ISCTE Business School
INSTITUTO UNIVERSITÁRIO DE LISBOA

# RISK AND RETURNS OF FINANCIAL STOCK MARKET INDICES: AN EMPIRICAL APPLICATION

Mariana de Sousa Magalhães Asseiceiro

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Supervisor: Professor Doutor José Dias Curto, Associate Professor,

ISCTE-IUL Business School, Quantitative Methods for Management and Economics Department

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## Abstract

In this dissertation it is presented an empirical study that focus the period from 3 January 2007 to 1 October 2018, about the interactions between stock markets of Europe, United States of America (USA) and Asia, by implementing a generalized vector autoregressive (VAR) model and a dynamic conditional correlation (DCC) model.

For this purpose, three different stock market indices (Euro Stoxx 50 - Europe, S&P 500 – USA, and Nikkei 225 – Asia) were chosen to be representative of each geography they concern, in order to inquire if the indices are related between each other or not.

In general, the empirical results allow to conclude that returns of S&P 500 and Euro Stoxx 50 returns depend on their own past returns. Additionally, Euro Stoxx 50 returns are influenced by past returns of S&P 500 and there is no evidence of causality relationship from Nikkei 225 returns to any of the other indices returns.

Moreover, the conditional analysis of the pairwise correlations reveals that these are positive. The results presented by the DCC model indicate that it provides an accurate description of the dynamics of the correlations between the time series analysed for the purpose of this dissertation.



**JEL classification:**

G10; C32; C51

**Keywords:** Correlations, Stock market returns co-movements, Volatility, VAR, DCC, Multivariate GARCH models

**Resumo**

No presente trabalho, é apresentado um estudo empírico com base no período entre 3 de Janeiro de 2007 e 1 de Outubro de 2018, acerca das interações entre os mercados de capitais da Europa, Estados Unidos da América (EUA) e Ásia, através da estimação do modelo VAR e do modelo DCC.

Para este propósito, foram escolhidos três índices de ações representativos da geografia a que dizem respeito (Euro Stoxx 50 – Europa, S&P 500 – EUA e Nikkei 225 – Ásia) de modo averiguar se os índices estão relacionados entre si ou não.

Em termos gerais, os resultados obtidos permitiram concluir que as taxas de rendibilidade dos índices S&P 500 e Euro Stoxx 50 dependem das suas rendibilidades passadas, as rendibilidades do Euro Stoxx 50 são influenciados pelas rendibilidades passadas do S&P 500 e não há evidências de causalidade nem do S&P 500 nem do Nikkei 225 para as rendibilidades dos restantes índices. Adicionalmente, a análise das correlações condicionais a pares revela que estas são positivas. Os resultados produzidos pelo modelo DCC revelam que este é um modelo apropriado para descrever as dinâmicas correlacionais entre as várias séries temporais em questão.

## Acknowledgements

There is a quote from George S. Patton that perfectly describes how I felt during this adventure of developing this dissertation: "*Accept the challenges so that you can feel the exhilaration of victory*". It feels good to come to an end.

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## Glossary and Acronyms

ACF - Autocorrelation Function

ADF - Augmented Dickey-Fuller

AR - Autoregressive

ARCH – Autoregressive Conditional Heteroskedasticity Model

CCC – Constant Conditional Correlation

CSV - Comma-separated values

DCC – Dynamic Conditional Correlation

DJIA - Dow Jones Industrial Average

GARCH – Generalized Autoregressive Conditional Heteroskedasticity Model

KPSS - Kwiatkowski–Phillips–Schmidt–Shin

LM - Lagrange Multiplier

MGARCH – Multivariate GARCH

OLS - Ordinary Least Squares

PACF – Partial Autocorrelation Function

SC - Schwarz Criterion

USA – United States of America

VAR – Vector Autoregressive

VAR-DCC – Vector Autoregressive Dynamic Conditional Correlations

**CONTENTS**

## List of Tables

## List of Figures

## 1. Introduction

Financial markets provide many opportunities, but it can also cause tremendous losses to investors. In order to be prepared to entry into this world, it is necessary to balance the reward and the risk. Therefore, it is very important to have knowledge about volatility since it is the main source of risk. To make more adequate investment choices, forecasting arises as a tool to create a better financial strategy

Although traditional research in financial economics has been concentrated on the conditional mean of stock market returns, the most recent developments in international stock markets have increased the interest and concern for investors, regulators and researchers towards the volatility of such returns. The number of crashes and the size of their effects have forced them all to look more carefully to the level and behaviour of volatility throughout time (Matei, 2009). Researchers are changing their attention towards development and the improvement of econometric models to produce accurate forecasts of returns' volatility. Moreover, it is known that forecast is highly sensitive to the choice of the volatility model. It is also known that volatility tends to cluster in periods: small changes tend to be followed by small changes, and vice versa. This event, when the standard deviation varies over time, is called conditional heteroscedasticity. Heteroscedasticity means fluctuating variance (Orskaug, 2009). In addition, volatility over time has shown to be autocorrelated, which means that today's volatility depends on the past volatility. The conditional heteroskedastic models developed for such purpose present special importance due to the extended concern in both the academic and applied literature for volatility modelling. Further investigations were conducted in this field and many models were constructed over time with the goal to create a model which estimates a more accurate and realistic value of the volatility in a conditional way. This is the challenge that motivates researchers to develop or modify the present available models.

Over the last few decades, a strong increase in globalization resulted in a greater integration in the dynamics of several asset prices. The worldwide financial crisis in 2008 demonstrated how correlations between different global equity markets changed over time. Considering financial innovations and enhanced connectivity between international stock exchanges, predicting and studying the prevailing co-movement in markets seems to assume an extreme importance when modelling equity prices and returns. While the first moment of equity prices (the mean) exhibits similar movements across markets, the co-movement in the volatilities also presents a large degree of similarity.

Volatility of an underlying asset return is represented by its standard deviation. Studying the volatility can have many applications in the financial domain, among which, for example, there is the analysis and calculation of the value at risk (VaR) of a financial position.

For this matter, traditional time series tools such as autoregressive moving average (ARMA) models (Box et al., 1970.) for the conditional mean were extended to essentially analogous models but for the conditional variance. According to the methods used to estimate the conditional volatility, the GARCH family models arise, being the most composed models.

There are multivariate models developed for ARMA and GARCH models, with the purpose of explaining the variations and changes over the time on the mean and volatility, respectively, for a set of multiple time series.

A method used for the multivariate analysis is the vector autoregression (VAR), a natural extension of the univariate autoregressive (AR) model to dynamic multivariate time series. This is one of the most effective and flexible models for the analysis of the mean behaviour of multivariate time series, and it is especially useful not only for explaining the dynamic behaviour of economic and financial time series but also to forecasting.

Another multivariate approach to study time series can be the extension from a univariate GARCH to a multivariate GARCH model. This models are used to understand the behaviour and possible patterns of volatility of the time series over the time. With this extension from univariate to multivariate analysis, there is a door opened to improve decision tools. The main challenge in constructing multivariate GARCH models is to make them parsimonious[1] enough, but still maintain the flexibility.

Among the GARCH multivariate models, one approach is to decompose the conditional covariance matrix into conditional standard deviations and a conditional correlation matrix – Constant Conditional Correlation (CCC) model (Bollerslev, 1990). In this model, the conditional correlation is assumed to be constant over time, and only the

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[1] Parsimonious: simplest model with the least assumptions and variables but with greatest explanatory power.

conditional standard deviation is time-varying. Assuming that conditional correlation is constant over time is not always reasonable, therefore an extension of the CCC-GARCH model, for which the conditional correlation matrix is designed to vary over the time arises with the name of Dynamic Conditional Correlation (DCC) model (Engle and Sheppard, 2001).

The key contribution of this study is to use the VAR-DCC, an econometric model approach, applied to financial markets, in order to model the dependency between three economical indices, geographically far. No study has used this model type to model these specific time series data as a system. It is a contribution to the literature, not only for using a VAR model but also to use univariate and multivariate GARCH models in the estimation over the same data set. This approach will allow us to observe volatility behaviour of each market – the individual volatility magnitude – but also between all the stock markets in study. Nowadays markets are connected and affect each other when certain events occur. This study allows us to understand and measure that dependency and the existent relationships among the three stock markets subject to analysis in this dissertation.

Indices are used by people to measure risk, since they work as benchmarks of, for example, industrial sectors and business performance in stock markets. Moreover, indices provide historical data which help investors to have a broader perspective of the tendencies in the markets and, from that, to take conclusions. As a result, it is possible to compare indices based on their historical and present prices or also on their sector and purpose. The main advantage by using stock indices is the simplified approach of tracking performances of stock markets without the need to measure each stock individually. By analysing the indices, we achieve dimensionality reduction of the series to be analysed.

The indices chosen to be analysed were S&P 500 (American index), EuroStoxx50 (European index) and Nikkei (Asian index). The choice of these indices was based on their influence in regional and global financial markets, so each of the index is representative of the respective region they belong. The S&P 500 Index is a market-capitalization-weighted index of the five hundred largest U.S. publicly traded nationally recognized companies by their value in the market. The index is widely regarded as the best reference of large-cap U.S. equities. The S&P 500 focuses on the U.S. market's large-cap sector. The Euro STOXX 50 Index is a market capitalization weighted stock index of the fifty largest European companies operating within Eurozone countries and its components are selected from the Euro STOXX Index. The Nikkei 225 is the leading index of Japanese stocks. It is a price-weighted index composed of Japan's top two hundred twenty-five nationally recognized companies traded on the Tokyo Stock Exchange. So and Tse (2009) concluded that Asian markets are becoming progressively more integrated and that evidence of their co-movements during periods of financial distress is growing in strength. Hyde, Nguyen and Bredin (2007) found evidence of significant increases in correlation during the Asian crisis is largely limited to crisis countries and that correlations with the US and Europe did not systematically increased throughout this period.

The data used in this study consists on the three mentioned indices daily Adjusted Closing Prices, in points. After collecting this data, it was transformed according to models' requirements, namely the continuously compound returns. The unit of measure for all the indices is points, in order to value the securities listed on each of them. Data series cover the period from 3 January 2007 to 1 October 2018 and were collected at a Bloomberg terminal, in Microsoft Office Excel CSV format. To process the data, it will be used RStudio, a software programme for statistical analysis that reads the CSV file input.

The stock markets in Europe, Asia and the U.S. were closed on different days as a result of holidays. To address this, adjustments were necessary, and as Wang and Firth (2004), we omitted the observations with missing values. Following this modification in the time series, there are a total of 2940 daily observations.

In this study, we modelled the three indices time series into a multivariate VAR-DCC model. This study is organised as follows: Section 2 addresses the Literature Review for the relevant models. Section 3 presents the methodology and introduces the models used in this study. Section 4 reports the empirical study and respective estimation results. Section 5 presents the final discussion of the results of the study, its limitations and eventual suggestions for future research on this topic.

*«The belief that market movements are loyal to each other turns into a self-fulfilling prophecy»*

Mehmet Dalkir

## 2. Literature review

In the last years ARMA and GARCH models have been used to model conditional mean and conditional variance. An autoregressive (AR) model, in its simplest form, is a model in which one uses the statistical properties of the past behaviour of a given variable to predict its behaviour in the future. Some of the most important volatility models are the autoregressive conditional heteroskedastic (ARCH) model proposed by Engle (1982), the generalized ARCH (GARCH) model developed by Bollerslev (1986*)*, the exponential GARCH (EGARCH), among many others.

### 2.1 VAR Models

Vector autoregressive models are included in a class that studies and describes the dependency of returns between different time series.

Multivariate simultaneous equations models were widely used for applied econometrics analysis. By that time, macroeconomic time series observations were longer and more frequent. Hence, the need of a model that could describe the dynamic structure of the variables was clear, and this was how this model emerged. Sims (1980) developed vector autoregressive (VAR) models. According to his paper, VAR models provide consistent and realistic approach to data description, forecasting, structural inference, and policy analysis.

The VAR framework provides a systematic way to capture powerful dynamics of more than one time series, i.e. a multivariate analysis with a simple and direct interpretation of the statistical output that came with this model estimations

Eun and Shim (1989), by estimating a VAR model, examined the international transmission mechanism of stock market movements. This study included nine markets, from December 1979 to December 1985. These authors found that U.S.A. fast growth contaminates other markets, but any foreign market can significantly explain U.S.A. market movements.

Liu et al. (1998), by applying a VAR, explored the dynamical structure of six different equity markets (U.S.A., Japan, Hong Kong, Singapore, Taiwan, and Thailand). These authors performed tests using as data the daily stock returns from January 1985 to

December 1990. Their results state that the U.S.A. market plays a dominant role in influencing the other markets as well as that both Japan and Singapore, together, have a substantial and persistent impact on the other Asian markets.

In general markets, developed and emerging, can move together over the short run. Janakiramanan and Lamba (1998) and Cha and Cheung (1998) investigate linkages between Asia Pacific and the USA equity markets using VAR models. These authors determined that the USA has a significant influence on these markets.

Arouri et al. (2011) used a VAR-GARCH approach to investigate the return linkage and volatility transmission among oil and stock markets in Gulf Cooperation Council countries.

Mensi et al. (2013) used a VAR-GARCH model to explore the return volatility transmission between the S&P 500 and commodity prices indices for energy, food, gold and beverages, from 2000 to 2011. The results for return and volatility spillovers exhibited significant transmission, with the S&P 500 strongly influencing the oil and gold markets. Moreover, this study observed that the highest conditional correlations were between the S&P 500 and commodity markets.

Selmi and Hachicha (2014) examined, for the period of 2004 until 2012, the role of oil prices, credit, financial and commercial linkages in the transmission of the industrial market crisis that occurred during the period of analysis. They used a VAR-DCC model and found that credit linkage had a significant role in the subprime, financial and global crises.

Bunnag (2015) examined co-movements and spillovers in oil futures using three types of multivariate VAR models - VAR (1)-diagonal VECH, the VAR (1)-diagonal BEKK and the VAR (1)-CCC models. Their results revealed that the estimates of the MGARCH parameters were statistically significant in almost all cases. In another study, Bunnag (2015) examined the oil futures and the carbon emissions futures volatility co-movements and spillovers for crude oil, gasoline and heat oil and also for carbon emissions. The data used was daily information from 2009 to 2014 and three MGARCH models - VAR (3)-diagonal VECH, the VAR (3)-diagonal BEKK and the VAR (3)-CCC. The results pointed for oil futures volatility to have an impactful effect on carbon emissions futures volatility.

## 2.2 ARCH Model

The Autoregressive conditional heteroskedasticity (ARCH) model is the first conditional heteroskedasticity model for time series data (Engle, 1982).

Bera and Higgins (1993, p.315) stated that “a major contribution of the ARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables.”

In ARCH literature, some interpretations of the process can be found. Lamoureux and Lastrapes (1990) point out that the conditional heteroskedasticity may be originated by a time dependence in the rate of information arrival to the market. They use the daily trading volume of stock markets as a representation for the information arrival and confirm its significance. Mizrach (1990) associates ARCH models with the errors of the economic agents’ learning processes.

Alternative models were further researched. One of them was developed by Bollerslev (1986), who proposed a useful extension of this model known as the generalized ARCH (GARCH).

## 2.3 GARCH Models

Since in empirical applications ARCH ($q$) models had many lags and numbers of parameters to estimate, Bollerslev (1986) proposed a more parsimonious specification when compared to ARCH model, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. A low order GARCH model can have the same properties of a higher order ARCH model without the problems related with the estimation of many parameters, subject to the non-negativity restrictions (Engle e Bollerslev, 1986) - which further explains the wide preference for its use in practice.

Financial volatilities move together more or less closely over time across assets and markets. Hence, it is essential to consider the dependence in the correlation of asset returns. The success of the autoregressive conditional heteroscedasticity (ARCH) model and the univariate GARCH model in capturing the time-varying variances of economic data in the univariate case have motivated many researchers to extend these models to the multivariate dimension. One method to estimate the covariance matrix between the assets is to extend the univariate GARCH into a multivariate GARCH (MGARCH) model. Some of these models not only give us the possibility to correlate the volatilities and the co-volatilities of the different series, but also allow us to analyse a large quantity of data of different series at the same time. The specifications for these kinds of models have been developing. Some of these specifications are present in a range of papers surveyed by Bollerslev, Engle, and Nelson (1994) and more recently by Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2008). Multivariate GARCH models specify equations for how the variances and covariances move over time (Orskaug, 2009). A disadvantage of the multivariate approach is that the number of parameters to estimate in the GARCH equation increases rapidly, which limits the number of assets that can be included. Thus, the main challenge of these kind of models is to make them parsimonious enough, but still maintain the flexibility.

Since correlations between asset returns and markets are important in many financial applications, multivariate volatility models were also extended to describe the time–varying feature of the correlations.

Further investigations of financial crises and contagion provided additional evidence that there is significant transmission across markets (Kaminsky et al, 1999; Bae et al., 2003). Worthington and Higgs (2004) presented evidence of the transmission of return and volatility between nine developed and emerging Asia-Pacific markets finding significant spillovers across markets using multivariate GARCH models.

Elder (2003) developed an analytical expression for an impulse-response function for a VAR with multivariate GARCH errors. He also presented the appropriate interpretation of an impulse-response function for such models and propose interesting empirical issues that can be addressed within the framework he developed.

Schröder and Schüler (2003) made an attempt to measure the Europe-wide systemic risk in banking. The existence of systemic risk justify why banks are regulated and supervised. As a measure of this kind of risk, they applied the conditional correlations between pairs of national bank stock indices of the European countries. The correlations measured the linear relationships amongst the residuals of the ARMA models and were estimated with GARCH models, which considered the influence of the national stock market index and a short-term interest rate as explanatory factors. Once these residuals mainly reflect bank specific factors, the authors concluded they were suitable to measure the systemic risk.

Valiani (2004) implemented a Multivariate GARCH (MGARCH) specification to estimate the time-varying correlations of underlying assets and related currency forwards so that it was possible to hedge the currency exposure risk in an international portfolio context. His empirical investigation indicates that the optimal multivariate GARCH dynamic hedging strategy can catch the currency fluctuations in its best possible way and over-performs the risk controlling process.

One approach to make the model more flexible and accurate is to decompose the conditional covariance matrix into conditional standard deviations and a conditional correlation matrix. The first model of this type was the CCC-GARCH model introduced by Bollerslev (1990). In this model, the conditional correlation is assumed constant over time, and only the conditional standard deviation is time varying, so the correlation matrix is time invariant. The assumption that the conditional correlation is constant over time is not always reasonable. One step forward was done by Engle and Sheppard (2001) with the introduction of the DCC-GARCH model, which is an extension of the CCC-GARCH model, for which the conditional correlation matrix is designed to vary over the time. Several specifications of correlation matrix have been suggested, but the most relevant one was the DCC-GARCH.

The idea behind models in this class is that the covariance matrix can be decomposed into conditional standard deviations and also in a correlation matrix, with both conditional standard deviations and correlation matrix are designed to be time-varying. The main advantage of this model is to be able to estimate large time-varying covariance matrices (Engle and Sheppard, 2001).

Engle (2002) claims that DCC model, when performed, offers more realistic empirical results. Tas (2008), concluded that the conditional correlation models are more viable in terms of estimation and interpretation of parameters. The DCC-GARCH model is more realistic than the CCC-GARCH model.

Engle (2002) developed an empirical study a bivariate DCC-MGARCH (between two series) to estimate conditional correlations between DJIA and NASDAQ indices, stock and bonds and exchange rates by using US daily time series data of ten years. His results lead to conclude that the DCC model is more accurate than other multivariate GARCH models, regardless the criteria chosen. He suggested in his paper that this version of DCC model provides a great approximation to a variety of time varying correlation processes. The comparison of this model with simple multivariate GARCH demonstrated that the DCC is the most precise model. Empirical examples from conventional financial applications are very encouraging as they reveal important time varying features that might otherwise be hard to quantify.

Another significant topic in financial econometrics is the asymmetric behaviour of the conditional variances. The rationale behind it, is that negative shocks, when compared to positive shocks of similar magnitude, have a different effect on the conditional variance evolution.

Ang and Chen (2002) examined the possibility of asymmetries in correlations among stock market in general and also on portfolios of domestic stocks in the USA. By using weekly data, from July 1963 and December 1998, they concluded that the correlations between the returns of portfolios and the market change, in general, from bear to bull market periods.

Cappiello et al. (2006), by using an asymmetric generalized-DCC (AG-DCC) specification, studied the evolution of asymmetries in conditional variances and correlations among three groups of countries (Europe, Australia and North America). Their results led to the conclusion that equity returns reveal strong evidence of asymmetries in conditional volatility. Also, they found evidence that the asymmetric conditional correlations increase more sharply in reaction to bad news in equity markets. Similarly, Aielli (2008) and Palandri (2009) extended the DCC model of Engle (2002) to an asymmetric DCC model, a generalization of the DCC model. These authors used several asymmetric variants of the DCC model.

Chiang et al. (2007) applied a DCC model to nine Asian stocks and confirm a contagion effect during the Asian crisis.

Ho et al. (2009) used several multivariate GARCH models in order to examine the evidence of asymmetry and time-varying conditional correlations between five sectors of Industrial Production of the USA.

Büttner and Hayo (2011) applied a bivariate DCC model so that they could extract dynamic conditional correlations between European stock markets.

Lahrech and Sylwester (2011) employed DCC multivariate GARCH models to examine the dynamic linkage between U.S. and Latin American stock markets. Their findings suggest an increase in the degree of co-movement between Latin American equity markets and U.S. equity ones.

Efimova and Serletis (2014) present on their paper an empirical application of various univariate and multivariate GARCH models – including DCC model – applied to energy markets in the USA. Their findings include the capacity, through DCC model, to observe relevant the interactions between three commodity and respective volatilities and the spillover effects direction. Since the dataset is large and the structure from multivariate models is strong, there is the possibility to study the magnitude of spillover effects and forecast performances.

## 3. Methodology

The key model of this study is a VAR-DCC model, whose theoretical aspects were presented before in more detail.

The data were collected from a relevant source and, after that, several tests were carried out on RStudio. The data was modified in order to fit the models used in the empirical part and we had to understand and conclude about the statistical significance of the tests in order to take conclusions to achieve the purpose of this study.

Before employing the VAR-DCC model, and after the needed transformations in data, there were several steps to perform.

As a first step, after obtaining the prices of each index, we applied the natural logarithm to compute the continuously compounded daily returns:

$$R_{i,t} = ln\left(P_{i,t}\right) - ln\left(P_{i,t-1}\right) = ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{1}$$

Where $P_{i,t}$ is the price of the stock market $i$ at moment $t$ and $t = 1, 2, 3, \dots, T$.
The returns will allow us to estimate the models relevant for this study, since having stationarity data is imperative to proceed.

From the moment that the data is properly transformed, time series can be used in the models referred before and we could initiate the estimation process.

The general characteristics of each model used in this empirical study are introduced in the following sub-sections. Furthermore, the features of the tests employed are presented in the Section 4.

### 3.1 From ARMA to ARCH model

The autoregressive moving-average (ARMA) models include the concepts of “AR” and “MA” models intending to keep the number of parameters small. The ARMA model was initially proposed by Box, Jenkins and Reinsel (1995). An ARMA model is a model in which one uses the statistical properties of the past behaviour of a variable $y_t$ to predict its pattern in the future. Thus, we can predict the value of the variable $y_{t+1}$ by considering the sum of the weighted values that $y_t$ took in the previous period and then add the error term $\varepsilon_t$.

This model is usually presented in the form of an ARMA (*p, q*) process, in which "*p*" is the order of the autoregressive part and "*q*" is the order of the moving average part.

The generalized ARMA (*p,q*) model can be defined as:

$$r_t = c + \varepsilon_t + \sum_{i=1}^{p} \emptyset_i \, r_{t-i} + \sum_{j=1}^{q} \varphi_i \varepsilon_{t-j} \tag{2}$$

Where $r_t$ is a given time series data, c is a constant, $\emptyset$ and $\varphi$ are the parameters of the model and random variables $\varepsilon_t$ and $\varepsilon_{t-i}$ are white noise error terms. Error terms are generally assumed to be independent identically distributed random variables sampled from a normal distribution with zero mean. In other words, it is assumed that each random variable has the same probability distribution as the other random variables, and all are mutually independent. However, this assumption may be considered unrealistic.

Engle (1982) developed the ARCH model. This model captures the tendency of financial variables to move between high and low volatility. Previous research either assumed the volatility to be constant or used simple methods to achieve approximations. As stated before, there was the need for a more realistic and consistent model to measure risk.

In this case, the error terms are split into a stochastic part and a time-dependent standard deviation. It has the following form:

$$\varepsilon_t = \sigma_t z_t \tag{3}$$

Where the random variable $z_t$ is a strong white noise process and $\sigma_t^2$ – conditional variance - is modelled by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \tag{4}$$

Where $\alpha_0$ , $\alpha_i$.> 0.

ARCH models concern to explain clustered errors, as well as nonlinearities. One characteristic of ARCH models is the power of forecast changes from one period to another.

### 3.2 From ARCH to GARCH model

Even though the ARCH model has a simple form, one of its weaknesses is that it requires many parameters to describe appropriately the volatility process. Therefore, alternative models were further developed, namely by Bollerslev (1986), who proposed an extension known as the generalized ARCH (GARCH) model. Comparatively with ARCH model, the Generalized Autoregressive Centralized Heteroskedastic model is more parsimonious, since GARCH estimation with less parameters can be as effective as a ARCH model with more parameters. For instance, a GARCH (1,1) can explain the same as an ARCH (16) model. In GARCH models, the number of parameters results from the values of p and q. As stated by Matei (2009), whilst ARCH includes the feature of autocorrelation observed in return volatility of most financial assets, GARCH improves ARCH by adding a more general feature of conditional heteroskedasticity. GARCH model comparably to ARCH model has been shown to perform better in explaining and predicting conditional volatilities than the ARCH model.

The conditional variance determined through GARCH is a weighted average of past errors, converging here with the same logical of ARCH model.

The GARCH ($p$, $q$) model is defined as:

$$r_t = \mu_t + u_t \tag{5}$$

$$u_t = \sigma_t z_t \tag{6}$$

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_i \, \sigma_{t-j}^2 \tag{7}$$

Where $r_t$ represents the log return of an asset at time $t$, $u_t$ is the conditional mean-corrected return of an asset at time $t$, $\mu_t$ is the expected value of the conditional $r_t$, $\sigma_t^2$ represents the conditional variance at time $t$ conditioned on the history. Finally, $\alpha_i$ and $\beta_i$ the parameters of the model and $z_t$ the sequence of independent and identically distributed standardized random variables.

The conditional variance fluctuates over time, dependent on the squared errors and past conditional variance. In case of the volatility is serially dependent, the time series will have periods of high volatility followed by periods of low volatility. This periodical dependence of volatility is often referred to as volatility clustering. The success of the autoregressive conditional heteroscedasticity (ARCH) model and the univariate GARCH model in capturing the time-varying variances of economic data in the univariate case have motivated many researchers to extend these models to the multivariate dimension.

“While univariate descriptions are useful and important, problems of risk assessment, asset allocation, hedging in futures markets and options pricing require a multivariate framework, since high volatilities are often observed in the same time periods across different assets” (Li and Fan, 2005, p.87). Another important weakness of GARCH model is that it does not distinguish between positive and negative movements in the market.

Some extensions of ARMA and GARCH models were developed in order to model the dependency between more than one time series of returns.

### 3.3 Multivariate GARCH models

Correlations between asset returns and markets are important in many financial applications. One of the most remarkable models for volatility is the class of multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models. These models allow us to specify a dynamic process for the entire time varying variance–covariance matrix of the time series thus jointly modelling the first and second moments (Orskaug, 2009).

This extension of a univariate GARCH model to an N-variate model requires allowing the conditional variance-covariance matrix of the N-dimensional zero mean random variables, $\varepsilon t$, depend on the elements of the information set.

The most challenging stage in MGARCH modelling may be to provide a realistic but parsimonious specification of the variance matrix ensuring its positivity. A drawback of this approach is that the number of parameters to be estimated in the GARCH equation increases rapidly, which limits the number of assets that can be addressed to the model.

The multivariate GARCH models is defined as:

$$r_t = \mu_t + \alpha_t \tag{8}$$

$$\alpha_t = H_t z_t \tag{9}$$

Where $r_t$ is a $n\ x\ 1$ vector of the logarithmic returns of $n$ assets at time $t$, $\alpha_t$ the $n\ x\ 1$ vector of mean-corrected returns of *n* assets at time *t*, $\mu_t$ the $n\ x\ 1$ vector of the expected value of the conditional $r_t, z_t$ the $n\ x\ 1$ vector of identically distributed errors and $H_t$ the $n\ x\ n$ matrix of conditional variances of $\alpha_t$ at time *t*.

As in the univariate case, $a_t$ is uncorrelated in time. However, this does not mean that there is no serial dependence, but that the dependence is non-linear. What remains to be specified is the conditional covariance matrix, $H_t$.

$H_t$ can have different specifications. This matrix increases very rapidly, as the dimension of $a_t$ increases. Since it is dependent of the time *t*, it must be inverted in each iteration, which makes the computation challenging - unless *n* is small. This fact causes difficulties in the estimation of the models. Thus, an important goal whilst building MGARCH models is to make them parsimonious enough, but still maintain the flexibility (Orskaug, 2009).

In order to turn the estimation more accurate, many multivariate GARCH have been studied and developed. These models distinguish each other's by the type of specifications, which can be divided into four classifications:

1. **Models of the conditional covariance matrix:** in this category the conditional covariance matrices, $H_t$, are modelled directly.

2. **Factor models:** in this class of models, the conditional covariance matrices are motivated by parsimony.

3. **Models of conditional variances and correlations:** models in this category are built on the idea of modelling the conditional variances and correlations instead of immediately modelling the conditional covariance matrix.

4. **Nonparametric and semiparametric approaches:** these models form an alternative to parametric estimation of the conditional covariance structure. The advantage of these models is that they do not impose a particular structure on the data.

Among the multivariate models developed, two of them are particularly important in the evolution of this type of model. They are the Constant Conditional Correlation (CCC-

GARCH) and Dynamic Conditional Correlation (DCC-GARCH) models which are explained in detail in the following section.

Bollerslev (1990) developed a relatively flexible model known as Constant Conditional Correlation (CCC) model. This approach allowed the combination of a univariate GARCH model, with the assumption of a constant correlation between time series over the time.

The conditional covariance matrix is decomposed into conditional standard deviations and a correlation matrix as:

$$H_t = D_t R_t D_t \tag{10}$$

where $D$ is the conditional standard deviation, and $R$ the correlation matrix.

Models of conditional variances and correlations can be classified in two groups: those with a constant correlation matrix and those when the correlation matrix is time-varying. In this case, the CCC model exhibit constant correlation matrix, therefore the correlation matrix is time invariant, then $R_t = R$. Hence it becomes:

$$H_t = D_t R D_t \tag{11}$$

Several specifications of $R_t$ have been suggested, but the most recent and significant one may be the DCC-GARCH. This model is a conventional parametric MGARCH model.

DCC model was introduced by Engle and Sheppard (2001). The idea of the DCC model is that the covariance matrix, $H_t$, can be decomposed into conditional standard deviations, $D_t$, and a correlation matrix, $R_t$ (Orskaug, 2009).

In the DCC-GARCH model both $D_t$ and $R_t$ are designed to be time-varying therefore the model is defined as:

$$H_t = D_t R_t D_t \tag{12}$$

This model formulates the volatilities of returns in one set of equations and the correlations between them in another set thus, treating them as independent stochastic processes, entailing more flexibility and different parameterizations.

The comparison of DCC model with some other simple multivariate GARCH, and several other estimators, revealed that DCC is often the most accurate (Peng and Deng, 2010).

This specification of time-varying correlations was widely studied by Engle and Sheppard (2001). The DCC model assumes that returns from *k* assets are conditionally multivariate normal with zero expected value and covariance matrix $H_t$.

$$r_t \mid F_{t-1} \sim N(0, H_t) \quad \text{where} \quad H_t \equiv D_t R_t D_t \tag{13}$$

R$_t$ is the time varying correlation matrix containing the conditional correlations, is made up from the time dependent correlations; it is called the time varying correlation matrix. $D_t$ is the diagonal matrix of time varying standard deviations implied by the estimation made with a univariate GARCH model.

The proposed elements of $D_t$ can be written as univariate GARCH model as:

$$h_{it} = w_i + \sum_{p=1}^{Pi} \alpha_{ip} r_{it-p}^2 + \sum_{q=1}^{Qi} \beta_{iq} r_{it-q}^2 \tag{14}$$

For $i = 1, 2, 3, \ldots, n$ with the usual GARCH restrictions for non-negativity and stationarity being imposed. The set $P$ and $Q$ for each series indicates the lag lengths chosen may not be the same as in GARCH (*p, q*) model.

The log likelihood function resulting from DCC can be expressed as:

$$r_t \mid F_{t-1} \sim N(0, D_t R_t D_t) \tag{15}$$

$$L = -\frac{1}{2} \sum\nolimits_t (n\, log(2\pi) + log|H_t| + r_t' H_t^{-1} r^t) \tag{16}$$

$$L = -\frac{1}{2} \sum\nolimits_t (n\, log(2\pi) + log|D_t R_t D_t| + r_t' D_t^{-1} R_t^{-1} D_t^{-1} r^t)$$

$$L = -\frac{1}{2} \sum\nolimits_t (n\, log(2\pi) + 2log|D_t| + log|R_t| + \varepsilon_t' R_t^{-1} \varepsilon_t)$$

Where $\varepsilon_t \sim N(0, R_t)$ are the errors standardized by their conditional standard deviations.

### 3.4 VAR Model

Essentially, the multivariate VAR model is an extension of the univariate autoregressive model. This model is useful for modelling the mean or the first order moment (mean) of the series. There are several multivariate time series models used for forecasting. Besides the ones referred above, this model is widely used. It was after the pioneering work of

Sims (1980), that VAR model have become one of the most popular to study correlated series. Some of the advantages of this model are its relative simplicity, flexibility, and ability to fit the data. It is also successful as a forecasting tool.

Additionally, besides data description and forecasting, the VAR model can also be used for structural inference and policy analysis. Hence, in order to have an improved understanding of the time series, modelling and forecasting volatility has been a major area of time series research for some years now. Traditional econometric models assume a constant one-period forecast variances, which in general is not a plausible assumption.

However, the model has some drawdowns namely data constraints (since it requires long time series), the fact of being computationally intensive.

According to Sims (1980), a simple Vector Autoregression (VAR) model involves a set of $K$ endogenous variables $y_t = (y_{1t}, y_{2t}, \dots, y_{kt})$ for $k = 1, \dots, K$.

Consider a VAR model of order $p$:

$$y_t = A_1 Y_{t-1} + _{\dots} + A_p y_{t-p} + Bx_t + \varepsilon_t \ , \tag{17}$$

Where $y_t$ is a $k$-vector of stationary variables; $x_t$ is a vector of exogenous variables; $A_1$, $A_2$,…, $A_p$ and $B$ are matrices of coefficients to be estimated and $\varepsilon_t$ is a vector of innovations that may be simultaneously correlated but uncorrelated with their own lagged values and also with all of the right-hand side variables. The coefficients of each VAR model equation are estimated by Ordinary Least Squares (OLS) method. (Curto, 2018), the most commonly used method to estimate the parameters in a linear regression model.

In general terms, a VAR model with $k$ time series consists of $k$ equations, one for each of the variables, where the regressors in all equations are lagged values of all the variables. (Curto, 2018). In our study, the VAR model contains three time series variables,$y_{1_t}$, $y_{2_t}$ and $y_{3_t}$, so it involves three equations. In one, the dependent variable is $y_{1_t}$; in the others the dependent variables are $y_{2_t}$ and $y_{3_t}$ respectively. Assuming that our VAR model contains one lagged value ($p = 1$), three endogenous variables ($k = 3$) and also letting a constant to be the only exogenous variable, the equations become:

$$y_{1_t} \ = \ c_1 + a_{11} y_{1_{t-1}} + a_{12} y_{2_{t-1}} + a_{13} y_{3_{t-1}} + \varepsilon_{1_t} \tag{18}$$

$$y_{2_t} \ = \ c_2 + a_{21} y_{1_{t-1}} + a_{22} y_{2_{t-1}} + a_{23} y_{3_{t-1}} + \varepsilon_{2_t} \tag{19}$$

$$y_{3_t} = c_3 + a_{31}y_{1_{t-1}} + a_{32}y_{2_{t-1}} + a_{33}y_{3_{t-1}} + \varepsilon_{3_t} \tag{20}$$

where $a_{ij}$ and $c_i$ are the parameters to be estimated.

The number of parameters to estimate in a VAR model increases with the number of variables ($k$) and the number of lags ($p$).

# 4. Empirical study

## 4.1 Relevance and purpose of the study

In this chapter we present an empirical study that involves three stock market indices with different geographic locations.

The main purpose of this study is to apply a VAR-DCC model in order to understand the dependency between three distinct financial markets, globally distributed. We chose this model in order to understand the transmissions of volatility among the three stock markets and its movement over the time. Furthermore, we would like to provide a complementary approach to the already existing empirical literature on the topic of transmissions in volatility through financial markets.

## 4.2 Stationarity of the Adjusted Closing Prices

Non-stationary data are unpredictable and cannot be forecasted. The results obtained when using non-stationary time series may be spurious and may indicate a relationship between variables where it does not even exist. In order to produce accurate results, the non-stationary data needs to be transformed into stationary data when the cointegration is not tested before. In the opposite side of non-stationary, the stationary process stands to a constant long-term mean and constant variance. Some of non-stationary processes are random walk with or without a drift (a slow steady change) and deterministic trends. A random walk - with or without a drift - can be transformed to a stationary process by differencing (taking the differences) and then the process becomes difference-stationary. In general, having non-stationary time series data in financial models can produce unreliable and spurious results and can lead to poor forecast. Hence, a way to overcome the problem is to transform the time series data so that it becomes stationary.

As stated before, the data collected are prices, that in general are non-stationary process. Therefore, we transformed the non-stationary data into stationary, by computing the logarithmic returns and then taking the first differences of the prices, in order to remove trends in the data. Figures 1, 2 and 3 (see Appendix) present the plot of the Adjusted Closing Prices, before data transformation.

A unit root process is a stochastic trend in a time series. If time series have a unit root, it indicates a systematic pattern that is unpredictable. The Augmented Dickey-Fuller (ADF) test is used to determine whether a unit root is presented in a model. This is one of the most widely used test to examine the stationarity of time series. In this test the null hypothesis points for non-stationary of time series, i.e. there is a unit root.

**Table 1: Augmented Dickey-Fuller Test Unit Root Test for Logarithmic Returns**

| | | | **Returns S&P 500** | |
|---|---|---|---|---|
| Test-Statistic value | | -43.0993 | 619.1839 | 928.7758 |
| Critical value for a significance level of: | | | | |
| | | 1pct | 5pct | 10pct |
| | tau3 | -3.96 | -3.41 | -3.12 |
| | phi2 | 6.09 | 4.68 | 4.03 |
| | phi3 | 8.27 | 6.25 | 5.34 |

| | | | **Returns Euro Stoxx 50** | |
|---|---|---|---|---|
| Test-Statistic value | | -41.0387 | 561.392 | 842.088 |
| Critical value for a significance level of: | | | | |
| | | 1pct | 5pct | 10pct |
| | tau3 | -3.96 | -3.41 | -3.12 |
| | phi2 | 6.09 | 4.68 | 4.03 |
| | phi3 | 8.27 | 6.25 | 5.34 |

| | | | **Returns Nikkei 225** | |
|---|---|---|---|---|
| Test-Statistic value | | -39.081 | 509.1109 | 763.6659 |
| Critical value for a significance level of: | | | | |
| | | 1pct | 5pct | 10pct |
| | tau3 | -3.96 | -3.41 | -3.12 |
| | phi2 | 6.09 | 4.68 | 4.03 |
| | phi3 | 8.27 | 6.25 | 5.34 |

After analysing the information of Table 1, we reject the null and conclude that the results point for all the time series returns to be stationary. Hence, we concluded that all the logarithmic returns of the time series were stationary, since the values of the Test-Statistic, for all the series, are greater than critical values, which led us to the rejection of the null hypothesis stated before.

Another test that can be used to test stationarity of time series is the KPSS test, developed by Kwiatkowski, Phillips, Schmidt and Shin (Kwiatkowski et al., 1992).

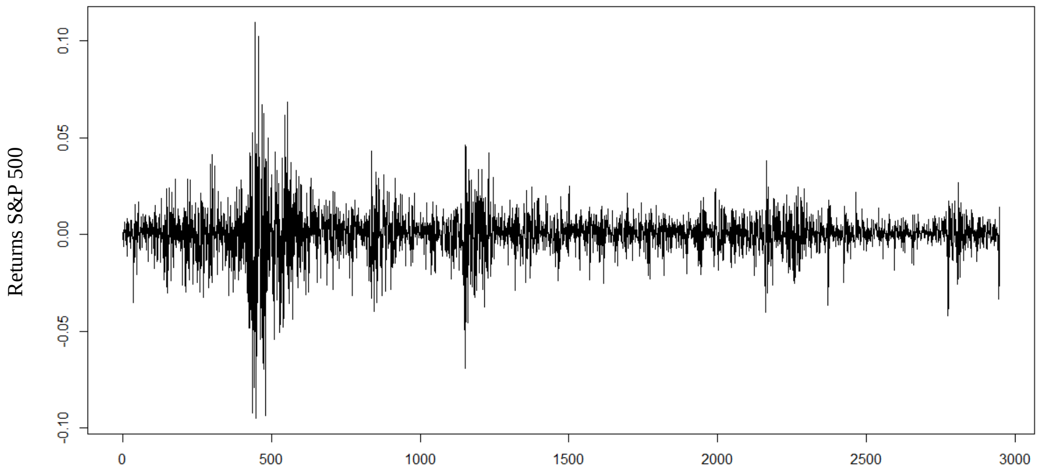
**Table 2: KPSS test for Logarithmic Returns**

| | Returns S&P 500 | Returns Euro Stoxx 50 | Returns Nikkei 225 |
|---|---|---|---|
| Test-Statistic value | 0.0752 | 0.055 | 0.064 |
| Critical value for a significance level of: | | | |
| 10pct | 0.119 | 0.119 | 0.119 |
| 5pct | 0.146 | 0.146 | 0.146 |
| 2.5pct | 0.176 | 0.176 | 0.176 |

After analysing the information of Table 2, we concluded that all the returns time series were stationary, since the values of the Test-Statistic, for all the series, were smaller than critical values, which led us to do not reject the null. Hence, the results of the test points for stationarity of time series and we found that the series are stationary after first order differencing.

### 4.2.1 Plots and summary statistics of the Daily Returns

Figures 1, 2 and 3 present the daily returns plots for the three indices. Daily returns exhibit volatility clustering. According to Mandelbrot (1963), volatility clustering pattern can be described as “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes.”




Ordered number of observations (from the oldest to the most recent, by date)

**Figure 4: Daily returns of S&P 500**

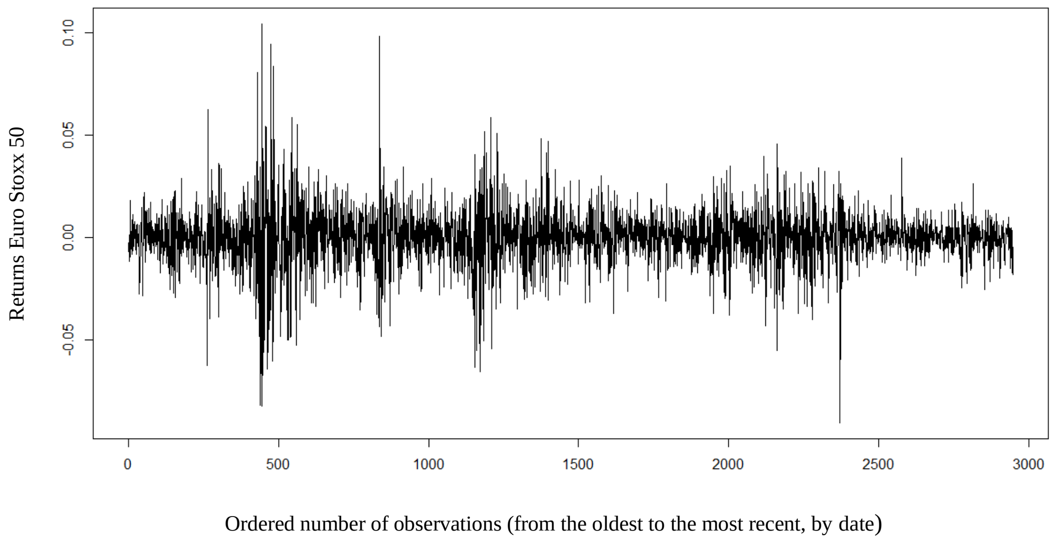



**Figure 5: Daily returns of Euro Stoxx 50**

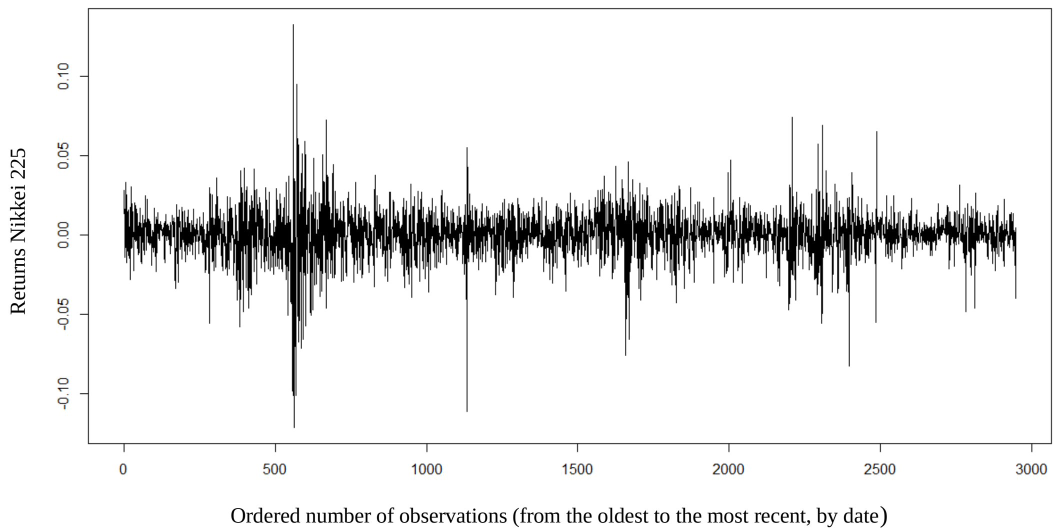



**Figure 6: Daily returns of Nikkei 225**

Figures 4, 5 and 6 represents graphically the plots of the daily returns of all the indices under analysis. Table 2 presents the stock market indices descriptive statistics. Average daily returns are positive across the three markets. The mean daily returns of S&P 500 is larger when compared to the others. The three markets display similar degrees of volatility, as indicated by their standard deviations. The standard deviations range between 1.2% and 1.6%. Nikkei 225 has the largest standard deviation. All series, except for S&P 500, have small negative skewness. Having negative skewness implies that large negative changes in the daily returns occur more often than positive ones. All time series of returns series display excess kurtosis, and when it occurs, it implies that large changes happen more often than what would be the case if the time series followed a normal distribution (Mohammadi and Tan, 2015).

**Table 3: Summary Statistics**

| | **Returns S&P 500** | **Returns Euro Stoxx 50** | **Returns Nikkei 225** |
|---|---|---|---|
| Mean | 0.0002271 | 0.0000000 | 0.0001526 |
| Median | 0.0006216 | 9.335e-05 | 0.0005685 |
| Maximum | 0.1095720 | 1.044e-01 | 0.1323458 |
| Minimum | -0.0946951 | -9.011e-02 | -0.1211103 |
| Std. Dev. | 0.0124818 | 0.01457082 | 0.01560029 |
| Skewness | -0.3734458 | -0.0615787 | -0.4906104 |
| Kurtosis | 10.92644 | 5.758 | 8.175313 |

### 4.2.2 ACF and PACF plots

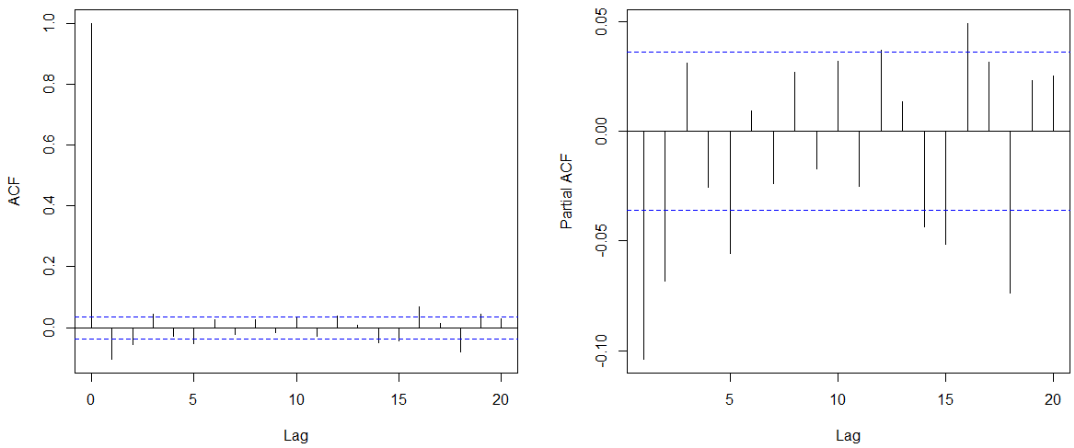
The estimated Autocorrelation Function (ACF) is a graphical representation based on estimates for the true values of the autocorrelation coefficients. The ACF measures the existent relationship between a variable and its lag values and it is generally used to check for white noise.

The Partial Autocorrelation Function (PACF) is the graphical representation of the partial autocorrelation coefficients. We can describe this plot as the autocorrelation of a variable with its final lag value with all intermediate lag values removed from the analysis.

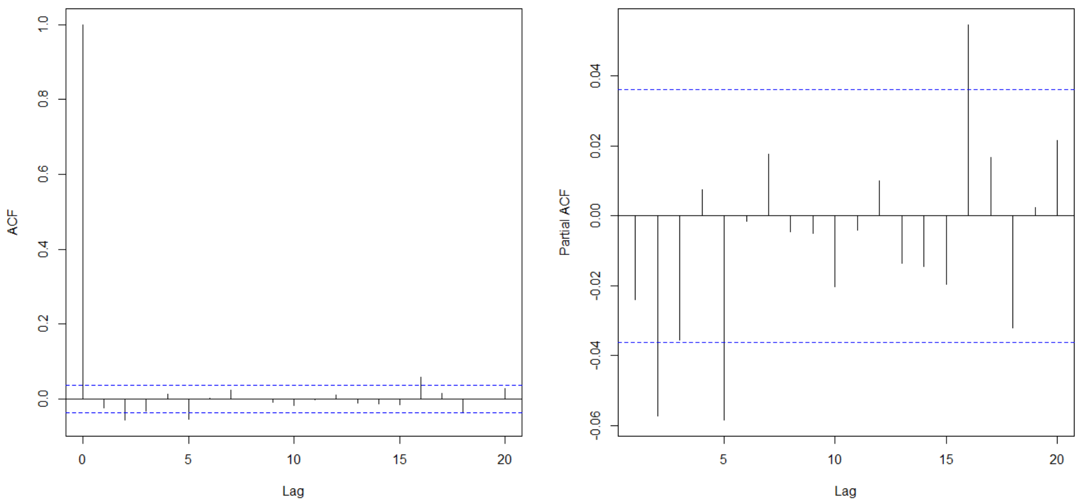
These plots graphically summarize the strength of a relationship with an observation in a time series with observations at prior time steps.

For the purpose of this study, we considered twenty lags for analysis, since time series have short-term memory, so it is an appropriate choice of lag number in order to find the structure of autocorrelation over time. The dashed horizontal lines represent the approximate 95% confidence intervals.

**S&P 500**




**Euro Stoxx 50**




**Nikkei 225**

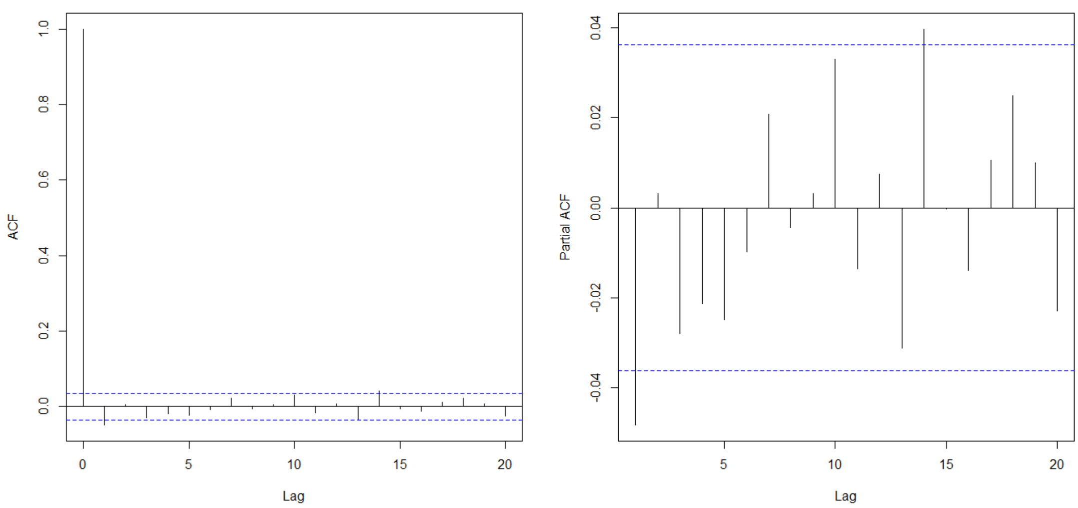



**Figure 7: ACF and PACF representations for time series logarithmic returns**

From the analysis of the autocorrelation (ACF) and partial autocorrelation (PACF) plots for the time series returns, we concluded that these data exhibit autocorrelation on some lags, at 5% level of significance, since some of the lags bars (vertical lines) lie outside the horizontal lines.

The procedure of analysing the plots of the ACF and PACF is subjective, relying on some assumptions such as no anomalies, level shifts, seasonal pulses or trends and assuming parameter and error variance constancy (error homogeneity) over the time. Hence, this process is only a predictor used for estimation and should not be used individually. In order to perform a more detailed analysis, in addition of looking at these plots, we can also perform a more formal test for the autocorrelation, the Ljung-Box test.

### 4.2.3 Ljung-Box statistics (Portmanteau tests)

The Ljung–Box test is a statistical tool to test whether all autocorrelation coefficients are zero (that is the null hypothesis), which means time series has been generated by a White-Noise process, i.e., when there is no linear relation between the time series observations.

**Table 4: Ljung-Box Test**

| | **Returns S&P 500** | **Returns Euro Stoxx 50** | **Returns Nikkei 225** |
|---|---|---|---|
| Probability value (p-value) | < 2.2e-16* | 0.001122* | 0.03786* |

Note: Significant values indicated with "*" at a 5% level, at least.

As p-values, for all the time series in question, are smaller than 0.05 (that is the level of significance defined in statistics as the probability of rejecting a null hypothesis), we reject the null hypothesis of no serial correlation for the series of returns. The results of this test do not point for a White Noise process, confirming that autocorrelation coefficients are not equal to zero.

### 4.2.4 Granger causality test

Correlation does not necessarily imply causation. In econometrics there are plenty of correlations, which are purely spurious or pointless.  Granger (1969) analysed the question of whether x causes y is to see how much of the current y can be explained by past values of $x$. Then, $y$ is said to be Granger-caused by $x$, if $x$ helps in the prediction of $y$. Thus, Granger causality test is a statistical hypothesis test for determining whether one time series is useful in forecasting another and seeks particularly for the direction of causality between pairs of time series. Granger causality measures precedence and information content but *per se* it does not indicate causality in the common use of the term. When the Granger Causality test is computed, the data must be stationary – as we confirmed before.

**Table 5 – Granger causality test**

| | |
|---|---|
| **Null hypothesis:** Returns of S&P 500 and Euro Stoxx 50 do not Granger-cause Returns of Nikkei 225 | |
| p-value | 0.3074 |
| **Null hypothesis:** Returns of S&P 500 and Nikkei 225 do not Granger-cause Returns of Euro Stoxx 50 | |
| p-value | < 2.2e-16* |
| **Null hypothesis:** Returns of Euro Stoxx 50 and Nikkei 225 do not Granger-cause returns of S&P 500 | |
| p-value | 0.6955 |

Note: Significant values indicated with "*" at a 5% level, at least

The results of Granger causality test are summarized in Table 5.

For the null hypothesis which states that returns of S&P 500 and Euro Stoxx 50 do not Granger-cause returns of Nikkei 225, since we do not reject the null.

For the null hypothesis which states that returns of S&P 500 and Nikkei 225 do not Granger-cause returns of Euro Stoxx 50, we reject the null, since the p-value is statistically significant. Hence, the returns of S&P 500 and Nikkei 225 Granger-cause returns of Euro Stoxx 50.

For the null hypothesis which states that returns of Euro Stoxx 50 and Nikkei 225 do not Granger-cause returns of S&P 500, we do not reject the null, then returns Euro Stoxx 50 and Nikkei 225 do not Granger-cause returns of returns of S&P 500.

### 4.3. Vector Autoregressive (VAR) model

One of the main uses of VAR models is forecasting. This model's structure provides information about a variable's or a group of variables' forecasting ability for other variables. If a certain variable (or a group of variables) is found to be helpful for predicting another variable (or group of variables) then the former is said to Granger-cause the latter; otherwise it is said to fail to Granger-cause the latter. Hence, after analysing the results of the Granger causality test, we concluded that it would make sense to proceed to the estimation of a VAR model and we were more confident to validate the conclusions of VAR model that will be reported below

In order to apply VAR models, the time series in study must be stationary. Hence, it is necessary to consider the logarithmic returns of the data series, and not the prices (levels). Therefore, by using this model we can only capture short-run dependencies between the three stock indices in analysis.

For the purpose of the analysis between the interactions of the three stock market indices returns, we estimated the VAR model with 1 lag. We chose one lag to study this data due to the results of the Information Criteria test results in Appendix 1 (see Appendix). Based on this statistic, we selected the model in which the Information Criteria was the smallest (SC - Schwarz Criterion), since the more we increase the number of parameters to estimate (increasing lags of estimation), the more we penalise the model. Hence, we choose the most parsimonious model.

An important concept to consider in this section is the returns spillover effects. In general, spillover effects can be described as the impact that apparently unrelated events in one country can have on the economies of other countries. There are positive spillover effects, but the most common case is the negative spillover effect (a negative domestic event impacts negatively other parts of the world). Spillover effects have been increasing since globalization and stock markets intensified the financial connections between economies.

As mentioned, we examined the causal relations between the three stock market indices using a VAR (1) model, where the lag length of one came from the results of the Schwarz information Criterion test. Hence, we regress the daily return in each market on one lag of itself, and also as one lag of returns in each of the two other markets.

**Table 6: VAR Model - Estimates**

| | | Dependent Variables | | |
|---|---|---|---|---|
| | | Returns S&P 500 | Returns Euro Stoxx 50 | Returns Nikkei 225 |
| **Past Returns** | Returns S&P 500 – lag 1 | -0.0919890 **(8.97e-05)*** | 0.4376083 **(<2e-16)*** | -0.0351172 **(0.23293)** |
| | Returns Euro Stoxx 50 – lag 1 | -0.0164060 **(0.414)** | -0.2569050 **(<2e-16)*** | -0.0003817 **(0.98791)** |
| | Returns Nikkei 225 – lag 1 | -0.0033716 **(0.818)** | 0.0014341 **(0.931)** | -0.0486358 **(0.00835)** |
| | Constant | 0.0002683 | -0.0001946 | 0.0001812 |

Note: Significant values indicated with "*" at a 5% level, at least.

Table 6 displays the results of the estimates on VAR model estimation among the stock markets returns, with p-values in parentheses.

The first column reports the response of S&P 500 returns to its own lag, as well as lag returns in the other three markets.

We could identify some patterns are evident from these results:

(1) Both S&P 500 and Euro Stoxx 50 returns depend on their own past returns, suggesting the existence of their own spillovers over time;

(2) Euro Stoxx 50 returns are influenced by past returns of S&P 500 (p-value is significant), but S&P current returns are not influenced by the returns in any of the other two markets;

(3) There is no evidence of causality from Nikkei 225 returns to any of the other indices returns.

### 4.3.1 ARCH Test

In this step, we checked whether there is volatility clustering in the residuals of VAR model – whether there is ARCH effect. In statistics, the residuals of a model are the differences between observed and predicted values of data. They are a diagnostic measure used when assessing the quality of a model.

The null hypothesis states that there is no ARCH effect in the data analysed. Hence, if there is ARCH effect, the null hypothesis is rejected. In general terms, the ARCH test display how residual squares are related with past observations.

**Table 7 – ARCH Test for VAR Model**

| | |
|---|---|
| p-value | < 2.2e-16* |

Note: Significant values indicated with "*" at a 5% level, at least.

Since the p-value is smaller than 0.05, we reject the null hypothesis, so this result points for the existence of ARCH effect.

### 4.3.2 Ljung-Box test

**Table 8 – Multivariate Ljung-Box Test for VAR Model**

| | |
|---|---|
| p-value | < 2.2e-16* |

Note: Significant values indicated with "*" at a 5% level, at least.

The test was applied on the residuals of the VAR model. The null hypothesis is the same as presented in Section 4.2.3.

As p-value is smaller than 0.05 (that is the level of significance defined in statistics as the probability of rejecting a null hypothesis) we reject the null. The results of this test do not point for a White Noise process, confirming that autocorrelation coefficients are not equal to zero.

The heteroscedasticity (or volatility clustering) phenomenon is observed and justifies the implementation of MGARCH models.

### 4.3.3 Breusch and Godfrey Lagrange Multiplier (LM) test

The test was applied on the residuals of the VAR model. For testing the lack of serial correlation in the errors of the model in question, the LM test proposed by Breusch and

Godfrey were implemented. The serial correlation LM test was used in order to test whether there is autocorrelation in the errors in our regression model. The null hypothesis states that the data has no serial correlation of any order up to $p$ (which is the lag – in our model, 1).

This test was carried out on the residuals obtained after fitting the VAR model on the three series.

**Table 9 – Breusch and Godfrey Lagrange Multiplier (LM) test**

| | **VAR Model residuals** |
|---|---|
| p-value | 6.98e-09* |

Note: Significant values indicated with "*" at a 5% level, at least.

The p-value is smaller than 0.05, so this result points for the existence of serial correlation, in other words, for autocorrelation in the errors of our model. The heteroscedasticity or volatility clustering phenomenon is observed and justifies the implementation of MGARCH models.

### 4.4 Estimation of the Dynamic Conditional Correlation (DCC) Model

In this study we a two-step procedure to estimate the DCC Multivariate GARCH model. The first step estimates a univariate GARCH (1,1) model – in this case we estimated three univariate GARCH (1,1) models since we have three time series under analysis - for each return series in the multivariate system. From this step, standardized residuals are generated, by maximizing the likelihood.  The obtained standardized residuals from the first step are then used to estimate the DCC parameters in the second step.

The GARCH (1,1) is a symmetric model. In symmetric models, both negative and positive news have the same impact on volatility. The choice of the parameters (1,1) usually is enough to catch dependency in the data. According to Bollerslev, Chou and Kroner (1992), GARCH (1,1) is, in general, satisfactory when modelling financial assets returns volatility. Hence, we defined for our study orders $p = 1$ and $q = 1$.

For the purpose of this study, in order to estimate the VAR-DCC model, we firstly specified and estimated an individual GARCH-type model for each time series. These univariate GARCH models were estimated based on the returns of the three series. We assumed and used the same univariate volatility model specification for each of the three stock indices, and then replicated it three times. Hence, we had to specify a GARCH (1,1) model – and then we needed to estimate its parameters, that is to estimate the model itself. The model specifications of the Univariate GARCH (1,1) model are reported in Appendix 2 (see Appendix) and the model estimation results in Appendix 3 (see Appendix).

For modelling all the time series together, we combined and modelled them with a VAR (1) model to undertake the causality relationship of the series among them.

For the purpose of this study, the estimation was conducted within a multivariate GARCH framework, which provides the interpretation of the conditional variance as a time-varying risk measure. The proposed dynamic correlation structure consists in:

$$Q_t = (1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n)\bar{Q} + \sum_{m=1}^{M} \alpha_m \, (\varepsilon_{t-m}\varepsilon'_{t-m}) + \sum_{n=1}^{N} \beta_n \, Q_{t-n} \qquad (21)$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \qquad (22)$$

Where $m, n \ = \ 1, 2$, α is the news coefficients and β is the decay coefficient.

To ensure a conditional correlation between -1 and 1, the model is mean reverting provided $\alpha + \beta \ < \ 1$. Additionally, $\bar{Q}$ is the unconditional covariance of the standardized residuals resulting from the first stage estimation and $Q_t^*$ is a diagonal matrix composed of the square root of diagonal elements of $Q_t$.

Equation 14 (see Section 3.3.) represents a standard univariate GARCH model, and Equations 21 and 22 are referred to a DCC $(m, n)$ model.

For the purpose of the estimation of the VAR-DCC model, we fit equations 14, 21 and 22 to the VAR (1) residuals. We chose to adopt the most parsimonious specification ($m = n = 1$).

The entire output that represents DCC GARCH Fit estimation is presented in Appendix 4 (see Appendix). From the analysis of Appendix 4 we found that twenty-six parameters were estimated by the model – twelve from VAR (1) model, nine for the univariate GARCH (1,1) models and only two parameters were required for the DCC.

The estimated values of all parameters related with the univariate GARCH and DCC models are summarized on Table 10. The model has nine parameters for the returns of the data series, of which six are significant. Furthermore, Joint dcca1 and dccb1 – the conditional correlation parameters - are presented and both significant. The correlation between the variables is governed by the scalar parameters dcca1 and dccb1.

**Table 10 – VAR-DCC model estimation**

| **VAR (1)** | | **Estimates** | |
|---|---|---|---|
| Optimal parameters: | **Returns S&P 500** | **Returns Euro Stoxx 50** | **Returns Nikkei 225** |
| Returns S&P 500 – lag 1 | -0.09198896 | -0.0164059731 | -0.003371551 |
| Returns Euro Stoxx 50 – lag 1 | 0.43760832 | -0.2569049835 | 0.001434094 |
| Returns Nikkei 225 – lag 1 | -0.03511718 | -0.0003816798 | -0.048635775 |
| | | **p-value** | |
| Returns S&P 500 – lag 1 | 8.748151e-05* | 0.0000000* | 0.232751710 |
| Returns Euro Stoxx 50 – lag 1 | 4.134430e-01 | 0.0000000* | 0.987903759 |
| Returns Nikkei 225 – lag 1 | 8.183654e-01 | 0.9307083 | 0.008299389 |

| **Univariate GARCH (1,1)** | **Estimates** | **p-value** |
|---|---|---|
| Optimal parameters: | | |
| Returns S&P 500 – Omega | 0.000002 | 0.246164 |
| Returns S&P 500 – Alpha 1 | 0.122662 | 0.000000* |
| Returns S&P 500 – Beta 1 | 0.860405 | 0.000000* |
| Returns Euro Stoxx 50 – Omega | 0.000003 | 0.228642 |
| Returns Euro Stoxx 50 – Alpha 1 | 0.095016 | 0.000008* |
| Returns Euro Stoxx 50 – Beta 1 | 0.890102 | 0.000000* |
| Returns Nikkei 225 – Omega | 0.000006 | 0.909184 |
| Returns Nikkei 225 – Alpha 1 | 0.131214 | 0.000074* |
| Returns Nikkei 225 – Beta 1 | 0.847615 | 0.000441* |
| **DCC (1,1)** | **Estimates** | **p-value** |
| Optimal parameters: | | |
| Joint dcca1 | 0.004974 | 0.001125* |
| Joint dccb1 | 0.991373 | 0.000000* |

Note: Significant values indicated with "*" at a 5% level, at least.

After analysing the p-values for the time series in Table 10, we were able to take some conclusions about the appropriateness of the model for the data under analysis.

The optimal parameters of Omega, Alpha 1 and Beta 1 for the three returns are estimates for the univariate GARCH (1,1) model. With these results we could have estimated the conditional variance of S&P 500, Euro Stoxx 50 and Nikkei 225 returns, respectively:

$$\sigma_t^2 = 0.000002 + 0.122662\mu_{t-1}^2 + 0.860405\ \sigma_{t-1}^2 \qquad (23)$$

$$\sigma_t^2 = 0.000003 + 0.095016\, \mu_{t-1}^2 + 0.890102\, \sigma_{t-1}^2 \tag{24}$$

$$\sigma_t^2 = 0.000006 + 0.131214\, \mu_{t-1}^2 + 0.847615\, \sigma_{t-1}^2 \tag{25}$$

As the parameters Alpha1 and Beta1 are jointly (and highly) significant for all the time series in question (returns of S&P 500, Euro Stoxx 50 and Nikkei 225), we can conclude that a GARCH (1,1), in terms of appropriateness to apply, is a model that fits the given time series for the purpose of the study. It is more accurate to look at these parameters simultaneously, rather than analyse only Alpha1.

The joint estimated dcca1 and dcca2 parameters concern to the analysis for the DCC model. These two parameters are also jointly and highly significant, supporting the time-varying nature of the conditional correlation. Hence, the DCC model is appropriate for the time series in question (rather than a model of constant conditional correlations), so it makes sense to analyse conditional correlations in a dynamic perspective.

The results of the DCC model were encouraging as all the relevant parameters to take relevant conclusions for this study were found to be significant at 5% level of significance. Thus, the DCC model provides a more accurate description of the dynamics of the correlations between the time series in question.

### 4.5 Dynamical Conditional Correlations Plots

The core of multivariate volatility models like the DCC is to allow for time-variation in the correlation between the financial assets – in this case, financial stock markets. Therefore, we were also able to extract the plots for the dynamical conditional correlations (Figures 7, 8 and 9) and for the conditional covariances (Figures 10, 11 and 12).

By observing the correlation structure of the plots of DCC model, we can conclude that there is a non-constant interaction of all the time series regarding conditional correlation. This interaction effect would be neglected if the three time-series of VAR residuals were only modelled in isolation, each with a univariate GARCH model.


**Figure 8 – Conditional Correlation between S&P 500 and Euro Stoxx 50**

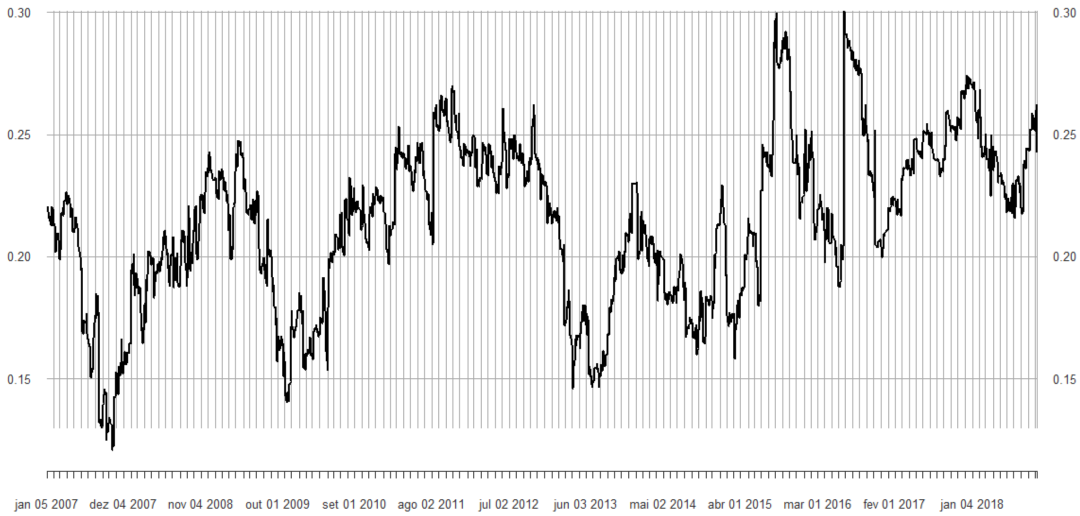



**Figure 9 – Conditional Correlation between S&P 500 and Nikkei 225**

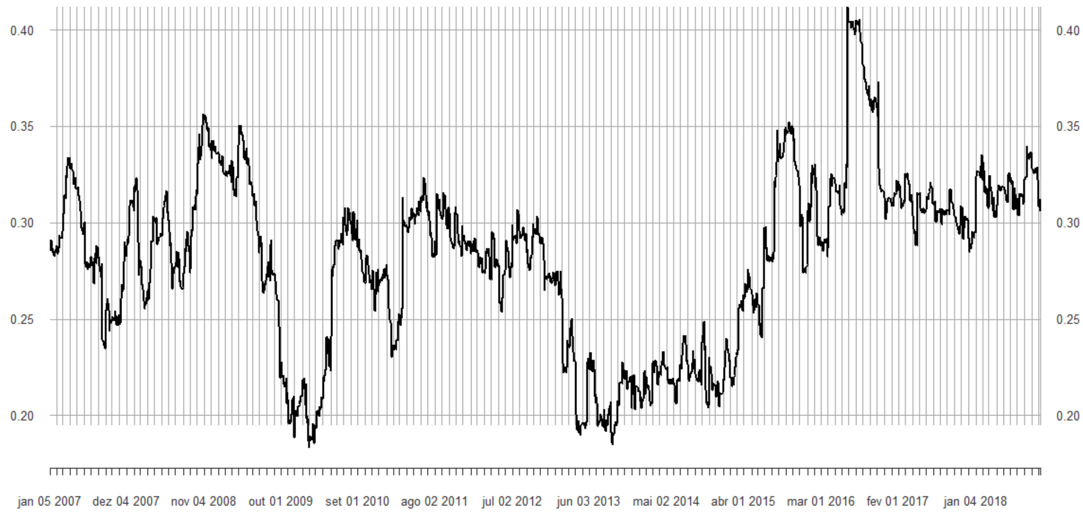



**Figure 10 – Conditional Correlation between Euro Stoxx 50 and Nikkei 225**

Finally, in order to conclude the empirical data analysis section, we evaluate the patterns of pairwise dynamic correlations over the recent thirteen years across the three markets.

The dynamic correlations exhibit two patterns:

(1) High conditional correlations between S&P 500 and Euro Stoxx 50 (Figure 8) with the existence of one sharp rise in November 2008.

(2) Medium correlation shown in Figures 9 and 10 with two maximum peaks occurring in February 2016 and February 2017 on the conditional correlation between S&P 500 and Nikkei 225 (Figure 9) and in early 2009 a pronounced drop in the conditional correlation between Euro Stoxx 50 and Nikkei 225 (Figure 10).

The values observed for S&P 500 and Euro Stoxx 50 are comprised between 0.52 and 0.72, for S&P 500 and Nikkei 225 between 0.10 and 0.30 and for Euro Stoxx 50 and Nikkei 225 between 0.18 and 0.42.

Still, the Financial Crisis (period that starts in the end of 2008) made the all correlations increase considerably due to instability and financial contagion.

## 5. Conclusions, limitations and future research

In this final section, we will review the contributions of this dissertation, as well as present its limitations and directions for future research.

In finance, people are concerned about risk. If it is known that the returns of an investor follow, in average, a process, there is also some kind of concern about the risk of it. So, it is useful not only to compute conditional return, but also to consider risk measures, by using a second moment: the conditional variance. Conditional variance is, in fact, a time varying measure that we must take into consideration. GARCH models will allow us to model a time-varying variance. Hence, the past is going to be used to estimate current observations. With GARCH models it is assumed that data that is being modelled follows a non-constant variance, i.e., conditional heteroscedasticity models – it changes from one time to another.

In this dissertation we analysed the relationship between three financial stock indices returns, geographically far, from 2007 to 2018. In order to conduct this study, a generalized VAR and dynamic conditional correlation models were implemented.

Our models were estimated, throughout the empirical study, on a set of stationary variables. These variables are returns of stock market prices for the Europe, United States of America and Asia. This methodology allowed us to gather some of the following results. With the Granger causality test, we were able to conclude that returns of S&P 500 and Euro Stoxx 50 do not Granger-cause returns of Nikkei 225, the returns of S&P 500 and Nikkei 225 Granger-cause returns of Euro Stoxx 50 and returns of Euro Stoxx 50 and Nikkei 225 do not Granger-cause returns of returns of S&P 500.

We initially employed a vector autoregressive (VAR) model to examine the relationship among stock market returns of the three geographical areas. With VAR results analysis we figured out that both S&P 500 and Euro Stoxx 50 returns depend on their own past returns; Euro Stoxx 50 returns are influenced by past returns of S&P 500 and there is no relevant evidence of causality from Nikkei 225 returns to any of the other indices returns. Janakiramanan and Lamba (1998) and Cha and Cheung (1998) examined possible linkages between Asia Pacific and the USA equity markets using VAR and they determined that the USA has a significant influence on these markets. In our study we were not able to converge our results with these authors conclusions.

As referred, we modelled all the time series together, combining them with a VAR (1) model in order to conclude about the causality relationship of the series among them.

In general terms, the DCC model was estimated with three steps:

1- Estimate VAR (1) model
2- Estimate univariate GARCH (1,1) model
3- Estimate multivariate DCC (1,1) model with the input from the steps 1. (residuals of VAR model) and 2. (three univariate GARCH (1,1) models).

After analysing the patterns of the pairwise dynamic correlations over the recent thirteen years across the three time series under analysis, we concluded that highest conditional correlations pattern was found between S&P 500 and Euro Stoxx 50. Furthermore, the financial crisis that started in 2008 made all correlations increase significantly – financial contagion effect. Moreover, the results presented by this model suggest that it provides an accurate description of the dynamics of the correlations between the time series in question. Following Engle and Sheppard (2001) approach, the main advantage of this model is to be able to estimate large time-varying correlations with a two-step process: firstly the estimation of univariate GARCH models for each asset, and using the residuals of the Univariate GARCH model (which are already transformed and generated in the first stage), estimate a conditional correlation estimator. Furthermore, we in our study we confirmed, as Engle (2002) stated, that DCC model provides a great approximation to a variety of time varying correlation processes, being the most precise model.

These results can have important implications and contributions for the existing literature, since it can give interesting insights on how these three markets have been related through time. Similarly, these results can inform institutions, corporations or individual investors to be aware and conscious of the existence of volatility spillovers in their decision making for important financial choices.

In terms of limitations, although we tried to make the best analysis with the models used, this study does not provide a complete picture of what is demonstrated in other studies about DCC model. We did not analyse, for example, the six equations that explain the variance and covariance between the time series, which may contribute to a more complete, descriptive and deep analysis of the relationships between the three financial stock indices. This could be a future research topic.

As a final remark, we point out that, despite the extensive literature in the VAR and Multivariate GARCH models, future research could tend to be based on the study of more than three time series. An extension of this work could also be done by incorporating some other aspects, namely the inclusion of more models, to enrich the content of the empirical study and its conclusions Another interesting topic could be, additionally to extract prices, to include in the study some economic variables of each country in analysis, such as inflation, interest rates and GDP (Gross Domestic Product).

Furthermore, besides the suggestions already mentioned, this dissertation lead to some findings that we did not anticipate from its beginning. As a suggestion for future research that could be used to explore such findings in future, we suggest the investigation of the analysis of the covariance matrix, the correlation matrix for the time $n$ (last observation before forecast) and the correlation matrix. From the current study it was possible to get the outputs with the estimation results for the referred matrices. By doing the VAR-DCC forecast for periods ahead (in this case we chose only one period ahead, so $n = 1$), it is possible to get these estimates.

As shown before in the Equation 12 (see Section 3.3.), in DCC-GARCH model $D_t$ and $R_t$ are be time-varying. By applying the forecast to the time series, we were able to estimate $H_t$ and $R_t$ for one period ahead (since $n = 1$). $H_t$ represents the covariance matrix decomposed and $R_t$ the correlation matrix. The first matrix below concerns to the $H_t$ matrix and the following one to the $R_t$ matrix. The returns of S&P 500, Euro Stoxx 50 and Nikkei 225 are represented, respectively, with the designations of r.SP500, r.SX5E and r.NIKKEI.

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{r.SP500} & \text{r.SX5E} & \text{r.NIKKEI} \end{array} \\
\begin{array}{c} \text{r.SP500} \\ \text{r.SX5E} \\ \text{r.NIKKEI} \end{array} &
\begin{bmatrix}
2.687685\text{e}-05 & 2.521359e-05 & 2.361756e-06 \\
2.521359\text{e}-05 & 7.391372e-05 & 6.061206e-06 \\
2.361756e-06 & 6.061206e-06 & 1.062656e-04
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{r.SP500} & \text{r.SX5E} & \text{r.NIKKEI} \end{array} \\
\begin{array}{c} \text{r.SP500} \\ \text{r.SX5E} \\ \text{r.NIKKEI} \end{array} &
\begin{bmatrix}
1.0000000 & 0.5656959\ 5 & 0.0441926 \\
0.5656959 & 1.0000000 & 0.068391 \\
0.0441926 & 0.068391 & 1.0000000
\end{bmatrix}
\end{array}
$$

In Appendix 5 (see Appendix) there are reported, among other estimation results, the forecast results for one period ahead or $Q_t$ which is the decomposition of Cholesky for the correlations and guarantees the matrix to be positive-definite.

These proposals for future research can be useful for further development on this topic.

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## 7. Appendices

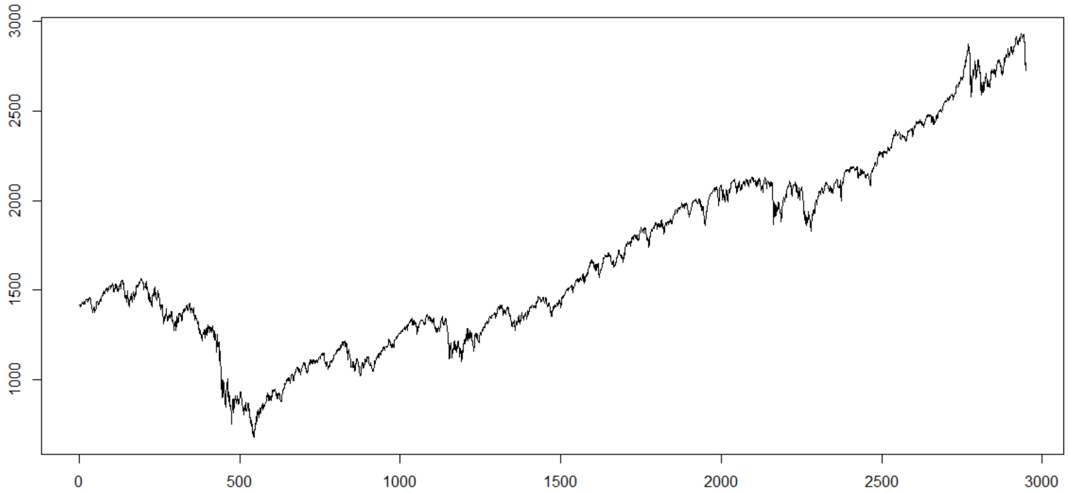


**Figure 1: Adjusted Closing Prices of S&P 500**

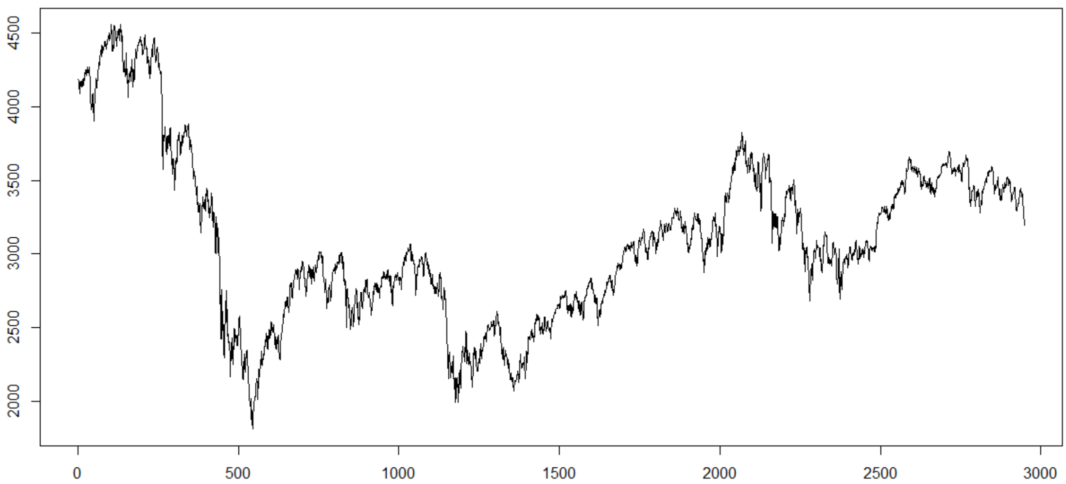


**Figure 2: Adjusted Closing Prices of Euro Stoxx 50**

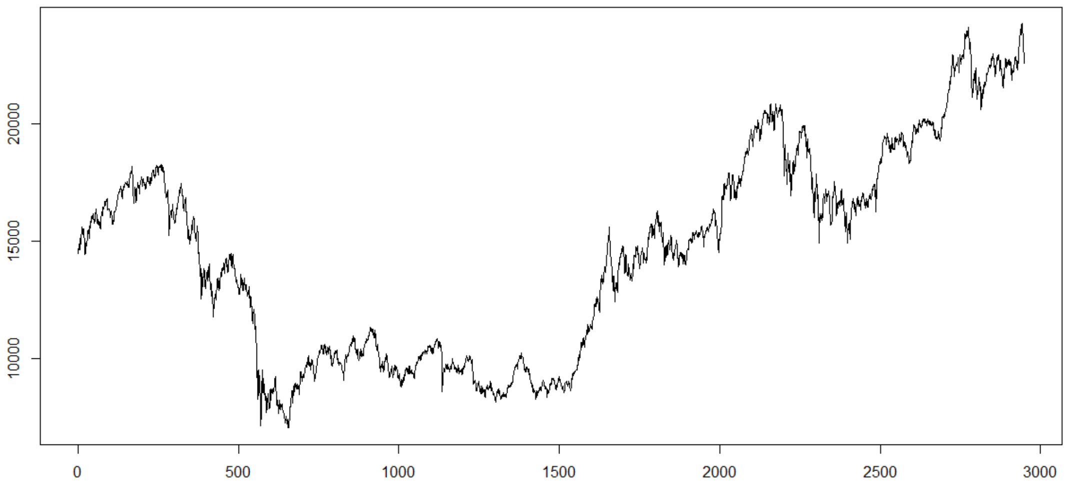


**Figure 3: Adjusted Closing Prices of Nikkei 225**

**Appendix 1: Information Criteria to choose lag for VAR model**

```
AIC(n)  HQ(n)  SC(n) FPE(n)
5      2      1      5

$criteria
1            2            3            4            5
AIC(n) -2.624622e+01 -2.625775e+01 -2.625844e+01 -2.625495e+01 -2.6260
48e+01
HQ(n)  -2.623740e+01 -2.624230e+01 -2.623637e+01 -2.622626e+01 -2.6225
18e+01
SC(n)  -2.622171e+01 -2.621486e+01 -2.619716e+01 -2.617529e+01 -2.6162
45e+01
FPE(n)  3.994020e-12  3.948243e-12  3.945539e-12  3.959316e-12  3.9374
65e-12
6            7            8            9            10
AIC(n) -2.625814e+01 -2.625781e+01 -2.625490e+01 -2.624951e+01 -2.6251
16e+01
HQ(n)  -2.621622e+01 -2.620926e+01 -2.619973e+01 -2.618772e+01 -2.6182
75e+01
SC(n)  -2.614172e+01 -2.612300e+01 -2.610171e+01 -2.607794e+01 -2.6061
21e+01
FPE(n)  3.946694e-12  3.948029e-12  3.959520e-12  3.980910e-12  3.9743
58e-12
```

**Appendix 2: Univariate GARCH (1,1) specifications**

```
*---------------------------------*
*       GARCH Model Spec          *
*---------------------------------*

Conditional Variance Dynamics
------------------------------------
GARCH Model             : sGARCH(1,1)
Variance Targeting      : FALSE

Conditional Mean Dynamics
------------------------------------
Mean Model              : ARFIMA(1,0,1)
Include Mean            : FALSE
GARCH-in-Mean           : FALSE

Conditional Distribution
------------------------------------
Distribution    :  norm
Includes Skew   :  FALSE
Includes Shape  :  FALSE
Includes Lambda :  FALSE
```

**Appendix 3: Univariate GARCH (1,1) model estimation**

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
Estimate  Std. Error  t value Pr(>|t|)
ar1     0.639554    0.168726   3.7905 0.000150
ma1    -0.670375    0.162688  -4.1206 0.000038
omega   0.000002    0.000001   3.2053 0.001349
alpha1  0.115605    0.008810  13.1221 0.000000
beta1   0.875780    0.009022  97.0709 0.000000

Robust Standard Errors:
Estimate  Std. Error  t value Pr(>|t|)
ar1     0.639554    0.194912  3.28125 0.001033
ma1    -0.670375    0.188376 -3.55871 0.000373
omega   0.000002    0.000004  0.56748 0.570386
alpha1  0.115605    0.034645  3.33688 0.000847
beta1   0.875780    0.040844 21.44205 0.000000

LogLikelihood : 26836.13
```

## Appendix 4: DCC-GARCH Fit

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  26
[VAR GARCH DCC UncQ] : [12+9+2+3]
No. Series           :  3
No. Obs.             :  2939
Log-Likelihood       :  27783.31
Av.Log-Likelihood    :  9.45

Optimal Parameters
-----------------------------------
                   Estimate  Std. Error   t value Pr(>|t|)
[r.SP500].omega    0.000002    0.000002   1.15972 0.246164
[r.SP500].alpha1   0.122662    0.021378   5.73787 0.000000
[r.SP500].beta1    0.860405    0.023944  35.93431 0.000000
[r.SX5E].omega     0.000003    0.000003   1.20386 0.228642
[r.SX5E].alpha1    0.095016    0.021228   4.47586 0.000008
[r.SX5E].beta1     0.890102    0.024230  36.73512 0.000000
[r.NIKKEI].omega   0.000006    0.000049   0.11407 0.909184
[r.NIKKEI].alpha1  0.131214    0.033118   3.96208 0.000074
[r.NIKKEI].beta1   0.847615    0.241189   3.51432 0.000441
[Joint]dcca1       0.004974    0.001527   3.25721 0.001125
[Joint]dccb1       0.991373    0.002862 346.37192 0.000000

Information Criteria
---------------------

Akaike       -18.889
Bayes        -18.836
Shibata      -18.889
Hannan-Quinn -18.870


Elapsed time : 7.851225
```

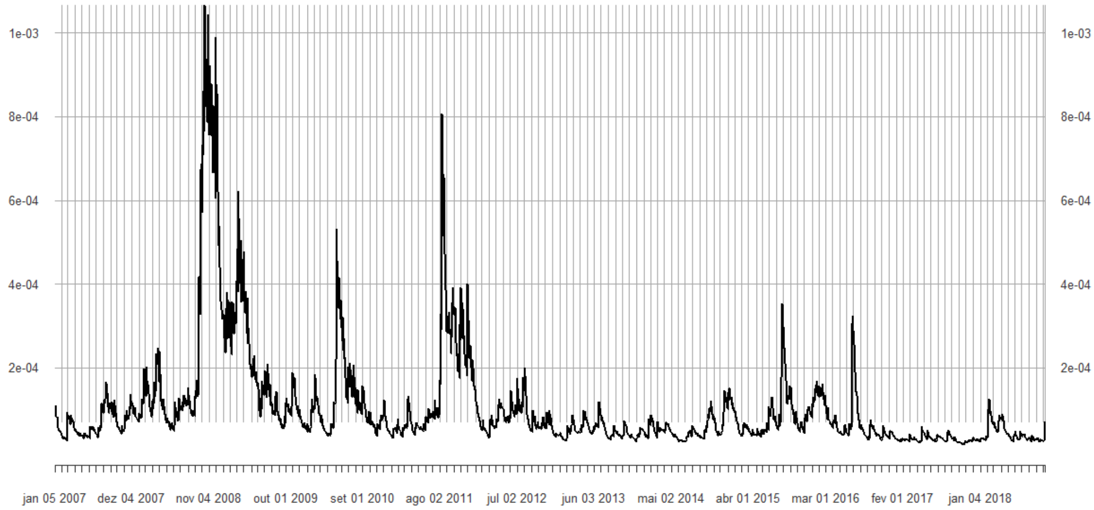


**Figure 11 - Conditional Covariance between S&P 500 and Euro Stoxx 50**

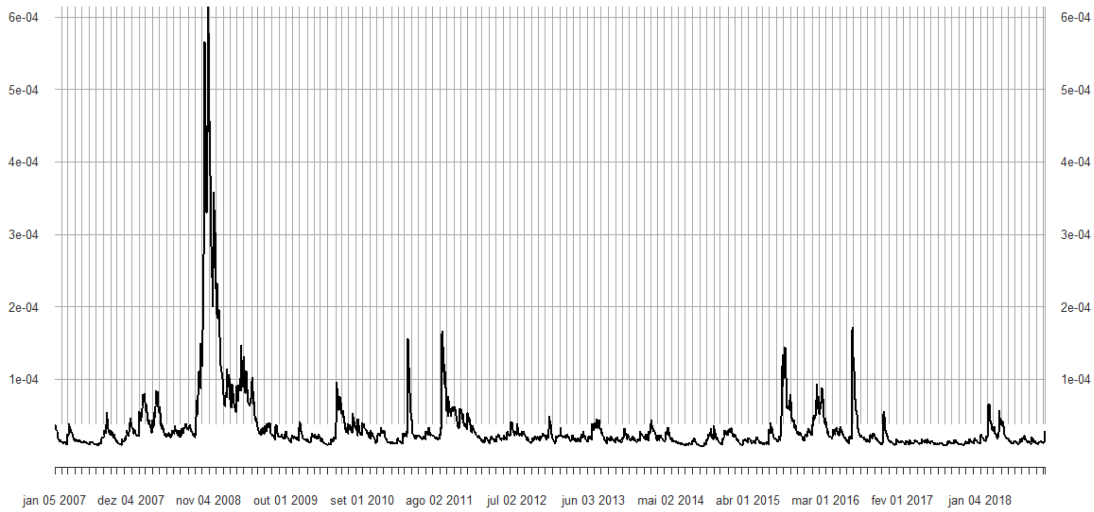


**Figure 12 - Conditional Covariance between S&P 500 and Nikkei 225**

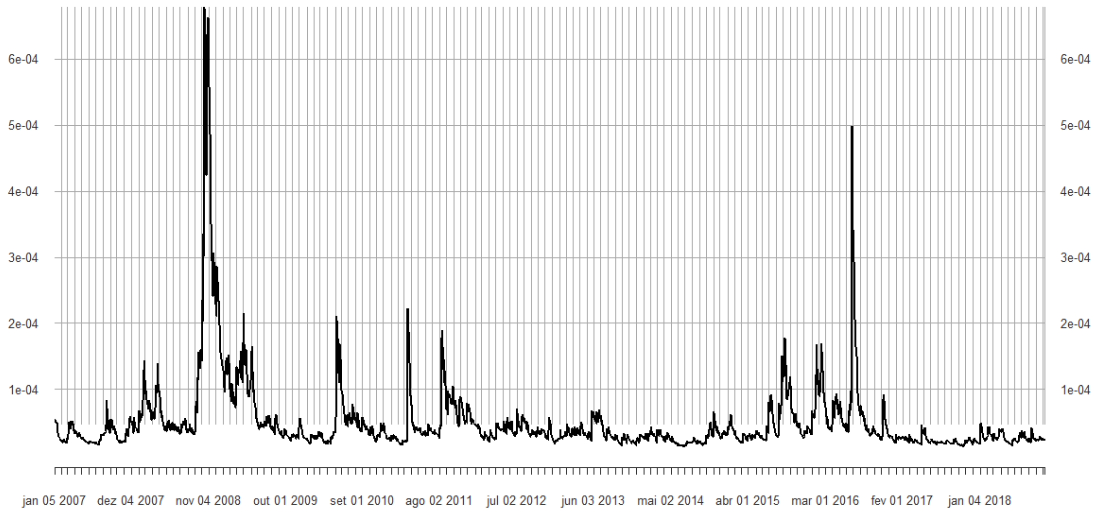


**Figure 13 - Conditional Covariance between Euro Stoxx 50 and Nikkei 225**

## Appendix 5: VAR-DCC Forecasting results

```
*       DCC GARCH Forecast        *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
Horizon              :  1
Roll Steps           :  0
-----------------------------------

0-roll forecast:
, , 1

        [,1]    [,2]    [,3]
[1,] 1.00000 0.56570 0.04419
[2,] 0.56570 1.00000 0.06839
[3,] 0.04419 0.06839 1.00000

> rcov(test)
$`2939-01-01`
, , T+1

              r.SP500       r.SX5E     r.NIKKEI
r.SP500  2.687685e-05 2.521359e-05 2.361756e-06
r.SX5E   2.521359e-05 7.391372e-05 6.061206e-06
r.NIKKEI 2.361756e-06 6.061206e-06 1.062656e-04

> rcor(test)
$`2939-01-01`
, , T+1

           r.SP500    r.SX5E  r.NIKKEI
r.SP500  1.0000000 0.5656959 0.0441926
r.SX5E   0.5656959 1.0000000 0.0683912
r.NIKKEI 0.0441926 0.0683912 1.0000000

> test@mforecast
$H
$H[[1]]
, , 1

             [,1]         [,2]         [,3]
[1,] 2.687685e-05 2.521359e-05 2.361756e-06
[2,] 2.521359e-05 7.391372e-05 6.061206e-06
[3,] 2.361756e-06 6.061206e-06 1.062656e-04
$R
$R[[1]]
, , 1

          [,1]      [,2]      [,3]
[1,] 1.0000000 0.5656959 0.0441926
[2,] 0.5656959 1.0000000 0.0683912
[3,] 0.0441926 0.0683912 1.0000000



$Q
$Q[[1]]
, , 1

           [,1]       [,2]       [,3]
```

```
[1,] 0.84586299 0.48386000 0.03859106
[2,] 0.48386000 0.86491486 0.06039125
[3,] 0.03859106 0.06039125 0.90151777



$Rbar
$Rbar[[1]]
             [,1]        [,2]         [,3]
[1,] 1.0000000000 0.658004731 0.0001665509
[2,] 0.6580047312 1.000000000 0.0038244994
[3,] 0.0001665509 0.003824499 1.0000000000


$mu
, , 1
              [,1]        [,2]          [,3]
[1,] -7.613911e-05 0.001413622 -0.0006161061
```