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# THE ASYMMETRY EFFECT ON VOLATILITY DURING THE GLOBAL FINANCIAL CRISIS

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Responsibility for any remaining errors lies with the author alone.

## II. Abstract

The main objective of this dissertation is to investigate the asymmetric effects of shocks on volatility during the Global Financial Crisis of 2007 – 2009. Using daily logarithmic returns, we estimate univariate EGARCH and GJR models assuming three different conditional distributions: the Gaussian normal, Student's t and Generalized Error Distribution. The stock indices under analysis, which include largest companies in the world, are S&P 500, NASDAQ, FTSE 100, DAX, CAC 40, NIKKEI 225 and HSI. The data ranges from September 15, 2006 to September 15, 2010, being split in two subsamples by the collapse of Lehman Brothers on September 15, 2008.

Our results suggest that asymmetric effects are present in all stock markets analysed. In most cases, the impact becomes weaker after the Lehman Brothers bankruptcy, indicating that the negative shocks did not raise volatility as much as they did before the bankruptcy. EGARCH model with fatter tailed distributions appears to be the best in-sample predictive model. Moreover, we test the statistical significance of the change between asymmetry coefficient estimates of the EGARCH model, and conclude that the majority are not statistically significant, suggesting that the asymmetry coefficients do not depend on the sample period.

**Keywords:** Volatility, Asymmetry effects, Global Financial Crisis, Stock Market Indices.

**JEL Classification:** C32; C55.

## III. Resumo

O principal objetivo desta dissertação é investigar os efeitos assimétricos dos choques na volatilidade durante a Crise Financeira de 2007 – 2009. Usando rendibilidades logarítmicas diárias, são estimados dois modelos univariados, EGARCH e GJR, que assumem três distribuições condicionais: distribuição Gaussiana normal, *Student's t* e *Generalized Error Distribution*. Os índices de ações analisados, que incluem grandes empresas mundiais, são S&P 500, NASDAQ, FTSE 100, DAX, CAC 40, NIKKEI 225 e HSI. O período temporal dos dados começa a 15 de setembro de 2006 até 15 de setembro de 2010, sendo dividido em dois sub-períodos pela falência do Lehman Brothers no dia 15 de setembro de 2008.

Os resultados sugerem que o efeito assimétrico está presente em todos os mercados acionistas que foram analisados. De um modo geral, o impacte torna-se mais fraco depois da falência do Lehman Brothers, indicando que os choques negativos não aumentam a volatilidade tanto como aumentam antes da falência. O modelo EGARCH com distribuições de caudas pesadas, é o melhor modelo para a previsão *in-sample*. Adicionalmente, é testada a significância estatística das diferenças entre as estimativas dos coeficientes de assimetria do modelo EGARCH. Concluiu-se que a maioria das diferenças não é estatisticamente significativa, sugerindo assim que os coeficientes de assimetria não dependem do período temporal dos dados.



**Palavras-Chave:** Volatilidade, Efeito assimétrico, Crise Financeira, Índices de ações.

**Classificação JEL:** C32; C55.

## IV. Index

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## VII. List of Abbreviations

BRIC - Brasil, Russia, India and China

CAPM - Capital Asset Pricing Model

CEEC - Central and Eastern European Countries

GDP - Gross Domestic Product

GED - Generalized Error Distribution

GFC - Global Financial Crisis

i.i.d. - independent and identically distributed

NIC - News Impact Curve

## 1. Introduction

The Global Financial crisis of 2007 - 2009 forever reshaped the markets across the world. One particular moment stands out – September 15, 2008. On this day, a major investment bank Lehman Brothers went bankrupt, making it the largest bankruptcy in history and impelling even more the Global Financial Crisis. Stock markets lost trillions of dollars in market capitalization, and the recession quickly affected the real economy. Given the uncertainty, this period is characterized by violent price drops. For instance, the second week of October 2008, (i.e. October 6 – 10, 2008) was the worst week for markets in 112 years (Chaudhury, 2014). Volatility reached high levels as extreme daily movement occurred more frequently, hindering investor's financial decisions in terms of risk modelling, hedging strategies, determination of cost of capital, portfolio selection and asset allocation, as well as pricing of primary and derivatives instruments.

In this context, and given that such market behaviour is rare, the present work is concerned with the study of volatility during the Global Financial Crisis. Furthermore, we make a particular emphasis on one of the stylized facts of asset returns: the asymmetry effect. Negative innovations have higher impact on volatility than positive innovations. The aim is to investigate the impact of the asymmetry, comparing the magnitude of the effect before and after the Lehman Brothers collapse.

The sudden risk aversion among investors within highly linked markets makes this crisis truly global. Taking this global integration into account, we examine the asymmetric effects (differentiating good and bad news) on volatility using data from seven different stock market indices. This allows us to compare the asymmetry, not only before and after the bankruptcy, but also across markets.

Different proxies and volatility models have been presented over the years, making the literature on this subject very extensive. Among them, the conditional heteroskedasticity models have become very popular and common in finance applications. According to Tsay (2013), conditional heteroskedastic models can be divided in two general classes: The autoregressive conditional heteroskedasticity models, such as ARCH and GARCH (Engle, 1982; Bollerslev 1986), which use an exact function to govern the evolution of volatility; while the second class is stochastic volatility models, which use a stochastic equation to describe volatility. We use two extensions of models belonging to the first category. The EGARCH

(Nelson, 1991) and GJR (Glosten et al., 1993) models which incorporate the asymmetric effects and have been widely used by researchers.

This dissertation contributes to the existing literature of asymmetry effects on volatility in several ways. The seven indices under analysis are split in subsamples and we use two different asymmetric models with three different conditional distribution specifications, allowing comparison from different perspectives. Additionally, we test the statistical significance of the asymmetry changes after the Lehman Brothers bankruptcy.

From the empirical study two main conclusions arise. The first one is that the asymmetric effect of shocks is present in the equity markets analysed. Results indicate that the effect is less pronounced after the bankruptcy of Lehman Brothers. However, testing the statistical significance of the changes, reveals that some of them are not statistically significant.

The reminder of this work is organized as follows. Section 2 focuses on the literature review. First, we provide a general overview of some important works on volatility, including stylized facts, and then narrow our attention to the asymmetry effects research. In Section 3 we briefly discuss the Global Financial Crisis. In Section 4 a preliminary analysis is performed on the data. In section 5 we present the methodology, including models and a short discussion of conditional distributions. Section 6 shows the empirical results. Lastly, Section 7 presents the main conclusions.

## 2. Literature Review

The behaviour of the stock market is not something that is easily addressed. Market participants face a lot of uncertainty that is intrinsic to it. The greater this uncertainty, the riskier it gets to make decisions. Volatility is related to risk, as it measures the spread of outcomes, either negative or positive. This market variable tells nothing about the direction of the stock price, but instead, it is a measure of how much asset returns fluctuate around its mean. Since the true volatility is unobservable (Tsay, 2013), different proxies have been developed over the years. Tsay (2013) divides volatility measures in three types: Volatility as the conditional standard deviation of daily returns; Implied volatility, derived from option contract prices; Realized volatility which consist in estimating daily volatility using high frequency financial data. Natenberg (1994) follows a different "real-word" option trader approach, and divides volatility in five types: Future volatility, which is the future distribution of prices of the underlying contract; Historical volatility, calculated based on the historical data; Forecast volatility, where based on stylized characteristics, volatility is estimated over a forecasting period; Implied volatility, which again, is obtained from theoretical option pricing model; Seasonal volatility, consisting in seasonal weather conditions that affect commodity prices.

Moreover, Tsay (2013) classifies conditional volatility models into two categories: models that use an exact function to calculate the evolution of the volatility and on the other hand models that use a stochastic equation to describe volatility. Thus, in this chapter we focus on literature on time-varying conditional volatility type models of the first category, in particular the applications of popular ARCH (Engle, 1982), GARCH (Bollerslev, 1986) and various extensions which attempt to improve volatility capturing. Compilation by Bollerslev (2010) provides a list of several models and their acronyms that have been used in the literature.

The importance comes from the fact that volatility can be used as parameter to numerous financial applications. In terms of pricing, Black and Scholes (1973) presented a theoretical valuation formula applicable to corporate liabilities and options. One of the inputs that plays a key role in the formula is the future volatility of the underlying asset during the life of the option. Prior to the Black-Sholes formula, pricing options required heavy mathematical calculations, that is why it is still a prevailing formula for option pricing until these days (Natenberg, 1994). For option traders, volatility sometimes is key for a successful strategy, as it dictates the likelihood for an option to end in-the-money or expire worthless. In the conditional volatility framework, several studies explored the option pricing using GARCH type models, which have been proven useful in correcting price biases of the Black-Scholes formula and due to easy implementation. (Duan, 1995, 1996).

In terms of portfolio selection and determination of cost of capital, volatility is essential to the investors and portfolio managers since it helps them to estimate the amount of risk they are comfortable with. Bollerslev, Engle and Wooldridge (1988) proposed a CAPM (Capital Asset Pricing Model) model that allowed the covariance matrix to vary over time following a GARCH process. Corhay and Rad (1996) estimate market model parameters adjusted for GARCH effects. In the same context, Bera, Bubnys and Park (1988) estimated individual securities and market portfolios betas based on ARCH model. They found out that this approach provided much more efficient beta estimates.

Analysing asset prices volatility is also useful when establishing monetary and exchange rate policy. Market expectations are fundamental in shaping Central Bank’s decisions, as market instability can have consequences for the real economy (Balder, 1997). Lastrapes (1989) suggested that the dollar exchange rate modelled as ARCH process is not independent of shifts in U.S. monetary policy regimes. Conditional volatility models have been proven appropriate in the investigation of inflation targeting, as a proxy for uncertainty (Kontonikas, 2004).

From a risk management perspective, the most well-know market risk measurement is the Value-at-Risk. The empirical research on this topic is enormous, and the discussion of such models is beyond the scope of this dissertation. Nevertheless, incorporating conditional volatility models such as GARCH can lead to better performing models in terms of estimating the worst loss (Hull and White, 1998). All in all, Engle (2001) argues that the ARCH and GARCH models have become standard tools where the volatility is the central issue.

Substantial amount of literature is focused on the predictive ability of the GARCH type models, and comparison between them. The general conclusion is that forecasting performance depends on the model’s specifications, asset type, market data that is analysed, its frequency and forecast horizon. Poon and Granger (2003) reviewed 93 published and working papers that study forecasting performance of various volatility models. They classified the reviewed papers into four categories: historical volatility models, which include random walk, historical averages, moving averages, exponential weights, autoregressive models and fractionally integrated autoregressive absolute returns models; Any member of ARCH/GRACH family models; Option implied volatility models; and Stochastic volatility models. Models that involved comparison between historical volatility and GARCH, 22 (56%) studies favoured historical volatility, against 17 (44%) that favoured GARCH family. Out of 18 papers that compare GARCH against implied volatility, only one paper found GARCH superior. Authors argue that the success of implied volatility results from the fact that these forecasts use a larger and more relevant information set when compared to other methods. One paper found GARCH better than stochastic volatility. An important remark made by the authors is that models that incorporate volatility asymmetry such as EGARCH and GJR usually perform better than GARCH.

## 2.1 Stylized Facts about Financial Market Volatility

Conventionally, it is believed that future price or returns of the financial assets are unpredictable. Nevertheless, financial data exhibit some persistent properties, which are common across assets, asset classes, markets and time periods. These statistical similarities are called Stylized Facts (Cont, 2001).

### 2.1.1 Fat-tails

Statistical validity of volatility depends on the distribution of the returns. Commonly, the distribution of the financial asset returns does not follow a normal distribution. It exhibits leptokurtosis or excess kurtosis, which means that the probability concentrates deeper into the tail when compared to the normal distribution. The returns distribution also tends to display a higher sharp centre (Praetz, 1972; Cont, 2001). The consequences of this departure are that the probability of very large returns, either positive or negative, are underestimated. In other words, violent price changes happen more often than captured. This has important implications from risk management perspective, as well as from empirical perspective (Fama, 1965). Mandelbrot (1963) argued that the tails of the returns are extraordinarily long, and suggested that, commodity such as cotton followed a Paretian stable distribution. Fama (1965) extended the analysis to stock prices and concluded that Mandelbrot hypothesis also applies. On the other hand, Praetz (1972) argued that a scaled t-distribution fits the stock market data better than Paretian stable and normal distribution. Lastrapes (1989) also verified that the unconditional distribution of foreign exchange growth rates is leptokurtic.

Nevertheless, the use of normal distribution in the finance is widespread. There are trade-offs between normal and non-normal distributions, but they usually not worth it, as long as the departure is not extreme. Conditional normality is often assumed due to ease of computation (Braun, Nelson and Sunier, 1995). Bollerslev (1987) presented a GARCH extension to allow for conditionally t-distributed errors. The author considered that the model fitted the data quite well. Nelson (1991) in his inaugural paper on EGARCH used the Generalized Error Distribution (GED) and the results showed that the conditional distribution of the errors had thicker tails than the normal. One general conclusion is that the asset returns distributions departs from normality, displaying leptokurtosis, and the common way to solve this problem is to adopt a conditional distribution with fatter tails (Bollerslev, Chou and Kroner, 1992). Ultimately, Fama (1965) suggested that investors are not concerned with the name of the distribution given by the researchers. The main interest is the shape of the distribution, how well it describes the relative frequency and what is the probability of gains and losses to be greater than a given amount.

### 2.1.2 Volatility Clustering

Volatility clustering refers to the persistence of the shocks through time. Mandelbrot (1963: 418) stated that "*(…) large changes tend to be followed by large changes* - *of either sign* - *and small changes tend to be followed by small changes (…)*", meaning that high volatility is likely to be followed by periods of high volatility, and low volatility tends to be followed by further low volatility. This property proves that there is some degree of regularity in the returns, and therefore the assumption that the returns are independent and identically distributed (*i.i.d.*) might be too strong. In practical terms, the market often has long periods of relatively low activity, followed by periods of high activity due to the information arrival that affects asset prices. It occurs in bunches rather than being evenly spaced over time (Praetz 1972; Brooks, 2008). According to Ding, Granger and Engle (1993) if the markets are efficient, arrival of new information would impact stock prices. Given that information comes in bunches, the distribution of the following return will depend on previous returns, but not necessarily implying that they are correlated. Fama (1965) suggested that volatility clustering is likely due to investors' ability to evaluate new information that comes into the market. It cannot always be evaluated precisely. Occasionally, new information will cause immediate large price movements, producing further reaction in the market. For example, during a market turmoil, participants are especially sensitive to new information, further increasing volatility. In other cases, price changes will not generate substantial reaction in the market, simply because participants will take time to adjust their expectations.

Overall, volatility is not constant over time and it has been one of the main incentives behind the development of ARCH and GARCH models. These are designed to deal with this issue (Engle, 2001).

### 2.1.3 Long Memory

Volatility is said to have long-memory, given that absolute and squared returns exhibit significant autocorrelation over long lags. It reflects long run dependencies between price changes. In line with the efficient market theory, asset returns contain little serial correlation, and dependence in asset returns is extremely slight or completely absent (Fama, 1970), but it is possible that they might be dependent. Cont (2001) argues that the absence of serial correlation in returns, does not imply the independence of the returns. Independence implies that any nonlinear functions of returns, either absolute or squared, will also have no autocorrelation. However, this property does not hold. Taylor (1986) argued that absolute returns has significant positive serial correlation over long lags. Autocorrelation can be found in both, daily squared and absolute returns, but it seems that absolute returns show much stronger effect (Granger, Ding and Spear, 2000). Ding, Granger and Engle (1993) suggest that the power transformation of the absolute returns has quite high autocorrelation for long lags, empirically proving that the property is strongest when exponent is around 1. This feature becomes weaker with weekly and monthly data. Granger, Ding and Spear (2000) found long memory property in stock indices, individual shares, commodity prices, interest rates and residuals of CAPM model.

According to Ding, Granger and Engle (1993) if asset returns are an *i.i.d*. process any transformation is also an *i.i.d.* process. But considering that the absolute and squared returns autocorrelation is usually well outside confidence intervals, asset returns are not an *i.i.d.* process. The authors suggest that one possible explanation for such serial correlation in the transformed series can be the heteroskedasticity of the data, which is changing over time. Cont (2001: 230) argued that such nonlinear dependence reflects "*(...) correlation in "volatility" of returns but not the returns themselves.*".

### 2.1.2 Asymmetry Effect

Another important stylized property of the financial data is the asymmetry effect. There is a negative correlation between current returns and future volatility. Two explanations for asymmetry are popular in the literature: leverage effect and volatility feedback hypothesis, latter also referred as risk premium effect. Leverage effects and asymmetry are often used as synonyms. According to Engle and Ng (1993) the name "leverage effect" is often used simply because it is popular among researchers. We attempt to explore this property with more detail, since asymmetry is the main point of this dissertation.

Early works of Black (1976) and Christie (1982) attributed asymmetry in the stock market volatility to the financial leverage. A drop in the equity value (negative return of a firm's stock), will cause its debt to equity ratio to rise. Shareholders, who bear residual risk, will perceive their future cash flow and the stock itself, as being more risky, further increasing the volatility. Cheung and Ng (1992) concluded that small firms are more sensitive to the leverage effect. Shocks with same magnitude will produce bigger effect on conditional volatility of smaller firms as compared to large firms, thus the impact of shocks on volatility varies inversely with the firm size. The reason is that the impact of shocks on prices of small firms creates more uncertainty regarding its stability, and therefore resulting in larger prices fluctuations. This property has also been found present in market indices (Cheung and Ng, 1992; Engle and Ng, 1993; Glosten, Jagannathan, and Runkle, 1993; Braun, Nelson and Sunier, 1995; Nelson, 1991). Schwert (1989) argued that although the leverage effect is more apparent during recessions periods, this factor explains only a small part of the variation of stock prices. Recently, McAleer (2014) and Caporin and Costola (2019) argued that asymmetry and leverage effect are two distinct phenomena. The authors state that leverage effect is a special case of asymmetry. Leverage effect imply that negative shocks lead to an increase in volatility, while positive shocks should lead to a decrease in volatility. On the other hand, asymmetry is when positive and negative shocks of the same size (in absolute terms) induce different magnitude changes in the conditional volatility. Furthermore, the authors argue that GJR and EGARCH models are in fact asymmetric but are not capable of showing leverage effects. Caporin and Costola (2019) concluded that TARCH and APARCH models also do not allow for leverage, while AGARCH allow for local leverage.

The second explanation for asymmetric volatility is based on the existence of time varying risk premium (Pindyck, 1984; French, Schwert and Stambaugh, 1987; Campbell and

Hentschel, 1992). Evidence shows that the expected market risk premium is positively related to market volatility, and if increases in market risk premia due to increased volatility are not offset by decrease in risk-free rate, then rising market volatility should lead to drops in the stock price (Braun, Nelson and Sunier, 1995). According to Wu (2001), if volatility is priced, an anticipated increase in volatility raises the required return on equity, resulting in an instant stock price decline. Bollerslev and Zhou (2006) noted that volatility feedback also implies negative correlation between current returns and future volatility, given that a shock to the volatility will require an immediate return adjustment to compensate higher risk in the future. Typically to account for volatility feedback, models within the GARCH-in-mean framework are used, where the conditional mean equation includes a parameter called the risk premium parameter (Tsay, 2013). Campbell and Hentschel (1992) argued that volatility feedback has little impact on returns but is more important during high volatility periods. According to Braun, Nelson and Sunier (1995) the impact of the volatility feedback depends on the strength of the link between market risk premium and market volatility, and whether the market risk premium is constant and how it evolves over time.

Both theories seem to explain the same property but the causality behind them is different. Leverage effect describes how negative returns increase volatility, while volatility feedback describes how increase in volatility negatively impacts stock returns. Wu (2001) suggested that both effects may be interacting. The author provides a hypothetical example, where an anticipated increase in volatility due to a foreign market turmoil, will raise traders' expectation of high volatility in the domestic market. When anticipating higher volatility and general uncertainty in the markets, traders will be hesitant to buy and willing to sell. The selling side will surpass the buying side, resulting in falling stock prices. As predicted by volatility feedback hypothesis, anticipated increase in volatility results in instant drop in the stock price. This stock price drop leads to an increase in the leverage ratio, as predicted by leverage effect hypothesis, consequently increasing volatility even more and a further drop in stock price.

Which effect is stronger and generates more asymmetry remains an open question. Bollerslev, Litvinova and Tauchen (2006) examined both effects using high-frequency five-minute S&P 500 futures returns. They found highly significant leverage effect at the intraday level, that lasts for several days. On the contrary, little or no evidence of volatility feedback was found. Sun and Wu (2018) studied the relationship between S&P 500 index returns and the squared VIX index (sometimes called as implied variance index) as the measure for volatility. Authors found evidence that nonparametric leverage effect is usually stronger than the nonparametric volatility feedback effect, except during calm market conditions. Bollerslev and Zhou (2006) developed a theoretical framework to assess the linkages between returns and realized and implied volatility. Using S&P 500 index data, authors concluded that the leverage effect, is always stronger for implied volatility than realized volatility. Regarding the volatility feedback effects, results are unclear. The correlation between returns and volatility depends on volatility proxy. For realized volatility the relationship with returns is negative, while for the implied volatility the relationship is positive, but the estimates are marginally significant. Bekaert and Wu (2000) investigated both effects using the market portfolio and three portfolios with different leverage ratio, constructed from the Japanese NIKKEI 225 stock index. Their results indicate that volatility feedback is stronger when compared to the leverage effect. Wu (2001) developed model where dividend growth and dividend volatility are two individual sources of uncertainty. Both asymmetry effects explanations are considered in the model. According to the author's results, volatility feedback and leverage effect are important in generating asymmetric volatility. Nevertheless, the author argues that the volatility feedback is the main determinant of asymmetry and can be very large during high volatility periods. Inkaya and Okur (2014) examined the interaction between leverage effect and volatility feedback rate for high-frequency five-minute ISE 30 index data. They suggest that both effects are significant during volatile periods and may be interacting. Authors also found evidence that increase in volatility does not always result in negative return, meaning that volatility feedback is not always present in the market behaviour. At the intraday level, the leverage effect also produced mixed results. The leverage parameter series alternated in sign, implying positive correlation between return and volatility.

### 2.2 Asymmetry Effects on Volatility during financial turmoil

During bearish market and financial crisis periods, overall volatility of the markets tends to rise, therefore making them more information-rich and attractive to analyse from asymmetry perspective. Thus, we further explore literature focused to some extent in addressing asymmetry effects within a crisis context.

Leeves (2007) investigated the conditional volatility for Indonesia during the Asian Financial crisis that began roughly in the summer of 1997. The crisis was provoked when the Thai baht was cut from being pegged to the U.S. dollar. Devaluation of Indonesian rupiah at the end of 1997 and mid 1998 is also pointed out as one of the main drivers of the crisis.

Significant spillover effect led to substantial losses all over southeast Asia and Japan. Author applied three asymmetric models (GJR, NGARCH and AGARCH) to Jakarta Stock Exchange index, for a period starting in 1990 until 1999. Although asymmetry estimates were statistically insignificant in all three models, the NIC (News Impact Curve) suggested some level of asymmetric response. Moreover, the author re-estimated the parameters using only 1997-1999 data, referring to it as the *crisis period*. In this scenario, asymmetry coefficients estimates become significant in all three models, indicating asymmetric response to shocks. Leeves further explore this period by obtaining parameters from a rolling regression of 400 observations. His results show that the asymmetry effect estimates in all three models increased (in absolute value) in late 1997 matching the devaluation of rupiah. However, by the end of 1999, the asymmetry estimates became negligible and even positive, suggesting that positive shocks started to have bigger impact on volatility. In the same historical context, Lim and Sek (2013) studied the Malaysian stock market. The authors use three subsamples to study the Asian financial crisis: January 1990 to June 1997 as the *pre-crisis* period, July 1997 to September 1998 as the *crisis* period and October 1998 to December 2010 as the *post-crisis* period. Results from the in-sample analysis indicate that simple GARCH outperforms the EGARCH and TGARCH models in the *pre-crisis* and *crisis* periods. The TGARCH model is superior in the *post-crisis* subsample. One important remark made by the authors is that the asymmetry coefficients estimates are not statistically significant in the *pre-crisis* period. The out-of-sample results indicate that TGARCH is superior in the *pre-crisis* and *post-crisis* periods, while GARCH outperforms in the *crisis period*. Still in the Asia-Pacific region, Nor and Shamiri (2007) examined Malaysian KLCI and Singaporean STI indices, from January 1991 to December 2004. Authors compared the performance of GARCH, EGARCH and GJR models using three different distributions: Gaussian normal, Student's *t* and Generalized Error Distribution. They found evidence of asymmetry effects in both markets. According to their results, asymmetric models with fatter tailed distributions outperform the symmetric GARCH. The best in-sample model for KLCI is the GJR with Student's t conditional distribution, while for the STI index, no clear result is obtained since EGARCH and GJR with Student's *t* provide equal log-likelihood.

Addressing more recent events, Olbrys (2013) employs a univariate EGARCH approach to four stock markets: S&P 500 index as a benchmark market and three biggest CEEC (Central and Eastern European Countries) markets, WIG (Warsaw), PX (Prague) and BUX (Budapest). Furthermore, there conditional distributions were assumed for the innovations: normal, Student's *t*, and skewed *t*. Author focuses on three different timeframes: January 2007 to December 2011; February 2007 to March 2009 labelled as the *down market*; and from March 2009 to March 2011 labelled as *up market*. The idea is to distinguish market moves during the Global Financial Crisis. The conclusions for the whole sample suggested that all four markets are more sensitive to negative returns than positive returns, confirming presence of asymmetry. Additionally, skewed *t* conditional distribution proved to be the most adequate. Regarding the *down market*, asymmetric effects are especially strong during this period for all markets. Volatility response to bad news is extremely pronounced for S&P 500 and WIG. As for the *up market*, model results are relatively poor, given the fact that most of the estimates are statistically insignificant. In general, these results demonstrate the connection between volatility asymmetry and the Global Financial Crisis. Kaur and Singh (2015) investigate the existence of asymmetry effects in BRIC countries after the Global Financial Crisis. They apply EGARCH-M and TGARCH-M models to the respective market indices, from July 2009 until June 2014. Furthermore, they separate their analysis between leverage and volatility feedback effect. TGARCH-M model results indicate that for the Brazilian Ibovespa index both leverage effects and volatility feedback coefficient estimates are statistically significant. Russian RTS and Indian CNX Nifty indices results only found leverage effect to be statistically significant, while Chinese SSE Composite index show no presence of leverage effect nor volatility feedback. For EGARCH-M model, results point out to presence of leverage effect in all markets. As for the volatility feedback, again only Ibovespa coefficient estimate is significant. Additionally, they compare the results with MSCI Frontier Markets and Emerging Market indices. Leverage effects were found to be present in both indices, while volatility feedback is present in the Frontier Markets index. One noteworthy finding in the TGARCH-M model for Brazilian and Russian markets is regarding the ARCH coefficients, which are not statistically significant. Similar approach was used by Birau and Trivedi (2013). Authors found presence of asymmetry in BRIC countries from 2003 to 2013. Výrost and Baumöhl (2009) analyse S&P 500 index volatility from July 2004 to August 2009. Like previous authors, they use EGARCH and TGARCH models to look for presence of asymmetric effects. They split the sample into two different timeframes with the same amount of observations. From July 2004 to January 2007 as the *pre-crisis* period and from February 2007 until August 2009 as the *crisis* period. According to the authors, February 2007 was chosen as split date based on first problem announcements in the subprime mortgage market by the HSBC. To compare the asymmetry effects, they apply the NIC. Both models reveal existence of asymmetry in the *pre-crisis* data: good news are followed by significantly lower variance. Regarding the *crisis* series, asymmetry effects were statistically significant in the EGARCH model, but the asymmetry coefficient estimate was lower (in absolute value) when compared to the *pre-crisis* period. When estimating the *crisis* series, TGARCH of higher order was applied, not allowing for a direct comparison with *pre-crisis* model specification. Nevertheless, the model also suggested asymmetry presence. Back to southeast Asia, Angabini and Wasiuzzaman (2011) tested the existence of asymmetry in the Malaysian KLCI index, covering the Global Financial Crisis. As previous authors, they split the data into two periods: from June 2000 until the end of 2007, thus not including the crisis, and from June 2000 to March 2010, which includes the Global Financial Crisis. Regarding the models applied, EGARCH and GJR both reveal presence of asymmetry regardless of the period. Authors report that the volatility is relatively constant from 2001 to 2007 and seems to increase in the middle of 2007 until 2009. When comparing the results between the series, asymmetry effect increased 11.5% and 18.5% in EGARCH and GJR estimates respectively, proving that crisis ultimately impacted volatility response to negative returns. Although asymmetry effects are statistically significant, their results indicate that a simple GARCH model outperformed the asymmetric models. Olbrys and Majewska (2017) investigate the asymmetry impact on volatility for major European stock markets by using a univariate EGARCH approach, with three different distributions: normal, Student's *t* and skewed *t*. Authors analysis cover a period from January 2003 to December 2016. Furthermore, they divide the data in three sub-samples, *pre-GFC* period, *GFC* period and *post-GFC* period, each one specific to its market. For FTSE 100 the *GFC* period ranges from October 2007 to February 2009; For CAC 40 the *GFC* period was defined from May 2007 to February 2009; Germany's DAX the *GFC* period lasted from December 2007 to February 2009. Whole sample results revealed that negative innovations increase volatility considerably more than positive, confirming the evidence of negative asymmetry effects for all markets. Skewed *t* distribution was found to be more appropriate. The subsample results indicate presence of asymmetry effects, regardless of the index and period. However, the asymmetry effect for FTSE 100 and CAC 40 is stronger in the *pre-GFC* period than *GFC* period. DAX results indicate that out of three periods, the asymmetry is stronger in the *GFC* period, nevertheless the coefficient becomes positive in the *post-GFC* period, meaning that positive innovations have bigger impact on volatility. Concerning the conditional distribution, the normal distribution prevails in six out of nine possible cases. The authors Slimane, Mehanaoui and Kazi (2013) examined the intraday volatility transmission for the European market during the Global Financial Crisis. As in the previous study, they use FTSE 100, CAC 40 and DAX indices. Firstly, they perform a structural brake test on S&P 500 daily index, to identify the structural brake date. According to their results, the date of the structural break is September 12, 2008, one trading day before the Lehman Brothers bankruptcy. Next, they apply a bivariate VAR EGARCH (Vector Autoregressive Exponential General Autoregressive Conditional Heteroscedasticity) model to the five-minute intraday data of the European markets, from July 1, 2008 to September 11, 2008 defined as *pre-turmoil* period and from September 12, 2008 to November 28, 2008 defined as *turmoil* period. Authors found evidence that during the *turmoil* period positive innovations may have bigger impact on volatility than negative innovations, as opposed to what happens during calm periods.

In sum, the results regarding the asymmetry effects during turbulent market conditions can be diverse, depending on the period under analysis, models applied and different statistical assumptions.

**3. Crisis Background**

The Global Financial Crisis of 2007 – 2009 is certainly one on the most severe and global crisis in modern history. Markets in nearly every country, sector and industry, lost large amount of value in a relatively short period. Among many repercussions, this period is characterized by rising unemployment rates, decline in investment and GDP (Gross Domestic Product) contractions. Investors faced difficulties in diversifying their portfolios and companies saw their revenues drop. The volatility levels increased significantly and large daily fluctuations occurred more frequently. Such unusual behaviour can potentially be of great importance for portfolio optimization and management, risk assessment, hedging strategies and so on (Chaudhury, 2014). Overall, is estimated that from October 2007 to February 2009 worldwide equity market alone, lost more than $29 trillion (Bartram and Bodnar, 2009).

The Subprime Mortgage crisis in early 2007 was the starting point. Banks, insurance companies and hedge funds created a large market for mortgage-backed securities and other complex derivatives that ultimately led to the bursting of the housing bubble. Mortgage crisis quickly spread to the banking industry as they were filled with these toxic products. Many financial institutions started to feel the pressure to find liquidity. During this period huge losses linked to subprime securities were reported and in March 2008, one of the United State's investment bank Bear Stearns is sold to JP Morgan Chase for $240 million. Equity markets were relatively stable during the first two quarters of 2008, and mainly the financial sector was affected. Serious downturn started in mid-September. On September 10, 2008, Lehman Brothers, one of the biggest investment banks puts itself for sale. Five days later, the institution filled for Chapter 11 bankruptcy protection. At the time of the filling, Lehman Brothers had approximately 25,000 employees and more than $690 billion in assets, making it the largest bankruptcy in history. Initially Barclays was interested in buying the distressed bank, to expand its operations, but at the end the deal failed, and the U.S. regulators didn't provide government bailout. Bank of America was also involved in the negotiations to buy Lehman Brothers, but instead purchased Merrill Lynch, another distressed institution, for $50 billion. On September 16, 2008, the Federal Reserve Bank lent AIG, the biggest insurance company in the world that also was seriously troubled, $85 billion and took control of 79.9% of the company (Chaudhury, 2014). At the end of 2008, the insurance company reported $99.3 billion loss, the biggest corporate loss in history. Barclays ended up buying the U.S. part of Lehman Brothers. At the beginning of October, U.S. president George Bush signed the *Emergency Economic Stabilization Act of 2008,* which allowed the United States Department of the Treasury to purchase troubled assets from financial institutions. The law established the *Troubled Asset Relief Program,* a $700 billion plan to help the financial system. Similarly, on October 8, 2008, United Kingdom Treasury announced a £500 billion bank rescue package. Bartram and Bodnar (2009) provided a detailed timeline of events, starting February 7, 2007 until February 27, 2009.

# 4. Preliminary Analysis

## 4.1 Data

In our analysis we use the September 15, 2008 as the dividing point between two nonoverlapping subsamples, defined as calm period and turmoil period. It is difficult to specify the exact date of the beginning of the crisis, thus we use the collapse of Lehman Brothers as the central event. Our analysis covers a period of four years. From September 15, 2006 to September 15, 2008 as the calm period and from September 16, 2008 to September 15, 2010 as the turmoil period.

The data was obtained from Yahoo Finance website and consists of seven equity indices from three different regions: S&P 500 and NASDAQ Composite from North American market; from the European market we selected United Kingdom's FTSE 100, German DAX and French CAC 40; finally, to analyse the effects in the Asia-Pacific region, we use the NIKKEI 225 representing the Japanese market and Hang Seng Index (HSI) from Hong Kong. Indices are a good proxy to represent the overall performance of the major public companies and the overall health of the economy. By using data from different regions, we analyse and compare volatility behaviour and how pronounced it is the asymmetry effect before and after the September 15, 2008.

Figure 1(a) illustrates the evolution of daily closing prices of the indices. Visually, two periods in all indices are very distinct. The equity market clearly entered downward trend after the Lehman's bankruptcy, and only started to recover in early March 2009. With exception of HSI during 2006, all markets had similar co-movement, suggesting strong linkages between them.

The closing prices were converted into daily logarithmic returns. The formula is given as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln\,(P_t) - \ln\,(P_{t-1}) \qquad (1)$$

for $t = 1, \dots, T$, in which $r_t$ denotes the return at time *t*, $P_t$ is the current price and $P_{t-1}$ the previous day's closing price. Figure 1(b) shows the returns over the sample period. Volatility rose after the dividing point, as all indices exhibit large fluctuations starting mid-September 2008, also evidencing the presence of volatility clustering. Large (small) price changes tend to

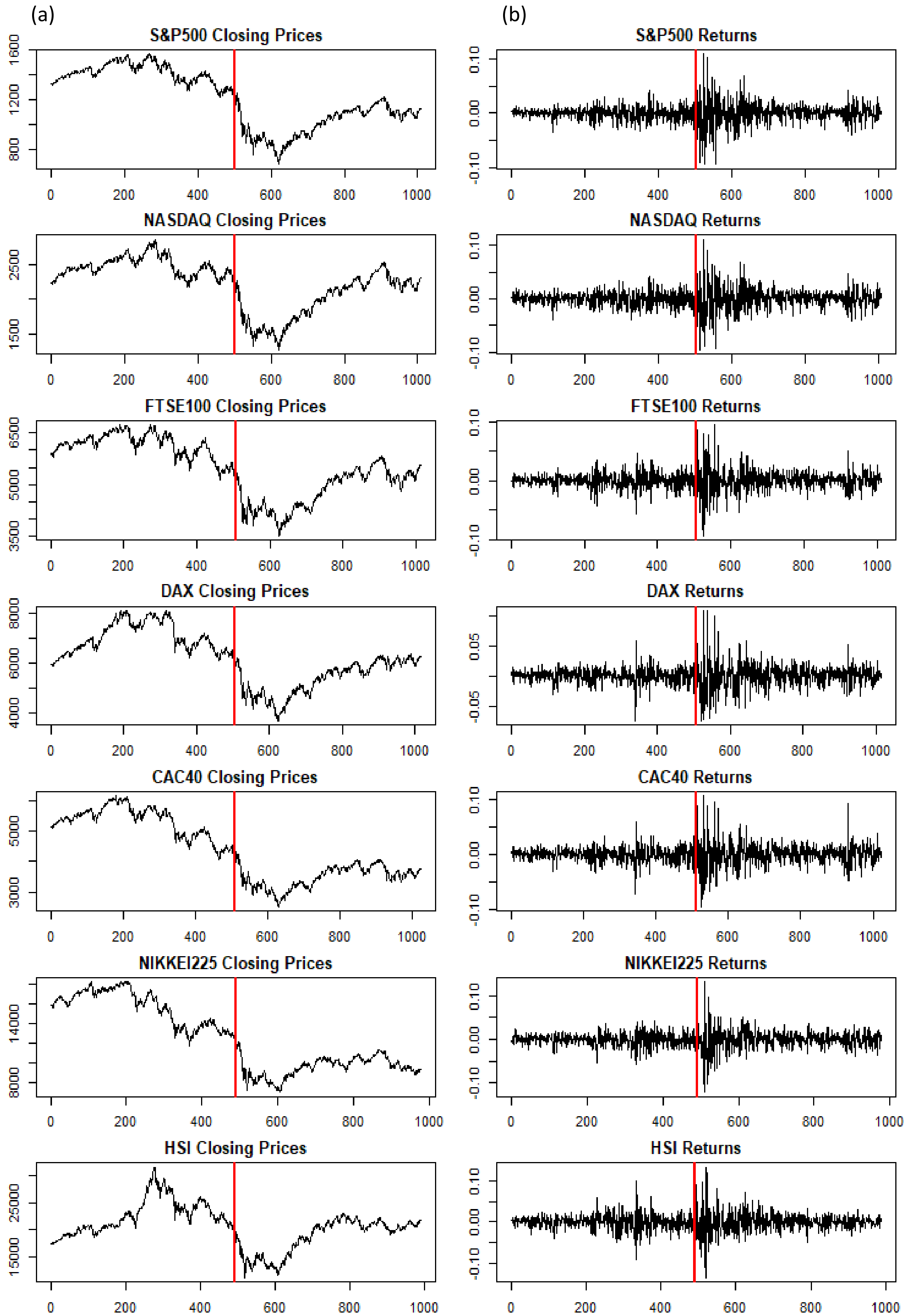


**Figure 1**. Time plots of: (a) Daily closing prices, and (b) Daily log returns of the indices under analysis. Red line indicates September 15, 2008.

be followed by large (small) changes of either sign, visually implying that market volatility changes overtime. Volatility clustering is the one of the foremost properties for the application of GARCH type models. These are specifically designed to deal with time varying volatility structure (Ding, Granger and Engle, 1993; Engle, 2001).

### 4.2 Descriptive Statistics

Table 1 presents the descriptive statistics for all indices, categorized for the calm period and turmoil period. The number of observations is different for each market and period due to differences in the trading days. The samples mean are all very close to zero and one sample *t*-test results show that all means are not statistically different from zero. S&P 500, CAC 40 and NIKKEI 225 have negative means in both periods. On the contrary, DAX and HSI display positive mean during all four years. The remaining indices display negative mean in the calm period and positive in the turmoil period. Regarding the maximum and the minimum returns, the difference between them is significantly larger in the turmoil period, and there is an increase in the standard deviations when comparing the two subsamples, indicating higher volatility in the turmoil period for all indices. Except for the DAX, CAC 40 and HSI in the turmoil period, skewness values are negative, meaning that there is a higher probability of negative returns. All indices, specially in the turmoil period, exhibit higher kurtosis than the standard value of normal distribution, which is 3, meaning that the extreme events are more likely to occur and suggesting that distributions with fatter tails might fit the data better. According to Jarque-Bera (1987) test, the normal distribution hypothesis is rejected for all indices, regardless of the period.

| | **S&P 500** | | **NASDAQ** | | **FTSE 100** | | **DAX** | | **CAC 40** | | **NIKKEI 225** | | **HSI** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period | Calm Period | Turmoil Period |
| Dates | 15/09/2006 15/09/2008 | 16/09/2008 15/09/2010 | 15/09/2006 15/09/2008 | 16/09/2008 15/09/2010 | 15/09/2006 15/09/2008 | 16/09/2008 15/09/2010 | 15/09/2006 15/09/2008 | 16/09/2008 15/09/2010 | 15/09/2006 15/09/2008 | 16/09/2008 15/09/2010 | 15/09/2006 12/09/2008 | 16/09/2008 14/09/2010 | 15/09/2006 12/09/2008 | 16/09/2008 15/09/2010 |
| Observations | 503 | 504 | 503 | 504 | 506 | 506 | 507 | 508 | 510 | 512 | 491 | 486 | 491 | 497 |
| Mean | -0.0001960 (0.6937) | -0.0001158 (0.9045) | -0.00004403 (0.9359) | 0.00010754 (0.9124) | -0.0002404 (0.658) | 0.00012912 (0.8773) | 0.00005167 (0.9207) | 0.00006315 (0.9444) | -0.0004044 (0.4704) | -0.0002039 (0.8302) | -0.0005624 (0.3756) | -0.0005410 (0.607) | 0.00012802 (0.879) | 0.00034588 (0.7499) |
| Minimum | -0.0482830 | -0.0946951 | -0.0393586 | -0.0958769 | -0.0563689 | -0.0926557 | -0.0743346 | -0.073355 | -0.0707737 | -0.0947154 | -0.0581568 | -0.1211103 | -0.0905132 | -0.1358202 |
| Maximum | 0.0415349 | 0.109572 | 0.04106064 | 0.1115944 | 0.04640907 | 0.09384339 | 0.05761049 | 0.1079747 | 0.05833491 | 0.1059459 | 0.04182331 | 0.1323458 | 0.1018394 | 0.1340681 |
| Std. Dev. | 0.01115711 | 0.02166424 | 0.01226366 | 0.02193248 | 0.01220507 | 0.01881105 | 0.01168698 | 0.0204136 | 0.01264334 | 0.02150299 | 0.01403817 | 0.02319387 | 0.01863649 | 0.02415429 |
| Skewness | -0.3648347 | -0.1586395 | -0.1658774 | -0.1289997 | -0.1980645 | -0.0444315 | -0.6000901 | 0.3502741 | -0.3677433 | 0.2768391 | -0.3919851 | -0.3352026 | -0.1113306 | 0.1798251 |
| Kurtosis | 4.779472 | 7.662545 | 3.7477458 | 6.75137 | 4.961886 | 8.311069 | 7.695607 | 7.895183 | 5.855861 | 7.613781 | 4.465363 | 8.902539 | 6.367114 | 8.951676 |
| Jarque-Bera | 79.02 (0.0000) | 464.64 (0.0000) | 14.512 (0.0007) | 301.18 (0.0000) | 86.113 (0.0000) | 602.28 (0.0000) | 502.45 (0.0000) | 524.14 (0.0000) | 187.68 (0.0000) | 466.59 (0.0000) | 57.567 (0.0000) | 724.95 (0.0000) | 237.04 (0.0000) | 743.69 (0.0000) |

**Table 1.** Descriptive statistics of daily logarithmic returns of S&P 500, NASDAQ Composite, FTSE 100, DAX, CAC 40, NIKKEI 225 and HSI indices divided by subsample. Numbers in parenthesis are *p*-values: For the mean is the conventional *t*-statistic; for the Jarque-Bera is the test statistic for the null hypothesis of normal distribution of sample log returns.

## 5. Methodology

In this section we present the models as well as the estimation techniques used in this dissertation. We selected GARCH-type models which capture volatility as the conditional standard deviation. It is very unlikely that within the financial time series framework, the volatility will be constant over time, and such models are suitable to deal with this behaviour. We extend our analysis by using different distributional assumptions.

Although not used in the estimation process, we discuss the general ARCH and GARCH models in order to provide a better insight of this class of models.

Let $F_{t-1}$ be the past information set of all relevant variables available up to time $t-1$ and $\mu_t$ conditional expected return, also expressed as $E(r_t|F_{t-1})$. The excess return, shock or innovation term at time $t$ than becomes:

$$\varepsilon_t = r_t - \mu_t = r_t - E(r_t|F_{t-1}),\ \ \varepsilon_t \sim N(0, \sigma_t^2) \tag{2}$$

Engle and Ng (1993) referred that a positive $\varepsilon_t$ shock suggests the arrival of good news, and on the other hand, a negative $\varepsilon_t$ suggest bad news, since the return is lower than expected.

The model for $r_t$ is called the mean equation, and typically follows an autoregressive process. The conditional variance of $r_t$ given the past information $F_{t-1}$ is then:

$$\sigma_t^2 = var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}] = var(\varepsilon_t|F_{t-1}) \tag{3}$$

so the rationale behind the ARCH model is to allow the conditional variance of innovations $\varepsilon_t$ to depend on previous values of squared innovations $\varepsilon_t^2$. The general structure of ARCH(*p*) model of Engle (1982), where variance depends on *p* lags of squared errors is represented by:

$$r_t = \mu_t + \varepsilon_t \tag{4.1}$$

$$\varepsilon_t = z_t \sigma_t \qquad z_t \sim N(0,1)$$

$$\sigma_t^2 = \omega + \sum_{t=1}^{p} \alpha_i \varepsilon_{t-1}^2\,, \tag{4.2}$$

where $\omega$ and $\alpha_i, i\,\epsilon[1, p]$, are nonnegative constants (i.e. $\omega > 0$, and $\alpha_i \geq 0$ for $i > 0$) to assure that the conditional variance is strictly positive. $\{z_t\}$ is a sequence of independently and identically distributed (*i.i.d)* random variables with mean 0 and variance 1. In general, the conditional distribution assumed for $z_t$ is standard normal, and it generates some degree of unconditional excess kurtosis in ARCH, but insufficient to fully account for the fat-tails characteristic of the asset returns (Bollerslev, Chou and Kroner, 1992). Nonetheless distributions like Student's *t* and Generalized Error Distribution (GED) are often assumed (Tsay, 2013).

Bollerslev (1986) introduced an extension to the model. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is essentially an infinite order ARCH. The general form of GARCH(*p*,*q*) is as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (5)$$

where $\omega$, $\alpha_i, i \epsilon[1,p]$, and $\beta_j, j \epsilon[1,q]$, are nonnegative constants. The $\alpha_i$ and $\beta_i$ coefficients are frequently referred as ARCH and GARCH terms respectively. The ARCH term indicates short-term persistence of shocks on volatility, while GARCH term represents the long-run persistence of shocks (Leeves, 2007). Usually, given its generalized form, GARCH model fits the data better than ARCH. Both models are good in capturing volatility clustering and rather simple specification made them widespread tools in the volatility modelling. The models also have some disadvantages. The ARCH responds slowly to large isolated shocks to the return series. Non-negativity constrains might be violated when the model is specified with large number of parameters. Both models are symmetrically constrained and fail to capture the asymmetric response of volatility to shocks. Bad news has the same impact as good news.

## 5.1 Asymmetric models

### 5.1.1 EGARCH

To overcome the symmetric weakness of the GARCH model, Nelson (1991) proposed the Exponential GARCH (EGARCH). The model incorporates a component that generates more volatility if the shock is negative. There are several ways to express the model, and one of the possible specifications is as follows:

$$ln\,(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{p} \beta_i \, ln\, \sigma_{t-i}^2 \quad (6)$$

where $\omega, \alpha_i, \beta_j$ and $\gamma_i$ are constant parameters. Due to its specification there is no non-negativity restriction on parameters. The logarithm of $\sigma_t^2$, ensures that the conditional variance remains positive. The $\gamma_i$ is the parameter responsible for allowing the conditional variance to respond asymmetrically to returns and it is tipically negative, allowing negative shocks to generate more volatility (Engle and Ng, 1993).

### 5.1.2 GJR

Another popular model used to capture asymmetry is the GJR (also known as Threshold GARCH). Proposed by Glosten, Jagannathan and Runkle (1993), it account for asymmetry by introducing an indicator or dummy variable. The conditional variance is now given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I_{\varepsilon_{t-i<0}})\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{7}$$

where $I_{\varepsilon_{t-i<0}} = \begin{cases} 1 \; if \; \varepsilon_{t-i} < 0, \\ 0 \; if \; \varepsilon_{t-i} > 0 \end{cases}$

and $\omega, \alpha_i, \gamma_i, \beta_j$, are nonnegative parameters satisfying same conditions as in GARCH model. When the $\varepsilon_{t-i}$ is positive, the indicator variable in (7) is 0, contributing $\alpha_i \varepsilon_{t-i}^2$. On the other hand, when the $\varepsilon_{t-i}$ is negative, the indicator variable in (7) is 1 and the contribution to the $\sigma_t^2$ is larger, $(\alpha_i \gamma_i)\varepsilon_{t-i}^2$, with $\gamma_i > 0$. The Threshold GARCH (or TGARCH) name comes from Zakoian (1994), who proposed a similar model. The difference between the two lies in the fact that the GJR models the conditional variance, while the TGARCH models the conditional standard deviation (Nor and Shamiri, 2007; Výrost and Baumöhl, 2009; Angabini and Wasiuzzaman, 2011).

There are other GARCH-type models that belong to the asymmetric class, notably the Asymmetric GARCH (AGARCH) of Engle (1990), Asymmetric Power ARCH (APARCH) model introduced by Ding, Granger, and Engle (1993) and Nonsymmetric GARCH (NGARCH) proposed by Engle and Ng (1993). For this dissertation we opted for EGARCH and GJR since they capture volatility in different ways and are predominant in the literature.

### 5.2 Conditional Distributions

Considering that GARCH-type models are non-linear, a technique called Maximum Likelihood Estimation (MLE) is used. It consists in maximizing the likelihood function (i.e. finding values of the parameters that maximize the log likelihood function). Furthermore, the likelihood function can take different forms, depending on the conditional distribution assumed for the innovations.

Since the test for normality of the returns was rejected for all periods under analysis, it may be reasonable to use different distributional assumptions for the innovations. This also allows to compare the asymmetry effects on volatility between different conditional distributions for the same period and model. In this study, we follow the suggestion of Tsay (2013) and Nor and Shamiri (2007), and consider three conditional distributions: the Gaussian normal distribution, the Student's *t* and the Generalized Error Distribution (GED).

#### 5.2.1 Normal Distribution

The Gaussian distribution, commonly known as Normal, is perhaps one of the most widely used to estimate GARCH-type models. Several financial models assume that asset returns are *i.i.d.*, therefore making their statistical properties tractable (Tsay, 2013). The conditional log-likelihood function for the innovations is given by:

$$\mathcal{L}_{Normal} = -\sum_{t=1}^{T}\left[-\frac{1}{2}\ln(\sigma_t^2) + \frac{1}{2}\frac{\varepsilon_t^2}{\sigma_t^2}\right] \tag{8}$$

#### 5.2.2 Student's *t*

Since extreme events occur more often than captured by the normal distribution, it might be appropriate to use a heavier-tailed distribution such as Student's *t*. The conditional log likelihood function is:

$$\mathcal{L}_{Student-t} = \prod_{t=1}^{T}\frac{\Gamma\left((v+1)/2\right)}{\Gamma(v\,/\,2)\sqrt{(v-2)\pi}}\frac{1}{\sigma_t}\left(1 + \frac{\varepsilon_t^2}{(v-2)\sigma_t^2}\right)^{-(v+1)/2} \tag{9}$$

where $v > 2$, are the degrees of freedom, which can be prespecified or estimated jointly with other parameters. As the $v$ increases, the Student's *t* gets closer to a Normal distribution. On the contrary, the smaller the value of $v$, the fatter the tails. The $\Gamma(x)$ is the gamma function (i.e.$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y}\, dy$).

### 5.2.3 Generalized Error Distribution

Finally, we assume the GED distribution for the innovations. Nelson (1991) when originally proposed the EGARCH model, assumed a GED distribution to account for nonnormality in the distribution of returns. The log likelihood function is as follows:

$$\mathcal{L}_{GED} = \sum_{t=1}^{T} \left\{ \log\left(\frac{v}{\lambda}\right) - \frac{1}{z}\left|\frac{\varepsilon_t}{\sigma_t \lambda}\right|^{v} - (1 + v^{-1}) log(2) - \log\left[\Gamma\left(\frac{1}{v}\right)\right] - \frac{1}{2}\log(\sigma_t^2) \right\} \quad (10)$$

where $\lambda = \left[\left[2^{(-2/v)} \Gamma(1 / v)(3 / v)\right]^{1/2}\right]$. Normal distribution is a special case of GED when $v = 2$. When $v < 2$, the distribution of innovations has thicker tails than the normal.

There are several alternatives for distributions densities that can be employed. These include: the Skew-Normal distribution, the Skew-Student's *t*, the Skew-Generalized Error distribution, the Normal Inverse Gaussian distribution, the Generalized Hyperbolic and the Johnson's SU distribution. Cont (2001) suggested that in order to be successful, a parametric model that describes the distribution of the stock returns must have at least four parameters: a location parameter, a scale parameter, a parameter that describes the decay of the tails and asymmetry parameter that allows different behaviours of left and right tail.

The estimation of the models can also be performed with two other methods: the Quasi Maximum Likelihood (QML) and the Generalized Method of Moments (Bollerslev and Wooldridge, 1992; Bollerslev, Chou and Kroner, 1992).

## 6. Empirical Results

Previously we introduced the dataset under analysis and the theoretical framework. This section is therefore dedicated to the application of asymmetric GARCH models and discussion of obtained results. We analyse seven different indices, split in two sub samples, employ EGARCH and GJR volatility models, and assume three conditional distributions for the innovations, which amounts to a total of 84 different specifications for the two asymmetric models.

In practice, the first step to build a volatility model consist in specifying the conditional mean equation. Despite not being the central issue, it can have some impact on the estimates for the conditional variance equation. Typically, the serial correlation in the returns is negligible, if existing at all. The model for the conditional mean should be chosen to reflect properly the presence of any serial correlation in the returns. In some cases, *demeaning* the return series (i.e. removing the sample mean from each return if the mean is statistically different from zero) is enough. In other cases, fitting an ARMA*(p,q)* model is necessary. On rare occasions more complex models with dummy variables to account for seasonality or other effects might be required (Tsay, 2013). To determine which model is more suitable one can use trial and error approach, selecting the model that gives the minimum value for information criteria. Graphically, ACF and PACF are also used to determine the appropriate specification for the conditional mean.

Bollerslev, Chou and Kroner (1992) noted that low order GARCH models seem sufficient to capture the variance dynamics over very long sample periods. Models with lag lengths $p$ and $q$ rarely exceed 2. In this dissertation all models are of $p = 1$, $q = 1$ order, allowing comparison between different types, indices and conditional distributions.

One common procedure before fitting a volatility model is to check for the existence of ARCH effects. This is done in order to make sure that this type of models is needed for the data. The idea is to check mean equation squared residuals for conditional heteroskedasticity (Tsay, 2013). Generally, two tests are used: the Ljung and Box (1978) test and the Lagrange Multiplier test (Engle, 1982). If the null hypothesis is rejected this suggests the presence of ARCH effects (i.e. presence of autocorrelation in $\varepsilon_t^2$ series). ACF and PACF are also helpful for graphical confirmation. Overall, such effects have been found highly significant in equity markets (Bollerslev, Chou and Kroner, 1992). See Appendix 2 for the results of ARCH effects test.

The computations were performed using RStudio computer software. Asymmetric GARCH-type models were estimated using the "rugarch" package (Ghalanos, 2019).

## 6.1 North American markets analysis

### 6.1.1 S&P 500

To build the conditional mean equation we first test the date subsets for autocorrelation. The Ljung-Box test statistic with 20 lags show that both periods exhibit some serial correlation in the returns. Both periods are also dependent, as the test for absolute returns show serial correlation as well. The results are summarized in the Table 2.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 32.554 (0.0377) | 50.725 (0.0002) |
| LB(20) of $\lvert r_t \rvert$ | 214.16 (2.2e-16) | 997.81 (2.2e-16) |

**Table 2**. Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of S&P 500, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values.

Figure 2 shows the ACF and PACF, which also suggest autocorrelation in minor lags (short term dependence).

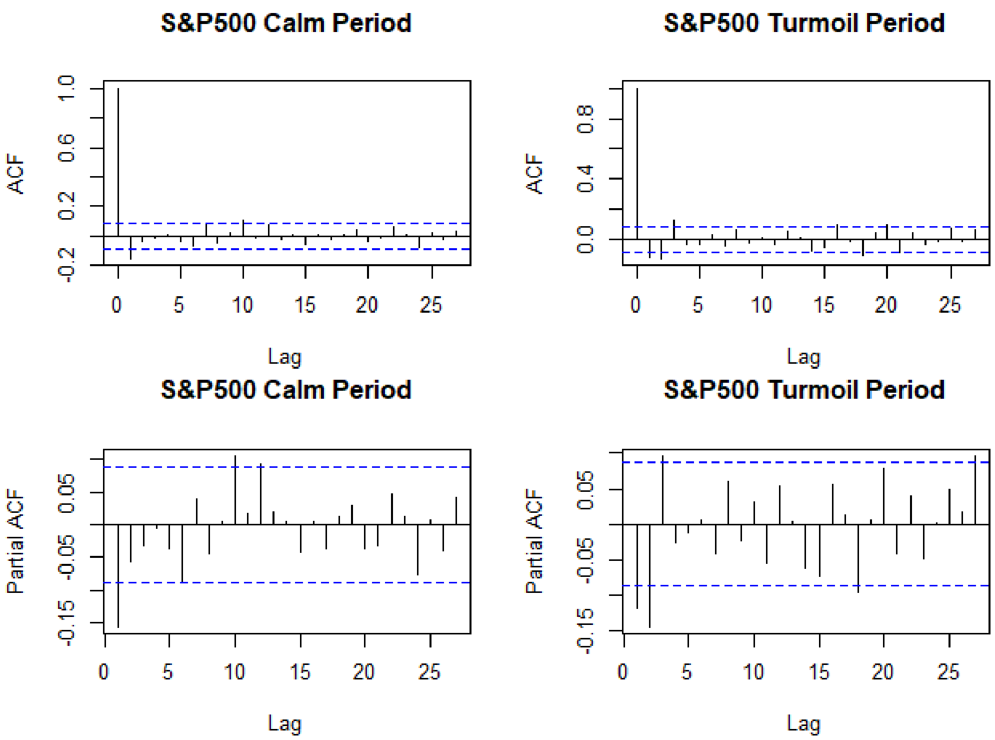


**Figure 2**. Sample autocorrelation and partial autocorrelation functions of the log returns of S&P 500.

Thus, for the S&P 500 log returns in the calm period we considered an AR(1) and for the turmoil period an AR(3). See Appendix 1 for the models tested.

The estimation results for GJR(1,1) and EGARCH(1,1) models are reported in Table 3.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000002 (0.0617) | 0.000001 (0.6154) | 0.000001 (0.6826) | -0.306474 (0.0000) | -0.144852 (0.0000) | -0.244741 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.040805 (0.1165) | 0.030030 (0.0067) | 0.046838 (0.1352) |
| $\beta_1$ | 0.912094 (0.0000) | 0.921256 (0.0000) | 0.916744 (0.0000) | 0.966458 (0.0000) | 0.985764 (0.0000) | 0.985938 (0.0000) |
| $\gamma_1$ | 0.139646 (0.0000) | 0.155487 (0.0058) | 0.147092 (0.0157) | -0.204362 (0.0000) | -0.240799 (0.0000) | -0.203513 (0.0000) |
| $v$ | - | 4.337417 (0.0000) | 1.077343 (0.0000) | - | 3.649605 (0.0001) | 1.109651 (0.0000) |
| Log-Likelihood | 1606.331 | 1627.977 | **1629.233** | 1615.774 | 1633.483 | **1633.94** |
| AIC | -6.3631 | -6.4452 | **-6.4502** | -6.4007 | -6.4671 | **-6.4689** |
| BIC | -6.3128 | -6.3865 | **-6.3915** | -6.3503 | -6.4084 | **-6.4102** |
| ARCH [5] | 0.4743 (0.8912) | 0.4020 (0.9121) | 0.5071 (0.8815) | 1.1786 (0.6808) | 0.8574 (0.7757) | 1.1838 (0.6793) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000003 (0.3967) | 0.000002 (0.5492) | 0.000002 (0.5262) | -0.169544 (0.0000) | -0.141204 (0.0000) | -0.146786 (0.0005) |
| $\alpha_1$ | 0.011218 (0.6430) | 0.006784 (0.8170) | 0.013133 (0.6789) | 0.150037 (0.0000) | 0.145343 (0.0180) | 0.150438 (0.0004) |
| $\beta_1$ | 0.907467 (0.0000) | 0.905398 (0.0000) | 0.902569 (0.0000) | 0.979822 (0.0000) | 0.984423 (0.0000) | 0.984080 (0.0000) |
| $\gamma_1$ | 0.127626 (0.0075) | 0.142650 (0.0047) | 0.132968 (0.0129) | -0.117291 (0.0000) | -0.135463 (0.0001) | -0.130852 (0.0001) |
| $v$ | - | 7.796495 (0.0078) | 1.328701 (0.0000) | - | 7.819388 (0.0666) | 1.331364 (0.0000) |
| Log-Likelihood | 1367.132 | 1372.514 | **1375.485** | 1368.369 | 1373.993 | **1376.756** |
| AIC | -5.3934 | -5.4108 | **-5.4226** | -5.3983 | -5.4166 | **-5.4276** |
| BIC | -5.3264 | -5.3354 | **-5.3472** | -5.3313 | -5.3412 | **-5.3522** |
| ARCH [5] | 1.8486 (0.5057) | 1.8997 (0.4938) | 1.8721 (0.5002) | 2.3568 (0.3976) | 2.3341 (0.4020) | 2.556 (0.3609) |

**Table 3.** EGARCH(1,1) and GJR(1,1) model estimation results for S&P 500. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

In the GJR model the estimated ARCH coefficients $\alpha_1$ are all very small and not statistically significant, meaning that there is no impact of squared errors on conditional volatility. The estimates for GARCH coefficients $\beta_1$ are all statistically significant meaning that current volatility is affected by past volatility. All asymmetry coefficients $\gamma_1$ estimates are significant and positive, therefore volatility is affected more by negative shocks. The value of the estimates becomes smaller in the turmoil period, indicating that bad news have smaller impact on volatility. Looking at the Log-likelihood and information criteria values, the best in-sample predictive model in both periods is the model with GED distribution. To verify the adequacy of the specification, ARCH LM test using 5 lags is performed on residuals. The null hypothesis of no ARCH effects for all models is not rejected, thus models are correctly specified.

Concerning the EGARCH results, the $\alpha_1$ coefficient estimates in the normal and GED distribution in the calm period is statistically insignificant. The asymmetry effect becomes less pronounced (i.e. the estimated values of $\gamma_1$ become less negative) in the turmoil period. These results are similar to the ones obtained by Výrost and Baumöhl (2009), which also reported lower absolute values of asymmetry coefficient estimates in the *crisis subsample.* Consistent with the GJR, distributions with fatter tails outperform the normal, being the GED distribution best in-sample predictive model. When compared against GJR, EGARCH models outperform for both periods. ARCH test on residuals indicates that the models are adequate.

### 6.1.2 NASDAQ Composite

To test the data for serial correlation and dependence, the Ljung-Box test was performed. According to the test results, only the turmoil period shows autocorrelation, since the null hypothesis of no autocorrelation is not rejected in the calm period. Concerning the dependence, both periods display autocorrelation in the absolute returns. See Table 4 for the results.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 24.726 (0.2121) | 43.34 (0.0018) |
| LB(20) of $\lvert r_t \rvert$ | 157.41 (2.2e-16) | 851.38 (2.2e-16) |

**Table 4.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of NASDAQ, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values**.**

The ACF and PACF of the NASDAQ log returns are shown in Figure 3. In the calm period the first and seventh lags appears to be significant. Regarding the turmoil period the first three lags are significant suggesting presence of serial correlation.

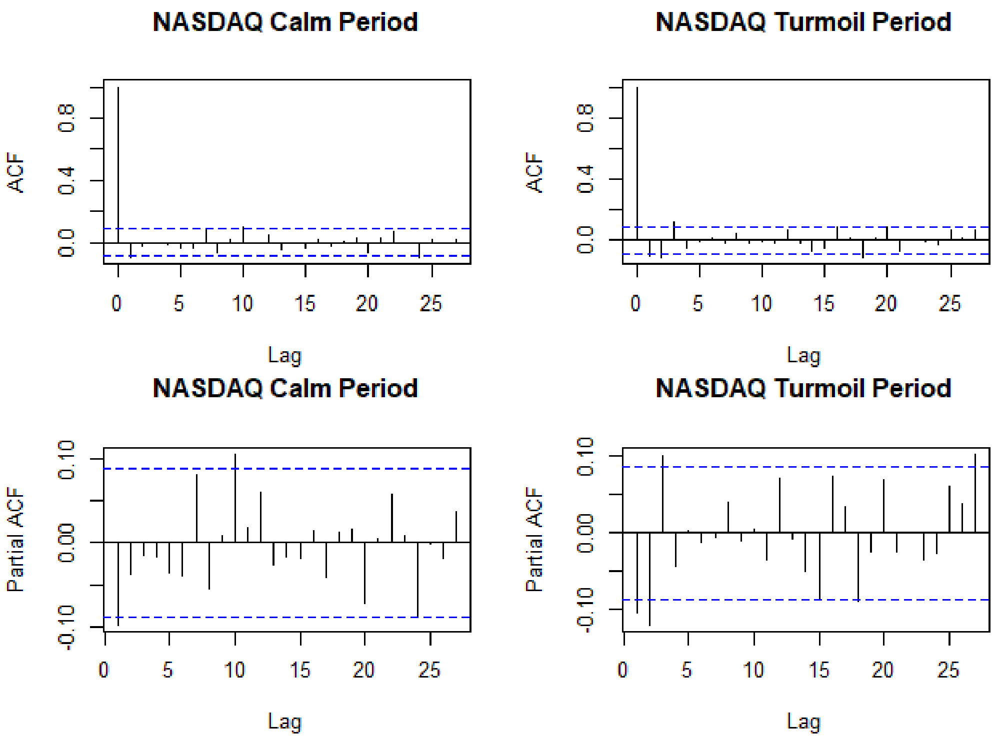


**Figure 3.** Sample autocorrelation and partial autocorrelation functions of the log returns of NASDAQ.

Based on our results, the specification for the conditional mean for the NASDAQ are equal to the ones of S&P 500 (i.e. an AR(1) for the calm period and an AR(3) for the turmoil period). Models tested are presented in Appendix 1.

Volatility model's estimation results are presented in the Table 5.

The obtained results for GJR show that none of the $\omega$ and $\alpha_1$ estimates are statistically significant, meaning that only the GARCH term is sufficient to predict the conditional volatility. Regarding the estimates for the asymmetry coefficient, only the in the normal distribution in the turmoil period, the estimate is not statistically significant. Unlike in the S&P 500 results, the impact of negative shocks is larger in the turmoil period, since the estimated values are greater.

In the EGARCH model, all the estimates for the coefficients are statistically significant at the 5 per cent level for both periods. In addition, all $\gamma_1$ coefficient estimates are more negative in the turmoil period, supporting the findings of the GJR model.

For checking the model adequacy, the ARCH test high *p*-values suggest no serial correlation in the squared residuals. The best in-sample predictive models are the ones using the Student's *t* distribution in the calm period, and GED distribution for the turmoil period. In both cases the EGARCH is superior to the GJR (based on maximum likelihood and information criteria).

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000002<br>(0.7281) | 0.000001<br>(0.4730) | 0.000001<br>(0.3953) | -0.205549<br>(0.0000) | -0.151168<br>(0.0000) | -0.177287<br>(0.0000) |
| $\alpha_1$ | 0.000000<br>(0.9999) | 0.000003<br>(0.9999) | 0.000000<br>(0.9999) | 0.058304<br>(0.0000) | 0.072763<br>(0.0000) | 0.066132<br>(0.0187) |
| $\beta_1$ | 0.944254<br>(0.0000) | 0.950445<br>(0.0000) | 0.943158<br>(0.0000) | 0.976596<br>(0.0000) | 0.983234<br>(0.0000) | 0.980420<br>(0.0000) |
| $\gamma_1$ | 0.082919<br>(0.0001) | 0.089055<br>(0.0135) | 0.089625<br>(0.0160) | -0.099430<br>(0.0000) | -0.105953<br>(0.0003) | -0.104551<br>(0.0002) |
| $v$ | - | 8.597277<br>(0.0026) | 1.479458<br>(0.0000) | - | 9.603244<br>(0.0084) | 1.528258<br>(0.0000) |
| Log-Likelihood | 1533.54 | **1539.24** | 1538.379 | 1537.406 | **1541.836** | 1541.09 |
| AIC | -6.0737 | **-6.0924** | -6.0890 | -6.0891 | **-6.1027** | -6.0998 |
| BIC | -6.0234 | **-6.0337** | -6.0302 | -6.0387 | **-6.0440** | -6.0410 |
| ARCH [5] | 0.1524<br>(0.9763) | 0.0139<br>(0.9992) | 0.0345<br>(0.9971) | 0.3174<br>(0.9356) | 0.0979<br>(0.9872) | 0.2064<br>(0.9640) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000003<br>(0.1761) | 0.000003<br>(0.4553) | 0.000003<br>(0.4640) | -0.184031<br>(0.0000) | -0.153799<br>(0.0000) | -0.161997<br>(0.0000) |
| $\alpha_1$ | 0.010081<br>(0.2241) | 0.005827<br>(0.8136) | 0.010243<br>(0.7059) | 0.135476<br>(0.0003) | 0.128265<br>(0.0000) | 0.132415<br>(0.0000) |
| $\beta_1$ | 0.903789<br>(0.0000) | 0.902414<br>(0.0000) | 0.900134<br>(0.0000) | 0.977904<br>(0.0000) | 0.982699<br>(0.0000) | 0.982007<br>(0.0000) |
| $\gamma_1$ | 0.137515<br>(0.1015) | 0.151311<br>(0.0089) | 0.144087<br>(0.0204) | -0.133010<br>(0.0000) | -0.152199<br>(0.0000) | -0.148096<br>(0.0000) |
| $v$ | - | 8.153712<br>(0.0069) | 1.371366<br>(0.0000) | - | 7.972033<br>(0.0045) | 1.375077<br>(0.0000) |
| Log-Likelihood | 1342.139 | 1347.463 | **1349.622** | 1343.482 | 1349.207 | **1351.001** |
| AIC | -5.2942 | -5.3114 | **-5.3199** | -5.2995 | -5.3183 | **-5.3254** |
| BIC | -5.2272 | -5.2360 | **-5.2445** | -5.2325 | -5.2429 | **-5.2500** |
| ARCH [5] | 1.4563<br>(0.6039) | 1.4781<br>(0.5981) | 1.5374<br>(0.5825) | 2.1578<br>(0.4374) | 2.4210<br>(0.3855) | 2.4027<br>(0.3889) |

**Table 5**. EGARCH(1,1) and GJR(1,1) model estimation results for NASDAQ. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

## 6.2 European markets analysis

### 6.2.1 FTSE 100

Similarly to the NASDAQ results for autocorrelation, the Ljung-Box test with 20 lags only detected autocorrelation in the turmoil period. Serial correlation is also present in the absolute log returns, meaning that they are dependent. The results are shown in the Table 6.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 26.549 (0.1484) | 58.983 (1.022e-05) |
| LB(20) of $\lvert r_t \rvert$ | 272.49 (2.2e-16) | 621.79 (2.2e-16) |

**Table 6.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of FTSE 100, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values**.**

Figure 4 shows the ACF and PACF of both subsamples of FTSE 100. Visually the calm period has significant autocorrelation in the first lag, and the turmoil period displays at lag 2, 3, 4 and 5, supporting the results of Ljung-Box test.

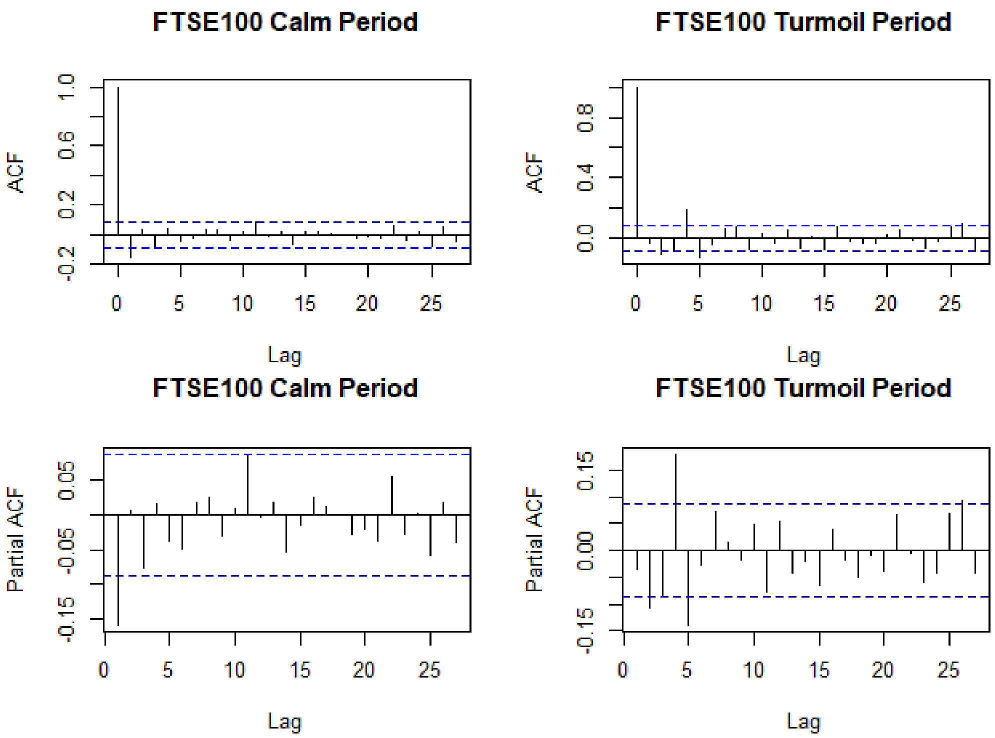


**Figure 4.** Sample autocorrelation and partial autocorrelation functions of the log returns of FTSE 100.

Based on model tested (see Appendix 1) the conditional mean equations for FTSE 100 are an AR(1) for the calm period, and an AR(5) for the turmoil period.

The Table 7 shows the results for the volatility models estimates.

In line with previous findings, the ARCH term in the GJR model is not statistically significant. The GARCH term is statistically significant and becomes greater in the turmoil period meaning that volatility persistence has increased (Leeves, 2007). The volatility response to negative shocks is smaller in the turmoil period, therefore negative innovations do not raise volatility as much as in the calm period.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000003 (0.0001) | 0.000003 (0.7567) | 0.000003 (0.0554) | -0.211340 (0.0000) | -0.198685 (0.0000) | -0.202694 (0.0000) |
| $\alpha_1$ | 0.000012 (0.9992) | 0.000003 (0.9999) | 0.000000 (0.9999) | 0.085040 (0.0000) | 0.067749 (0.0229) | 0.079368 (0.0000) |
| $\beta_1$ | 0.883817 (0.0000) | 0.882328 (0.0000) | 0.882232 (0.0000) | 0.976252 (0.00000) | 0.978420 (0.0000) | 0.977742 (0.0000) |
| $\gamma_1$ | 0.189237 (0.0000) | 0.196971 (0.1526) | 0.192548 (0.0000) | -0.143839 (0.0000) | -0.175685 (0.0000) | -0.154195 (0.0000) |
| $\upsilon$ | - | 10.289048 (0.0180) | 1.632411 (0.0000) | - | 9.920458 (0.0024) | 1.642567 (0.0000) |
| Log-Likelihood | 1585.203 | **1589.506** | 1587.808 | 1588.731 | **1593.517** | 1591.106 |
| AIC | -6.2419 | **-6.2550** | -6.2483 | -6.2559 | **-6.2708** | -6.2613 |
| BIC | -6.1918 | **-6.1965** | -6.1898 | -6.2057 | **-6.2123** | -6.2028 |
| ARCH [5] | 2.3236 (0.4040) | 2.2339 (0.4218) | 2.2685 (0.4149) | 1.2308 (0.6659) | 0.8395 ( 0.7811) | 1.0364 (0.7221) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000004 (0.6337) | 0.000004 (0.5163) | 0.000004 (0.3991) | -0.190356 (0.0000) | -0.195275 (0.0000) | -0.189764 (0.0000) |
| $\alpha_1$ | 0.000000 (1.0000) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.118455 (0.0000) | 0.107694 (0.0000) | 0.106933 (0.0006) |
| $\beta_1$ | 0.904432 (0.0000) | 0.906733 (0.0000) | 0.905334 (0.0000) | 0.978060 (0.0000) | 0.977755 (0.0000) | 0.978744 (0.0000) |
| $\gamma_1$ | 0.149191 (0.0149) | 0.138839 (0.0142) | 0.141553 (0.0056) | -0.157212 (0.0000) | -0.149847 (0.0000) | -0.151831 (0.0000) |
| $\upsilon$ | - | 8.594310 (0.0071) | 1.441965 (0.0000) | - | 9.830562 (0.0144) | 1.478604 (0.0000) |
| Log-Likelihood | 1415.31 | 1420.836 | **1421.38** | 1418.517 | 1422.588 | **1423.366** |
| AIC | -5.5546 | -5.5725 | **-5.5746** | -5.5673 | -5.5794 | **-5.5825** |
| BIC | -5.4711 | -5.4806 | **-5.4827** | -5.4837 | -5.4875 | **-5.4906** |
| ARCH [5] | 2.9702 (0.2941) | 3.3949 (0.2374) | 3.555 (0.2188) | 4.238 (0.1536) | 4.392 (0.1417) | 4.539 (0.1312) |

**Table 7.** EGARCH(1,1) and GJR(1,1) model estimation results for FTSE 100 AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

Analysing the EGARCH model we can see that all the estimates for the coefficients are significant at conventional levels. Comparing the subsamples in terms of asymmetry it is rather inconclusive. For Student's *t* and GED distributions, the coefficient estimates are more negative in the calm period, which is not the case for normal distribution. Consequently, the models with Student's *t* and GED conditional distribution are less sensitive to negative shocks in the turmoil period. Olbrys and Majewska (2017) found similar results. In their analysis, the *pre-GFC* period has higher asymmetry coefficient estimates than *GFC* period.

No serial correlation was detected by the ARCH test performed on residuals and the models that performed best are the same as in North American indices. One particular aspect worth notice, is that the asymmetry coefficient in the GJR model with Student's *t* conditional distribution in the calm period, is not statistically significant.

### 6.2.2 DAX

Running the Ljung-Box test to check for autocorrelation in the log returns of the German market, show presence only in the turmoil period. Like with the other indices, both subsamples are dependent. Table 8 presents the results of the test.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 21.121 (0.3900) | 33.078 (0.0331) |
| LB(20) of $\|r_t\|$ | 109.94 (2.01e-14) | 385.6 (2.2e-16) |

**Table 8.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of DAX, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values.

Looking at the ACF and PACF shown in Figure 5, no significant serial correlation is found in the calm period, except for small one at lag 10. On the other hand, turmoil period suggests autocorrelation at lags 2, 4, 18 and 25.

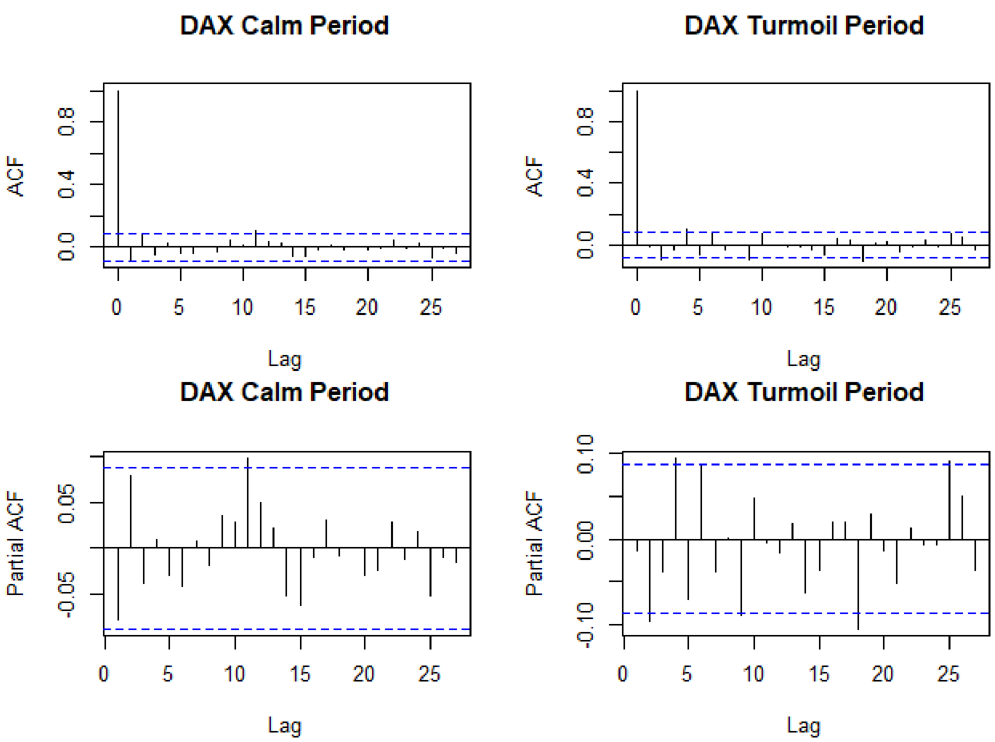


**Figure 5.** Sample autocorrelation and partial autocorrelation functions of the log returns of DAX.

Therefore, the DAX index log returns in the calm period is a white noise series without drift (i.e. the sample mean is not removed from the series). As for the turmoil period, we considered an AR(4). See Appendix 1 for the models tested.

The results for conditional volatility equation are presented in the Table 9.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000006 (0.0000) | 0.000005 (0.0000) | 0.000006 (0.0000) | -0.625978 (0.0000) | -0.537267 (0.0000) | -0.592589 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.072968 (0.0000) | 0.081061 (0.0001) | 0.076646 (0.0492) |
| $\beta_1$ | 0.852881 (0.0000) | 0.877259 (0.0000) | 0.865429 (0.0000) | 0.930707 (0.0000) | 0.940607 (0.0000) | 0.934591 (0.0000) |
| $\gamma_1$ | 0.198975 (0.0000) | 0.172973 (0.0002) | 0.186357 (0.0001) | -0.190195 (0.0000) | -0.185480 (0.0000) | -0.190600 (0.0000) |
| $v$ | - | 8.629507 (0.0021) | 1.468714 (0.0000) | - | 9.174399 (0.0067) | 1.518027 (0.0000) |
| Log-Likelihood | 1586.573 | **1594.793** | 1593.283 | 1594.366 | **1600.131** | 1599.304 |
| AIC | -6.2429 | **-6.2714** | -6.2654 | -6.2736 | **-6.2924** | -6.2892 |
| BIC | -6.2095 | **-6.2297** | -6.2237 | -6.2403 | **-6.2507** | -6.2475 |
| ARCH [5] | 1.9112 (0.4912) | 1.9966 (0.4720) | 1.9505 (0.4823) | 2.2004 (0.4286) | 2.3989 (0.3896) | 2.2914 (0.4103) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000004 (0.6102) | 0.000004 (0.4615) | 0.000004 (0.5082) | -0.142554 (0.0000) | -0.155892 (0.0000) | -0.148103 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.138371 (0.0000) | 0.135746 (0.0028) | 0.133543 (0.0000) |
| $\beta_1$ | 0.909536 (0.0000) | 0.907682 (0.0000) | 0.908851 (0.0000) | 0.982748 (0.0000) | 0.981587 (0.0000) | 0.982798 (0.0000) |
| $\gamma_1$ | 0.152203 (0.02633) | 0.154301 (0.0184) | 0.151980 (0.0250) | -0.127954 (0.0000) | -0.139435 (0.0000) | -0.136618 (0.0000) |
| $v$ | - | 13.679730 (0.1267) | 1.537058 (0.0000) | - | 12.069553 (0.0014) | 1.517976 (0.0000) |
| Log-Likelihood | 1358.041 | 1359.405 | **1361.343** | 1357.787 | 1359.647 | **1361.539** |
| AIC | -5.3112 | -5.3126 | **-5.3202** | -5.3102 | -5.3136 | **-5.3210** |
| BIC | -5.2362 | -5.2293 | **-5.2370** | -5.2352 | -5.2303 | **-5.2377** |
| ARCH [5] | 4.8433 (0.1117) | 4.9814 (0.1038) | 5.032 (0.1010) | 6.8544 (0.0377) | 6.946 (0.0359) | 6.8995 (0.0368) |

**Table 9.** EGARCH(1,1) and GJR(1,1) model estimation results for DAX. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

No particular differences are found in the German market. The $\beta_1$ estimates of GJR are statistically significant regardless of the period and higher after the Lehman Brothers bankruptcy. Again, the conditional volatility response to negative shocks is smaller in the turmoil period. The best specification for the in-sample prediction in the GJR framework is the Student's *t* distribution for the calm period and GED distribution for the turmoil period.

The asymmetry effect in the exponential model follows the same pattern. The estimates become less negative in the turmoil period. Once again, EGARCH outperforms GJR, being the model with Student's *t* distribution best fit for the calm period, and GED distribution for turmoil period.

The presence of ARCH effect is detected for EGARCH model in the turmoil period, meaning that conditional heteroskedasticity present in the data is not completely captured. Olbrys e Majewska (2017) also reported similar difficulties regarding the DAX index. According to authors, the quality of EGARCH model in their analysis is rather low.

### 6.2.3 CAC 40

The autocorrelation findings in the French market are like in the previous markets. Only turmoil period log returns exhibit autocorrelation. The full series is dependent as strong autocorrelation is present in the absolute log returns. The results of the Ljung-Box test with 20 lags are shown in the Table 10.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 23.622 (0.2593) | 47.462 (0.0005) |
| LB(20) of $\|r_t\|$ | 189.97 (2.2e-16) | 394.44 (2.2e-16) |

**Table 10.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of CAC 40, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values**.**

Figure 6 gives the ACF and PACF graphs of the CAC 40 log returns, both subsamples exhibit some autocorrelation. In the calm period the first lag appears to be significant. In the turmoil period serial correlation is present at least at lags 2, 4 and 5.

After checking several models (see Appendix 1), the conditional mean equations considered for the CAC 40 are AR(1) for the calm period, and AR(5) for the turmoil period. These specifications are the same as in the FTSE 100 index, suggesting similar behaviour in these markets before and after September 15, 2008.

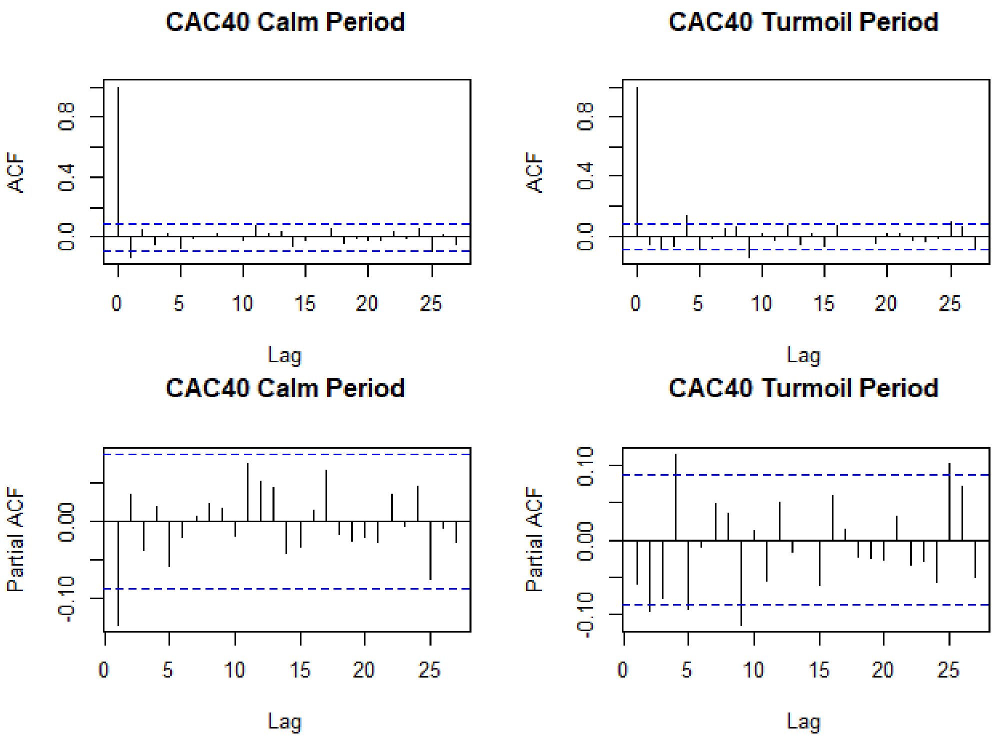


**Figure 6.** Sample autocorrelation and partial autocorrelation functions of the log returns of CAC 40.

GJR (1,1) and EGARCH (1,1) estimation results for the French market are reported in Table 11.

Except for the ARCH term in the GJR models, all other GJR and EGARCH coefficients estimates are statistically significant. Both models have greater GARCH term in the turmoil period, pointing to an increase in the long-run persistence of volatility in the French market.

Asymmetry estimates are relatively similar in both subsamples, but when comparing in terms of model, the results are conflicting. GJR estimates are smaller in the turmoil period (i.e. the impact of negative shocks is smaller), whereas the EGARCH are more negative (i.e. bigger impact of negative shocks). The results of the EGARCH are opposing the ones presented by Olbrys and Majewska (2017). Authors report stronger asymmetry effects in the *pre-GFC* period.

Once more the EGARCH provided better results in terms of in-sample predictive ability than GJR and the null hypothesis of the ARCH test on residuals is not rejected for all specifications, meaning there is no heteroskedasticity problem present in the residuals.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000005 (0.0000) | 0.000004 (0.0000) | 0.000005 (0.0000) | -0.366086 (0.0000) | 0.349195 (0.0000) | -0.354866 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.032232 (0.0286) | 0.038216 (0.0201) | 0.032497 (0.0098) |
| $\beta_1$ | 0.865614 (0.0000) | 0.873740 (0.0000) | 0.868614 (0.0000) | 0.959215 (0.0000) | 0.961593 (0.0000) | 0.960906 (0.0000) |
| $\gamma_1$ | 0.201841 (0.0000) | 0.195145 (0.0000) | 0.198297 (0.0000) | -0.197041 (0.0000) | -0.201383 (0.0000) | -0.200112 (0.0000) |
| $v$ | - | 11.965798 (0.0174) | 1.638776 (0.0000) | - | 13.097304 (0.0499) | 1.695434 (0.0000) |
| Log-Likelihood | 1569.235 | **1573.217** | 1571.732 | 1578.123 | **1580.691** | 1579.699 |
| AIC | -6.1303 | **-6.1420** | -6.1362 | -6.1652 | **-6.1713** | -6.1674 |
| BIC | -6.0805 | **-6.0839** | -6.0781 | -6.1154 | **-6.1132** | -6.1093 |
| ARCH [5] | 3.508 (0.2241) | 3.5071 (0.2242) | 3.6043 (0.2133) | 1.873 (0.5000) | 2.2325 (0.4221) | 2.118 (0.4458) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000007 (0.0000) | 0.000008 (0.0000) | 0.000008 (0.0000) | -0.258934 (0.0000) | -0.281502 (0.0000) | -0.285187 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.110514 (0.0000) | 0.101369 (0.0000) | 0.106252 (0.0004) |
| $\beta_1$ | 0.882153 (0.0000) | 0.883359 (0.0000) | 0.878970 (0.0000) | 0.968223 (0.0000) | 0.966025 (0.0000) | 0.965753 (0.0000) |
| $\gamma_1$ | 0.190324 (0.0030) | 0.184531 (0.0006) | 0.193877 (0.0007) | -0.205555 (0.0000) | -0.217316 (0.0000) | -0.216678 (0.0000) |
| $v$ | - | 9.095452 (0.0062) | 1.428916 (0.0000) | - | 9.521861 (0.0115) | 1.444106 (0.0000) |
| Log-Likelihood | 1341.349 | 1345.92 | **1347.745** | 1345.418 | 1349.65 | **1351.256** |
| AIC | -5.2008 | -5.2145 | **-5.2217** | -5.2165 | -5.2291 | **-5.2354** |
| BIC | -5.1180 | -5.1235 | **-5.1306** | -5.1337 | -5.1380 | **-5.1443** |
| ARCH [5] | 2.756 (0.3271) | 3.2218 (0.2592) | 3.167 (0.2664) | 2.5377 (0.3642) | 2.9510 (0.2970) | 2.9196 (0.3017) |

**Table 11.** EGARCH(1,1) and GJR(1,1) model estimation results for CAC 40. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

### 6.3 Asian markets analysis

### 6.3.1 NIKKEI 225

Contrasting with previous findings, the null hypothesis of the Ljung-Box test is not rejected, meaning that NIKKEI 225 log returns are not autocorrelated, regardless of the period. The null hypothesis is rejected when testing absolute log returns. Therefore, the daily log returns of the Japanese index are serially uncorrelated but dependent. Results are provided in Table 12.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 18.256 (0.5706) | 19 (0.5218) |
| LB(20) of $\lvert r_t \rvert$ | 256.41 (2.2e-16) | 936.41 (2.2e-16) |

**Table 12.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of NIKKEI 225, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values**.**

Visually, with exception of the lag 21 in the turmoil period, both subsamples suggest no serial correlation, supporting the Ljung-Box test results. The ACF and PACF are given in Figure 7. Considering no serial correlation in NIKKEI 225 log returns and to keep the models simple, the mean equation of both periods is as white noise process without drift. See Appendix 1 for models examined.

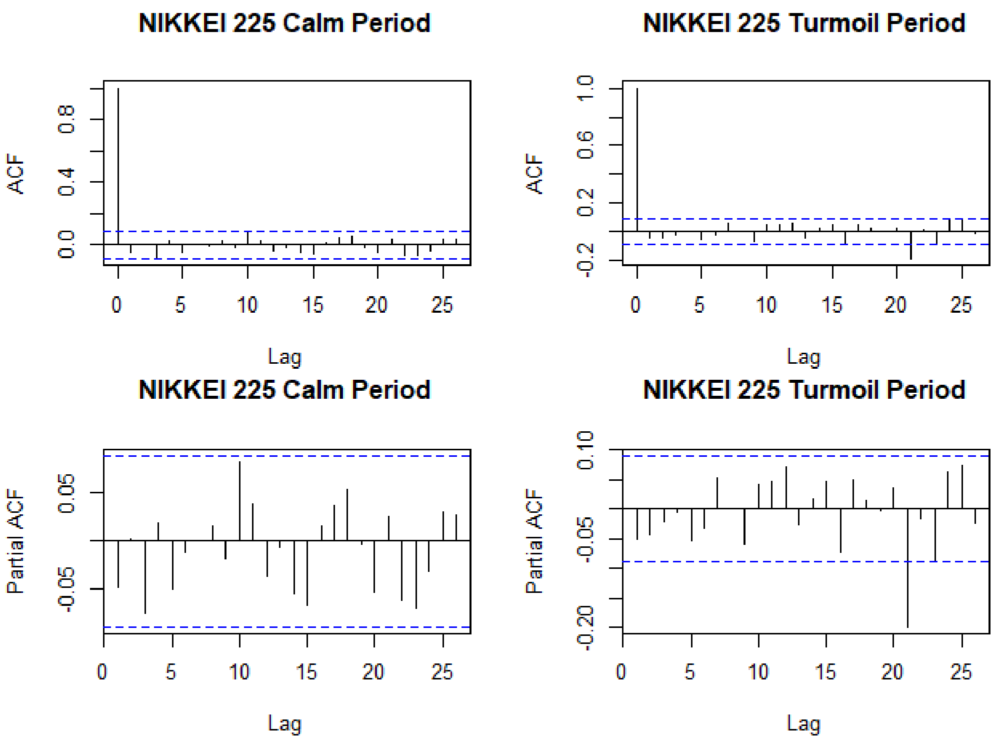


**Figure 7.** Sample autocorrelation and partial autocorrelation functions of the log returns of NIKKEI 225.

The Table 13 summarizes the results for volatility models.

The $\beta_1$ estimates in the GJR decrease in the turmoil period, meaning that past conditional volatility has lesser impact on today's conditional volatility. Regarding the EGARCH this is only verified in the Student's *t* distribution case.

The asymmetry behaviour is rather incoherent. EGARCH points out that the effect is less pronounced in the turmoil period, while GJR show an increase in $\gamma_1$ estimates with normal and GED distribution, and a decrease with Student's *t* distribution.

Unlike in previous cases, the EGARCH with Student's *t* distribution outperforms GJR in the calm period, but information criteria and Log-Likelihood point out that GJR with Student's *t* distribution is better for in-sample prediction in the turmoil period. ARCH test results indicate that the best model for the calm period rejects the null hypothesis suggesting that there are ARCH effects that remained in the residuals.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\omega$ | 0.000003 (0.0283) | 0.000002 (0.6288) | 0.000003 (0.5077) | -0.259811 (0.0000) | -0.163127 (0.0000) | -0.212376 (0.0000) |
| $\alpha_1$ | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.000000 (0.9999) | 0.086062 (0.0000) | 0.063445 (0.0061) | 0.078167 (0.0000) |
| $\beta_1$ | 0.904746 (0.0000) | 0.919014 (0.0000) | 0.914452 (0.0000) | 0.970810 (0.0000) | 0.982030 (0.0000) | 0.976428 (0.0000) |
| $\gamma_1$ | 0.142457 (0.0000) | 0.131159 (0.0030) | 0.133791 (0.0144) | -0.148747 0.0000) | -0.148300 (0.0000) | -0.146558 (0.0000) |
| $v$ | - | 12.748668 (0.0533) | 1.563245 (0.0000) | - | 11.428656 (0.0167) | 1.571504 (0.0000) |
| Log-Likelihood | 1451.136 | 1453.199 | **1454.002** | 1453.491 | **1456.484** | 1456.344 |
| AIC | -5.9067 | -5.9110 | **-5.9143** | -5.9163 | **-5.9244** | -5.9239 |
| BIC | -5.8724 | -5.8682 | **-5.8715** | -5.8820 | **-5.8816** | -5.8811 |
| ARCH [5] | 1.3919 (0.6212) | 2.182 (0.4324) | 1.858 (0.5035) | 4.169 (0.1592) | 8.588 (0.0145) | 5.663 (0.0720) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000008 (0.0002) | 0.000007 (0.0077) | 0.000008 (0.0001) | -0.17674 (0.0000) | -0.17678 (0.0000) | -0.17841 (0.0001) |
| $\alpha_1$ | 0.012615 (0.2312) | 0.005631 (0.5933) | 0.009984 (0.3900) | 0.16436 (0.0000) | 0.16116 (0.0000) | 0.15873 (0.0001) |
| $\beta_1$ | 0.886688 (0.0000) | 0.903311 (0.0000) | 0.887980 (0.0000) | 0.97843 (0.0000) | 0.97848 (0.0000) | 0.97842 (0.0000) |
| $\gamma_1$ | 0.144476 (0.0000) | 0.131032 (0.0002) | 0.146149 (0.0002) | -0.10985 (0.0000) | -0.11018 (0.0000) | -0.11072 (0.0000) |
| $v$ | - | 56.145315 (0.5943) | 1.784430 (0.0000) | - | 66.18218 (0.2707) | 1.79951 (0.0000) |
| Log-Likelihood | 1267.18 | **1270.508** | 1267.765 | 1266.16 | 1266.146 | **1266.647** |
| AIC | -5.1876 | **-5.1972** | -5.1859 | -5.1834 | -5.1792 | **-5.1813** |
| BIC | -5.1532 | **-5.1542** | -5.1429 | -5.1490 | -5.1362 | **-5.1383** |
| ARCH [5] | 2.8843 (0.3070) | 4.0819 (0.1667) | 3.048 (0.2829) | 4.137 (0.1619) | 4.317 (0.1474) | 4.443 (0.1380) |

**Table 13.** EGARCH(1,1) and GJR(1,1) model estimation results for NIKKEI 225. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

### 6.3.2 HSI

The Table 14 shows the Ljung-Box test for the HSI index. Again, autocorrelation is found to be significant only in the turmoil period, although the null hypothesis is rejected by a small margin (considering 5% significance level). Regarding the absolute returns, the null hypothesis is rejected as in all other indices.

| | Calm Period | Turmoil Period |
|---|---|---|
| LB(20) of $r_t$ | 14.207 (0.8199) | 32.315 (0.0401) |
| LB(20) of $\|r_t\|$ | 256.41 (2.2e-16) | 936.41 (2.2e-16) |

**Table 14.** Ljung Box test results for serial correlation in daily log returns and daily absolute log returns of HSI, distributed as $\chi^2(20)$. Numbers in parenthesis are *p*-values**.**

The ACF and PACF of the HSI index (Figure 8) suggest minor serial correlation in lag 1 for the calm period. For the turmoil period, serial correlation is significant at lags 9 and 10.

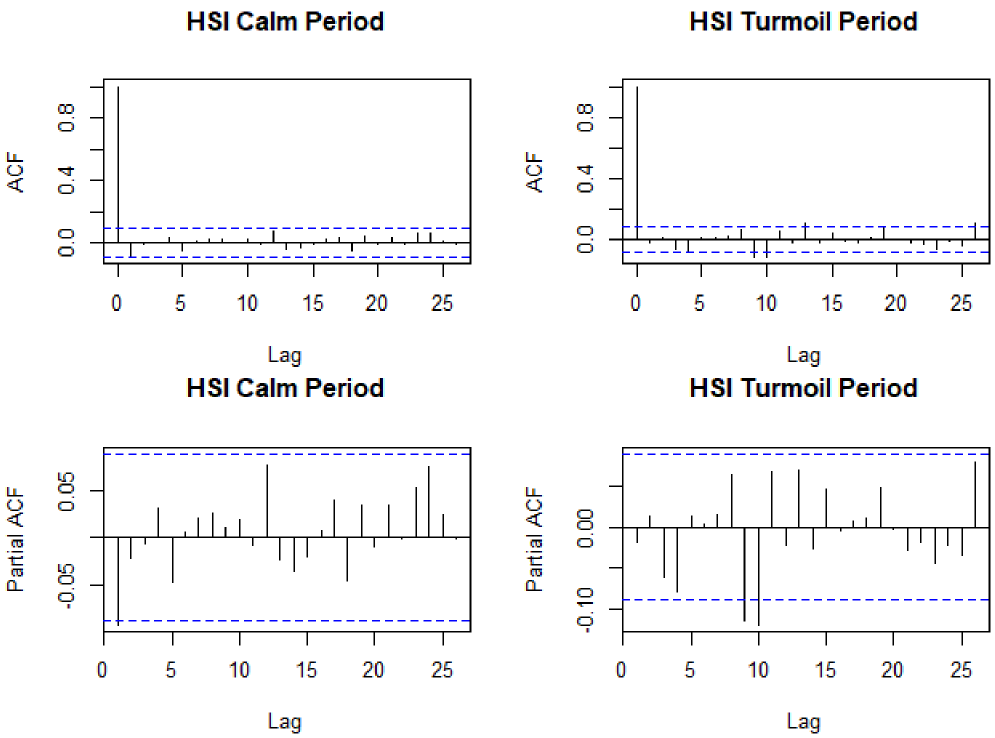


**Figure 8.** Sample autocorrelation and partial autocorrelation functions of the log returns of HSI.

Appendix 1 shows the mean equations considered for the HSI. Similar to the NIKKEI 225, we keep the models simple, and find a white noise process without drift to be appropriate specification, outlining the fact that, in our analysis, the Asian markets are relatively distinct from North American and European.

Table 15 gives the results for the GJR and EGARCH models.

| | GJR(1,1) | | | EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student-t | GED | Normal | Student-t | GED |
| $\omega$ | 0.000012 (0.0000) | 0.000009 (0.0001) | 0.000010 (0.0000) | -0.40987 (0.0067) | -0.34587 (0.0001) | -0.36116 (0.0026) |
| $\alpha_1$ | 0.062592 (0.0000) | 0.061177 (0.0011) | 0.059971 (0.0005) | 0.24873 (0.0000) | 0.24547 (0.0000) | 0.24381 (0.0000) |
| $\beta_1$ | 0.815337 (0.0000) | 0.827816 (0.0000) | 0.828980 (0.0000) | 0.94899 (0.0000) | 0.95712 (0.0000) | 0.95557 (0.0000) |
| $\gamma_1$ | 0.184406 (0.0000) | 0.184903 (0.0006) | 0.181884 (0.0005) | -0.11233 (0.0007) | -0.11442 (0.0006) | -0.11384 (0.0012) |
| $v$ | - | 12.893038 (0.0214) | 1.532515 (0.0000) | - | 12.84180 (0.0976) | 1.54044 (0.0000) |
| Log-Likelihood | 1340.337 | 1341.951 | **1343.52** | 1341.827 | 1343.527 | **1344.936** |
| AIC | -5.4323 | -5.4348 | **-5.4411** | -5.4383 | -5.4412 | **-5.4469** |
| BIC | -5.3981 | -5.3921 | **-5.3985** | -5.4042 | -5.3985 | **-5.4042** |
| ARCH [5] | 5.390 (0.0835) | 4.966 (0.1046) | 5.153 (0.0947) | 5.627 (0.0735) | 5.141 (0.0953) | 5.331 (0.0861) |
| | Turmoil Period | | | | | |
| $\omega$ | 0.000002 (0.5992) | 0.000002 (0.6432) | 0.000002 (0.6436) | -0.059626 (0.0000) | -0.058479 (0.0000) | -0.059645 (0.0000) |
| $\alpha_1$ | 0.029500 (0.1803) | 0.030222 (0.2043) | 0.029817 (0.2064) | 0.130211 (0.0000) | 0.130023 (0.0000) | 0.130567 (0.0000) |
| $\beta_1$ | 0.914485 (0.0000) | 0.915089 (0.0000) | 0.914358 (0.0000) | 0.993042 (0.0000) | 0.993230 (0.0000) | 0.993121 (0.0000) |
| $\gamma_1$ | 0.099576 (0.0175) | 0.097581 (0.0261) | 0.099805 (0.0262) | -0.079231 (0.0000) | -0.078771 (0.0005) | -0.079553 (0.0004) |
| $v$ | - | 43.284663 (0.5096) | 1.863235 (0.0000) | - | 56.400117 (0.6030) | 1.885984 (0.0000) |
| Log-Likelihood | 1275.321 | 1275.561 | **1275.577** | 1277.137 | 1277.275 | **1277.315** |
| AIC | -5.1263 | -5.1232 | **-5.1233** | -5.1336 | -5.1301 | **-5.1303** |
| BIC | -5.0924 | -5.0808 | **-5.0809** | -5.0997 | -5.0877 | **-5.0879** |
| ARCH [5] | 0.6716 (0.8321) | 0.67397 (0.8314) | 0.6744 (0.8312) | 0.6887 (0.8269) | 0.6935 (0.8254) | 0.6892 (0.8267) |

**Table 15.** EGARCH(1,1) and GJR(1,1) model estimation results for HSI. AIC and BIC are the Akaike Information Criteria and Bayesian information Criteria respectively. ARCH [5] denotes the ARCH LM test at lag 5. Numbers in parenthesis are *p*-values**.**

For the GJR model, all estimates for coefficients are statistically significant at 5% level in the subsample prior to September 15, 2008 meaning that current volatility is affected not only by past volatility but also by immediate impact from shocks. In the turmoil period, GJR ARCH term is not statistically significant. The GARCH term estimates increase in both models.

The asymmetry estimates do not show strong differences in both models, and the effect becomes weaker, meaning that negative shocks do not increase volatility as much as in the calm period.

EGARCH is superior to GJR and using the GED distribution for estimation provides the best results for in-sample prediction. The adequacy of the models is verified by the ARCH test. The null hypothesis of no serial correlation in the residuals is not rejected in all estimated models.

## 6.4 Individual Analysis of the Asymmetry Effects

Except for the NIKKEI 225 in the turmoil period, the results indicate that the EGARCH is better in-sample predictive model than GJR. Overall, based on Log-likelihood and information criteria values, the fit of the models is poorer in the turmoil period, suggesting that the log returns behaviour after the Lehman Brothers collapse is harder to capture and describe. Table 16 and 17 summarizes the estimates of the asymmetry coefficients.

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Normal | Student's t | GED | Normal | Student's t | GED |
| S&P500 | 0.139646 [0.021018] (0.0000) | 0.155487 [0.067047] (0.0058) | 0.147092 [0.060882] (0.0157) | 0.127626 [0.047746] (0.0075) | 0.142650 [0.050501] (0.0047) | 0.132968 [0.053486] (0.0129) |
| NASDAQ | 0.082919 [0.014691] (0.0001) | 0.089055 [0.036065] (0.0135) | 0.089625 [0.037221] (0.0160) | 0.137515 [0.083981] (0.1015) | 0.151311 [0.057868] (0.0089) | 0.144087 [0.062130] (0.0204) |
| FTSE 100 | 0.189237 [0.034317] (0.0000) | 0.196971 [0.137717] (0.1526) | 0.192548 [0.034801] (0.0000) | 0.149191 [0.061299] (0.0149) | 0.138839 [0.056644] (0.0142) | 0.141553 [0.051115] (0.0056) |
| DAX | 0.198975 [0.043890] (0.0000) | 0.172973 [0.045814] (0.0002) | 0.186357 [0.048786] (0.0001) | 0.152203 [0.068521] (0.02633) | 0.154301 [0.065475] (0.0184) | 0.151980 [0.067788] (0.0250) |
| CAC 40 | 0.201841 [0.040638] (0.0000) | 0.195145 [0.043864] (0.0000) | 0.198297 [0.044338] (0.0000) | 0.190324 [ 0.047582] (0.0030) | 0.184531 [0.053952] (0.0006) | 0.193877 [0.057185] (0.0007) |
| NIKKEI 225 | 0.142457 [0.019840] (0.0000) | 0.131159 [0.044223] (0.0030) | 0.133791 [0.054675] (0.0144) | 0.144476 [0.036943] (0.0000) | 0.131032 [0.035418] (0.0002) | 0.146149 [0.039360] (0.0002) |
| HSI | 0.184406 [0.046753] (0.0000) | 0.184903 [0.053823] (0.0006) | 0.181884 [0.052461] (0.0005) | 0.099576 [0.041911] (0.0175) | 0.097581 [0.043853] (0.0261) | 0.099805 [0.044877] (0.0262) |

**Table 16.** Summary of the asymmetry coefficient estimates of GJR(1,1). Numbers in brackets are standard errors. Numbers in parenthesis are *p*-values

The GJR asymmetry coefficient estimates, in general, decrease in the turmoil period. The exception is the NASDAQ index and NIKKEI 225 with normal and GED conditional distributions, where the effect is stronger. In terms of statistical significance, NASDAQ with normal distribution in the turmoil period and FTSE 100 with Student's *t* conditional distribution in the calm period, $\gamma_1$ estimates are the only ones that are not statistically significant.

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Normal | Student's t | GED | Normal | Student's t | GED |
| S&P500 | -0.204362 [0.029083] (0.0000) | -0.240799 [0.050671] (0.0000) | -0.203513 [0.009780] (0.0000) | -0.117291 [0.016321] (0.0000) | -0.135463 [0.035365] (0.0001) | -0.130852 [0.035770] (0.0001) |
| NASDAQ | -0.099430 [0.013688] (0.0000) | -0.105953 [0.029395] (0.0003) | -0.104551 [0.027945] (0.0002) | -0.133010 [0.027045] (0.0000) | -0.152199 [0.026163] (0.0000) | -0.148096 [0.030156] (0.0000) |
| FTSE 100 | -0.143839 [0.015905] (0.0000) | -0.175685 [0.013919] (0.0000) | -0.154195 [0.025157] (0.0000) | -0.157212 [0.025286] (0.0000) | -0.149847 [0.029801] (0.0000) | -0.151831 [0.025833] (0.0000) |
| DAX | -0.190195 [0.023157] (0.0000) | -0.185480 [0.023108] (0.0000) | -0.190600 [0.003616] (0.0000) | -0.127954 [0.025667] (0.0000) | -0.139435 [0.033448] (0.0000) | -0.136618 [0.030303] (0.0000) |
| CAC 40 | -0.197041 [0.023236] (0.0000) | -0.201383 [0.026884] (0.0000) | -0.200112 [0.025420] (0.0000) | -0.205555 [0.036656] (0.0000) | -0.217316 [0.040024] (0.0000) | -0.216678 [0.035234] (0.0000) |
| NIKKEI 225 | -0.148747 [0.016359] 0.0000) | -0.148300 [0.023584] (0.0000) | -0.146558 [0.025835] (0.0000) | -0.10985 [0.017006] (0.0000) | -0.11018 [0.023956] (0.0000) | -0.11072 [0.015560] (0.0000) |
| HSI | -0.11233 [0.032971] (0.0007) | -0.11442 [0.033389] (0.0006) | -0.11384 [0.035167] (0.0012) | -0.079231 [0.005724] (0.0000) | -0.078771 [0.022606] (0.0005) | -0.079553 [0.022425] (0.0004) |

**Table 17.** Summary of the asymmetry coefficient estimates of EGARCH(1,1). Numbers in brackets are standard errors. Numbers in parenthesis are *p*-values

Additionally, ARCH coefficient estimates are all not statistically significant, indicating no immediate impact of innovations on conditional volatility. Kaur and Singh (2015) found similar results for Brazilian Ibovespa and Russian RTS indices with the TGARCH-M model.

Concerning the asymmetry effects of the EGARCH model, all $\gamma_1$ estimates are statistically significant, meaning that the negative shocks increase conditional volatility more than positive shocks of the same magnitude, regardless of the period. Nevertheless, the S&P500, DAX, NIKKEI 225, HSI and FTSE 100 with Student's *t* and GED conditional distribution estimates suggest that the effect becomes weaker in the turmoil period.

Analysing the subsamples in terms of conditional distribution, log-likelihood and information criteria indicate that distributions with fatter tails outperform the normal in all cases. In general, the Student's *t* conditional distribution better fitted the data from the calm period, while GED is better for the turmoil period. The exception are S&P 500 and HSI, where GED conditional distribution is superior in all cases. Models with Student's *t* conditional distribution usually have higher asymmetry estimates (in absolute value). Comparing the asymmetry effects between indices, French CAC 40 has the highest estimates in the GJR model, with exception of the Student's *t* in the calm period, in which FTSE 100 has the highest estimate, but it is not statistically significant, being French market estimate the second highest. S&P 500 shows the highest asymmetry estimates (i.e. most negative estimates) in the calm period for EGARCH model, while CAC 40 has the most negative estimates in the turmoil period.

Although we found that, in the literature, dividing the series in subsamples and modelling its volatility is a common procedure, no attention is given to test if the difference between the coefficient estimates of the subsamples is statistically significant. Having this in mind, we attempt to close this gap by employing a *z*-test on the difference between the asymmetry estimates of the superior EGARCH model. The idea is to understand whether the increase or decrease of the asymmetry effect of two independent samples after the September 15, 2008, is statistically significant. Therefore, the hypothesis tested is the equality of two coefficients and consists on the following:

$$\begin{cases} H_0: \gamma_C = \gamma_T \\ H_a: \gamma_C \neq \gamma_T \end{cases}$$

where the null hypothesis is that $\gamma_C$ (asymmetry coefficient in the calm period) is equal to the $\gamma_T$ (asymmetry coefficient in the turmoil period). The formula of the test is as follows:

$$Z = \frac{\hat{\gamma}_C - \hat{\gamma}_T}{\sqrt{(SE\hat{\gamma}_C)^2 + (SE\hat{\gamma}_T)^2}} \quad (11)$$

where $(SE\hat{\gamma}_C)^2$ and $(SE\hat{\gamma}_T)^2$ are the $\hat{\gamma}_C$ and $\hat{\gamma}_T$ are the variances for the coefficient estimators, respectively. The comparison is performed considering the conditional distribution of the coefficients estimates. Table 18 summarizes the *z*-test results.

| | Normal | | Student's *t* | | GED | |
|---|---|---|---|---|---|---|
| | *z* score | *p*-value | *z* score | *p*-value | *z* score | *p*-value |
| S&P500 | -2.61086 | 0.009054 | -1.70469 | 0.088381 | -1.95942 | 0.050113 |
| NASDAQ | 1.107827 | 0.267948 | 1.175192 | 0.239955 | 1.059145 | 0.289554 |
| FTSE 100 | 0.447673 | 0.654442 | -0.78556 | 0.432454 | -0.06556 | 0.772161 |
| DAX | -1.80047 | 0.071861 | -1.13261 | 0.257634 | -1.76886 | 0.077061 |
| CAC 40 | 0.196174 | 0.844532 | 0.330458 | 0.741098 | 0.381295 | 0.703055 |
| NIKKEI | -1.64838 | 0.099353 | -1.13395 | 0.257214 | -1.1883 | 0.234833 |
| HSI | -0.98909 | 0.322663 | -0.88411 | 0.376696 | -0.82206 | 0.411077 |

**Table 18.** *z*-test results.

Only one difference between asymmetry coefficients estimates is statistically significant at 5% level. The EGARCH with normal distribution for the S&P index. For the remaining cases the null hypothesis is not rejected. If we relax the standard significance level assumption, and consider 10% significance level, the null hypothesis is rejected in five more cases. The differences in the S&P 500 index for all conditional distributions become statistically significant. The remaining cases are DAX with normal and GED conditional distributions and NIKKEI 225 with normal conditional distribution.

Overall, we conclude that the asymmetry effect is present in the stock market indices, regardless of the period. Bad news has bigger impact on volatility than good news. In general, the effect becomes weaker (except for NASDAQ and CAC 40) in the turmoil period, and one probable explanation for this is that after the collapse of the Lehman Brothers, the volatility was extremely high, consequently the bad news did not impact as much the already high volatility. Nevertheless, in most cases, the changes in the asymmetry coefficient estimates for the EGARCH model are not statistically significant, implying that the effect could remain the same. Asymmetry effect does not depend on the subsample period.

## 7. Conclusion

The main objective of this dissertation was to investigate the asymmetry effects on volatility during the Global Financial Crisis. In order to obtain a broader view, we analyse seven major stock market indices from three different regions: The North American market, composed by S&P 500 and NASDAQ Composite indices; The European market, composed by FTSE 100, DAX and CAC 40 indices; and NIKKEI 225 and Hang Seng (HSI) indices from Asia-Pacific region.

Regarding the sample periods, different approaches can be found in the literature. Using statistical methods such as structural break tests or graphically identifying market trends are among possible techniques. We use September 15, 2008 as the central event of the crisis. This date is better known as the collapse of the large investment bank Lehman Brothers. Thus, our analysis covers two non-overlapping subsamples of each index. The period of two years prior to the bankruptcy is labelled as *clam period,* while the *turmoil period* corresponds to two years after the bankruptcy.

We estimate two univariate conditional volatility models based on daily logarithmic returns. The EGARCH and GJR models are considered standard tools to assess asymmetry effects on volatility and extensive literature proves the popularity among researchers. Furthermore, we assume three conditional distribution for the innovations: the Gaussian normal, the Student's *t* and Generalized Error Distribution.

According to the results obtained, it is possible to conclude that asymmetry effects are present in the indices analysed, regardless of the subsample. These findings are in line with the literature review and empirical studies. In general, the asymmetry coefficient estimates decrease in the turmoil period (except for NASDAQ and CAC 40), suggesting that the impact of negative shocks during turbulent markets is weaker when compared to calm market period. In the literature, mixed findings concerning asymmetry effect changes are reported. Given the leptokurtosis of the financial data, models with heavy tailed conditional distributions provides a better fit. Results also indicate that the Student's *t* conditional distribution better fits the data prior to the Lehman Brothers bankruptcy, while GED conditional distribution provides better results for the subsample after the bankruptcy. Moreover, asymmetry estimates are generally higher in models that assume Student's *t* distribution. Based on Log-likelihood and Information Criteria, the EGARCH model is superior to GJR for in-sample prediction.

To some extent, this dissertation can be considered as a study that compiles several techniques to examine asymmetry effects on volatility. A total of 84 different specifications are possible which allows to compare the effects from several perspectives. Perhaps, the key contribution of this work to the finance literature is that we test the statistical significance of the changes in the coefficient estimates using a *z*-test. We found evidence that overall, the change in the asymmetry estimates of the EGARCH model are not statistically significant, suggesting that the impact of the asymmetry effect does not change when comparing calm and turmoil periods. At 5% significance level only the change of S&P 500 with normal conditional distribution was statistically significant.

Nevertheless, this work has several limitations. Although we chose the collapse of Lehman Brothers as reference point for the crisis, the period under analysis is limited to four years, meaning that extending or decreasing the period, can lead to different results. Another limitation is that we only analyse daily data of stock indices, and all of them can be considered mature markets. Nowadays, other more robust and up to date models can be applied to evaluate asymmetry effects. In fact, the limitations referred above can be taken as suggestions for further research. Extending the period under analysis to include the European debt crisis that started roughly in the beginning of 2009, reaching peaks in early 2012, is one of the possibilities. Instead of analysing non-overlapping subsamples, rolling regression approach as used by Leeves (2007) may be interesting. Using more complex models with different conditional distributions can also be relevant. The methodology can be applied to individual stocks, commodities such as gold or oil, currency pairs, bonds and other securities. Selecting high frequency intra-day data can provide unique insights, not evident in daily or weekly data. Furthermore, testing the statistical significance of the changes in asymmetry coefficient estimates on volatility may open completely new questions.

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## 9. Appendices

**Appendix 1 –** Determination of the conditional mean equations.

Appendix 1 shows the conditional mean equations specified for each index returns. The order determination was assessed based on log-likelihood, Akaike information criterion, Bayesian information criterion, as well as ACF and PACF functions.

- **S&P 500**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(1) | 1554.48 | **-3102.97** | **-3090.31** | 1220.24 | -2434.48 | -2421.81 |
| AR(2) | **1555.36** | -3102.72 | -3085.84 | 1225.51 | -2443.02 | **-2426.13** |
| AR(3) | - | - | - | **1227.92** | **-2445.84** | -2424.73 |

**Table 19.** Conditional mean equation specifications tested for S&P 500 index.

The conditional mean equations considered for the joint estimation are AR(1) for the calm period and AR(3) for the turmoil period.

| | AR(*p*)-GJR(1,1) | | | AR(*p*)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\mu_1$ | 0.000045 (0.8739) | 0.000608 (0.0274) | 0.000930 (0.0000) | -0.000110 (0.0000) | 0.000542 (0.0169) | 0.000864 (0.0000) |
| $\phi_1$ | -0.140276 (0.0037) | -0.088671 (0.0329) | -0.096092 (0.0005) | -0.136362 (0.0037) | -0.082692 (0.0413) | -0.091516 (0.0000) |
| | Turmoil Period | | | | | |
| $\mu_1$ | 0.000492 (0.3369) | 0.001016 (0.0373) | 0.001194 (0.0289) | 0.000482 (0.1015) | 0.001022 (0.1637) | 0.001163 (0.0449) |
| $\phi_1$ | -0.058164 (0.2389) | -0.068128 (0.1315) | -0.066314 (0.1572) | -0.067188 (0.0002) | -0.067704 (0.1510) | -0.069463 (0.2100) |
| $\phi_2$ | -0.058379 (0.2219) | -0.055705 (0.2313) | -0.025240 (0.6175) | -0.049097 (0.1658) | -0.041618 (0.3781) | -0.015052 (0.7614) |
| $\phi_3$ | -0.025353 (0.5963) | 0.004808 (0.9164) | 0.021809 (0.6574) | -0.003234 (0.6078) | 0.016421 (0.7209) | 0.032636 (0.5039) |

**Table 20.** Conditional mean equation estimation results for S&P 500 index. $\phi_i$ denotes AR coefficients. Numbers in parenthesis are *p*-values.

- **NASDAQ**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(1) | 1502.94 | **-2999.89** | **-2987.23** | 1213.21 | -2420.42 | -2407.757 |
| AR(2) | **1503.29** | -2998.58 | -2981.70 | 1216.84 | -2425.68 | **-2408.79** |
| AR(3) | - | - | - | **1219.47** | **-2428.95** | -2407.836 |

**Table 21.** Conditional mean equation specifications tested for NASDAQ index.

The conditional mean equations considered for the joint estimation are AR(1) for the calm period and AR(3) for the turmoil period.

| | AR($p$)-GJR(1,1) | | | AR($p$)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\mu_1$ | 0.000110 (0.8051) | 0.000535 (0.1890) | 0.000608 (0.1536) | -0.000055 (0.9159) | 0.000353 (0.4791) | 0.000417 (0.2223) |
| $\phi_1$ | -0.083060 (0.0747) | -0.069088 (0.1171) | -0.057411 (0.2078) | -0.083477 (0.0083) | -0.071006 (0.2174) | -0.062552 (0.1431) |
| | Turmoil Period | | | | | |
| $\mu_1$ | 0.000700 (0.31507) | 0.001196 (0.0309) | 0.001358 (0.0162) | 0.000689 (0.0908) | 0.001200 (0.0206) | 0.001346 (0.0013) |
| $\phi_1$ | -0.026298 (0.59248) | -0.034919 (0.4407) | -0.022504 (0.5247) | -0.053665 (0.2721) | -0.044497 (0.3150) | -0.030480 (0.4231) |
| $\phi_2$ | -0.049532 (0.31945) | -0.060759 (0.1907) | -0.041012 (0.3250) | -0.041396 (0.1783) | -0.048168 (0.2940) | -0.030895 (0.1047) |
| $\phi_3$ | -0.009421 (0.84025) | 0.027970 (0.5418) | 0.029813 (0.4345) | 0.017041 (0.7070) | 0.046375 (0.2909) | 0.045628 (0.2654) |

**Table 22.** Conditional mean equation estimation results for NASDAQ index. $\phi_i$ denotes AR coefficient. Numbers in parenthesis are $p$-values.

- **FTSE 100**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(0) | 1511.91 | -3019.81 | -3011.36 | - | - | - |
| AR(0)* | 1511.81 | -3021.61 | -3017.39 | - | - | - |
| AR(1) | **1518.40** | **-3030.81** | **-3018.13** | 1293.34 | -2580.67 | -2568.00 |
| AR(2) | - | - | - | 1296.32 | -2584.65 | -2567.74 |
| AR(3) | - | - | - | 1298.21 | -2586.41 | -2565.28 |
| AR(4) | - | - | - | 1306.86 | -2601.72 | -2576.36 |
| AR(5) | - | - | - | **1311.83** | **-2609.65** | **-2580.07** |

**Table 23.** Conditional mean equation specifications tested for FTSE 100 index. * Denotes the conditional mean equation without drift.

The conditional mean equations considered for the joint estimation are AR(1) for the calm period and AR(5) for the turmoil period.

| | AR($p$)-GJR(1,1) | | | AR($p$)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\mu_1$ | -0.000149 (0.6748) | 0.000120 (0.8409) | 0.000034 (0.9220) | -0.000351 (0.3292) | -0.000079 (0.424657) | -0.000152 (0.63203) |
| $\phi_1$ | -0.074301 (0.1100) | -0.062613 (0.1680) | -0.064416 (0.1574) | -0.061598 (0.1704) | -0.048558 (0.213711) | -0.052064 (0.24081) |
| | Turmoil Period | | | | | |
| $\mu_1$ | 0.000537 (0.5181) | 0.000734 (0.2709) | 0.000861 (0.1136) | 0.000373 (0.4868) | 0.000535 (0.2957) | 0.000661 (0.0153) |
| $\phi_1$ | -0.013568 (0.7836) | -0.006778 (0.8802) | 0.001281 (0.9714) | -0.003626 (0.9394) | -0.002662 (0.9527) | 0.003480 (0.7321) |
| $\phi_2$ | 0.001173 (0.9807) | -0.006778 (0.7989) | -0.025560 (0.6085) | 0.010902 (0.8090) | -0.005844 (0.8974) | -0.017099 (0.7623) |
| $\phi_3$ | -0.045524 (0.3452) | -0.026150 (0.5718) | -0.011981 (0.7893) | -0.036135 (0.4188) | -0.020604 (0.6475) | -0.007259 (0.6484) |
| $\phi_4$ | 0.037472 (0.4093) | 0.048490 (0.2707) | 0.057411 (0.1741) | 0.046357 (0.2916) | 0.055423 (0.2029) | 0.062390 (0.0063) |
| $\phi_5$ | -0.006889 (0.8781) | -0.052572 (0.2549) | -0.043775 (0.2727) | 0.003529 (0.9332) | -0.033720 (0.4539) | -0.032115 (0.1422) |

**Table 24.** Conditional mean equation estimation results for FTSE 100 index. Numbers in parenthesis are *p*-values.

- **DAX**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(0) | 1536.88 | -3069.77 | -3061.31 | - | - | - |
| AR(0)* | 1536.88 | **-3071.76** | **-3067.53** | - | - | - |
| AR(1) | **1538.44** | -3070.88 | -3058.20 | 1256.64 | -2507.28 | **-2494.59** |
| AR(2) | - | - | - | 1259.02 | -2510.04 | -2493.11 |
| AR(3) | - | - | - | 1259.38 | -2508.76 | -2487.60 |
| AR(4) | - | - | - | **1261.67** | **-2511.34** | -2485.96 |

**Table 25.** Conditional mean equation specifications tested for DAX index. * Denotes the conditional mean equation without drift.

The conditional mean equations considered for the joint estimation are white noise without drift for the calm period and AR(4) for the turmoil period.

| | AR(*p*)-GJR(1,1) | | | AR(*p*)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| | White Noise without Drift | | | | | |
| | Turmoil Period | | | | | |
| $\mu_1$ | 0.000233 (0.2013) | 0.000415 (0.5474) | 0.000484 (0.4689) | 0.000194 (0.3754) | 0.000351 (0.4671) | 0.000437 (0.0002) |
| $\phi_1$ | -0.007865 (0.8619) | 0.001461 (0.9743) | 0.006159 (0.8883) | 0.005817 (0.4061) | 0.015198 (0.7294) | 0.017411 (0.1093) |
| $\phi_2$ | -0.055219 (0.2326) | -0.048960 (0.2938) | -0.033357 (0.4925) | -0.039694 (0.3692) | -0.035185 (0.4351) | -0.021608 (0.0001) |
| $\phi_3$ | -0.008860 (0.8431) | 0.002650 (0.9546) | 0.010504 (0.8153) | 0.005530 (0.7826) | 0.015237 (0.7330) | 0.022353 (0.3948) |
| $\phi_4$ | 0.027152 (0.5360) | 0.029549 (0.4977) | 0.029950 (0.4822) | 0.014570 (0.6400) | 0.025264 (0.5661) | 0.027148 (0.0000) |

**Table 26.** Conditional mean equation estimation results for DAX index. $\phi_i$ denotes AR coefficients. Numbers in parenthesis are *p*-values.

- **CAC 40**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(1) | 1510.52 | **-3015.05** | **-3002.35** | 1240.78 | -2475.56 | **-2462.84** |
| AR(2) | **1510.85** | -3013.71 | -2996.77 | 1243.21 | -2478.42 | -2461.47 |
| AR(3) | - | - | - | 1244.84 | -2479.68 | -2458.49 |
| AR(4) | - | - | - | 1248.45 | -2484.89 | -2459.46 |
| AR(5) | - | - | - | **1250.77** | **-2487.53** | -2457.86 |

**Table 27.** Conditional mean equation specifications tested for CAC 40 index.

The conditional mean equations considered for the joint estimation are AR(1) for the calm period and AR(5) for the turmoil period.

| | AR(*p*)-GJR(1,1) | | | AR(*p*)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| $\mu_1$ | -0.000158 (0.6878) | 0.000067 (0.8587) | 0.000034 (0.9289) | -0.000314 (0.3228) | -0.000108 (0.7437) | -0.000135 (0.6997) |
| $\phi_1$ | -0.079410 (0.0886) | -0.083419 (0.0644) | -0.083667 (0.0554) | -0.072010 (0.1137) | -0.076585 (0.0976) | -0.077987 (0.0442) |
| | Turmoil Period | | | | | |
| $\mu_1$ | 0.000159 (0.7914) | 0.000347 (0.5665) | 0.000316 (0.6158) | -0.000382 (0.5506) | -0.000136 (0.8293) | -0.000128 (0.0002) |
| $\phi_1$ | -0.028286 (0.5480) | -0.009000 (0.8398) | -0.005916 (0.8888) | -0.002722 (0.9538) | 0.017660 (0.6012) | 0.012986 (0.6110) |
| $\phi_2$ | -0.013848 (0.7669) | -0.019952 (0.6602) | -0.011353 (0.8159) | 0.014256 (0.7544) | 0.004582 (0.9332) | 0.010486 (0.6047) |
| $\phi_3$ | -0.029694 (0.5279) | -0.018944 (0.6736) | -0.014459 (0.7499) | 0.008744 (0.8486) | 0.009926 (0.8276) | 0.012240 (0.2990) |
| $\phi_4$ | 0.023989 (0.5732) | 0.037665 (0.3745) | 0.034760 (0.3435) | 0.039954 (0.3357) | 0.057555 (0.1837) | 0.053052 (0.0242) |
| $\phi_5$ | 0.006237 (0.8874) | -0.016667 (0.7045) | -0.016978 (0.6883) | 0.024914 (0.5485) | 0.004035 (0.9286) | -0.004297 (0.6059) |

**Table 28.** Conditional mean equation estimation results for CAC 40 index. $\phi_i$ denotes AR coefficients. Numbers in parenthesis are *p*-values.

- **NIKKEI 225**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(0) | 1395.55 | -2787.1 | -2778.71 | 1142.48 | -2280.96 | -2272.59 |
| AR(0)* | 1395.15 | **-2788.31** | **-2784.12** | 1142.35 | **-2282.7** | **-2278.51** |
| AR(1) | **1396.12** | -2786.24 | -2773.66 | **1143.09** | -2280.18 | -2267.61 |

**Table 29.** Conditional mean equation specifications tested for NIKKEI 225 index. * Denotes the conditional mean equation without drift.

The conditional mean equations considered for the joint estimation are white noise without drift for both periods.

| | AR(*p*)-GJR(1,1) | | | AR(*p*)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| | White Noise without Drift | | | | | |
| | Turmoil Period | | | | | |
| | White Noise without Drift | | | | | |

**Table 30.** Conditional mean equation for NIKKEI 225 index.

- **HSI**

| | Calm Period | | | Turmoil Period | | |
|---|---|---|---|---|---|---|
| | Log likelihood | AIC | BIC | Log likelihood | AIC | BIC |
| AR(0) | 1261.84 | -2519.68 | -2511.28 | 1143.46 | -2282.92 | -2274.51 |
| AR(0)* | 1261.83 | **-2521.65** | **-2517.46** | 1143.41 | **-2284.82** | **-2280.61** |
| AR(1) | **1263.96** | -2521.52 | -2509.33 | **1143.54** | -2281.08 | -2268.46 |

**Table 30.** Conditional mean equation specifications tested for HSI index. * Denotes the conditional mean equation without drift.

The conditional mean equations considered for the joint estimation are white noise without drift for both periods.

| | AR(*p*)-GJR(1,1) | | | AR(*p*)-EGARCH(1,1) | | |
|---|---|---|---|---|---|---|
| | Calm Period | | | | | |
| | Normal | Student's t | GED | Normal | Student's t | GED |
| | White Noise without Drift | | | | | |
| | Turmoil Period | | | | | |
| | White Noise without Drift | | | | | |

**Table 31.** Conditional mean equation for HSI index.

**Appendix 2 –** Testing for ARCH effects.

Appendix 2 shows the results of the Ljung-Box and ARCH Lagrange Multiplier test on squared residuals of mean equation with 20 lags.

| | LB(20) of $\varepsilon_t^2$ | | ARCH LM (20) of $\varepsilon_t^2$ | |
|---|---|---|---|---|
| | Calm Period | Turmoil Period | Calm Period | Turmoil Period |
| S&P500 | 95.382<br>(8.369e-12) | 626.76<br>(2.2e-16) | 38.893<br>(0.006875) | 185.35<br>(2.2e-16) |
| NASDAQ | 101.38<br>(7.127e-13) | 546.17<br>(2.2e-16) | 24.096<br>(0.2382) | 216.66<br>(2.2e-16) |
| FTSE 100 | 129.1<br>(2.2e-16) | 374.03<br>(2.2e-16) | 36.753<br>(0.01253) | 227.85<br>(2.2e-16) |
| DAX | 99.664<br>(1.447e-12) | 343.04<br>(2.2e-16) | 63.772<br>(1.828e-06) | 202.37<br>(2.2e-16 |
| CAC 40 | 98.173<br>(2.671e-12) | 288.88<br>(2.2e-16) | 37.566<br>(0.01) | 64.896<br>(1.213e-06) |
| NIKKEI | 229.02<br>(2.2e-16) | 778.29<br>(2.2e-16) | 59.942<br>(7.271e-06) | 377.05<br>(2.2e-16) |
| HSI | 227.43<br>(2.2e-16) | 546.34<br>(2.2e-16) | 101.58<br>(6.568e-13) | 139.63<br>(2.2e-16) |

**Table 32.** Summary of results of the Ljung-Box and ARCH LM tests with 20 lags. Numbers in parenthesis are *p*-values.

ARCH effects of order 20 are present in all subsamples. The only exception is the NASDAQ index in the calm period, where ARCH LM test results do not reject the null of no ARCH effects. Nevertheless, Ljung-Box test results reject the null.

**Appendix 3** – List of RStudio packages used for auxiliary analysis.

- '**tseries**' by Trapletti, and Hornik (2019).
- '**psych**' by Revelle (2019)
- '**fDMA**' by Krzysztof (2018)