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BUNDLING PRICING FOR INFORMED AND UNINFORMED CONSUMERS
-
and their relationship with Profit Equilibrium

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Dissertation submitted as partial requirement for the conferral of

Master in Economics

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October 2018

BUNDLING PRICING FOR INFORMED AND UNINFORMED CONSUMERS – AND THEIR RELATIONSHIP WITH PROFIT EQUILIBRIUM

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Acknowledgements

I would like to give special thanks...

to my coordinator, Professor Miguel Carvalho, for his continuous guidance and patience throughout this journey;

to the professors of ISCTE, who had the availability to take my questions;

to the community of MatlabCentral for their hints on the proper use of MATLAB;

to Jean Burrows for reviewing the text;

to my friends and family for their support and encouragement.

I assume full responsibility for any and all possible errors present in this thesis.

Abstract

Based on the concept of profit equilibrium, we studied which strategies two firms (one of which is an incumbent, while the other is a challenger) should follow when disputing the same market of two individual services across a four-period timeline. Three key features characterized these Markets: the possibility of ***Mixed Bundling****, the presence of both* ***Informed and Uninformed Consumers****, and the existence of* ***Switching Costs****.*

To study this model, we performed numerical simulations through the usage of two custom-built .m files that allowed us to define a Benchmark Case and several subsequent simulations, in which we could test the alteration of certain parameters. Finally, we compared their different results, noted some limitations of the study and proposed some possibilities for future applications of the model.

We were able to verify that the obtained results from these simulations, namely how bundle practises can be beneficial for firms and consumers, how switching costs can be easily exploited by firms, and how the informed-to-uninformed consumer ratio relates to the profit of firms, are all compatible results with the existing scientific literature.

Keywords: Mixed Bundling, Uninformed Consumers, Switching Costs, Profit Equilibrium

JEL: L13, L19

Resumo

Com base no conceito de equilíbrio de lucro, estudámos as estratégias que duas empresas devem seguir (sendo uma das quais incumbente, e a outra entrante) ao disputar o mesmo mercado de dois serviços individuais, ao longo de quatro períodos de tempo. Três características principais definem este Mercado: a possibilidade de ***"Bundling" Misto****, a presença de* ***Consumidores Informados e Desinformados*** *e a existência de* ***Custos de Transição****.*

Para estudar este modelo, realizámos várias simulações numéricas através do uso de dois ficheiros .m customizados que nos permitiram definir um Caso de Referência e algumas simulações subsequentes, nas quais pudemos testar a alteração de um conjunto de parâmetros definidos. Por fim, comparámos os diferentes resultados obtidos, notámos algumas limitações do estudo e propusemos hipóteses para futuras aplicações do modelo.

Verificámos que os resultados obtidos a partir dessas simulações que avaliam o impacto de todas as três características em simultâneo, ou seja, como as práticas de "Bundling" podem ser benéficas para empresas e consumidores, como os custos de transição podem ser facilmente explorados, e como se relaciona o rácio de consumidores informados e desinformados com o lucro de empresas, são todos resultados compatíveis com a literatura científica existente.

Palavras-chave: Bundling Misto, Consumidores Desinformados, Custos de Transição, Equilíbrio de Lucro

JEL: L13, L19

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Introduction

As is widely known, the main purpose behind the setting up of a firm is to make a profit. In this thesis, we will not study the maximization of profit, but rather its rationalization, more specifically, the concept of profit equilibrium.

It is from this perspective that we will develop our idea, attempting to demonstrate that profit equilibrium can be affected by the induction of several arbitrary components.

We therefore develop a set of features that serve as the crucial point of our model. These features are the implementation of bundling, the existence of switching costs, and the context of a market with informed and uninformed consumers. It is by merging these concepts that we form our core idea.

Starting from a benchmark case, we submit it to several numerical simulations (here called "experiments"), which consist of the arbitrary alteration of the values of certain predefined parameters, and draw our conclusions from the resulting comparisons.

To perform the experiments, we will utilize two .m files, custom-built for this thesis: one of them is used to set the values of the parameters, the other then attempts to achieve a profit maximizing Nash Equilibrium in accordance with said parameters.

This thesis will therefore study the interactions between two firms across two different markets. At a certain point in time, one of these firms is the monopolist of a service (in this case, television provision), as well as a duopolist in another market (telephone provision), and bundles these two services together. At the same point in time, the second firm is the other duopolist. The latter firm will expand into the television providing market and begin providing their own bundle in order to directly compete with the former firm in all markets.

It is crucial to note that these two services are not meant to represent "Television provision" and "Telephone provision" on a literal basis. These terms are meant to serve as stand-ins for two non-complementary, non-substitute services. As such, all discrepancies related to the nature of television or telephone provision within this thesis compared to the real world (historical inaccuracies, for instance), will be ignored.

Similarly, it is also necessary to highlight that throughout this thesis, the term "firm(s)" is used to denote firms in general, or when describing any examples of firms that do not belong to the case study. Meanwhile, the term "**Firm(s)**" is used when specifically mentioning the two subjects of the case study, those being **Firm 1** and/or **Firm 2**.

Taking into account certain market variables, particularly the pricing of services, the role of informed and uninformed consumers, and the switching costs that these consumers face, this thesis will attempt to deduce the strategies that each **Firm** should choose in order to ensure a profit maximizing Nash Equilibrium.

It will therefore pose the following questions:

1. How can the profit equilibrium of the two Firms be altered due to the deviation of the informed-to-uninformed ratio of consumers that can fluctuate within and/or between the two Firms?

2. How can the profit equilibrium of the two Firms be altered due to the implementation and existence of switching costs?

3. In which scenarios can bundling be advantageous for the Firms?

We therefore follow a path through a **Literature Review**[1] section, where we first provide the basic definitions for the keywords that serve as crucial points for the model which we test in the Body of Work, and then showcase the scientific literary backing for each of those crucial points.

We continue with the **Methodology**[2], where through a timeline of events, we present the nature of the two Firms, the services which they provide, and the features of the model, as a way to fully contextualize it.

Afterwards, we advance to the **Body of Work**[3]. Here, we introduce two custom-built **.m files** (or "**scripts**") for **MATLAB©,** which we specifically created for the purpose of this

[1] pp. 5-10.
[2] pp. 11-31.
[3] pp. 33-45.

thesis, to test the possible profit equilibria within several scenarios. The limitations we were able to identify with these .m files are also stated.

We showcase a "Benchmark Case", which serves as our point of comparison for other scenarios, known as "experiments". In each experiment, we alter at least one of certain variables (namely the Monopoly prices, the values of the switching costs, and the initial numbers of informed and uninformed consumers). We subsequently draw our conclusions from each of these experiments, and finalize with an overall **Synthesis of Benchmark Case and Experiments**.

Finally, we go through the **Discussion of Results,**[4] where we study the obtained results, and seek to understand the type of information we can extract, and see if that information can answer the previously formulated questions.

We finish with our **Final Conclusions.**[5]

[4] pp. 47-50.
[5] p. 51.

Literature Review

1) Introduction

The purpose of this section is twofold: first, it contains the basic definitions of each of the keywords that serve as crucial points for the model. Second, it serves to show the literary backing of each keyword.

We must stress that is not uncommon to find scientific literature studies about the impact that individual features, such as the practise of bundling,[6] or the role of informed and uninformed consumers[7], or the presence of switching costs[8], have in markets. It is also possible to find papers that study two of these features in simultaneous[9]. However, we did not find any literature that seeks to evaluate the simultaneous impact of all three features, as we attempt to do so in this thesis.

2) Bundling and Types of Bundling

The reason for the focus on bundling is quite simple: it is a factor that a hefty portion of papers in the area of industrial economics often neglect, instead only focusing on the role of individual goods or services provided by single or multiple firms. This is not particularly compatible with reality as bundling is often used (i.e. selling shaving cream in conjunction with sets of razor blades) or is even the norm in several markets (i.e. computers being sold with exclusively-compatible software).

The often-referenced, quintessential template for studying bundling by (Adams & Yellen, 1976), states that in the context of a monopoly bundling is a possible consequence of a monopolist's wish to charge to their consumers reservation costs[10] for each of their

[6] I.e. (Adams & Yellen, 1976).
[7] I.e. (Deck & Wilson, 2006).
[8] I.e. (Crawford & Cullen, 2007).
[9] I.e. (Aloysius et al., 2012).
[10] The maximum cost that a customer is willing to pay for a specific good or service.

products, and that bundling could cause oversupply or undersupply of certain goods, which could lead to the wrong people consuming each good.[11] This suggests that the practice of bundling can be detrimental to consumers.

This idea is supported by (Crawford & Cullen, 2007), who claim that in a scenario where consumers have heterogeneous preferences for multiple products, the application of bundling would punish them for their heterogeneous preferences. That would mean that while firms earn a greater profit through bundling, as opposed to individually selling their individual goods at individual prices, consumers are forced to purchase products that they have little or no interest in, resulting in a decline of consumer welfare.[12]

This concept was developed further by (Aloysius, Deck, & Farmer, 2012), who re-contextualized it in an non-monopolistic scenario with informed and uninformed consumers[13]. Within this scenario, they find that when bundling is combined with a high proportion of informed consumers, it leads to overly competitive prices, which helps both informed and uninformed consumers,[14] unlike in a monopoly (where bundling is usually seen as an anti-competitive tactic), This is supported by (Sun, 2014), who concluded that, if consumers have homogeneous preferences for the bundle, and if there is a high level of consumer information, bundling has one negative and two positive effects on consumer welfare.[15]

It is possible to distinguish two specific types of bundles, which should be made very clear: *"In (pure bundling), the services are available only in bundled form - they cannot be purchased separately. Mixed bundling, in contrast, enables the consumer either to purchase one or more of the services individually or to purchase the bundle.".*[16]

Pure bundles are more commonly used for goods and services that have a higher level of complementary between them (i.e.: shaving cream and razors), as it is usually assumed that consumers need one good in order to fully utilize the other. Mixed bundles are more commonly used for goods and services that do not have a high level of complementary – or even between neutral goods and services. An example of this would be the relationship between bread and water. As for goods and service that have a substitution effect between them (i.e.: butter and margarine), it would be considered counterproductive to form any sort of bundle (especially of the pure variety).

[11] (Adams & Yellen, 1976), p. 495.
[12] (Crawford & Cullen, 2007), p. 380.
[13] Both of these concepts will be explained below.
[14] (Aloysius et al., 2012), p. 669.
[15] (Sun, 2014), pp. 19-20.
[16] (Guiltinan, 1987) , p. 75.

The specific type of bundle that is used throughout our model will be thoroughly explained in the Methodology section of this paper.

3) Informed and Uninformed Consumers

Being titular concepts, these keywords are a crucial component in our model. Unlike the standard market analysis in this field, we do not assume that every single consumer is knowledgeable in terms of the prices practised by all providers in the market. Regardless of how big or small a particular market is, it is unrealistic for all potential consumers to always be aware of all prices present in that market.[17]

Within this scenario, an arbitrary fraction of consumers is informed, with the others being uninformed; or *"Departing from the standard setting of full information and responsive expectations for all users, we assume that some consumers (...) are informed about all subscription prices and hold responsive expectations, whereas the remaining consumers are uninformed about (...) subscription prices and hold passive expectations."*.[18] In other words, **uninformed consumers** join a random firm depending on their own reservation costs and expectations, while **informed consumers** tend to flock to the firm that sets the lowest price[19].

Regarding a market with uninformed consumers and with firms producing goods at homogenous costs and of homogenous quality, (Aloysius, Deck, & Farmer, 2012) state that, unlike a scenario of full information, the optimal strategy may not be as simple as to slightly undercut the prices set by the competition.[20]

Depending on the ratio of informed to uninformed consumers, in some situations (i.e.: if uninformed consumers greatly outnumber their informed counterparts) it could be more beneficial for a duopolist to set the Monopoly price, maximizing their profits from their unaware customers. Thus, it is possible for a firm to have a higher profit than its' homogeneous competitor(s), despite the former setting a higher price and the latter having a larger number of consumers.

[17] Assuming that it is not a monopoly.
[18] (Sun, 2014), p. 3.
[19] That is, assuming that the products that the firms produce are homogeneous.
[20] (Aloysius et al., 2012), p. 664.

Also according to (Sun, 2003), within a non-monopoly, a higher fraction of informed consumers makes bundling a more effective means to stimulate consumer demand. This is because an increased relative number of informed consumers will tighten price competition between firms.[21]

4) Search Costs

Uninformed consumers only know the prices of firms that they are subscribed to; however they are also aware that they can pay a cost (commonly known as the **Search Cost**) to gain knowledge of prices of other firms, thus becoming informed consumers in the process.

According to (Bakos, 1997), "*In differentiated markets, seller profits decrease smoothly as search costs are reduced. If search costs become low enough, buyers will look at all product offerings and will purchase the one best serving their needs (...). At the other extreme, very high search costs lead to search and allocation inefficiencies, and eventually cause the market to break down".*[22]

5) Switching Costs

According to (Klemperer, 1987), this factor can be seen as *"consumers who switch between different companies (being) penalized relative to those who remain with a single firm.".*[23]

Switching costs are defined by (Burnham, Frels, & Mahajan, 2003) as *"the one-time costs that customers associate with the process of switching from one provider to another. While switching costs must be associated with the switching process, they need not be incurred immediately upon switching. Furthermore, switching costs need not be limited to*

[21] (Sun, 2014), p. 3.
[22] (Bakos, 1997), p. 1690.
[23] (Klemperer, 1987), p. 376.

objective, 'economic' costs.".[24] Their role is, in a sense, to increase a firm's consumer retention.

They also define eight distinct facets of switching costs, which would be subsequently categorized under three clear-cut types of switching costs:

I) procedural (related to expenditure of time and effort),

II) financial (involving the loss of financially quantifiable resources),

III) relational (associated with psychological or emotional discomfort due to the loss of identity and the breaking of bonds).[25]

It is worth noting that much of (Burnham et al., 2003) relates to the heterogeneity and perceived complexity of the products of each firm. This is not the case in the model proposed in this thesis, where the services of both firms are homogeneous.[26]

6) Vendor Lock-In

Vendor lock-in can be seen as a possible direct consequence of switching costs. It is defined as *"consumers' decreased propensity to search and switch after an initial investment. Lock-in is driven by a preference to minimize immediate costs and an underweighting of the impact of future switching costs.".*[27] In short, switching costs can be so high that consumers that wish to switch from one provider to another know they would end up having large costs by switching, and so they stay with the original provider. In other words, as they are subject to high switching costs, consumers are essentially "locked-in" inside a firm with much higher prices.

This situation seems to be more noticeable, and therefore more studied, in the field of technology. (Zauberman, 2003) realized this and made a more generalized study of lock-in coupled with the inter-temporal dynamics of consumers. The proposed model from that paper studies multi-faceted aspects of this situation (mainly from the perspective of the consumer), from consumers' tendency to minimize present costs to their underestimation of future costs into their decisions, to the exaggeration of their own propensity to initiate a search for

[24] (Burnham et al., 2003), p 110.
[25] (Burnham et al., 2003), p. 112.
[26] This may translate into some of the switching costs types presented by (Zauberman, 2003), having diminished roles and impact in this thesis.
[27] (Zauberman, 2003), p. 405.

alternatives. In short, it emphasizes how easy it is a customer to become "locked-in" (even when it pertains to small investments), as well as the financial and psychological inter-temporal complexities that prevent them from escaping.

7) Profit Maximizing Nash Equilibrium

The all-encompassing definition of a Nash Equilibrium is described as *"an array of strategies, one for each player, such that no player has an incentive (in terms of improving his own payoff) to deviate from his part of the strategy array."*.[28]

Profit equilibrium, being a key concept of this thesis, is a type of Nash Equilibrium relevant to achieving profit through the choice of prices. The term "profit equilibrium" will be used throughout this thesis as meaning "the outcome of a profit maximizing Nash Equilibrium".

[28] (Kreps, 1990), p. 34.

Methodology

1) Introduction

The purpose of this section is to contextualize our model, describing the two Firms (framed through a timeline of events) and the services they provide, and explaining how the features of the market will be implemented.

1.1) Timeline of Events

The set of events described below thoroughly explains the basic timeline of events in our model. These events are assumed to be fixed, meaning that they will always occur regardless of the price choices of the **Firms**.

In **period** $t = 0$, **Firm 1** is established as the monopolist in the provision of Television services. This is because it is the first firm to create this service, and thus becomes the monopolist in a market with a high entry cost. The totality of consumers in the market (100 consumers) joins **Firm 1**.[29] Afterwards, no new consumers join the Television provision market.

Later on, in **period** $t = 1$, a new market is formed: Telephone provision services. **Firm 1**, spotting the opportunity to become the monopolist in yet another market, decides to enter. Simultaneously, however, a new **Firm** also enters the market. This other entrant is **Firm 2**. **Firms 1** and **2** therefore become non-cooperative duopolists in the market of Telephone service provision and begin competing with one another in this specific market. Any other potential entering firms, knowing about the intentions of these two large **Firms** to enter the market, opt to stay out. Now, the same 100 consumers of Firm 1's Television provision choose whether to subscribe to Firm 1 or Firm 2 for the provision of Telephone services.[30] After this occurs, no new consumers join the Telephone provision market.

[29] That is, assuming Firm 1 does not set a price so high that it exceeds the reservation cost of consumers. However, they will not opt to do so.
[30] Once again, assuming both Firms do not set prices so high that they exceed the reservation cost of consumers.

Soon afterwards, in **period $t = 2$**, **Firm 1** is still ambitious to be the sole provider in both markets. Although all Television consumers in the market have subscribed to them, many of these Television subscribers have also subscribed to **Firm 2**'s Telephone services. This is when **Firm 1** introduces their bundle, a package deal where consumers can purchase both **Firm 1**'s Television and Telephone services at a price lower than their combined individual prices. It is meant to be an incentive for **Firm 2**'s Telephone subscribers to desert their provider and join **Firm 1** in full.

During this time, **Firm 2** is not able to venture into the Television provision market for an unspecified reason. Let us suppose that this reason is a legal obstacle that prohibits them from entering said market until the beginning of $t = 3$. In other words, we assume that from the beginning of $t = 1$ until the beginning of $t = 3$, their entry cost into the Television provision market is valued as infinite ($+\infty$), a value that ensures they will not expand until the beginning of $t = 3$. At the beginning of $t = 3$, the value of the entry cost for this market will be reduced to zero for **Firm 2** only. For any possible entering firm, the same legal obstacle is applied to both markets until after the end of $t = 3$.

As described above, at the beginning of **$t = 3$**, **Firm 2** will enter the Television provision market, and consequentially create their own bundle to compete with **Firm 1** in both markets. Because of the already tight grip that **Firm 1** has on its monopoly, the high individual influence that both **Firms** exert over the duopoly and the aforementioned legal obstacle, the possibility of other entering firms joining either market before the end of **period $t = 3$** is null.

1.2) The Case Study

The two **Firms** will compete in a variation of a non-cooperative Bertrand pricing competition, lasting from period $t = 0$ and ending in $t = 3$. This competition will take into account certain arbitrary market variables, more specifically the monopoly prices of the individual services and bundles, the ratio of informed and uninformed consumers, and the value of search and switching costs that those consumers face. Their objective is to maximize their individual profits in the final period, $t = 3$.[31] In order to do so, both **Firms** are required

[31] In other words, this is a sub-game where both **Firms** will attempt to maximize their respective profits in $t = 3$.

to reach a profit maximizing Nash Equilibrium, which will necessitate the employment of backward induction.

2) Defining the Market and Competition

In short, we have so far presented:

At a particular point in time**, Firm 1** is the monopolist of a service (in this case, provision of Television services), as well as a duopolist in another market (Telephone service), and is bundling these two services together. **Firm 2** is the other duopolist in the second market mentioned.

The latter **Firm** will venture into the provision of Television services market and introduce their own bundle to directly compete with the monopolist in the two markets (three, if we count the bundling of the two services as a market in itself). No other firms will enter either market due to legal reasons.

It is worth noting that aside from the differing starting positions and divergences across the timeline detailed above, the two **Firms** and the quality of their respective services are homogeneous between them, and their marginal costs of operation and production are normalized to zero.

It is also worth noting that no consumer will want to subscribe to duplicate services. Consumers will not, for example, be simultaneously subscribed to both telephone services of **Firms 1** and **2**. Consumers can, however, cancel one or more of these services and acquire others (either from the same **Firm** or from their rival). In other words, a consumer who has both television and telephone provided by **Firm 1** can cancel these two services and purchase their bundle; a subscriber to **Firm 1**'s television service and **Firm 2**'s telephone service can cut their ties with **Firm 2** and subscribe to **Firm 1**'s Telephone service. The output from every **Firm** in every service will always match the demand of consumers.

The Bertrand price competition in our case study will span all four periods, with each period being divided into three phases:

1. The beginning of the period. New services are always introduced at this point in a period. Both **Firms** (or only **Firm 1** during $t = 0$) will set their optimal

prices, until both **Firms** find their optimal prices. These prices cannot be altered until the beginning of the next period.

2. The middle of the period. Consumers react to the current prices set by the **Firms**. Uninformed consumers can decide whether to become informed or not, and informed consumers can decide whether or not to switch services and/or providers.
3. The end of the period. Nothing occurs in this phase during periods $t = 0$ to $t = 2$. At the end of $t = 3$, however, the profits of both Firms during this period are tallied.

We will now present each of the concepts that will be introduced in this model, including their implementation and purpose within the model, and how they affect the interactions between the two **Firms**, between the **Firms** and consumers, and also how these concepts could affect one another.

3) Bundling

3.1) Implementation

Within the context of this thesis, the type of bundle that both **Firms** will practice will be **mixed.**

There are three major reasons for this:

i) Television and Telephone provision are neither complements nor substitutes. As non-substitutes, a bundle of these two services would not be counterproductive. But since they are also not complements, pairing them as part of a pure bundle would be considered horizontal tying.[32] This is not the

[32] Horizontal Tying is a practice wherein a customer wishes to purchase a good/service from a firm, but the firm requires its consumers to purchase an unrelated good/service in order to make the transaction. It is considered illegal in many countries (for example, in the United States, under the Clayton Antitrust Act of 1914, and Brazil, under "Lei Nº 12.529", dated November 30th, 2011). Exceptions to this would be if goods/services were, in fact, complementary (i.e.: a car dealership selling a car that requires brand-specific tyres), or if the tie-in good/service was not mandated for the consumer to purchase (for example, as part of a mixed bundle).

case if consumers have the option of not purchasing these goods or services separately (i.e.: as part of a mixed bundle).

ii) Some consumers have subscribed to **Firm 1** and **Firm 2** for different services. If for example, **Firm 1** introduced a pure bundle of their two services, what would happen to these consumers? As stated in the previous point, **Firm 1** cannot force consumers to purchase their bundle of non-complementary services without giving them an option to subscribe to individual services. If **Firm 1** introduced a mixed bundle, however, some of these consumers could abandon **Firm 2** and subscribe to **Firm 1**'s bundle.

iii) If both **Firms** did, in fact, implement pure bundles comprised of their two services, the bundle would be considered as the only service that each **Firm** provides. The model during the entirety of period $\boldsymbol{t = 3}$ would devolve into a single-good (or in this case, a single-service) price competition, negating the purpose of the bundling aspect altogether.

3.2) Purpose of Bundles

The reason for introducing of the bundling aspect is quite simple. A Bertrand pricing game between two firms in a single-good market (and also in two-good markets) has been extensively studied. The introduction of another good for each firm would not add much more complexity to the competition. But an additional "good" (or in this case a bundle of goods) that can make the purchase of other goods redundant for consumers, would form a new layer of depth to the "game". In this case, we now have a third "good" that serves as a substitute for the other two. This creates a dynamic where both **Firms** will set their prices having to not consider the prices of their competitor, but also how one **Firm's** prices for each service relate to each other.

4) Informed and Uninformed Consumers

The added concept of informed and uninformed consumers is intended to alter the more simplistic and traditional dynamics of Bertrand price competition, shifting both the

relationships between consumers and firms, and subsequently between competing firms themselves.

To better distinguish how these two types of consumers lead to shifts in these dynamics, let us suppose that, in a duopoly scenario, two homogenous firms (α and β) compete with one another, selling the same homogeneous good, in a market without switching costs, during a single period.

4.1) A Market of Informed Consumers

If the ratio of informed consumers to the whole market is 100% (meaning that all consumers are informed), the firms will engage in a classic Bertrand price competition; where the one that sets the lower price will attract all consumers, and the other will get no one. Should the prices be equal, the firms will share the market equally. Symmetrically between the two firms:

$$\pi_\alpha(P_\alpha) = \begin{cases} (P_\alpha - c_\alpha) * \left(Q(\boldsymbol{P_\alpha})\right) & if\ P_\alpha < P_\beta \\ (P_\alpha - c_\alpha) * \left(\frac{1}{2} Q(\boldsymbol{P_\alpha})\right) & if\ P_\alpha = P_\beta \\ 0 & if\ P_\alpha > P_\beta \end{cases} ; \quad (1)$$

Each firm would then attempt to undercut the price of the other by a small amount while still attempting to maintain profit. This is because each firm knows that if their rival sets a lower price for their good, informed consumers will immediately "jump ship" to their competitor.

This would lead to successive undercutting by each firm:

$$P_\alpha = \begin{cases} P_\beta - \varepsilon & , \quad if\ P_\beta > c_\alpha \\ Stay\ out\ of\ the\ market & , \quad if\ P_\beta < c_\alpha \end{cases} ; \quad (2)$$

with ε representing a very small amount.

This process would be repeated until the only possible Nash Equilibrium is reached: the firms would undercut each other until

$$P_\alpha = P_\beta = c \Longrightarrow \pi_\alpha = \pi_\beta = 0 \ , \quad (3)$$

s.t. $c = c_\alpha = c_\beta$

The individual profits of each firm would be null. This situation is known as the Bertrand Paradox, where in a market with two firms, none can get any profit.[33] This is because a high ratio of informed consumers increases price competition.[34] This situation is in fact a Nash Equilibrium, albeit a no-profit one.

4.2) A Market of Uninformed Consumers

Uninformed consumers in a market are generally normalized to have an equal presence in each firm. If a specific market has two firms, then each firm will attract half of all uninformed consumers. This is represented by the formula $\frac{\theta_u}{n}$, with $\boldsymbol{\theta_u}$ defining the ratio of uninformed consumers relative to all consumers in this market, and n representing the number of firms within the same market. $\theta_u \in [0;1] : \theta_u \in \mathbb{R}$

For this type of consumer, the price that they are willing to pay for a good (their reservation price) is higher than that of informed counterparts. Thus, the higher the ratio of uninformed consumers in the market, the higher firms will set their prices. In a scenario opposite to the one above, the optimal solution for both firms would be to set the monopoly price to maximize their profits. Assuming again that α and β have homogeneous costs:

$\pi_\alpha = \pi_\beta = \frac{\theta_u}{n} * Q(\boldsymbol{P^M}) * (P^M - c) \quad ; \quad (4)$

where $\theta_u = 1;\ n = 2;$ and $P_\alpha = P_\beta = P^M$.

In this scenario, unlike the previous one, the act of undercutting the price of the competitor is not an optimal strategy, as doing so would not secure a firm any new customers, and it would simply decrease its profit. The high ratio of uninformed consumers reduces the price competition. The scenario described is indeed a Nash Equilibrium.

4.3) A Market with Informed and Uninformed Consumers

Let us now take the previous example, but now both informed and uninformed consumers exist in the market.

[33] (Durlauf & Blume, 2008), p.476
[34] (Sun, 2014), p. 3.

This type of scenario will usually lead one firm into setting a lower price, attracting the informed portion of consumers, and their rival to set a higher price in order to maximize the profits from their uninformed consumers.

One firm would set $P^M - \varepsilon$ and get all informed consumers and half of the uninformed ones. The other would set P^M and get the remaining half of the uninformed consumers. This results in the first firm having more customers, but profits less from individual consumers; while the second profits more from individual consumers but has less customers. Equilibrium is given as:

$$\pi_\alpha = \pi_\beta \Leftrightarrow \left(\frac{\theta_u}{2} + \theta_i\right) Q(\boldsymbol{P^M} - \boldsymbol{\varepsilon})(P^M - \varepsilon - c) = \frac{\theta_u}{2} Q(\boldsymbol{P^M})(P^M - c) \ ; \ (5)$$

where $\boldsymbol{\theta_i}$ represents the ratio of informed consumers and is calculated as $\theta_i \equiv 1 - \theta_u$.

The higher the value of θ_i, the lower $(P^M - \varepsilon)$ will be and thus, the higher ε will be. This is because, in the above equation, the respective profits of both firms are equal, in order to maintain equilibrium.[35] By attempting to further undercut β, α would only decrease their profits since it would already have all the informed consumers on their side and would not gain any new consumers.

4.4) Purpose of Informed and Uninformed Consumers

Several studies within the area of microeconomics assume that all consumers are informed. As stated above, this kind of consumer base would always lead to a classic Bertrand pricing competition between two firms and harshly limit the possible strategies of each firm. The exact opposite scenario (if all consumers were uninformed) would be equally limiting for both firms, as they would simply set the monopoly price.

This is why the introduction of and distinction between informed and uninformed consumers matter in this thesis - a market that has both types of consumers would require more dynamic and complex strategies from both competitors.

[35] Both firms having equal profit is not a pre-requisite for profit equilibrium. This is only applied in very specific cases (i.e. if both firms, as well as their goods, are absolutely homogeneous in all aspects).

5) Bundling with Informed and Uninformed Consumers

5.1) Pricing and Profits

In this sub-section, we return to our case study for period $t = 3$. **Firms 1** and **2** are at this point providing services **A** (Television provision), **B** (Telephone provision), and **C** (bundle of A and B).

$\boldsymbol{P_{k,j,t}}$ represents the price charged by **Firm** $\boldsymbol{k}$ ($k = \{\mathbf{1}; \mathbf{2}\}$) for service $j = \{A; B; C\}$, during period $t = \{0; 1; 2; 3\}$.

$\boldsymbol{\omega_i}$ is the number of informed consumers within the market, and $\boldsymbol{\omega_u}$ is the number of their uninformed counterparts.

$\boldsymbol{\omega_{i,k,j,t}}$ is the number of informed consumers that subscribe to **Firm** $\boldsymbol{k}$*'s* good j, during period t. I.e.: $\boldsymbol{\omega_{i,1,A,t=3}}$ is the number of informed consumers that subscribe to **Firm 1**'s A during period $t = 3$.

$$\omega_{i,1,t} = \omega_{i,1,A,t} + \omega_{i,1,B,t} + \omega_{i,1,C,t}; \quad (6)$$

$$\omega_{i,A,t} = \omega_{i,1,A,t} + \omega_{i,2,A,t}$$

The same principle is also applied to uninformed consumers, although with exceptions to three specific services: due to the nature of informed and uninformed consumers, and the timeline of events relating to our model, $\omega_{u,1,C,t} \equiv \omega_{u,2,A,t} \equiv \omega_{u,2,C,t} \equiv 0$.

θ_i is the ratio of informed consumers within the market, and θ_u is the ratio of uninformed consumers within the market.

$\theta_{i,t} + \theta_{u,t} \equiv 1;$

where $\theta_{i,t}\,; \theta_{u,t} \in [0; 1]$: $\theta_{i,t}\,; \theta_{u,t} \in \mathbb{R}$

$$\theta_{i,t} = \frac{\omega_{i,t}}{\omega_{i,t}+\omega_{u,t}} \implies \theta_{i,k,j,t} = \frac{\omega_{i,k,j,t}}{\omega_{i,k,j,t}+\omega_{u,k,j,t}} \quad (7)$$

$$\theta_{u,t} = \frac{\omega_{u,t}}{\omega_{i,t}+\omega_{u,t}} \implies \theta_{u,k,j,t} = \frac{\omega_{u,k,j,t}}{\omega_{i,k,j,t}+\omega_{u,k,j,t}}$$

Likewise,

$\theta_{u,1,C,t} \equiv \theta_{u,2,A,t} \equiv \theta_{u,2,C,t} \equiv 0$

5.2) Consumer Overlaps

It is very important to stress that the sub-sections of consumers described above are not all independent of one another. After the formation of the service B market in period $t = 1$, all $\omega_{1,A,t=1}$ will acquire service B from either **Firm 1** or **2**. This means that there will be overlaps of consumer bases. For instance, a consumer can simultaneously be part of $\omega_{1,A,t}$ and of $\omega_{1,B,t}$ or simultaneously part of $\omega_{1,A,t}$ and of $\omega_{2,B,t}$. The eventual expansion of **Firm 2** at the beginning of period $t = 3$ could further lead to more overlaps, specifically for $\omega_{2,A,t}$ with $\omega_{2,B,t}$ and for $\omega_{2,A,t}$ with $\omega_{1,B,t}$.

Note that since consumers do not want to have repeated services, there cannot be any consumer overlap between the same services for different Firms. Nor can there be any consumer overlap between any of the bundles and any other service. This is because, as established previously, bundles act as substitutes to any other service in the market of the case study.

If we denote G = {A;B}, we have:

$$\left(\theta_{k,G,t}\right) \cap \left(\theta_{k,G-1,t}\right) + \left(\theta_{k,G,t}\right) \cap \left(\theta_{k-1,G-1,t}\right) \equiv 1; \qquad (8)$$

where $\left(\theta_{k,G,t}\right) \cap \left(\theta_{k,G-1,t}\right)$; $\left(\theta_{k,G,t}\right) \cap \left(\theta_{k-1,G-1,t}\right) \in [0; 1]$

Also,

$$\left(\theta_{k,G,t}\right) \cap \left(\theta_{k-1,G,t}\right) \equiv 0 ; \qquad (9)$$

$$\left(\theta_{k,C,t}\right) \cap \left(\theta_{k,G,t}\right) \equiv 0 ;$$

$$\left(\theta_{k,C,t}\right) \cap \left(\theta_{k-1,G,t}\right) \equiv 0 ;$$

$$\left(\theta_{k,C,t}\right) \cap \left(\theta_{k-1,C,t}\right) \equiv 0.$$

6) Search Costs and Switching Costs

6.1) Search Costs

As previously established, uninformed consumers can pay a charge, known as the Search Cost to find out the prices set by other firms in the markets which they are not subscribed to. It needs to be stated that search costs are not necessarily of a monetary nature;

they can also be associated with the time, effort, patience, *et cetera*; that they will lose in acquiring this information.

If an uninformed consumer believes, according to their expectations, that another firm can provide the same service at a lower price, they will be tempted to learn how much that firm is charging. If said price is indeed lower, they would leave their current provider and subscribe to the competitor.

6.1.1) Example

Returning to one of the examples of firms α and β where uninformed consumers are present in the market, uninformed consumers of α would be constantly faced with three choices:

$$\begin{cases} pay\ search\ cost\ and\ become\ informed, & if\ P_\alpha > E(P_\beta) + c_s \\ not\ pay\ search\ cost\ and\ remain\ uninformed, & if\ P_\alpha < E(P_\beta) + c_s \\ be\ indifferent, & if\ P_\alpha = E(P_\beta) + c_s \end{cases}; \quad (10)$$

where $\boldsymbol{c_s}$ represents search costs.

The uninformed consumers of β would be faced with the same, albeit symmetrical choices.

6.1.2) Implementation

In our case study, consumers can be subscribed to different services provided by different **Firms**, and uninformed consumers are only aware of the prices of services that they are subscribed to. So, when they pay the search cost, they become aware of all the prices practised by the two **Firms**. Also note that any possible monetary facets of the search cost that the consumers pay are not being paid to either of the **Firms**.

6.2) Search Costs and Expectations – "Stretching the Rope"

As stated before, in a price competition between two homogeneous firms selling homogeneous good, informed consumers will flock to the one that sets the lower price. The other firm will attract half of the uninformed consumers.

So, if one firm were to increase their prices, a certain percentage of their uninformed consumers (the ones with lower expectations) would pay the search cost and become informed.

6.2.1) Definition and Basic Implementation

For our case study, this results in the establishment of a linear proportion between the price increases of a **Firm**, and the ratio of uninformed consumers that would pay the search cost, and consequently be faced with the choice of cutting ties with said **Firm** or not.

Uninformed consumers will be uniformly distributed by their expectations. The lower the expectations of a consumer, the more easily they will be tempted to pay the search cost in response to smaller price increases.

$P^o_{k,j}$: price of comparison that **Firm** $\boldsymbol{k}$ charged for service j when uninformed consumers joined. Its initial default price is the original price of said service by said **Firm**

$P^o_{k,j}\left(P_{k,j,t}\right)$: the current price of comparison of future prices of the same service by the same **Firm**

$P^N_{k,j,t+m}(P_{k,j,t+m})$: price that **Firm** $\boldsymbol{k}$ charges for service j in a future period

P^M: Monopoly price

$\varphi_{k,j,t+m}$: the percentage of uninformed consumers of service j from **Firm** $\boldsymbol{k}$ that will pay search costs when $P_{k,j,t+m}$ is set. $\varphi_{k,j,t+m} \in [0;1] : \varphi_{k,j,t+m} \in \mathbb{R}$.

$$\begin{cases} \varphi_{k,j,t+m} = \frac{P^N_{k,j,t+m}(P_{k,j,t+m}) - P^o_{k,j}\left(P_{k,j,t}\right)}{P^M - P^o_{k,j}\left(P_{k,j,t}\right)}, & if \quad P^N_{k,j,t+1}(P_{k,j,t+1}) > P^o_{k,j}\left(P_{k,j,t}\right) \\ \varphi_{k,j,t+m} = 0\,, & if \quad P^N_{k,j,t+1}(P_{k,j,t+1}) \leq P^o_{k,j}\left(P_{k,j,t}\right) \end{cases} \tag{11}$$

If $P^N_{k,j,t+m}(P_{k,j,t+m}) > P^o_{k,j}\left(P_{k,j,t}\right)$ occurs, then $P^o_{k,j} = P^o_{k,j}(P_{k,j,t+m})$. In other words, if a Firm sets a new price that is higher than the price of comparison, the new price becomes the current price of comparison to be compared with all future prices.

The more a **Firm** increases their price and the more it approaches the Monopoly price of its respective service, the larger the percentage of uninformed consumers that will pay the search cost and become informed, and thus the larger will the number of consumers a **Firm** risks losing to their competitor.

Additionally, whenever a Firm utilizes this mechanism (setting a new price higher than the original), the new price that they have set is thereafter declared the original price to be compared with their future prices.

6.2.2) Implementation within our Model

Within our model, this mechanism will only be applied to services that can have uninformed consumers, these being 1A, 1B, and 2B. In other words, it is only applied to uninformed consumers belonging to the overlaps $(\omega_{1,A} \cap \omega_{1,B})$ and $(\omega_{1,A} \cap \omega_{2,B})$.

Thus, any increase of $P_{1,B}^{N} > P_{1,B}^{o}$ (assuming $P_{1,A}$ and $P_{2,B}$ remain unaltered or lower than their respective original prices) will inevitably alter the uninformed-to-informed consumer ratios in $\omega_{1,B}$ and $(\omega_{1,A} \cap \omega_{1,B})$, meaning that will alter both $\downarrow \omega_{1,u,B} \Longrightarrow\uparrow \omega_{1,i,B}$ and $\downarrow \omega_{1,u,A} \Longrightarrow\uparrow \omega_{1,i,A}$. More specifically, this decrease will be

$$\omega_{u,1,B,t+m} * \varphi_{1,B,t+m} = \omega_{u,1,B,t+m} * \frac{P_{1,B}^{N}(P_{1,B,t+m}) - P_{1,B}^{o}}{P_{B}^{M} - P_{1,B}^{o}} \quad (12)$$

for the consumers of 1B, and

$$\left(\omega_{u,1,A,t+m} \cap \omega_{u,1,B,t+m}\right) * \varphi_{1,B,t+m} = \left(\omega_{u,1,A,t+m} \cap \omega_{u,1,B,t+m}\right) * \frac{P_{1,B}^{N}(P_{1,B,t+m}) - P_{1,B}^{o}}{P_{B}^{M} - P_{1,B}^{o}} \quad (13)$$

for the overlapped consumers of 1 A and 1 B.

Similar effects will occur in a symmetrical way when discussing a scenario where $P_{2,B,t+m}^{N}(P_{2,B,t+m}) > P_{2,B}^{o}$ (once again assuming $P_{1,A}$ and $P_{1,B}$ remain unaltered or lower than their respective original prices), altering the uninformed-to-informed consumer ratios in $\omega_{2,B,t+m}$ and $(\omega_{1,A,t+m} \cap \omega_{2,B,t+m})$, with the decrease of each ratio being dependent on $\varphi_{2,B,t+m}$.

What if $P_{1,A}$ is the only one to increase beyond its original price?

The answer for this is quite simple: an increase of $P_{1,A}^{N} > P_{1,A}^{o}$ will lead to the alterations of the uninformed-to-informed ratios of $(\omega_{1,A,t+m} \cap \omega_{1,B,t+m})$ as well as $(\omega_{1,A,t+m} \cap \omega_{2,B,t+m})$, with the decrease of each ratio being dependent on $\varphi_{1,A,t+m}$.

But what if more than one of these three prices increase beyond their respective original prices?

If both $P_{1,B}$ and $P_{2,B}$ increase beyond their respective original prices, it will lead to the decrease of $\omega_{1,u,B,t+m}$ and $(\omega_{u,1,A,t+m} \cap \omega_{u,1,B,t+m})$ through the effect of $\varphi_{1,B,t+m}$, as well as the decrease of $\omega_{u,2,Bt+m}$ and $(\omega_{u,2,A,t+m} \cap \omega_{u,2,B,t+m})$ through the effect of $\varphi_{2,B,t+m}$.

However, if for example, $P_{1,A}$ and $P_{2,B}$ both increase beyond their respective original prices, $\omega_{u,1,B,t+m}$ and $(\omega_{u,1,A,t+m} \cap \omega_{u,1,B,t+m})$ will both decrease through the effect of $\varphi_{1,A,t+m}$.

Meanwhile, the inevitable decreases of $\omega_{2,u,B,t+m}$ and $(\omega_{u,1,A,t+m} \cap \omega_{u,1,B,t+m})$ will be dependent on whichever price has relatively increase the most towards the monopoly price, meaning whichever value among $\varphi_{1,A,t+m}$ and $\varphi_{2,B,t+m}$ is the highest. Similar effects occur symmetrically when discussing the increases of $P_{1,A}$ and $P_{1,B}$.

If $P^N_{1,A,t+m} > P^o_{1,A}$, $P^N_{1,B,t+m} > P^o_{1,B}$ and $P^N_{2,B,t+m} > P^o_{2,B}$, then $\omega_{1,u,B,t+m}$ and $(\omega_{1,u,A,t+m} \cap \omega_{1,u,B,t+m})$ will both decrease through the effect of the highest value between $\varphi_{1,A,t+m}$ and $\varphi_{1,B,t+m}$; while $\omega_{u,2,B,t+m}$ and $(\omega_{u,1,A,t+m} \cap \omega_{u,2,B,t+m})$ will decrease through the effect of the highest value between $\varphi_{1,A,t+m}$ and $\varphi_{2,B,t+m}$.

6.2.3) Purpose of the "Stretch the Rope" mechanism

This factor will implement another layer of strategy for the pricing competition, especially in the **Firm** that would set the higher price and focus on the uninformed section of consumers. Both **Firms** will be placed in a situation where they would need to be cautious of how much they are willing to "stretch" the proverbial "rope" (how much they can increase their prices) until it "breaks" (until enough newly-informed consumers abandon them in favour of their competitor, decreasing their profits).

6.3) Switching Costs

6.3.1) Implementation

Switching costs paid by consumers can be separated into two facets: **exit costs**, which are the costs associated with leaving their current provider; and **entry costs**, the ones associated with joining a new provider.

$\boldsymbol{c_{st}}$: switching costs

$\boldsymbol{c_{en}}$: entry costs

$\boldsymbol{c_{ex}}$: exit costs

$$c_s = c_{en} + c_{ex} \text{ ; (14)}$$

As with many other aspects of our case study, both the exit and the entry costs associated with entering and exiting the two **Firms** are homogeneous.

Neither **Firm** can implement artificial exit costs (meaning any costs other than the ones intrinsic to the exit of a consumer) to hinder or outright prevent their consumers from leaving. Besides not being able to create artificial exit costs, neither **Firm** can charge exit or entry fees to exiting or entering customers. Aside from gaining or losing consumers (therefore gaining or losing profit), **Firms** neither directly gain nor lose anything if a consumer abandons or joins them.

6.3.2) Example – Vendor Lock-In

Let us now assume that the two homogeneous firms from our previous example (α and β) provide a single homogeneous service, but are now in a market with only informed consumers, and they have set exactly the same prices as each other. They share the market, with each firm providing half of the consumers. Now, let us suppose that α increases their price by ε. In a situation consistent with the previous examples of pricing competition, all the consumers in α would leave and join β, due to the latter boasting a lower price. However, if we introduce entry and exit costs in this scenario, this may not always be the case.

$$\begin{cases} consumer\ leaves\ \alpha\ and\ joins\ \beta & if \quad P_\alpha + \varepsilon > P_\beta + c_{ex} + c_{en} \\ consumer\ remains\ in\ \alpha & if \quad P_\alpha + \varepsilon < P_\beta + c_{ex} + c_{en} \\ consumer\ is\ indifferent & if \quad P_\alpha + \varepsilon = P_\beta + c_{ex} + c_{en} \end{cases} , (15)$$

, where $\boldsymbol{c_{ex}}$ represents the cost of exiting one firm and $\boldsymbol{c_{en}}$ the cost of entering the other.

In the case where $P_\alpha + \varepsilon < P_\beta + c_{ex} + c_{en}$, or since $P_\alpha = P_\beta$; $\varepsilon < c_{ex} + c_{en}$, firm α, despite setting the higher price in a market comprised of exclusively informed consumers, still maintains all of their consumers. This is because for consumers, the cost of switching from α to β out-weights the difference between $P_\alpha + \varepsilon$ and P_β. This situation, where informed consumers are purchasing from the firm that sells at a higher price due to the existence of switching costs, is an example of ***vendor lock-in***.[36]

If β were to set a new price ($P_\beta - \tau$), low enough such that

$$P_\alpha + \varepsilon > P_\beta - \tau + c_{ex} + c_{en} \quad , (16)$$

or since $P_\alpha = P_\beta$,

[36] (Zauberman, 2003), p. 405.

$\varepsilon + \tau > c_{ex} + c_{en}$,

all the consumers of α would pay the switching costs and join β.

Once again, this would lead to successive undercutting from each firm, which would eventually lead to a very similar situation to a Bertrand Paradox. While this is not a no-profit Nash Equilibrium, the maximum price for one firm can only be as high as the switching costs required for their consumers not to leave.

6.3.3) Purpose of Switching Costs

With the existence of switching costs, the practice of slightly undercutting the competitor in order to attract their informed consumers can be prejudicial to a firm, as they would only decrease their own profit. An undercut large enough to make it worthwhile for consumers to switch would be needed to attract these consumers.

On the other hand, the act of setting a slightly higher price than the competitor, which in a normal Bertrand pricing competition would be considered nonsensical, is now a viable move for a firm to make. In this scenario, a firm that wishes to undercut their competitor would want the price difference between the two firms to be significant enough for them to attract new consumers. Meanwhile, the other firm would want that difference to be as large as possible, provided that it is still lower than the value of the switching costs so as to keep their consumers locked-in.

7) Switching with Search Costs

We can now merge the concepts of search and switching costs together in the following manner:

All consumers (informed and uninformed) are aware that if they want to switch providers, they will be required to pay switching costs. Taking into account the market in the example above, we already know that informed consumers will leave α for β if:

$P_\alpha + \varepsilon > P_\beta + c_{en} + c_{ex}$; (17)

$s.t.\ P_\alpha + \varepsilon > P_\beta$

Also taking into account the example provided in the sub-section on search costs, we know that uninformed consumers will only pay the search cost if they believe that the price of the competitors' respective service is lower than the one from charged current provider.

Now, an uninformed consumer of firm α will be faced with:

$$\begin{cases} pays\ search\ cost & if \quad P_\alpha + \varepsilon > E(P_\beta) + c_s + c_{ex} + c_{en} \\ does\ not\ pay\ search\ cost & if \quad P_\alpha + \varepsilon < E(P_\beta) + c_s + c_{ex} + c_{en} \\ indifference & if \quad P_\alpha + \varepsilon = P_\beta + c_s + c_{ex} + c_{en} \end{cases} ; (18)$$

By paying the search cost, an uninformed consumer becomes an informed one, and will subsequently be faced with the previous dilemma:

$$\begin{cases} consumer\ leaves\ \alpha\ and\ joins\ \beta & if \quad P_\alpha + \varepsilon > P_\beta + c_{ex} + c_{en} \\ consumer\ remains\ in\ \alpha & if \quad P_\alpha + \varepsilon < P_\beta + c_{ex} + c_{en} \\ consumer\ is\ indifferent & if \quad P_\alpha + \varepsilon = P_\beta + c_{ex} + c_{en} \end{cases} ; (19)$$

8) Internal Switching Costs

8.1) Definition

In a previous section, we discussed the role of switching costs in a market where two firms compete in selling one good.

In the same vein as when breaking down switching costs into entry costs and exit costs, we can further break down each type of cost into joining **Firms** and acquiring services (for entry costs), as well as leaving **Firms** and cancelling services (for exit costs).

We can contextualize this within the narrative of the model as the **Firms** having to manage the creation and deletion, as well as the addition and cancellation of services in their database of client files for each consumer that wishes to switch **Firms**, switch services, or variations of both.

8.2) Basic Implementation

Let us now return to our case study in period $t = 3$, where **Firms 1** and **2** each provide the services A, B and C, where C is a bundle of A and B, and therefore a substitute for both A and B. A customer of either **Firm** that already has A and B will not want C unless they relinquish their right to have A and B. Similarly, as long as they have C, no consumer would want A and/or B in addition to C.

8.2.1) Entry Costs

When a consumer that is new to **Firm k** wants to join them, they would need to go through **Firm k**'s bureaucratic process to create a client file. This serves as a part of the entry costs.

After that same consumer creates their client file, they can "add" service(s) by going through an additional bureaucratic process for each new service that they want. If a new consumer wants to have both A and B services, they would need to pay an entry cost create a client file, and pay another entry cost to individually "add" A and B.

8.2.2) Exit Costs

Conversely, consumers can also cancel one or more of the services they have from **Firm k**. Consumers would need to pay an exit cost if they are subscribers to A and B and desire to cancel B, for example. Similarly to the case above, consumers must pay individual exit costs to cancel multiple services.

If a consumer cancels all of their services from **Firm k** to join their competitor, they will need to delete their client file. They must pay the individual exit costs of each service they have cancelled, as well as pay the exit cost of deleting the file.

The deletion process does not apply if a consumer cancels all of their services from **Firm k**, but adds new services from the same **Firm**. For example, a consumer of **Firm k** can cancel A and B, and acquire C without deleting their client file. This is also the case if a subscriber of k's A and $k-1$'s B simultaneously switches to k's B and $k-1$'s A, where they will not be required to create or delete their client files from either **Firm**.

8.3) Implementation and Denotation of Variables

Within the narrative of the case study, the reason for the deletions is a legal requirement that prohibits the **Firms** from retaining client files of former consumers, and each file must be deleted by its respective client. To ensure this, each **Firm** implements a policy whereby consumers who are no longer subscribed to their services are required to delete their files, under the penalty of a hefty fine valued as infinite ($+\infty$).

We will define the variables that represent the actions of consumers as such:

$\boldsymbol{c_{en,w,x}}$: the entry cost of joining **Firm(s)** $\boldsymbol{k}$ and acquiring service(s) j

$\boldsymbol{c_{cr}}$: the entry cost of creating one client file – joining an individual Firm

$\boldsymbol{c_{ad}}$: the entry cost of adding an individual service to an existing client file.

$\boldsymbol{w}$: number of **Firms** that a client joins (number of client files a consumer creates). $w \in [0;1] : w \in \mathbb{N}$

$\boldsymbol{x}$: number of services that a consumer wishes to add from a **Firm** where they have a client file. $x \in [1;2] : x \in \mathbb{N}$

$\boldsymbol{c_{en,w,x}} \equiv wc_{cr} + xc_{ad}$

$\boldsymbol{c_{ex,y,z}}$: the exit cost of leaving y **Firm(s)** and cancelling z service(s)

$\boldsymbol{c_{de}}$: the exit cost of deleting one client file – leaving an individual **Firm**.

$\boldsymbol{c_{re}}$: the exit cost of removing an individual service to an existing client file.

$\boldsymbol{y}$: number of **Firms** that a client leaves (number of client files a consumer deletes). $y \in [0;1] : y \in \mathbb{N}$

$\boldsymbol{z}$: number of services that a consumer wishes to cancel from a **Firm** where they have a client file. $z \in [1;2] : z \in \mathbb{N}$

$\boldsymbol{c_{ex,y,z}} \equiv yc_{de} + zc_{re}$

$$(wc_{cr} + xc_{ad}) + (yc_{de} + zc_{re}) \equiv c_{en,w,x} + c_{ex,y,z} \equiv c_{st,w,x,y,z} \; ; \quad (20)$$

Thus, $c_{st,w,x,y,z}$ represents the total switching costs of a consumer that joins w **Firms**, adds x services, leaves y **Firms**, and cancels z services.

For example, a subscriber to **Firm** $\boldsymbol{k}$'s A and B who wishes to switch to **Firm** $\boldsymbol{k-1}$'s C would have their switching cost valued as:

$$c_{st,1,1,1,2} = c_{en,1,1} + c_{ex,1,2} = 1c_{cr} + 1c_{ad} + 1c_{de} + 2c_{re} \; ; \quad (21)$$

Meanwhile, the cost for a subscriber of **Firm** $\boldsymbol{k}$'s C who wants to switch to **Firm** $\boldsymbol{k}$'s A and B would be:

$$c_{st,0,2,0,1} = c_{en,0,2} + c_{ex,0,1} = 2c_{ad} + 1c_{re} \; ; \quad (22)$$

8.4) Purpose of Internal Switching Costs

The role of this expansion of switching costs is to further emphasize the effects prevailing after the introduction of the bundling aspect: they are meant to increase the caution that each **Firm** should use when setting the price of one of their services relative to the prices of their other services.

9) Conclusion of Methodology

In sum, we can say that the two **Firms** compete in a variation of a non-cooperative Bertrand pricing competition, lasting from period $t = 0$ to the end of $t = 3$. Their objective is to set their prices across all four periods in order to maximize their individual profits in the final period, $t = 3$; this requires them to reach a profit maximizing Nash Equilibrium through the employment of backward induction.

π_k represents the profits of **Firm** $\boldsymbol{k}$ at the end of period $t = 3$, with the respective profit for each **Firm** being given as:

$$\pi_1 = P_{1,A,t=3}(\omega_{i,1,A,t=3} + \omega_{u,1,A,t=3}) + P_{1,B,t=3}(\omega_{i,1,B,t=3} + \omega_{u,1,Bt=3}) + P_{1,C,t=3}(\omega_{i,1,C,t=3})$$

$$\pi_2 = P_{2A,t=3}(\omega_{i,2,A,t=3}) + P_{2,B,t=3}(\omega_{i,2,B,t=3} + \omega_{u,2,B,t=3}) + P_{2,C,t=3}(\omega_{i,2,C,t=3})\ ; \quad (23)$$

The market in question has a fixed number of consumers (100), all present in $t = 0$, and in this competition the two **Firms** must take into account certain arbitrary market variables, more specifically the monopoly prices of the individual services and bundles, the initial ratio of informed-to-uninformed consumers (although the actions of the Firms can lead to an increase of this ratio), and the value of search and switching costs that those consumers face.

The behaviour of informed consumers is consistent across all four periods, although will slight deviations in some of them.

In $t = 0$, informed consumers will join **Firm 1** and acquire A if and only if $P_{1,A,t=0} + c_{st,1,1,0,0} \leq P_A^M$. Otherwise, they will not join either **Firm** for the entire duration of the timeline.

In $t = 1$, they will want the combinations of services that will result in the lowest possible total cost[37] between $P_{1,B,t=1} + c_{st,0,1,0,0}$ or $P_{2,B,t=1} + c_{st,1,1,0,0}$, provided that the lowest value of these is still equal to or lower than P_B^M.

In $t = 2$ and $t = 3$, they will want to be subscribed to the combination of individual services or bundle that requires the lowest total cost in that respective period, provided that these costs respect the limits of the monopoly prices of each service.

[37] Meaning the price of the services plus the switching costs necessary to acquire them.

Uninformed consumers, will act exactly like their informed counterparts in $t = 0$, but afterwards their behaviour will diverge from the one displayed by informed consumers.

In $t = 1$ they will randomly join a **Firm** if their total cost is lower than the monopoly price. We normalized this behaviour as being each **Firm** getting half of the total uninformed consumers. If one Firm sets a price where the total cost is higher than the monopoly price, they will get no uninformed consumers, and these consumers will go to their competitor instead. If both **Firms** set prices where the total cost is higher than the monopoly price, neither **Firm** gets uninformed consumers.

In $t = 2$ and $t = 3$, uninformed consumers will remain in the situation they chose in $t = 1$. They will consider the option of switching services and/or providers if they become informed, which requires for at least one of the **Firms** to trigger the "Stretch the Rope" mechanism.

It is necessary to stress that in this model, **Firms** perform their actions in accordance to their desired outcome in $t = 3$. They do not set prices without considering future consequences. Consumers, however, act in the moment without considering future outcomes or consequences.

Main Body

1) Introduction

For the purpose of testing profit maximizing Nash Equilibrium within our model across several different scenarios, we created two custom-built .m files (or "scripts") for programs such as MATLAB©.

We defined a scenario called the "Benchmark Case" to serve as a point of comparison for other scenarios called "experiments". In each experiment, we tweaked variables such as the monopoly prices, the values of switching costs, and the informed-to-uninformed ratios; and subsequently drew our conclusions for each of them.

Therefore, we have built two scripts – "Parameters" and "Equilibrium". These two scripts enables us to set specific parameters scenarios, and find a possible profit maximizing Nash Equilibrium (if there is one) in accordance with the set parameters, to test several distinct scenarios.

1.1) "Parameters"

The first script is called "Parameters". As the name implies, here is where we set the values for our parameters (specifically the *monopoly price* of each service, all the possible *switching costs*, and the *number of informed and uninformed consumers*). These values can be altered for the study of different scenarios.

We also defined static values for each of the fourteen $P_{k,j,t}$. For the sake of simplicity, we defined the all fourteen $P_{k,j,t}$ to be random values between zero and the services' respective monopoly price. The reason for implementing these static values is made clear when describing the second file.

1.2) "Equilibrium"

Our second script, "Equilibrium", initiates a process that attempts to find a profit maximizing Nash Equilibrium while taking into account the values inputted in "Parameters".

In the simulations performed with this script, both Firms and consumers will act in their respective manners, which were previously described.[38]

The main mechanism behind this script, in short, is to individually target each of the fourteen $P_{k,j,t}$, and alter any given $P_{k,j,t}$ to a value which leads to the highest profit for the respective k of that specific $P_{k,j,t}$. This process then continuously loops itself until profit maximizing Nash Equilibrium is achieved.

A more detailed explanation of this mechanism is described below:

- The script calculates the profits of **Firm 1** (π_1) and **Firm 2** (π_2)[39], taking into account the arbitrary values attributed in "Parameters", as well as the randomized prices from the same file.
- The script targets the first of the fourteen $P_{k,j,t}$, which are displayed in a pre-arranged order[40]. The first of the fourteen $P_{k,j,t}$ (in this case, $P_{1,A,t=0}$) is re-defined from a static value to a dynamic variable between zero and the value of the services' monopoly price (in this case, P_A^M), divided into increments of 0.01.
- The program then calculates which value should the targeted $P_{k,j,t}$ (in this case, $P_{1,A,t=0}$) be in order to maximize π_k (in this case, π_1).[41] The targeted $P_{k,j,t}$ becomes a static value once again.
- The program yields three values: the maximized $P_{k,j,t}$-dependent π_k (in this case, π_1), the subsequent $P_{k,j,t}$-dependent π_{k-1} (in this case, π_2), and the optimized $P_{k,j,t}$ (in this case, $P_{1,A,t=0}$).
- The script then performs the previous three steps, but now targeting the next $P_{k,j,t}$ (i.e.: $P_{1,A,t=1}$) in the order. This process is continued until all fourteen $P_{k,j,t}$ have been optimized. A sequential optimization of all fourteen prices is known as an *iteration*.

[38] pp. 30-31.

[39] π_1 and π_2 refer to the profits of Firm 1 and Firm 2 exclusively obtained in period $t = 3$. While profits from previous periods exist, the profits obtained in $t = 3$ represent the long-term profits for both **Firms**. Thus, they will be the only one we will take into account.

[40] After many tests in which the script targets the prices in different orders, the obtained results do not seem to differ in any way. We therefore concluded that the order at which the fourteen prices are targeted is inconsequential. For clarification, the sequence utilized in our script was $P_{1,A,t=0}$, $P_{1,A,t=1}$, $P_{1,B,t=1}$, $P_{2,B,t=1}$, $P_{1,A,t=2}$, $P_{1,B,t=2}$, $P_{1,C,t=2}$, $P_{2,B,t=2}$, $P_{1,A,t=3}$, $P_{1,B,t=3}$, $P_{1,C,t=3}$, $P_{2,A,t=3}$, $P_{2,B,t=3}$, $P_{2,C,t=3}$.

[41] For each $P_{k,j,t}$, their optimization process can lead to an increase of π_k through three factors: (1) – the alteration of the positions of consumers in current and future periods. (2) – the alteration of the informed-to-uninformed consumer ratio in services where the usage of the "Stretch the Rope" mechanism is possible. (3) – the optimized price is a $P_{k,j,t=3}$, and the increase of π_k is unrelated to factors (1) and (2).

- The previous four steps are then repeated in a continuous loop, with each *iteration* of this loop providing fourteen price-dependent π_k, fourteen price-dependent π_{k-1}, and the respective optimized prices.
- The loop will automatically cease if it finds an *iteration* where the fourteen price-dependants π_k are equal, and the fourteen price-dependant π_{k-1} are likewise homogeneous. It will then output the equilibrium prices of all services, the equilibrium profits of both **Firms**, and the number of informed and uninformed consumers in each service across all four periods.
- The file also possesses a countermeasure that ensures that the loop will stop if a profit maximizing Nash Equilibrium has not been found after 5000 *iterations*. For comparison, most successful simulations (ones where a profit maximizing Nash Equilibrium is reached) take between 1000 to 2000 *iterations* to produce a result.

1.3) Limitations of the .m Files

The .m files that we specifically created for the performance of our numerical simulations, although efficient and precise, are not perfect and have a few limitations that are difficult to get around. These limitations may eventually be surpassed, and the program could serve as a basis for further studies.

The first of these limitations is that the fourteen $P_{k,j,t}$ can only take values in the range of zero and their respective monopoly prices to a maximum of two decimal places.[42] This makes it impossible to find a solution for scenarios where an equilibrium requires at least one of the $P_{k,j,t}$ to have more than two decimal spaces. Moreover, it becomes impossible to distinguish scenarios that have solutions that require more than two decimal places from scenarios that do not have solutions at all. Although it is possible to increase the number of decimal spaces allowed, it is impossible to work around the absence of values with infinite repetends.

[42] The limitation to two decimal spaces was self-imposed. This is because each additional decimal space would have increased the performance time of the simulations by tenfold.

Another limitation of this script is that it can only calculate and optimize one $P_{k,j,t}$ at a time, so while a **Firm** can, for example, optimize their $P_{k,A,t=3}$ and $P_{k,B,t=3}$ individually and sequentially, they cannot optimize both of them simultaneously.

The final limitation of this script is that it will only find **one** equilibrium that respects the set parameters. The outputted results merely represent a possible equilibrium within the selected parameters, meaning that each scenario may have multiple possible equilibria that result in the same profit equilibrium as the obtained solution.

2) Benchmark Case and Experiments

2.1) Benchmark Case

The Benchmark Case which will serve as point of comparison will have the following values assigned to its parameters:

- $P_A^M = 50;$
- $P_B^M = 50;$
- $P_C^M = 102;$
- $c_{cr} = c_{ad} = c_{de} = c_{re} = 1;$
- $\omega_{i,t=0} = 35;$
- $\omega_{u,t=0} = 65$
- $\theta_{i,t=0} = \frac{\omega_{i,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .35;$
- $\theta_{u,t=0} = \frac{\omega_{u,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .65;$

Inputting these values into "Parameters" and then running them through "Equilibrium", we attempt to attain a state of equilibrium.[43]

The full timeline of events then takes the following shape:

- Period $t = 0$. **Firm 1** sets $P_{1,A,t=0}$ as 48 (or $P_A^M - c_{st,1,1,0,0}$). All consumers in the market join them. $\omega_{i,1,A,t=0} = 35$ and $\omega_{u,1,A,t=0} = 65$.

[43] Figure 1.

- Period $t = 1$. **Firm 1** sets $P_{1,A,t=1} = 50$, and $P_{1,B,t=1} = 2.01$, while **Firm 2** sets $P_{2,B,t=1} = 30.44$. Half of the uninformed consumers subscribe to each of the **Firms**' service B, while all the informed subscribe to 1's B. $\omega_{i,1,A,t=1} = 35$ and $\omega_{u,1,A,t=1} = 65$ to 1,A; while $\omega_{i,1,B,t=1} = 35$, $\theta_{i,2,B,t=1} = 0$, and $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 32.5$. Uninformed consumers do not choose to become informed because they are aware that there is no substitute for service A at this point, and they are unaware of the services that will be introduced in the future.
- Period $t = 2$. **Firm 1** sets $P_{1,A,t=2} = 3.01$, $P_{1,B,t=2} = 0$, and $P_{1,C,t=2} = 0$, while **Firm 2** sets $P_{2,B,t=2} = 0$. Neither Firm decides to take advantage of the "Stretch the Rope" mechanism to increase the informed-to-uninformed ratio. All informed consumers change to 1,C. $\omega_{i,1,A,t=2} = \omega_{i,1,B,t=2} = \omega_{i,2,B,t=2} = 0$, and $\omega_{i,1,C,t=2} = 35$, while $\omega_{u,1,A,t=2} = 65$, $\omega_{u,1,B,t=2} = \omega_{u,2,B,t=2} = 32.5$.
- Period $t = 3$. **Firm 1** sets $P_{1,A,t=3} = 48$, $P_{1,B,t=3} = 2.01$, and $P_{1,C,t=3} = 3.99$; while **Firm 2** sets $P_{2,A,t=3} = 0$, $P_{2,B,t=3} = 2.01$, and $P_{2,C,t=3} = 0$. Once again, neither Firm takes use of the "Stretch the Rope" mechanism.

  The final numbers of consumers for each service are $\omega_{i,1,A,t=3} = \omega_{i,1,B,t=3} = \omega_{i,2,B,t=3} = \omega_{i,2,A,t=3} = \omega_{i,2,B,t=3} = \omega_{i,2,C,t=3} = 0$ and $\omega_{i,1,C,t=3} = 35$ for informed consumers; and $\omega_{u,1,A,t=3} = 65$, $\omega_{u,1,B,t=3} = \omega_{u,2,B,t=3} = 32.5$ for uninformed consumers.

  The equilibrium profits for each Firm in this scenario are 3325 for **Firm 1** and 989.3 for **Firm 2**.

2.2) Experiment I – Increase P_B^M

All parameters are equal to those used in the Benchmark Case, with two exceptions: P_B^M increases by 50 and takes a value of 100; P_C^M also increases by the same amount and is now valued at 152.

The full timeline of events then takes the following shape:[44]

- Period $t = 0$. An identical outcome to the $t = 0$ of the Benchmark Case. $P_{1,A,t=0} = 48$, $\omega_{i,1,A,t=0} = 35$ and $\omega_{u,1,A,t=0} = 65$.

[44] Figure 2.

- Period $t = 1$. All aspects regarding the position and number of consumers are identical to the ones seen in $t = 1$ of the Benchmark Case. The values of $P_{k,B,t=1}$, meanwhile, differ from the Benchmark Case, with $P_{1,B,t=1}$ =26.5 and $P_{2,B,t=1} = 59.41$. This translates into $\omega_{i,1,B,t=1} = 35$ and $\omega_{i,2,B,t=1} = 0$, while $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 32.5$.
- Period $t = 2$. A very similar outcome to the $t = 2$ of the Benchmark Case. $P_{1,B,t=2} = P_{1,C,t=2} = P_{2,B,t=2} = 0$, while $P_{1,A,t=2} = 4.01$. The positions and numbers of both informed and uninformed consumers are identical to the ones in $t = 2$ of the Benchmark Case; in other words, $\omega_{i,1,A,t=2} = \omega_{i,1,B,t=2} = \omega_{i,2,B,t=2} = 0$, and $\omega_{i,1,C,t=2} = 35$, while $\omega_{u,1,A,t=2}$ =65, $\omega_{u,1,B,t=2} = \omega_{u,2,B,t=2} = 32.5$.
- Period $t = 3$. Once again, a very similar outcome to the $t = 3$ of the Benchmark Case. The only discrepancies between the two scenarios are the prices of $P_{k,B,t=3}$, with $P_{k,B,t=3} = 26.5$ and $P_{k,B,t=3} = 59.41$.

The equilibrium profits for each Firm in this scenario are 5349.1 for **Firm 1** and 1829.1 for **Firm 2**.

2.2.1) Conclusions from Experiment I

In this experiment the higher P_B^M leads to a higher ceiling in all the $P_{k,B,t}$, which in turn allows both **Firms** to charge their respective uninformed consumers higher prices. This naturally translates into higher profits for each Firm.

2.3) Experiment II – Increase P_A^M

Contrary to Experiment I, P_A^M is now the value increased to 100, while P_B^M remains at its base value of 50. Like Experiment I, however, P_C^M is once again valued at 152.

In this scenario, the timeline of events will be as follows:[45]

- Period $t = 0$. As expected, **Firm 1** sets a $P_{1,A,t=0}$ that reflects the increase of P_A^M. This price is $P_{1,A,t=0} = P_A^M - c_{st,1,1,0,0} = 100 - 2 = 98$. Like the previous experiment, $\omega_{i,1,A,t=0} = 35$ and $\omega_{u,1,A,t=0} = 65$.

[45] Figure 3.

- Period $t = 1$. Moderately different from the Benchmark Case, in several ways. As expected, $P_{1,A,t=1}$ takes the value of the defined monopoly price for A, which is consistent with the Benchmark Case. The values associated with service B, meanwhile, fall more in line with Experiment I than with the Benchmark Case; with $P_{1,B,t=1} = 37.34$ and $P_{2,A,t=1} = 28.99$. This translates into $\omega_{i,1,B,t=1} = 0$ and $\omega_{i,2,B,t=1} = 35$, while $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 32.5$.
- Period $t = 2$. An identical outcome to the $t = 2$ of Experiment I. In this scenario, $P_{1,A,t=2} = 4.01$, $P_{1,B,t=2} = P_{1,C,t=2} = P_{2,B,t=2} = 0$, $\omega_{i,1,C,t=2} = 35$, $\omega_{i,1,A,t=2} = \omega_{i,1,B,t=2} = \omega_{i,2,B,t=2} = 0$, $\omega_{u,1,A,t=2}$ =65, $\omega_{u,1,B,t=2} = \omega_{u,2,B,t=2} = 32.5$.
- Period $t = 3$. A similar outcome to the $t = 3$ of the Benchmark Case. With the exceptions of $P_{1,A,t=3} = 98$, $P_{1,B,t=3} = 37.34$ and $P_{2,B,t=3} = 28.99$; all prices, and positions and numbers of informed and uninformed consumers are unaltered.

The equilibrium profits for each Firm in this scenario are 7723.2 for **Firm 1** and 942.18 for **Firm 2**.

2.3.1) Conclusions from Experiment II

In this experiment the higher P_A^M leads to a higher ceiling on all the $P_{k,A,t}$. This will overwhelmingly benefit **Firm 1** more than **Firm 2**, since $\omega_{i,t=3} = \omega_{i,1,C,t=3}$, and $P_{1,A,t=3}$ exclusively targets uninformed consumers. This increase of P_A^M will also indirectly lead to the market of service B becoming more competitive, resulting in decreased profits for **Firm 2**.

2.4) Experiment III – Increase Informed-to-Uninformed Ratio

All parameters are set to the default values of the Benchmark Case, with two exceptions: $\omega_{i,t=0} = 60$, and $\omega_{u,t=0} = 40$. This leads to $\theta_{i,t=0} = \frac{\omega_{i,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .6$, and $\theta_{u,t=0} = \frac{\omega_{u,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .4$.

With these alterations, the timeline of events follows this path:[46]

- Period $t = 0$. As per usual, **Firm 1** sets $P_{1,A,t=0} = 48$, and all consumers join them. $\omega_{i,1,A,t=0} = 60$ and $\omega_{u,1,A,t=0} = 40$.

[46] Figure 4.

- Period $t = 1$. Similar to the Benchmark Case, **Firm 1** sets $P_{1,A,t=1} = 50$ and $P_{1,B,t=1} = 2.01$. **Firm 2**, meanwhile, sets $P_{2,B,t=1} = 27$. Informed consumers subscribe to B of **Firm 1**, while each Firm gets half of the uninformed consumers for B. $\omega_{i,1,A,t=1} = \omega_{i,1,B,t=1} = 60$, $\omega_{i,2,B,t=1} = 0$, $\omega_{u,1,A,t=1} = 40$, and $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 20$.
- Period $t = 2$. Identical to the $t = 2$ of the Benchmark Case in terms of prices and consumers' location. The only discrepancies relative to the Benchmark Case are $\omega_{u,1,A,t=2}$ =40, $\omega_{u,1,B,t=2} = \omega_{u,2,B,t=2} = 20$, and $\omega_{i,1,C,t=2} = 60$.
- Period $t = 3$. Similar to the outcome of $t = 3$ in the Benchmark Case, barring the value of $P_{2,B,t=3}$, which in this experiment takes the value 27; and the respective values of informed and uninformed consumers, which are $\omega_{i,1,C,t=3} = 60$ for informed consumers, and $\omega_{u,1,A,t=3} = 40$, and $\omega_{u,1,B,t=3} = \omega_{u,2,B,t=3} = 20$ for their uninformed counterparts.

The equilibrium profits for each Firm in this scenario are 2199.6 for **Firm 1** and 540 for **Firm 2**.

2.4.1) Conclusions from Experiment III

The higher ratio of informed consumers will inevitably lead to tighter price competition between the two **Firms** (especially in the sub-section of the market pertaining to service B), caused by the decreased number of uninformed consumers that both **Firms** will have during $t = 3$, and resulting in lower profits for both.

2.5) Experiment IV – Decrease Informed-to-Uninformed Ratio

In this experiment, we go in the opposite direction to Experiment III, by decreasing the informed-to-uninformed ratio in the Benchmark Case. We set $\omega_{i,t=0} = 20$, and $\omega_{u,t=0} = 80$, leading to $\theta_{i,t=0} = \frac{\omega_{i,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .2$, and $\theta_{u,t=0} = \frac{\omega_{u,t=0}}{\omega_{i,t=0}+\omega_{u,t=0}} = .8$.

These alterations mean that the timeline of events is:[47]

[47] Figure 5.

- Period $t = 0$. **Firm 1** sets $P_{1,A,t=0} = 48$, and all consumers join them. $\theta_{i,1,A,t=0} = 20$ and $\omega_{u,1,A,t=0} = 80$.
- Period $t = 1$. A similar outcome to the $t = 1$ of the Benchmark Case occurs. **Firm 1** sets $P_{1,A,t=1} = 50$ and $P_{1,B,t=1} = 2.01$, while **Firm 2** sets $P_{2,B,t=1} = 40.01$. Half of the uninformed consumers subscribe to service B of each Firm, while all the informed acquire **Firm 1**'s B. In this scenario, $\omega_{i,1,A,t=1} = \omega_{i,1,B,t=1} = 20$, $\omega_{i,2,B,t=1} = 0$, $\omega_{u,1,A,t=1} = 80$, and $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 40$.
- Period $t = 2$. Once again, a very similar outcome to the $t = 2$ of the Benchmark Case, with the only substantial discrepancies being the values of informed and uninformed consumers, which are $\omega_{u,1,A,t=2}$ =80, $\omega_{u,1,B,t=2} = \omega_{u,2,B,t=2} = 20$, and $\omega_{i,1,C,t=2} = 20$.
- Period $t = 3$. In a very similar fashion to Experiment III, the outcome of this period is nearly identical to the $t = 3$ of the Benchmark Case, barring the values of $P_{2,B,t=3}$ and the number of informed and uninformed consumers (their positions remain unaltered). In this specific scenario, $P_{2,B,t=3} = 40.01$, , $\omega_{u,1,A,t=3} = 80$, and $\omega_{u,1,B,t=3} = \omega_{u,2,B,t=3} = 40$, and $\omega_{i,1,C,t=3} = 20$.

The equilibrium profits for each Firm in this scenario are 4000.2 for **Firm 1** and 1600 for **Firm 2**.

2.5.1) Conclusions form Experiment IV

Contrary to Experiment III, the decrease in the ratio of informed consumers will result in a looser price competition (once again, especially for the sub-section of the market pertaining to service B). A higher number of uninformed consumers in $t = 3$ translates into higher profits for both **Firms**.

2.6) Experiment V – Slightly Increased Switching Costs

This specific experiment consists of slightly increasing the value of the individual components of switching costs (c_{cr}, c_{ad}, c_{de}, c_{re}) to 4. In this particular scenario, the lowest possible switching cost in this timeline is $c_{st,0,1,0,0} = 4$, while the highest is $c_{st,1,2,1,2,} = 24$.

Considering these changes, the timeline of events is as follows:[48]

- Period $t = 0$. **Firm 1** sets $P_{1,A,t=0} = 42$ $(P_A^M - c_{st,1,1,0,0})$, taking into account the altered switching costs. The number of consumers is unaltered, with $\omega_{i,1,A,t=0} = 35$ and $\omega_{u,1,A,t=0} = 65$.
- Period $t = 1$. **Firm 1** sets $P_{1,A,t=1} = 50$ and $P_{1,B,t=1} = 8.03$, while **Firm 2** sets $P_{2,B,t=1} = 31.54$. This results in **Firm 1** getting the informed consumers for their service B. The values of consumers at this stage are $\omega_{i,1,A,t=1} = \omega_{i,1,B,t=1} = 35$, $\omega_{i,2,B,t=1} = 0$, $\omega_{u,1,A,t=1} = 60$, and $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 32.5$.
- Period $t = 2$. The values and positions of all consumers are identical to those in $t = 2$ of the Benchmark Case. $P_{1,B,t=2}$, $P_{1,C,t=2}$ and $P_{2,B,t=2}$ also remain valued at zero, while $P_{1,A,t=2}$ now assumes a value of 12.01.
- Period $t = 3$. The values and positions of all consumers are once again identical to those showcased in $t = 3$ of the Benchmark Case. Most prices in this period, however, are different from those found in $t = 3$ of the Benchmark Case. In this specific scenario, $P_{1,A,t=3} = 42$, $P_{1,B,t=3} = 8.03$, $P_{1,C,t=3} = 15.99$, $P_{2,A,t=3} = 0$, $P_{2,B,t=3} = 31.54$, and $P_{2,C,t=3} = 0$.

  The equilibrium profits for each Firm in this scenario are 3550.6 for **Firm 1** and 1025 for **Firm 2**.

2.6.1) Conclusions from Experiment V

The slightly increased values for the components of switching costs will lead to higher $P_{1,C,t=3}$ and $P_{2,B,t=3}$. Compared with the Benchmark Case, **Firm 1** profits more from their informed consumers (although their profits from uninformed consumers is slightly decreased), while **Firm 2** profits more from their uninformed consumers. Overall, this alteration results in profit increases for both **Firms**.

[48] Figure 6.

2.7) Experiment VI – Greatly Increased Switching Costs

As an extension of the previous experiment, we test a heftier increase of the individual switching cost components (c_{cr}, c_{ad}, c_{de}, c_{re}), which now each take a value of 8. Here the lowest possible switching cost is $c_{st,0,1,0,0} = 8$, while the highest is $c_{st,1,2,1,2,} = 48$.

With these new values, the respective timeline of events is:[49]

- Period $t = 0$. **Firm 1** sets $P_{1,A,t=0} = 34$ ($P_A^M - c_{st,1,1,0,0}$), taking into account the altered switching costs. Thus, all consumers join them, with $\omega_{i,1,A,t=0} = 35$ and $\omega_{u,1,A,t=0} = 65$.
- Period $t = 1$. **Firm 1** sets $P_{1,A,t=1} = 50$ and $P_{1,B,t=1} = 0$, while **Firm 2** sets $P_{2,B,t=1} = 0$. This leads to $\omega_{i,1,A,t=1} = \omega_{i,2,B,t=1} = 35$, $\omega_{i,1,B,t=1} = 0$, $\omega_{u,1,A,t=1} = 60$, and $\omega_{u,1,B,t=1} = \omega_{u,2,B,t=1} = 20$.
- Period $t = 2$. Both **Firms** set all of their prices as zero. There is no change in the position or number of consumers compared with the previous period. As for the bundle, $\omega_{i,1,C,t=2} = 0$.
- Period $t = 3$. **Firm 1** sets $P_{1,A,t=3} = 23.99$, $P_{1,B,t=3} = 19.66$, and $P_{1,C,t=3} = 19.66$; while **Firm 2** sets $P_{2,A,t=3} = 0$, $P_{2,B,t=3} = 26.83$, and $P_{1,C,t=3} = 18.83$. By setting $P_{1,B,t=3} > P_{1,B,t=2} = P_{1,B,t=1}$, **Firm 1** uses the "Stretch the Rope" mechanism, which increases $(\omega_{i,1,A} \cap \omega_{i,1,B})$. By setting $P_{2,B,t=3} > P_{2,B,t=2} = P_{2,B,t=1}$, **Firm 2** also utilizes the "Stretch the Rope" mechanism, increasing $(\omega_{i,1,A} \cap \omega_{i,2,B})$. The final amounts of consumers in each service are $\omega_{i,1,A,t=3} = 65.219$, $\omega_{i,1,B,t=3} = 47.779$, $\omega_{i,1,C,t=3} = 0$, $\omega_{i,2,A,t=3} = 0$, $\omega_{i,2,B,t=3} = 17.44$, $\omega_{i,2,C,t=3} = 0$, $\omega_{u,1,A,t=3} = 34.781$, $\omega_{u,1,B,t=3} = 19.66$, and $\omega_{u,2,B,t=3} = 15.06$.

The equilibrium profits for each Firm in this scenario are 3726.1 for **Firm 1** and 871.98 for **Firm 2**.

[49] Figure 7.

2.7.1) Conclusions from Experiment VI

The exuberant value of the switching costs set in this experiment causes the optimal recurring strategy of the previous experiments to be seen as non-viable by the **Firms**, leading both to adopt a different strategy. In this scenario, the two **Firms** make use of the switching costs, as well as the "Stretch the Rope" mechanism in order to implement uncharacteristically harsh vendor lock-in situations.

Although slight profit decreases occur for both **Firms** compared to the Benchmark Case, this specific strategy adopted in this experiment seems to be the optimal one to achieve profit maximizing Nash Equilibrium.

3) Synthesis of Benchmark Case and Experiments

Despite the possibility of using differing short-term strategies from those used in the **Benchmark Case**, we can see that the optimal long-term strategies employed in **Experiments I through V** are largely unchanged.

– In period $t = 0$, **Firm 1** sets $P_{1,A,t=0} = P_A^M - c_{st,1,1,0,0}$, with $P_{1,A,t=0}$ also being equal to the eventual $P_{1,A,t=3}$.

– In the next period, **Firm 1** sets $P_{1,A,t=1} = P_A^M$. This does not "Stretch the Rope" because uninformed consumers are aware that there is no substitute for service A at this point, but they are also unaware of the services that will be introduced in the future. Both **Firms** will then set each of their $P_{k,B,t=1}$, with each of these values being equal to the respective $P_{k,B,t=3}$. The matter of who gets $\omega_{i,B,t=1}$ is irrelevant in the long run.

– In period $t = 2$, **Firm 1** will funnel all $\omega_{i,A,t=2}$ (in other words, all the informed consumers present in the market at this point in time) into being $\omega_{i,C,t=2}$. Neither **Firm** will opt to "Stretch the Rope", because that will only tighten the price competition and eventually lead to lower profit.

– Finally, in period $t = 3$, **Firm 1** will set the highest possible value for $P_{1,C,t=3}$ which guarantees that $\omega_{i,C,t=3}$ remains unchanged from the previous period. The two **Firms** will respectively set $P_{1,A,t=3}$, $P_{1,B,t=3}$, and $P_{2,B,t=3}$ as equal to $P_{1,A,t=0}$, $P_{1,B,t=1}$, and $P_{2,B,t=1}$ (meaning that neither one will opt to "Stretch the Rope") so as to exploit their uninformed consumers as much as possible. **Firm 2** will also use their $P_{2,A,t=3}$ and $P_{2,C,t=3}$ in a way that limits the options of **Firm 1**.

Experiment VI seems to be the exception to the rule, with both **Firms** adopting long-term strategies different from those deployed until now.

– In period $t = 0$, **Firm 1** once again opts to set $P_{1,A,t=0} = P_A^M - c_{st,1,1,0,0}$.

– In period $t = 1$, not only does **Firm 1** actively seek to attract all the $\omega_{i,B,t=1}$, but both **Firms** set $P_{k,B,t=1} = 0$.

– In $t = 2$, we see that **Firm 1** opts not to funnel $\omega_{i,A,t=2}$ into being $\omega_{i,C,t=2}$.

– Ultimately, in period $t = 3$, we see both **Firms** using the "Stretch the Rope" mechanism, while setting $P_{1,C,t=3}$, $P_{2,A,t=3}$, and $P_{2,C,t=3}$ to values that will deter informed consumers from performing switches (even from switching within the same Firm). The exceptionally high switching costs characteristic of this experiment lead to particularly draconian situations of vendor lock-in for the informed consumers of both **Firms**. Although Vendor Lock-In is commonplace in all the previously observed scenarios, the outcome of this particular experiment actively abuses it to an unprecedented degree.

Discussion of Results

1) Analysis and Discussion of Results

The outcomes of our experiments seem to implement the scientific literature consensus regarding many of the **features** detailed in the Literature Review section of this thesis.

We can see throughout the Benchmark Case, and in Experiments I through V, that the presence of **bundles** is beneficial to consumers in a **non-monopolistic** scenario,[50] as the joint prices of $P_{1,A,t=3}$ and $P_{1,B,t=3}$, as well as $P_{1,A,t=3}$ and $P_{2,B,t=3}$ tend to be higher than the prices of bundles with consumers ($P_{1,C,t=3}$). Admittedly, the bundles only appear to benefit informed consumers, but we should keep in mind that within our model, uninformed consumers only perform switches (and therefore may only switch to bundles) if they first become informed.

We can also verify in Experiment III that within a non-monopoly, a higher ratio of **informed consumers** will tighten price competition, leading to **Firms** suffering a drop in profits.[51] Additionally, we see the opposite effect in Experiment III's counterpart, Experiment IV, with a higher ratio of **uninformed consumers** leading to looser price competition between the two **Firms**.

It is also important to highlight that within all scenarios barring Experiment VI, both **Firms** avoided triggering the "Stretch the Rope" mechanism, as that would cause an increase the informed-to-uninformed ratio within the market. This would induce a tighter price competition and subsequently lower profits for each **Firm**, reinforcing the idea that generally, markets of exclusively uninformed consumers are more beneficial to firms than markets with exclusively informed ones.

The implementation and gradual increase of **switching costs** also demonstrate the ease of consumers being subjected to **vendor lock-in**. This type of situation seems to be relatively

[50] (Aloysius et al., 2012), p. 669.
[51] (Sun, 2014), p. 3.

innocuous in most scenarios, but in Experiment V and especially in Experiment VI, consumers are punished with exuberantly high prices and even higher overall costs should they decide to switch providers, or leave the market in general.

2) Conclusion

As mentioned in the Introduction, this thesis set out to explore the strategies that each Firm should enact in order to ensure the desired profit equilibrium, taking into account several arbitrary market variables such as the role of informed and uninformed consumers, the possibility of bundling, as the existence of switching costs.

The results obtained from the created model seem to answer the previously posed questions.

How can profit equilibrium be altered with the variation of the initial informed-to-uninformed ratio?

Experiment III suggests that a higher informed-to-uninformed consumer ratio would cause the price competition to tighten, expressed in lower prices and therefore lower profits and a lower profit equilibrium.

Experiment IV confirms the opposite effect. A lower informed-to-uninformed ratio loosens price competition, leading to higher prices and therefore higher profits and a higher profit equilibrium.

In both of these experiments, as well as in the Benchmark Case, we can verify that both **Firms** actively avoid triggering the "Stretch the Rope" mechanism, which would increase the informed-to-uninformed consumer ratio. This in turn would have tightened price competition and led to lower profits. The two **Firms** were aware of this, and thus neither decided to take advantage of it.

How can profit equilibrium be offset by the variation of switching costs?

Experiment V, having a slight increase in switching costs, allows a higher ceiling for prices, translating into higher profits for both **Firms**, and a higher profit equilibrium.

Experiment VI, with its extremely high switching costs, meanwhile, is a very different scenario from all other experiments as well as the Benchmark Case.

While in all the other scenarios the option of informed consumers in $t = 1$ was irrelevant (since **Firm 1** would always be able to funnel them into their bundle in $t = 2$), **Firm 1** will instead opt to keep $(\omega_{i,1,A,t=3} \cap \omega_{i,1,B,t=3})$ as high as possible. Thus, it becomes imperative for **Firm 1**'s optimal strategy to get all the $\omega_{i,t=1}$. This, by extension, requires **Firm 1** to set $P_{1,B,t=1}$ as low as possible, and then set $P_{1,A,t=3}$ and $P_{1,B,t=3}$ at the highest possible value that will still keep their informed consumers from switching to **Firm 2**, as well as setting a $P_{1,C,t=3}$ that will keep these informed consumers from switching to their bundle, since $P_{1,A,t=3} + P_{1,B,t=3} > P_{1,C,t=3}$, fully exploiting the vendor lock-in situation that their consumers are in. This results in the usage of the "Stretch the Rope" mechanism, which increases the informed-to-uninformed ratio of consumers.

Firm 2, meanwhile, would also utilize the "Stretch the Rope" mechanism to exploit their Locked-In consumers. Now, contrary to all other scenarios, **Firm 2** will have $\omega_{i,2,B,t=3} > 0$, but would still need to keep an eye on the other prices in the market. Thus, they set $P_{2,C,t=3}$ at a high enough value to keep their informed consumers Locked-In, and set $P_{2,A,t=3} = 0$ so as to limit the options of **Firm 1**.

In Experiment VI, while the presence of such high switching costs might have resulted in draconian situations of vendor lock-in, the increase of the informed-to-uninformed consumer ratio brought about by the usage of the "Switch the Rope" mechanism actually tightened price competition (and therefore, led to a lower profit equilibrium) relative to Experiment V.

In what scenarios can the usage of bundling be advantageous for firms?

Within most scenarios, the bundles seem to be advantageous to the **Firms** in one of two ways. The first way is as a service to directly profit from (this is seen primarily for **Firm**

1). The second is as a means to threaten a competitor and limit their price options (this is seen primarily for **Firm 2**).

In Experiment VI, meanwhile, bundles do not seem to be very useful in either way. Here, **Firms** can more easily profit from their consumers by abusing the high switching costs and individual services (in the case of **Firm 1**), and using a low price for an individual service to threaten the competitor seems to be more effective (in the case of **Firm 2**).

Final Conclusions

- We can affirm that the outcomes of our experiments seem to enforce the scientific literature consensus regarding the features detailed in the Literature Review section of this thesis.
- In spite of the observed limitations (such as the restricted number of experiments performed and the aforementioned flaws of the .m files), we have also concluded that the results obtained by our experiments seem to answer the previously mentioned questions, namely that higher ratios of informed-to-uninformed consumers in a market cause a decrease in profit equilibrium, higher switching costs (up to a certain degree) can increase profit equilibrium, and that the practice of bundling will only be advantageous for consumers if it is also advantageous for firms (which will only occur if bundles are a component of a Firm's optimal strategy).
- This thesis describes the creation of a model which involves for the performing of simulations for this specific market. The model eases the acquisition of clearer, more concise information about the optimal strategies that firms can choose. Furthermore, we believe that this model can be refined in the future to overcome the aforementioned limitations of the .m files.[52] This would allow for the inclusion of other distinct variables, and make it possible to study how different features can impact one another.
- The present thesis could equally serve as a baseline for future developments which seek more extensive and diverse studies, such as implementing non-universal reservation costs for different consumers, or implementing subjective, personal switching costs dependant on the preferences of individual consumers.

[52] p. 35-36.

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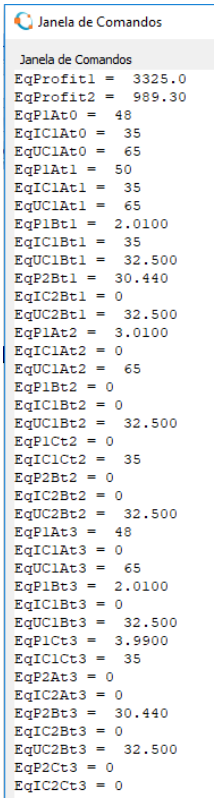
Appendix 1: Variable Denomination

- P_j^M – Monopoly price of an individual service or bundle. In this context, it represents the reservation cost for consumers of a specific service or bundle.
- $P_{k,j,t}$ – Price set by **Firm** k for service or bundle j in period t.
- ω_t – Total number of consumers in period t.
- $\omega_{i,t}$ – Total number of informed consumers in period t.
- $\omega_{u,t}$ – Total number of uninformed consumers in period t.
- $\omega_{i,k,j,t}$ – Total number of informed consumers of service/bundle j in **Firm** k during period t.
- $\omega_{u,k,j,t}$ – Total number of uninformed consumers of service/bundle j in **Firm** k during period t.
- $\theta_{i,t}$ – Ratio of informed consumers in period t.
- $\theta_{u,t}$ – Ratio of informed consumers in period t.
- $P_{k,j}^o$ – The price of comparison regarding a **Firm's** service to be compared to future prices of the same service for the same **Firm**.
- $P_{k,j}^o(P_{k,j,t})$ – The current price of comparison regarding a **Firm's** service to be compared to future prices of the same service for the same **Firm**.
- $P_{k,j,t+m}^N(P_{k,j,t+m})$ – A future price of a **Firm's** service that is compared to $P_{k,j}^o(P_{k,j,t})$.
- $\varphi_{k,j,t+m}$: The percentage of uninformed consumers of service j of **Firm** $\boldsymbol{k}$ that will pay search costs when $P_{k,j,t+m}$ is set and compared to $P_{k,j}^o(P_{k,j,t})$.
- c_{cr} –The entry cost for a consumer to join an individual **Firm.**
- c_{ad} – The entry cost for a consumer to acquire an individual service from a **Firm** which they are subscribed to.
- c_{de} – The exit cost for a consumer to leave an individual **Firm.**
- c_{re} – The exit cost for a **Firm** to cancel the provision of a service.
- $c_{st,w,x,y,z} \equiv w * c_{cr} + c_{ad} * x + c_{de} * y + c_{re} * z$ – The switching cost of joining w **Firms**, acquiring x services, leaving y **Firms**, and cancelling z services.
- π_k – Profits of Firm k in period $t = 3$.

Appendix 2: Experiment Outputs

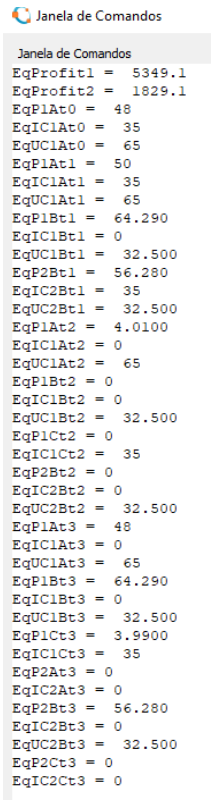


Janela de Comandos

Janela de Comandos

```
EqProfit1 =  3325.0
EqProfit2 =  989.30
EqP1At0 =  48
EqIC1At0 =  35
EqUC1At0 =  65
EqP1At1 =  50
EqIC1At1 =  35
EqUC1At1 =  65
EqP1Bt1 =  2.0100
EqIC1Bt1 =  35
EqUC1Bt1 =  32.500
EqP2Bt1 =  30.440
EqIC2Bt1 = 0
EqUC2Bt1 =  32.500
EqP1At2 =  3.0100
EqIC1At2 = 0
EqUC1At2 =  65
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  32.500
EqP1Ct2 = 0
EqIC1Ct2 =  35
EqP2Bt2 = 0
EqIC2Bt2 = 0
EqUC2Bt2 =  32.500
EqP1At3 =  48
EqIC1At3 = 0
EqUC1At3 =  65
EqP1Bt3 =  2.0100
EqIC1Bt3 = 0
EqUC1Bt3 =  32.500
EqP1Ct3 =  3.9900
EqIC1Ct3 =  35
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  30.440
EqIC2Bt3 = 0
EqUC2Bt3 =  32.500
EqP2Ct3 = 0
EqIC2Ct3 = 0
```

^Figure 1 - Benchmark Case



Janela de Comandos

Janela de Comandos

```
EqProfit1 =  5349.1
EqProfit2 =  1829.1
EqP1At0 =  48
EqIC1At0 =  35
EqUC1At0 =  65
EqP1At1 =  50
EqIC1At1 =  35
EqUC1At1 =  65
EqP1Bt1 =  64.290
EqIC1Bt1 = 0
EqUC1Bt1 =  32.500
EqP2Bt1 =  56.280
EqIC2Bt1 =  35
EqUC2Bt1 =  32.500
EqP1At2 =  4.0100
EqIC1At2 = 0
EqUC1At2 =  65
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  32.500
EqP1Ct2 = 0
EqIC1Ct2 =  35
EqP2Bt2 = 0
EqIC2Bt2 = 0
EqUC2Bt2 =  32.500
EqP1At3 =  48
EqIC1At3 = 0
EqUC1At3 =  65
EqP1Bt3 =  64.290
EqIC1Bt3 = 0
EqUC1Bt3 =  32.500
EqP1Ct3 =  3.9900
EqIC1Ct3 =  35
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  56.280
EqIC2Bt3 = 0
EqUC2Bt3 =  32.500
EqP2Ct3 = 0
EqIC2Ct3 = 0
```

^Figure 2 - Experiment I - Increased MPB and MPC

Janela de Comandos

```
Janela de Comandos
EqProfit1 =  7723.2
EqProfit2 =  942.18
EqP1At0 =  98
EqIC1At0 =  35
EqUC1At0 =  65
EqP1At1 =  100
EqIC1At1 =  35
EqUC1At1 =  65
EqP1Bt1 =  37.340
EqIC1Bt1 = 0
EqUC1Bt1 =  32.500
EqP2Bt1 =  28.990
EqIC2Bt1 =  35
EqUC2Bt1 =  32.500
EqP1At2 =  4.0100
EqIC1At2 = 0
EqUC1At2 =  65
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  32.500
EqP1Ct2 = 0
EqIC1Ct2 =  35
EqP2Bt2 = 0
EqIC2Bt2 = 0
EqUC2Bt2 =  32.500
EqP1At3 =  98
EqIC1At3 = 0
EqUC1At3 =  65
EqP1Bt3 =  37.340
EqIC1Bt3 = 0
EqUC1Bt3 =  32.500
EqP1Ct3 =  3.9900
EqIC1Ct3 =  35
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  28.990
EqIC2Bt3 = 0
EqUC2Bt3 =  32.500
EqP2Ct3 = 0
EqIC2Ct3 = 0
```

^Figure 3 - Experiment II - Increased MPA and MPC

Janela de Comandos

```
Janela de Comandos
EqProfit1 =  2199.6
EqProfit2 =  540
EqP1At0 =  48
EqIC1At0 =  60
EqUC1At0 =  40
EqP1At1 =  50
EqIC1At1 =  60
EqUC1At1 =  40
EqP1Bt1 =  2.0100
EqIC1Bt1 =  60
EqUC1Bt1 =  20
EqP2Bt1 =  27
EqIC2Bt1 = 0
EqUC2Bt1 =  20
EqP1At2 =  3.0100
EqIC1At2 = 0
EqUC1At2 =  40
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  20
EqP1Ct2 = 0
EqIC1Ct2 =  60
EqP2Bt2 = 0
EqIC2Bt2 = 0
EqUC2Bt2 =  20
EqP1At3 =  48
EqIC1At3 = 0
EqUC1At3 =  40
EqP1Bt3 =  2.0100
EqIC1Bt3 = 0
EqUC1Bt3 =  20
EqP1Ct3 =  3.9900
EqIC1Ct3 =  60
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  27
EqIC2Bt3 = 0
EqUC2Bt3 =  20
EqP2Ct3 = 0
EqIC2Ct3 = 0
```

^Figure 4 - Experiment III - Increase Informed-to-Uninformed Ratio

Janela de Comandos

```
Janela de Comandos
EqProfit1 =  4000.2
EqProfit2 =  1600.4
EqP1At0 =  48
EqIC1At0 =  20
EqUC1At0 =  80
EqP1At1 =  50
EqIC1At1 =  20
EqUC1At1 =  80
EqP1Bt1 =  2.0100
EqIC1Bt1 =  20
EqUC1Bt1 =  40
EqP2Bt1 =  40.010
EqIC2Bt1 = 0
EqUC2Bt1 =  40
EqP1At2 =  3.0100
EqIC1At2 = 0
EqUC1At2 =  80
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  40
EqP1Ct2 = 0
EqIC1Ct2 =  20
EqP2Bt2 = 0
EqIC2Bt2 = 0
EqUC2Bt2 =  40
EqP1At3 =  48
EqIC1At3 = 0
EqUC1At3 =  80
EqP1Bt3 =  2.0100
EqIC1Bt3 = 0
EqUC1Bt3 =  40
EqP1Ct3 =  3.9900
EqIC1Ct3 =  20
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  40.010
EqIC2Bt3 = 0
EqUC2Bt3 =  40
EqP2Ct3 = 0
EqIC2Ct3 = 0
```

^Figure 5 - Experiment IV - Decrease Informed-to-Uninformed Ratio

Janela de Comandos

```
Janela de Comandos
EqProfit1 =  3550.6
EqProfit2 =  1025.0
EqP1At0 =  42
EqIC1At0 =  35
EqUC1At0 =  65
EqP1At1 =  50
EqIC1At1 =  35
EqUC1At1 =  65
EqP1Bt1 =  8.0300
EqIC1Bt1 =  35
EqUC1Bt1 =  32.500
EqP2Bt1 =  31.540
EqIC2Bt1 = 0
EqUC2Bt1 =  32.500
EqP1At2 =  12.010
EqIC1At2 = 0
EqUC1At2 =  65
EqP1Bt2 = 0
EqIC1Bt2 = 0
EqUC1Bt2 =  32.500
EqP1Ct2 = 0
EqIC1Ct2 =  35
EqP2Bt2 = 0
EqIC2Bt2 = 0
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EqP1At3 =  42
EqIC1At3 = 0
EqUC1At3 =  65
EqP1Bt3 =  8.0300
EqIC1Bt3 = 0
EqUC1Bt3 =  32.500
EqP1Ct3 =  15.990
EqIC1Ct3 =  35
EqP2At3 = 0
EqIC2At3 = 0
EqP2Bt3 =  31.540
EqIC2Bt3 = 0
EqUC2Bt3 =  32.500
EqP2Ct3 = 0
EqIC2Ct3 = 0
```
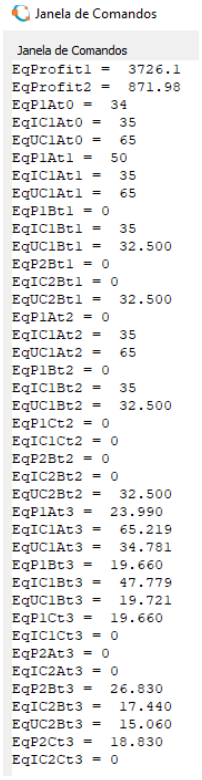

^Figure 6 - Experiment V - Slightly Increased of Switching Costs



^Figure 7 - Experiment VI - Greatly Increased of Switching Costs