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PLANNING OF MENTAL HEALTH SERVICES IN PORTUGAL UNDER UNCERTAIN CONDITIONS

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Abstract

The demand for mental health care services is increasing significantly in the World and in Europe. For a country like Portugal, that is one of the countries with the largest prevalence of mental illnesses in Europe and with a level of supply that is not enough for the level of demand that exists nowadays, the urgency to be able to present a mental health care network able to respond to the expected increase in the demand for mental health services is higher and higher.

In this thesis, a mathematical programming model - MHCU model - is presented in order to assist the decision makers to plan a mental health network that can respond to the current and future situation of the mental health care in Portugal. The model focus in the Great region of Lisbon and considers the different services provided and multiple objectives relevant in the mental health sector like the minimization of the cost or the maximization of the different equities values that are used in the model. The MHCU model is a stochastic model in order to be able to take into consideration the uncertainty associated with the mental health sector in different parameters like the demand for service and the length of stay in the network for each patient.

Keywords: Mental Health Care; Mathematical Programming Models; Stochastic Model; Planning under uncertainty.

JEL Classification System codes:

C63 Computational Techniques • Simulation Modeling

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Resumo

A procura por serviços da rede de saúde mental está a aumentar significativamente no mundo e na Europa. Para um país como Portugal, que é um dos países com maior número de doentes mentais na Europa e com um nível de oferta deste tipo de serviços que não é suficiente para corresponder ao nível de procura que existe. A urgência de conseguir reformular a rede de saúde mental em Portugal de forma a que consiga responder ao expectável aumento da procura é cada vez maior.

Nesta tese, é apresentado um modelo matemático – modelo MHCU – como forma de assistir os responsáveis pela gestão da saúde mental em Portugal a tomar decisões que permitam reformular a rede de saúde mental em Portugal de forma a que esta consiga responder a atual e futura realidade deste sector em Portugal Este modelo é focado na grande região de Lisboa e considera os diferentes serviços e diferentes objetivos que são relevantes para o sector da saúde mental, como minimizar o custo ou maximizar as diferentes equidades que são utilizadas no modelo. O modelo MHCU é um modelo estocástico de forma a que consiga ter em consideração a incerteza que se encontra associada ao sector da saúde mental em diferentes parâmetros como a procura pelos serviços e o tempo de permanencia nos serviços por parte de cada paciente.

Palavras – chave: Cuidados de Saúde Mental; Modelos de Programação matemática, Modelo Estocástico; Planeamento em condições de incerteza

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1 – Introduction

In this section, a brief description of the importance of mental health care in the society is made as well as an overview of the balance between the level of mental health care that is needed and the level of mental health care that is provided to people and the motives that can lead to this.

Context in Mental Health in Europe and Portugal

Mental Health and well-being are fundamental in order to enable people to experience life as meaningful and to be a creative and active citizen. (World Health Organization, 2005).

Although this is true, the provision of mental health care in the World is still scarce, with the current supply being far from meeting the needs of all the people that need it. In fact, according to the WHO (World Health Organization), between 76 % and 85% of the people with severe mental disorders are not receiving treatment at all in low-income countries and middle-income countries (World Health Organization, 2013). And between 35 % and 50% of people in need are not receiving mental health treatment in countries with high incomes. This may suggest that a significant part of the people that suffers from mental problems will not receive the care that they need in order to be able to live as an active citizen. This reality can be explained due to the insufficient number of specialized and general health workers dealing with mental health, as well as to the low annual spending in mental health care. Experts advise that governments should be spending between $2 and $3 per person in mental health care in low income countries (LICs), which is 10 to 15 times the current level of investment. In lower middle-income countries, the investment should be between $3 to $4. (Caddick et al. 2016).

With this being said, it is clear that the planning of the supply of mental health care is a key cornerstone of any country. Since it is through planning that it will be possible to provide to people the degree of mental care that they need. Muijen and McCulloch described Mental health care as a “wicked” public policy issue that requires detailed coordination of efforts across many government departments or ministries which highlights the importance that planning has when talking about mental health care.

A patient with mental health care necessities usually require support in different areas other than health care, such as support across housing, employment, social care, education, employment and welfare (Petrea and McCulloch, 2013). This means that, in order to rehabilitate a mental health care patient, the investment that needs to be made is not just on the health care per se but also in all the other fields that will be necessary in order to give stability for the patient to get back to its stable life. Unfortunately, many countries still struggle to develop the truly cross-government mental health policies that are needed in order to provide this relation between government departments and ministries (Petrea and McCulloch ,2013). All of this relation between the different types of support that a mental health patient needs, makes the planning of the provision of supply of health care even more challenging and important.

The importance of planning is even larger when considering the current trend of de deinstitutionalization (Franz, L.et al., 1984). Deinstitutionalization consists in the transference of patients from psychiatric hospitals to community-based centres. This transference has required planning for the effective (re)distribution of services and resources.

Together with planning, it is also important to have a reliable way to measure the real results and compare it with the planning. In this way, it is important to have reliable indicators that can measure the impacts in mental health care.

In the case of the mental health sector, the existence of these indicators are also a challenge, especially when compared with the physical health indicators that are much more developed (Petrea and McCulloch , 2013).

One of the biggest difficulties is the fact that when talking about mental health, the indicators that should be used are the ones that represent improvements in symptoms, behaviour, functioning/impairment and social integration. And this type of indicators are not very common, because most countries do not have systems in place to collect this kind of data on such outcomes. And the ones that use it, have difficulties in validating this data. Since Service users, carers and professionals attach different meanings to outcome measures in mental health, and often the measurements made by these groups differ significantly.

When considering the particular case of Portugal, this planning is especially relevant, due to the low supply of mental health care that is in place in the country, and also due the fact that Portugal is one of the countries with the largest prevalence of mental illnesses in Europe (Programa Nacional para a Saúde Mental, 2017).

According to the Health at a Glance: Europe 2018 State of Health in the EU cycle by OCDE, in the year of 2016, 18,4 % of the Portuguese population has a mental health problem. This are divided in five disease groups (Anxiety Disorders 5,70%; Depressive disorders 5,70%; Substances Abuse disorders 1,75%; Bipolar disorders and Schizophrenia 1,75 % and others corresponding to 3,50%. Portugal is the fifth country with a higher prevalence of mental health disorders in Europe, the highest is Finland with 18,8 %.

The high level of demand and the low level of supply results in a high percentage of people with serious mental diseases without access to mental health care. All of this indicates that a deep and urgent change in some key aspects of mental health policies and services needs to be done in the Portuguese context, in order to bring Portugal to the right path.

Also, since it is not possible to foresee with total confidence how the demand for this particular type of health care service will evolve in the future, an adequate planning would account for such uncertainty. In fact, for a long time, the planning community has maintained an interest in decision-theoretic planning, where the focus was on efficient algorithms for planning under quite restrictive assumptions. But recently, a resurgence in working on planning under uncertainty came up. This new way of planning can bring some leverage to the planning that is being done in Portugal, by providing planning information that can be useful today and also in the future.

Research Question

The research question explored in this dissertation is as follow: How can the current provision of mental health care services in Portugal be improved in order to satisfy the needs of patients with mental health disorders under uncertain conditions?

Objectives

The aim of this thesis is to develop a planning model to support the planning of the Portuguese mental health care services in the coming years when accounting for the underlying uncertainty.

Scope

This thesis will be focused in the mental health care provided in the metropolitan area of Lisbon, since it is the biggest and most populated region in Portugal. according with Pordata. From this region, 9 different regions were selected (that represent the 9 “municípios” of the Lisbon region) to take in consideration in this model: Amadora, Cascais, Lisboa, Loures, Mafra, Odivelas, Oeiras, Sintra and Vila Franca de Xira.

Methodology

In respect to the methodology that will be used during this thesis, the following will be made: i) a description of the current provision of mental health care services, in order to understand the current situation in respect to the provision of mental health care that is provided to the Portuguese population; ii) Identifying the key policy concerns to be considered while planning a network of mental health care services in Portugal; iii) Developing a planning model to support the planning of the already existing mental health care network using the General Algebraic Modeling System (GAMS); iv) Defining different planning scenarios with relevance for the Portuguese context in order to be able to find different interpretation to the model; v) Applying this planning model to the different scenarios and analyse the obtained results in order to be able to present recommendations in the way to plan the network of mental health care in Portugal.

Thesis Structure

The thesis will be structured in the following way. Chapter 2 presents the literature review on the existing methods to plan the delivery of the different health care services, where we can find (i) the difference between different planning levels: Strategical, Tactical and Operational (ii) different mathematic models that have been used in the health sector and in the mental health sector and (iii) the different mathematic models that take uncertainty into consideration.

Chapter 3 presents the process used to predict the demand and the execution of a stochastic model and a way to apply this model in an accessible way. At last, the main conclusions and final remarks, as well as the future work that might need to be developed are described in Chapter 4.

2 - Literature Review

This literature review was based on a research using different databased, such as ISI Web of Knowledge, Google Scholar, Science Direct, B-on and Springer. The keywords used (individually or in combination) within the research where: Health Care; Mental Health Care; Tactical Planning; Strategical Planning; Planning under uncertainty; Stochastic Model; Optimization; Multi-objective.

This section is organized as follows. To start with, an explanation of the different planning levels existent in the health care supply chain, followed by the existing optimization models that are used and corresponding solution methods and finally the different mathematical programming methods and its differences.

2.1 – Health Care Supply Chain planning

Similarly, to any supply chain, the planning of a health care supply chain comprises the operational, tactical and strategic planning levels.

Strategic planning is designed to help public and non-profit organizations or communities to respond effectively to their new situations. It is a disciplined effort to produce fundamental decisions and actions shaping the nature and direction of an organization or other entity activities within legal bounds (Bryson et all. 2018). The strategic planning consists in designing, dimensioning and developing a process in order to accomplish the organization´s mission. It has a long planning horizon and it is based on highly aggregated information and forecast (Hulshof et al. 2012).

Examples of strategic planning are for example: determining the facility’s location, dimensioning resource, capacities (number of MRI machines, staffing, others) and deciding on the service and case mix. (Hulshof et al. 2012).

For example, the article from Mestre et al. 2012 that proposes the location and supply of hospital services in the South region of Portugal taking into consideration the decision makers strategy to maximize patients’ geographical access to a hospital network.

On the other hand, the Tactical planning occurs after a strategic plan has been created and it transforms strategic planning decisions into guidelines that must be taken in order to reach the strategic plan goals and that will facilitate operational planning decisions (Hulshof et al. 2012). This type of planning is usually used at medium term planning horizon. (Anjomshoa et al. 2018 & Cardoen et al.2010).

As a first stage in tactical planning, patient groups are characterized based on different criteria's like disease type/diagnose, urgency and resource requirements. As a second step, the available resource capacities, settled at the strategic level, are allocated to the different patient groups. In this way, blueprints for the operational planning are created that allocate resources to different tasks, specialties and patient groups. (Hulshof et al. 2012).

Examples of tactical planning are allocation of staff like for example staff shift scheduling or the cyclic surgical block schedule that allocates operating time capacity to patient groups.

One good example of this type of Tactical planning can be found in the article of Hulshof et al. 2012, where they create a tactical plan that allocates available resources to various care processes and determine the selection of patients to be served that are at a particular stage of their process.

Finally, Operational Planning is the link between strategic objectives and the implementation of the activities (Burch, S., 2010). Operational planning involves the short-term decision making related to the execution of the health care delivery process. Inside the operational planning it exists the online and the offline operational planning: the offline operation planning reflects the planning of operations that happens before the situations really happen (i.e before schedule execution), like scheduling an appointment or assigning shifts to the staff; and the online operation planning deals with the reaction to acute events in real time (i.e during schedule execution), like for example, to reschedule an appointment because an emergency patient requires immediate attention. (Cardoen et al. 2010).

All three of the different planning levels presented are important and depend on each other, as we can understand in the article of (Guerriero & Guido, 2011) where it is possible to see the way the different planning levels interact with each other while planning an operating theatre, has demonstrated below:

Strategical Planning: It is defined how much operating room time is assigned to the different surgical groups.

Tactical Level: Based on the assigned time for each of the surgical groups, it is defined a scheduling of the several types of surgeries to a given planning period (usually from one month to a year).

Operation Planning: Taking into consideration the scheduling of surgeries it is construted the Patients scheduling for the short planning horizon.

With that being said, it is possible do understand that this thesis aims to support the strategic and tactical planning of mental health care services in order to guarantee a better resources allocation. In particular, this planning will imply making decisions on services location, capacity planning and allocation of patients to services.

2.2 – Healthcare strategic and tactical planning methods

In order to make decisions in topics like the services location, capacity planning and allocation of patients, different methods have been proposed, such as computer simulation, Markov processes and mathematical programming (Kuno, et al., 2005 & Leff et al., 2009 & Brailsford,S. and Vissers, 2011).

That can be solved using two different approaches, heuristic simulation and exact method approach.

Simulation

Computer Simulation is the reproduction of a real-world system reaction to conditions that are not easily or safely reproduced, using a computer to simulate the outcomes of a mathematical model. It is usually based on assumptions regarding the operation in the form of mathematical or logical relations between the objects of interest in the system (Law and Kelton, 2006 & Winston, 2003). Simulation models are very useful for what-if analysis, which explores the best course of action in different circumstances.

One of the main motivations to use computer simulation is that it is easier to apply than analytical methods, due to the flexibility that this method can provide in order to make changes in the model to try different scenarios and understand the impacts that a change can make in the results of the model. Analytical methods would make us consider some assumptions in order to simplify the model, computer simulation models have less restrictions than analytical methods. (Winston, 2003 & Heermann, 1990).

However, these methodologies have some limitations. In particular, it is not adequate to determine the best solution for the configuration of a health network for planning purposes, as they do not allow to calculate optimal solutions.

An example of Computer simulation can be found in the article proposed by Kuno et al. (2005), where they use a simulation-based approach for mental health system planning of the

Philadelphia mental health system that is trying to combine hospitals and residential services as a decision support tool.

Markov models

Markov models are a stochastic model used to model randomly changing systems. This model assumes the Markov property. This property says that the future (evolution of the process), depends only on the current state and not on the events that already occurred before it (Tijms, 2013). These methods allow to divide the population into discrete states of health and evaluate the transition between these states over time, according to pre-defined transition probabilities. That is the case of Leff et al. (2009), who developed a deterministic first order Markov simulation model for mental health planning. The model allows planners to assign service packages to functional level groups, that describe the states through which patient pass in the course of mental illness.

Similarly, to Computer Simulation, Markov models are simulation models that do not guarantee that an optimal solution is found.

In the paper written by Long and Meadows (2018), where were identified 160 papers written about simulation modelling in mental health care, it was identified that two- thirds of the papers written were using Markov modelling.

Mathematical programming

Finally, mathematical programming allows to obtain the optimal solution of optimization models by maximizing or minimising an objective function, respecting a set of constraints that circumscribe the decisions variables (Jensen, and Bard, 2003)

Thus, mathematical programming models can represent real world interactions through mathematical relationships (such as equations, inequalities and logical dependencies), mostly independent of the data in the model. These models can be classified as linear programming (LP) models, non-linear programming (NLP) models and integer programming (IP) models (Williams, H.P., 2013). Within LP there is a particular class of models, the so-called mixed integer linear programming (MILP) models, capable of modelling several types of problems and characterized by linear mathematical expressions containing both continuous (real) and discrete (integer) variables.

Mathematical programming approaches have been increasingly used to support the optimal planning of health care services, by providing guidance on location selection and capacity planning decisions in the health care sector (Brailsford,S. and Vissers, 2011).

Solution Approaches

All the optimization models referred above can be solved using two different solution methods: Exact method solution or Heuristic method solution.
On one hand, exact methods solutions are guaranteed to find an optimal solution for the model, although the more complex the problem is, the more time it will take to be solved. On the other hand, Heuristic methods can help to obtain a fair solution in less time but don´t offer an optimal solution to the model.

Heuristics method

Heuristics methods are systematic methods used to solve optimization problems by improving presented solutions when exact approaches take too much computation time. Although useful for quick resolutions, they do not guarantee that an optimal solution is found as well (Aarts et al., 2003 & Winston, 2003)

An example of this method can be found in the article of Anderson et al. (2017) where it is proposed a mathematical model in order to ensure an improved distribution of existing beds in a hospital in Denmark. In order to do this, a continuous – time Markov chain model was built and solved by the heuristic method in just under 30 minutes. Once the result of the heurist was a low value, it was also done a complete enumeration of the search space in order to reach the optimal solution. The process to reach the optimal solution was 5 days and 9 hours.

Exact Method

The exact method allows the finding of optimal solutions but are often extremely time-consuming when solving real -world problems.

The exact method solution approach can be found in the article of Testi and Tànfani (2009), where in order to solve an Operating room planning problem it is used an exact method approach that reaches to an optimal solution to this problem.

Within the context of this thesis, mathematical programming models seems to be the one with the most potential to be used, since unlike simulation and markov models that are not adequate to determine the optimal solution, mathematical programming models can reach an optimal solution what is important in the case of mental health care planning.

The following section thus presents the main features considered relevant for health care planning and provides examples of studies using mathematical programming models for planning the delivery of health care services.

2. 3 Mathematical programming models for health care planning

Mathematical programming models have been widely used in the health care planning literature (Brailsford and Vissers, 2011).

The existing models in this area of research differ from each other in several aspects like: the planning propose; in the number and types of services accounted for; in the number and types of objectives pursued; and in the consideration or not of uncertainty aspects.

However, an adequate planning of health care networks must take into consideration: (i) the multi-service nature of health care systems; (ii) the joint effect of multiple objectives relevant in the health care sector; and (iii) the impact of uncertainty on planning decisions.

2.3.1 – Single and multi-service planning models

It exists two different types of planning models in the health care planning literature in what concerns the number of services to be planned. The ones that take in consideration just a single service are the most commonly referred in the literature, like it is the case of Oliveira and Bevan [14] and Ben Abdelaziz and Masmoudi [18]. On the other hand, the multi-service models have been more explored in the last few years, but there is still little research on the topic. Examples of multi-service planning models are proposed by Teshebaeva and Jain (2007) Santibáñez et al. (2009) Mestre et al. (2012) and Cardoso et al. (2016) who all developed mathematical programming models in health care while studying the delivery of multiple health care services (such as inpatient, outpatient and emergency care services).

2.3.2 - Single and multi-objective planning models

Models found in the health care literature can also be either single-objective studies or multi-objective studies.

Multi objective planning model have the advantage to better reflect the goals of the decision makers and give them the opportunity to understand the different trade-offs they can obtain with the different objectives. A drawback of the multi objective approach is that the increasing number of objectives chosen also results in an increase in the computational time of the model. (Decerle et al, 2019).

According to Buchanan, et al. (1996), the possibility of the decision makers to understand the trading off a reduction in one objective for a gain in another is one of the basis of multi-objective planning models.

Most of the existing studies are single objective (Stummer et al., 2004) due to the challenge that is the elaboration of a multiple objective study. Nevertheless, there has been an increasing interest in the development of multi-objectives approaches. Examples of multi-objective mathematical programming models can be found in Mestre et al. (2012); Cardoso et al (2016); Oliveira and Bevan (2006); Drezner and Drezner (2011); Mitropoulos et al. (2006); Syam and Côté (2010); Sun et al. (2014); Stummer et al. (2004) and Koyuncu and Erol (2010).

In terms of objectives, the most widely used within the health care literature are equity, efficiency, cost and health gains.

In mental- health sector, the objectives are especially in the areas of cost -effective analysis and epidemiology (Long & Meadows, 2018). As an example, we can see the paper of Cardoso et al. (2015).

2.3.3. Deterministic models and models accounting for uncertainty

There are mathematical programming models who do not take uncertainty into consideration, and these are called deterministic models that are based on initial conditions and parameters with no uncertainty associated. This means that the output of the model will always be the same if run multiple times, once the output is only dependent on the input data and the structure of the model (Marino et al. 2008).

When it comes to account for uncertainty in mathematical programming models, different approaches can be followed. Like for example, stochastic models, robust models (Snyder, 2006; Verderame et al., 2010) or can simply use sensitive analysis (Owen and Daskin, 1998).

Sensitive analysis is the simplest way to face uncertainty, and is the process of recalculating outcomes by varying the values of parameters that are recognized as uncertain parameters (Cardoso et al. 2015).

An example of this method can be found in the article of Mestre et. al (2015), where it is used to handling uncertainty in two locations – allocation models with the aim to inform how the hospital networking system may be reorganized in order to improve geographical equity while minimizing costs.

Accordingly, to the existing literature, stochastic models have been applied a lot in allocation problems and capacity planning. All Stochastic Programming problems are divided in two stages. The first stage is to resolve variables that do not need to take uncertainty into consideration and the second stage is to resolve variables that can only be solved after uncertainty has been resolved. (Snyder, 2006). According to Piedro (2019), there are three different main sources of uncertainty used in the stochastic programming problems: Demand, Process and Supply.

As a limitation of the stochastic model we can find the need to define different scenarios each with a probabilitie assigned and the fact that the number of scenarios choose (usually a small number) will be a limitation to the future results that the model can assume.

An example of stochastic models can be found in the work of Abdelaziz and Masmoudi (2012), Mestre et al. (2015), Cardoso et al. (2015) and Shi et al. (2018) who all proposed stochastic models for health care planning. Abdelaziz and Masmoudi (2012) propose a multi-objective stochastic model to assign beds to hospital departments when the demand for beds is random. Mestre et al. (2015) developed two stochastic location–allocation models to assist hospital network planning under uncertain conditions. The study focuses on uncertainty associated to the demand for hospital services (more specifically, the impact of demand changes from populations that are ageing and experiencing increases in life expectancy), modelled through a set of discrete scenarios that illustrate future possible realizations of the uncertain parameters.

Cardoso et al. (2015) suggests the reorganization of a long-term care network, based on a stochastic mixed integer linear programming model. The model aims at minimizing the total costs while considering demand uncertainty.

At last, Shi et al. (2018) proposed a Stochastic Programming model with Recourse (SPR model) has a solution to the Home Health Care (HHC) routing problem with stochastic travel and service times, which comes from the logistics practice of HHC companies.

Another way to deal with uncertainty is through the development of a robust model and through the robust counterpart approach (a new approach developed from the robust model).

The robust optimization (RO) has been used to immunize deterministic optimization problems against infeasibility, caused by perturbations in model parameters, while simultaneously preserving computational tractability. (Goh & Sim, 2010)

The robust model is used has an alternative to the stochastic model when no probability information about the uncertainty parameters is known and it is embedded within a rolling horizon framework to capture parameter values that may change over time.

The robust model also differs from the stochastic model in the objective, normally stochastic models have the objective to reduce expected costs while robust modelling aim at minimizing the possible lost for a worst-case scenario.

According to Snyder (2006), robust models are sometimes overly conservative and sometimes reckless, once it presents the solution of the worst possible situation has the optimal solution.

A robust model example can be found in the article of Luke Muggy and Jessica L. Heier Stamm (2018) where it was developed a dynamic, robust optimization framework to locate post-disaster health care service facilities. The model incorporates measures of accessibility and equity, both important in the context of health care services.

Accordingly, to the existing literature that has been referred throughout this chapter, we can conclude that when planning the supply of mental health care, it should be approached with a multi service perspective due to the fact that it is a sector that offers multiple services like for example inpatient, outpatient and emergency care services. It should also include multi – objectives due to the importance that this brings to the decision makers and that also takes uncertainty into consideration, since there are some parameters that are uncertain like for example the demand of this type of services.

2.3.4 - Mental Health supply Chain planning

After exploring the Health Care literature, it is now important to look at what is being done in the Mental Health Care field specifically.

According to the article of Kuno et al. (2015), the mental health sector is in an advanced era of deinstitutionalization and face the challenge of trying to combine residential, ambulatory treatment and support services in order to reach the optimal service system.

Several researchers developed mathematical programming models to aid planners with the deinstitutionalization process, for instance Wolpert and Wolpert (1976), Muraco et al. (1977) and Leff et al (1983). Wolpert and Wolpert (1976) developed an assignment model in order to reallocate released patients from mental hospitals to residential communities. The model attempts to maximize favourable treatment outcomes for patients by evaluating the distribution of patients across treatment modalities and re-assigning them, when necessary, in order to achieve a more favourable therapeutic outcome. Muraco et al. (1977) proposed an allocation model applied to the Lucas County (Ohio) mental health delivery system, in order to analyse the impact of decentralized and centralized strategies when demand is geographically concentrated. The objective considered was the minimization of patients travelling distance subjected to service constraints. And finally, Leff et al (1987) proposed a multi-period mathematical programming model for resource planning and policy evaluation to aid health planners making resource allocation decisions in the mental health care sector. The model developed optimizes the decisions of allocating resources to programs and assigning programs to patients.

At this point, little research has been found in what concerns applying mathematical programming in the mental health sector. This is an area where few models exist (according to the literature review developed, the only studies that were found are the three studies described above), and these few models only account for one single objective and no uncertainty analysis was performed.

The models refereed before have some disadvantages when compared with other types of models referred in the previous section of this literature review. First, all the models referred before only account for one single objective. Wolpert and Wolpert (1976) focuses in attempting to maximize favourable treatment outcomes for patients, while Muroco et al. (1977) tries to minimize the distance cost to patients and Leff et al. (1987) focuses in different objectives like the maximization of the total improvement in patients' functional levels and minimizing the number of patients with minimal level. The model of Leff et al. (1987) does not count as being multiple objective, once the different objectives are analysed separately in different scenarios by a single objective model. Another disadvantage is the fact that neither of the models presented account for uncertainty and because of that they do not represent correctly the reality, since mental health planners operate in a very uncertain environment (Franz, L. S. Rakes et al., 1984). Finally, in terms of number of services, all the previous studies only analyse one single service which is not the recommended since mental health care patients require a multiple response of different types of service like for example inpatient care, outpatient care and home-based care. In neither of these models this necessity is presented as it should.

After analysing the studies that exists and the improvements needed, it is possible to understand that a gap in the literature was found, since in regard to the Mental Health Sector, no studies that respect all the necessities of the sector have been identified. In particular, no stochastic model has been developed although has seen before is one of the most indicated to due to being able to consider uncertainty, can analyse multiple objective and can present an optimal solution to the model.

2.4 Conclusions

In order to resume the research presented in this literature review, the following table presents a summary on the key planning issues that should be considered when developing planning models in the health care sector in general, and in the mental health care sector in particular. With this table it is easy to identify a gap in the existing literature, since no study exists proposing a planning model that considers the specificities of the mental health care sector, while aiming at planning its multiple services, accounting for a multiplicity of policy objectives and also while accounting for the impact of uncertainty in planning decisions.

Table 1 – Specifications of the different studies using mathematical programming models in the health care services in general and more particularly in the mental health care.

Studies	Multiple Services	Multiple Objectives		Planning under uncertainty	Mental Health Care Sector Application	NHS
		Cost	Equity			
[4]						X
[7]		X		X		
[15]	X					X
[17]			X			X
[18]	X			X		X
[19]	X	X		X		X
[20]					X	
[25]						X
[28]	X	X		X	X	
[41]	X				X	
[44]					X	
[45]				X	X	
[49]	X	X	X	X		X
[51]		X				
[55]				X		
[62]						X
[63]		X				
[66]	X	X		X		X
[68]	X	X			X	
[70]		X	X			
[79]				X		
MHCU	X	X	X	X	X	X

In conclusion, this thesis major focus is to fill the existing gap in the literature, by developing a mathematical programming model that looks at the multi-service nature of the mental health care sector, while taking uncertainty into consideration and also taking into consideration multiple objectives.

3 - Planning background

In this section, a description of the way the Portuguese mental health network is organized is presented.

3.1 – Mental Health Care

In the year of 2001, the OMS recommended in its World Health Report, for the developed countries to close the Psychiatric Hospitals and to develop alternative residences at the local level, in order to assure the best possible quality of mental health care.

Following this recommendation, the Portuguese government created the National Plan of mental health for the years of 2007 to 2016, with the objective to meet with the recommendation of the WHO. (OMS, 2001)

Nowadays, the Portuguese mental health network is organized at the regional and local level, where each facility is responsible for a specific geographical area. All of this geographical area are part of the national health system (SNS) (Xavier et al., 2017).

The basic units of mental health care are Local Mental Health Services (SLSM – Serviços Locais de Saúde Mental), articulating with primary health care (ACES and USF) as well as two autonomous psychiatric hospitals.

The Local Mental Health Services are responsible for ensuring the provision of mental health care, both on an outpatient and inpatient population of a given geographical area through a network of programs and services ensure continuity of care.

The ACES (Agrupamento de Centros de Saúde) are public health services with administrative autonomy whose mission is to ensure the provision of primary health care to the population of a given geographical area. Nowadays there are 55 ACES spread across Portugal mainland according to the 2017 Annual Report on Health Care Access in NHS established and agreed entities. (Relatório Anual de acesso a cuidados de Saúde nos estabelecimentos do SNS e entidades convencionadas, 2017).

The USF (Unidade de Saúde Familiar) is a functional unity of both ACES and ULS (Unidades Locais de Saúde), that provides health care, based on a multidisciplinary team of doctors, nurses and clinical secretaries and who develops their activity with autonomy organizational, functional and technical, integrated in a network logic with the other units.

According to the Nation Plan of mental health for the years 2007 to 2016 referred before, the Psychiatric Hospital have the mission to continue to ensure the local level care in there geographical are for which are responsible as long as no local services are created in these areas.

The Substance Abuse disorder is the only type of disease group which the care is provided outside of the previously presented network, being provided by the ARSs (Regional Administrations of Health) and the SICAD (Service for Intervention in Addictive Behaviours and Dependencies).

This network is also supported by other non-governmental institutions that give support along.

With that being said, the Portuguese Mental Health care is composed by three Psychiatric Hospitals (Centro Hospitalar do Porto,EPE; Centro Hospitalar Universitário de Coimbra; EPE, Centro Hospitalar de Lisboa Central, EPE). Along with 44 general hospitals that provide integrated psychiatric services. 12 entities with agreement with the Portuguese SNS to provide mental health care. And finally, 26 Non- Governmental organization.

Table 2.1: Number of NHS establishments with mental health care by ARS

	ARS North	ARS Center	ARS Lisboa and Vale do Tejo	ARS Alentejo	ARS Algarve	Total
Psychiatric Hospitals	1	1	1	0	0	3
Local Services	16	10	13	3	2	44
IPSS	4	2	5	1	0	12
NGOs	4	5	13	1	3	26

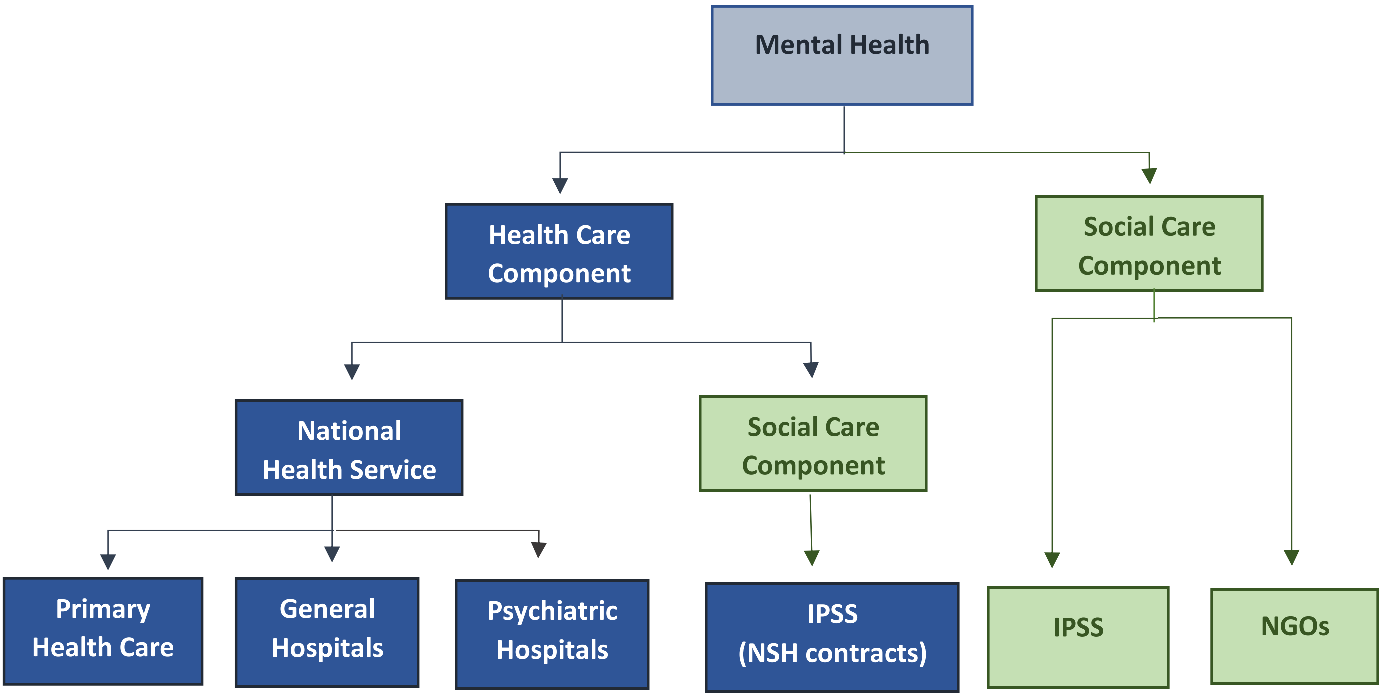
ARS- Regional Health Administration

3.2 – Mental Health Care Organization in Portugal

The Mental Health in Portugal is divided in two main sectors, the Health care component where it is included all National Health sector services and the Social Care Component (which are private institution services provided by NGO´s) and IPSS (Instituições Particulares de solidariedade social).

Inside the National Health Sector services, there are also IPSS contracted by the NHS,Primary Health Care Centers as well as General Hospitals, Psychiatric Hospitals and IPSS subcontracted by the NHS (Figure.1).

Figure 1 – Mental Health Care Structure



Inside the Mental Health Sector in Portugal, different services are provided to the population. There are institutional care (IC), ambulatory care (AC), home – based care (HBC), occupational unit (OU) and residential unit (RU). These services are delivered by multidisciplinary teams which include physicians, nurses, social assistants, general practitioners and occupational therapists, among others.

Institutional Care is provided to acute patients (short term patients) and to chronic patients (long term patients). The public sector deliveries this service mainly to acute patients while the social sector provides it both to the chronic patients as well as to acute patients, although it focuses more on the chronic patients.

Ambulatory Care is a specialised mental health service that aims to provide services to patients who are not currently admitted to a hospital or residential service. This is provided in general Hospitals, Psychiatric Hospitals and IPSS (including day hospitals/ day center´s).

Home – based care is provided in the scope of primary health care services but is also delivered by general and psychiatric hospitals teams.

The occupational unit is aimed at patients with low and moderate psychosocial disability. This service is the only one that des not involve the residence of the patient and it is aimed to help in the reinsertion and integration in professional training programs.

The residential unit provides accommodation for patients that are stable at a clinical level but that don´t have the needed support at home. This residential units are divided in three different levels accordingly to the degree level of psychosocial disabilities: Life autonomous unit (low degree), Life protected unit (moderate degree), Life support unit (high degree).

Each type of institution referred previously provides different types of services as showed in the figure 3.

Table 2.2 - Services provided by Institution

	IC	AC	HBC	OU	RU
National Health Sector					
Psychiatric Hospitals	X	X	X	X	X
Local Services	X	X	X		
PHCCs			X		
Social Sector					
IPSS	X	X		X	X
NGOs				X	X

3.3 – Mental Health Care Capacity in Portugal

According to the article of Xavier (2017), the number of beds in Portugal mainland in the year of 2016 has decreased 8 % when compared to the year of 2005 due to the decrease in the number of public beds.

In the public sector, the number of beds for acute patients decreased from 1383 in 2005 to 1108 in 2016 and for chronic patients has decreased from 1364 in the year of 2005 to 479 in 2016. This reduction in the public sector was balanced by an increase in the number of private beds subcontracted by the NHS.

When looking at the number of beds we can see that in psychiatric hospitals in 2016 the percentage of beds has decreased more than 70 % when compared with the year of 2005. The other way around happened with the number of beds in general hospitals that have raised 8 % in 2016 when compared with the year of 2005. This raise in the number of beds in general hospitals was not enough to compensate the decrease in the number of beds in the psychiatric hospitals.

The decrease in the number of beds in psychiatric hospitals is aligned with the Portuguese National Plan of mental health of 2007 to 2016 referred before, with the aim to decrease the importance of psychiatric hospitals with the intention to decentralize the mental health care system.

In terms of human resources, there was a raise in the number of psychiatrics both in adult hospitals and in children and adolescents' hospitals. Unfortunately, the same did not happened with the nurses and social workers, which the reduction continues to be an alarmistic once it can affect the functioning of the mental health sector.

3.4 – Planning Decisions

Several planning decisions may need to be made when planning a network of mental health care services. Particularly, the following decisions need to be considered:

- The existing locations/institutions that should be opened or closed?
- Should the locations/institutions that are open provide all the different type of services (Institutional Care, Ambulatory Care, Residential unit Care and Occupational care) that are capable to provide?
- How many beds should exist in the service? How many need to be added? And reallocated?
- Which patients should be allocated to which location?

3.5 – Policy Objectives

The WHO mental health action plan for the years of 2013 to 2020, presents the universal health coverage as one of the principle approaches to its action plan. Using the principle of equity and aiming to ensure that every patient receives the necessary mental health care services regardless of their age, sex, socioeconomic status, race, ethnicity or sexual orientation. (World Health Organization, 2013).

The Portuguese government also published the "Modelo de Governação a 2020, Plano Nacional de Saúde e programas de Saúde Prioritários" where similarly to the WHO action plan, it focusses the importance of having services in the community accessible for everyone (Direção Geral de Saúde, 2017).

However, with the budget restrictions that exists in a country like Portugal, it is neither possible neither recommended to do the necessary investment in order to be able to provide universal health care. The Joint Action on Mental Health and Well-being (2015) suggest taking decisions that are cost-effective.

Once the optimal solution can not be reached do to this economic constraint, it is necessary to find a "satisficing" path although not being the optimal solution. (Simon, 1956)

The following equity objectives were chosen due to being key policy objectives (Barros, et al. 2011):

1) **Equity of access:** Patients should receive the treatment needed as close as possible to their residence in order to guarantee the most easy and comfortable dislocation of the patient. A maximum travel time value should be defined (according to the DM´s point of view).

2) **Geographical Equity:** There should not exist discrepancies in the way the services are provided in the different regions. It is important to guarantee that people in different areas of residence are getting access to the same type of care. This can be fixed by imposing a minimum service level value across regions (according to the DM´s point of view).

3) **Equity of service utilization:** It is also important to make sure that some services are not ignored when compared with others, due to being more expensive to treat then others

for example. In order to assure this, it can be interesting to fix a minimum service value for each type of service (according to the DM´s point of view).

4) **Equity of disease utilization:** The situation explained before with the different types of services can also be applied to patients with different diseases types having a different access to care. It is important to give to the decision makers the possibility to assign a specific value to the minimum service level to each disease.

3.6 - Structuring Uncertainty

Due to the unpredictability associate with the mental health care, that depends on factors like epidemiological factors or demographic profiles that cannot be predicted in an exact manner. It was decided to build a model that takes uncertainty into consideration in order to mitigate the risk.

Using the scenario tree approach (Bierge and Louveneaux,1997), a probability scenario tree was built with all the possible number of scenarios represented. Assigning a value of demand and a value of length of stay for each branch and each branch with a probability of happening associated with it.

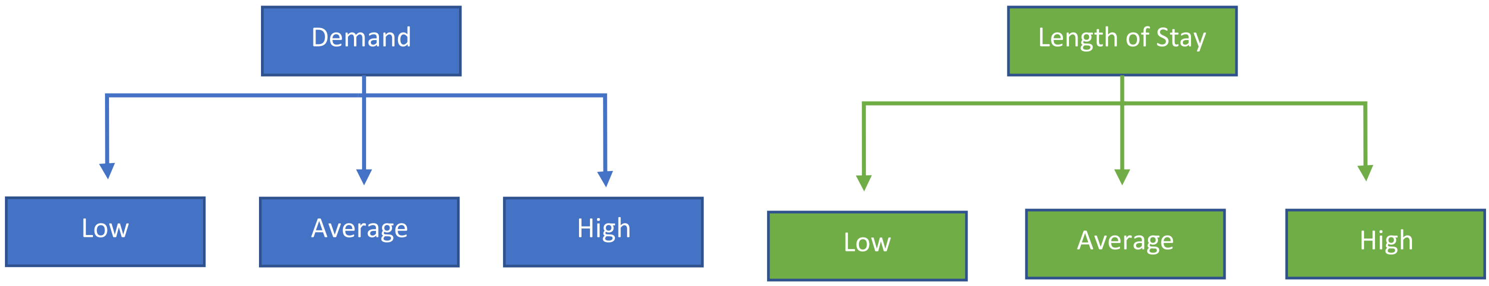
The parameters where demand and length of stay since this values cannot be predicted with total confidence.

The extended Pearson Tukey specifies what three outcomes to choose and the three probabilities to assign to each outcome (Clemen and Reilly, 2003). Following this method, for each one of the parameters (demand and length of stay), three different values were taking into consideration according to the probability of each one to occur.

The values of demand and length of stay can either be low, average or high. Each one with a different value associated with it.

In terms of the probability tree, branches with average values of demand and length of stay have a higher opportunity to occur, compared with extreme values like low or high., like explained in the extended Pearson Turkey method (Clemen and Reilly, 2003).

Figure 2 – Probability Tree Scenario



4 – Model

This section presents the details of the MHCU model used during this thesis. It is presented the notation used (section 4.1), the objective function (section 4.2) and finally the constraints used (section 4.3).

The MHCU model was built using as a starting point the model presented by Monteiro (2016).

The model of Monteiro (2016) is a MILP (mixed integer linear problem) used to reformulate the mental health care network in the region of Lisbon.

The MHCU model it is a stochastic model applied to the mental health care network in the region of Lisbon. All the changes done to the model will be presented in the following sections.

4.1 – Notation

The list of indices and sets used are going to be presented in the following tables:

Table 3.1: List of Indices

Indices	Description
t, t´	Time period
d	Demand Points
s, s´	Mental Health Care Services
l, l´	Location for services
p	Patient groups
a	Age groups
j, j´	Type of mental health care provider
n	Scenario tree nodes

Table 3.2: List of Sets

Sets	Description
T	Set of time periods
D	Set of demand points
P	Set of patient groups
A	Set of age groups
NT	Set of scenario tree nodes
$S = S^1 \cup S^2 \cup S^3 \cup S^4 \cup S^5$	Set of mental health care services divided into subsets S^1 (subset of IC services, $s, s' \in S^1 \subseteq S$), S^2 (subset of AC services, $s, s' \in S^2 \subseteq S$), S^3 (subset of HBC services, $s, s' \in S^3 \subseteq S$), S^4 (subset of RU services, $s, s' \in S^4 \subseteq S$) and S^5 (subset of OU services, $s, s' \in S^5 \subseteq S$)
$L = L^1 \cup L^2 \cup L^3 \cup L^4$	Set of location for services divided into subsets L^1 (subset of locations for IC services, $l, l' \in L^1 \subseteq L$), L^2 (subset of locations for AC services, $l, l' \in L^2 \subseteq L$), L^3 (subset of locations for HBC services, $l', l' \in L^3 \subseteq L$) and L^4 (subset of locations for RU and OU services, $l, l' \in L^4 \subseteq L$)
$M = \{(s, l) : s \in S, l \in L\}$	Set of services s that can be provided in locations l
$V = \{(s, l) : s \in S, l \in L\}$	Set of services s provided in locations l at the beginning of the planning horizon
$\overline{V} = \{(s,l) : s \in S, l \in L\}$	Set of services s not provided in locations l at the beginning of the planning horizon
$C = \{(l, j) : l \in L, j \in J\}$	Set of locations l of services delivered by provider j
$F = \{(d, l) : d \in D, l \in L\}$	Set of demand points d that can receive care in locations l

Table 3.3: List of Parameters

Parameters	Description
ni_{dpastn}	Number of individuals from demand point d, patient group p, age group a requiring service s at t in scenario tree node n
niD_{dtn}	Number of individuals from demand point d requiring care at t in scenario tree node n
niS_{stn}	Number of individuals requiring service s at t in scenario tree node n
niP_{ptn}	Number of individuals from patient group p requiring care at t in scenario tree node n
ni_s^{min} / ni_s^{max}	Minimum/Maximum number of individuals allowed per AC, HBC and OU service s(s ∈ (S^2 ∪ S^3 ∪ S^5) ⊆ S)
nb_{sl}	Number of beds available in IC and RU service s(s ∈ (S^1 ∪ S^4) ⊆ S) located in l(l ∈ (L^1 ∪L^4) ⊆ L) at t=0
LOS_{sn}	Average length of stay, measured by the number of days in IC and RU service s(s ∈ (S^1 ∪S^4) ⊆ S) in scenario tree node n
nb_s^{min} / nb_s^{max}	Minimum /maxium bed capacity allowed per IC and RU service s(s ∈ (S^1 ∪S^4) ⊆ S)
oc_{st}	Operational cost per service s per period t (in €)
Ic_{st}	Investment cost per new bed installed in IC and RU service s(s ∈ (S^1 ∪ S^4) ⊆ S) at t (in €)
rc^{dif}_{st}	Cost of reallocating a bed to IC and RU service s(s ∈ (S^1 ∪ S^4 ⊆ S) from a service delivered in a different location at t (in €)
rc^{same}_{st}	Cost of reallocating a bed to IC and RU service s(s ∈ (S^1 ∪ S^4 ⊆ S) from a service delivered in the same location at t (in €)
T_{dl}	Travel Time between demand point d and service location l (in minutes)
T^{max}	Maximum travel time allowed for patients to access services (in minutes)
T_{tn}^{tot}	Maximum total travel time at t (in minutes) in scenario tree node n
ϵ_s	Efficiency factor associated with the provisiono f service s
α	Number of days per time period
B_t	Minimum level of satisfied demand per service s at t
Pn	Probability of scenario tree node n
T^{EA}, T^{GE}, T^{ESU},T^{EDU}	Equity targets set by DM at the end of the planning horizon (corresponding to the desired levels of achievement)

Table 3.4: List of Variables

Variables	Description
Binary variables	
X_{sljt}	Equal to 1 if service s is located in l delivered by provider j at t; 0 otherwise
A_{siljt}	Equal to 1 if at least one bed is reallocated to IC and RU service s(s ∈ (S^1 ∪S^4) ⊆ S) located in l(l ∈ (L^1 ∪L^4) ⊆ L) delivered by provider j at t; 0 otherwise
Integer variables	
ID_{dtn}	Number of individuals from demand point d receiving care at t in scenario tree node n
IS_{stn}	Number of individuals receiving s at t in scenario tree node n
IP_{ptn}	Number of individuals from patient group p receiving care at t in scenario tree node n
$NB_{dpasljtn}$	Number of beds to be made available for individuals from demand point d, patient group p and age group a receiving IC and RU service s (s ∈ (S^1 ∪ S^4) ⊆ S) located in l(l ∈ (L^1 ∪ L^4) ⊆ L) delivered by provider j at t in scenario tree node n
NAB_{sljtn}	Number of additional beds to invest in for IC and RU service s(s ∈ (S^1 ∪ S^4) ⊆ S) located in l(l ∈ (L^1 ∪ L^4) ⊆ L) delivered by provider j at t
$NRB^{in}_{sljs'l'j't}$	Number of beds reallocated from IC and RU service s (s ∈ (S^1 ∪ S^4) ⊆ S) located in l (l ∈ (L^1 U L^4) delivered by provider j from IC and RU service s´ (s ∈ (S^1 ∪ S^4) ⊆ S) located in l (l ∈ (L^1 U L^4) delivered by provider j in scenario tree node n
$NRB^{out}_{sljs'l'j't}$	Number of beds reallocated to IC and RU service s (s ∈ (S^1 ∪ S^4) ⊆ S) located in l (l ∈ (L^1 U L^4) delivered by provider j to IC and RU service s´ (s ∈ (S^1 ∪ S^4) ⊆ S) located in l (l ∈ (L^1 U L^4) delivered by provider j in scenario tree node n
Positive Variables	
$Q_{dpasljtn}$	Proportion of individuals from demand point d, patient group p and age group a receiving service s in location l delivered by provider j at t in scenario tree node n
TT_{tn}	Total travel time (in minutes) at t in scenario tree node n
TO_{tn}	Total operational cost at t in scenario tree node n
TI_t	Total investment cost at t
$f^{EA}, f^{EDU}, f^{ESU}, f^{EDU}$	Multiple equity objectives (EA, GE, ESU and EDU)

4.2 – Defining the multi objective function

The objective functions of the model consist in minimizing the cost and maximizing the different equity objectives in order to meet with the policy objectives.

In order to obtain the maximization of the different equities, we need to look for the minimization of its value, like presented in the following functions:

Equity of access

In order to guarantee that patients receive care as close as possible to its residence, we aim to minimize the equity of access value (f^{EA}). The equity of access value is obtained by dividing the total travel time (TT_{tn}) by the maximum total travel time (T_{tn}^{tot}). The result will be presented in a common [0,1] scale.

$$\boldsymbol{Min\, f^{EA} = \frac{\sum_{n \in NT} \sum_{t \in T} TT_{tn}}{\sum_{n \in NT} \sum_{t \in T} T_{tn}^{tot}}} \tag{4.1}$$

The equation that define the total travel time (TT_t) and the maximum total travel time (T_t^{tot}) are also presented:

$$\boldsymbol{TT_{tn} = \sum_{d \in D} \sum_{p \in P} \sum_{a \in A} \sum_{s \in S} \sum_{\substack{l \in L \\ l:(s,l) \in M \\ l:(d,l) \in F}} \sum_{\substack{j \in J \\ j:(l,j) \in C}} Q_{dpasljtn} \times ni_{dpastn} \times T_{dl} \; \forall\, t \in T, n \in NT} \tag{4.2}$$

$$\boldsymbol{T_{tn}^{tot} = \sum_{d \in D} \sum_{p \in P} \sum_{a \in A} \sum_{s \in S} ni_{dpastn} \times T^{max} \; \forall\, t \in T,\ n \in NT} \tag{4.3}$$

Geographical Equity

The geographical equity aims at not existing discrepancies in the way the services are provided in the different regions. It is calculated as a percentage of patients in the geographical area that receives the less amount of care and is then compared with the other regions. It is also represented in a common [0,1] scale. This means that a 0.2 value indicates that 20% of the people that live in the geographical area that receives less amount of care are not receiving care.

$$\boldsymbol{Min\, f^{GE} = max_d \left(1 - \frac{\sum_{n \in NT} \sum_{t \in T} ID_{dtn}}{\sum_{n \in NT} \sum_{t \in T} niD_{dtn}} \right)} \tag{4.4}$$

This objective ensures the maximum provision of care in the geographical area, with the lowest level of provision.

Equity of service utilization

The equity of service utilization aims at guaranteeing that some services are not ignored when compared with others. Similarly, to the geographical equity, it is calculated by maximizing the number of patients that receive care of the service with the lowest level of provision. This equity is also represented in a common [0,1] scale. A 0.2 value means that care is not provided to 20% of the patients is need of the service with the lowest level of provision.

$$\boldsymbol{Min\ f^{ESU}} = \boldsymbol{max_s}\left(\mathbf{1} - \frac{\sum_{n\in NT}\sum_{t\in T}\boldsymbol{IS_{stn}}}{\sum_{n\in NT}\sum_{t\in T}\boldsymbol{niS_{stn}}}\right) \tag{4.5}$$

Equity of disease utilization

The equity of disease utilization follows the same logic as the geographical equity and the equity of service, aiming at guaranteeing that a type of disease is not neglected when compared with others. It is calculated by maximizing the number of patients that receive care for the disease type with the lowest level of provision. This equity is also represented in a common [0,1] scale and follows the same logic as the geographical equity and the equity of service.

$$\boldsymbol{Min\ f^{EDU}} = \boldsymbol{max_p}\left(\mathbf{1} - \frac{\sum_{n\in NT}\sum_{t\in T}\boldsymbol{IP_{ptn}}}{\sum_{n\in NT}\sum_{t\in T}\boldsymbol{niP_{ptn}}}\right) \tag{4.6}$$

Cost

The minimization of the total cost is also one of the objectives of the model. The total cost is obtained by the sum of the operational cost (Eq.4.7) and the investment cost (Eq.4.8).

$$TO_{tn} = \sum_{d\in D}\sum_{p\in P}\sum_{a\in A}\left(\sum_{s\in(S^1\cup S^4)}\ \sum_{\substack{l\in(L^1UL^4)\\ l:(s,l)\in M\\ l:(l,j)\in F}}\ \sum_{\substack{j\in J\\ (l,j)\in C}} \boldsymbol{NB_{dpasljtn}} \times \boldsymbol{oc_{st}} + \sum_{s\in(S^2\cup S^3\cup S^5)}\ \sum_{\substack{l\in(L^2UL^3\cup L^5)\\ l:(s,l)\in M\\ l:(d,l)\in F}}\ \sum_{\substack{j\in J\\ j:(l,j)\in C}} \boldsymbol{Q_{dpasljtn}} \times \boldsymbol{ni_{dpastn}} \times \boldsymbol{oc_{st}}\right) \forall\, t\in T, n \in NT \tag{4.7}$$

$$TI_t = \sum_{s\in(S^1\cup S^4)} \sum_{\substack{l\in(L^1UL^4)\\ l:(s,l)\in M\\ l:(l,j)\in F}} \sum_{\substack{j\in J\\ (l,j)\in C}} \left(NAB_{sljt} \times ic_{st} \right. \tag{4.8}$$

$$+ \sum_{s\in(S^1\cup S^4)} \sum_{\substack{l'\in(L^1UL^4)\\ t':(s',l')\in M\\ l'\neq l}} \sum_{\substack{j'\in J\\ j':(t',j')\in C}} NRB_{sljs'l'j't} \times rc_{st}^{diff}$$

$$\left. + \sum_{\substack{s'\in(S^1\cup S^4)\\ s':(s',l)\in M}} NRB_{sljss'l'j't} \times rc_{st}^{same} \right) \forall\, t \in T, n \in NT$$

The operational cost (Eq.4.6) is obtained by the operational cost associated with the number of beds in IC and RU services and with the cost associated with the provision of the other services (AC, HBC, OU). The investment cost is obtained by the investment in new beds and with the cost associated with the reallocation of the beds that already exist.

The minimization of the total cost is given by:

$$Min\, f^C = \sum_{n\in NT} \sum_{t\in T} (TO_{tn} + TI_t) \tag{4.9}$$

4.3 – Constraints

In the following section, the several constraints present in the model are described: i) Assignment of patients, ii) Open and closure of services, iii) Capacity; iv) Resources requirements; v) Resources realocation; vi) Minimum service level

Once again, the constraints used in this model were built from the model presented by Monteiro (2016) and the model of Cardoso et al. 2015 and modified in accordance to the needs of this model.

Assignment of patients

To start with, it was defined that patients could only be assigned to services s in location l where the services needed was being provided ($X_{sljt} = 1$), represented in Constraint (4.10).

$$Q_{dpasljtn} \leq X_{sljt} \forall p \in P, a \in A, s \in S, l \in L: (s,l) \in M, (l,j) \in C, (d,l) \in F, t \in T, n \in NT \quad (4.10)$$

In order to guarantee that patients would receive care in the closest location to them possible in order to avoid patients to travel a long distance to receive care, the constraint (4.11) was built.

$$X_{sljt} + Q_{dpasl'jtn} \leq 1 \; \forall p \in P, a \in A, s \in S, (l,l') \in L: (s,l) \in M, (s,l') \in M, (l,j) \in C, (l',j) \in C, (d,l) \in F, t \in T, n \in NT, \tau dl < \tau dl', l \neq l' \quad (4.11)$$

The model also ensure with constraint (4.12) that if the only available location where patients can receive care is located beyond the maximum travel time defined in the model (T^{max}) then the service should not be provided.

$$Q_{dpasljtn} = 0 \; \forall p \in P, a \in A, s \in S, l \in L: (s,l) \in M, (l,j) \in C(d,l) \in F, t \in T, n \in NT_{,Tdl} > T^{max} \quad (4.12)$$

Open and closure of services

The following constraints were built in order to limit the possibility of services opening and closing over the planning horizon.

Constraint (4.13) and (4.14) are only applicable to IC,RU and OU services. Constraint (iv) states that if a service is open at the beginning of the planning horizon and it closes, then it must stay closed till the end of the planning horizon.

$$X_{sljt'} \leq X_{sljt} \forall s \in (S^1 U S^4 U S^5), l \in (L^1 U\, L^4): (s,l) \in (M \cap V), (l,j) \in C, (t,t') \in T, t' > t \quad (4.13)$$

Constraint (v) assures that if a service is closed at the beginning of the planning horizon and then it opens, then it should be opened until the end of the planning horizon.

$$X_{sljt'} \leq X_{sljt} \forall s \in (S^1 U S^4 U S^5), l \in (L^1 U\, L^4): (s,l) \in (M \cap \overline{V}), (l,j) \in C, (t,t') \in T, t' > t \quad (4.14)$$

This ensures that services are not opening and closing multiple times during the planning horizon, what brings high costs and does not make sense in a short planning horizon with only 3 years.

As stated before and represented in constraint (4.6), AC and HBC services are not applicable to the constraint (4.13 and 4.14), since this services are partially provided by the primary care network.

$$\boldsymbol{X_{sljt} = 1} \; \forall \boldsymbol{s} \in (\boldsymbol{S^2 U S^3}), \boldsymbol{l} \in (\boldsymbol{L^2 U\, L^3}) : (\boldsymbol{s}, \boldsymbol{l}) \in \boldsymbol{M}, (\boldsymbol{l}, \boldsymbol{j}) \in \boldsymbol{C}, \boldsymbol{t} \in \boldsymbol{T} \tag{4.15}$$

<u>Capacity</u>

It was also important to define the available services that can be provided to patients according to the capacity of the existing network. In order to do that, two different constraints were made:

For IC and RU services, the capacity of services provided to patients will depend on the number of available beds that exist. In order to face this necessity, constraint (4.16) determines that services can only be open if at least a minimum number of beds exist (nb_s^{min})and that the maximum number of beds cannot exceed a defined number (nb_s^{max}).

$$\boldsymbol{nb_s^{min}} \times \boldsymbol{X_{sljt}} \leq \sum_{\boldsymbol{d}:(\boldsymbol{d},\boldsymbol{l}) \in \boldsymbol{F}} \sum_{\boldsymbol{p} \in \boldsymbol{P}} \sum_{\boldsymbol{a} \in \boldsymbol{A}} \boldsymbol{NB_{dpasljtn}} \leq \boldsymbol{nb_s^{max}} \times \boldsymbol{X_{sljt}} \forall \boldsymbol{s} \in (\boldsymbol{S^1 U S^4}), \boldsymbol{l} \in (\boldsymbol{L^1} \cup \boldsymbol{L^4}) : (\boldsymbol{s}, \boldsymbol{l}) \in \boldsymbol{M}, (\boldsymbol{l}, \boldsymbol{j}) \in \boldsymbol{C}, \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{n} \in \boldsymbol{NT} \tag{4.16}$$

The other services (AC, HBC and OU) are not restricted to the number of beds available in order to provide services, since they are not necessary. So, the capacity of this services is given by the minimum (ni_s^{min}) and maximum (ni_s^{max}) number of patients that can be assigned per service.

Constraint (4.17) has been made in order to limit the maximum capacity of these three services.

$$\boldsymbol{ni_s^{min}} \times \boldsymbol{X_{sljt}} \leq \sum_{\boldsymbol{d}:(\boldsymbol{d},\boldsymbol{l}) \in \boldsymbol{F}} \sum_{\boldsymbol{p} \in \boldsymbol{P}} \sum_{\boldsymbol{a} \in \boldsymbol{A}} \boldsymbol{Q_{dpasljtn}} \times \boldsymbol{ni_{dpastn}} \leq \boldsymbol{ni_s^{max}} \times \boldsymbol{X_{sljt}} \forall \boldsymbol{s} \in \left(\boldsymbol{S^2} \cup \boldsymbol{S^3} \cup \boldsymbol{S^5}\right), \boldsymbol{l} \in \left(\boldsymbol{L^2} \cup \boldsymbol{L^3} \cup \boldsymbol{L^5}\right) : (\boldsymbol{s}, \boldsymbol{l}) \in \boldsymbol{M}, (\boldsymbol{l}, \boldsymbol{j}) \in \boldsymbol{C}, \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{n} \in \boldsymbol{NT} \tag{4.17}$$

Resources allocation

The following constraint aims at defining the number of beds needed at t for IC and RU services (constraint 4.18). The number of beds will depend on the Length of Stay of the patients in the services, and an efficiency factor is (ϵ_s) is also associated with this constraint since it might be necessary to restrict the number of beds, since that in the mental health sector a full occupancy of services is not expected.

Once the Length of stay is faced by the uncertainty of the model, this constraint will be affected by the uncertainty associated with the stochastic model.

$$\boldsymbol{NB_{dpasiljtn}} = \frac{\boldsymbol{LOS_{sn}}}{\boldsymbol{\alpha}} \times \boldsymbol{Q_{dpasljtn}} \times \boldsymbol{ni_{dpastn}} \times \frac{\boldsymbol{1}}{\boldsymbol{\epsilon_s}} \forall \boldsymbol{p} \in \boldsymbol{P}, \boldsymbol{a} \in \boldsymbol{A}, \boldsymbol{s} \in (\boldsymbol{S^1} \cup \boldsymbol{S^4}), \boldsymbol{l} \in (\boldsymbol{L^1} \cup \boldsymbol{L^4}): (\boldsymbol{s}, \boldsymbol{l}): \in \boldsymbol{M}, (\boldsymbol{l}, \boldsymbol{j}) \in \boldsymbol{C}, (\boldsymbol{d}, \boldsymbol{l}) \in \boldsymbol{F}, \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{n} \in \boldsymbol{NT} \quad (4.18)$$

Constraint (4.19) states that the total number of beds result in a balance between new beds, reallocated beds and existing beds. The existing number of beds for $t = 1$ are the beds that were available at the beginning of the planning horizon, while for $t \geq 1$ they correspond to the number of beds in place in the previous period.

$$\sum_{d:(d,l)\in F} \sum_{p\in P} \sum_{a\in A} \boldsymbol{NB_{dpasljtn}} = \boldsymbol{NAB_{sljtn}} + \sum_{\substack{l'\in(L^1 U L^4) \\ l':(s',l')\in M l':(l',j')\in C}} \sum_{j'\in J(l',j')\in C} \boldsymbol{NRB^{in}_{sljs'l'j't}} - \boldsymbol{NRB^{out}_{sljs'l'j't}} + \begin{cases} \boldsymbol{nb_{sl}}, & \boldsymbol{t = 1} \\ \sum_{d:(d,l)\in F} \sum_{p\in P} \sum_{a\in A} \boldsymbol{NB_{dpasiljtn\,(t-1)}}, & \boldsymbol{t \geq 1} \end{cases} \forall \boldsymbol{s} \in (\boldsymbol{S^1} \cup \boldsymbol{S^4}), \boldsymbol{l} \in (\boldsymbol{L^1} \cup \boldsymbol{L^4}): (\boldsymbol{s}, \boldsymbol{l}) \in \boldsymbol{M}, (\boldsymbol{l}, \boldsymbol{j}) \in \boldsymbol{C}, \boldsymbol{n} \in \boldsymbol{NT} \quad (4.19)$$

Resources reallocation

The reallocation of beds between the IC service and the RU service needs to follow some rules that are guaranteed by the following constraints:

First, ensures that the number of beds reallocated to a new service s, located in l and provided by j is the same as the number of beds that are removed from another service s´, located in l´ and provided by j´, as demonstrated by the constraint (4.20).

$$NRB^{in}_{sljs'l'j'tn} = NRB^{out}_{s'l'j'sljtn} \forall\, (s,s') \in (S^1 \cup S^4), (l,l') \in (L^1 \cup L^4): (s,l) \in M, (s',l') \in M, (l,j) \in C, (l',j') \in C, t \in T, n \in NT \qquad (4.20)$$

In order not to allow the reallocation of beds by different providers, constraint (4.21) was build. This constraint needed to exist in order for the model not to permit the reallocation of beds from the social sector to the public sector, and the other way around.

$$NRB^{in}_{sljs'l'j'tn} = 0 \; \forall (s,s') \in (S^1 \cup S^4), (l,l') \in (L^1 \cup L^4): (s,l) \in M, (s',l') \in M, (l,j) \in C, (l',j) \in C, j \neq j', t \in T, n \in NT \qquad (4.21)$$

The reallocation of beds is permitted in between the same service or between two different services during all the years of the planning horizon (IC services with IC Services, RU services with RU services or IC service with RU service and vice-versa). However, the reallocation of services provided in different locations is only possible to be done in the first year of the planning horizon, after that the reallocation of beds can only be done inside the same location between whatever service as represented in constraint (4.22).

This decision is justified by the benefit that having the possibility to transfer beds between locations brings in the first year of the planning horizon, in a network that is not the most adequate and to the advantage of bringing stability to the mental health care network after the first year of the planning horizon.

$$NRB^{in}_{sljs'l'j'tn} = 0 \; \forall (s,s') \in (S^1 \cup S^4), (l,l') \in (L^1 \cup L^4): (s,l) \in M, (s',l') \in M, (l,j) \in C, (l',j') \in C, l \neq l', t > 1 \in T, n \in NT \qquad (4.22)$$

Finally, constraint (4.23) defines the maximum number of beds that can be reallocated from a service in each year of the planning horizon.

The limitation presented is that the number of beds reallocated must be lower that the number of existing beds in that service in the previous period.

This constraint is useful in order for the model not to suggest the reallocation of all the beds in one particular service, what would only make sense if the service is to be closed.

$$\sum_{s'\in(S^1\cup S^4)} \sum_{\substack{l'\in(L^1 U L^4) \\ l':(s',l')\in M l':(l',j')\in C}} \sum_{j'\in J} NRB^{out}_{sljs'l'j't} \tag{4.23}$$

$$\leq \begin{cases} nb_{sl}, & t = 1 \\ \sum_{d:(d,l)\in F} \sum_{p\in P} \sum_{a\in A} NB_{dpsalj\,(t-1)}, & t > 1 \end{cases} \forall\, s \in (S^1 \cup S^4), l \in (L^1 \cup L^4): (s,l) \in M, (l,j) \in C$$

Minimum Service level

In order to make sure that at least a part of the patients are receiving treatment, a minimum level of satisfied demand per service s (represented by β_t) is demonstrated in constraint (4.24).

$$\frac{IS_{sn(t=|T|)}}{niS_{sn(t=|T|)}} \geq \beta_t \forall\, s \in S, t \in T, n \in NT \tag{4.24}$$

5 – Case Study

In this section, the data and assumptions taken into consideration in the MHCU model will be presented.

5.1. Dataset used

The mental Health Care provision in the region of Lisbon is ensured in 9 different regions (Amadora, Cascais, Lisboa, Loures, Mafra, Odivelas, Oeiras, Sintra and Vila Franca de Xira). Each region is characterized by, different levels of demand.

According to (Health at a glance: Europe 2018), five different diseases are recognized as being related to a high level of demand for mental health care services. This different disease are: Anxiety, Mood Swings (depressive), Impulse Control, Substances Abuse and Schizophrenia and are the ones we focus on this study.

For each disease group, the population was divided in two different age groups, both with different levels of demand. The first group composed by people in the 15-64 years age group and the other age group with people above 65 years old. This division was made do to the fact that health patients with different ages requiring different types of care.

In order to understand the weight of each disease in each age group, table 4 was made.

Table 4: Percentage of the population in each disease group divided by age group.

Mental Disorders	2020		2021		2022	
	15-64	65+	15-64	65+	15-64	65+
Anxiety	51,101	29,949	51,090	30,512	51,093	31,045
Depressive	24,467	14,339	24,461	14,609	24,463	14,864
Impulse Control	10,840	6,353	10,837	6,472	10,838	6,585
Substances use	4,955	2,904	4,954	2,959	4,955	3,010
Schizopherenia	2,323	1,361	2,322	1,387	2,322	1,411

There are five different mental health service typologies provided in Portugal: Institutional Care (IC), Ambulatory Care (AC), Home-base Care (HBC), Residential Unit (RU), Occupational Unit (OU).

There are 30 locations where mental Health Service are provided in the region of Lisbon. The Institutional Care (IC) can only be provided in 18 locations. The other types of service can be provided in all of the 30 locations.

From the 30 locations that provide mental health care services in Lisbon, 7 of them are public hospitals, 14 are social institutions and 9 are primary health care centres.

Probability Tree Scenario

The demand and length of stay of patients in the services depends on different factors like epidemiological and demographic factors, which are not easy to predict.

As a result, a high level of uncertainty is associated when trying to predict the values of demand and length of stay.

In order to deal with the uncertainty associated with it, a probability scenario tree was built, where the two sources of uncertainty as referred before are: Demand and Length of stay (LOS).

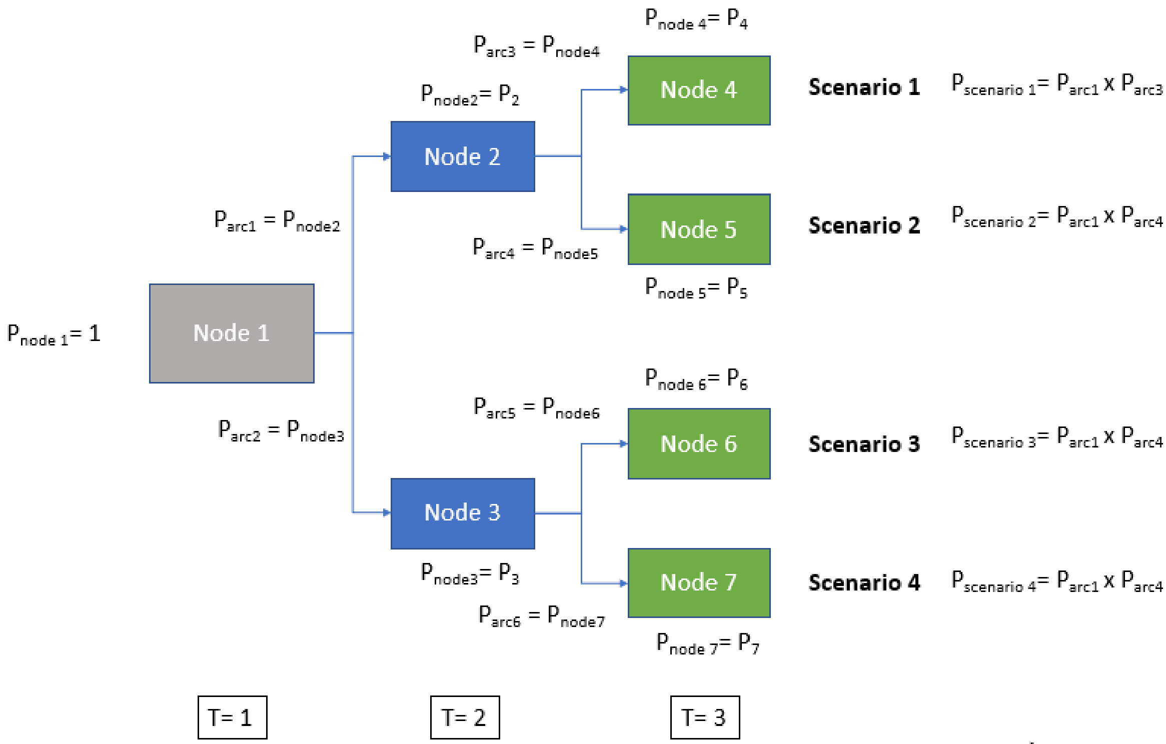
A scenario tree is composed by a set of nodes and arcs (Wallace and Klassen, 2012). Each node represents a possible outcome of the parameter that is uncertain. In the case of this study, there are two parameters that are uncertain (demand and LOS) which means that each node can assume the following 9 different combinations:

1) Low Demand & Low LOS
2) Low Demand & Average LOS
3) Low Demand & High LOS
4) Average Demand & Low LOS
5) Average Demand & Average LOS
6) Average Demand & High LOS
7) High Demand & Low LOS
8) High Demand & Average LOS
9) High Demand & High LOS

Each one of this 9 combinations have a probability associated with it. As expected, the most extreme values (low and high levels of demand and LOS) have a lower possibility to happen has opposed to the average levels which have a higher probability to happen. The probability of each node is given by the product of the probability of the level of demand to happen with the probability of the level of LOS to happen.

The arcs demonstrate the evolution between nodes from one initial stage to its final and each arc also have a probability associated with it. The probability of each branch of the scenario tree is the product of the different arcs. In the case of this study, the planning horizon is composed by three different time periods where each time period corresponds to a decision point (nodes), as demonstrated in the figure 3.

Figure 3 – Standard scenario tree



5.2. Planning contexts under analysis

In this model, three different planning contexts are used, in order to obtain different perspectives aligned with the planning objectives:

Planning Context A - The objective is to ensure the minimum total cost while accounting for multiple equity objectives that guarantee that the service provided to the different patients is as equitable as possible in different aspects like geography, travel time, types of services delivered and a minimum number of patients receiving care.

Planning Context B – The objective is to ensure the minimum total cost while guaranteeing that all the patients receive the care they need.

Planning Context C – The objective of the model is to reduce the total travel time to the minimum possible while guaranteeing that all the patients receive the care they need.

Depending on the planning context, different target values at the end of the last year of planning were used in order to present to the decision makers the possibility to reach the objectives of minimizing cost or minimizing the travel time but also making sure that some minimum target values were obtained.

Table 5 – Planning context´s predefined equities and objectives.

		Planning Context A	Planning Context B	Planning Context C
	Objective	Minimize Cost	Minimize Cost	Minimize travel time
Targets	Equity of access	0,625	0,625	0,625
	Geographical Equity	0,300	0,300	0,300
	Equity of service utilization	0,300	0,300	0,300
	Equity of disease utilization	0,600	0,600	0,600
	Minimum Demand satisfaction	50 %	100%	100%

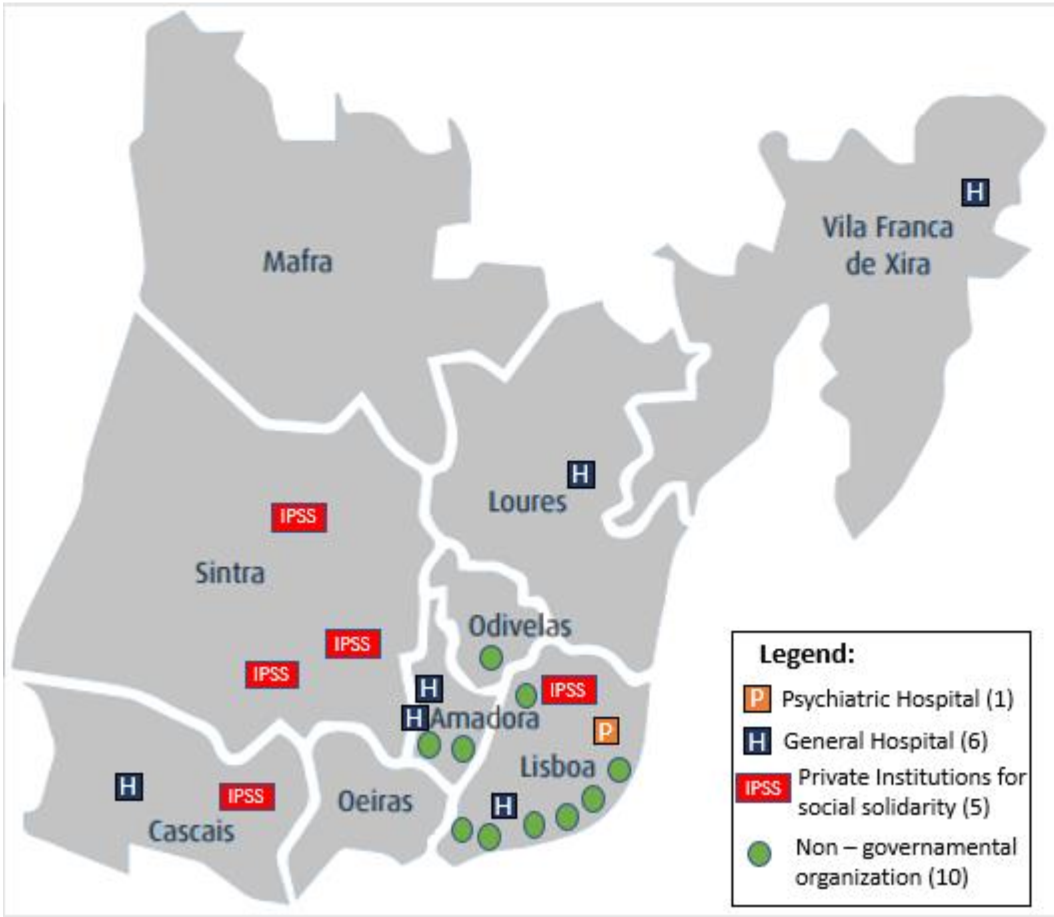
Service Location

As refered before, there are five different types of service provided in Lisbon: Institutional Care (IC), Ambulatory Care (AC), Home – based Care (HBC), Residential Unit (RU), Occupational Unit (OU).

In figure 4.1, we can see a map that shows all the location where the different types of services can be provided in the region of Lisbon. Institutional Care and ambulatory Care can be provided in 12 different location (1 psychiatric hospital, 6 general hospitals and 5 IPSS (Private Institutions for social solidarity). Both residential unit care and occupational unit care can be provided in any one of the 22 locations (1 psychiatric hospital, 6 general hospitals, 5 IPSS and 10 non-governmental organizations).

The Home – based Care is a service provided at the home of the patient and in our model were attributed 9 locations to this service that correspond to the 9 Lisbon counties (Lisboa, Amadora, Oeiras, Cascais, Odivelas, Loures, Sintra, Vila Franca de Xira and Mafra).

Figure 4.1 – Location of the mental health institutions in the region of Lisbon.



6 – Results

In this section, the key results of the models will be presented in the following way: (i) the location and type of services provided in the region of Lisbon; (ii) additional capacity of beds and respective investment; (iii) satisfied demand and finally (iv) the cost associated with the changes suggested.

The analyse of the results will be focused in the planning context A and B and some important highlights will be given about planning context C. A comparison between the scenario at the beginning of the planning horizon and the suggestions of the model for each planning context will also be done.

6.1- Planning Context A

Service Location

For each one of the types of service existing in Portugal, a map with the location of the different institutions providing services at the end of the planning horizon will be presented.

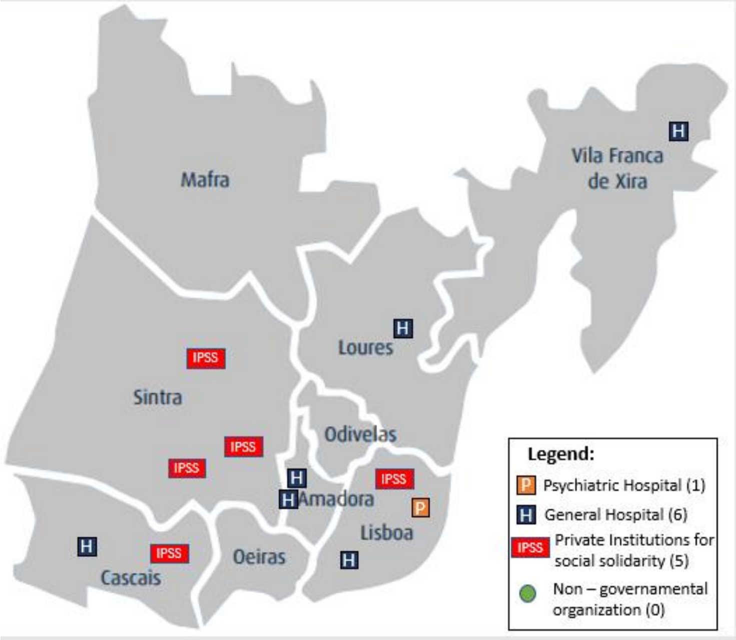
Institutional Care

Institutional Care is provided to acute patients (short term patients) and to chronic patients (long term patients). The public sector deliveries this service mainly to acute patients while the social sector provides it both to the chronic patients and to the acute patients, although it focuses more on the chronic patients.

As referred before, the institutional care services can be provided in 12 different institutions in the region of Lisbon (1 psychiatric hospital, 6 general hospitals and 5 IPSS).

According to the results of the model, the 12 institutions should be opened throughout the 3 years of the planning horizon providing institutional care, has demonstrated in figure 5.1.

Figure 5.1 – Institutions providing Institutional Care is provided in the year of 2020, 2021 and 2022 at the planning horizon A.

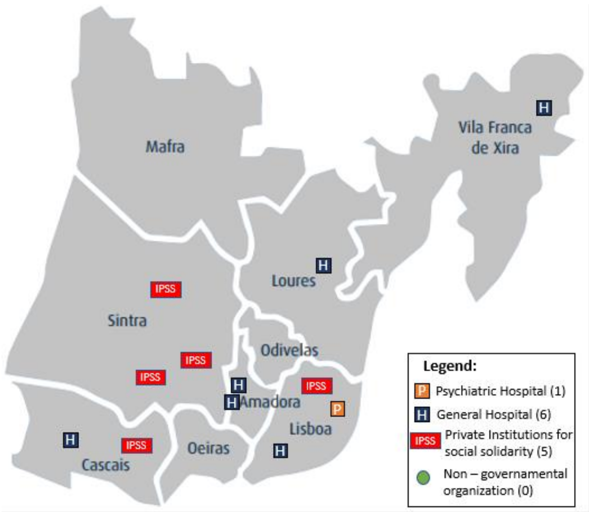


Ambulatory Care

Ambulatory Care is a specialised mental health service that aims to provide services to patients who are not currently admitted to a hospital or residential service. This service is provided in general Hospitals, Psychiatric Hospitals and IPSS (including day hospitals/ day centers).

Similarly to the institutional care, the ambulatory care can be provided in the exact same twelve locations. Once again, according to the results of the model, all of them should be open during the entire planning horizon in order to provide ambulatory care services to the population.

Figure 5.2 – Institutions providing Ambulatory Care in the year of 2020, 2021 and 2022 at the planning horizon A.

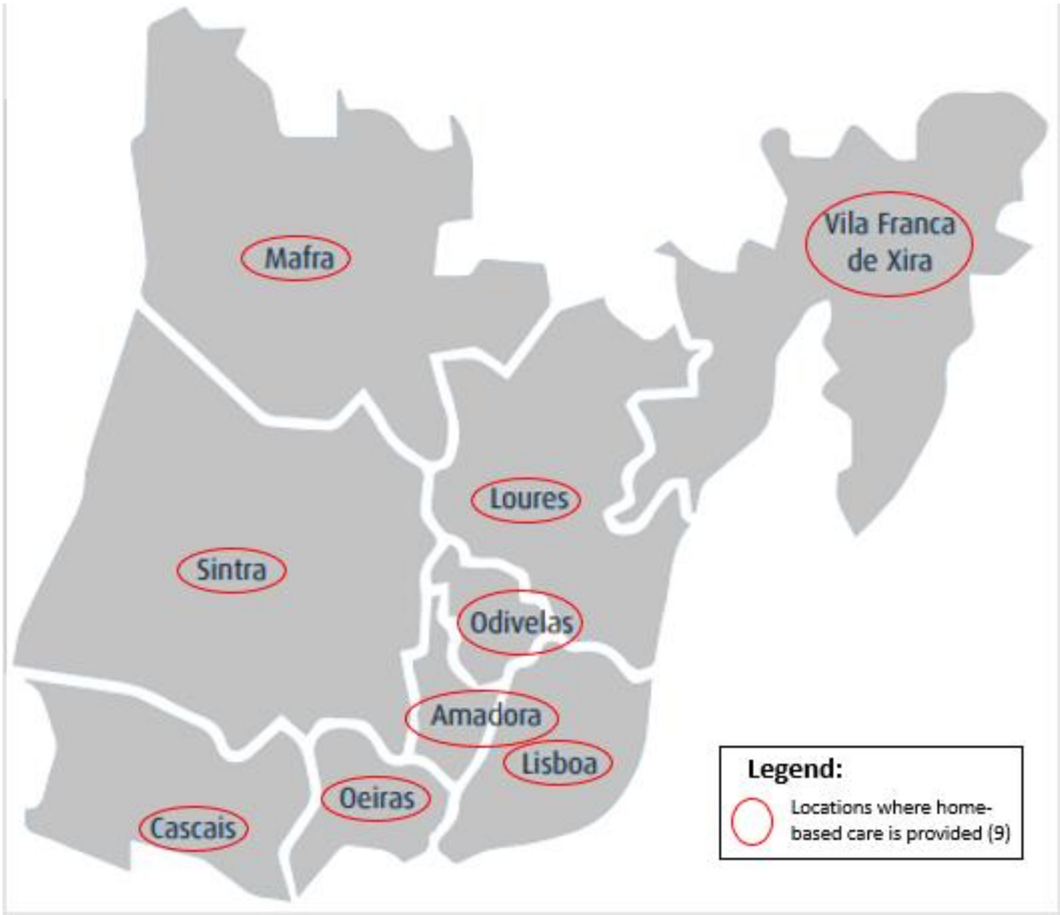


Home-based Care

In what respects to home-based care, this service can be provided in the different locations where patients live. In order to simplify the number of possibilities, in the model where identified 9 different locations that correspond to the 9 different "concelhos" where home-based care services can be delivered to the patients, as demonstrated in figure 5.3.

Patients can only receive care in the location they live in. Not existing patients living in Vila Franca de Xira and receiving home-based care at Mafra, for example.

Figure 5.3 – Locations where Home-based Care is provided in 2020, 2021 and 2022 at the planning horizon A.



Residential Unit

The residential unit provides accommodation for patients that are stable at a clinical level but that do not have the needed support at their residence. This residential units are divided in three different levels accordingly to the degree level of psychosocial disabilities: Life autonomous unit (low degree), Life protected unit (moderate degree), Life support unit (high degree).

The residential unit care can be provided in all of the 22 existing institutions. In the first year of the planning horizon, the model suggests that only 18 of the location should be opened with 4 of the general hospitals not providing this type of service, has demonstrated in figure 5.4.

This suggestion changes in the second and third year with all of the 22 locations providing residential unit services, has demonstrated in figure 5.5.

Figure 5.4 – Institutions providing residential unit care in the year of 2020 at the planning horizon A.

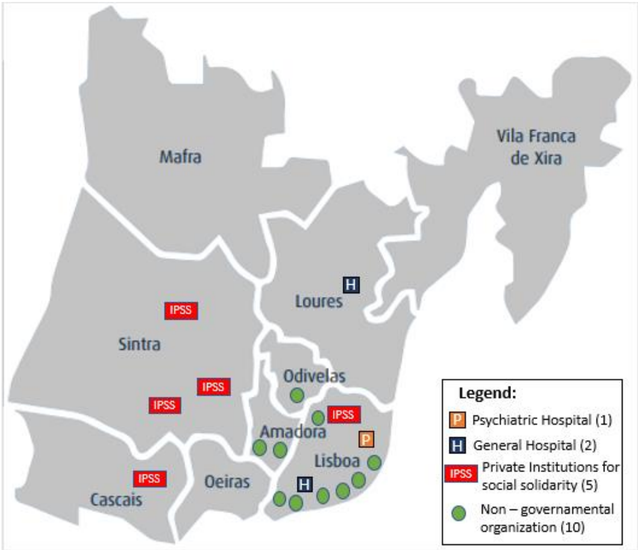
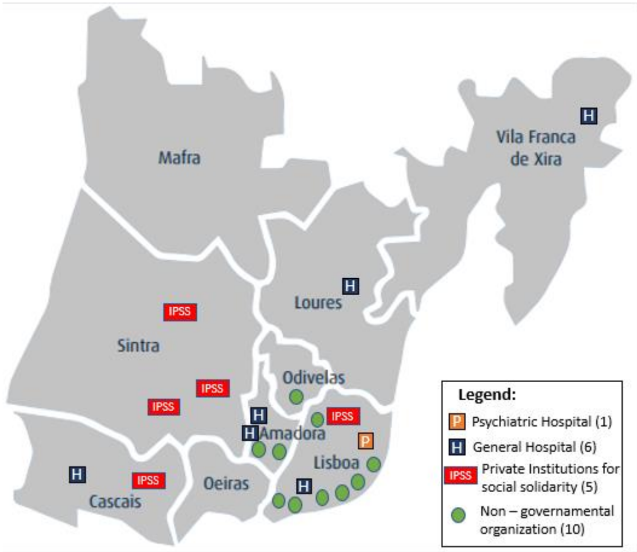


Figure 5.5 – Institutions providing residential unit care in the year of 2021 and 2022 at the planning horizon A.



Occupational Unit

The occupational unit is aimed at patients with low and moderate psychosocial disability. This service is the only one that does not involve the residence of the patient in the institution and it is aimed to help in the reinsertion and integration in professional training programs.

In the region of Lisbon, this service can be provided in all of the 22 locations. In the first year of the planning horizon, only one of the 22 locations were not providing this type of service, as demonstrated in figure 5.6. In the second and third year, all of the 22 locations are providing the service (figure 5.7).

Figure 5.6 – Institutions providing occupational unit care in the year of 2020 at the planning horizon A.

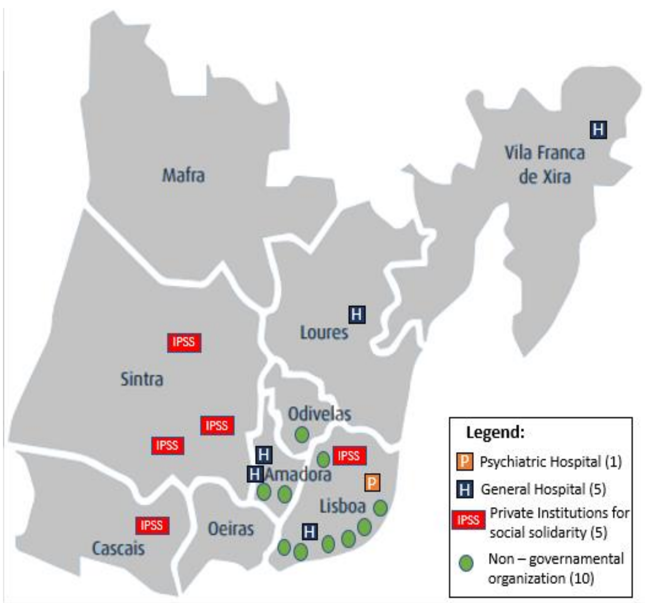
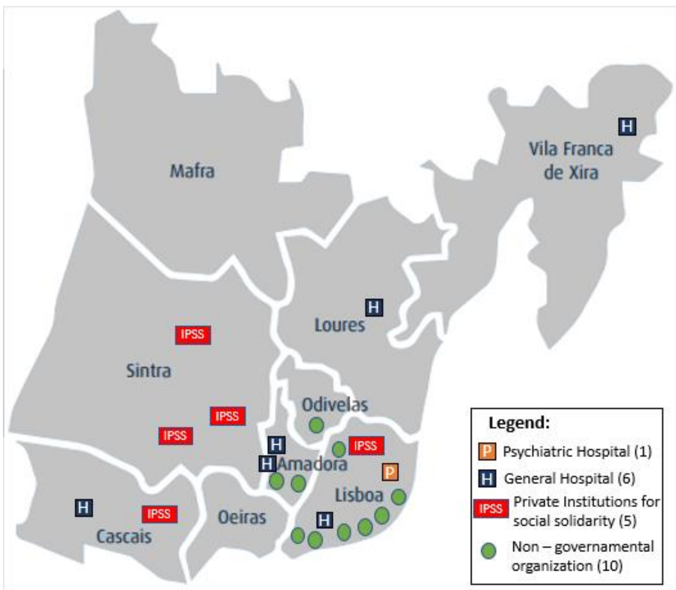


Figure 5.7 – Institutions providing occupational unit care in the year of 2021 and 2022 at the planning horizon A.



Beds Capacity and Cost

There are two different services that are subjected to the number of available beds. This are institutional care and residential units.

The number of available beds will depend on the value of the parameters that are affected by the uncertainty associated with the model: Demand and Length of Stay.

Once the number of beds available will vary on the levels of these two parameters, a minimum, average and maximum number of beds available will be presented for each of the services.

In table 6.1 is presented the number of beds in the beginning of the planning horizon, for each of the services.

Table 6.1 – Number of beds available at planning horizon A

Type of Service	**Institutional Care**	**Residential Unit**
Number of beds at the beginning of the planning horizon	1592	362
Number of additional beds	0	0
Number of reallocated beds in to the service	438	177
Number of reallocated beds out of the service	405	210
Number of beds at the end of the planning horizon	1625	329

In planning horizon A, no beds were added, what results in no investment in terms of additional beds.

The number of existing beds is the same as the number existing at the beginning of the planning horizon.

Satisfied Demand

The value of satisfied demand as well as the number of beds that was analyzed before will be affected by the uncertainty considered while building this model. The average value of satisfied demand per year are the following:

Table 6.2 – Satisfied Demand in planning horizon A

	2020	2021	2022
Satisfied Demand	0,515	0,515	0,700

Cost

The cost associated with this planning context is approximately 465 M € for the three years of the planning horizon has demonstrated in the table 5.10.

Table 6.3 - Cost of the planning horizon A

Costs (M€)	Year 1	Year 2	Year 3	Total
Investment	-	-	-	-
Operational	139,229	138,956	186,809	464,994
Total	139,229	138,956	186,809	464,994

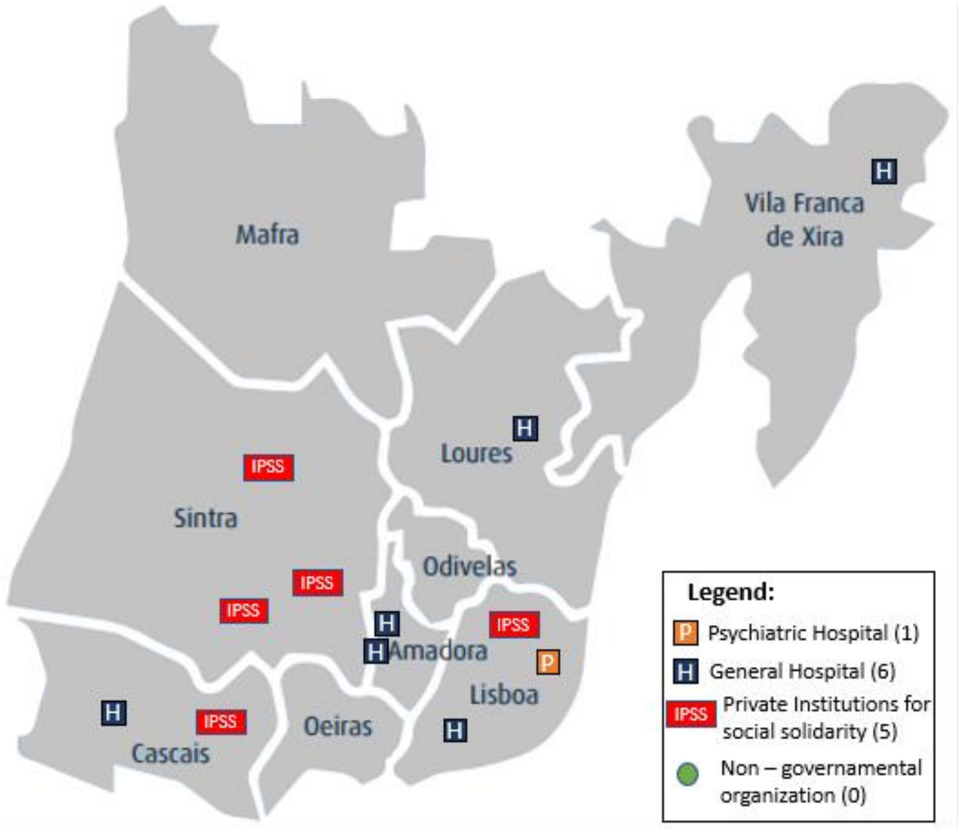
6.2- Planning Context B

Service Location

Institutional Care and Ambulatory Care

As well as planning context A, all the twelve institutions that provide institutional care services and ambulatory care services are providing those services in all of the twelve possible locations as demonstrated in figure 6.1.

Figure 6.1 - Institutions providing institutional and ambulatory care in the year of 2021 and 2022 at planning horizon B.



Home-based Care

In respect to home-base care, the number of available institutions follows the same logic of the previous planning context, with 9 possible locations where home-based care services can be provided. All of them with services being provided during this planning context.

Residential Unit

In the first year of the planning context B, there will be 14 out of the 22 possible institutions providing residential unit care (1 Psychiatric Hospital, 3 general hospitals, 3 private institutions for social solidarity and finally 7 non-governmental institutions). The distribution of this institutions in the map are presented in figure 6.2.

In the next two years, the model presents that all of the 22 possible locations should be providing this service, has showed in figure 6.3.

Figure 6.2 – Institutions providing residential unit care in the year of 2020.

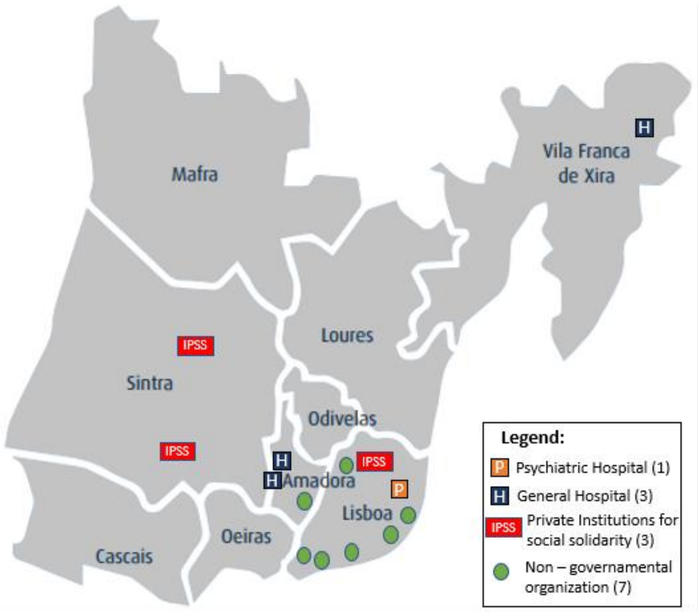
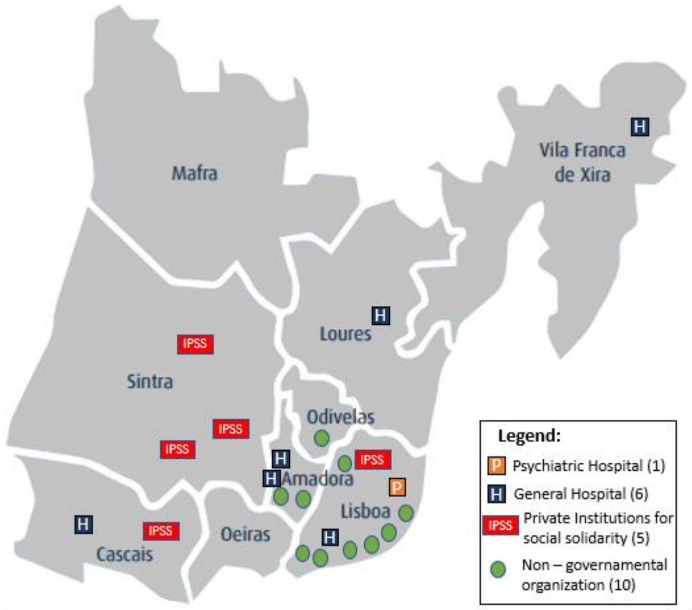


Figure 6.3 – Institutions providing residential unit care in the year of 2021.



Occupational Unit

In the planning context B, the institutions that provide this service are not the same in every year. The number of institutions and their locations are presented in figures 6.4 - 6.6.

1) **Year of 2020:** 1 Psychiatric hospital, 1 general hospital, 2 IPSS and 9 non-governmental organizations.
2) **Year of 2021:** 1 Psychiatric hospital, 2 general hospitals, 2 IPSS and 9 non-governmental organizations.
3) **Year of 2022:** 1 Psychiatric hospital, 6 general hospitals, 5 IPSS and 4 non-governmental organizations.

Figure 6.4 – Institutions providing occupationall unit care in the year of 2020.

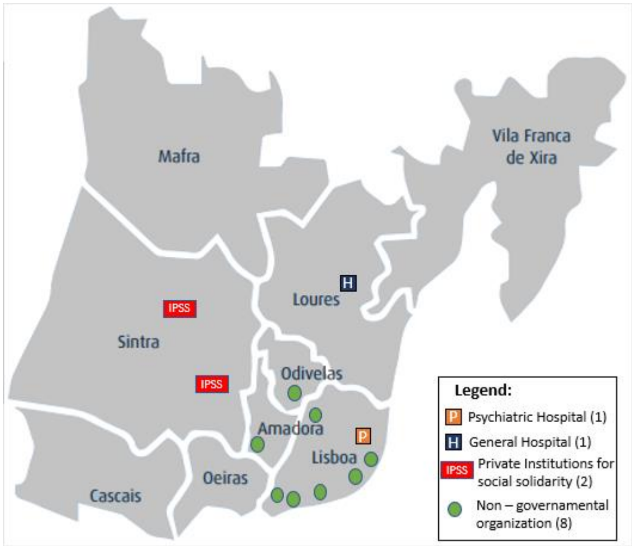


Figure 6.5 – Institutions providing occupational unit care in the year of 2021.

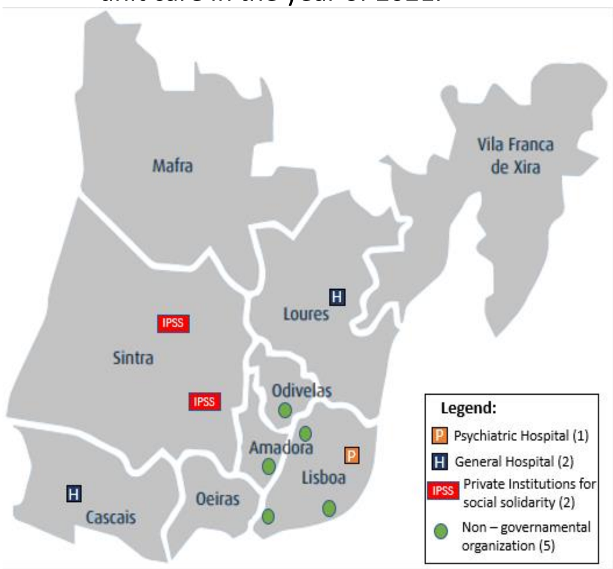
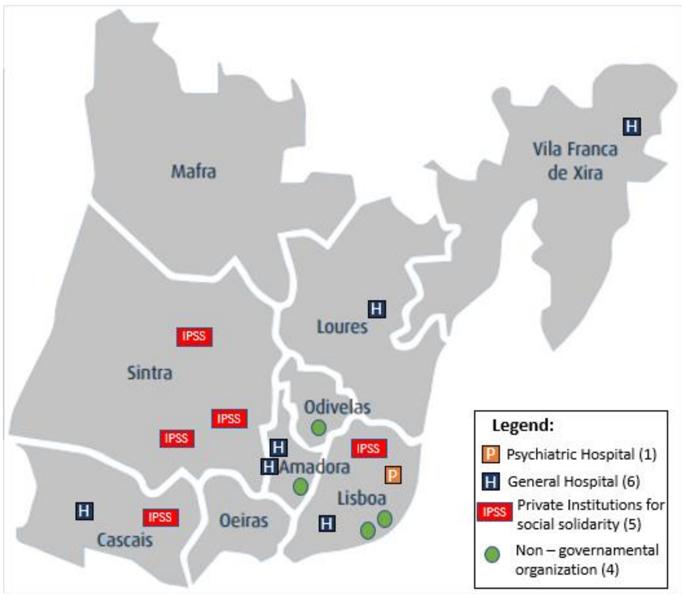


Figure 6.6 – Institutions providing occupational unit care in the year of 2021.



Beds Capacity and Cost

In the planning horizon B, the number of beds existing at the begging of the planning horizon and added in order to correspond to the demand are presented in table 7.1. As well as the total number of beds at the end of the planning horizon.

Table 7.1 – Number of beds available at planning Horizon B

Type of Service	**Institutional Care**	**Residential Unit**
Number of beds at the beginning of the planning horizon	1592	362
Number of additional beds	911	43
Number of reallocated beds in to the service	265	102
Number of reallocated beds out of the service	189	178
Number of beds at the end of the planning horizon	2 579	329

The 911 beds added to Institutional Care are divided by six different institutions and the 5 beds added to residential unit services are all added in one institution. The investment cost corresponding to the number of added beds is approximately 19 M €.

Satisfied Demand

The value of satisfied demand as well as the number of beds that was analyzed before will be affected by the uncertainty considered while building this model. The average value of satisfied demand per year are presented in table 7.2:

Table 7.2 - Satisfied Demand in planning horizon B

	2020	**2021**	**2022**
Satisfied Demand	1,000	1,000	1,000

Cost

The cost associated with this planning context is approximately 866 M € for the three years of the planning horizon has demonstrated in the table 7.3.

Table 7.3 – Cost of the planning horizon B

Costs (M€)	**Year 1**	**Year 2**	**Year 3**	**Total**
Investment	19.080	-	-	19.080
Operational	279.674	283.008	284.358	847.040
Total	285 268	269.522	270.872	866.120

Travel Time

The total travel time in the planning context B was 37 M minutes.

6.3 - Planning Context C

Beds Capacity and Cost

In the planning horizon C, the number of additional beds existing at the begging of the planning horizon and added in order to correspond to the demand are presented in table 8.1. As well as the total number of beds at the end of the planning horizon.

Table 8.1 – Beds available at Planning Horizon C

Type of Service	**Institutional Care**	**Residential Unit**
Number of beds at the beginning of the planning horizon	1592	362
Number of additional beds	949	5
Number of reallocated beds in to the service	780	241
Number of reallocated beds out of the service	742	279
Number of beds at the end of the planning horizon	2579	329

The 949 beds added to Institutional Care are divided by three different institutions and the 5 beds added to residential unit services are all added in one institution. The investment cost corresponding to the number of added beds is approximately 19 M €.

Satisfied Demand

The value of satisfied demand as well as the number of beds that was analyzed before will be affected by the uncertainty taken in consideration while building this model. The average value of satisfied demand per year are presented in table 8.2:

Table 8.2 - Satisfied Demand in planning horizon C

	2020	**2021**	**2022**
Satisfied Demand	1,000	1,000	1,000

Cost

The cost associated with this planning context is approximately 825 M € for the three years of the planning horizon has demonstrated in table 8.3.

Table 8.3 – Cost of the planning horizon C

Costs (M€)	**Year 1**	**Year 2**	**Year 3**	**Total**
Investment	19.080	-	-	-
Operational	266.188	269.522	270.872	806,582
Total	285 268	269.522	270.872	825.662

Travel Time

The total travel time in the planning context C was 21 M minutes. The total travel time in this planning context in lower than in the planning context C has predicted, since this planning context had the objective to minimize the total travel time, in order to provide a better experience for the patients.

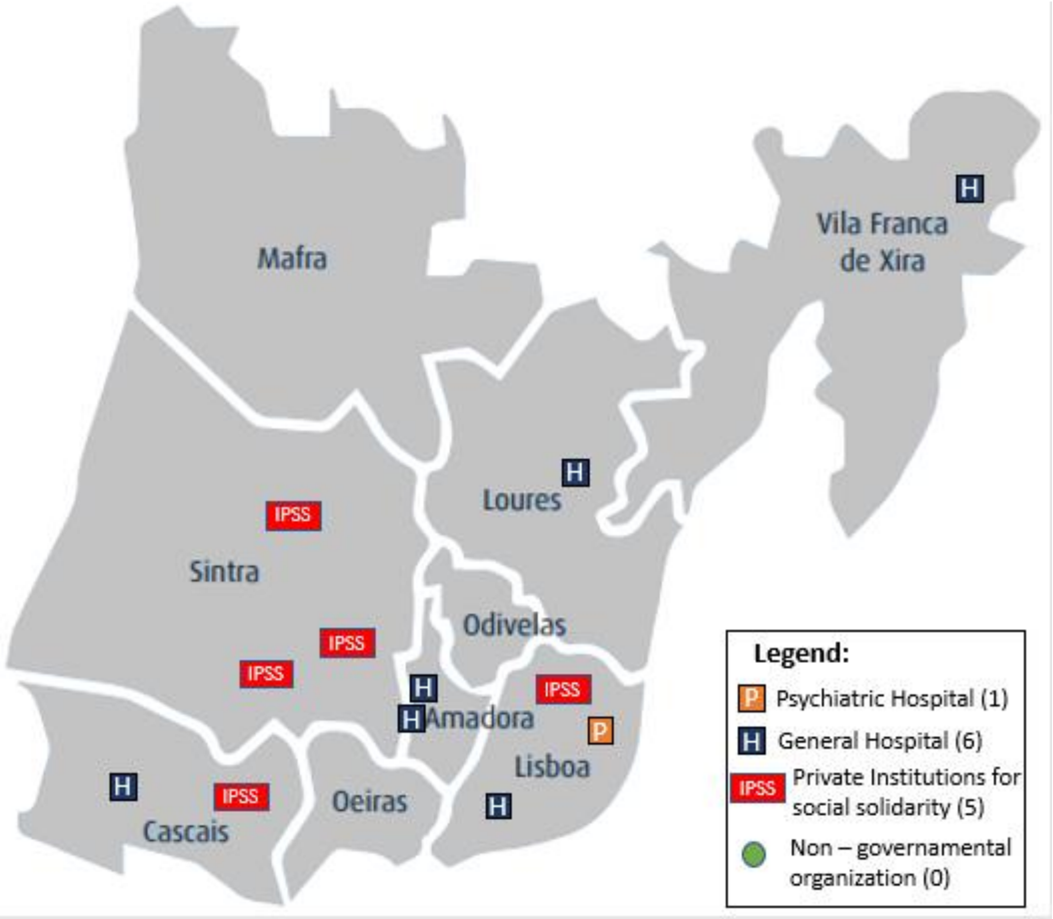
6.4 - Comparison between the different planning contexts and the initial scenario

In this section, it will be presented the different institutions that provide care in the different planning context. A map with the locations where each of the services was being provided at the beginning of the planning horizon will be presented.

Institutional Care

As presented in figure 7.1, in the beginning of the planning horizon, all the twelve possible locations were providing this service.

Figure 7.1 - Institutions providing Institutional Care at the beginning of the planning horizon.



In all the planning horizon analysed, the number of institutions providing this service were the same twelve location except for the planning horizon C that only had eleven location opened.

Although the lower number of locations opened, it was in the planning horizon C that the biggest number of beds were in use, has demonstrated in table 9.1.

Table 9.1 – Number of beds in each planning horizon providing Institutional Care services

Type of Service	Planning context A	Planning context B	Planning context C
Number of beds at the beginning of the planning horizon	1592	1592	1592
Number of additional beds	0	911	949
Number of reallocated beds in to the service	438	265	780
Number of reallocated beds out of the service	405	189	742
Number of beds at the end of the planning horizon	1625	2 579	2579

Once the demand was completely satisfied in both the planning context B and C, we can assume that the minimum number of beds needed in order to fulfil the demand for these two services is 2579 beds.

<u>Ambulatory Care</u>

Similar to what was presented for the institutional care service, a map with the locations that provide ambulatory care at the beginning of the planning horizon will be presented in figure 7.2.

Figure 7.2 - Institutions providing Ambulatory Care at the beginning of the planning horizon.

In all the other planning horizon analysed, the twelve locations that can provided this type of services are all open.

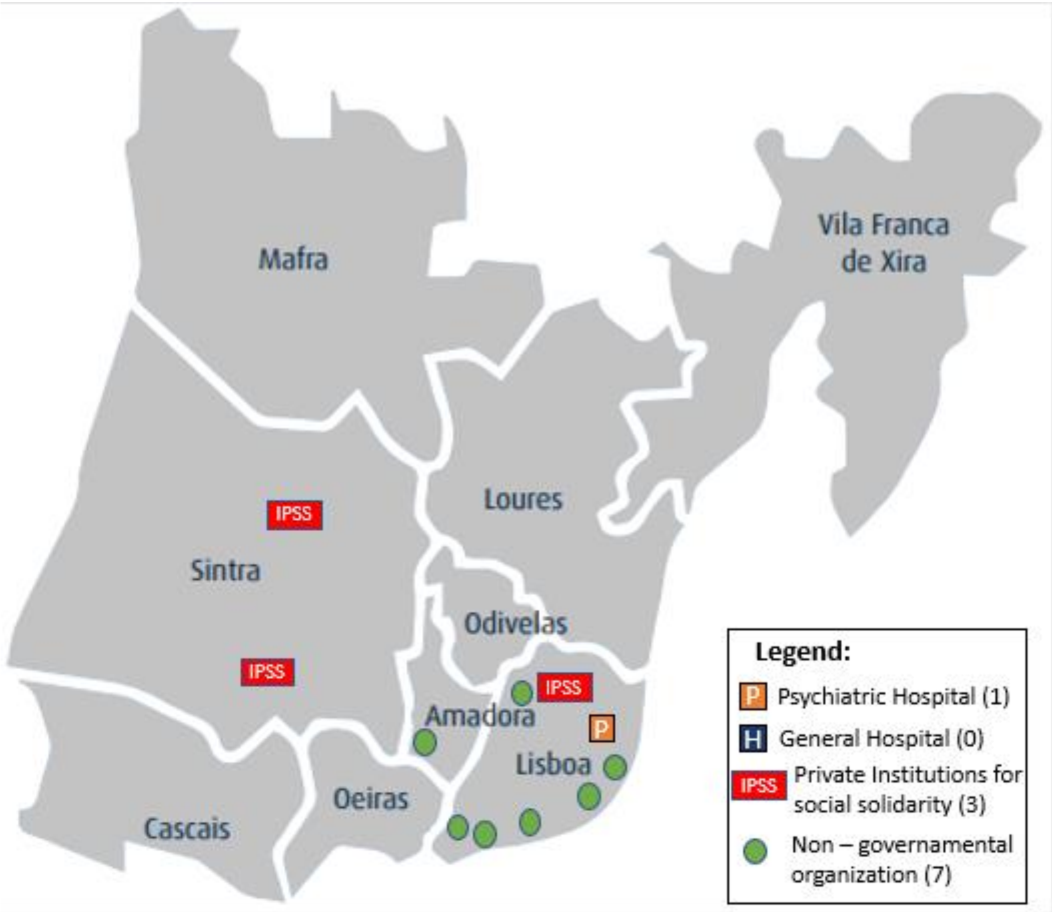
Home-based Care

In respect to home-based care services, the map with the locations that present the locations where this service is provided will not be presented since that as well as in the other planning horizons, the locations were home-based care is provided do not change in the different scenarios and are only indicative of the different locations were this service can be provided.

Residential Unit Care

In the beginning of the planning horizon, the residential unit care services were only provided in 11 locations, as represented in figure 7.3.

Figure 7.3 - Institutions providing Residential Unit Care at the beginning of the planning horizon.



It is when comparing the number of institutions that were providing residential unit care services in the beginning of the planning horizon and the institutions that the model suggests that should be open and providing this service that we see the biggest differences. In the beginning of the planning horizon, only eleven of the possible twenty-two locations were providing unit care service. In all the planning contexts, the model suggests that at the end of the planning horizon, all the twenty - two locations should be providing unit care services.
The impact of this change is especially visible in the case of general hospitals where this type of service was not being provided at all any of the 6 existing general hospitals.

Table 9.2 – Number of beds in each planning horizon providing Residential Unit Care services

Type of Service	Planning context A	Planning context B	Planning context C
Number of beds at the beginning of the planning horizon	362	362	362
Number of additional beds	0	43	5
Number of reallocated beds in to the service	177	102	241
Number of reallocated beds out of the service	210	178	279
Number of beds at the end of the planning horizon	329	329	329

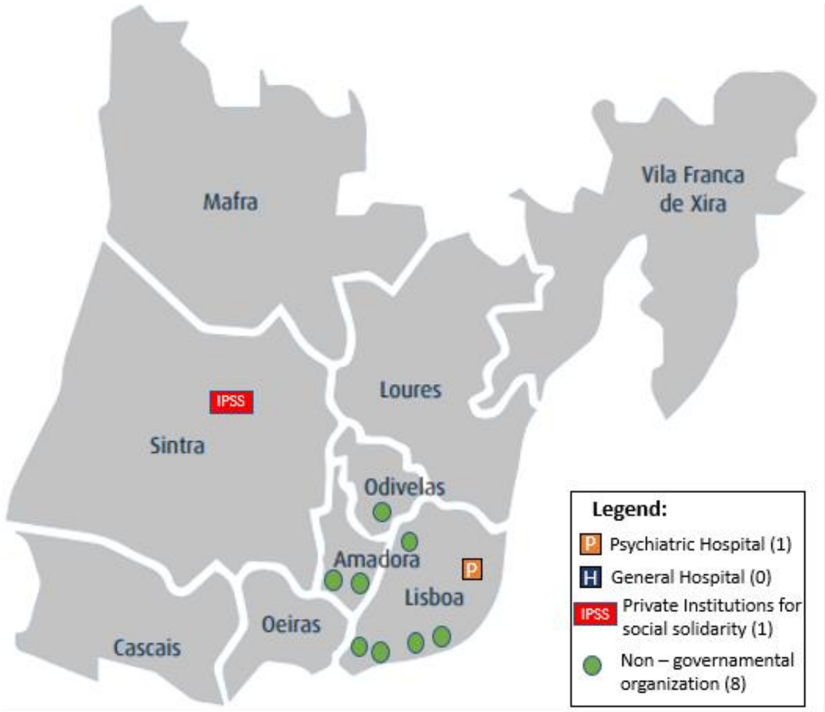
According to the results of the model, the number of necessary beds to provide residential unit care to every patient is 329 beds, since this is the number of beds at the end of the planning horizon in the planning context B and C, were the demand is totally satisfied.

It is then possible to conclude that although the number of institutions providing this type of service are more than at the beginning of the planning horizon, the existing capacity of the service (number of beds) is now less than it was before, what indicates that the existing capacity was enough to the existing demand.

Occupational Unit

In the beginning of the planning horizon, the occupational unit services were being provided in only 10 different locations, as demonstrated in figure 7.4.

Figure 7.4 - Institutions providing Occupational Unit Care at the beginning of the planning horizon.



The following table represents the number of institutions that should be opened according to the MHCU model for each one of the planning contexts.

The MHCU model suggests that in the planning context A and planning context C, all of the possible 22 locations should be providing care. In regard to planning context B, the model suggests that only 16 locations should be opened.

This means that it is only necessary to have 16 locations opened in order to be able to satisfy all the demand and that in planning context C, the suggestion of having all the institutions opened should be explained do to the necessity of minimizing the total travel time.

6.5 – Computational results

The computational results of the MHCU model implemented in GAMS (General Algebraic Modelling System), when applied to the different planning contexts, are represented in the Table 20.

The results of this model were reached with an Intel Core® i5-8250U CPU 1.60 with 12 GB RAM.

Table 10 – Computational Results

Planning context	Total Equations	Total Variables	Integer Variables	Iterations	CPU(s)	GAP (%)
A	525,165	2,608,542	3,325	76621	29.390	0
B	565,547	3,174,586	3,325	53345	30.296	0
C	565,547	3,165,406	3,325	39249	30.375	0

7 – Conclusions

It is clear the high impact that mental health diseases are having in the population across the World and especially in Portugal, and the increasingly importance that this type of disease will continue to bring in the future especially due to the continue aging of the population.

With the increase demand for this type of services and a network of services that even nowadays is not capable to face with the needs of its population, the importance of a restructuration of the mental health care in Portugal is a reality where the different types of mathematical programming models that are used to aid health care planners in general can come in hand.

In order to support this idea, the aim of this thesis was to develop a mathematical programming model – MHCU model – in order to assist the mental health care planners in the context of NHS – based countries both at a strategical and tactical level in order to restructure the Mental Health Care in the great region of Lisbon.

Do to the uncertainty associated with the mental health care sector, this model takes uncertainty into consideration both in the demand of mental health care services and in the length of stay of patients in the network. It also takes into consideration the multi-service nature of the sector and provides an analyse to each on of the main services that are provided in the sector. Finally, this model also takes into consideration a multiple objective approach.

Different scenarios align with the different policy objectives were built for policy makers to be able to adapt the model to there needs and vision. The three scenarios presented were: i) reducing total cost while accounting for multiple equity objectives (equity of access, geographical equity, equity of service utilization and equity of disease utilization), ii) reducing total cost while assuring the total satisfaction of demand, iii) reducing the total travel time while assuring the total satisfaction of demand.

This thesis add to the literature by i) proposing a mathematical programming approach to support planning decisions in the mental health sector, a health sector not widely studied in the literature, ii) taking uncertainty into consideration, iii) considering the multiple services used in the mental health sector, iv) considering multi-objectives aligned with the strategic needs of the different strategic health care policies.

According to the obtained results in the model, we can assume that the current provision of mental health care is not enough when compared with the demand for service, due to the number of locations that the model suggests that should be providing care is much higher than the number of institutions providing this care at the beginning of the planning horizon as well as the high number of additional beds that the model suggests and that will increase significantly the investment cost associated with the mental health network suggested.

In the planning context A, it is possible to obtain the minimum cost possible while making sure that part of the demand is being satisfied and that the equity values defined are being respected.

Planning context B helps to understand the necessary capacity in order to obtain a total demand satisfaction and the total level of investment and cost associated with it. Once it aims at minimizing the total cost of the planning horizon while assuring for predefined levels of equity and a complete satisfaction of the demand. This is probably the most useful scenario for a

country like Portugal that has economical struggles but at the same time has a high level of demand for mental health care services that needs to be fulfil.
Planning context C, is useful to understand the additional effort that would be needed in order to provide to its citizens a total demand satisfaction but while also making sure that the travel time that its patients will need to go through is the minimum possible, in order to provide the most comfortable access to the services that the patient's needs.

In conclusion, we can assume that the model is appropriate to be run in computational terms, what also goes according to the possibility of having decision makers using it to support their decisions.
After running the model for each different scenario, it is possible to say that the uncertainty associated with the model is performing in the expected way, with different results being presented according to the level of demand or length of stay that the model assumes.
This model can now be adapted by the decisions makers in order to go according to their needs or only to simulate different scenarios and understand the output received. Like for example changing the levels of equity desired or the minimum level of satisfied demand in order to understand the impact that this change will have in the output of the model and if it has the desired result or if same adjustments might be necessary.

8 – Future Research

In respect to future work that can be done in this field of research, the following are suggested:

- Expand the utilization of the MHCU model to other regions in Portugal or to other countries where the model might make sense to be adapted.

- Create a model similar to the MHCU model adapted to the necessities of the child´s and adolescents mental health care network in order to improve the mental health sector in Portugal as a whole.

- Prove the applicability of the MHCU model in a real-life scenario and analyse with the decision makers possible alternative objectives that can be added to the model in order to improve it.

- Apply the MHCU model to other planning contexts, with other equity objectives like for example: Maximization of equities or maximization of health gains for the patients.

- Develop a decision support tool in order to be able to provide to Decision Makers (that do not possess mathematical knowledge) the possibility to use the model in an easy way with no need of mathematical knowledge.

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