SIMULATED DISCOUNTED CASH FLOW VALUATION FOR A FINANCIAL INSTITUTION: MILLENNIUM BCP

Ana Carolina Silva Almeida

Project submitted as partial requirement for the conferral of
Master of Finance

Supervisor:
Prof. Pedro Manuel de Sousa Leite Inácio
Assistant Professor, Department of Finance, ISCTE Business School

September 2019

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Abstract

Valuing a bank is not an easy challenge. The only similarities with the valuation of other companies are the different approaches and models that can be used to valuate both. That variety, frequently, leads to different outcomes and/or interpretations and, thus, there is not a consensual agreement between financial analysts. This report aims to valuate a financial institution using the discounted cash flow valuation model, known to be the most appropriate and used amongst the financial and academic community.

Knowing the particularities of a financial institution bank, the purpose of this report is to valuate Millennium BCP, one of the biggest Portuguese private banks, determining a share price on 31th December, 2018, and then compare it to the real price. To remind the sensitivity of banking institutions and to consider an eminent economic and financial crisis, the values will be estimated using probabilistic distributions and statistical simulations.

Firstly, the present project presents a theoretical information regarding the discounted cash flow model and the simulation process, as well as presenting all probability distributions. Secondly, to contextualize the reader a brief presentation about Millennium BCP business model and an economic presentation about the countries where is mainly present is made. Following, all the assumptions, results and interpretations are be presented.

In the end of this report, it was concluded that Millennium BCP's shares were traded above its real price, at our valuation date.

Keywords: Simulation, Discounted Cash Flow, Equity, Probabilistic Distributions

JEL Classification: G21 – Banks

G32 – Value of Firms

Resumo

Avaliar um banco não é um desafio fácil. As únicas parecenças com o processo de avaliação de outras empresas são os diferentes métodos e modelos que podem ser usados para as avaliarem. Frequentemente, essa variedade leva a diferentes resultados e/ou interpretações, não sendo por essa razão uma decisão consensual entre os analistas financeiros. Este relatório propõe avaliar uma instituição financeira usando o método de avaliação de Fluxo de Caixa Descontados, conhecido por ser o mais apropriado e usado entre a comunidade financeira e académica.

Sabendo as particularidades de uma instituição financeira bancária, o propósito deste relatório é de avaliar o Millennium BCP, um dos maiores bancos privados portugueses, determinar o preço da ação no dia 31 dezembro de 2018 e compará-lo com o preço real. Para relembrar a sensibilidade das instituições bancárias e para considerar uma crise económica e financeira eminente, os valores vão ser estimados usando distribuições probabilísticas e simulações estatísticas.

Primeiramente, este projeto apresenta informação teórica sobre o método de avaliação de Fluxo de Caixa Descontados e processo de simulação, bem como a apresentação de todas as distribuições probabilísticas. Segundamente, para contextualizar o leitor, será feita uma breve apresentação sobre o modelo de negócios do Millennium BCP e uma apresentação económica sobre os principais países onde está presente. Para concluir, todas as premissas, resultados e interpretações serão apresentadas.

No final deste relatório, concluiu-se que as ações do Millennium BCP estavam a ser negociadas acima do seu preço real, à data da presente avaliação.

Palavras-Chave: Simulação, Fluxos de Caixa Descontados, Capitais Próprios, Distribuições probabilísticas

JEL Classification: G21 – Banks

G32 – Value of Firms

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To my family, especially to my parents, brother and grandparents, for all the unconditional love and support. To my dog Luna, who will never read this, for all the love and company during the lonely periods while creating this project.

To my friends, for all the laughs, love, support and patience.

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List of Abbreviations

Δ BVE	Delta Book Value of Equity
BVE	Book Value of Equity
CAPEX	Capital Expenditures
CAPM	Capital Asset Pricing Model
CDF	Cumulative Density Function
CPI	Consumer Price Index
DCF	Discounted Cash Flow
ECB	European Central Bank
ECF	Equity Cash Flow
FCFE	Free Cash Flow to the Equity
FCFF	Free Cash Flow to the Firm
GDP	Gross Domestic Product
NWC	Net Working Capital
OCI	Other Comprehensive Income
PDF	Probability Density Function
WACC	Weighted Asset Cost of Capital

1. Introduction

This project aims to provide an equity valuation of Banco Comercial Português S.A. (a.k.a. throughout this project as "BCP", "Millennium BCP" and "Group") with the goal of informing the current and future shareholders about the equity valuation of the bank in 2018, as well as its valuation per share.

Millennium BCP's group is the one of the largest Portuguese private banks, serving more than 2.3 million clients worldwide. According to its 2018 Annual report, its total assets are valued in €75,923.049 million, with a shareholder net income of €301.065 million. As of December 2018, BCP shares were listed in more than 50 national and international stock indexes, with a total number of shares of 15,113,989,952, in which naturally BCP has the highest weight in PSI-20 market index.

Currently, the bank offers an extensive variety of financial products and services to its Portuguese and international clients, by having subsidiaries in Portugal, Poland and Mozambique, a private banking platform in Switzerland, an on-shore branch in Macao and commercial agreements in countries with a large presence of Portuguese emigrants.

To achieve the goal of valuating Millennium's 2018 equity, every variable needed for its calculation entered in a probability fitting process, using historical annual data from 2008-2018 and software R Studio. After finding the best probability distribution for each variable and considering its characteristics and restrictions, a simulation of 5000 iterations was made, starting in 2019 and ending in 2023. The equity value in 2018 was obtained using the average yearly values of the three variables used in the Discounted Cash Flow approach. Considering the complexity of this process, made to consider any unpredictable crisis or extreme changes, each instrument, concepts and assumptions should be very well understood. Having this said, as mentioned in Martins (2016), the corporate valuation is a very important financial field, since it helps the investors to make important and correct decisions in matters such as investment decisions, purchasing or selling financial instruments or mergers and acquisitions. Thus, as previously explained, the motivation and goal of this project was to carry a meticulous analysis and create a methodology, that could cover several scenarios to achieve a reliable equity valuation.

To achieve the mentioned goals and explain it in the clearest way possible, the structure of this project will be the following:

The theoretical research will be separated into two different sub-chapters, the 2.1 and 2.2. In sub-chapter 2.1, named 'Company Valuation' there will be a description of the different valuation models used and an explanation on why the financial service firms have a different valuation method from other companies. On the following sub-chapter, named 'Simulation Process', we will have a theoretical explanation helped with graphics for every probabilistic distribution used in the fitting process, as well as every information and visual criterion and goodness-of-fit tests to help decide the best distribution for each variable.

In chapter 3, named 'Millennium BCP', there will be a brief description of the bank's history, structure and the location of every financial operation worldwide. In addition, we present an analysis concerning its financial performances, profitability, efficiency and ratings. In the end, it is presented a sub-section regarding the goals to achieve until 2021.

On the following chapter, named 'Countries' economic and financial analysis', we will make a brief review on the most important economical and social variables for Portugal, Poland and Mozambique, as well as analysing their respective net interest income/margin values throughout the years.

On chapter 5, intituled as 'Data and Methodology', it will be presented the characteristics and definitions of the data for each variable and respective methodology, ending with a table with all distributions used in each variable in the fitting process. The following chapter, 'Discussion of results', follows the same structure as the previous one, since the simulated data for each variable is presented and analysed. This chapter ends with the finding of the equity value for 2019, the goal of this report.

Chapter 7 is the conclusion of the project, followed by the references and appendixes.

2. Theoretical Research

In this section, some theoretical framework about Equity valuation methods and Probabilistic Simulations will be presented. This will serve as a base for the model and valuation made in the next sections.

2.1. Company valuation

According to Damodaran (2012), valuation models can be sub-divided into discounted-cash, relative and contingent claim valuation, depending on the expected and available data and the type of the company. On the first, the value of the company is estimated by discounting the future predicted cash flows or dividends, using a discount rate to make its values backdated to the present. The second model has the same goal, where its market values are predicted by analysing publicity firms of the same industry, in the form of multiples, such as book value and cash flows. The third, called contingent claim valuation, uses option pricing models to calculate the value of other similar assets.

Even though the second model is more common to be used in non-publicity firms due to its difficulty of getting financial data or in publicity listed companies where it is very difficult to calculate cash flows, the most used model is the first one since it is considered that the value of a firm or an asset can be related to the returns that the investor expects to happen (Pinto et. al, 2010).

Regardless of the type of valuation this process is always similar and involves five steps, according to Pinto et. al (2010): the first step, the most important and necessary one, has the purpose of constructing a basis for forecasting the performance of the company, with an understanding of the business and its industry by analysing its financial documents, such as income statements and balance sheets; after that, a forecast of financial categories must be done to provide inputs needed for the valuation; the third and fourth steps regard the choice of the type of valuation model, depending on the characteristics of the company, its estimations and the analysis of the results. Here, the same authors also defend several checks to test if the results make sense considering the company, its industry and the robustness of the chosen valuation model, in a way to reduce the possibility of errors. The fifth step ends up with conclusions and recommendations.

2.1.1. Discounted Cash Flow Valuation Model

Discounted Cash Flow (DCF) model is, according to Pinto et al. (2010), a valuation model calculated with the intention of producing an estimation using future cash flows discounted at a specific discount rate related with the risk of the asset. The model is based on detailed and reliable yearly forecasts of each financial categories related with the calculation of the cash flows based on historical data and future expected values (Fernandez, 2017). Thus, we can divide this model in two sub models: Free Cash Flow to the Firm (FCFF) and Free Cash Flow to the Equity (FCFE).

2.1.1.1. Free Cash Flow to the Firm

Free Cash Flow to the Firm, most known as FCFF, consists on the amount of cash available to the investors of the firm after all investments and operating costs are paid.

The expenses in investments have the goal to grow the asset base and revenues of the company and possibly the future cash flows. It can be split into investments in working capital, such as inventory, and investments in fixed capital, such as property and equipment (Nurminen, 2016).

As the previous author also notes, after everything is paid the cash flows can be used to pay dividends, buy shares or to fund acquisitions or mergers. Its formula is the following:

$$\boldsymbol{FCFF = EBIT * (1 - t) + Dep + Amort - CAPEX - \Delta NWC} \qquad (\mathbf{1})$$

Where: EBIT = earnings before interest and taxes;

t = tax rate

Dep = Depreciation

Amort = Amortization

CAPEX = Capital expenditures

ΔNWC = Difference between Net Working Capital of year t with t-1

The EBIT, main category in this equation, is obtained by the subtraction of revenues with costs of goods sold (COGS) and other operating costs such as research and development and general and administration;

Depreciation and amortization are tax deductions related with the aging of an asset, like equipment and buildings, throughout its useful lifetime.

Net Working Capital (NWC) represents the current assets available to cover, during a certain year, the amount of short-term obligations. Thus, it is calculated through the difference between the amounts of current assets with current liabilities. If it is positive it means that the company has the capacity to pay all the short-terms obligations and help in future investments (Nurminen, 2016).

2.1.1.2. Free Cash Flow to the Equity

Free Cash Flow to Equity, most known as FCFE, consists on the remaining cash flow distributed as dividends to equity shareholders after all operating costs, CAPEX and payments to debtholders. In general, it can be calculated by subtracting after-tax interest expenses and adding net debt borrowings to FCFF, as seen in this equation (Pinto et.al 2010):

$$\boldsymbol{FCFE = FCFF - Interest * (1 - t) + Net\ Debt\ Borrowings} \quad (2)$$

$$\boldsymbol{= Net\ income - CAPEX - \Delta NWC + Net\ Debt\ Borrowings} \quad (3)$$

Net debt borrowings consist on the difference between the debt issued and repaid during a certain year.

Net income consists on the income available to all shareholders, who decide if it should be distributed as dividends and/or for reinvestment (Koller et. al, 2010). It is calculated through this equation:

$$\boldsymbol{Net\ income = EBT * (1 - t)} \quad (4)$$

If the investor wants to know the value of the equity per share it should divide the obtained equity per the number of shares outstanding.

Subsequently, to calculate the Equity DCF there are three most common equations which may be used, depending on the company growth's characteristics (Pinto et al. (2010))[1].:

1. The general idea is to determine the value of the equity using this equation

$$\boldsymbol{Equity = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1 + r)^t}} \quad (5)$$

If the company has no perspective of growth the FCFE may then be constant;

2. If the company grows at a constant rate (g) its equity valuation model is called Constant Growth FCFE and its equation is the following:

[1] To calculate Firm DCF the equations are similar, with the exception of using FCFF, instead of FCFE

$$Equity = \frac{FCFE_0 * (1 + g)}{r - g} \qquad (6)$$

3. When a company grows, during a certain period, higher than the growth rate of the economy, leading to an advantage over other similar companies but then settles to a stable rate during a long period of time, its equity value is calculated using two stages – one for the high growth period and other for the stable growth, called terminal value. Therefore, this model is called Two-Stage FCFE and its equation is the following:

$$Equity = PV\ of\ FCFE + PV\ of\ Terminal\ Value \qquad (7)$$

Since it is not possible to estimate all cash flows until infinity, analysts decided to create the definition of terminal value. This value contains the discounted value in perpetuity of the first cash flow after the last period (period n) with known value and a constant growth rate, discounted for n periods (Viebig et. al, 2010).

Therefore, its equation and "total equity" are obtained like this:

$$FCFE_{n+1} = FCFE_n * (1 + g) \qquad (8)$$

$$Terminal\ Value = \frac{FCFE_{n+1}}{(r - g)} * \frac{1}{(1 + r)^n} \qquad (9)$$

$$Equity = \sum_{t=1}^{n} \frac{FCFE_t}{(1+r)^t} + \frac{FCFE_{n+1}}{(r-g)} * \frac{1}{(1+r)^n} \qquad (10)$$

2.1.1.3. Discount Rate

In the case of FCFF calculation the discount rate (r) – rate used to find the present value of future cash flows - is obtained through the Weighted Average Cost of Capital (WACC), which is determined by this equation:

$$WACC = \frac{Equity}{Equity + Debt} * re + \frac{Debt}{Equity + Debt} * rd * (1 - t) \qquad (11)$$

However, since the goal in this project is to calculate the equity of a financial service firm the only rate needed is the cost of equity (re). This rate is determined by three variables, when the estimated cash flows are in the same currency or country as the domestic currency of the country in question. The model which will be used to estimate it is the Capital Asset Pricing Model (CAPM), represented by the following equation (Agarwal and Mukhtar (2010) and Pinto et.al (2010)):

$$re = rf + \beta * [E(R_m) - rf] \qquad (12)$$

Where: rf is the risk-free rate

β (beta) that represents the stock's risk sensitivity to its index market;

$E(R_m)$ is the expected return of the index market;

$[E(R_m) - rf]$ consists on the market risk premium;

$\beta * [E(R_m) - rf]$ is the equity risk premium.

In this model the risk-free rate and market risk premium are common to all companies of the same index market; whereas beta varies depending on the chosen company.

2.1.1.3.1. Beta

Damodaran (2012) defends the existence of two main approaches to calculate beta – the stock's risk sensitivity to its index market: the historical and the fundamental methods.

The conventional historical approach mentions the estimation of beta using an econometric regression of historical returns of the asset against the historical returns of the respective index market, like the one below:

$$\boldsymbol{R_j = a + b * R_m} \quad \textbf{(13)}$$

Where: R_j are the stock returns

R_m are the market returns

b is the slope of the equation and beta of the stock. It is obtained by $\frac{Cov(R_j, R_m)}{\sigma_m^2}$

However, this approach has the severe problem of creating betas too noisy or skewed to be considered as the beta of a stock. Besides that, the historical betas are influenced by events that affected the index markets, like economic crisis and national or international incidents.

The second approach is the fundamental method, where the beta is affected by three conditions: the type of business(es) in which the firm acts, the level of operating leverage and financial leverage. The unlevered or asset beta is determined by the two first conditions, whereas the levered or equity beta is determined by the riskiness of the sector the firm operates in and the amount of financial leverage risk. The levered beta is, normally, the beta used for the calculation of cost of equity, since it considers the debt of the firm.

Its calculation is the following:

$$\beta_L = \beta_U \left[1 + (1 - t) * \frac{D}{E}\right]$$

(14)

Where: β_L – Levered beta for the equity of the fir

β_U – Unlevered beta of the firm

t – Corporate tax rate

D/E – Debt/Equity ratio

2.1.1.3.2. Risk-Free Rate

Koller et. al (2010) defines the risk-free rate, symbolized as rf in the equation of the cost of equity, as the return of a portfolio or security with beta null. In rule, to determine this rate the 10-year government default-free bonds are analysed since the government controls the printing of currency paper (Damodaran 2012) and "long-term government bonds make interim interest payments, causing their effective maturity to be shorter than their stated maturity" (Koller et. al, 2010: 241).

For example, when estimate a cash flow in USD the 10-year US government default-free bonds rates are used. In Europe, when estimate a cash flow in EUR the 10-year German zero-coupon Eurobond is usually used, since they "have higher liquidity and lower credit risk than bonds of other European countries" (Koller et. al, 2010: 241).

2.1.1.3.3. Market Risk Premium

The market risk premium – obtained by the difference of annual index market's expected return and risk-free rate – is one of the most discussed variables in finance, since it is very hard to observe the behaviour of an index market's expected return. That uncertainty led to the creation of several calculation models (Koller et. al, 2010). Damodaran (2012) presented two different models to estimate it: the modified historical premium and implied equity approach.

In the modified historical premium approach, Damodaran stated the need to add a new variable to equity risk premium, called country risk premium, for countries more socially and financially risky than the ones whose 10-year government bonds rates are usually used as risk-free rate (Germany and United States). Thus, the equation is the following:

$$\textit{Equity Risk Premium} = \textit{Base premium for mature equity market} + \textit{country premium} \quad (15)$$

The second approach, named implied equity, does not require historical data nor the country premium variable, since it assumes that the financial markets are correctly priced. Considering the equation below, if all variables, except cost of equity, are known is possible to obtain cost of equity.

$$\textit{Total Market capitalization} = \frac{\textit{Expected dividends}}{re - g} \qquad \textbf{(16)}$$

After obtaining the cost of equity, the equity risk premium is calculated through the subtracting of cost of equity with risk-free rate.

2.2.1. Institutional Finance Firms' Special Valuation Cases

According to Damodaran (2012), a financial service business can be categorized in four groups depending on how its profit is created. The first is a bank, who makes money on the difference (spread in financial language) between the interests offered in products, like bank deposits, to the clients and the interests that the clients pay every time they request products to the institution, like borrowings. Insurance companies create their income in two ways: the first is through periodic payments received from clients with insurance products from that company; the other is income gained by the company through some investment portfolios. Investment banks gain their money by advising and providing products to non-financial firms to make them raise their capital from financial markets or through acquisitions or merger deals. Finally, investment firms offer investment advices and management portfolio services for investors, that pay for the required services.

In financial service firms the valuation is not as simple as in the other companies, due to their unique characteristics, which brings a challenge for an analyst who tries to value them. According to several authors, throughout the years, Equity DCF has been defined as the best DCF model for valuing a financial institution, and not Firm DCF, which is the most used model to obtain a valuation.

The first reason for that can be seen in Koller et al. (2010) where the authors defend that operational and financial cash flows cannot be separated, like it is done in Firm DCF, since the banks create value from funding and lending operations, besides the fact that most of their assets are financial instruments like bonds and securitized obligation that are active in the market (Damodaran, 2009).

Secondly, these institutions are deeply regulated such as having to comply with the maintenance of mandatory minimum capital ratios to prevent these institutions from lending above their means and having restricted rules for investing their funds, making very difficult for new firms to enter the market (Damodaran (2012, 2009)). For example, the Basel III solvency agreement made in 2010 demanded that, starting in 2015, the minimum core capital (Tier 1) raised to 6%. This implies that under this rule banks and other financial firms can only lend up to 16.6 times their amount of equity.

The third reason, also defended by the previous author, is related with the similarities of debt with "raw material" rather than a source to get capital, in the sense that it is moulded into other products and sold by the bank to a higher price. Also, and since the definition of debt in the financial firms is so inconstant there are doubts about the category of, for example, deposits made by customers into their bank accounts. If it is considered as debt, the operating income for the bank is measured prior to interest paid to depositors, which would bring a problem since interest expenses are the largest cost for the bank. All of this makes it questionable to use debt to calculate the WACC and enterprise value.

Lastly, the definitions of reinvestment and NWC are not clear, making it difficult to estimate the expected future growth rate. Contrarily to other firms, banks (re)invest mainly in intangible assets (i.e. brand name and human capital), leading to small amounts of CAPEX and depreciation in their balance sheets, since, as said before, the investments are considered operating expenses in accounting statements (Damodaran, 2012).

Regarding the NWC, if its value is defined as the difference between current assets and current liabilities a large part of the bank would fall into one of the categories. Therefore, changes in this number might be large and volatile and without any indication of the need to reinvest and grow (Damodaran, 2012).

2.2.Simulation Process

According to Chance (2009), simulation is a process where random numbers are generated using defined probabilities that reflect the uncertainty and risk of the variables, such as quantity of sales, interest and exchange rates or total earnings. It the end, the outcomes associated with those random experiences are analysed to obtain results and its associated risk.

Bélaïd and Wolf (2009) point that the purpose of simulation is the determination of wished variable's values, using other variables that affect its value. If for example, the equity value is a function of three variables X, Y, Z then the relation between the four can be described by this equation:

$$Equity = f(X, Y, Z) \quad (17)$$

If the variables X, Y, Z had exact values then the calculation of equity would have a simpler and deterministic calculation, since it would not exist any uncertainty. However, the financial industry is still too volatile to determine a fixed value for any associated variable, which makes simulation the best process. In this process, the variables X, Y and Z have each a probability distribution that represent the interval of possible values and its respective probabilities.

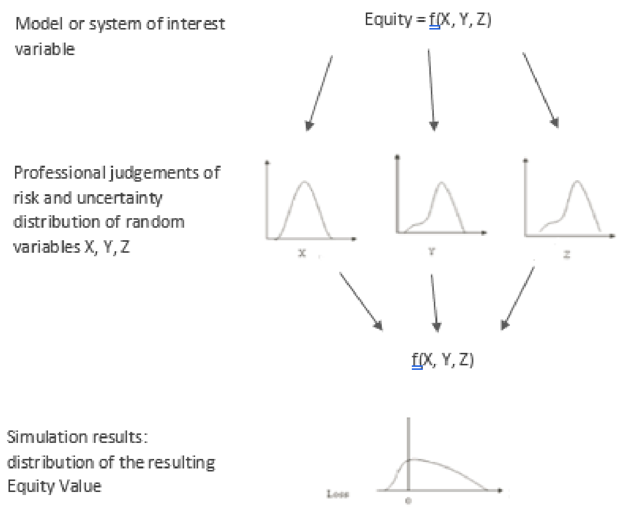


Figure 1 - Simulation Process
Source: Bélaïd and Wolf (2009)

In addition to that, the same authors also pointed out that even though some variables might be well defined mathematically there are some problems that are only solved using simulation:

I. Each probability distribution will have a density function. If we determined them for each random variable and then insert those functions in the equation 17 to discover equity value, then the solution would be very hard to find due to its complexity.

II. Another option could be the addition of the average values of the variables in the equation 17 to obtain the average value of equity value. However, that is not correct because it is not possible to obtain the average value of a dependent variable by replacing the average values of each independent variable in the equation.

2.2.1. Probabilistic Distributions

A probability distribution can be discrete or continuous. A discrete probability distribution can assume a value with a certain probability. As many authors mention, including Walpole et. al (2007: 100), "the set of ordered pairs (x, f(x)) is a probability function, probability mass function, or probability distribution of the discrete random variable X, respectively, if, for each possible outcome x,

1. $f(x) \geq 0$ **(18)**
2. $\sum_x f(x) = 1$ **(19)**
3. $P(X = x) = f(x)$" **(20)**

According to Ieong (2006: 7), a continuous distribution is a distribution where "the random variable of the underlying distribution can take on infinitely many different values (or that the outcome space is infinite)." Besides that, a continuous random variable has a probability of zero of assuming any exact value.

Its cumulative distribution function (CDF) and probability of a random variable X distributed between a and b, where a < b, respectively, are the following:

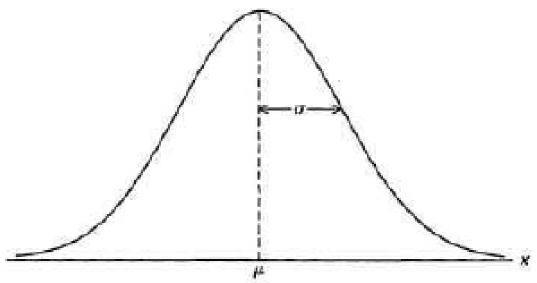
- $F(a) = P(X \leq a) = \int_{\infty}^{a} f(x)dx$ **(21)**
- $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx = F(b) - F(a)$ **(22)**

2.2.1.1. Normal Distribution

The normal distribution was defined by Gauss (1809), even though it was first mentioned by Abraham DeMoivre (1733, 1738). It is the most used distribution nowadays.

Its probability density function (PDF) equation and graph are obtained by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < +\infty \quad \textbf{(23)}$$



Graph 1 - Probability density function of Normal distribution
Source: Walpole et. al (2007)

Its CDF is defined as:

$$F(x) = \frac{1}{2}\left[1 + e^{\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)}\right], -\infty < x < +\infty \quad \textbf{(24)}$$

Where $-\infty < \mu < +\infty$ and $0 < \sigma^2 < +\infty$ are, respectively, the average and the variance parameters.

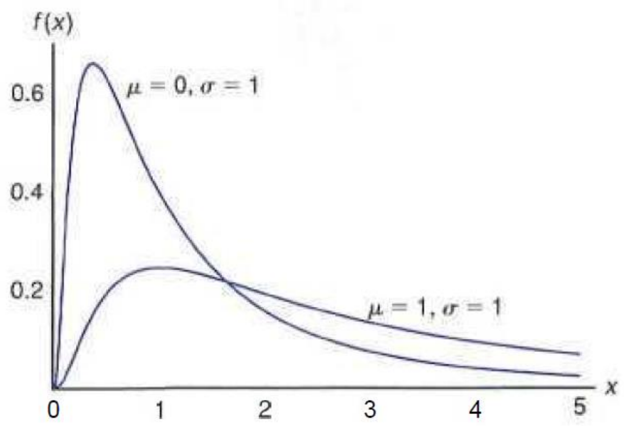
2.2.1.2. Lognormal Distribution

The lognormal distribution was introduced by Hazen (1930) as a way to apply normal distribution in cases with the need of reducing the skewness of a sample. In economics, this distribution is known as Cobb-Douglas distribution (Sahoo, 2013).
A random variable X is logarithm normally distributed, then $Y = ln(X)$ is normally distributed and X's PDF and CDF are, respectively, given by:

$$f(X) = \begin{cases} \frac{1}{X\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{\ln(X)-\mu}{\sigma}\right)^2\right] & , if\ 0 < x < \infty \\ 0 & , otherwise \end{cases} \quad \textbf{(25)}$$

$$F(X) = \frac{1}{2} + \frac{1}{\sqrt{\pi}}\int_0^X \exp - \left[\frac{\ln(X)-\mu}{\sqrt{2}\sigma}\right]^2 dX \quad \textbf{(26)}$$

Where $-\infty < \mu < +\infty$ and $0 < \sigma^2 < +\infty$ are, respectively, the average and the variance parameters.
The following graph shows how lognormal PDF changes when only the average changes, but the standard variation maintains its value:



Graph 2 - Probability density function of log-normal distribution for $\mu = 0, \sigma = 1$ and $\mu = 1, \sigma = 1$
Source: Walpole et. al (2007)

2.2.1.3. Cauchy Distribution

The Cauchy distribution, named after August Cauchy, was firstly mentioned by Poisson (1824), in a study in which Cauchy became associated due to his developments.

This distribution is not defined by average nor by variance parameters but it is, instead, defined by scale and location parameters.

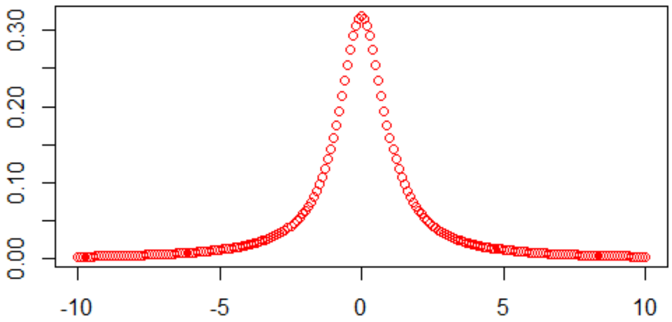
For a random variable X this distribution has the following PDF and CDF, respectively:

$$\boldsymbol{f(X, xo, \gamma) = \frac{1}{\pi}\left[\frac{\gamma}{(x-xo)^2+\gamma^2}\right]} \quad \textbf{(27)}$$

$$\boldsymbol{F(X, xo, \gamma) = \frac{1}{\pi}\arctan\left(\frac{x-xo}{\gamma}\right)+\frac{1}{2}} \quad \textbf{(28)}$$

Where $-\infty < x < +\infty$ and xo and γ symbolize the location and scale, respectively.

In the graph below is it possible to see the graph of Cauchy distribution's PDF when location is null, and scale is equal to 1:



Graph 3 - Cauchy distribution's PDF when location=0 and scale=1

2.2.1.4. Weibull Distribution

This distribution is named after the originators Waloddi Weibull (1939) and it can be defined by two or three parameters.

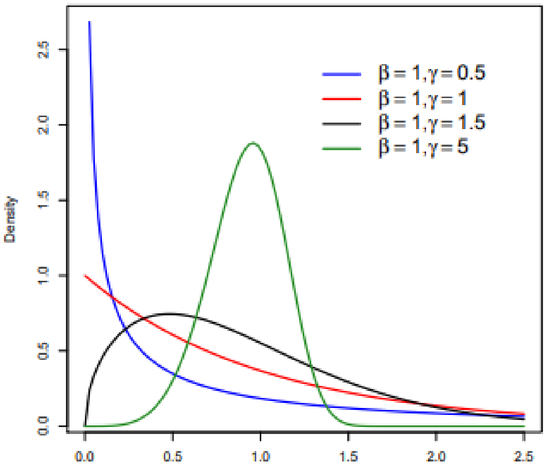
The two parameters of this distribution are scale and shape, symbolized by β *and* γ, respectively (Nielson, 2011).

Its PDF and CDF for a random and positive variable X are, respectively:

$$\boldsymbol{f(X) = \frac{\gamma}{\beta} * \left(\frac{X}{\beta}\right)^{\gamma-1} * e^{-\left(\frac{X}{\beta}\right)^{\gamma}}} \quad \textbf{(29)}$$

$$\boldsymbol{F(X) = 1 - e^{-\left(\frac{X}{\beta}\right)^{\gamma}}} \quad \textbf{(30)}$$

The PDF's graph for Weibull's distribution for various values of γ and $\beta = 1$ is showed in the following graph:



Graph 4 - Probability density function of weibull distribution for several shape values and $\beta = 1$
Source: Nielson (2011)

2.2.1.5. Gamma Distribution

The gamma distribution is a shaped and scaled distribution represented by α and θ, respectively. A random variable X has a gamma distribution if its values are bigger than zero and its PDF and CDF are, respectively:

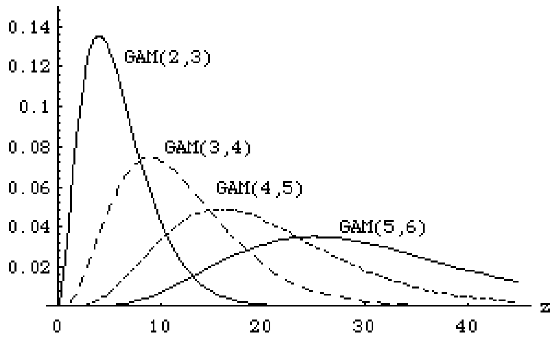
$$f(X,\alpha,\theta) = \frac{X^{\alpha-1} * e^{\frac{-X}{\theta}}}{\theta^{\alpha} * \Gamma(\alpha)}, for\ X,\alpha,\theta > 0 \qquad (31)$$

$$F(X,\alpha,\theta) = \int_0^y f(X,\alpha,\theta)dX = \frac{\gamma\left[\alpha,\frac{X}{\theta}\right]}{\Gamma(\alpha)} \qquad (32)$$

Where: $$\Gamma(\alpha) = \int_0^{\infty} X^{\alpha-1} * e^{-X}dX \qquad (33)$$

$$\gamma\left[\alpha,\frac{X}{\theta}\right] = \int_0^{X/\theta} t^{\alpha-1} * e^{-t}dt \qquad (34)$$

The graph below shows the gamma PDF for given values of α and θ.



Graph 5 - Probability density function of gamma distribution for several shape and scale values
Source: Sahoo (2013)

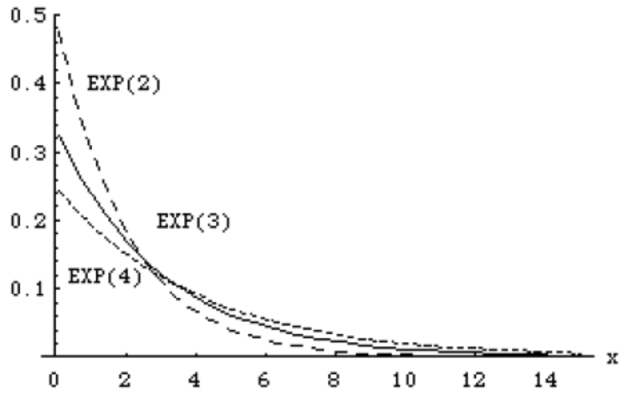
2.2.1.6. Exponential Distribution

Th exponential distribution is a particular case of gamma distribution, since it is assumed that $\alpha = 1$. The only parameter is called rate or inversed scale and symbolized by λ (Sahoo, 2013). A random variable x is exponentially distributed if its values are bigger than zero and its PDF and CDF are, respectively:

$$\boldsymbol{f(x,\lambda) = \lambda e^{-\lambda x}} \quad \textbf{(35)}$$

$$\boldsymbol{F(x,\lambda) = 1 - e^{-\lambda x}} \quad \textbf{(36)}$$

In the graph below, it is possible to see the exponential PDF for given values of λ.



Graph 6 - Probability density function of exponential distribution for several rates
Source: Sahoo (2013)

2.2.2. Fitting the data

According to Vose (2010), the principle behind fitting distributions is to obtain the type of probability distribution and value of respective parameters that create the best forecasts for the observed data, since there are many distributions and few that can be suitable to each variable. The first and easiest step should be the visual comparison, using the Probability-Probability Plot (PP-Plot), Quantile-Quantile Probability (QQ-Plot) and the density probability function with a histogram, where it is possible the see the existence of a data pattern.

The other tests are the information criteria and goodness-of fit tests. The information criteria tests do not give a true measure of the probability that the data comes from a given distribution. Instead, the result is the probability that random data, generated from a certain distribution, produces parametric values as low as the observed data. Vose (2010) uses the following information criterion:

1. Akaike Information Criterion (AIC)
2. Bayesian Information Criterion (BIC)

In the goodness-of-fit tests, represented in this thesis by Kolmogorov-Smirnov (K-S) Test, is given the probability of the data given the parameters estimated through the maximum likelihood estimators.

2.2.2.1. PP-Plot and QQ-Plot

PP-Plot is a plot in which is showed the cumulative probabilities plot of a given variable against the ones from a probability distribution.

QQ-Plot is a plot where the quantile probabilities of a given variable and a probability distribution are showed and plotted against each other.

In both graphs, if the sampled data, represented by points on the plot, matches the distribution the points will coincide with the diagonal of the theoretical distribution (Jin-yang et. al, 2016).

2.2.2.2. Akaike and Bayesian Information Criterion

The AIC (Akaike 1974) and BIC (Schwarz 1978) are indicators that estimate the amount of information lost in a given estimated model and are, nowadays, the most used criterion to compare the fit of different models.

These statistics are based on the calculation of the log-likelihood of the distribution that is being fitted with the produced observations and in the number of parameters of the model. Its conclusions about the fit are obtained by its statistical value, since the lower the value the better the fit.

If the final value is too big then the distributions with many parameters are being prejudiced and should be re-modelled to prevent over-fitting (Camelo, 2010; Vose 2010).

The equations that define its values are the following:

$$\boldsymbol{AIC = \frac{n-2k+2}{n-k+2} - 2\ln(L_{Max})} \quad \textbf{(37)}$$

$$\boldsymbol{BIC = k\ln(n) - 2\ln(L_{Max})} \quad \textbf{(38)}$$

Where:

a) k is number of parameters to be estimated;
b) n is the number of observations;
c) L_{Max} is the maximized value of log-likelihood for the estimated model.

2.2.2.3. Kolmogorov-Smirnov Test

The K-S Test is a statistical test that measures the maximum vertical distance between the CDF of a given data with any CDF of a fitted continuous distribution. Its statistical value is obtained by the data value with largest difference, excluding the lack of fit throughout the rest of the distribution (Vose, 2010).

The K-S statistic is defined by:

$$\boldsymbol{D_n = sup\left[F_x(n) - \hat{F}(x)\right]} \tag{39}$$

Where:

- n – total number of data;
- $\hat{F}(x)$ – the hypothesized distribution;
- N_x – number of N_i's lower than x;
- $F_x(n) = \frac{N_x}{n}$.

The K-S statistical interpretation is the following:

H0: The distribution is a good fit

H1: The distribution is not a good fit

3. Banco Comercial Português (Millennium BCP)

3.1. Company's history

Banco Comercial Português (a.k.a. as Millennium BCP or just BCP) is a Portuguese financial institution created in 17th of June 1985, following the deregulation of the Portuguese banking system. Millennium BCP has passed through several organic and inorganic growth phases. During its organic growth phase, throughout its first years of development, multiple strategic acquisitions were made to create and solidify a position in the Portuguese market aiming to attract new clients with the increase in the offer of services and financial products. In 1989, the group launched an innovative bank, directed to young population, named Nova Rede offering several financial products and services to its clients within an extensive geographical coverage. In 1995, BCP acquired the whole share capital of Banco Português do Atlântico, which was, at the time, the largest private Portuguese bank. Five years later, the two banks, Atlântico and BCP, merged. In the same year, BCP acquired insurance company Império and banks Mello and Pinto & Sotto Mayor. In 2004, BCP sold Império and other insurance companies to Caixa Geral de Depósitos group and entered in a joint venture with Ageas, named Millenniumbcp Ageas, which ended in 2014 due to priority restructuring of its strategic plans.

After consolidating its position in Portugal, the group started to develop its retail activities to new international locations, mainly in countries with high growth prospects in external markets and a close relationship with Portugal and with large Portuguese communities (such as Angola, Mozambique, Canada, United States, France and Macao), as well as in countries like Greece, Poland and Greece where the Portuguese format could be exported and adapted. These banking segments acted autonomously under various brand names, until 2003 when the group started to rename the segments acting in Portugal to Millennium BCP, completing the international rebranding in 2006.

Throughout 2012 and 2013, the bank decided for a management restructure and introduced the plan to the Portuguese government, as required by Portuguese and European laws. The plan consisted on the improvement of the profitability of the group by focusing on its core activities and reducing costs, without Portuguese government financial support.

Nowadays, just like the image below shows, the goal is to continue to be the leading private bank in Portugal, by transforming its business to attract new clients and adapting to the needs of the existing ones.

As of 30th May 2018, Nuno Amaro, former CEO of the group, was defined as Chairman of the board of directors with Miguel Maya replacing him.

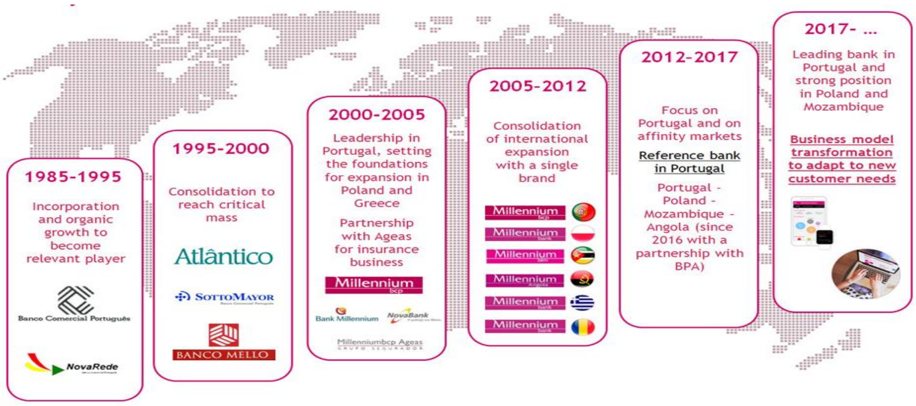


Figure 2 - Diverse business lines throughout BCP's history.
Extracted from Millennium BCP's website

3.2. Group's Structure

3.2.1. Portugal

In Portugal, the group has its focus on the retail and companies' markets, by offering its financial products and services through several subsidiaries and distribution networks.

Currently, the BCP group operates under three different brands:

a) Millennium BCP – The largest private banking institution in Portugal, specialist in various financial products, market segments and services. In 2018, this bank had 7,095 employees distributed by 546 branches, having suffered a decrease of 1.3% and 5.5%, respectively, compared with 2017;
b) Activobank – Bank directed to young and technologic customers who prefer simple banking services and innovative products and services;
c) Interfundos – Gestão de Fundos de Investimento imobiliário, S.A. – This institution is a legal entity who manages real estate funds and administrate them, by representing the clients, if asked. According to Associação Portuguesa de Fundos de Investimento, Pensões e Patrimónios, the entity was managing thirty-six real estate funds, in 2018.

For the first two brands, their activities focus "on Retail Banking, which is segmented in order to best serve Customer interests, both through a value proposition based on innovation and speed targeted at Mass-Market Customers, and through the innovation and personalised management of service for Prestige, Business Customers, Companies, Corporates and Large Corporates.". Besides that, there are also the offering of remote banking communication channels, by telephone or online, to help and inform about new innovative products and services.

3.2.2. International

As mentioned on section 3.1., throughout the recent years the bank has redefined its priorities on what considers to be core operations. As part of the restructuring plan, the bank has terminated some of its international operations (in France, Luxembourg, United States, Canada, Turkey and Romania), despite of retaining commercial agreements with Portuguese communities in some specific markets.

As of December 2013, the banking operations, in Greece, under the name "Millennium Greece" were terminated due to a sell and merger to Piraeus Bank. In 2016, a similar financial operation occurred when the bank operating in Angola under the name "Banco Millennium Angola" merged with Banco Privado Atlântico resulting on Banco Millennium Atlântico, the second-largest private Angolan bank. Since then, BCP only holds 20% of its stake.

Nowadays, the bank possesses a private banking platform in Switzerland (Millennium Banque Privée BCP), which provides services to clients in other international locations. The group is also present in Macao, an on-shore branch since 2010, due to its location and historical Portuguese roots.

In addition to that, the group has subsidiaries in Portugal, Poland, as Bank Millennium, and in Mozambique, as Millennium Bim. In those three locations, each institution managed to become a reference at international level by focusing on the retail distribution of financial services and products. Considering that, the international operations of the bank represented, in 2018, 555 of the 1,101 branches (50.5%) and 8,834 of the 15,039 (58.8%) employees of Millennium group.

3.3. Shareholder Structure

BCP shares are mainly listed in Euronext Lisbon and WIG30, a polish stock index market composed by the 30 biggest companies transitioning in Warsaw Stock Exchange (WSE). At the end of 2018's fiscal year, the average number of shares outstanding were 15,113,989,952 , resulting in earnings per share of €0.02.

The bank's shareholder structure is much divided and only 4 institutions hold 20% or more of the share capital. As shown below, as of December 2018, the Fosun Group was the biggest shareholder with 27.25% of share capital.

Shareholder Structure	**Number of shareholders**	**% of share capital**
Individual Shareholders		
Group Employees	2,781	0.24%
Others	152,17	22.67%
Companies		
Institutional	323	22.79%
Qualified Shareholders	4	52.22%
Other companies	4,392	2.07%
Total	159,67	100%

Table 1 – Total shareholder structure as of 31th December, 2018.
Source: Bank's Annual Report

Shareholder Structure	**Number of shareholders**	**% of share capital**	**% of voting rights**
Chiado (Luxembourg) S.a.r.l., an affiliate of Fosun, whose parent company is	4,118,502,618	27.25%	27.25%
Sonangol – Sociedade Nacional de combustíveis de Angola, EP, directly	2,946,353,914	19.49%	19.49%
BlackRock	512,328,512	3.39%	3.39%
EDP Group Pensions Fund	315,336,362	2.09%	2.09%
Total	7,892,521,406	52.22%	52.22%

Table 2 – Distribution for company shareholders as of 31th December, 2018.
Source: Bank's Annual Report

Portugal continues to be main nationality of BCP's shareholders. However, in December 2018 there was a decrease on the total number of shareholders to 11.2 thousand, when compared to December 2017.

In May 2019, it was approved for the first time in nine years the payment of €0.002 per share for all shareholders, using 30 million euros of group's net income (and thus obtaining a pay-out ratio of 10%) and €227,979.90 of Portuguese subsidiary's earnings.

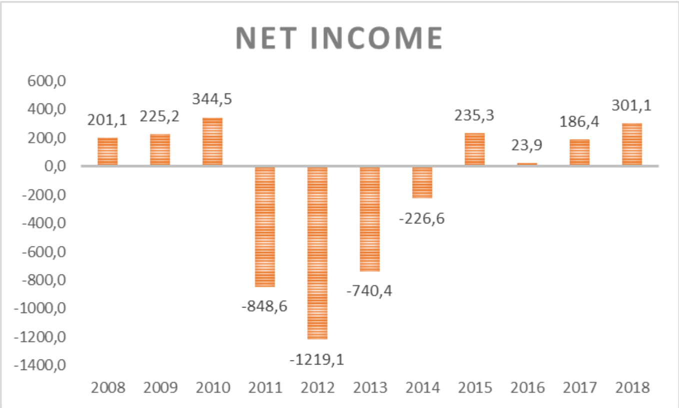
3.4. Financial performance

3.4.1. Profitability

The magnitude of the 2008 financial crisis, that started to affect in an irreversible way the Portuguese economy in 2010, had a significant impact on financial institutions' results worldwide but particularly in the financial institutions with sensitive and weak economies like the Portuguese and Greek ones.

Despite of 2008 financial crisis, in 2010, the group showed positive signs of recovery due to global "reconstruction" of financial markets by growing its net income in 119 million euros. However, on 2011 the results decreased more than 1 Billion euros and in 2012 more than 1.5

Billion, when compared to 2010, thanks to the severe financial and economic crisis in Portugal and Greece.



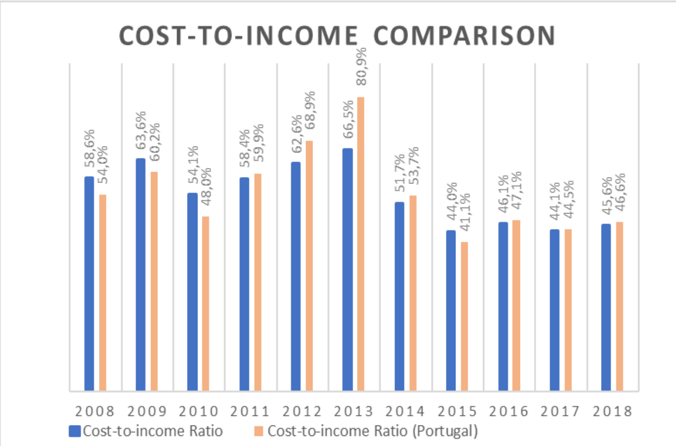
Graph 7 – Group's net income since 2008-2018, in millions of euros.
Source: Group's Annual Reports

This negative result in 2012, the lowest in the years in analysis, was related to the negative net income generated by the greek branch, creating a loss of €693.6 million, and with the recognition of impairment losses and provisions values in a total of €1,236 million in the portuguese branch.

After 2012, the evolution of the net income has been positive, reaching its eight year maximum in 2018 (€301.1 million). This result was obtained mainly due to huge reduction in the group's credit and other impairments and provisions and due to a sharp increase in charged commissions, other net operating income and net interest income categories. However, a higher increase on the net income was stopped by low values of net trading income and high costs with the staff and tax derivated from high net income.

3.4.2. Efficiency

Concerning operating costs, composed by staff costs, other administrative cost, amortization and depreciation, the group registered a total of €1,027.2 million, a raise of €73 million when compared to 2017. This raise was, mainly, due to evolution in staff costs in Portugal (€53.7 million), caused by the impact on the reposition of wages and due to general increases in international activities, particularly in Poland subsidiary that registered an increase of €11.9 million, and Mozambique's, that registered an increase of €6.4 million.



Graph 8 – Group and Portugal's cost-to-income since 2008-2018.
Source: Group's Annual Reports

Looking at graph 8, it is possible to conclude that the group's cost-to-income ratio – ratio that evaluates the cost of a bussiness compared to its operating income – increased 1.5% between 2017 and 2018. Portugal registered a cost-to-income of 46.6% in 2018, an increase of 2.1% when compared to 2017.

In addition, it is possible to conclude that in the past three years (2016, 2017 and 2018), Portugal has been less efficient than the international operations, which might suggest that Portugal has not yet discovered the best way to decrease its operational costs, since it is something that has happened before (in 2011, 2012, 2013 and 2014).

3.4.3. Ratings

When supported by national goverments, mainly on capital, liquidity and insurance of assets, banks are very exposed to sovereign risk, besides the risk associated with international capital markets.

In 2018, there was an improvement of portuguese macroeconomic indicators, which reflected the positive devolpments in stabilize its economic and financial behaviours and reduce its external vulnerability. This development achieved in 2018 was recognised by the rating agencies when Fitch, Moody's and Standard & Poor's (S&P) decided to give or maintain Portugal's rating classification on the lowest investment rate.

Even though during the same year, the portuguese banks continued to operate within European Central Bank (ECB)'s restricted rules, they managed to improve their asset quality, through the reduction of Non-Performing Exposures, and strengthened their profitability and capital levels. However, the amount of problematic assets possessed by the banks continues high, which concers the main rating agencies.

BCP was rewarded by three rating agencies due to its successeful strategic plan, which allowed an upgrate of its long-term deposits and senior unsecured debt's rating by one notch.

Moody's		
	Short Term	Long Term
Counterparty Risk Assessement	Baa3	P-3
Counterparty Risk	Ba1	Non Prime
Deposits	Ba3	Non Prime
Subordinated Debt	B2	
Covered Bonds	Aa3	

Table 3 – Moody's rating classification for BCP as of 31th December, 2018.
Source: Bank's Annual Report

Standard & Pool's		
	Short Term	Long Term
Counterparty Credit Rating	Baa3	P-3
Subordinated Debt	Ba1	Non Prime
Issuer Credit Rating	-	B

Table 4 – Standard & Poor's' rating classification for BCP as of 31th December, 2018.
Source: Bank's Annual Report

Fitch		
	Short Term	Long Term
Deposits	BB	B
Subordinated Debt (Tier 2)	-	BB-
Covered Bonds	-	BBB+

Table 5 – Fitch's rating classification for BCP as of 31th December, 2018.
Source: Bank's Annual Report

3.5. Future Strategies

The group has successfully accomplished the recapitalization plan stablished in 2012, by reinforcing its financial and capital positions in its core business despite the financial, social and economic Portuguese crisis.

Millennium's ambition is to create a cycle of profitable growth, requiring innovative capabilities and strategies to secure its position in the Portuguese market. Based on that, it was created a new strategic vision: "Partnering with our customers to create and share value.", which reflects "the desire to generate more value for Millennium's customers (through a convenient, personal, and mobile experience enabling human-centric solutions), for the Bank (by expanding the customer base and strengthening relationships), for shareholders (by achieving strong performance through a resilient business model), and for employees (by promoting a teamwork-driven model and strengthening satisfaction, compensation, and capabilities)".

To help its achievement the bank has defined five priorities for the future:

a) Mobile-centric digitalization, as an effort to increase and improve the relationship between the customers and technology, by redesigning the digital models and transform operations using artificial intelligence;
b) Talent mobilization, by recruiting new talent and allocating them to their respective area. In addition to that, there will be a merit-growth model to reward the work of the employees and promote new capacities;
c) Business model sustainability, by improving the group's credit portfolio quality and risk and compliance analysis to create a sustainable growth with a low risk profile.
d) Growth and leadership in Portugal by holding the bank's unique position in the Portuguese market and achieve the full potential of ActivoBank to potentially reach international markets.
e) Growth and international footprint with the goal of capitalizing the opportunities offered by international markets where the bank is present. As example, there are the raise of customer business bases and the exploration of new markets.

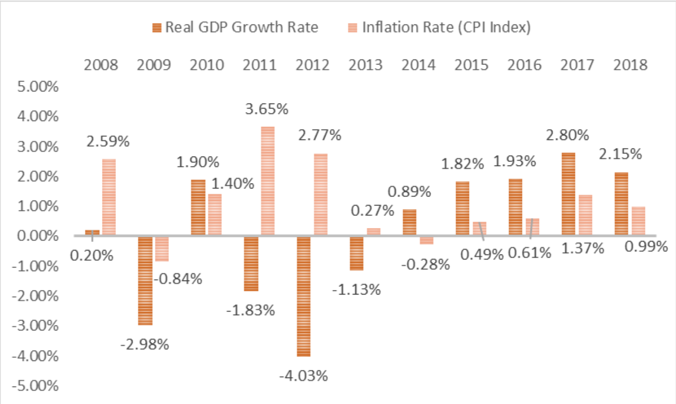
If those priorities are successfully achieved they will help the group to accomplish its goals for 2021: franchise growth by having more than 6 million active customers; future readiness by growing the percentage of digital and mobile customers from 45% to 60% and from 26% to 45%, respectively; create and maintain a sustainable business model and create attractive returns for shareholders (such as, increasing the pay-out ratio to 40%).

4. Countries' economic and financial analysis

Despite having financial operations and/or subsidiaries scattered around the world, in this section we will focus our analysis into the group's main markets, with lots of subsidiaries and a high possibility of growth, as of 2018: Portugal, Poland and Mozambique.

4.1. Portugal

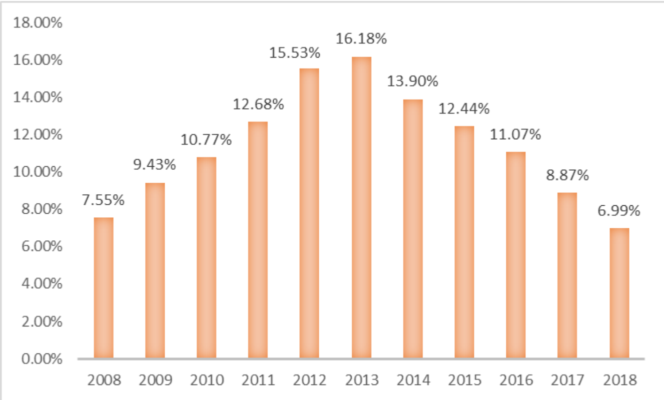
The Portuguese economy has been through some rough periods, with a severe recession that lasted three years, which led to financial and economic adjustment policies by the Portuguese government. Due to the relation between real Gross Domestic Product (GDP) growth rate and inflation rate we can assume based on graph 9 that the latter influences negatively the real GDP growth rate when its value is higher; whereas starting in 2014 the inflation rate contributes positively for the real GDP growth rate. The evolution between the inflation rate, obtained through the consumer price index (CPI), extracted from OECD data source, and the real GDP growth rate, whose data was extracted from World Bank, is displayed in the graph below:



Graph 9 – Portuguese Real GDP Growth and Inflation Rate (CPI Index), from 2008-2018
Source: World Bank and OECD Data

In 2018, there was a slowdown in the real GDP growth rate, the first since 2012. According to Financial Stability Report of the Bank of Portugal (June, 2019: 35), this deceleration was related with a decrease in exports and investment not compensated by the increase in private and public consumption.

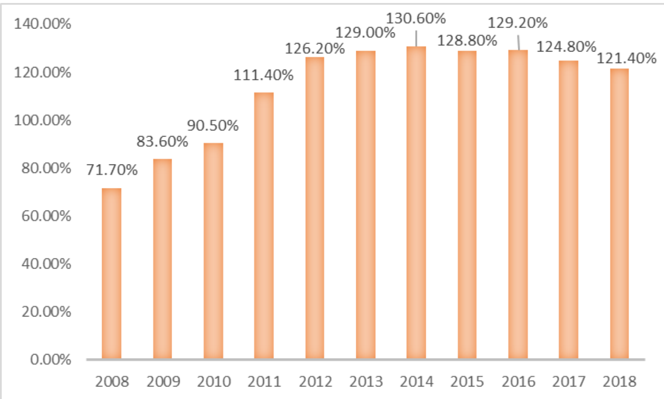
Back to 2011, the strictness of the adjustment policies affected not only the real GDP growth rate but also the unemployment rate, due to restrictions in banking credit and decrease in consumer purchasing power. As proven by the data extracted from OECD database and exhibited on graph 10, the unemployment rate reached its maximum in 2013, due to the measures taken. In 2018, the unemployment rate reached a new minimum (6.99%), due to a decrease in long-term and youth unemployment and improved labour market conditions, according to Financial Stability Report of the Bank of Portugal (June, 2019: 36).



Graph 10 - Portuguese Unemployment rate, from 2008-2018
Source: OECD Data

As expected, during the years of the economic and financial crisis there was a general government gross debt – formed by, according to IMF, by "all liabilities that required payment(s) of interest and/or principal by the debtor to the creditor at a date or dates in the future" – increase, probably due to the government's inability to pays its dues and the increase of the GDP, as proved on graph 11.

Even though the worst years of the crisis are gone, the most recent years show a low recovery to the values registered before the crisis, which may indicate an increase of the debt to stimulate the economy growth and in the aggregate demand.

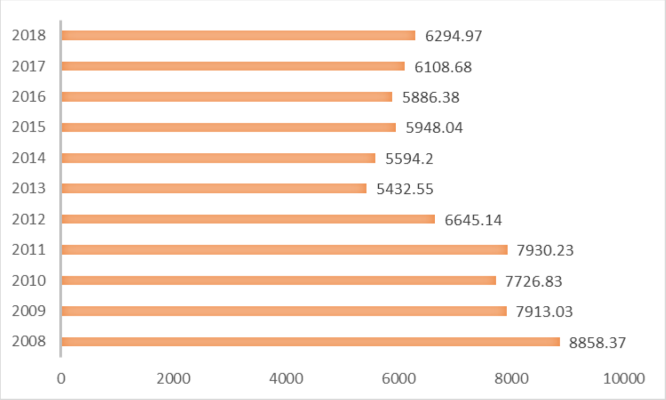


Graph 11 - Portuguese General government gross debt, as percentage of GDP, from 2008-2018
Source: IMF

The net income interest – difference between the revenues from banking assets, such as mortgages and commercial loans, and the expenses with banking liabilities, such as deposits – has an irregular behaviour, unlike the previous variables.

Despite the growth in the banking sector in the last few years, the Portuguese banking sector has yet to achieve the values registered before the crisis. However, to try to achieve higher values the banking sector has been decreasing the interest rates of the loans but not as much as the deposits', to increase the spread and create more net income interest.

For 2018, the highest value since 2012, as seen on graph 12, Bank of Portugal justifies its value on June's to Financial Stability Report that it did not grow as expected due to "low volumes of new credit and interest margin in new business".



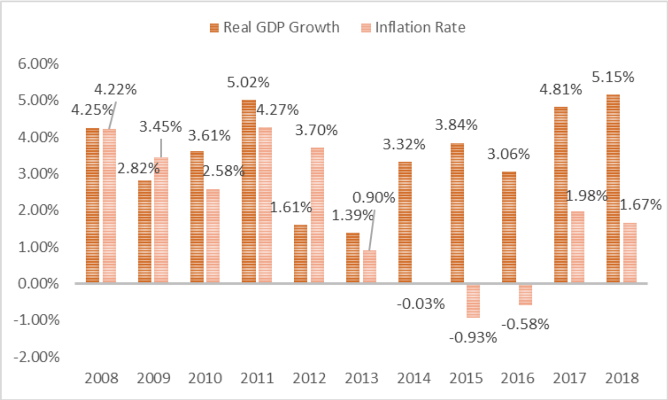
Graph 12 - Portuguese Net Interest Income, in millions of euros, from 2008-2018
Source: Banco de Portugal

4.2. Poland

Considering the economic and financial European crisis and its high exposure to more developed European countries, the polish economy has been performing well with no economic recession from, at least, 2008. However, it is worth noting that in most of years the polish nominal GDP growth rate was higher than the real growth rate, due to the decrease effect that the inflation rate made on the latter.

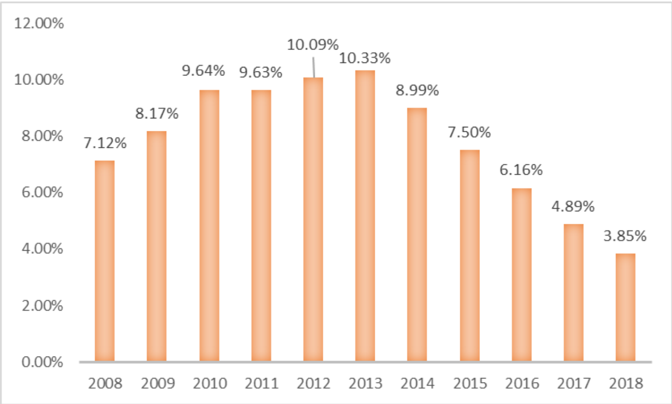
In 2018, the real GDP growth rate registered its highest value (5.15%), for which contributed mainly the increase of consumer demand, a decrease in the unemployment rate, an increase of wages and in the consumer's confidence in the system, according to the Financial Stability Report of the Bank of Poland (June, 2019: 13).

The relation between the inflation rate, obtained through the CPI data and extracted from OECD data source, and the real GDP growth rate, whose data was extracted from World Bank, is displayed in the graph below.



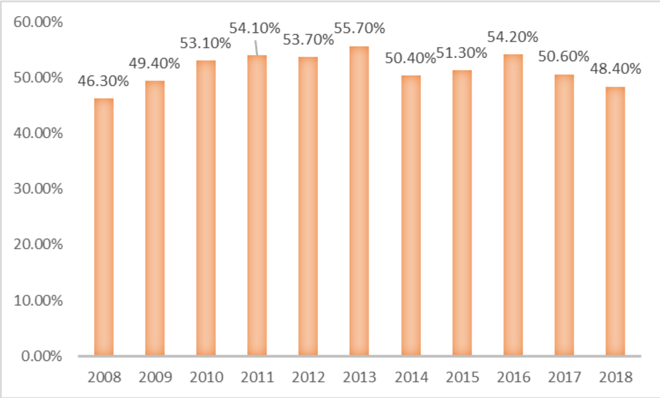
Graph 13 – Polish Real GDP Growth and Inflation Rate (CPI Index), from 2008-2018
Source: World Bank and OECD Data

Concerning unemployment rate, with data extracted from OECD Data source and exposed in graph 14, there was an irregular behaviour until 2013, the year where the unemployment rate started to decrease, until reaching its minimum (3.85%), caused by higher wages and a general economic growth, as said on previous paragraphs.



Graph 14 - Polish Unemployment rate, from 2008-2018
Source: OECD Data

As predicted by the previous graphs, the great polish economic and financial situation allows the country to have, in average, a general government gross debt half the amount of the yearly GDP. As exposed in graph 15, if Poland paid its debt every year it would still have half of its GDP to maintain and improving public services and invest in its economy.

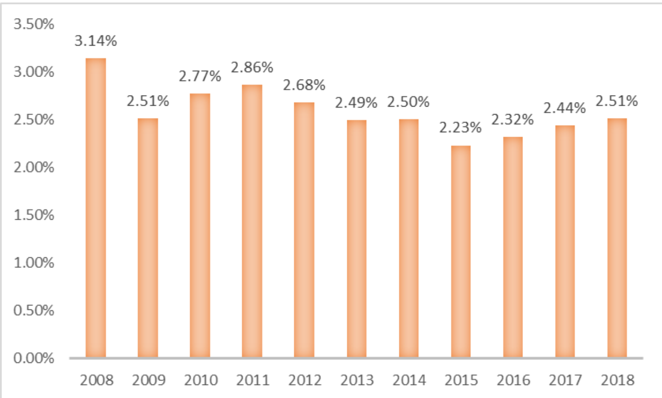


Graph 15 - Polish General government gross debt, as percentage of GDP, from 2008-2018
Source: IMF

Passing now to the analysis of the polish banking sector, the net interest margin – defined as "the ratio of net interest income in a given period to average assets in this period", in accordance with Financial Stability Report of the Bank of Poland (June, 2019: 133) – has an irregular path, formed by consecutive increases, interrupted by a decrease.

2018 was the fourth consecutive year with a rising rate, caused by changes in credit liabilities and portfolios. According to the same report, the main charges were: repayment in house loans in foreign currency and creation of new loans in the domestic currency with higher margins – which generate larger interest incomes – and a change of deposit terms from fixed term, with lower rates, to current deposits.

The net interest margin for polish banking sector is represented below:



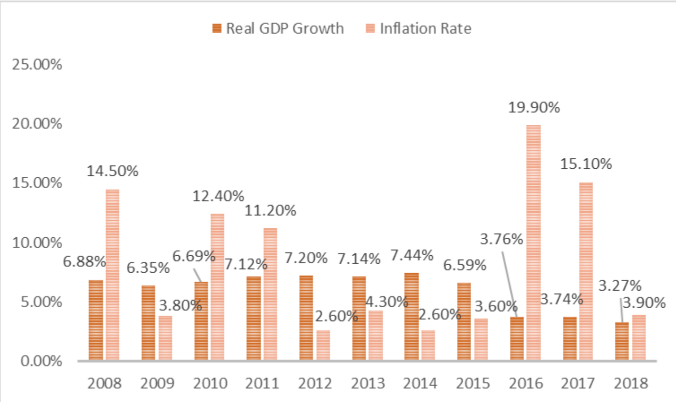
Graph 16 – Polish Net Income Margin, from 2008-2018
Source: National Bank of Poland

4.3. Mozambique

As a developing economy, Mozambique had a sustained and homogenous growth, until 2016, where its GDP growth grew almost half the percentage of the previous year, due to the "scandal of the hidden debt" – a crisis created after three public companies failed to repay a debt of millions of euros. As seen on graph 17, the country has still not recovered from the fall, since in 2018 it registered an increase of 3.27% - half of the growth percentage registered in 2015 -, due to a lower dynamism of the agriculture, extractive, transportation and communication sectors, according to the 2018 annual report of National Bank of Mozambique.

Following the real GDP growth rate, the inflation rate has also been suffering a decrease to 3.9% since reaching its maximum value in 2016. According to 2018 annual report of National Bank of Mozambique, the main reasons for 2018 inflation rate are the suppressed aggregate demand and a stable behaviour of the Mozambican currency when compared with other business countries, contributing to stable costs of imported goods and services.

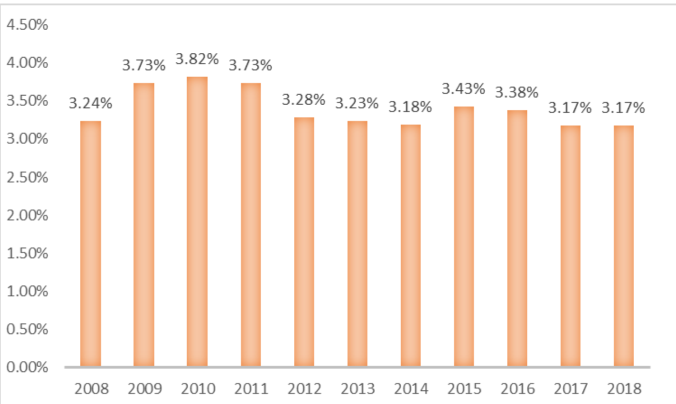
The relation between the inflation rate and the GDP real growth rate is display on graph 17. Both data were extracted from World Bank data source.



Graph 17 – Mozambican Real GDP Growth and Inflation Rate (CPI Index), from 2008-2018
Source: World Bank

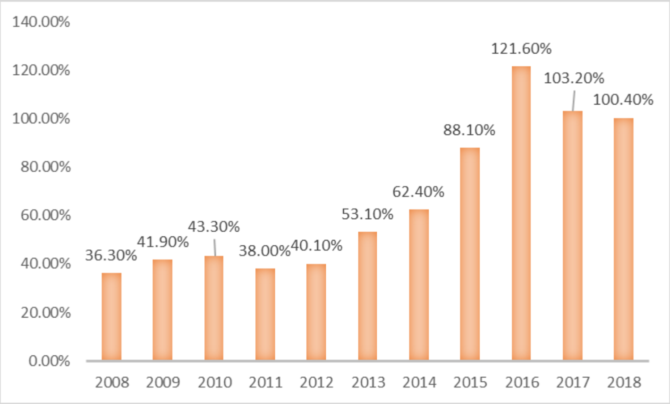
Concerning the unemployment rate, it did not suffer large changes throughout the years. In 2018, the rate was equal to the previous year (3.17%), which can show a stabilization despite the recovery period that the economy is facing.

The unemployment rate evolution is displayed in the graph below, whose data was extracted from World Bank data source.



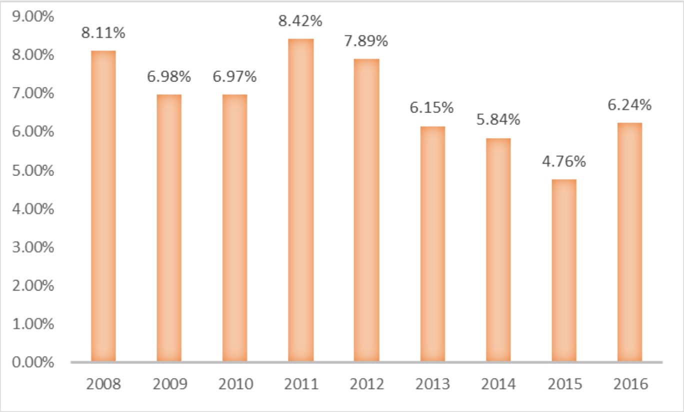
Graph 18 – Mozambican Unemployment rate, from 2008-2018
Source: World Bank

As seen on the graph below, formed by data extracted from IMF, the Mozambican general government gross debt had acceptable values before 2015. On the following year, the ratio raised, for the first time, to more than 100%, due to the "scandal of the hidden debt", as mentioned on previous paragraphs. Since that and according to 2018 annual report of National Bank of Mozambique, the government started a consolidation and fiscal restructuring process, with positive results.



Graph 19 - Mozambican General government gross debt, as percentage of GDP, from 2008-2018
Source: IMF

The net interest margin, extracted from FRED Statistic Data, was only available until the first day of 2016. Even though it is not updated, in graph 20 it is possible to see that, starting in 2011, the banking sector's net income did become less profitable. That behaviour finished on 2016, leaving the impression that during 2015 the banking sector changed its net interest spread to create more profit or that the net average assets decreased their value.



Graph 20 – Mozambican Net Income Margin, from 2008-2016, as of 1st January
Source: Fred Statistic Data

5. Data and Methodology

After reviewing the literature in which this report was based, we now explain and show the data used besides the methodology applied.

5.1. Equity Equation

The goal in this project is to calculate the 2018's equity value for group BCP, using a horizon period of four years (2019-2022) with 2023 as terminal value, from which is assumed a constant and infinite growth, as showed in the equation below:

$$\boldsymbol{Equity_{2018}} = \frac{\boldsymbol{ECF_{2019}}}{(\boldsymbol{1+Re})^{\boldsymbol{1}}} + \frac{\boldsymbol{ECF_{2020}}}{(\boldsymbol{1+Re})^{\boldsymbol{2}}} + \frac{\boldsymbol{ECF_{2021}}}{(\boldsymbol{1+Re})^{\boldsymbol{3}}} + \frac{\boldsymbol{ECF_{2022}}}{(\boldsymbol{1+Re})^{\boldsymbol{4}}} + \frac{\boldsymbol{ECF_{2023}}}{(\boldsymbol{1+Re})^{\boldsymbol{4}}(\boldsymbol{re-g})} \qquad \textbf{(40)}$$

However, as discussed on section 1.2., the valuation of financial institutions firms is not measured in the same way as other firms, which makes necessary change the Equity Cash Flow (ECF) calculation. Based on Koller et.al (2010), it was obtained an ECF equation, showed below, considering these restricted and special conditions.

$$\boldsymbol{Equity\ Cash\ Flow = Net\ income - \Delta BVE + OCI} \qquad \textbf{(41)}$$

With: ΔBVE = Delta Book Value of Equity

OCI = Other Comprehensive Income

The forecast period for ECF variables, cost of equity and growth rate will include financial data, extracted from the group's annual reports, from 2008-2018. That data will be fitted through several probability distributions, previously explained in section 2.1. - Normal, lognormal, Cauchy, Weibull, Gamma and Exponential –, using software R Studio, which will be explained with more detail in section 5.5. After the fitting process, there will be a simulation with 5,000 iterations, considering the best probability distribution and parameters obtained with it. The ECF for the year in question will be the average of the 5000 iterations, using all variables in every iteration; and the net income will be calculated using the average yearly value of ECF, growth rate and cost of equity.

Considering a variable t symbolizing all years in horizon period (2019 – 2022), for the probability fitting of year $t + 1$ in every variable used, the value obtained in year t, symbolized by the average value, will be added to the historical data list to add consistency to the fitting process and obtain the most feasible and adapted results to the year in question.

5.2. Free Cash Flow to the Equity Variables

5.2.1. Net Income

Previously, in section 1.1.2 the meaning of net income was explained as the income available to all shareholders who decide if it should be distributed as dividends and/or for reinvestment. In this particular case, due to the high quantity of variables used for the calculation of the net income, there are no fitting process nor simulation for the net income, but instead for the group of variables needed for its calculation. Every variable was tested and fitted probabilistically and simulated, using the historical data from 2008 – 2018.

Thus, there will be 5000 net income values for each year $t = 2019, \dots, 2022$ (and 2023), obtained in the end of each iteration and considering the probability distribution and parameter levels of each variable.

Hence, for every iteration i, with $2 \ll i \ll 5000$:

$$\boldsymbol{NI_i = (1 + NORGR_i) * NOR_{i-1} - OC_i + LI_i + OIPAG_i + IT_i + IAFDO_i + OI_i - NCOMI_i} \quad \textbf{(42)}$$

With:

NI = Net income;

NORGR = Net operating revenues growth rate;

NOR = Net Operating Revenues;

OC = Operating Costs;

LI = Loans impairment;

OIPAG = Other impairments, provisions and goodwill;

IT = Income Tax;

IAFDO = Income arising from discontinued operations;

OI = Other Items;

NCOMI = Non-controlling or minority interests.

In the following sub-sections, more details regarding the meaning and data used for each variable will be given. (For more information, please see exhibit 1.)

Important to notice that the variable Net Operating Revenues will not be directly simulated in any iteration, since its values will be influenced by the simulated values of Net Operating Revenues Growth Rate in order to mirror the uncertainty of the financial world, as defended in Fernandez (2001), Nurminen (2016) and Damodaran (2012). Knowing that, the first iteration will be different from the following ones with NOR_t being the Net Operating Revenue obtained in the year t's annual report.

$$NI_1 = NORGR_1 * NOR_t - OC_1 + LI_1 + OIPAG_1 - IT_1 + IAFDO_1 + OI_1 - NCOMI_1 \quad (43)$$

5.2.1.1. Net Operating Revenues Value and Growth Rate

Net operating revenues are revenues generated from a company's primary business.

The net operating revenues data, adapted from BCP's annual report, are stated in table 6[2].

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Net interest income	1721.0	1334.155	1516.9	1579.274	997.96	848.089	1116.151	1301.575	1230.126	1391.275	1423.631
Dividends from equity instruments	36.816	3.336	35.906	1.379	3.84	3.68	5.888	11.941	7.714	1.754	0.636
Net fees and commission income	740.417	731.731	811.581	789.372	655.087	662.974	680.885	692.862	643.834	666.697	684.019
Net gains or losses arising from trading and hedging activities	280.203	249.827	367.28	204.379	391.874	80.385	154.247	173.698	101.827	45.346	29.113
Net gains or losses arising from available for sale financial assets	-262.104	-24.457	72.087	3.253	44.871	184.065	302.407	421.746	138.54	103.03	0.0
Net gains or losses arising from financial assets held to maturity	0.0	0.0	0.0	0.0	-0.022	-0.278	-14.492	0.0	0.0	0.0	0.0
Net gains or losses from derecognition of financial assets at fair value through other comprehensive income	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	49.435
Net gains from insurance activity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.966	4.212	8.477
Other operating income	57.58	41.137	17.476	-22.793	-43.689	-55.627	-53.3	-110.519	-104.547	-110.606	-135.878
Other net income from non banking activity	17.39	16.233	16.55	26.974	20.093	20.502	19.278	18.856	0.0	0.0	0.0
Net Operating Revenues	**2591.302**	**2351.962**	**2837.78**	**2581.838**	**2070.014**	**1743.79**	**2211.064**	**2510.159**	**2022.46**	**2101.708**	**2059.433**
Net Operating Revenues Growth Rate	**0.052**	**-0.092**	**0.207**	**-0.09**	**-0.198**	**-0.158**	**0.268**	**0.135**	**-0.194**	**0.039**	**-0.02**

Table 6 – Net operating revenues, in million of euros, and net operating revenues growth rate, in euros, since 2008 and until 2018.
Source: Group's annual reports

As seen in table 6, the group has yet to achieve the net operating revenues values registered before and during the worst years of the Portuguese economic and financial crisis, despite of the inconsistent behaviour in the yearly growth rates since the beginning of the sample data.

5.2.1.2. Operating Costs

Operating costs, generally composed only by cost of labour and cost of materials used to fabricate products sold in the same period, are expenses who offer benefits in the same period within the company (Damodaran, 2012).

The operating costs data, adapted from BCP's annual report, are stated in table 7.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Staff costs	915.307	865.337	831.168	953.649	751.466	767.463	635.616	616.07	356.602	526.577	592.792
Other administrative costs	642.641	570.177	601.845	584.459	501.725	459.653	448.451	423.833	373.57	374.022	376.676
Amortization and Depreciation	112.843	104.736	110.231	96.11	68.05	68.123	65.543	66.623	49.824	53.582	57.745
Operating Costs	1670.791	1540.25	1543.244	1634.218	1321.241	1295.239	1149.61	1106.526	779.996	954.181	1027.213

Table 7 – Operating costs, in millions of euros, since 2008 and until 2018.
Source: Group's annual reports

[2] Even though, 2007's results are not used in this report, the growth rate for 2008 was obtained using 2007 and 2008's net operating revenues

Observing table 7, it is possible to realise the attempts in decreasing the operating costs, mainly during and after the financial crisis, without having a very long-lasting success since the operating costs restarted to increase after 2016, the year with the lowest value, due to staff costs.

After comparing the net operating revenues with the operating costs, it is possible to conclude that the profit before provisions and impairments was always positive. That behaviour is especially observed since 2015 because the operating costs are less than half the net operating revenues registered in the same year, allowing a larger margin for impairment and provisions expenses and a higher probability of a positive net income.

Regarding the fitting and simulation process, some conditions must be created due to the characteristics of the variable and its importance on influencing negatively the net income and, potentially, the equity. The simulated values will vary within an interval with upper (maximum) and down (minimum) defined limits. Until 2022, the maximum simulated value will be 1,500 – assuming that, the group will not have expenses as the ones registered before the crisis – decreasing to 1,200 in 2023, the perpetuity year; whereas, the minimum will be always 200 million, since it is assumed that the group has continuous fixed costs, in order to continue its operations. Considering those restrictions, the Normal and Cauchy distributions will be considered in this process, despite its definitions allow negative values. Also, in spite of being presented in millions of euros, the parameters and results of this category will be obtained based on operated costs divided by ten since they were too high to obtain a feasible conclusion. In the end, all simulated values will be multiplied by 10 to return to the original valuation, millions of euros.

5.2.1.3. Loans Impairment and Other Impairments, Provisions and Goodwill

According to 2018's annual report, the only that explains the meaning of most accounting categories, loans impairment is defined as "impairment of financial assets at amortised cost for loans and advances of credit institutions, for loans to customers and for debt instruments related to credit operations". On the other hand, the category other impairments, provisions and goodwill is defined by the "impairment of financial assets, other assets impairment, in particular provision charges related to assets received as payment in kind not fully covered by collateral, investments in associated companies and goodwill of subsidiaries and other provisions.".

BCP's impairment data, adapted from the annual report of the chosen period, are the following:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Loans impairment	-544.699	-560.029	-713.256	-1331.91	-969.6	-820.827	-1106.99	-833.024	-1116.92	-623.708	-465.468
Other impairments, provisions and goodwill	-44.524	-97.356	-227.79	-825.085	-349.641	-465.766	-209.274	-161.289	-481.076	-301.104	-135.634

Table 8 – Loans impairment and other impairments, provisions and goodwill, in million of euros, since 2008 and until 2018.
Source: Group's Annual reports

After the 2008-2012 financial crisis, the banking sector has been trying to diminish the values of impairments by applying more restricting rules and conditions to credit, since most of the companies, families and individuals with financial difficulties have a high probability of not paying their loans, what creates a loss for the financial institution. In this case, as seen on table 7, those changes started to have effect after 2017, even though it is too close to the present day to line a conclusion.

Just like the previous variable, the fitting and simulation process for these variables will be similar. In "loans impairment", the maximum value that a simulated value can take its 0, since the definition of the variable does not allow any positive numbers; whereas the minimum will be -800 – assuming that a raise on the amount of paid impairments is the most probable scenario. On the other hand, the simulated values for the variable "other impairments, provisions and goodwill" will be between -750 million and 400 million, assuming that the goodwill will have, by himself, large positive values, combined with a decrease of the losses.

5.2.1.4. Income Tax

Income tax, tax charged by the government based on the earnings of a company or individual, is divided, in this case, by current and deferred tax. The first, according to 2018's annual report, is defined as the "value that determines the taxable income for the year, using tax rates enacted or substantively enacted by authorities at Balance Sheet dare and any adjustment to tax payable in respect of previous years". Whereas the calculation of deferred taxes is based on the "liability method on the balance sheet, considering temporary differences, between the carrying amount of assets and liabilities and the amounts used for taxation purposes using the tax rates approved or substantially approved at the balance sheet date and that is expected to be applied when the temporary difference is reversed.".

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Current Tax	-44.001	-65.634	-54.158	-66.857	-81.286	-115.635	-100.995	-99.746	-113.425	-102.113	-105.559
Deferred Tax	-39.997	19.417	39.814	525.714	213.343	326.434	198.67	43.349	495.292	71.954	-32.458
Income tax	**-83.998**	**-46.217**	**-14.344**	**458.857**	**132.057**	**210.799**	**97.675**	**-56.397**	**381.867**	**-30.159**	**-138.017**

Table 9 – Income Tax, in millions of euros, since 2008 and until 2018.
Source: Group's annual reports

As observed in table 9, only in 2008 and 2018, the deferred tax had a negative value, causing a liability, since its amount of income was inferior to the taxable amount. In the years in between, those results were inverted, causing a tax deduction. In the group's worst years (2011-2014) the deduction was larger than the current tax, leading to an income tax reimbursement.

5.2.1.5. Income Arising From Discontinued Operations

Damodaran (2012) considers income arising from discontinued operations as a measure for the income (or loss) during the phase-out period and during the selling process of the operations. However, the same author notices that for this category to be considered the operations have to be separable from the main firm.

Looking at table 10, this category only started to be mentioned on 2013 annual report, the year of the sell and merger of Millennium BCP Greece with Piraeus Bank, which led to a change in the previous' year annual report for comparison reasons. 2012's value also contains the income arising from the sale of the group's operations in Romania and expenses related with the restructuring of asset management business.

In 2016, the income increased since it contained the profit before tax of Banco Millennium Angola before and during the process of sell and merger.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Income arising from discontinued operations	0	0	0	0	-730.267	-45.004	-40.83	14.648	45.228	1.225	-1.318

Table 10 – Income arising from discontinued operations, in millions of euros, since 2008 and until 2018.
Source: Group's annual reports

5.2.1.6. Other Items

This category was formed to comprise two others named "share of profit associated under the equity method" and "gains/losses arising from sales of subsidiaries and other assets". Although, these categories are as important as the previous ones to define net income there is not much information regarding its meanings. In spite of that, the last can be defined as "losses arising from the sale of assets of the group classified as non-current assets held for sale and gains/losses arising on sales and revaluations on investment properties", as mentioned on 2018's annual report.

Below in table 11, it is shown the historical data for both variables.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Share of profit associated under the equity method	19.08	66.262	67.661	14.62	55.659	62.26	35.96	23.528	80.525	91.637	89.175
Gains/losses arising from sales of subsidiaries and other assets	-8.407	74.93	-2.978	-26.872	-24.193	-36.759	45.445	-30.138	-6.277	4.139	37.916
Other Items	10.673	141.192	64.683	-12.252	31.466	25.501	81.405	-6.61	74.248	95.776	127.091

Table 11 - Other Items, in million of euros, since 2008 and until 2018.
Source: Group's annual reports

5.2.1.7. Non-Controlling or Minority Interests

The last category for the calculation of the net income is defined in the 2018's annual report as the "portion attributable to third parties of net income of subsidiary companies, consolidated under the full method, where in group BCP does not hold, directly or indirectly, the entirety of their share capital.".

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Non-controlling or minority interests	56.829	24.085	59.307	85.853	81.843	93.702	110.060	125.617	121.877	103.166	117.809

Table 12 – Non-controlling or minority interests, in million of euros, since 2008 and until 2018.
Source: Group's annual reports

Similarly to previous variables, this one will also have an interval of possible value to assume in the simulated data, during its fitting process. In each year, the values will be around 15 and 130 million, 5 million higher than the maximum historical value (in 2015). The goal is to avoid a negative net income.

5.2.2. Other Comprehensive Income

Other Comprehensive Income (OCI) is an income category that includes revenues and expenses not included in the calculation of net income. Examples of OCI are "net unrealized gains and losses on certain equity and debt investments, hedging activities, adjustments to the minimum pension liability, and foreign-currency translation items." (Koller et. all, 2010: 770-771).

As explained on section 5.1., the OCI interval for 2019-2023, will be determined by 5,000 iterations obtained randomly, considering the probability distribution and parameters obtained from it. The average values of 2019-2022 will be assumed as historical data to help in the probability fitting of the following year to obtain a more reliable, consistent and feasible result. The historical data from 2008-2018 used for the determination of 2019's probability fitting and respective iterations is showed in table 13.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Fair value reserves	-12035	-106276	-285888	-274409	494881	21279	94380	-107017	-237551	298533	66240
Effect on equity of Bank Millenium Atlântico, SA	0	0	0	0	0	0	0	0	0	28428	14914
Exchange differences arising on consolidation	-85567	-34747	18426	-40190	25083	-48782	10604	-150948	-228902	54808	-131345
Actuarial gains/losses for the year	0	0	0	-36755	-164191	-215447	-478359	-110692	-303705	33129	-95094
Taxes	8130	-14557	25767	56770	-72295	182000	24084	121367	130177	-114147	-36748
Others	0	0	0	0	0	0	0	0	151892	-3965	-2737
Other comprehensive income	-89472	-155580	-241695	-294584	283478	-60950	-349291	-247290	-488089	296786	-184770

Table 13 - Other comprehensive income, in millions of euros, since 2008 and until 2018.
Source: Group's annual reports

5.2.3. Δ BVE

Delta Book Value of Equity (***Δ***BVE) symbolizes the difference between the BVE for a company in a certain year, obtained from the subtraction between the total assets with its total liabilities (Woerd, 2011), and the value registered in the previous one.

However, when considering the FCFE equation adapted in this report is possible to see that the other comprehensive and the net income will be considered in a double way, since both belong to the equity part, in the balance sheet. To avoid this issue, instead of assuming the ***Δ***BVE as the difference between the BVE in consecutive years, it will be assumed as 10% of the difference between the total asset values in consecutive years - an adaptation of Basel III minimum total capital requirements' which obligate the financial institutions to retain, at least, 8% of its risky-weighted asset as equity (Maia, n.a.[3]).

As previously explained, the BVE is one of the variables that will have its interval values for 2019-2023 determined by 5,000 iterations obtained randomly, considering the probability distribution and parameters obtained from it. The average value of each interval until 2022 will be assumed as historical data to help in the probability fitting of the following year to obtain a better result.

For 2019's BVE, the historical data will be formed by data from 2008-2018, showed in table 14[4].

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Total Assets	94.424	95.550	98.547	93.482	89.744	82.007	76.361	74.885	71.265	71.939	75.923
Δ Book Value Equity (BVE)	0.626	0.113	0.300	-0.506	-0.374	-0.774	-0.565	-0.148	-0.362	0.067	0.398

Table 14 – Δ Book value of equity, in billion of euros, since 2008 and until 2018.
Source: Group's annual reports

In spite of having a similar probability fitting and iteration processes as the previous variables, this category is in billion of euros to facilitate the distribution fitting process since the values were too high to obtain a feasible conclusion. After the simulation process, its values will be multiplied by 1000, to return to the "standard" measure of this report – millions of euros.

5.3. Cost of equity

As explained previously in section 2.1.1.3. the most adequate discount rate for equity calculation is the cost of equity. To estimate it, CAPM, the most used model amongst financial analysts, was applied. Given that BCP is composed by several locations it was calculated a

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[3] This document was extracted from Ordem de Economistas' Website and does not have a defined publishing date. For more informations, please see section "References"

[4] Even though, 2007's results are not used in this report, the ΔBVE for 2008 was obtained using 2007 and 2008's asset value

separated cost of equity for each one of them, by applying different values for each variable, taking in consideration its financial and economic characteristics and data available. The cost of equity for each location will be helpful for the calculation of the group's cost of equity as will be demonstrated in section 5.3.4.

5.3.1. Portugal and Poland

The risk-free rate used for the Portuguese and polish operations was the ten-year German Government Bonds, since they are the riskless European bonds in the present moment.

Even though the beta's conventional historical approach is not considered the most correct one amongst analysts, it is, according to Damodaran (2012), one of the most appropriate for the evaluation of a financial service firm since its estimations are more precise than in other sectors. However, this permission only applies if the regulatory restriction does not change over the estimation period. Knowing that and since Portuguese and Polish subsidiaries are listed in the stock markets, the beta values assumed were the Bloomberg's adjusted betas obtained by a regression between BCP PL and Portuguese stock index (PSI20) historical returns, in case of Portugal, and between MIL PW and historical returns of the thirty largest companies in Warsaw stock exchange (WIG30), in the case of Poland, for each respective complete year with a daily frequency.

For Portugal, the market risk premium was obtained through the subtraction of PSI20 annual expected market return with the 10-year German Government Bonds rate; while, for Poland, it was through the subtraction of WIG30 annual expected market return with 10-year German Government Bonds rate.

Hence, the Portuguese results for 2008 – 2018 were the following.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Beta	0.965	1.227	1.258	1.269	1.188	1.7	1.7	1.416	1.695	1.895	1.38
Risk free rate (10y German)	2.95	3.39	2.96	1.83	1.32	1.93	0.54	0.63	0.21	0.43	0.24
Expected market return	12.36	11.58	11.67	15	16.89	13.5	13.11	10.1	11.41	11.27	10.41
Market risk premium	9.41	8.19	8.71	13.17	15.57	11.57	12.57	9.47	11.2	10.84	10.17
Cost of equity	12.031	13.439	13.917	18.543	19.817	21.599	21.909	14.040	19.194	20.972	14.275

Table 15 – Portugal's cost of equity, from 2008-2018.
Source: Bloomberg

As expected, the Portuguese cost of equity during 2011-2014 stayed between 18.543% and 21.909%, very high values when considering Portugal's position in Europe. However, these values are understandable when considering the fragile financial and economic situation that the country faced during those years, which led the investors require a bigger return to compensate the risk of investing in a weak economy.

In 2016 and 2017, the cost of equity was between the intervals registered in 2011-2014. In the first year, it was due to the decrease in the German risk-free rate, stimulated by the Brexit referendum and with the entry of more money into the economy, a decision made by the European central bank. In 2017, the beta value was the main responsible for the cost of equity since the stock's in the Portuguese equity index were more volatile than the index itself, probably due to the decision of cashier reinforcement in the beginning of the year, which led to a big equity selling by investors that did not want to participate.

The Polish results were the following:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Beta	1.116	0.805	1.061	1.008	0.981	1.1	1.015	1.158	1.167	1.106	0.948
Risk free rate (10y German)	2.95	3.39	2.96	1.83	1.32	1.93	0.54	0.63	0.21	0.43	0.24
Expected market return	13.5	13.96	12.09	13.56	9.5	10.79	10.22	9.28	10.08	12.58	14.72
Market risk premium	10.55	10.57	9.13	11.73	8.18	8.86	9.68	8.65	9.87	12.15	14.48
Cost of equity	14.724	11.899	12.647	13.654	9.345	11.676	10.365	10.647	11.728	13.868	13.967

Table 16 – Poland's cost of equity, from 2008-2018.
Source: Bloomberg

Poland's cost of equity, throughout the years in the sample, had a smaller ampleness as well as smaller values since its financial and economic health was not as affected as the Portuguese one. However, in this case its cost of equity was not immune to all economic, financial and social European decisions since the risk-free rate considered was the 10-year German government bond rate.

5.3.2. Greece

For Greece, the calculation of the cost of equity is different from the two previous European countries. As mentioned before, the banking operations were terminated due to a sell, in June 2013, and a merger, six months later, leading to a reallocation of its 2013's results to "Income arising from discontinued operations" category in the same year's annual report. Due to that, its cost of equity will be considered only until 2012, inclusive.

Considering that, until its dismantlement the institution was private the beta was obtained through the average of Bloomberg's adjusted betas of three Greece public banks with the historical returns of FTSE/Athens Stock Exchange Large Cap (FTASE), since they had similar characteristics and financial products (Pessanha, 2013).

Its results were obtained through Bloomberg by using the same process for Polish and Portuguese betas.

	2008	2009	2010	2011	2012
Attica Bank	0.936	0.805	0.967	1.29	1.361
Alpha Eurobank	1.033	1.191	1.301	1.51	1.756
Piraeus Bank	1.107	1.181	1.174	1.342	1.716
Beta Millennium Greece	1.025	1.059	1.147	1.381	1.611

Table 17 – Calculation for Greek Beta, from 2008-2018
Source: Bloomberg

Therefore, there are no results available to evaluate the expected market return. Hence, the expected market return values, and consequently, the market risk premium rates are assumed equal to the Portuguese ones due to resemblance between its financial, economic and social results within the considered period.

The risk-free rate considered continued to be the 10-year German government bonds rate.

The cost of equity for the years 2008-2012 is demonstrated in the table below:

	2008	2009	2010	2011	2012
Beta Millennium Greece	1.025	1.059	1.147	1.381	1.611
Risk free rate (10y German)	2.95	3.39	2.96	1.83	1.32
Expected market return	12.36	11.58	11.67	15	16.89
Market risk premium	9.41	8.19	8.71	13.17	15.57
Cost of equity	**12.598**	**12.063**	**12.953**	**20.013**	**26.403**

Table 18 – Greece’s cost of equity, from 2008-2018.
Source: Bloomberg

When comparing the computed Greek values, in table 18, with the ones created from normal conditions it is possible to conclude that the first are massive and only justified by the bigger influence that the financial, economic and social crisis had in the Baltic country than in Portugal.

5.3.3. Mozambique and Angola

For these two African countries, the calculation method for cost of equity is different from previous countries. While Mozambique’s cost of equity is calculated until 2018, Angola’s is only calculated until 2015, due to Banco Millennium Angola’s merger with Banco Privado Atlântico in 2016.

Since both banks were or still are private the calculation of the betas is obtained using the average value of Bloomberg’s adjusted betas of three Botswanan public banks with the historical returns of Botswana Gaborone market index (BGSMDC) and three Kenyan public banks with the historical returns of Nairobi Securities Ltd 20 market index (KNSMIDX), since the four countries have similar social, economic and financial characteristics (Pessanha, 2013). Their names and beta values and Angola and Mozambique’s betas are demonstrated in the table below.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
First National Bank Botswana	1.476	0.753	1.125	1.151	1.091	2.005	0.653	0.385	0.596	2.092	0.471
Barclays Bank (Botswana)	0.843	0.552	0.795	0.747	0.513	0.642	0.933	0.402	0.381	0.814	0.543
Standard Chartered Bank Botswana	1.035	0.275	0.836	0.54	0.285	0.447	0.63	0.41	1.191	1.573	0.393
Barclays Bank of Kenya	0.894	0.616	0.682	1.069	0.671	0.82	0.604	0.789	0.983	1.105	0.639
Equity bank (Kenya)	1.52	0.929	0.832	0.894	0.499	1.298	1.142	1.271	1.338	1.1	1.128
Kenya Commercial Bank	1.14	0.739	0.714	1.143	0.572	0.975	0.967	0.887	1.291	0.921	0.887
Millenium Angola Beta	1.151	0.644	0.831	0.924	0.605	1.031	0.822	0.691			
Millenium BIM Beta	1.151	0.644	0.831	0.924	0.605	1.031	0.822	0.691	0.963	1.268	0.677

Table 19 – Calculation for Angola from 2008-2015, and Mozambique’s betas, from 2008-2018
Source: Bloomberg

For market risk premium variable was used Damodoran's database[5] , where is considered the expected annual market return of S&P500 with the risk-free rate, the 10-year Treasury bond rate.

Since the risk-free rate is in a different currency and to incorporate the social, economic and politic situations in both countries, it was added a country risk premium variable to cost of equity equation. For Angola, starting in 2010, the country risk premium values came from Damodaran's database[6]. Since there was no data available for the two previous years, it is assumed that its values are equal to 2010's one. For Mozambique, since, until 2012, there is no data available in Damodaran's database it is assumed that its value is superior to Angola's in 0.15% since its economy has a weaker straight (Pessanha, 2013).

Angola's cost of equity and variables values are demonstrated in the table below:

	2008	2009	2010	2011	2012	2013	2014	2015
Beta	1.151	0.644	0.831	0.924	0.605	1.031	0.822	0.691
Risk-free rate (10y Treasury-bond)	2.21	3.84	3.29	1.88	1.76	3.03	2.17	2.27
Market Risk Premium	6.43	4.36	5.2	6.01	5.78	4.96	5.78	6.12
Country risk premium Angola	6	6	6	4.88	4.88	5.4	4.5	7.07
Cost of equity Angola	15.613	12.648	13.609	12.313	10.138	13.545	11.418	13.567

Table 20 – Angola's cost of equity, from 2008-2015
Source: Bloomberg, Damodaran

The Mozambican values were the following:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Millenium BIM Beta	1.151	0.644	0.831	0.924	0.605	1.031	0.822	0.691	0.963	1.268	0.677
Risk-free rate (10y Treasury-bond)	2.21	3.84	3.29	1.88	1.76	3.03	2.17	2.27	2.44	2.41	2.68
Market Risk Premium	6.43	4.36	5.2	6.01	5.78	4.96	5.78	6.12	5.69	5.08	5.96
Country Risk premium	6.15	6.15	6.15	5.03	5.03	6.75	6.75	11.77	14.21	11.52	13.87
Cost of equity Moçambique	15.763	12.798	13.759	12.463	10.288	14.895	13.668	18.267	22.131	20.369	20.584

Table 21 – Mozambique's cost of equity, from 2008-2018
Source: Bloomberg, Damodaran

The cost of equity for Angola in 2015, its last year on sample, was 13.567%, mainly due to market and country risk premium, which reflected its social and financial instable situation.

The cost of equity for Mozambique in 2018 was 20.584%, mainly influenced by the country risk premium that since 2015 has very high values. The reasons for that might be the decrease in the GDP's growth rate, which has been reaching minimal values, increase of the inflation rate and the depreciation of MZN against the USD, which increases the investment and social risk when compared with the United States'.

[5] Database is in an excel file named "Implied ERP (Annual) from 1960 to Current"
[6] Database is separated by years in a compilation of excel files called "Risk Premiums for other markets"

5.3.4. Group's cost of equity

Keeping in mind that valuation of the group is obtained as a "whole", the cost of equity must follow that reasoning. The first step is not only to consider all subsidiaries existent in the year in question, but also all the representation offices (mainly in countries with large Portuguese communities), private banking platform, in Switzerland, and an on-shore-branch in Macao.

Due to the difficulty of calculating the influence of every contributor, since only the net income of each subsidiary is available, the influence of each location's cost of equity in the group's was defined considering the general influence they have within it. Hence, the group's cost of equity will be composed by a fixed percentage of each country/group times its respective cost of equity. The chosen percentage of each influence was the following:

- Portugal's cost of equity will have an influence of 50%, considering it was the country where the group started, and it is where the bank has its headquarters and more offices;
- Poland's cost of equity will influence by 25%, since it is the second most influential country in the daily basis;
- Remaining countries with subsidiaries (Mozambique, Angola, until 2016 and Greece, until 2012) will have an equal influence in a total of 20%;
- The group formed by the location where the group only has financial operations and representation offices will have an influence of 5%, however, its cost of equity will be null.

Thus, the group's historical data for cost of equity will be the following:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Portugal	12.031	13.439	13.917	18.543	19.817	21.599	21.909	14.04	19.194	20.972	14.275
Poland	14.724	11.899	12.647	13.654	9.345	11.676	10.365	10.647	11.728	13.868	13.967
Greece	12.598	12.063	12.953	20.013	26.40	-	-	-	-	-	-
Angola	15.613	12.648	13.609	12.313	10.138	13.545	11.418	13.567	-	-	-
Mozambique	15.763	12.798	13.759	12.463	10.288	14.895	13.668	18.267	22.131	20.369	20.584
Other Locations	0	0	0	0	0	0	0	0	0	0	0
Group's Cost of equity	**12.628**	**12.195**	**12.808**	**15.671**	**15.367**	**16.562**	**16.054**	**12.865**	**16.955**	**18.027**	**14.746**

Table 22 – Group's cost of equity calculation, from 2008-2018

Even though this variable is measured in percentage, the process to get the future cost of equity values is similar to the ones previously explained, since for the following years, after 2019, the fitting process will be formed by data from 2008-2018, as demonstrated in table 22, and the average of the cost of equity from the previous year(s) obtained from the probabilistic fitting and simulation processes.

For the equity calculation, there will be a programming of a simulation with 5,000 iterations, considering the probability distribution, its respective parameters. The average for each group of iterations in each year will be the cost of equity respective in the historical data group.

5.4. Growth Rate

For the group's yearly growth rate, we assumed it as the nominal GDP growth rate – defined as the change, in percentage, of the value of goods and services produced by a country in a certain year, at current prices, compared with the results of the previous year. To obtain it, we added to the GDP real growth rate the inflation rate for each country or group, with the goal of harmonizing the real growth rate and turning it into a nominal one, as showed in the equation below.

$$GDP's\ Nominal\ GrowthRate_t = GDP's\ Real\ Growth\ Rate_t + Inflation\ Rate_t \qquad (44)$$

(For more details, please see exhibit 2.)

The final growth rate for each year will be obtained by each country's own value weighted identically as done in the calculation of the cost of equity.

The GDP real growth rates for all countries were extracted from the Worldbank data source; for the group of countries/regions formed by all with financial operations was assumed the global GDP real growth rate, by using the average value of all countries.

The results for all countries and for the group are showed in the table below:

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Portugal	0.199	-2.978	1.899	-1.827	-4.028	-1.130	0.893	1.822	1.926	2.795	2.145
Poland	4.250	2.820	3.607	5.017	1.608	1.392	3.318	3.839	3.063	4.814	5.149
Greece	-0.335	-4.301	-5.479	-9.132	-7.300						
Angola	11.166	0.859	4.859	3.472	8.542	4.955	4.823	0.944			
Mozambique	6.876	6.352	6.687	7.118	7.198	7.142	7.444	6.594	3.763	3.737	3.265
Other countries	8.759	2.929	3.285	4.805	3.731	2.613	2.239	1.405	1.429	2.177	2.514

Table 23 -Real GDP Growth Rate for all countries with BCP's subsidiaries and group of countries with financial operations, in percentage, since 2008-2018.
Source: Worldbank data source

For inflation, a more difficult variable to define due to its different methods of calculation and assumptions, it was assumed the definition from the ECB, who says inflation consists on "a general increase in consumer prices and is measured by an index which has been harmonised across all EU Member States: Harmonised Index of Consumer Prices (HICP). The HICP is the measure of inflation which the Governing Council uses to define and assess price stability in the euro area as a whole in quantitative terms.".

Bearing in mind the goal of having the group's equity valuated in euros, the inflation rate assumed was the medium-term inflation goal determined by ECB and with an equal value for all countries/regions. The inflation rate chosen for this report was 2% to create results more accurate despite the opinion of the ECB, who considers that the medium-term inflation goal in all euro-countries should be below 2%, but close to.

After summing the two variables to its respective country, the real GDP growth rate for each country is obtained. For the calculation of the group's growth rate, we decided to apply the same methodology as the calculation of the group's cost of equity, with the results as showed in table 24.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Portugal	2.199	-0.978	3.899	0.173	-2.028	0.870	2.893	3.822	3.926	4.795	4.145
Poland	6.250	4.820	5.607	7.017	3.608	3.392	5.318	5.839	5.063	6.814	7.149
Greece	1.665	-2.301	-3.479	-7.132	-5.300						
Angola	13.166	2.859	6.859	5.472	10.542	6.955	6.823	2.944			
Mozambique	8.876	8.352	8.687	9.118	9.198	9.142	9.444	8.594	5.763	5.737	5.265
Other countries	10.759	4.929	5.285	6.805	5.731	4.613	4.239	3.405	3.429	4.177	4.514
Group's Real Growth Rate	**4.780**	**1.556**	**4.420**	**2.678**	**1.137**	**2.587**	**4.073**	**4.310**	**3.784**	**4.692**	**4.436**

Table 24 – Nominal Growth Rate for all countries with BCP's subsidiaries and group of countries with financial operations and for group BCP in percentage, since 2008-2018

As previous variables, there will be a programming of a simulation with 5,000 iterations, considering the probability distribution, its respective parameters. The average for each group of iterations in each year will be the respective growth rate on the historical data group.

5.5. R's Code Development

The fitting and simulation process would not be possible without the code development in R Studio, a professional software for R language[7].

The first step to achieve the goal of having the best and most reliable results possible is to test all possible distributions, using the historical data and taking into account its range, using the maximum likelihood method – method that estimates values that create the maximum combined probability density for the observed data (Vose, 2010).

Considering that this fitting process can be difficult, since there are few data inputs that fit perfectly into a probability distribution there are some assumptions that can be made (Damodoran, 2009; Nurminen, 2016 and Talevski and Lima 2009):

a) If none of the distributions available for the goodness of fit provide a satisfying fit, we should accept the one that best describes it.
b) The costs will never be less than zero, therefore any distribution, like the Normal distribution, that accepts and requires the variable to take negative values must be ruled out. In those cases, we can use the lognormal distribution;
c) The Normal distribution is the best for the variables with high concentration around the mean, symmetrical and with low probability of having extreme values.

[7] The code was made by myself and it is not present in this report due to copyright reasons

Hence, to avoid errors and the creation of unnecessary coding some probability distributions were excluded from the fitting process for some variable, due to incompatible characteristics. Thus, in table 25 is exposed the probability distributions used for each fitting process:

	Normal	Exponencial	Log-Normal	Gamma	Weibull	Cauchy
Net operating revenues growth rate	√					√
Operating costs	√	√	√	√	√	√
Loans impairment	√					√
Other Impairment, provisions and goodwill	√					√
Income tax	√					√
Income arising from discontinued operations	√					
Other items	√					√
Non-controlling or minority interests	√	√	√	√	√	√
Other comprehensive income	√					√
Delta BVE	√					√
Cost of Equity		√	√	√	√	
Growth rate		√	√	√	√	

Table 25 – Probability distributions chosen for each fitting process
Source: Author

In the case of two or more possible distributions, the next step is to visualize, in the same graph, the historical data with all chosen distributions to help line a pre-conclusion on which one fits the better. Based on Ricci (2005), there will be demonstrated three different graphs: PP-Plot, QQ-Plot and density plot, whose results will determine the 2 to 3 best possible probabilistic distributions followed by a probabilistic simulation with 5,000 iterations, using the parameters of the historical data

Lastly, the AIC and BIC results will be known using the simulated data, in which the lower its results the better since there is less information lost. The KS-Test will also be applied, but just for continuous distributions without a truncated simulation range, since this limitation can affect the random parameters in a big way, leading to a rejection of the null hypothesis. Even without this restriction, the test is only applied lastly because, according to Vose (2010), it can only be applied to simulated data without any repeated data, otherwise its critical region and results will no longer be valid. In the case of one distribution has the smallest AIC and/or BIC values but not the highest p-value, the distribution with the first characteristics is chosen.

As previously pointed before, after the whole process is done, the average value of all 5,000 simulations will be calculated and used for the fitting process of the following year.

6. Discussion of the results

In this chapter, we will describe and present all estimation criteria results for every variable, whose outputs made possible to find the best probability distribution and its respective parameters and, consequently, the simulated probabilistic data.

As it was mentioned earlier, the historical data used in each variable to produce these results began in 2008 and finished on the previous year just before the year in question.

6.1. Net Operating Revenues Value and Growth Rate

As mentioned on table 25, the only two distributions tested for the fitting of variable net operating revenues growth rate were the Normal and Cauchy distributions, due to the presence of negative values in historical data.

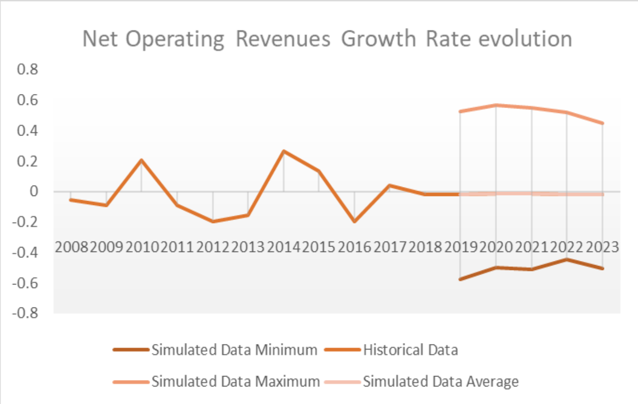
In each year, after simulating data for both distributions based on the historical data parameters it was tested the AIC and BIC estimation criteria and KS-Test and, as stated in bold in table 26, the best distribution was always the same: Normal distribution.

	Top Distributions	AIC	BIC	Ks-test
2019	Cauchy	887.2238	900.2582	0.7071
	Normal	**-4802.861**	**-4789.826**	**0.8895**
2020	Cauchy	540.1893	553.2237	0
	Normal	**-5077.719**	**-5064.685**	**0.4653**
2021	Cauchy	-1186.195	-1173.161	0.8235
	Normal	**-5327.646**	**-5314.611**	**0.6645**
2022	Cauchy	-1818.898	-1805.864	0.9948
	Normal	**-6016.681**	**-6003.647**	**0.9654**
2023	Cauchy	-3445.824	-3432.789	0.8322
	Normal	**-6069.235**	**-6056.201**	**0.9754**

Table 26 – Criteria decision results for the best distribution for Net operating revenues growth rate, from 2019 - 2023

In table 27, it is possible to see the Normal distribution parameters, the limits of the simulated data and its average value – which is used to represent the value of the year in question for the fitting process of the upcoming ones.

In the same table, it possible to conclude that the group, in the upcoming years, will have an average decrease of, at least, 1.3%, in its net operating revenues.



Graph 21 - Simulation data average and limits, from 2008 – 2023, for net operating revenues growth rate

Table 27 - Simulation data characteristics and limits, from 2019 – 2023, for net operating revenues growth rate

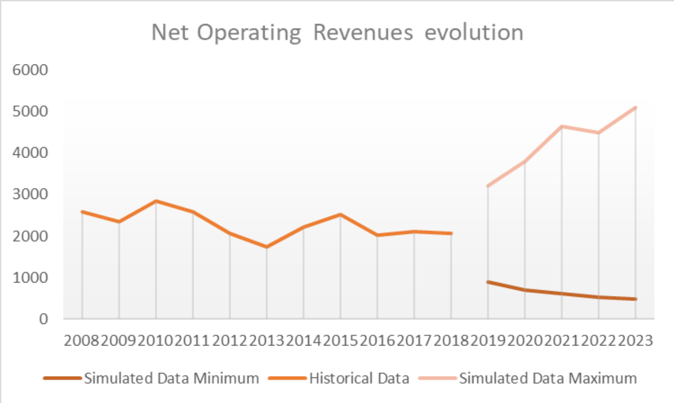
	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	mean = - 0.01974 sd = 0.14962	-0.57341	0.52607	-0.01974
2020	mean = - 0.01504 sd = 0.14557	-0.49972	0.56517	-0.01504
2021	mean = - 0.01308 sd = 0.14198	-0.51085	0.55031	-0.01308
2022	mean = - 0.01676 sd = 0.13252	-0.44646	0.52187	-0.01676
2023	mean = - 0.01702 sd = 0.12183	-0.50598	0.44835	-0.01702

In comparison with the historical data, as showed in graph 21, it is also possible to conclude that the average future results do not vary as much as the past ones, due to their calculation type. However, the amplitude within the maximum and minimum values open the possibility of big changes in the net growth rate in consecutive years, as just observed in previous years. Even though, the net operating revenues are not simulated statistically as the other variables its results are still needed for the net income calculation.

The following graph and table show the evolution of net operating revenues since 2008, the initial year. Even though the predictions, in graph 21, show a decrease in the maximum simulated growth rate, starting in 2020, in graph 22, it is possible to observe that the decrease in the maximum simulated operating revenues only happens in 2022.

However, the inverse behaviour is observed in for the minimum simulated data. While in graph 21 its behaviour is very irregular, in graph 22 the simulated data reach new minimum at each year, since the growth rate is still negative, causing, consequently, a decreasing effect on the net operating revenues.

Although in table 27, the average growth rate in net operating revenues does not change as much as in the past years, the increase of the maximum simulated value and the decrease of the minimum simulated value lead to a greater uncertainty on which will be the net operating revenues for the upcoming years, since the range of possibilities becomes larger every year.



Graph 22 - Simulation data average and limits, from 2008 – 2023, for net operating revenues

Table 28 – Net operating revenues data limits, from 2019 - 2023

	Data Minimum	Data Maximum
2019	899.3396	3217.2677
2020	697.1790	3792.3695
2021	615.7810	4651.5846
2022	533.76892	4483.35557
2023	477.37436	5108.98265

6.2. Operating Costs

Contrarily with the previous category, the constraints in this decision process are related to the fact that the variable can only assume positive values.

As a consequence of the creation of upper and down limit, the Normal and Cauchy distributions could be considered to the process, and as per observed in the table below the first was always considered one the strongest possibilities. However, in every year, the distribution with the best AIC and BIC results was Gamma.

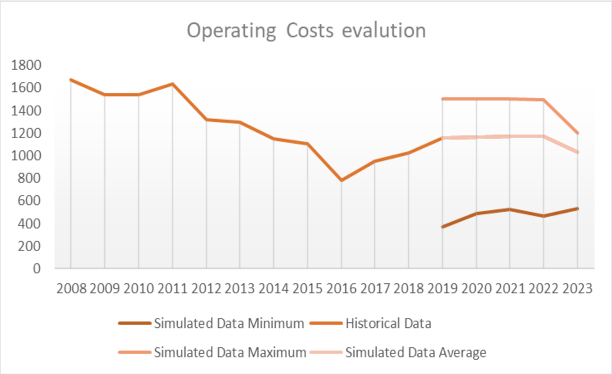
	Top Distributions	AIC	BIC
2019	Normal	44797.97	44811
	Gamma	**44447.13**	**44460.16**
	Weibull	44683.96	44967
2020	Normal	44494.24	44507.28
	Gamma	**44206.86**	**44219.89**
	Weibull	44914.53	44927.56
2021	Normal	44307.19	44320.23
	Gamma	**44058.95**	**44071.98**
	Weibull	44426.07	44439.1
2022	Normal	44057.92	44070.96
	Gamma	**43851.4**	**43864.43**
	Weibull	44705.99	44719.02
2023	Normal	40631.22	40644.25
	Gamma	**39497.85**	**39510.85**
	Weibull	41136.95	41149.99

Table 29 – Criteria decision results for the best distribution, from 2019 – 2023, for operating costs

(The graphs – PP-Plot, QQ-Plot and density plot – what helped to decide the top three distributions in table 29 can be seen in exhibit 3).

Following this decision, the simulated parameters and limits were obtained and are presented in the table below.

Even though, the average is not a necessary parameter for the simulation of these values it is still significant for comparison with the historical data, since it is the most balanced and usable parameter, and it will perform as operating cost of the year in question in the fitting process for the following years. Hence, it is important to refer that in, until 2022, there is an average yearly increase on the costs between 3 and 6 million, - which can be explained by the closeness between the historical data and the upper limit -, followed, in 2023, by a decrease of 140 million, motivated by a change on the down limit.



Graph 23 - Simulation data average and limits, from 2008 – 2023, for operating costs

Table 30 - Simulation data characteristics and limits, from 2019 – 2023, for operating costs

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	shape = 31.03843 rate = 0.26751	372.38065	1499.97445	1160.24935
2020	shape = 32.95056 rate = 0.28247	488.38361	1499.64820	1166.46165
2021	shape = 34.14982 rate = 0.29197	524.06385	1499.91067	1169.62585
2022	shape = 35.78755 rate = 0.30529	469.46944	1499.47870	1172.23660
2023	shape = 66.93057 rate = 0.64815	535.76316	1199.99898	1032.61709

When adding the graph 23 to the analysis it is possible to better observe the trajectory for both minimum and maximum simulated data of each year. While the maximum operating costs has a stable behaviour due to the presence of an upper limit; the minimum simulated costs have an irregular pattern, due to the lowest limit, achieving its minimum on 2019 (372.38065 million).

6.3. Loans Impairment

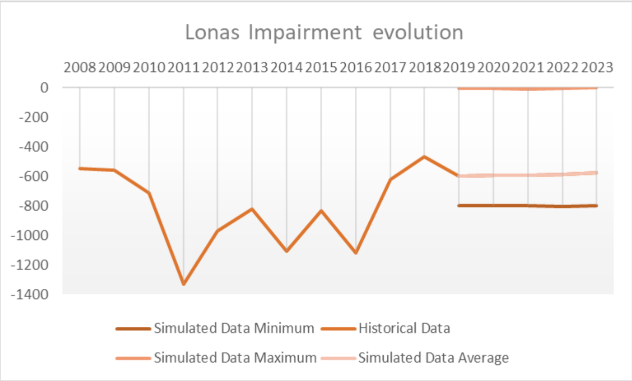
As mentioned on section 5.2.1.3, this variable had special conditions, because it is defined to only assume negative or null values. Despite having a defined range for the simulated values, the only two distributions which met every requirement were Cauchy and Normal.

In the table below, in bold, it is possible to see that in every year, expect 2023, the best distribution was the Normal one, changing after to Cauchy.

	Top Distributions	**AIC**	**BIC**
2019	Cauchy	66691.74	66704.77
	Normal	**64617.79**	**64630.83**
2020	Cauchy	66498.17	66511.21
	Normal	**64552.05**	**64565.09**
2021	Cauchy	66064.44	66077.47
	Normal	**64456.41**	**64469.45**
2022	Cauchy	65070.47	65083.5
	Normal	**64650.06**	**64633.1**
2023	**Cauchy**	**63798.31**	**63811.34**
	Normal	64548.22	64561.25

Table 30 – Criteria decision results for the best distribution, from 2019 – 2023, for loans impairment

The results were the following:



Graph 24 - Simulation data average and limits, from 2008 – 2023, for loans impairment

Table 31 - Simulation data characteristics and limits, from 2019 – 2023, for loans impairment

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	mean = - 597.0915 sd = 154.8474	-799.80271	-0.46000	-597.09146
2020	mean = - 593.6084 sd = 153.8329	-799.88702	-3.20679	-593.60837
2021	mean = - 594.0684 sd = 152.3686	-799.98243	-8.14513	-594.06845
2022	mean = - 587.7392 sd = 155.348	-799.98804	-3.59231	-587.73920
2023	mean = - 587.7147 sd = 153.7739	-799.86889	-0.21329	-573.36602

According to the simulated results in both graph 24 and table 31, until 2023, the group will never have a null value of loans impairment. This means that there will always be an investor, firm or other financial institution whose loan will not be paid due to its lack of cash and/or credit worthiness.

The impairment in loans reaches its minimum value in 2022 – of 799.98804 million –, in spite of the range between the minimum simulated values is around 0.18 million, a very small value when compared to other variables. From 2019 to 2021, there is a tendency of increase in the maximum value – which indicates a higher default -, followed by two consecutive decreases – which may indicate that the debt owners on the previous years gained enough credit to pay their debts, even though, the bank considered them as a permanent loss. Before the decreases, in 2021, the simulated data reaches its maximum of -8.14513 million.

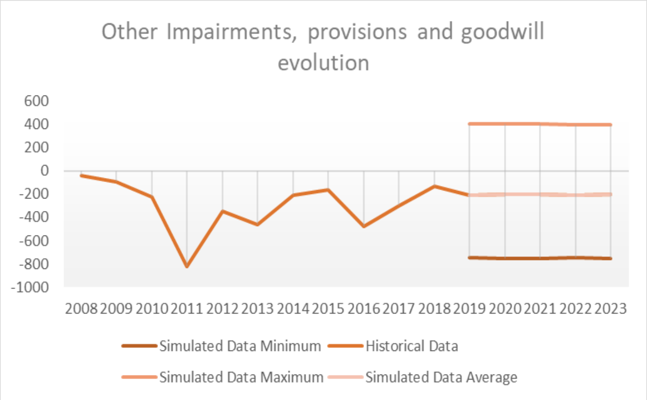
6.4. Other Impairments, Provisions and Goodwill

The fifth variable needed for the calculation of the net income will have the same characteristics as the first two, unlike the name indicates, since the provisions and goodwill can assume a positive value by themselves. Considering that it can also assume a general negative value, the upper and down limit were composed by a positive and a negative value, respectively, leading to the only two possibilities for the best probability distribution being Cauchy and Normal. Its criteria results were the following:

	Top Distributions	AIC	BIC
2019	**Cauchy**	**66483.54**	**66496.58**
	Normal	67186.17	67199.21
2020	**Cauchy**	**65184.45**	**65197.48**
	Normal	66980.47	66993.51
2021	**Cauchy**	**64431.27**	**64444.3**
	Normal	66718.02	66731.06
2022	**Cauchy**	**63010.08**	**63023.12**
	Normal	66604.78	66617.82
2023	**Cauchy**	**61567.8**	**61580.83**
	Normal	66264.13	66277.17

Table 33 – Criteria decision results for the best distribution, from 2019 – 2023, for other impairments, provisions and goodwill

As indicated in bold, the Cauchy distribution was the best choice in every year. The simulated parameters and limits are shown in the table and the graph below:



Graph 25 - Simulation data average and limits, from 2008 – 2023, for other impairments, provisions and goodwill

Table 34 - Simulation data characteristics and limits, from 2019 – 2023, for other impairments, provisions and goodwill

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	location = - 218.8079 scale = 83.9377	-747.27589	399.81033	-212.66925
2020	location = - 212.2814 scale = 72.1474	-749.68556	398.34255	-206.21584
2021	location = - 209.1023 scale = 65.6982	-749.59148	399.67907	-206.28152
2022	location = - 209.0555 scale = 55.4086	-746.73398	394.87009	-206.49831
2023	location = - 206.3856 scale = 46.3547	-749.84403	395.86283	-203.93004

The range between the average values is around 8.5 million, a low value considering the total amount. Regarding the extremes, both have irregular patterns, but its values are always close to the upper or down limit. The minimum simulated values register its lowest value in 2020 of -749.68556 million euros, whereas the maximum is registered in 2019, with a value of 399.81033 million.

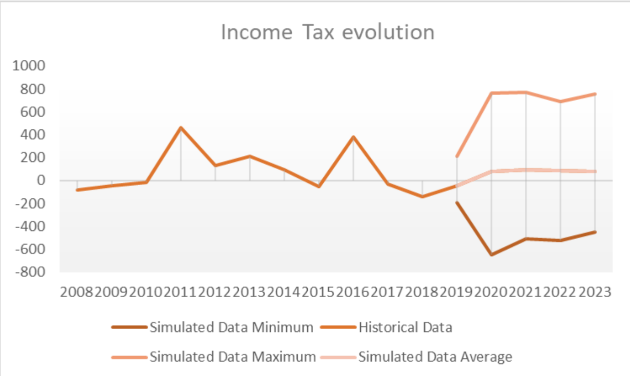
6.5. Income Tax

In income tax, the distributions considered were the Cauchy and Normal, when considering the presence of negative and positive values in historical data. In the fitting process, the best distribution was the Normal in all year, as shown in bold in table 35:

	Top Distributions	AIC	BIC	Ks-test
2019	Cauchy	69332.03	69345.06	0.9565
	Normal	**66443.62**	**66456.65**	**0.9352**
2020	Cauchy	71328.7	71341.73	0.7393
	Normal	**66187.24**	**66200.28**	**0.624**
2021	Cauchy	71116.33	71129.36	0.8438
	Normal	**65735.02**	**65748.05**	**0.9029**
2022	Cauchy	70602.09	70615.12	0.9243
	Normal	**65269.82**	**65282.86**	**0.9146**
2023	Cauchy	69408.12	69421.15	0.8548
	Normal	**65085.54**	**65098.58**	**0.4108**

Table 35 – Criteria decision results for the best distribution, from 2019 – 2023, for income tax

Below it is possible to see that, even though the minimum simulated values are very low, the average still manages to have a very positive way, meaning that the group will have a possibility of more than 50% of having to pay taxes. The maximum simulated tax has an irregular pattern, but the maximum that the group should be charged is around 766.764001 million, in 2020. On the other hand, if the group has negative revenues its income tax will be negative – i.e., the group will receive tax – in a minimum – that turns into the maximum amount received, in this case – of - 808.25676 million.



Graph 26 - Simulation data average and limits, from 2008 – 2023, for income tax

Table 36 - Simulation data characteristics and limits, from 2019 – 2023, for income tax

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	mean = 83.93726 sd = 185.86551	-191.08256	209.39862	83.93726
2020	mean = 79.95972 sd = 181.16092	-648.20504	766.76401	79.95972
2021	mean = 91.14745 sd = 173.1509	-508.25676	773.74583	91.14745
2022	mean = 83.27835 sd = 165.28045	-524.69601	688.03846	83.27835
2023	mean = 81.92942 sd = 162.26255	-448.30069	754.17291	81.92942

6.6. Income Arising From Discontinued Operations

For this variable, two probabilistic distributions were pre-selected for the fitting process, given they were the only ones that allow negative values. However, there is a failure in R Studio when processing the code for a Cauchy fitting process, using the maximum likelihood method, which indicated that there is a high probability of the data not following a Cauchy distribution (Carrasco, 2017). As alternative, the methods of maximum goodness-of-fit estimation and moment matching estimation were used but without any success, given that the first does not accept data with null values and the second does not exist because the distribution does not have matching moment fit function defined.

Thus, the distribution assumed for this variable was the Normal distribution. Its parameters and simulation limits and average were the following:

	2019	2020	2021	2022	2023
Distribution Parameters	mean = -71.06909 sd = 211.0651	mean = -68.84087 sd = 204.1976	mean = -70.03046 sd = 194.6152	mean = -76.3715 sd = 187.848	mean = -71.49665 sd = 179.192
Simulated Data Minimum	-912.08437	-867.93726	-766.15456	-882.84191	-644.20625
Simulated Data Maximum	626.1284	616.7455	812.4752	580.7651	589.2352
Simulated Data Average	-71.0691	-69.8409	-70.0305	-76.3715	-71.4967

Table 37 - Simulation data characteristics and limits, from 2019 – 2023, for income arising from discontinued distributions

Here, the minimum simulated data has an increasing pattern, except in 2022, where it decreases but does not reaches the smallest value, registered in 2019, of -912.08437 million euros. On the other hand, the maximum simulated data has an irregular pattern of decreases followed by increases. Its higher value is registered in 2021 with a value of 812.4752 million. The average has a range of 7 million but is always around the 70 million euros.

6.7. Other Items

In this category, there were only two options for the best distribution, Cauchy and Normal. As observed in table 38, the latter was the best distribution in all years.

	Top Distributions	AIC	BIC	Ks-test
2019	Cauchy	61470.26	61483.29	0.8788
	Normal	**53295.56**	**53308.59**	**0.9827**
2020	Cauchy	59906.1	59919.13	0.999
	Normal	**52811.41**	**52824.44**	**0.5603**
2021	Cauchy	58919.52	58932.56	0.9952
	Normal	**52408.88**	**52421.92**	**0.8724**
2022	Cauchy	57546.3	57559.33	0.959
	Normal	**52137.33**	**52150.36**	**0.8346**
2023	Cauchy	56032.96	56045.99	0.5775
	Normal	**51678.64**	**51691.67**	**0.9962**

Table 38 – Criteria decision results for the best distribution, from 2019 – 2023, for other items

In table 39, it is possible to observe that the range in the average simulated data is about 1.5 million, which leave us, once again, with the conclusion that the average ranges in the Normal distribution do not change very much. Both the maximum and the minimum values are registered in the first year, 2019, with the respective values of 245.43345 and -159.13034 million euros.

	2019	2020	2021	2022	2023
Best Distribution	Normal	Normal	Normal	Normal	Normal
Distribution Parameters	mean = 57.97559 sd = 49.90978	mean = 57.6022 sd = 47.55098	mean = 56.76362 sd = 45.67495	mean = 56.36935 sd = 44.4513	mean = 58.23173 sd = 42.45842
Simulated Data Minimum	-159.13034	-127.41130	-96.56793	-93.46572	-83.00074
Simulated Data Maximum	245.43345	215.22059	213.14743	223.20060	237.46036
Simulated Data Average	57.97559	57.60220	56.76362	56.36935	58.23172735

Table 39 - Simulation data characteristics and limits, from 2019 – 2023, for other items

6.8. Non-Controlling or Minority Interests

In the fitting process for this variable, all distributions were considered, even though they had upper and down limits for the simulated data.

With all six of them being tested, the best three possibilities were very hard to choose, but with a more careful analysis, the Weibull distribution was considered the best until 2022, where it changed to Cauchy, as pointed out in bold in table 40:

	Top Distributions	AIC	BIC
2019	Cauchy	46447.8	46460.84
	Normal	46271.27	46284.31
	Weibull	**45986.67**	**45999.71**
2020	Cauchy	46239.3	46252.33
	Normal	45994.74	46007.77
	Weibull	**45869.91**	**45882.94**
2021	Cauchy	45770.21	45783.24
	Normal	45973.51	45986.55
	Weibull	**45731.14**	**45744.18**
2022	**Cauchy**	**44988.38**	**45001.42**
	Normal	45874.94	45887.97
	Weibull	45390.77	45403.81
2023	**Cauchy**	**43684.47**	**43697.51**
	Normal	45407.3	45420.34
	Weibull	45450.77	45463.8

Table 40 – Criteria decision results for the best distribution, from 2019 – 2023, for non-controlling or minority interests

(The graphs – PP-Plot, QQ-Plot and density plot – what helped to decide the top three distributions in table 40 can be seen in exhibit 4).

In table 41, it is possible to see that the yearly simulated average has a very small variation, with a range of only 1.6 million euros, despite not being a formal parameter for each of the distributions and not having the same distribution for every year.

	2019	2020	2021	2022	2023
Best Distribution	Weibull	Weibull	Weibull	Cauchy	Cauchy
Distribution Parameters	shape = 4.0083 scale = 93.2796	shape = 4.1247 scale = 94.5379	shape = 4.21296 scale = 94.777	location = 88.3989 scale = 10.1258	location = 87.4542 scale = 8.7664
Simulated Data Minimum	15.36900	15.38938	16.28240	15.07611	15.00820
Simulated Data Maximum	129.97171	129.94539	129.97260	129.96387	129.97043
Simulated Data Average	84.42760	85.70546	86.02674	85.89924	85.60537

Table 41 - Simulation data characteristics and limits, from 2019 – 2023, for non-controlling or minority interests

The minimum simulated valued increases until 2021, to 16.2824 million, the furthest of the down limit, whereas the maximum simulated data has an irregular pattern, since an increase is followed by a decrease on the following year. In 2021, the predicted maximum will be 129.9726 million euros.

6.9. Net Income

After obtaining the simulated values for all variables belonging to net income, it is time to determine the net income. The calculations were obtained using each iteration from the variables, using the equation 42 for the first iteration, and the equation 43 for the remaining. The extreme values for each year are registered in table 42. All results seem to lead to a positive net income average, except in 2019. For that year, two of the most probable causes for a negative average result might be the registration of the lowest simulated data minimum in all years in the variables "other items" and "income arising from discontinued operations".

	2019	2020	2021	2022	2023
Minimum Value	-2001.3	-1809.03	-1983.73	-2024.95	-1945.89
Maximum Value	1886.369	2834.131	2593.537	2998.579	3388.648

Table 42 - Simulation data limits, from 2019 – 2023, for net income

6.10. Other Comprehensive Income

On the first fitting and simulation process for a variable outside net income, the only two probability distributions appropriated for OCI's characteristics were Cauchy and Normal. As observed in bold in table 43, the best distribution in every year was the Normal one.

	Top Distributions	AIC	BIC	Ks-test
2019	Cauchy	71984.58	71997.62	0
	Normal	**69158.41**	**69171.45**	**0.6336**
2020	Cauchy	71301.55	71314.58	0.9489
	Normal	**68762.31**	**68775.34**	**0.4535**
2021	Cauchy	70208.98	70222.02	0.3099
	Normal	**68556.86**	**68569.89**	**0.7474**
2022	Cauchy	68599.09	68612.12	0.8547
	Normal	**68005.38**	**68018.42**	**0.9779**
2023	Cauchy	67734.99	67748.03	0.5814
	Normal	**67521.69**	**67534.72**	**0.9766**

Table 43 – Criteria decision results for the best distribution, from 2019 – 2023, for other comprehensive income

In table 44, it is possible to notice that both the simulated minimum and maximum values have an irregular behaviour during these five years, with irregular increases and decreases. The maximum simulated value is registered in 2020 (820.35965 million), while the minimum is registered in 2019 (-1061.5991). Yet, these irregular changes do not affect the average in a significant way, since its range is only around 4 million during those five years.

	2019	2020	2021	2022	2023
Best Distribution	Normal	Normal	Normal	Normal	Normal
Distribution Parameters	mean = -145.2091 sd = 243.8377	mean = -149.9242 sd = 234.3679	mean = -146.7519 sd = 229.602	mean = -146.8172 sd = 217.2828	mean = -149.9745 sd = 207.0231
Simulated Data Minimum	-1061.59910	-922.04822	-955.04025	-922.73617	-886.19922
Simulated Data Maximum	743.71878	820.35965	644.93055	554.76688	579.08558
Simulated Data Average	-145.20905	-149.92421	-146.75186	-146.81720	-149.97451

Table 44 - Simulation data characteristics and limits, from 2019 – 2023, for other comprehensive income

6.11. Delta Book Value of equity

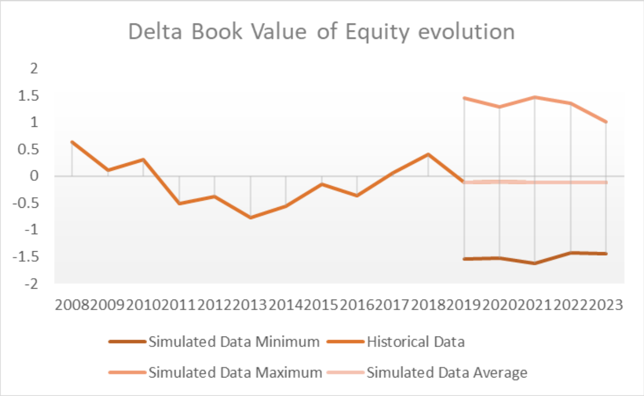
For this variable, the two possible probability distributions were Cauchy and Normal, considering that its historical data has both negative and positive values. As point out in the table below, in bold, the best probability distribution is the Normal, in every year.

	Top Distributions	AIC	BIC	Ks-test
2019	Cauchy	13972.28	13985.31	0.9778
	Normal	**5715.7**	**5728.74**	**0.2649**
2020	Cauchy	13308.34	13321.37	0.5947
	Normal	**5072.04**	**5085.07**	**0.999**
2021	Cauchy	11059.35	11072.38	0.3354
	Normal	**5013.72**	**5026.76**	**0.3923**
2022	Cauchy	10619.66	10632.7	0.9991
	Normal	**4600.37**	**4613.41**	**0.9828**
2023	Cauchy	8529.91	8542.95	0.7813
	Normal	**4054.99**	**4068.03**	**0.5738**

Table 45 – Criteria decision results for the best distribution, from 2019 – 2023, for ΔBVE

Both in graph 27 and table 46 are exposed the parameters and limits of the simulated results, using Normal distribution. The average values follow a linear pattern, unlike the minimum and maximum simulated values that have irregular patterns. In 2021, both the smallest and maximum simulate values are achieved with a respective value of -1.51627 million and 1.46344 million euros.

It is also important to notice that in every year there is the probability of almost 50% that the group registers a negative ΔBVE, meaning that the value of the assets diminish from one year to another.



Graph 27 - Simulation data average and limits, from 2008 – 2023, for ΔBVE

Table 46 - Simulation data characteristics and limits, from 2019 – 2023, for ΔBVE

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	mean = - 0.12097 sd = 0.4284	-1.54679	1.45675	-0.12097
2020	mean = -0 .1057 sd = 0.4017	-1.52051	1.28806	-0.10565
2021	mean = - 0.1142 sd = 0.3993	-1.61627	1.46344	-0.11420
2022	mean = - 0.1132 sd = 0.3832	-1.43218	1.35447	-0.11324
2023	mean = - 0.1226 sd = 0.3628	-1.43707	1.00374	-0.12258

6.12. Cost of equity

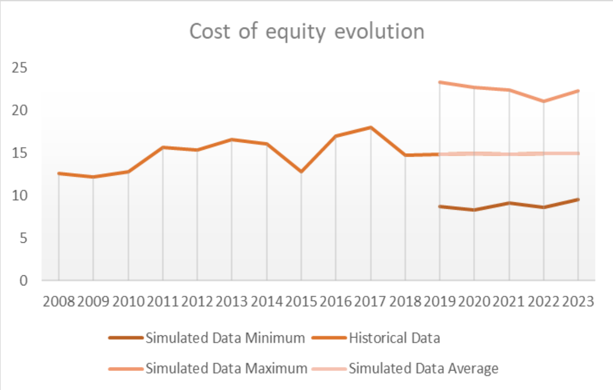
In cost of equity, all distributions, except Cauchy and Normal, were considered for the fitting process, given that the variable is a rate associated with a cost and, therefore, cannot be negative. As noted in bold in table 47, for all future five years, the best distribution is the Gamma.

	Top Distributions	AIC	BIC	Ks-test
2019	**Gamma**	**20669.39**	**20682.42**	**0.8276**
	Log Normal	20653.25	20666.28	0.00000
	Weibull	20946.63	20959.67	0.5461
2020	**Gamma**	**20211.77**	**20224.8**	**0.9728**
	Log Normal	20095.13	20108.17	0
	Weibull	20632.75	20645.78	0.8787
2021	**Gamma**	**19871.05**	**19884.09**	**0.3383**
	Log Normal	19719.17	19732.2	0.00000
	Weibull	20090.08	20103.12	0.7904
2022	**Gamma**	**19412.75**	**19425.78**	**0.9933**
	Log Normal	19571.82	19584.85	0
	Weibull	19886.44	19899.48	0.000434
2023	**Gamma**	**19204.79**	**19217.82**	**0.6675**
	Log Normal	19145.46	19158.49	0
	Weibull	19547.21	19560.25	0.5366

Table 47– Criteria decision results for the best distribution, from 2019 – 2023, for cost of equity

(The graphs – PP-Plot, QQ-Plot and density plot – who helped decide the top three distributions in table 47 can be seen in exhibit 5).

As showed in the table and graph below, the minimum value for simulated data has an irregular pattern during the five years in analysis, reaching its lowest value in 2020 of 8.32352%; while the maximum simulated data decreases until 2023, reaching, thus, its maximum four years before with a value of 23.32361%. Despite these changes and not being a formal parameter in Gamma distribution, the average rate continues not to vary much as excepted, having only a range of around 0.02%. The average rate also represents its respective year in the historical data list when estimating the values for the following years.



Graph 28 - Simulation data average and limits, from 2008 – 2023, for cost of equity

Table 48 - Simulation data characteristics and limits, from 2019 – 2023, for cost of equity

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	shape = 60.134 rate = 4.03557	8.77717	23.32361	14.90101
2020	shape = 66.19226 rate = 4.43492	8.32352	22.67924	14.92521
2021	shape = 70.73203 rate = 4.74483	9.15530	22.46970	14.90733
2022	shape = 77.62797 rate = 5.20575	8.62408	21.08842	14.91198
2023	shape = 81.08919 rate = 5.43404	9.51940	22.32416	14.92235

6.13. Growth Rate

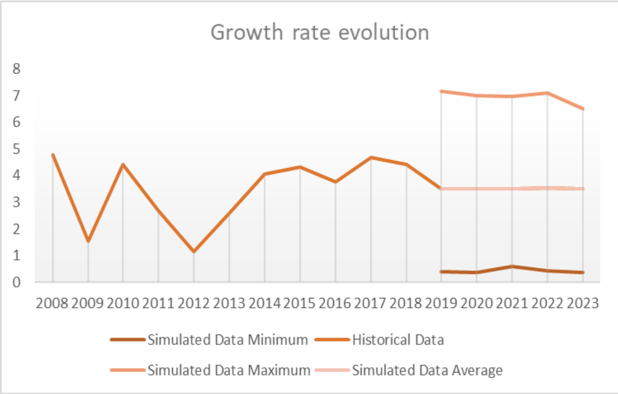
Despite the possibility of negative values in both parcels of growth rate, we only considered in the fitting process non-negative distributions, since we assumed that it follows the behaviour of the economy of each country that has an expected tendency of long-term growth.
As exposed in the table below, the Weibull distribution was the best choice in all years.

	Top Distributions	AIC	BIC	Ks-test
2019	Gamma	17208.2	17293.23	0.6403
	Log Normal	18194.66	18207.69	0
	Weibull	**15302.03**	**15315.06**	**0.8587**
2020	Gamma	17041.98	17055.01	0.9192
	Log Normal	23209.01	23222.04	0
	Weibull	**15001.21**	**15014.24**	**0.6967**
2021	Gamma	16618.35	16631.39	0.6148
	Log Normal	17468.4	17481.44	0
	Weibull	**14493.51**	**14506.55**	**0.497**
2022	Gamma	16219.42	16232.45	0.8013
	Log Normal	17118.31	17131.34	0
	Weibull	**14048.13**	**14061.16**	**0.5375**
2023	Gamma	15819.16	15832.19	0.6499
	Log Normal	16801.84	16814.87	0
	Weibull	**13745.97**	**13759.01**	**0.3639**

Table 49 – Criteria decision results for the best distribution, from 2019 – 2023, for growth rate

(The graphs – PP-Plot, QQ-Plot and density plot – who helped decide the top three distributions in table 49 can be seen in exhibit 6).

Both in graph 29 and table 50, the minimum value for simulated data only rises in 2021 since on the previous year it reaches minimum (0.3656%); while the maximum simulated data registers its maximum (7.17568%) in 2019, since it is followed by to decrease on the following years. Despite these changes and not being a formal parameter in Weibull distribution, the average rate continues to not vary much as excepted, having only a range of around 0.04%. The average rate also represents its respective year in the historical data list when estimating the values for the following years, except for 2023.



Graph 29 - Simulation data average and limits, from 2008 – 2023, for cost of equity

Table 50 - Simulation data characteristics and limits, from 2019 – 2023, for growth rate

	Distribution Parameters	Simulated Data Minimum	Simulated Data Maximum	Simulated Data Average
2019	shape = 3.46854 scale = 3.90701	0.41243	7.17568	3.51323
2020	shape = 3.56258 scale = 3.87568	0.36560	7.00179	3.49045
2021	shape = 3.76644 scale = 2.87231	0.60577	6.95611	3.49749
2022	shape = 3.98312 scale = 3.89286	0.44147	7.10176	3.52854
2023	shape = 4.08197 scale = 3.85223	0.37726	6.52824	3.49711

6.14. Final Result

After presenting all results, equation 40 can, finally, be solved. Remembering it from section 5.1., it was just like this:

$$\boldsymbol{Equity_{2018}} = \frac{\boldsymbol{ECF_{2019}}}{(1+\boldsymbol{Re})^1} + \frac{\boldsymbol{ECF_{2020}}}{(1+\boldsymbol{Re})^2} + \frac{\boldsymbol{ECF_{2021}}}{(1+\boldsymbol{Re})^3} + \frac{\boldsymbol{ECF_{2022}}}{(1+\boldsymbol{Re})^4} + \frac{\boldsymbol{ECF_{2023}}}{(1+\boldsymbol{Re})^4(\boldsymbol{re}-\boldsymbol{g})} \quad \textbf{(45)}$$

Now,

$$\boldsymbol{Equity_{2018}} = \frac{58.7465}{(1+0.1490)^1} + \frac{7.2495}{(1+0.1492)^2} + \frac{-1.4319}{(1+0.1491)^3} + \frac{-47.9468}{(1+0.1491)^4} + \frac{86.2462}{(1+0.1492)^4(0.1492-0.035)} \quad (46)$$

$$= 460.9442 \text{ million}$$

Despite having a positive value, the 2021 and 2022 ECF's should be pointed out. In that case, the company net income does not allow for full coverage of the needed equity reinforcement, therefore, the company will be compelled to make seasoned equity offerings. This situation is likely to happen due to the Basel IV rules, who start in 2022. In 2023, the perpetuity year, that problem should be solved, possibly by this reinforcement of equity capital.

In 31th December 2018, the equity of the group available to shareholders is €460.9442 million and with 15,113,989,952 shares outstanding, the Equity Per Share (EPS) is €0.03. Considering that the real price of the same share was €0.23, the same is overvalued and the recommendations for the investors should be to sell it.

If we consider the share price of €0.23, the market value of equity will be around €3,476.22 million. In 2019, to overcome the average return of 14.90101% that the shareholders expect over the results, formed by the sum of net income and OCI, since there is no payment of dividends, they should be more than €517.992 million. Hence, the net income should have a minimum value of €663.2 million, which would have just a 13.48% of possibility, considering the range of 2019's net income.

7. Conclusion

In this report, we attempted to valuate Millennium BCP's equity and reach a reliable and accurate share price, at the end of 2018. For that purpose, we used the Discounted Cash Flow valuation method, using an equity cash flow equation adapted to the characteristics of financial institutions, like Millennium BCP. The future values were obtained using probabilistic and simulation methods, calculated by R Studio software, whose results were based on historical data from 2008-2018.

After performing all fitting and simulation processes, the discounted cash flow approach valuation obtained was €460.9442 million, resulting on a share price of €0.03. By considering the closing price in 31th December 2018, marked at €0.23 per share, the most accurate recommendation to the investor is to sell.

When comparing our valuation with the real one, with share prices distanced by €0.2 (or 20 cents), showing that the results are not in line, possibly due to the different methods to calculate the variables, added to the existence of some limitations and choices made that could have impacted our result, such as:

- The possibility of having more fitted and adequate results with other probabilistic distributions not considered in this report;
- Not having restricted limits for the future simulated values in all variables, in order to consider possible future extreme events, such an unexpected decrease of the growth rate or on the net operating revenues' growth rate. On the other hand, the existence of variables with restricted simulated results did not allow the possibility of any extreme change outside those limits;
- Not considering future purchases or sales of the Millennium BCP's main assets, which could increase the equity, in the case of a large sale, or diminish it in the case of a good purchase deal, nor considering any strategic exit from any location;
- For the calculation of delta book value of equity, in all future years, the ratio of assets to be retained as equity considered was always 10%, 2% more than the Basel III minimum total capital requirements. However, starting in 2022, Basel IV will enter in practice, where the minimum total capital requirements will be between 10% and 15%;
- The assumption of a cost of equity of 0% for all countries that do not have subsidiaries but still contribute to the results of the group, due to lack of financial information from the bank;

- The possibility of future dividends paid to shareholders. As of 2018, the group has not paid dividends for eight consecutive years, however, if the board decides to restart the payment the delta Book Value of Equity will reduce in the same amount because the money will be extracted from that section.

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9. Appendixes

Exhibit 1 – Milllenium BCP's consolidated income statement, from 2008- 2018

Please note that the sections from each income statement were adapted, since throughout the years there has been some changes caused by the change in the version of the international financial reporting stardards (IFRS) in force. All values and respective sections and definitions of a year with a change were review based on the results from the previous year, since the group did not change the accounting sections when comparing the consecutive years.

To conclude, this table was created with the purpose of maintain the definitions and consistency while being the smallest one possible, to allow the evaluation and simulation of the minimum variables possible filled as much as possible.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Net interest income	**1721.0**	**1334.2**	**1516.9**	**1579.3**	**998.0**	**848.1**	**1116.2**	**1301.6**	**1230.1**	**1391.3**	**1423.6**
Dividends from equity instruments	36.8	3.3	35.9	1.4	3.8	3.7	5.9	11.9	7.7	1.8	0.6
Net fees and commission income	740.4	731.7	811.6	789.4	655.1	663.0	680.9	692.9	643.8	666.7	684.0
Other operating income	280.2	249.8	367.3	204.4	391.9	80.4	154.2	173.7	101.8	45.3	29.1
Net gains or losses arising from trading and hedging activies	-262.1	-24.5	72.1	3.3	44.9	184.1	302.4	421.7	138.5	103.0	0.0
Net gains or losses arising from available for sale financial assets	0.0	0.0	0.0	0.0	0.0	-0.3	-14.5	0.0	0.0	0.0	0.0
Net gains or losses arising from financial assets held to maturity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	49.4
Net gains or losses from derecognition of financial assets at fair value through other comprehensive income	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0	4.2	8.5
Net gains from insurance activity	57.6	41.1	17.5	-22.8	-43.7	-55.6	-53.3	-110.5	-104.5	-110.6	-135.9
Other net income from non banking activity	17.4	16.2	16.6	27.0	20.1	20.5	19.3	18.9	0.0	0.0	0.0
Net operating revenues	**2591.3**	**2352.0**	**2837.8**	**2581.8**	**2070.0**	**1743.8**	**2211.06**	**2510.2**	**2022.5**	**2101.7**	**2059.4**
Staff costs	915.3	865.3	831.2	953.6	751.5	767.5	635.6	616.1	356.6	526.6	592.8
Other administrative costs	642.6	570.2	601.8	584.5	501.7	459.7	448.5	423.8	373.6	374.0	376.7
Amortization and Depreciation	112.8	104.7	110.2	96.1	68.1	68.1	65.5	66.6	49.8	53.6	57.7
Operating costs	**1670.8**	**1540.3**	**1543.2**	**1634.2**	**1321.2**	**1295.2**	**1149.6**	**1106.5**	**780.0**	**954.2**	**1027.2**
Profit before impairment and provisions	**920.5**	**811.7**	**1294.5**	**947.6**	**748.8**	**448.6**	**1061.5**	**1403.6**	**1242.5**	**1147.5**	**1032.2**
Loans impairment	-544.7	-560.0	-713.3	-1331.9	-969.6	-820.8	-1107.0	-833.0	-1116.9	-623.7	-465.5
Other impairment and provisions (including goodwill)	-44.5	-97.4	-227.8	-825.1	-349.6	-465.8	-209.3	-161.3	-481.1	-301.1	-135.6
Profit before income tax	**331.3**	**154.3**	**353.5**	**-1209.4**	**-570.5**	**-838.0**	**-254.8**	**409.3**	**-355.5**	**222.7**	**431.1**
Income tax	-84.0	-46.2	-14.3	458.9	132.1	210.8	97.7	-56.4	381.9	-30.2	-138.0
Income arising from discontinued operations	0.0	0.0	0.0	0.0	-730.3	-45.0	-40.8	14.6	45.2	1.2	-1.3
Other items	10.7	141.2	64.7	-12.3	31.5	25.5	81.4	-6.6	74.2	95.8	127.1
Non-controlling interests	56.8	24.1	59.3	85.9	81.8	93.7	110.1	125.6	121.9	103.2	117.8
Net income to shareholders	**201.1**	**225.2**	**344.5**	**-848.6**	**-1219.1**	**-740.4**	**-226.6**	**235.3**	**23.9**	**186.4**	**301.1**

Exhibit 2 – Growth Rate

Knowing that the real GDP measures the pure growth of an economy during a certain period, while nominal GDP measures the total growth of the economy including the growth due to the change in prices, therefore there is the need to add a new variable – called deflator.

Thus, the nominal GDP is calculated in the following way:

$$Real\ GDP_{\ t} = \frac{Nominal\ GDP_t}{Inflation\ Deflator_t} \qquad (47)$$

After applying logarithms to that equation with the purpose of turning the variables into growth rates the result is the following:

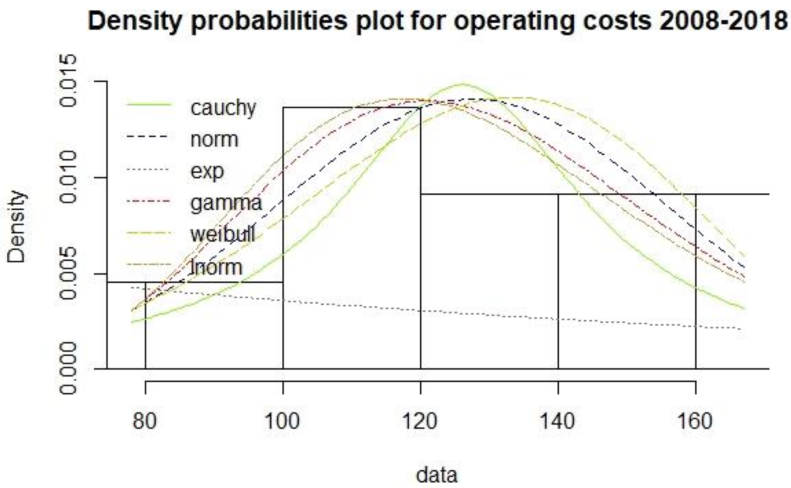
$$\ln(\text{Real GDP}_{t}) = \ln\left(\frac{\text{Nominal GDP}_t}{\text{Inflation Deflator}_t}\right) \Rightarrow \quad (48)$$

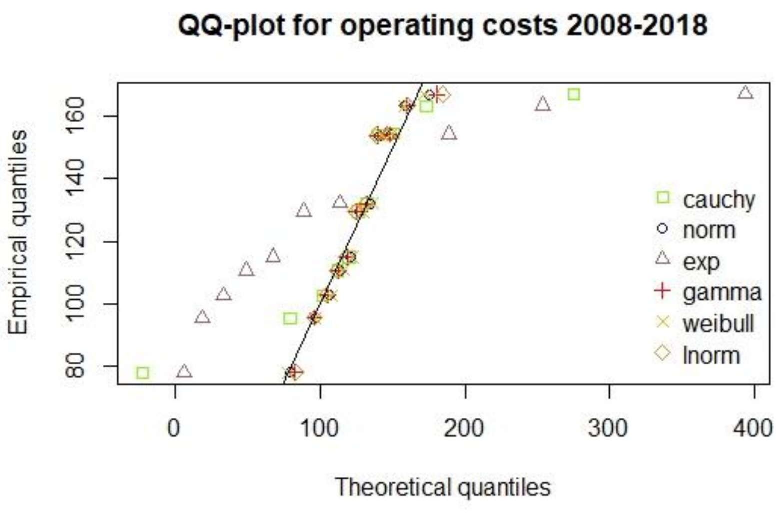
$$\ln(\text{Real GDP}_{t}) = \ln(\text{Nominal GDP}_t) - \ln(\text{Inflation}_t) \Rightarrow \quad (49)$$

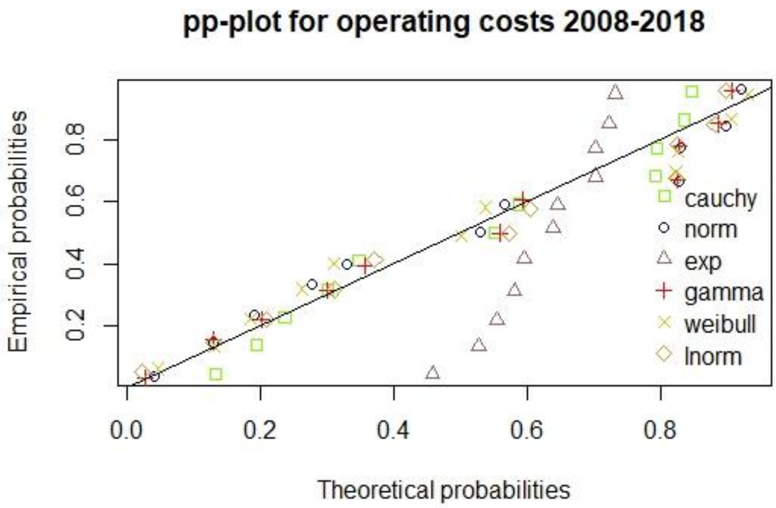
$$\text{GDP's Nominal GrowthRate}_t = \text{GDP's Nominal Growth Rate}_t + \text{Inflation Rate}_t \quad (50)$$

Exhibit 3 – Fitting Process for Operating Costs' all possible distributions

<u>For 2019 Fitting Process</u>







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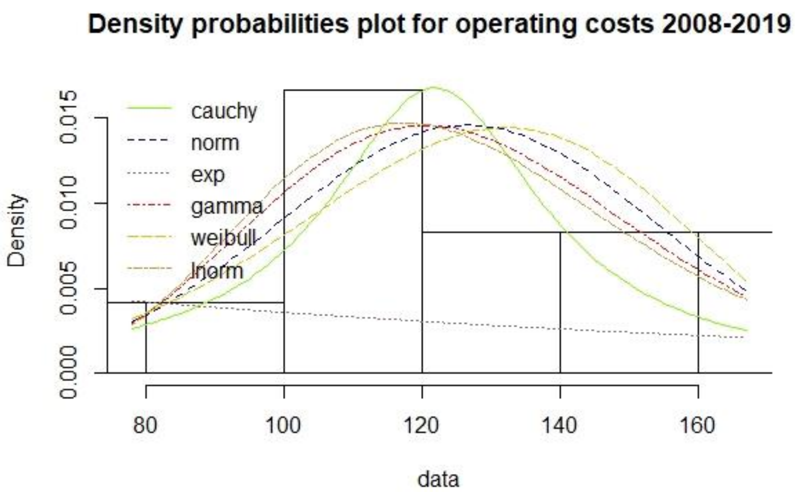

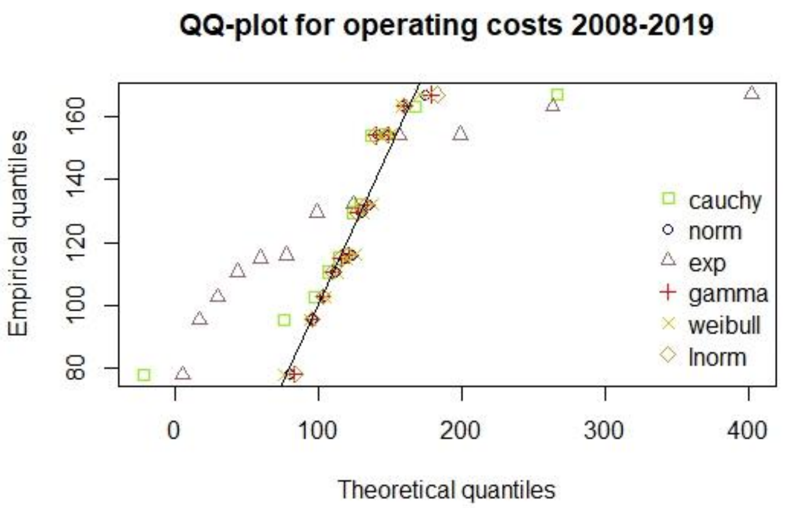

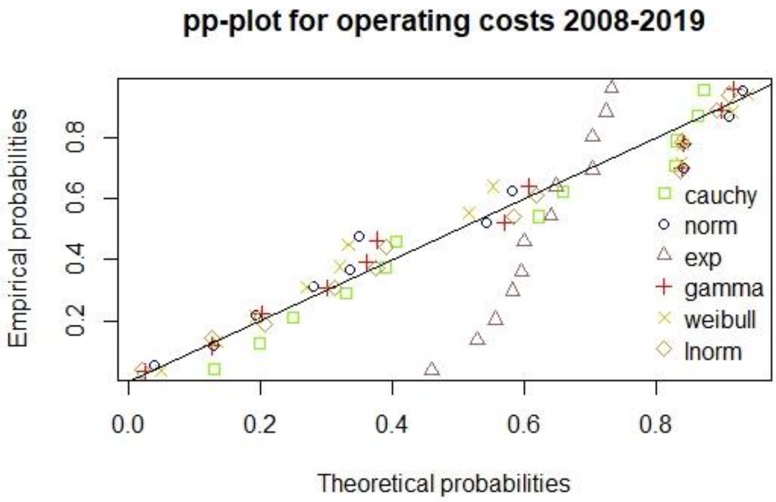

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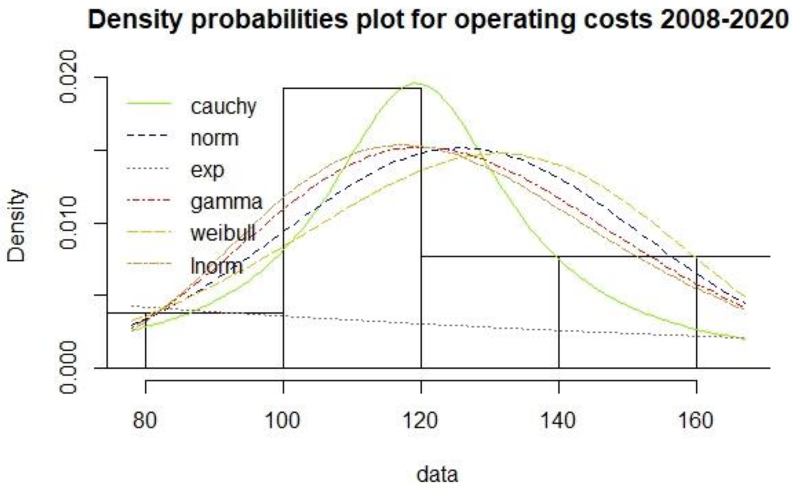
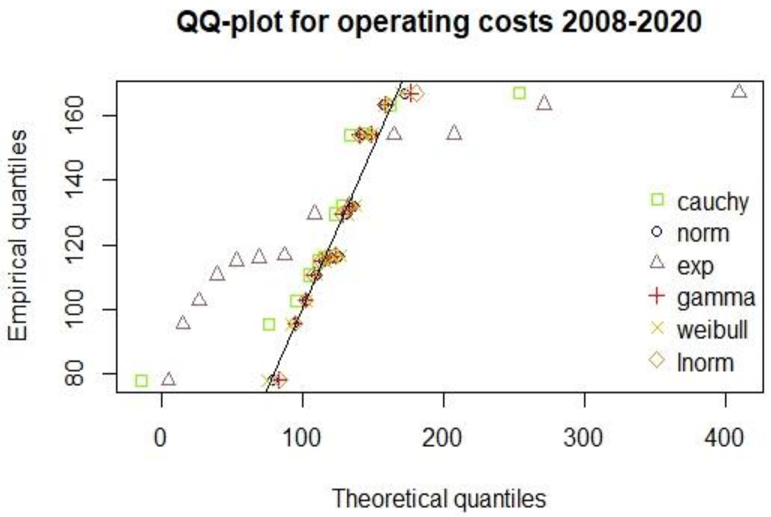
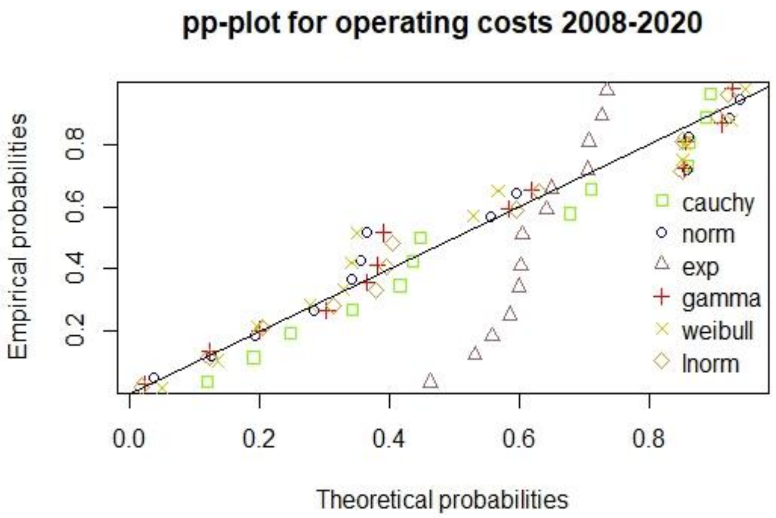
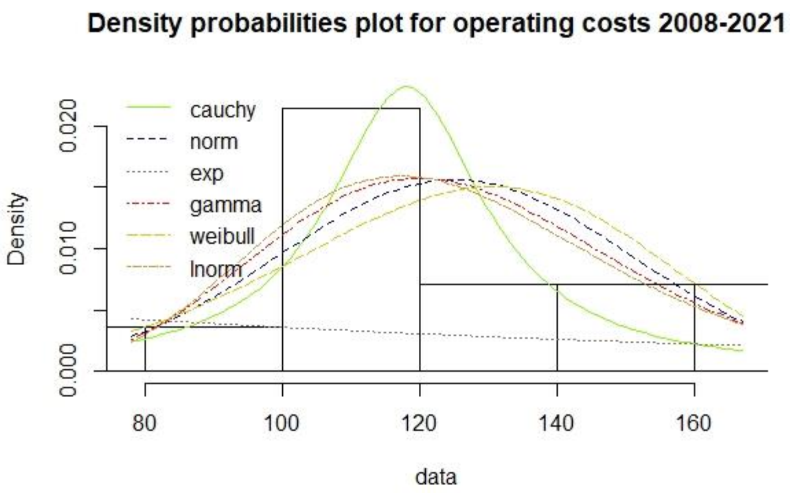
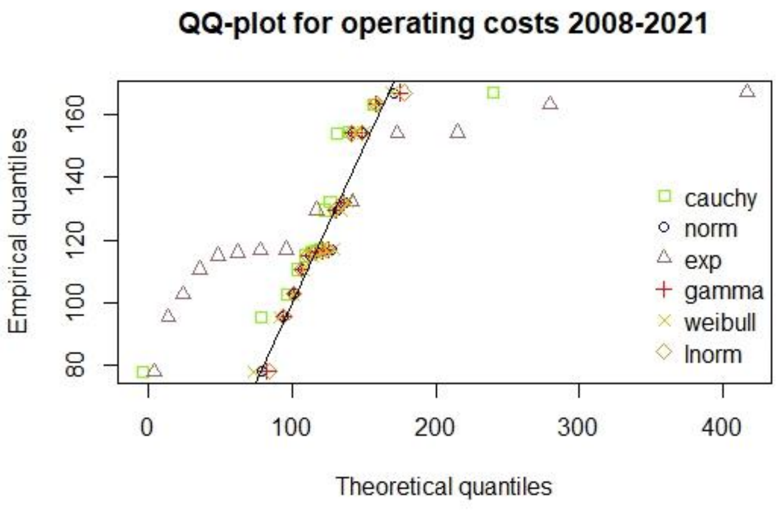


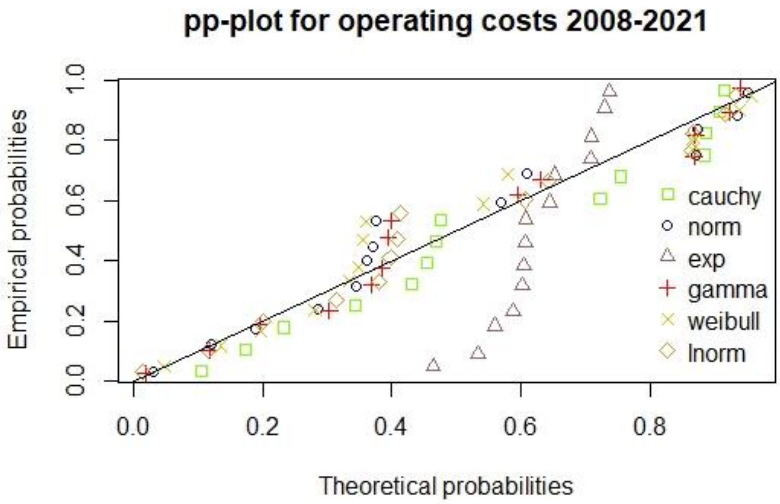




For 2022’s Fitting Process







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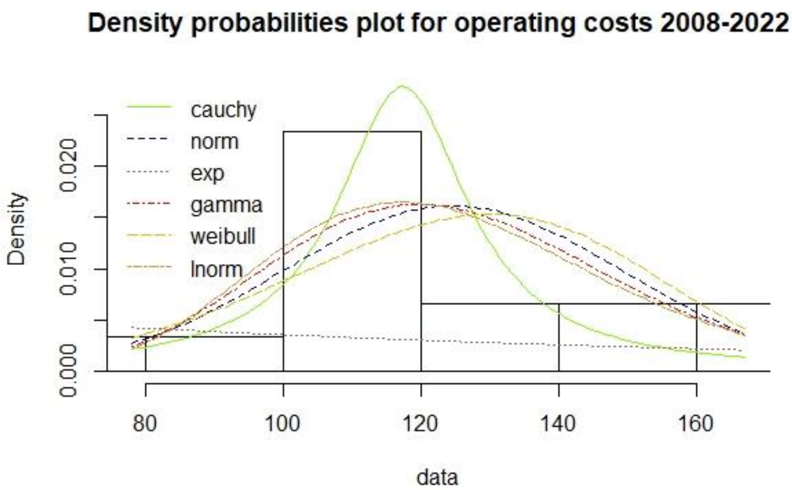
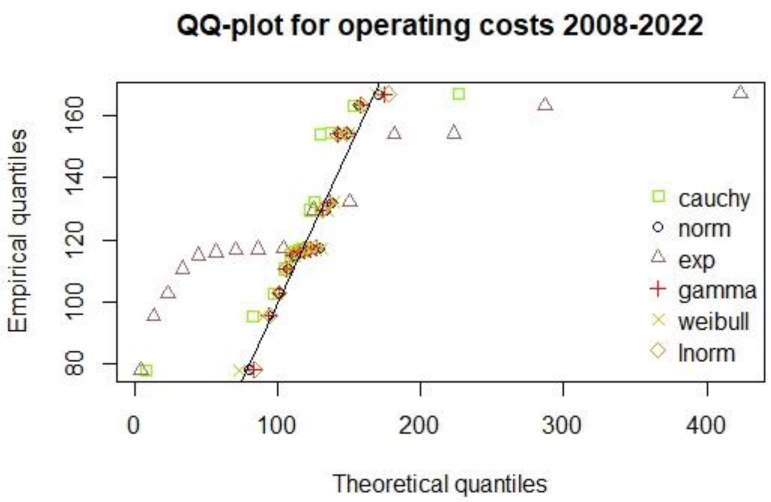




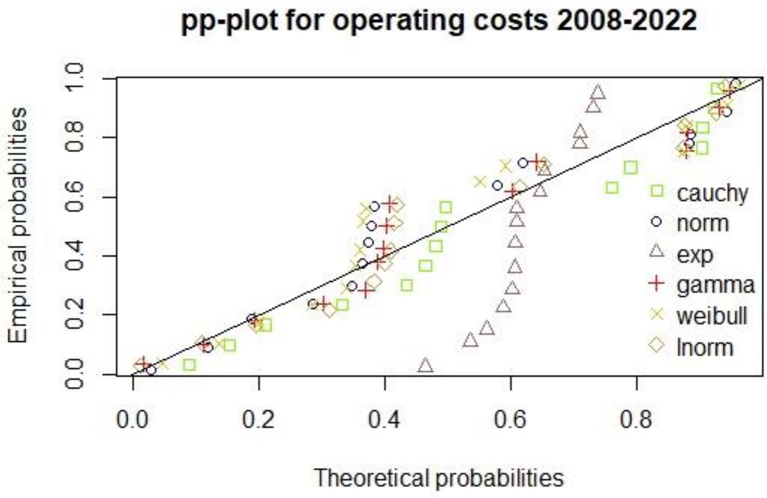
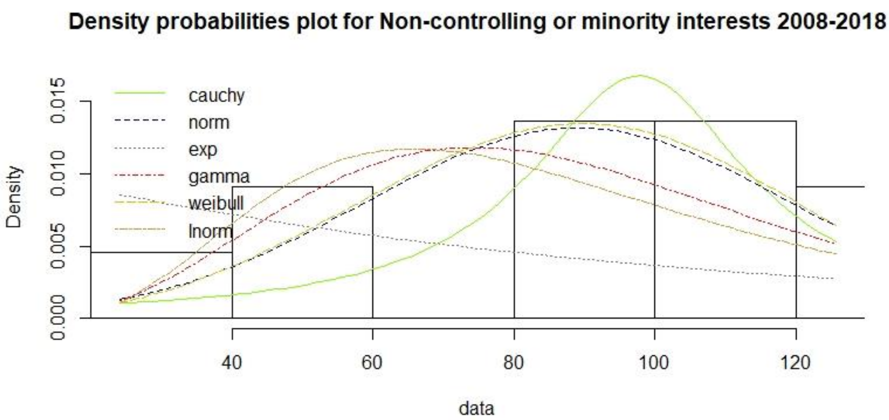
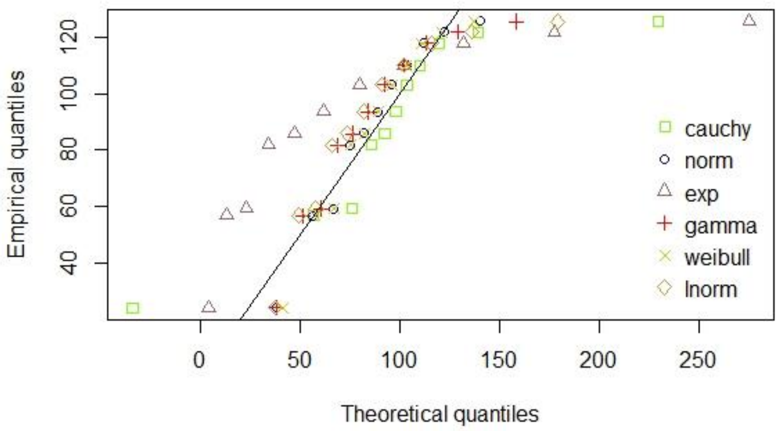


Exhibit 4– Fitting Process for Non-controlling or minority interests' all possible distributions

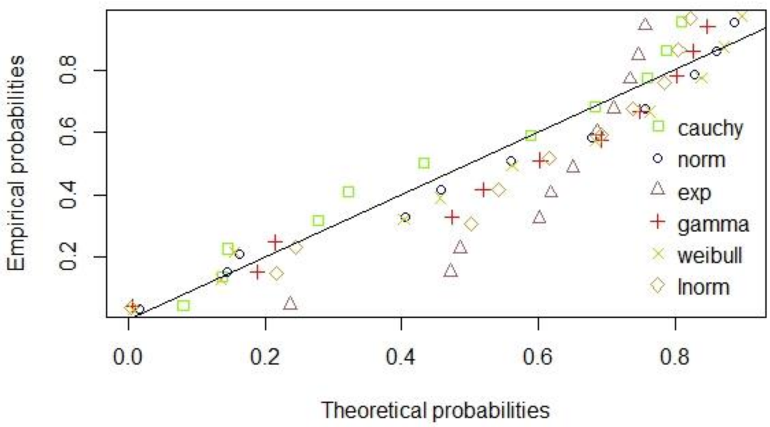
<u>For 2019's Fitting process</u>



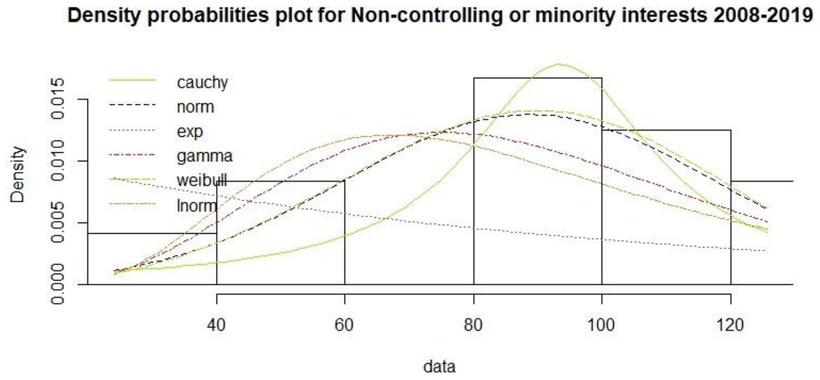
QQ-plot for Non-controlling or minority interests 2008-2018

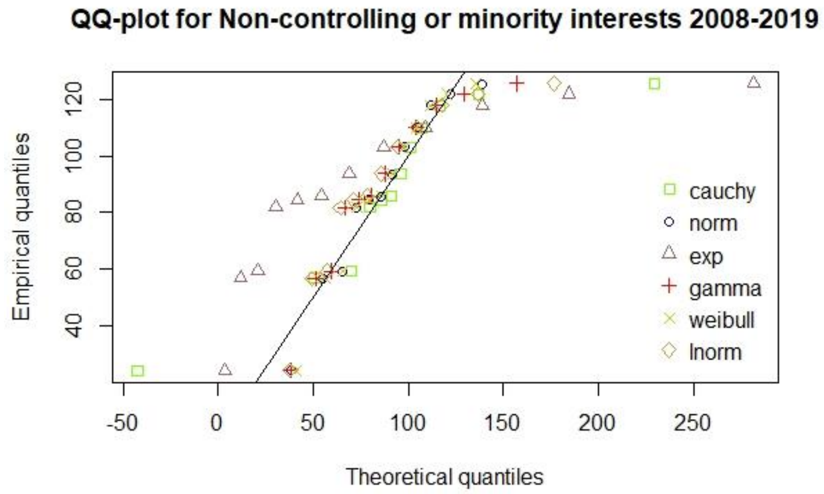


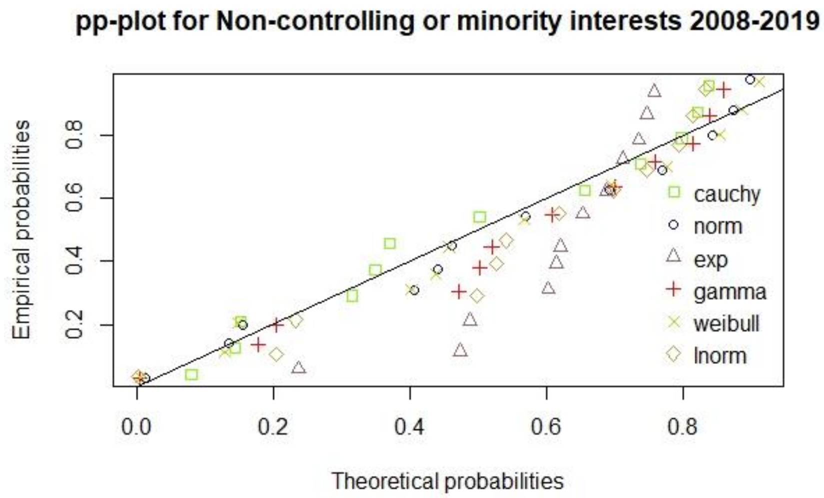
pp-plot for Non-controlling or minority interests 2008-2018



For 2020's Fitting Process







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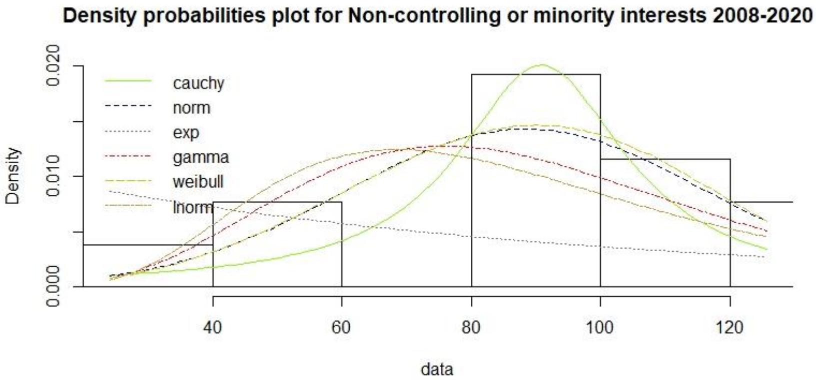
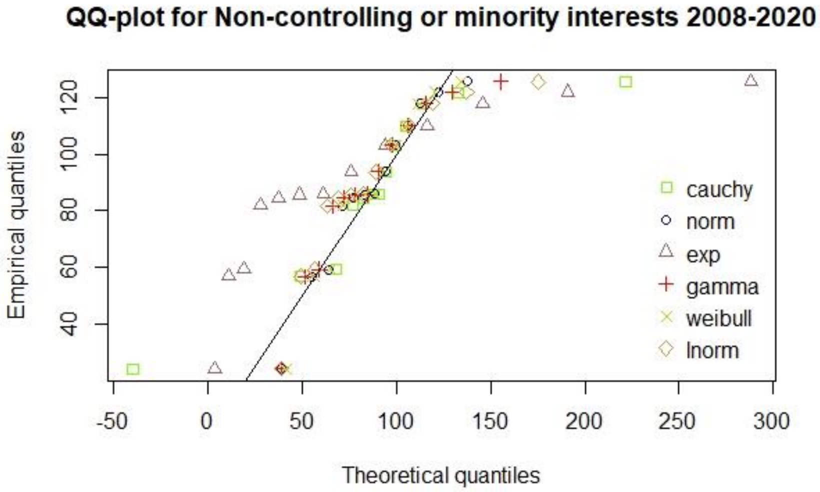
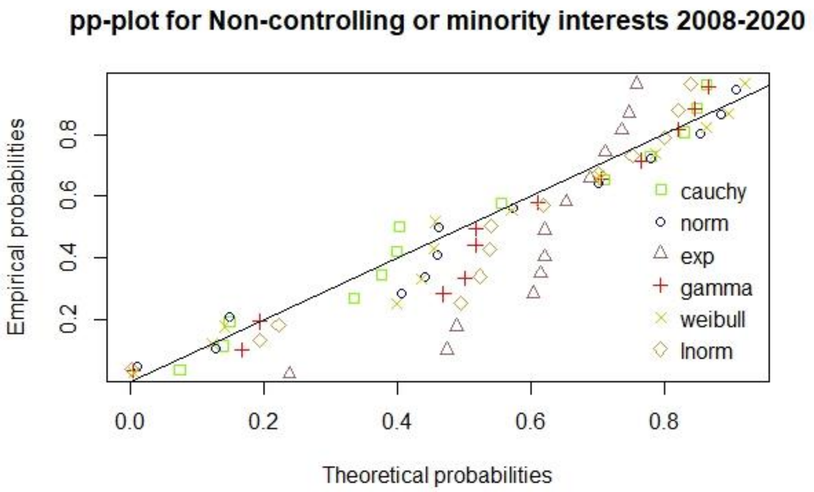
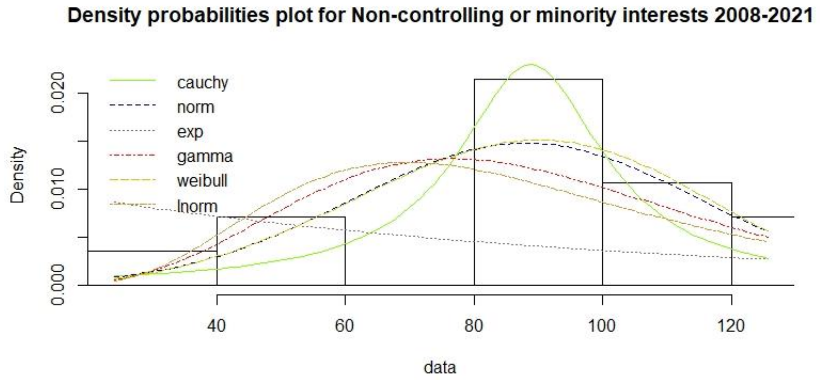
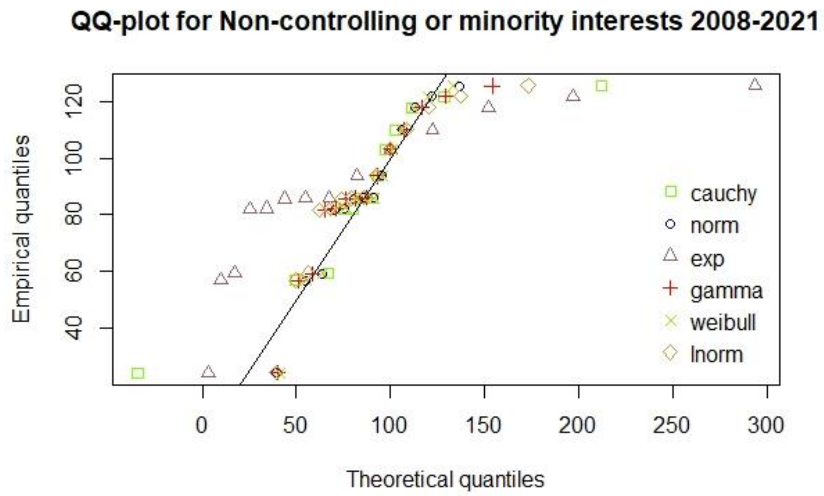


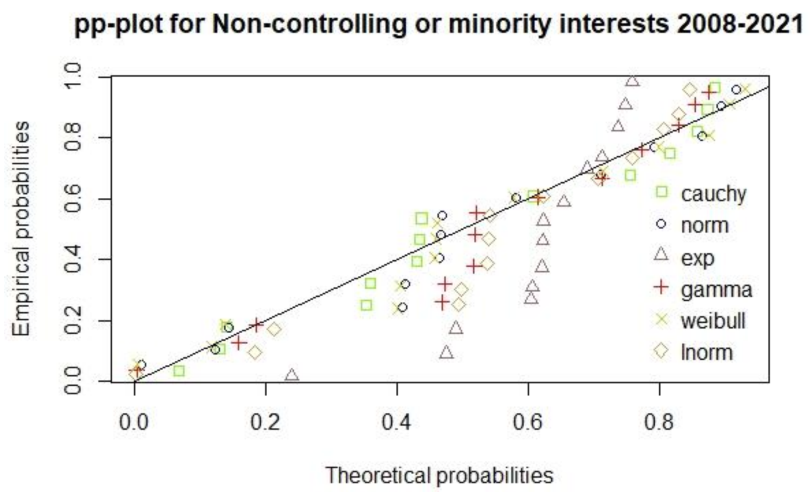




For 2022's Fitting Process







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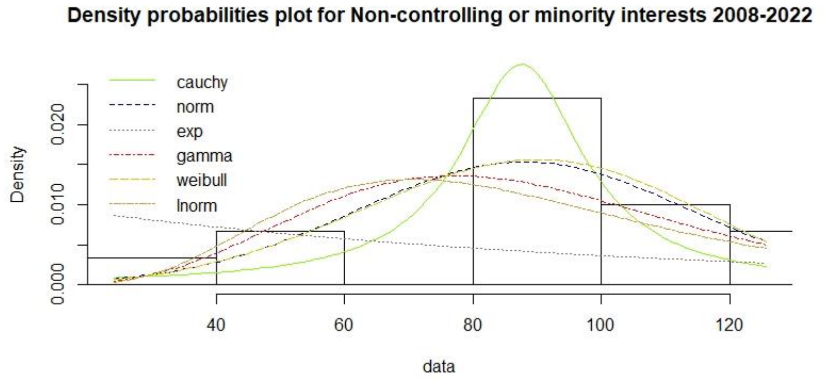
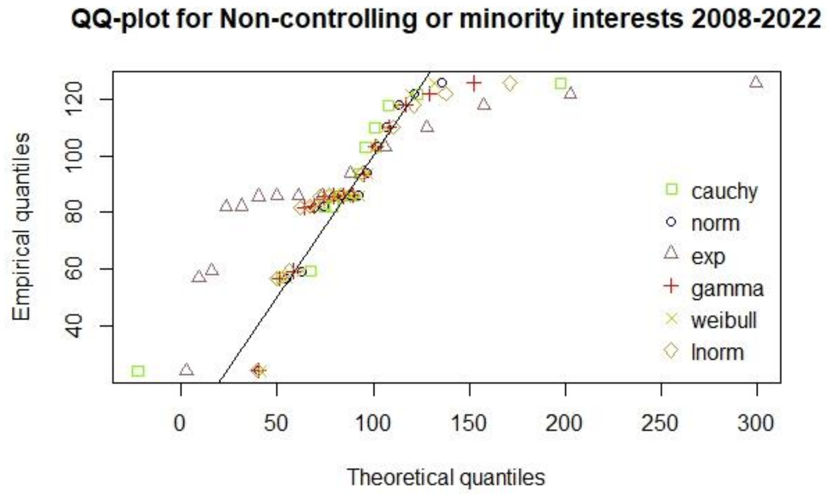




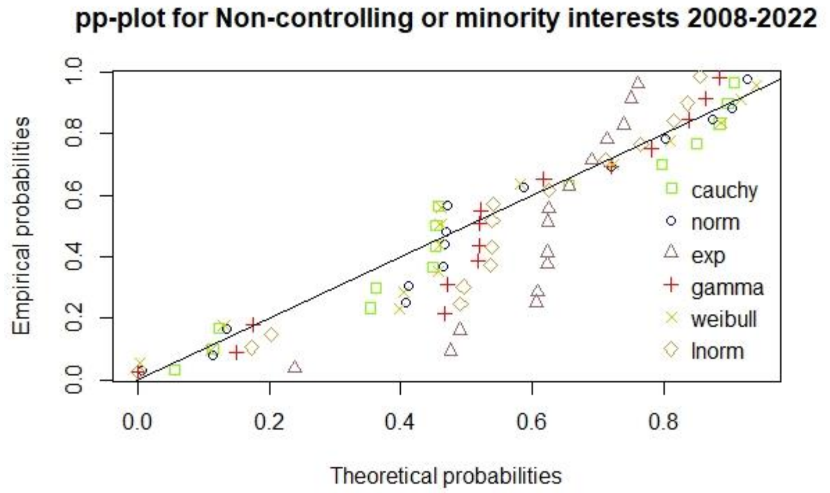
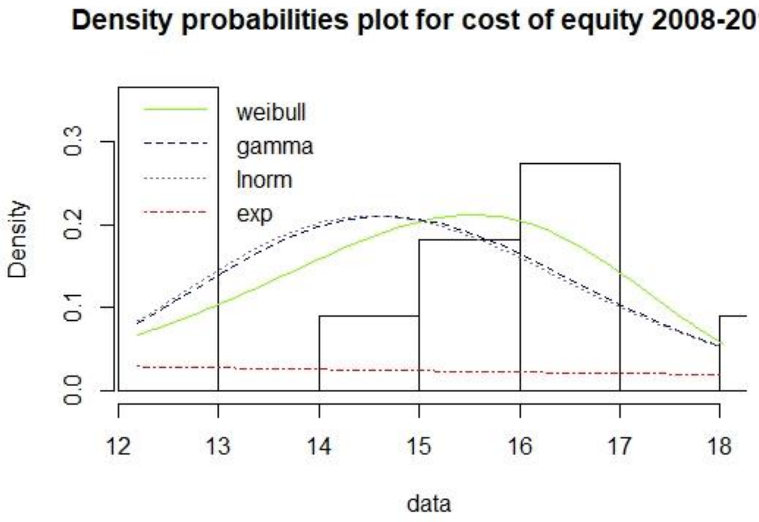
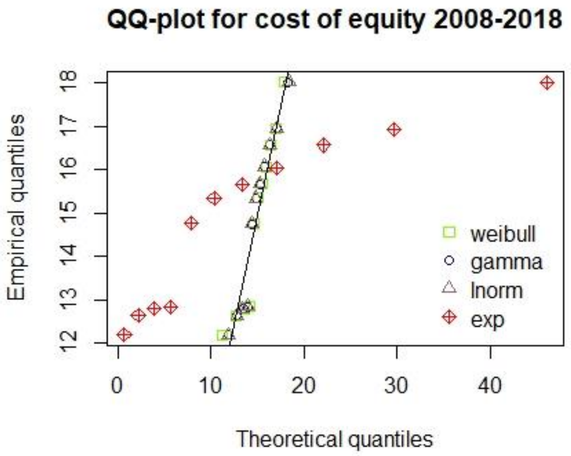
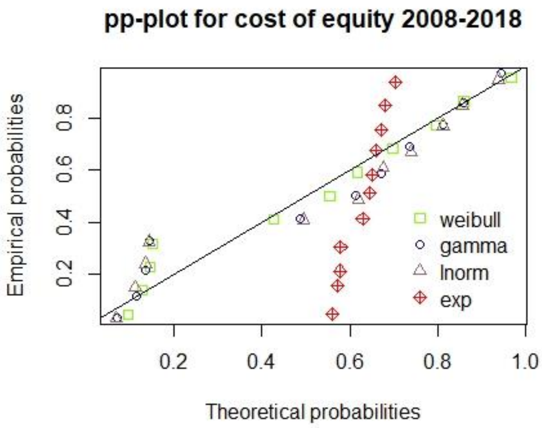


Exhibit 5 – Fitting Process for cost of equity's all possible distributions

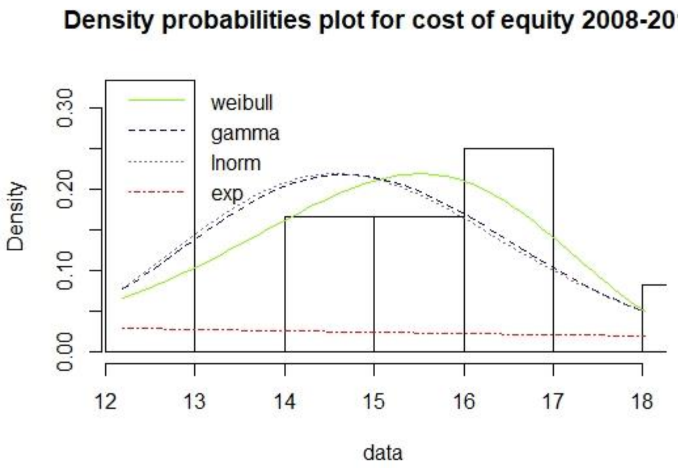
For 2019's Fitting process

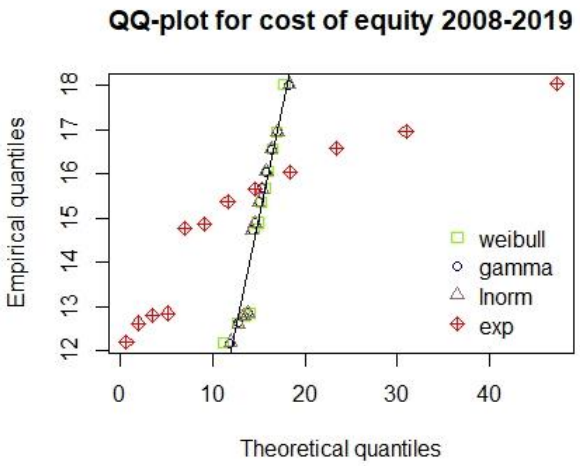


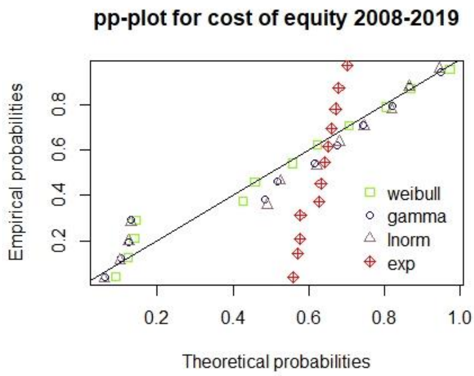




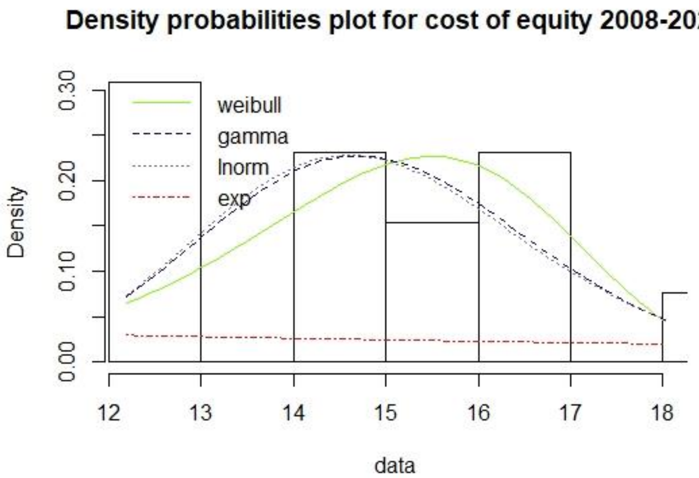
For 2020's Fitting process

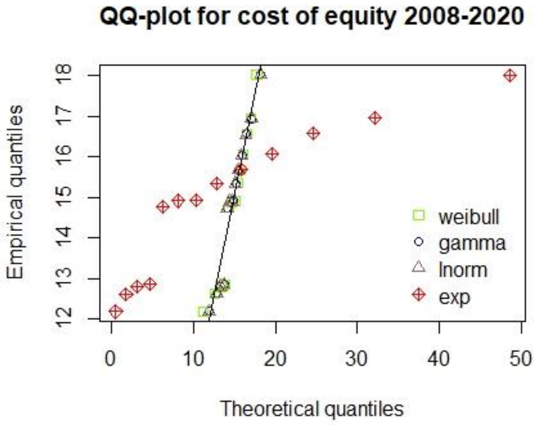


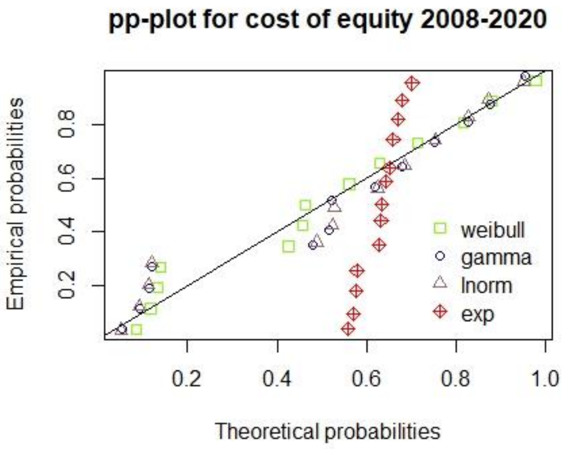




2021's Fitting Process







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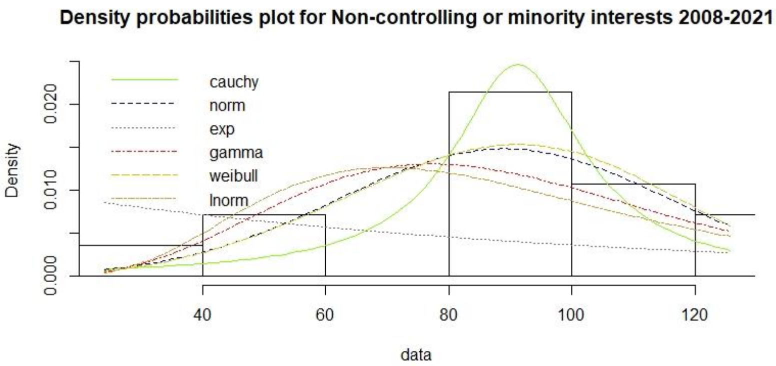

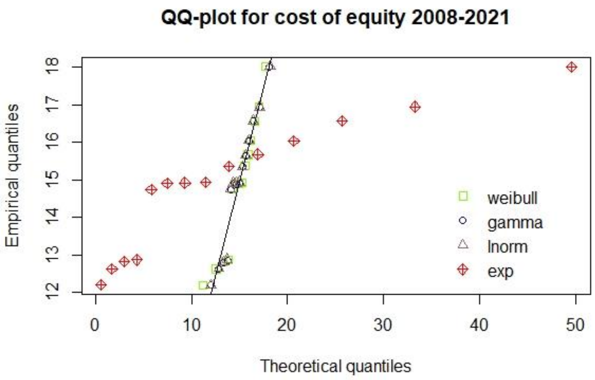

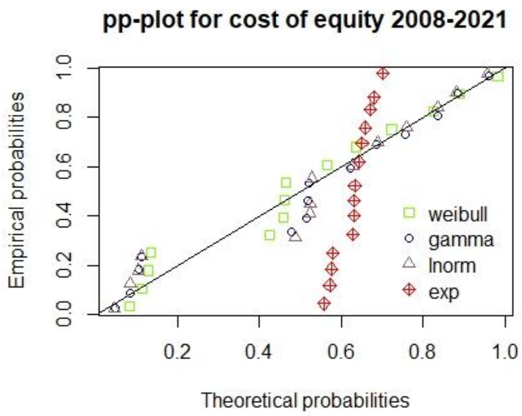

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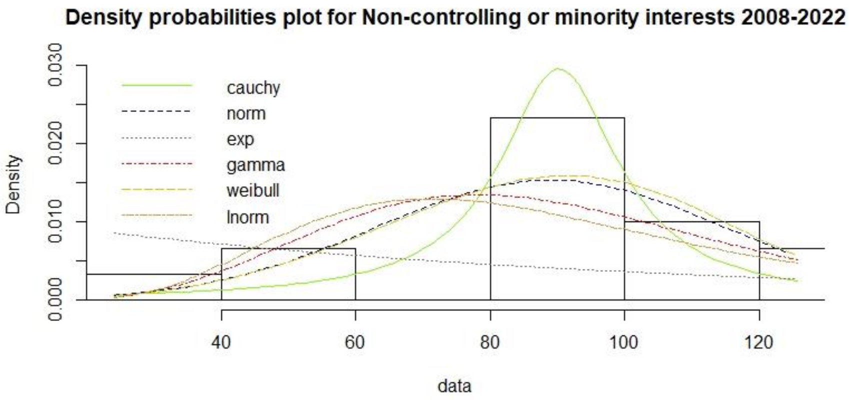
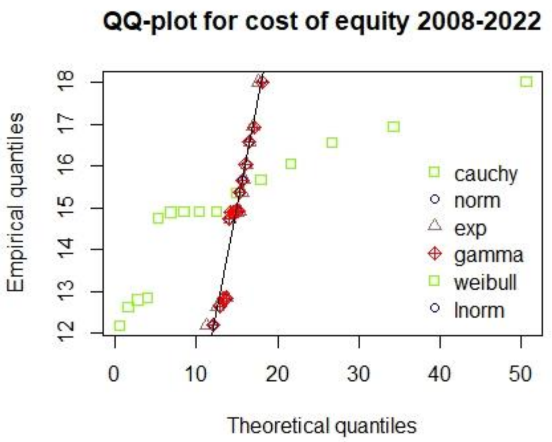




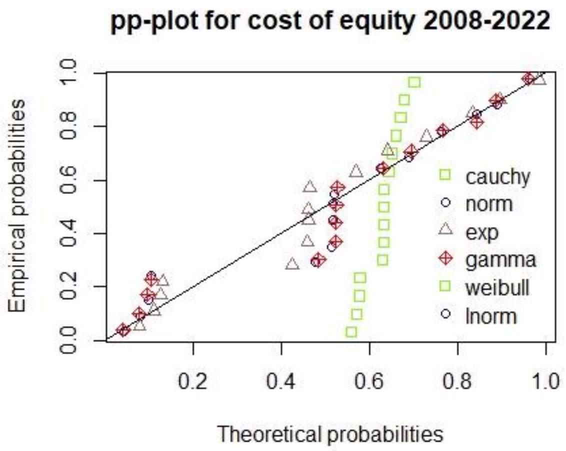
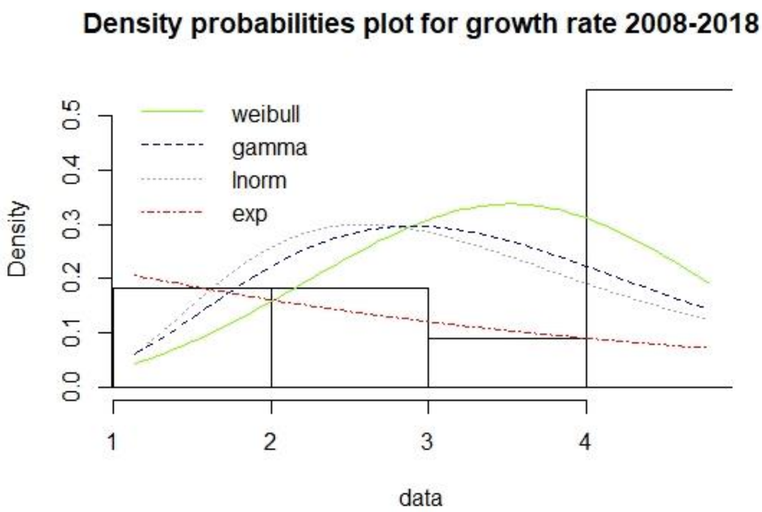
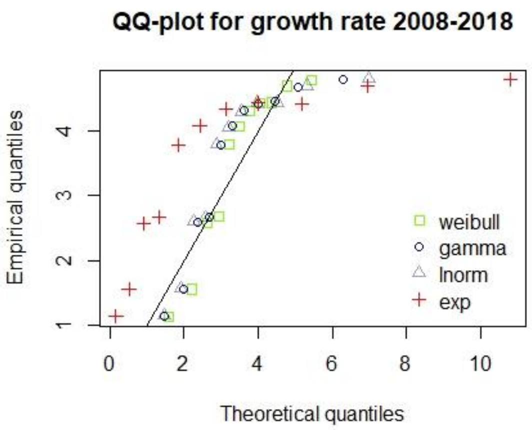
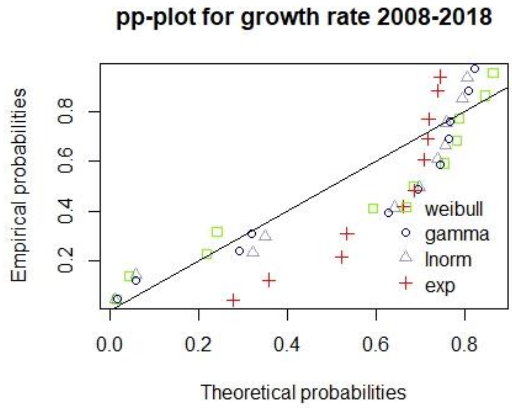


Exhibit 6 – Fitting Process for growth rate's all possible distributions

For 2019's Fitting process







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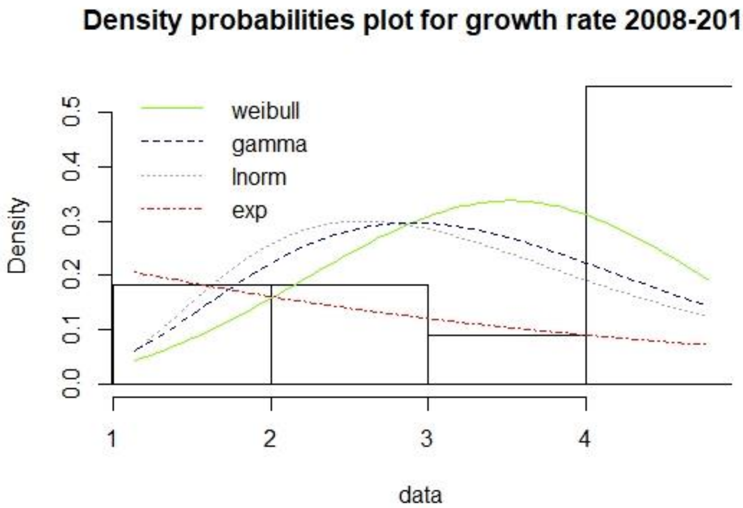
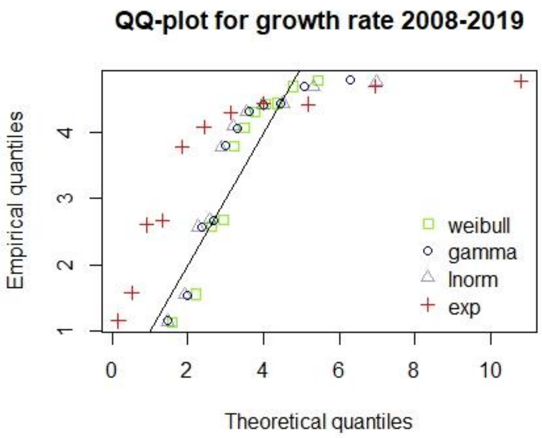
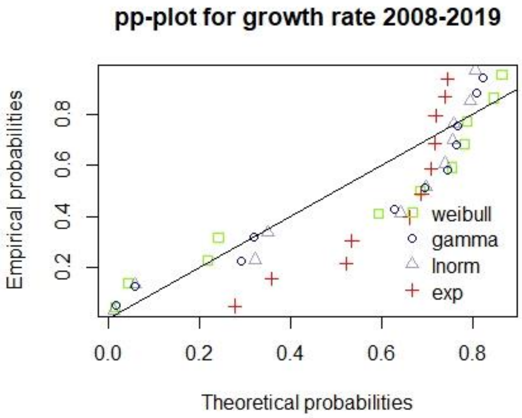
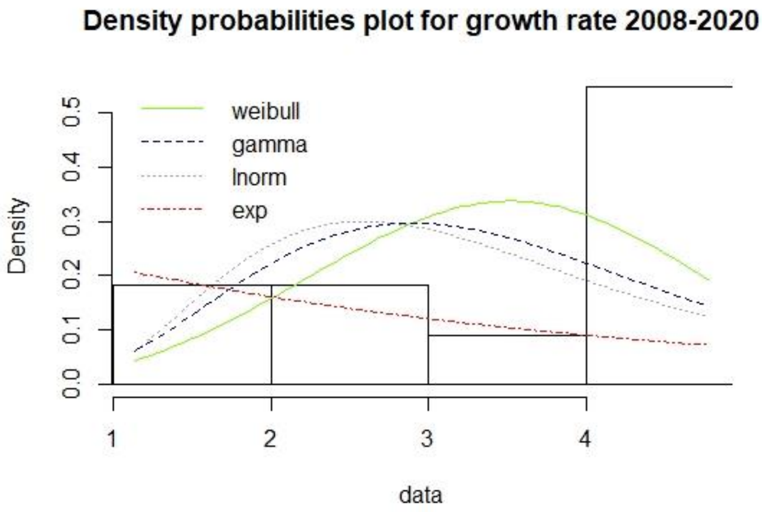
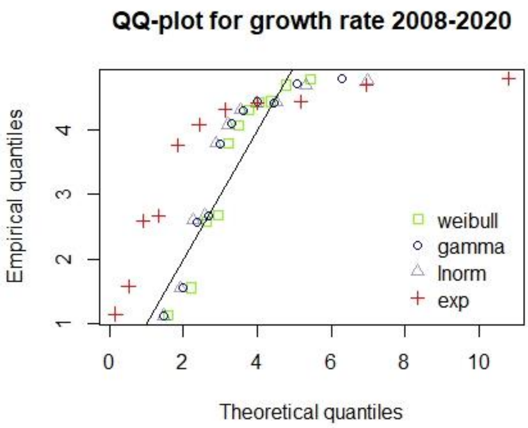


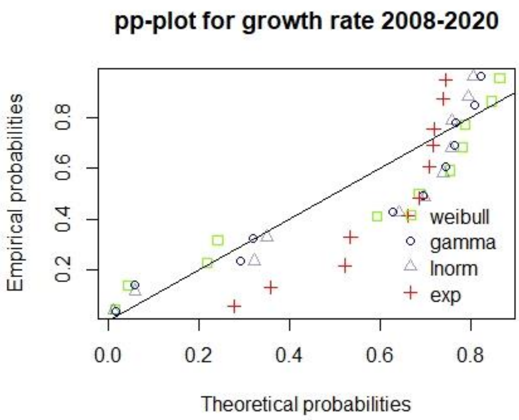




For 2021’s Fitting process







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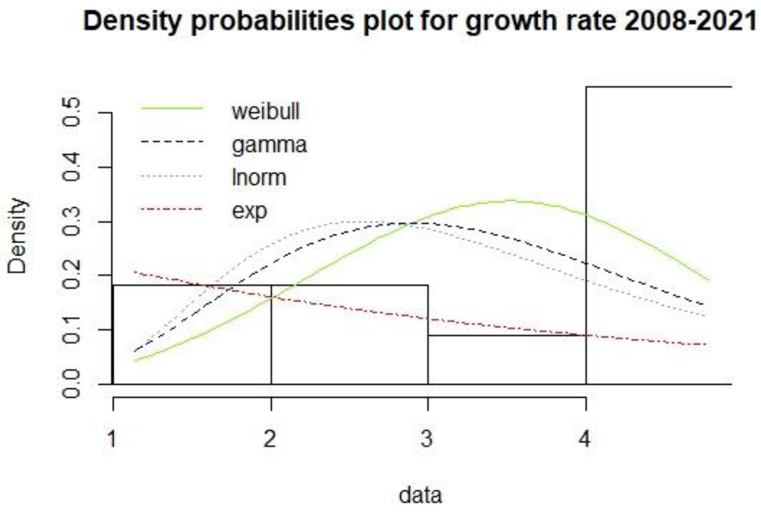

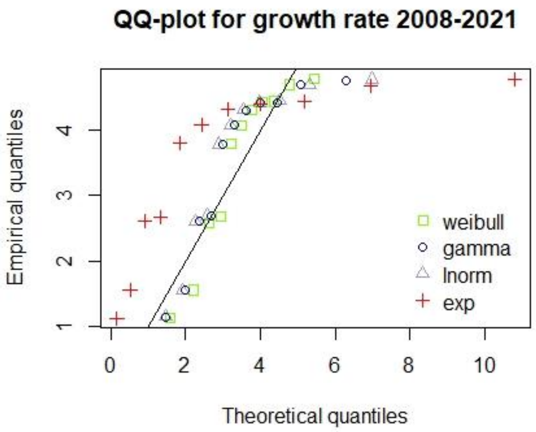

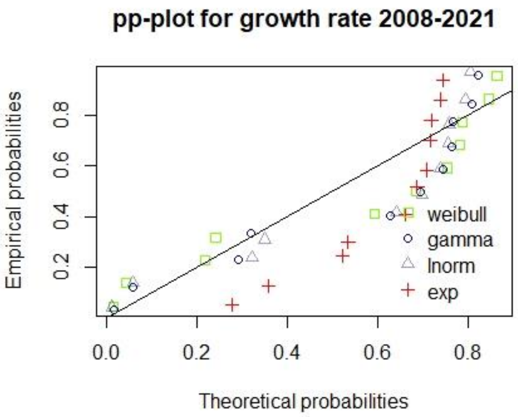

For 2023's Fitting process