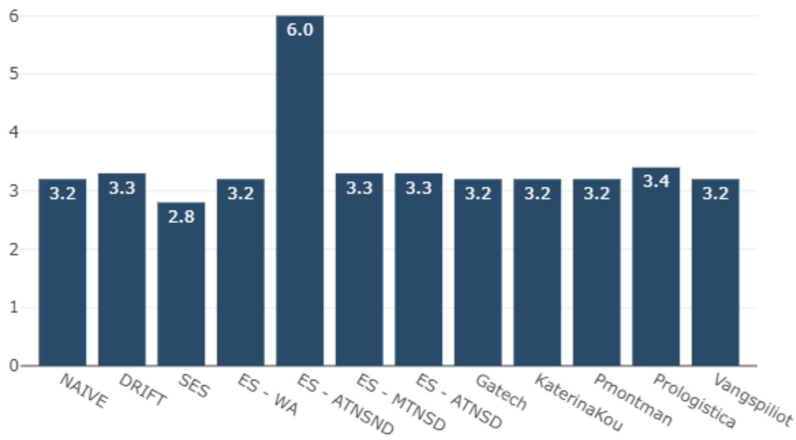

FIGURE 4.32: Accuracy measured using MASE network access request by UE's

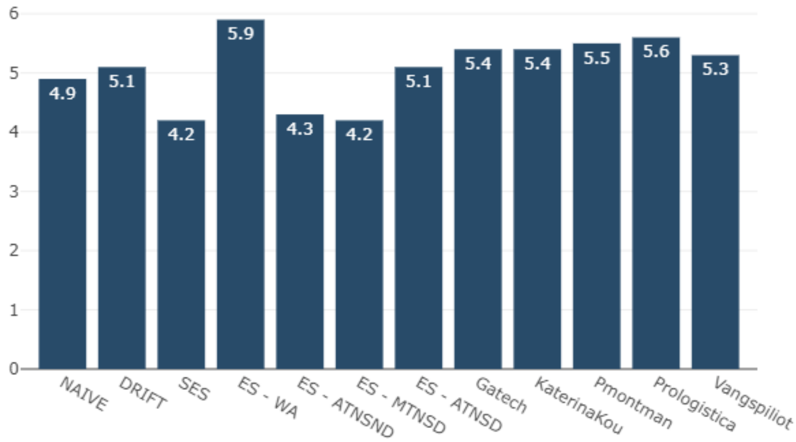

FIGURE 4.33: Accuracy measured using sMAPE network access request to by UE's

The obtained results for an horizon of ten days are apparently to similar to the previous results, SES continues to have an overall best performance compared to the remaining models, despite the blunt differences recorded.

Model	sMAPE	MASE
NAIVE	4.9	3.2
DRIFT	5.1	3.3
SES	**4.2**	**2.8**
ES - WA	5.9	3.2
ES-MTNSD	4.3	6
ES-ATNSD	4.2	3.3
ES-ATNSND	5.1	3.3
Gatech	5.4	3.2
KaterinaKou	5.4	3.2
Pmontman	5.5	3.2
Prologistica	5.6	3.4
Vangspiliot	5.3	3.2

Table 4.15: Seasonal results of sMAPE and MASE, with a ten day horizon, for KPI network access request to by UE's

Fourteen Days Horizon

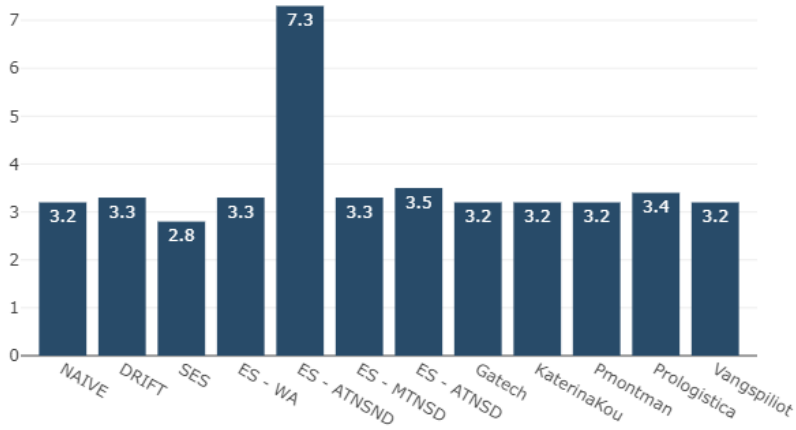

FIGURE 4.34: Accuracy measured using MASE network access request by UE's

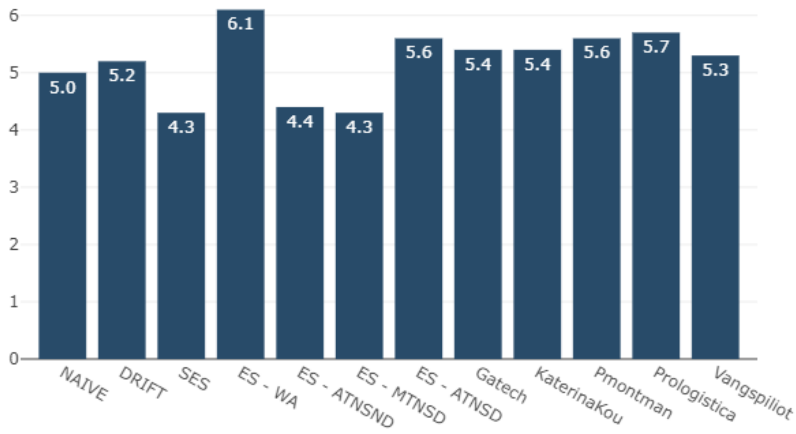

FIGURE 4.35: Accuracy measured using sMAPE network access request to by UE's

Observing the overall results of the present Key Performance Indicators (KPI), the results are similar independent of the analyzed horizon, the differences between a large majority of the used models can be characterized as blunt and is almost irrelevant. And like what happened in the previous horizons here the best performance model is also Simple Exponential Smoothing (SES).

Model	sMAPE	MASE
NAIVE	5	3.2
DRIFT	5.2	3.3
SES	**4.3**	**2.8**
ES - WA	6.1	3.3
ES-MTNSD	4.4	7.3
ES-ATNSD	4.3	3.3
ES-ATNSND	5.6	3.5
Gatech	5.4	3.2
KaterinaKou	5.4	3.2
Pmontman	5.6	3.2
Prologistica	5.7	3.4
Vangspiliot	5.3	3.2

Table 4.16: Seasonal results of sMAPE and MASE, with a fourteen day horizon, for KPI network access request to by UE's

- Allocation of the transport channel for downlink

  One Day Horizon

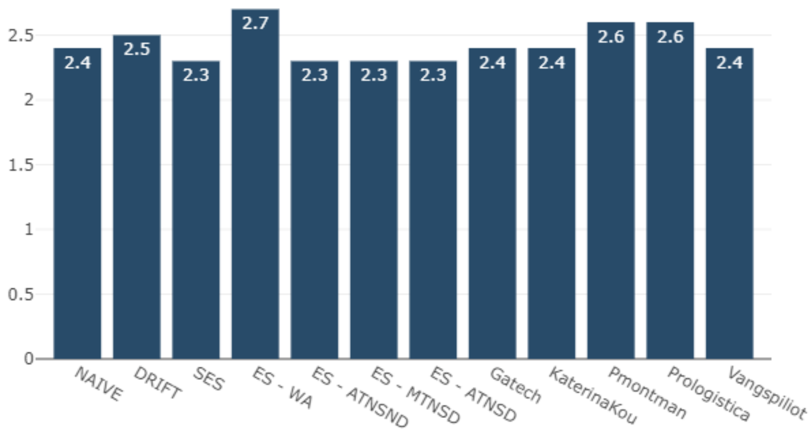

FIGURE 4.36: Accuracy measured using MASE with a one day horizon forecast

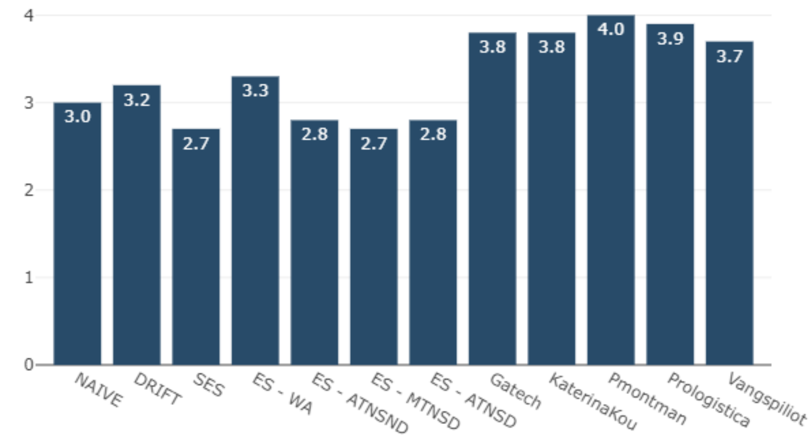

FIGURE 4.37: Accuracy measured using sMAPE with one day horizon forecast

Table 4.21 contains the values previously represented in the graphics (above), for sMAPE and MASE.

By evaluating the values it is observed a notorious similarity in the results, and not much of a difference in the complete set of models. SES and ES-MTNSD are equal, and they are the best performing models for a horizon of one day.

Model	sMAPE	MASE
NAIVE	3	2.4
DRIFT	3.2	2.5
SES	**2.7**	**2.3**
ES - WA	3.3	2.7
ES-MTNSD	**2.7**	**2.3**
ES-ATNSD	2.8	2.3
ES-ATNSND	2.8	2.3
Gatech	3.8	2.4
KaterinaKou	3.8	2.4
Pmontman	4	2.6
Prologistica	3.9	2.6
Vangspiliot	3.7	2.4

Table 4.17: Seasonal results of sMAPE and MASE, with horizon equal to one day, for the KPI allocation of the transport channel for user data downlinks

Five Days Horizon

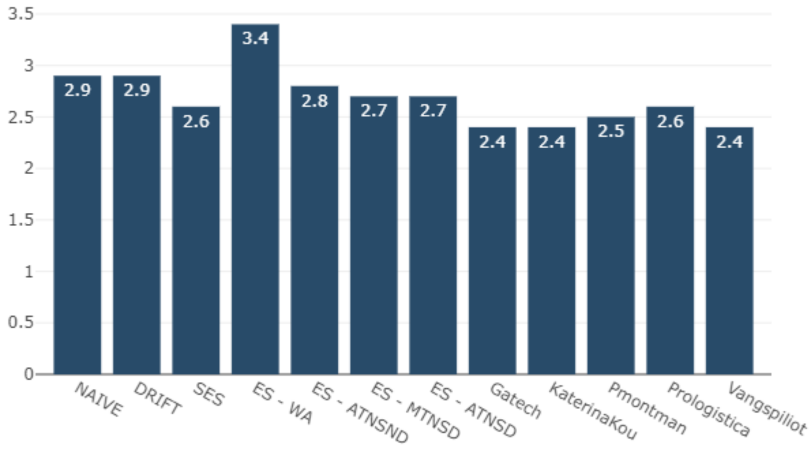

FIGURE 4.38: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with a five day horizon forecast

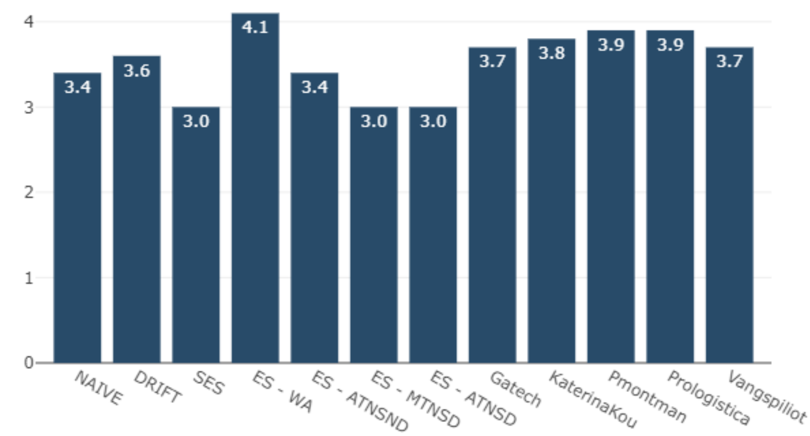

FIGURE 4.39: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with a five day horizon forecast

The values present on the table 4.22, do not have considerable differences between them, this is similar to the previous analyzed horizon. SES continues to have a better performance, even though the difference is irrelevant when the results are compared to ES-ATNSD.

Model	sMAPE	MASE
NAIVE	3.4	2.9
DRIFT	3.6	2.9
SES	**3**	**2.6**
ES - WA	4.1	3.4
ES-MTNSD	3	2.7
ES-ATNSD	3	2.7
ES-ATNSND	3.4	2.8
Gatech	3.7	2.4
KaterinaKou	3.8	2.4
Pmontman	3.9	2.5
Prologistica	3.9	2.6
Vangspiliot	3.7	2.4

Table 4.18: Seasonal results of sMAPE and MASE, with an horizon equal to five days, for the KPI allocation of the transport channel for user data downlink

Ten Days Horizon

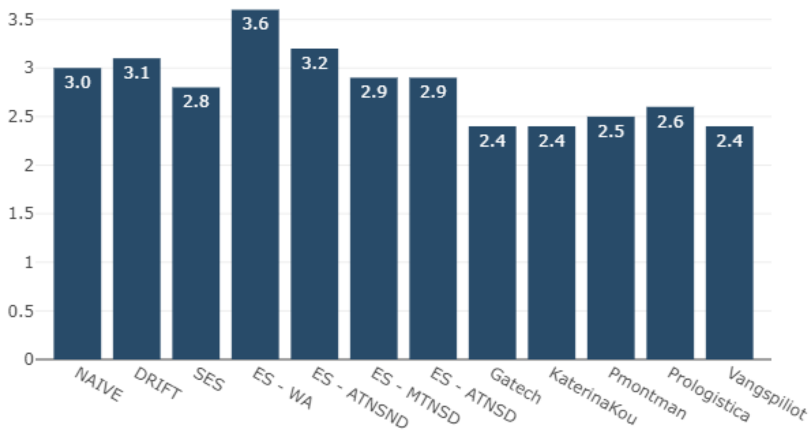

FIGURE 4.40: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with ten day horizon forecast

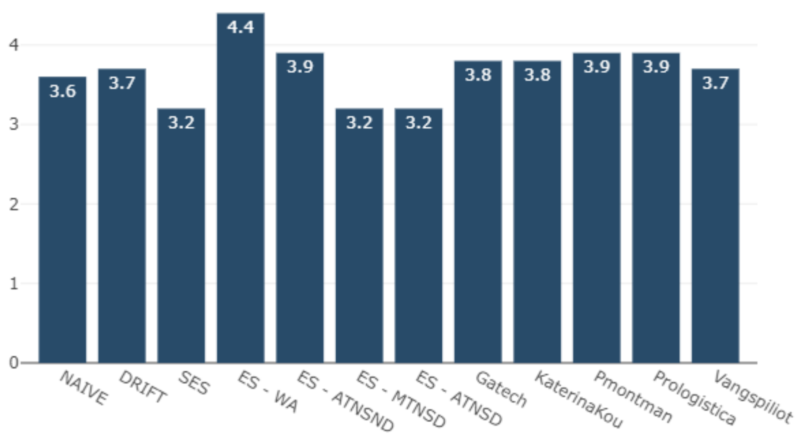

FIGURE 4.41: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with ten day horizon forecast

The values of MASE on the M4 models are consistently better than in all other models, although there is not much a difference in the overall standings, realistically the models seem to have really similar behaviours. Still in this case both KaterinaKou and Vangspiliot have a small advantage from the rest.

Model	sMAPE	MASE
NAIVE	3.6	3
DRIFT	3.7	3.1
SES	3.2	2.8
ES - WA	4.4	3.6
ES-MTNSD	3.2	2.9
ES-ATNSD	3.2	2.9
ES-ATNSND	3.9	3.2
Gatech	3.8	2.4
KaterinaKou	**3.8**	**2.4**
Pmontman	3.9	2.5
Prologistica	3.9	2.6
Vangspiliot	**3.7**	**2.4**

Table 4.19: Seasonal results of sMAPE and MASE, with horizon ten day, for KPI Maximum Number of Connected UE's

Fourteen Days Horizon

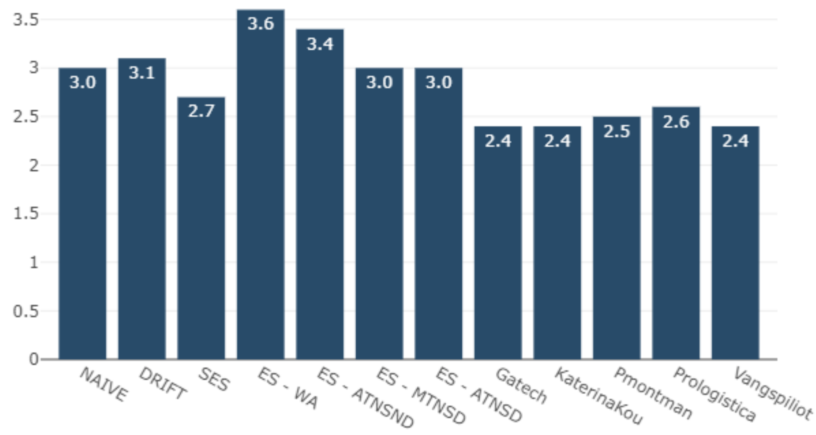

FIGURE 4.42: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with fourteen day horizon forecast

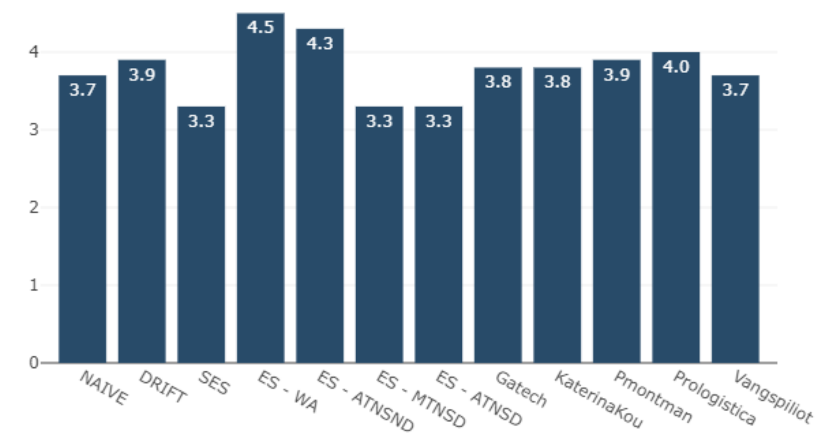

FIGURE 4.43: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with fourteen day horizon forecast

Table 4.24 the results of both sMAPE and MASE for a fourteen day horizon are available.

Like in the previous cases for this KPI there is not a great difference in the values. In such cases it is difficult to say which is the best performing model. Once again the MASE on the M4 models is lower than the calculated for the rest of the models. The best performing M4 model is Vangspiliot, but not by a margin that would declare it as the best.

Model	sMAPE	MASE
NAIVE	3.7	3
DRIFT	3.9	3.1
SES	3.3	2.7
ES - WA	4.5	3.6
ES-MTNSD	3.3	3
ES-ATNSD	3.3	3
ES-ATNSND	4.3	3.4
Gatech	3.8	2.4
KaterinaKou	3.8	2.4
Pmontman	3.9	2.5
Prologistica	4	2.6
Vangspiliot	**3.7**	**2.4**

Table 4.20: Seasonal results of sMAPE and MASE, with horizon fourteen day, for the KPI - allocation of the transport channel for user data downlink

- Allocation of the transport channel for uplink

  One Day Horizon

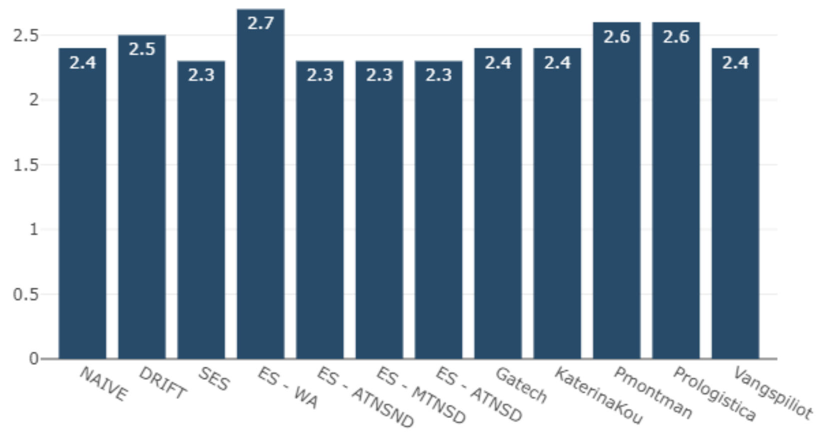

FIGURE 4.44: Accuracy measured using MASE with a one day horizon forecast

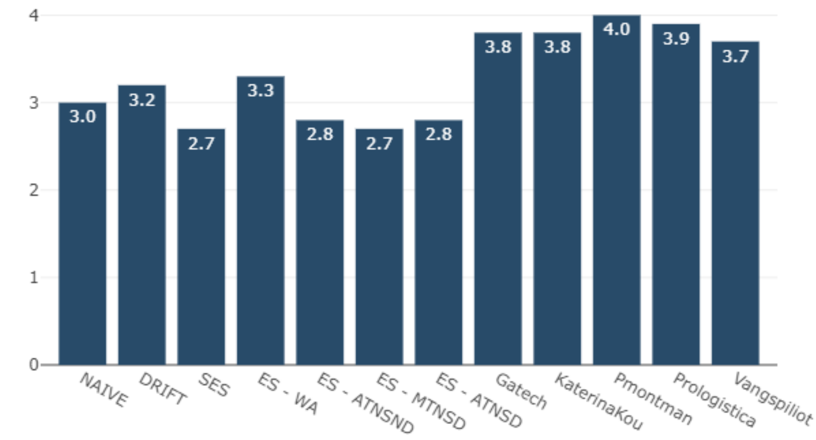

FIGURE 4.45: Accuracy measured using sMAPE with one day horizon forecast

Table 4.21 contains the values previously represented in the graphics (above), for sMAPE and MASE.

By evaluating the values it is observed a notorious similarity in the results, and not much of a difference in the complete set of models. SES and ES-MTNSD are equal, and they are the best performing models for a horizon of one day.

Model	sMAPE	MASE
NAIVE	3	2.4
DRIFT	3.2	2.5
SES	**2.7**	**2.3**
ES - WA	3.3	2.7
ES-MTNSD	**2.7**	**2.3**
ES-ATNSD	2.8	2.3
ES-ATNSND	2.8	2.3
Gatech	3.8	2.4
KaterinaKou	3.8	2.4
Pmontman	4	2.6
Prologistica	3.9	2.6
Vangspiliot	3.7	2.4

Table 4.21: Seasonal results of sMAPE and MASE, with horizon equal to one day, for the KPI allocation of the transport channel for user data downlinks

Five Days Horizon

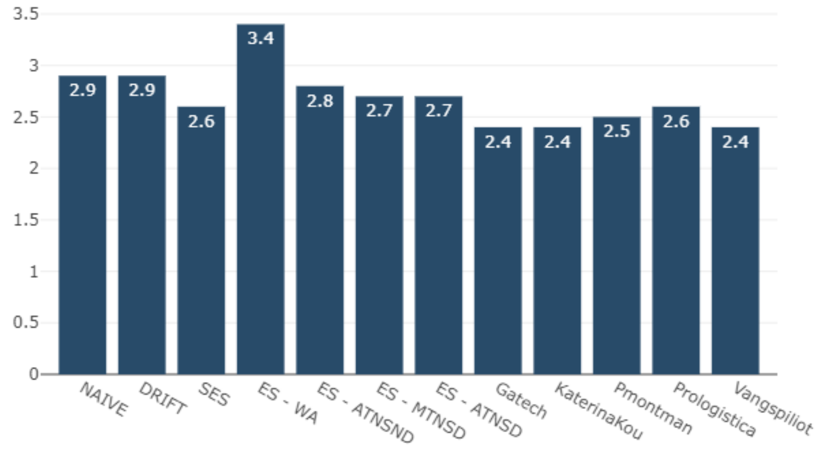

FIGURE 4.46: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with a five day horizon forecast

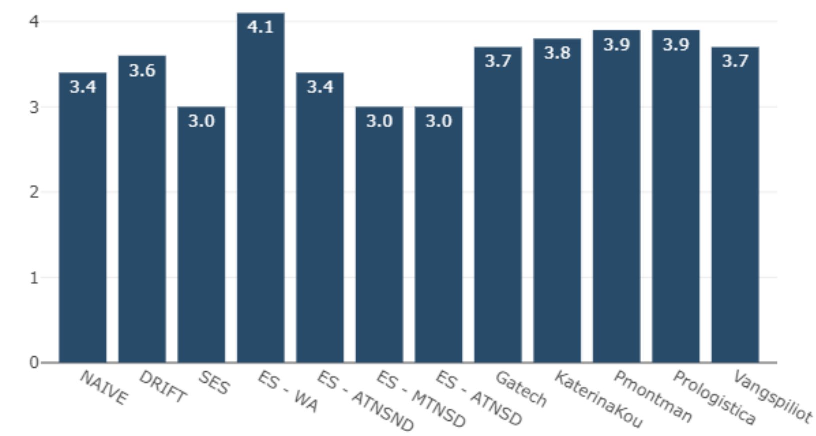

FIGURE 4.47: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with a five day horizon forecast

The values present on the table 4.22, do not have considerable differences between them, this is similar to the previous analyzed horizon. SES continues to have a better performance, even though the difference is irrelevant when the results are compared to ES-ATNSD.

Model	sMAPE	MASE
NAIVE	3.4	2.9
DRIFT	3.6	2.9
SES	**3**	**2.6**
ES - WA	4.1	3.4
ES-MTNSD	3	2.7
ES-ATNSD	3	2.7
ES-ATNSND	3.4	2.8
Gatech	3.7	2.4
KaterinaKou	3.8	2.4
Pmontman	3.9	2.5
Prologistica	3.9	2.6
Vangspiliot	3.7	2.4

Table 4.22: Seasonal results of sMAPE and MASE, with an horizon equal to five days, for the KPI allocation of the transport channel for user data downlink

Ten Days Horizon

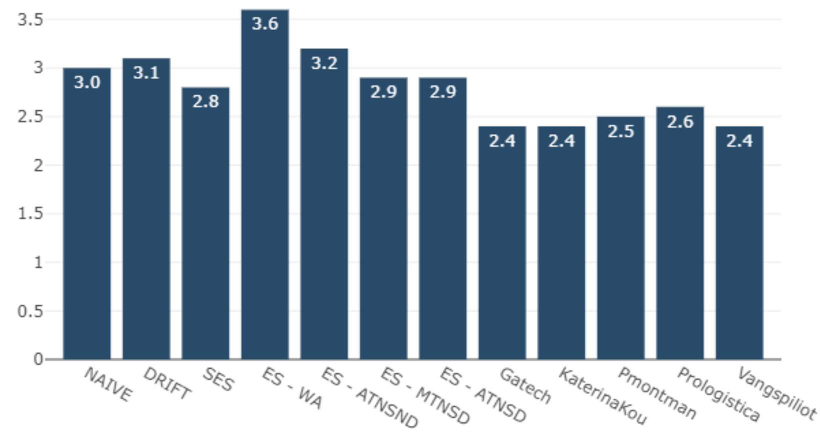

FIGURE 4.48: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with ten day horizon forecast

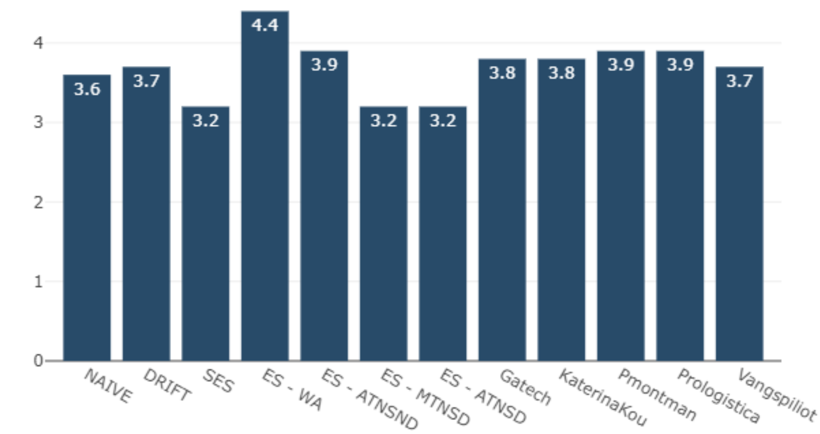

FIGURE 4.49: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with ten day horizon forecast

The values of MASE on the M4 models are consistently better than in all other models, although there is not much a difference in the overall standings, realistically the models seem to have really similar behaviours. Still in this case both KaterinaKou and Vangspiliot have a small advantage from the rest.

Model	sMAPE	MASE
NAIVE	3.6	3
DRIFT	3.7	3.1
SES	3.2	2.8
ES - WA	4.4	3.6
ES-MTNSD	3.2	2.9
ES-ATNSD	3.2	2.9
ES-ATNSND	3.9	3.2
Gatech	3.8	2.4
KaterinaKou	**3.8**	**2.4**
Pmontman	3.9	2.5
Prologistica	3.9	2.6
Vangspiliot	**3.7**	**2.4**

TABLE 4.23: Seasonal results of sMAPE and MASE, with horizon ten day, for KPI Maximum Number of Connected UE's

Fourteen Days Horizon

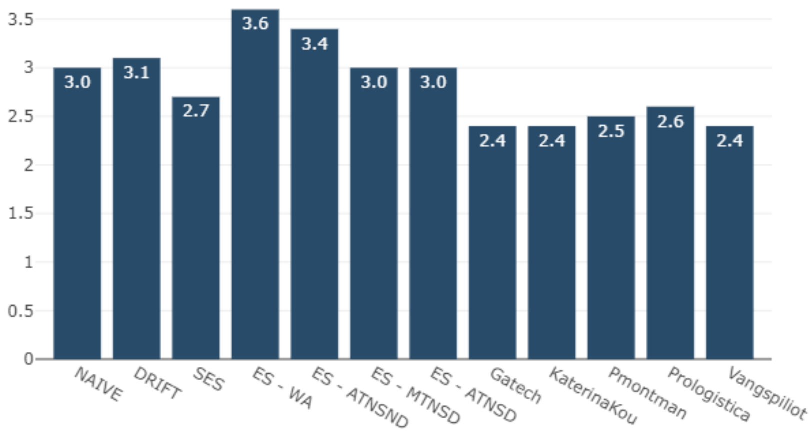

Figure 4.50: Accuracy measured using MASE for the Allocation of the transport channel for user data downlink KPI, with fourteen day horizon forecast

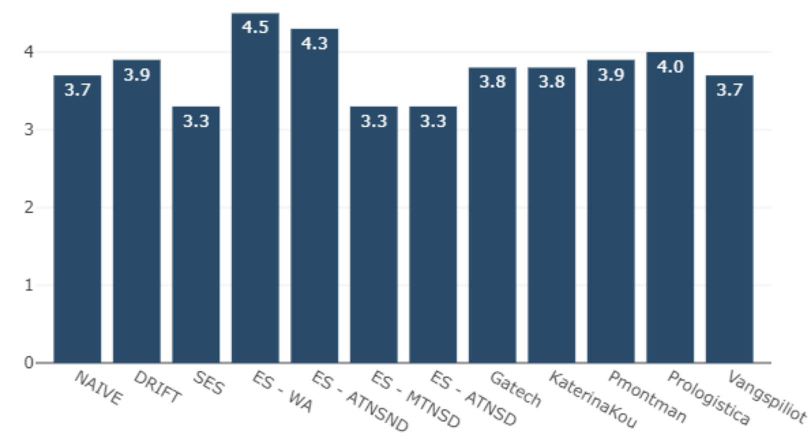

Figure 4.51: Accuracy measured using sMAPE for the Allocation of the transport channel for user data downlink KPI, with fourteen day horizon forecast

Table 4.24 the results of both sMAPE and MASE for a fourteen day horizon are available.

Like in the previous cases for this KPI there is not a great difference in the values. In such cases it is difficult to say which is the best performing model. Once again the MASE on the M4 models is lower than the calculated for the rest of the models. The best performing M4 model is Vangspiliot, but not by a margin that would declare it as the best.

Model	sMAPE	MASE
NAIVE	3.7	3
DRIFT	3.9	3.1
SES	3.3	2.7
ES - WA	4.5	3.6
ES-MTNSD	3.3	3
ES-ATNSD	3.3	3
ES-ATNSND	4.3	3.4
Gatech	3.8	2.4
KaterinaKou	3.8	2.4
Pmontman	3.9	2.5
Prologistica	4	2.6
Vangspiliot	**3.7**	**2.4**

TABLE 4.24: Seasonal results of sMAPE and MASE, with horizon fourteen day, for the KPI - allocation of the transport channel for user data downlink

4.3.3 Results and Analysis Summary

Table 4.25 summarizes the previous presented results in terms of best choice according with the analyzed Key Performance Indicators (KPI) and the intended horizon.

As it is noticeable the horizons within the same KPI affect model performances. For seasonal KPIs (the top three on the table), the models ES-NTASND and ES-NTMSND outperformed other models in most cases, and something similar happened in the case of non seasonal KPIs (the bottom three on the table), the models SES, KaterinaKou and Vangspiliot cover most of these cases.

	Horizon (days)			
KPI	1	5	10	14
Average of UE's connected	ES-NTASND ES-NTMSND Theta	ES-NTASND ES-NTMSND	ES-NTASND ES-NTMSND	ES-NTASND
Maximum number of active UE's per cell	ES-MTMSD	ES-ATASD ES-MTMSD	ES-ATASD ES-MTMSD	KaterinaKou
Maximum number of connected UE's per cell	ES-MTMSD	Theta	Theta	Theta
Requests to network access made by UE's during RACH	ES-MTMSD ES-ATNSD	SES	SES	SES
Allocation of the transport channel for downlink	SES ES-MTNSD	SES	KaterinaKou Vangspiliot	Vangspiliot
Allocation of the transport channel for uplink	SES ES-MTNSD	SES	KaterinaKou Vangspiliot	Vangspiliot

TABLE 4.25: Summary of model choice by KPI and horizon

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Chapter 5

Conclusion

The continuously ever growing of mobile networks generated new problems, specially considering network management decisions, where forecasts became not only a useful tool, but rather a mandatory one. The purpose of the present work has been to discover a model or multiple models that can better perform the task of forecasting to a given data set.

Even though forecasts and their precision rely on many different factors that include the nature of the data, the prediction horizon and of course the used model, the results obtained in the present study are nonetheless somehow surprising, specially when comparing them to the M4 competition.

In the present study, the best performing models from M4 (KaterinaKou and Vangspiliot) did not have the best standings in that competition, of course that are factors for that fact - the most plausible is the size of the used data set, but not limited to it - however the results were unexpected.

The use of several different horizons was helpful to determining the best models for a KPI, despite in general the difference between the majority of the used models was not really substantial to determine the best model for a given KPI and horizon.

On the seasonal data, although the analysis is made KPI by KPI, both Theta, ES-NTASND and ES-MTMSD did perform well in comparison with the competition. The ES-NTASND is better suited on the average User Equipment's connected, Theta achieved better results the Maximum Number of Connected User Equipment's and ES-MTMSD has better performance on Maximum number of active User Equipment's per cell Key Performance Indicators.

Regarding non seasonal data the results revealed SES as the overall best model for the request to network access by UE's and the allocation of the transport channel for downlink KPIs. Whilst for the allocation of the transport channel for uplink KPI on shorter horizons SES continued to have the best results, for longer horizons the models KaterinaKou and Vangspiliot did perform better.

However it is worth to mention that the KPI that addresses the request to network access by UE's, has the worst results of all analysed data sets - this is explained by the volatile behaviour recorded for such Key Performance Indicators - which also increases the difficult of its analysis.

Overall the results do not contain clear advantages for either model, the results are similar across the whole data set - some models already identified did have better performances, yet not with clear distinctions.

5.1 Future Work

The most important future activity is to validate the veracity of the obtained results. That can be achieved by applying the Diebold Mariano Test (addressed in this study on 2.3.2).

The used data in this study is from a northern European country, consequently characterized by the habits, culture and way of living there, if the data was from other regions it would be characterized differently hence it would be interesting to test these results, with other sets from the same KPIs.

As future work it is also important to use a bigger data set, not by adding more cells, or KPIs, but by increasing the historic data, instead of using a whole year, it would definitely improve the results if the historical data was composed of several years.

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