ISCTE IUL
Instituto Universitário de Lisboa
Lisbon University Institute

IUL School of Social Sciences
Department of Political Economy

Volatility Derivatives – Expected Option Returns

Dissertation submitted as partial requirement for the conferral of
Master in Monetary and Financial Economics

by

Hugo António Figueiredo Matias

Supervisor:
José Carlos Dias, Ph.D., Associate Professor with Aggregation,
ISCTE

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Resumo

Este trabalho teve como principal preocupação estabelecer a ligação entre os retornos das opções e a volatilidade do índice subjacente. Por outras palavras, compreender se e como ambos os componentes se influenciam. Foram estudados diferentes tipos de opções, como a opção de compra, a opção de venda e a opção *straddle* (conjugação de ambas), tendo como base o índice *Standard & Poor's 500.*

A elaboração deste estudo foca-se maioritariamente no Modelo de Precificação de Ativos Financeiros, mais conhecido por *Capital Asset Pricing Model* e, ao longo do mesmo, diversos factos que contradizem conclusões já alcançadas por outros autores para este tema foram possíveis de provar diversos factos que contradizem conclusões já alcançadas por outros autores para este tema.

Os resultados obtidos, especialmente para as opções *straddle* beta-zero, contrariam as premissas de *Black-Scholes*/Modelo de Precificação de Ativos Financeiros uma vez que os retorns esperados obtidos foram negativos. Desta forma, os mesmos indicam que, para além do risco de mercado, existe outro tipo de risco associado ao preço dos contratos das opções.

Palavras-chave:

S&P 500, Instrumentos Financeiros, Derivados, Volatilidade, Retornos das Opções.

Classificação JEL: C1, G12, G23.

Abstract

This thesis establishes how option returns are influenced by the underlying index volatility. To elaborate it, call, put and straddle options of the Standard & Poor's 500 index were object of study.

The elaboration of this study focused, mainly, on the Black-Scholes/Capital Asset Pricing Model, and, along it, was found some curious facts that contradict previous conclusions collected for this theme.

Either way, for zero-beta at-the-money straddle options the expected returns obtained were negative, contradicting Black-Scholes/Capital Asset Pricing Model assumptions. The results indicate that in addition to market risk there is another risk associated with option contracts pricing.

Keywords:

S&P 500, Financial Instruments, Derivatives, Volatility, Option Returns.

JEL Classification: C1, G12, G23.

Index

Tables Index

Figures Index

Glossary of Acronyms

AMEX – American Express Company

ARCH - Autoregressive Conditional Heteroskedasticity

ATM - At-the-money

CAPM - Capital Asset Pricing Model

CBOE - Chicago Board of Options Exchange

ETNs - Exchange Traded Notes

GARCH - Generalized Autoregressive Conditional Heteroskedasticity

GBM - Geometric Brownian Motion

ITM - In-the-money

OTM - Out-of-the-money

NYSE – New York Stock Exchange

S&P - Standard & Poor's

SPX - S&P 500 index

VIX - Volatility Index

1. Introduction

The present thesis has the main objective of study the option returns and if or how these returns are influenced or not by the underlying index volatility. This study is executed relying on the research question: "What is the relation between expected options returns and the volatility of the underlying assets?" that is supported by several sub-questions that will be introduced further in the study. The importance of the question is related with the prominence of understanding how options returns vary in accordance with their type and level of moneyness, not excluding the fact that options allow investors to diminish their risks on the market, therefore, investigating with more precision these financial assets will always have a positive outcome.

In general, all authors that have studied this theme recognise the importance of it, since options have become one of the most important financial assets to investors as they have remarkable risk-return characteristics. Additionally, all authors agree with the fact that there is another factor influencing options returns.

The elaboration of this thesis will bring a most up-to-date analysis of the theme and, also, it will be possible to see if the results that will be obtained are similar to the results already obtained by other authors or if different conclusions are achieved.

For the creation of the present thesis it will be used the time-series data of daily returns of European call and puts options on the S&P 500 index, which will be examined over a one-year period, between January 2016 and December 2016. Additionally, the short-term interest rate, the S&P 500 volatility and dividend yield will also be used to obtain the results desired. The present thesis will focus, especially, on the Black-Scholes/Capital Asset Pricing Model[1].

The present thesis is divided in 4 chapters. In chapter 2, the theoretical framework and the literature review are presented, providing a brief contextualization of the theme and a summary of the scientific articles that are more related with the theme. Chapter 3 contains the methodology used for the elaboration of the study and the data used. In chapter 4, the results obtained are

[1] CAPM stands for Capital Asset Pricing Model and describes the link between systematic risk and expected assets return, especially stocks.

exposed, as well as the answers obtained for the research question and sub-questions. In the last chapter, it is made a conclusion of the thesis.

2. Theoretical Framework and Literature Review

In the present section, firstly, the theoretical framework will be presented which will provide a contextualization of the theme, followed by a brief review of the scientific articles that are more related with the research question with the objective of providing a theoretical base for the theme that this thesis approaches and expose the results that other authors reached on the topic.

Before presenting the theoretical framework and the scientific articles that were chosen, it is worth reminding that the research question of the present thesis is “What is the relation between expected options returns and the volatility of the underlying securities?”, with the sub-questions to support it being the following “What is the real meaning of expected option returns?”, “Which type of option is expected to generate the highest returns?”, “Do Out-of-the-money options have expected positive returns?” and “Do moneyness level influence expected option returns?”.

2.1 Theoretical Framework

The investigation will be focused on the asset pricing theory. To contextualize what will be the object of study in the present thesis, a brief definition/explanation of the main intervenients will be performed before advancing to a deeper investigation.

What is the index S&P 500?

The S&P 500, that stands for Standard and Poor, is a stock market index that tracks the stock’s performance of the 500 largest U.S. companies.

What are options and what are they used for?

Options are a type of financial instruments, known as derivatives that have their value derived from the underlying asset. An option is a type of contract that gives the opportunity to the buyer to buy or sell the underlying asset, depending on the type of the contract held. There are two types of options, call options that allow the holder to buy the asset at the accorded price on a pre-determined date and put options that allow the holder to sell the asset at the accorded price on a pre-determined date. Options contracts are used by investors, mainly for two reasons: they allow them to leverage position in an asset by spending less than buying shares and they can also be used to reduce the risk exposure.

What is volatility?

Based on the general definition, volatility is a statistical measure that measures the risk of a security. Usually the risk is directly proportional to volatility, meaning that the higher the volatility the riskier the security.

The standard model for valuing options is the Black and Scholes (1973) Model, since it will have a main role on this thesis, a brief explanation of it and its limitations is provided below.

The Black-Scholes Model was first revealed by Fischer Black and Myron Scholes in 1973, being further developed by Merton (1973). The model considers that the price of high traded assets follows a geometric Brownian motion with continuous volatility and drift.

Despite its popularity, the model is constructed under some unrealistic assumptions about the market. These assumptions can be found below:

- As already stated above, the model assumes that volatility is constant over time. This assumption does not match with the reality, since volatility can be relatively constant in very short terms, however in longer terms it will never be constant.
- The stock prices are calculated based on several economic factors that do not have all the same weight on affecting the movement of stock prices.
- The model assumes that there are no commissions and transaction costs for buying and selling options and stocks.
- It also assumes that markets liquidity is perfect and that it is possible to trade any amount of stock or options at any given time.

As listed above, the limitations present in the model are related to fundamental aspects of the market.

2.2 Review of the Literature

In this chapter a brief resume of the articles that are more related with the theme in study is made, showing the answers that other authors have reached on the theme.

Carr and Wu (2006) presented a first look on the major differences between the old and the new volatility indexes, and on the pricing of VIX futures and options. The Chicago Board of Options Exchange (CBOE) first introduced the CBOE Volatility Index (VIX) in 1993, based on near-the-money Black-Scholes implied volatilities of options data on the S&P (Standard & Poor's) 100 index (OEX). In 2003, the CBOE changed the definition and calculation method of the VIX, calculating the new VIX based on market option prices instead of implied volatilities. Moreover, the new VIX uses the S&P 500 index (SPX) replacing the OEX as the underlying stock index. The authors perform an estimation using GARCH (1,1) process on the S&P 500 index return innovation with an AR (1) assumption on the return process[2]. A comparation between GARCH volatility and the VIX index is also made to predict the realized return variances. The final results conclude that the VIX can predict movements in future realized variance and that GARCH volatilities do not provide extra details. Additionally, VIX futures quotes provide information about risk-neutral mean and combine it with information from the SPX options market to expose the risk-neutral variance of the VIX that can be used to price VIX options.

Coval and Shumway (2001) published the first scientific article focused on the theoretical and empirical nature of option returns. The scientific article starts with a theoretical analysis, based on Black-Scholes (1973)/CAPM assumptions that call options always have positive returns greater than the underlying security, since call options betas are larger in absolute value. For put options, the opposite happens, that is expected returns should be below the risk free-rate. On the second part, to test the implications mentioned on the first point, weekly returns of European-

[2] The ARCH (Autoregressive Conditional Heteroskedasticity) method first introduced by Engle (1982) grants a way to model a shift in variance in a time series that is time dependent. The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) allows the method to support changes in the time dependent volatility. An autoregressive (AR) model predicts future behaviour based on past performance.

style call and put options on the S&P 500 index are examined. To provide an independent verification for the obtained results, daily returns of American-style call and put options on the S&P 100 index are examined as well. The results obtained on the third point seem to be consistent with the Black-Scholes/CAPM model, even though both calls and puts returns are too low. To obtain more information, these issues are investigated in detail. In conclusion, the results obtained in the article, strongly suggest that there is another important factor, besides market risk for pricing the risk associated with option contracts. Through the analysis of zero-beta straddle returns, that are basically determined by innovations in market volatility, the results imply that systematic stochastic volatility may be an important factor for pricing assets.

Bakshi, Madan and Panayotov (2010) created a scientific article with the purpose of showing how U-shaped pricing kernels, who are hinged on asset pricing theory, can explain various dimensions on the data. For the elaboration of this article, they focused on an empirical investigation, with the S&P 500 index call returns as base and the average call returns of the larger international equity markets, digital calls[3], upside variance contracts[4] and kernel calls to support it. With the development of this article, the authors obtain evidence that U-shaped pricing kernels are capable of explaining the observed negative average returns of out-of-the-money (OTM) index calls, upside variance contracts declining in strike, average returns of digital calls and OTM put returns highly negative, growing in strike. The theoretical evidence between short-selling, the slope of the pricing kernel and expected call options returns is also tested, and its relation is statistically significant, suggesting that U-shaped pricing kernels help to untangle the return patterns of claims with pay-out on the upside. Contrary to the conclusion that Coval and Shumway (2001) arrived on their article, Bakshi, Madan and Panayotov (2010) obtained results that reject the assumption that call options always have positive returns greater than the underlying security and increasing with strike price, as the returns that they obtained for OTM call options are negative.

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[3] A digital call option is a type of financial derivative with a fixed pay-out if the underlying asset passes a predetermined strike price. There is an upfront fee called the premium, which is the maximum loss for the option.

[4] A variance contract or variance swap is a type of financial derivative that is used to hedge or speculate on the level of a price movement of an underlying asset. In resume, the variance is the discrepancy between an anticipated result and the actual result.

Eraker and Wu (2017) have written an article to study the returns of investing in VIX futures, VIX Exchange Traded Notes (ETNs), and variance swaps. It is documented a substantial negative return premium for these assets. The first objective is to provide descriptive statistics on the average returns to VIX futures positions and the associated ETNs. The second objective is to verify if the negative average returns are consistent with returns from a present value-based equilibrium model. On the second point, evidence of statistical behaviour of VIX futures and ETNs is exposed. The sample data used is at daily frequency from January 2006 to May 2013. Between this period the VIX futures averaged returns were negative for all contracts and the returns tend to increase with maturity. On the third part, an outline to understand and analyse the returns to VIX futures, their ETNs and variance swap is made with the objective of explain how VIX futures earn high negative expected returns. In conclusion, the article shows that average returns earned on volatility and variance derivatives are very negative. The conclusion obtained on this article matches with the one mentioned on the above article: both articles conclude that the obtained returns are too low.

Ang, Hodrick, Xing and Zhang (2006) examined the aggregated price of volatility risk in the cross-section of stock returns, they found that stocks with high sensitivities to innovations in aggregated volatility and with high idiosyncratic volatility have low average returns. The article focuses in two main objectives: determine how the stochastic volatility of the market is priced in the cross-section of expected stock returns and the relation between idiosyncratic volatility and expected returns. Based on economic theories, idiosyncratic volatility should be positively related to expected returns, since investors will demand compensation for not being able to diversify risk. Then agents will request a premium for holding stocks with high idiosyncratic volatility. However, the results obtained on this article are the opposite. The authors conclude that stocks with high idiosyncratic volatility have low average returns. In summary, through the examination of returns of a set of assets, sorted by idiosyncratic volatility, based on the Fama and French (1993) model, the stocks with highest idiosyncratic volatility are the ones with lower returns.

Carr and Wu (2009) proposed a new method for quantifying the variance risk premium on financial assets. They use the difference between the realized variance and synthetic variance swap rate to calculate the variance risk premium. To perform this study, the authors synthesized variance swap rates on 5 stock indexes and 35 individual stocks over a period of seven years. Evidence that variance risk premiums are strongly negative for the S&P and Dow indexes is found. Through the use of CAPM, the negative variance risk premiums that were obtained can be clarified by the negative correlation that exists between index returns and volatility, this correlation generates a negative beta. To sum up, the negative variance risk premiums indicate that investors see market volatility going up as an unfavourable shock, which will lead them to pay a large premium to hedge against market volatility going up.

Glosten, Jagannathan and Runkle (1993) performed an investigation using the GARCH-M model with more general specifications to test the negative statistical significance relation between conditional variance and expected returns. To perform the investigation, the authors focus their attention on the volatility information of the variables: nominal interest rate, October and January seasonal dummies and the unanticipated part of the excess return on stocks. To estimate the relation between risk and return, Campbell's Instrumental Variable Model or a variety of Modified GARCH-M and EGARCH-M models are used. After analysing the obtained results, the conclusion that the authors make is that using a modified GARCH-M model, allowing positive and negative unanticipated returns to have different impacts on the conditional variance, there is a negative relation between the conditional mean and the conditional variance of the excess return on stocks.

Christensen and Prabhala (1998) performed a study to verify if implied volatility in index option prices predicts ex-post realized volatility. To perform this study, the authors focus on the S&P 100 index options. To elaborate it, the relation between implied volatility and realized volatility for the OEX options market is examined. Differently from other studies that were done, the volatility data sample used implies a longer period of time and the implied and realized volatility series have a lower frequency, that enable the authors to construct volatility series with nonoverlapping data. Based on the obtained results, two conclusions were possible to extract. The first one is that after the October 1987 stock market crash, implied volatility has become a better predictor of future volatility. The second conclusion is that implied volatility predicts the future realized volatility individual or in conjunction with the history of past realized volatility. The results obtained in this article match with the ones already mentioned. However, the interesting point on it is the conclusion made about stock crash and the use of nonoverlapping data, that will lead to cleaner results.

Bali and Hovakimian (2009) present a paper with the main objective of investigating if realized and implied volatilities of individual stocks can predict cross-sectional variation in expected returns. To perform this study, the data used is the stock return data from the Center for Research in Security Prices monthly and daily return files and the implied volatilities for options from NYSE, AMEX and NASDAQ between February 1996 and January 2005, focusing on the market's expectation of future volatility on individual stocks. Through the use of the Fama-MacBeth (1973) methodology, by using the realized, call and put implied volatility as measures, the results suggest that although the level of volatilities cannot predict future stock returns, it can offer a significant relation between volatility spreads and the cross-section of expected returns. In conclusion, even though this article is focused on the expected stock returns, the results obtained on it match with the ones obtained on the articles focused on options. It supports the other articles presented in this thesis.

Broadie, Chernov and Johannes (2009) elaborated an article to investigate the significance of index option returns. To obtain the results, the authors rely on two strategies: analytical formulas for expected returns and Monte Carlo simulation[5] to assess statistical significance, on which the models are standardised to suit the realized historical behaviour of the underlying index returns over the sample period. The sample used is the daily S&P 500 index returns from 1987 to 2005. Several interesting findings are reached. For instance, the authors conclude that put option returns do not provide any particularly information about potential option mispricing. On the other hand, it is found evidence that option portfolios are very informative due to the fact that they are almost neutral to movements in the underlying.

To conclude, Jackwerth (2000) published an article with the objective of recovering risk aversion empirically from risk-neutral and subjective probability distributions. To perform this study, the risk-neutral distributions are recovered from option prices based on a variation of the method of Jackwerth and Rubinstein (1996), and the subjective distributions are approximated by the actual return distribution of a broad index. The data used contains quotes of S&P 500 from January 1928 through December 1995. The obtained results match with the standard assumptions made in economic theory during the pre-crash period. However, partially negative and partially increasing risk aversion functions during the post-crash period are found. The most logical explanation for this, is the mispricing of options in the market.

[5] Monte Carlo simulations are used to calculate the chances of different conclusions to happen in a process that cannot simply be predicted due to the interference of random variables. It is an approach used to figure out the impact of risk and uncertainty in prediction and forecasting models.

Based on the scientific articles referred above, it is possible to verify that the authors arrive to similar conclusions: that implied volatility helps predicting the future realized volatility and that Black-Scholes/CAPM assumptions are not completely accepted. The Black-Scholes model follows the Geometric Brownian Motion (GBM)[6] assumption with constant drift and constant volatility when modelling stock prices. Since it follows the GBM, it is not able to fit the leverage effect and the volatility smile effect[7] on its results, which led it to be rejected by the authors of the articles, as when testing the Black-Scholes/CAPM assumptions the evidences achieved were quite the opposite.

After analysing the above articles, it makes sense to perform this study since the results that will be obtained can be compared with the ones of other authors. The research question is important because options have remarkable risk-return characteristics. In this way, obtaining an answer for it, will, at least, provide an update of the results and if a final answer for the research question is obtained it will establish how the volatility of securities affect the expected option returns.

[6] A Geometric Brownian Motion (GBM) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with drift. It is used, particularly, in mathematical finance to model stock prices in the Black–Scholes model.

[7] Leverage effect aims to quantify how much business risk a given company is currently experiencing, basically, how sensitive net income is to changes in revenues. Volatility smile effect is created by implied volatility changing as the underlying asset moves more in-the-money (ITM) or out-of-the-money (OTM). The more an option is ITM or OTM, the greater its implied volatility becomes.

3. Methodology and Data

3.1 Methodology

Different methods were performed by the previous authors that studied this theme. Based on the aforementioned articles selected to support the elaboration of this thesis, the following methods were the ones used with more regularity: the Generalized Autoregressive Conditional Heteroskedastic in mean (GARCH-M)[8], Exponential Generalized Autoregressive Conditional Heteroskedastic in mean (EGARCH-M)[9] models, the Generalized Method of Moments (GMM)[10] and the Student's T-Test, always based on the Black-Scholes/CAPM asset-pricing theory.

It is with ground on the above tests that this thesis will be elaborated, with special focus on the Student's T-Test. Therefore, the obtained results can be compared to verify if they are in accordance with the literature chosen or if contrary conclusions are reached.

A brief explanation of the methods that will be performed to get the results is presented next.

The Student's T-test, or simply T-test, is used to compare the relation between two means, telling if they are different from each other and if they are, how significant that difference is. The result obtained, also known as t-score, represent the difference within the groups compared, the bigger the t-score the greater difference there is between groups and vice-versa. With every t-score, there is a p-value associated that gives the probability of the results obtained occurring by chance, in this case, if the p-value is low (<5%) it indicates that the results did not occur by chance.

[8] The GARCH model is an econometric term created by Engle (1982) to estimate volatility in financial markets. It assumes that positive and negative error terms have symmetric effect on the volatility. The GARCH-M model computes a heteroscedasticity term into the mean equation.

[9] The EGARCH-M model differs from the GARCH model structure due to the log of the variance.

[10] The GMM is a statistical method, formalized by Hansen (1982), which mixes observed economic data with the information in population moment conditions to create estimates of the unknown parameters of the economic model in question. As described, one of the factors that explain the popularity of this method is the fact that the GMM estimators can be constructed without specifying the full data generating process what can be exploited to analyse economic models which the data is not full available/specified.

Following the Black-Scholes/CAPM asset-pricing theory, the betas of call options are always bigger in value than the respective underlying asset, and hence, it is expected that on market indices, call options have positive long-run returns bigger than those of their underlying securities. By contrast, it is expected that returns on put options to be below the risk-free rate[11].

Specifically, as introduced by Coval and Shumway (2001), the following two theorems produce similar results to the above claims. To produce their results, they assume the existence of a stochastic discount factor[12] that prices all assets according to the relation:

$$\boldsymbol{E[R_i \cdot m] = 1}\,, \tag{1}$$

where:

- E: Expectation operator;
- R_i: Gross return of any asset;
- m: Strictly positive stochastic discount factor.

They provide assumptions on how the stochastic factor is related to the options underlying asset returns, since it is known that this type of element exists when there is no arbitrage.
In the case of call options, the following proposition is introduced:

PROPOSITION 1: *If the stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any call option on that security will have a positive expected return that is increasing in the strike price.*

Proof: Let the expected gross return from now until maturity on a call option with a strike price of K on an underlying security whose price has a distribution $f(y)$ be expressed as:

[11] The risk-free rate is the hypothetical rate of return of an investment with null risk. It represents the rate that an investor would expect from an absolutely risk-free investment over a determined period of time.

[12] The stochastic discount factor, also known as pricing Kernel, forms the relationship between asset's payoffs in different future states of the world and its current price.

$$\boldsymbol{E[R_c(K)]} = \frac{\int_{s=K}^{0}(s-K)f(s)\partial s}{\int_{q=0}^{0}\int_{s=K}^{0} q(s-K)f(s,q)\partial s\partial q}, \tag{2}$$

where:

- $f(y,z)$: joint distribution of the security price and the stochastic discount factor.

The expected net return, $E[r_c(K)] = E[R_c(K)] - 1$, can be reported as:

$$\boldsymbol{E[R_c(K)]} = \frac{\int_{s=K}^{0}(s-K)[1-E[\boldsymbol{m}|\boldsymbol{S}]]f(s)\partial s}{\int_{s=K}^{0}(s-K)E[\boldsymbol{m}|\boldsymbol{S}]f(s)\partial s}, \tag{3}$$

where:

- m: Stochastic discount factor.

With respect to the strike price, the derivative of expected net returns can be presented as:

$$\frac{\int_{s=K}^{0}(s-K)f(s)\partial s\cdot\int_{s=K}^{0}E[\boldsymbol{m}|\boldsymbol{S}]f(s)\partial s-\int_{s=K}^{0}(s-K)E[\boldsymbol{m}|\boldsymbol{S}]f(s)\partial s\cdot\int_{s=K}^{0}f(s)\partial s}{[\int_{s=K}^{0}(s-K)E[\boldsymbol{m}|\boldsymbol{S}]f(s)\partial s\,]^2}. \tag{4}$$

By defining $F(s)$ as the cumulative density function that corresponds to $f(s)$ and reordering it gives:

$$\frac{\partial E[r_c(K)]}{\partial K} = \frac{\int_{s=K}^{0}(s-K)\frac{f(s)}{1-F(K)}\partial s\cdot\int_{s=K}^{0}E[\boldsymbol{m}|\boldsymbol{S}]\frac{f(s)}{1-F(K)}\partial s-\int_{s=K}^{0}(S-K)E[\boldsymbol{m}|\boldsymbol{S}]\frac{f(s)}{1-F(K)}\partial s}{[\int_{s=K}^{0}(s-K)E[\boldsymbol{m}|\boldsymbol{S}]\frac{f(s)}{1-F(K)}\partial s\,]^2}. \tag{5}$$

Since the numerator of the above equation is the negative of the covariance of $s-K$ and m, restricted on the option being ITM:

$$\boldsymbol{-Cov[E(m|s), s-K|s>K] = E[m|s>K]\cdot E[s-K|s>K] - E[E(m|s)(s-K)|s>K]}. \tag{6}$$

The above expression will be positive for any m that is negatively correlated with the security price. Therefore, any call option for which the price is negatively correlated with the stochastic discount factor is expected to have positive returns and they should increase as the option strike price rises. Since there is no asset-pricing theory that allows a positive correlated stochastic discount factor with the market level, the results for the call options that will be obtained on this thesis are expected to be positive and increasing with the strike price.

For put options, the corresponding proposition is the following:

PROPOSITION 2: *If a stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any put option on that security will have an expected return below the risk-free rate that is increasing in the strike price.*

Proof: Let the net expected return on a put option with a strike price of K on an underlying security whose price has a distribution $f(y)$ be expressed as:

$$\boldsymbol{r_p(K) = \frac{\int_0^{s=K}(K-s)[1-E[m|S]]f(s)\partial s}{\int_0^{s-K}(K-s)E[m|S]f(s)\partial s}}, \tag{7}$$

where:

- $f(y,z)$: joint distribution of the security price and the stochastic discount factor;
- m: stochastic discount factor.

Therefore, the derivative of net returns with respect to strike price can be stated as:

$$\boldsymbol{\frac{\int_0^{s=K}(K-s)E[m|S]f(s)\partial s\cdot\int_0^{s=K}f(s)\partial s-\int_0^{s=K}(K-s)f(s)\partial s\cdot\int_{s=K}^{0}E[m|S]f(s)\partial s}{[\int_0^{s=K}(K-s)E[m|S]f(s)\partial s]^2}}. \tag{8}$$

As performed in Proposition 1, the numerator in equation Z is proportional to the covariance between $K-s$ and m, restricted on the option being ITM:

$$-\boldsymbol{Cov}[\boldsymbol{E}(\boldsymbol{m}|\boldsymbol{s}), \boldsymbol{K}-\boldsymbol{s}|\boldsymbol{s}<\boldsymbol{K}] = \boldsymbol{E}[\boldsymbol{E}(\boldsymbol{m}|\boldsymbol{s})(\boldsymbol{s}-\boldsymbol{K})|\boldsymbol{s}<\boldsymbol{K}] - \boldsymbol{E}[\boldsymbol{m}|\boldsymbol{s}<\boldsymbol{K}] \cdot \boldsymbol{E}[\boldsymbol{s}-\boldsymbol{K}|\boldsymbol{s}<\boldsymbol{K}]\ . \quad \textbf{(9)}$$

The above expression will be positive for any m that is negatively correlated with the security price. A put option with an infinitive strike price has an expected net return equal to the risk-free rate, since the net returns are increasing with the option strike price, all put options are expected to have returns below the risk-free rate.

3.2 Data

To perform this study, the time-series data of daily returns of European call and puts options of the S&P 500 index are examined over a one-year period, between January 2016 and December 2016. Due to the difficulty of finding data for this study, one year of it was the maximum that was possible to obtain. Nevertheless, even though the sample period is not the desired, it will be possible to have an idea of the behaviour of the options returns and compare it with the results obtained by other authors. It is important to highlight that the data used on this thesis was provided by Professor José Carlos Dias, without him and his data, the tests performed would not be possible.

The sample period was chosen based on the economic events that occurred during the year impacting the global economy, so their direct impact on the market can be inspected. The events that had more impact on the world economy in 2016 were: China's stock market crash where it fell 18 per cent in 11 days, the Brexit, on which the United Kingdom voted to leave the European Union leading to an increase of the US dollar against the euro and the United States presidential elections that as studied by Oehler, Walker and Wendt (2013) have influence on the stock market. The method used to filter the options is based on the same method used by Coval and Shumway (2001) on their article. The options that are to expire during the following calendar month, and therefore are roughly between 20 and 50 days to expiration are selected, since they will be the ones with highest trading volume and hence, they will provide more accurate results. Finally, the midpoint of the bid-ask quotes[13], used as a proxy for the market price of the option contract, is taken to calculate the daily holding-period returns for each option. The return of the call/put options position is calculated via the formula used by Bakshi, Madan and Panayotov (2010) as:

$$\boldsymbol{r_{t,T}^{C}[y] = \frac{(S_t e^{R} - yS_t)^{+}}{C_{t,T[y]}} - 1} \qquad \textbf{(10)}$$

$$\boldsymbol{r_{t,T}^{P}[y] = \frac{(yS_t - S_t e^{R})^{+}}{P_{t,T[y]}} - 1}\ , \qquad \textbf{(11)}$$

where:

- $S_t: Stock\ price\ at\ time\ t;$
- $K = Strike\ price;$
- $R: \ln(\frac{S_{t+T}}{S_t});$
- $y = K/S_t;$
- $C_{t,T[y]}: price\ of\ call\ option\ with\ time\ to\ expiration\ T\ (midpoint\ Bid - Ask\ quotes);$
- $P_{t,T[y]}: price\ of\ put\ option\ with\ time\ to\ expiration\ T\ (midpoint\ Bid - Ask\ quotes).$

$(S_t e^R - yS_t)^+$ is the realized payoff of the contract at maturity, meaning that if it is negative the option contract will not be exercised. By other words, all the contracts that were not exercised will have expected returns of -100%, which, especially in options out-of-the-money (OTM) will be reflected in a big percentage of the data.

To better classify the options returns, the options are divided in five different groups based on their level of moneyness in comparison with the underlying index, S&P 500. A total of 11460 call options and 9546 put options were observed to elaborate this study.

In summary, the moneyness classification of the options is the evaluation between the spot price of the underlying asset, in this case, the S&P 500 index, and the strike price of the

[13] The midpoint bid-ask quotes is used as a reference price, it is the average of the quoted bid and ask prices that expresses the general market value of a determined asset.

respective option contract. To elaborate this classification, it is used the price of the underlying asset, S&P 500 index that is associated to each option and the strike price of the respective option contract. The data selected was restrained to levels of moneyness between -5% and 10% so the results obtained will not be influenced by abnormal values.

The moneyness level informs if the option contract is in-the-money (ITM), at-the-money (ATM) or out-of-the-money (OTM). An option is ITM when the intrinsic value is positive, ATM when its intrinsic value is zero and OTM when the payoff that it generates if exercised is negative. The intrinsic value of each option contract can be calculated using the following formulas.

For call options:

$$\boldsymbol{Intrinsic\ Value = Spot\ Price\ (S_t) - Strike\ Price\ (K)}\ . \tag{12}$$

For put options:

$$\boldsymbol{Intrinsic\ Value = Strike\ Price\ (K) - Spot\ Price\ (S_t)}\ . \tag{13}$$

Following the above explanation, it is possible to classify the options as follows:

Option Type	**ITM**	**ATM**	**OTM**
Call	$S_t > K$	$S_t = K$	$S_t < K$
Put	$S_t < K$	$S_t = K$	$S_t > K$

Table 3.1. - Options Classification (Moneyness)

The Underlying Index:

To verify if the daily returns of the call and put options are aligned with the returns of its underlying asset, the daily returns of the S&P 500 index were calculated based on the quotes made available by the S&P Dow Jones Indices.

On the year 2016, the S&P 500 index had an average daily return of 0.0328%, which translates in a monthly return of 0.984% or an annual return of 11.96%. The maximum daily return verified during the year was 2.476%, when the minimum was -3.592%.

The graph shown in Figure 1 resumes how the accumulated daily returns changed during the year, on which it is possible to verify that until March the returns obtained on the S&P 500 index were negative.

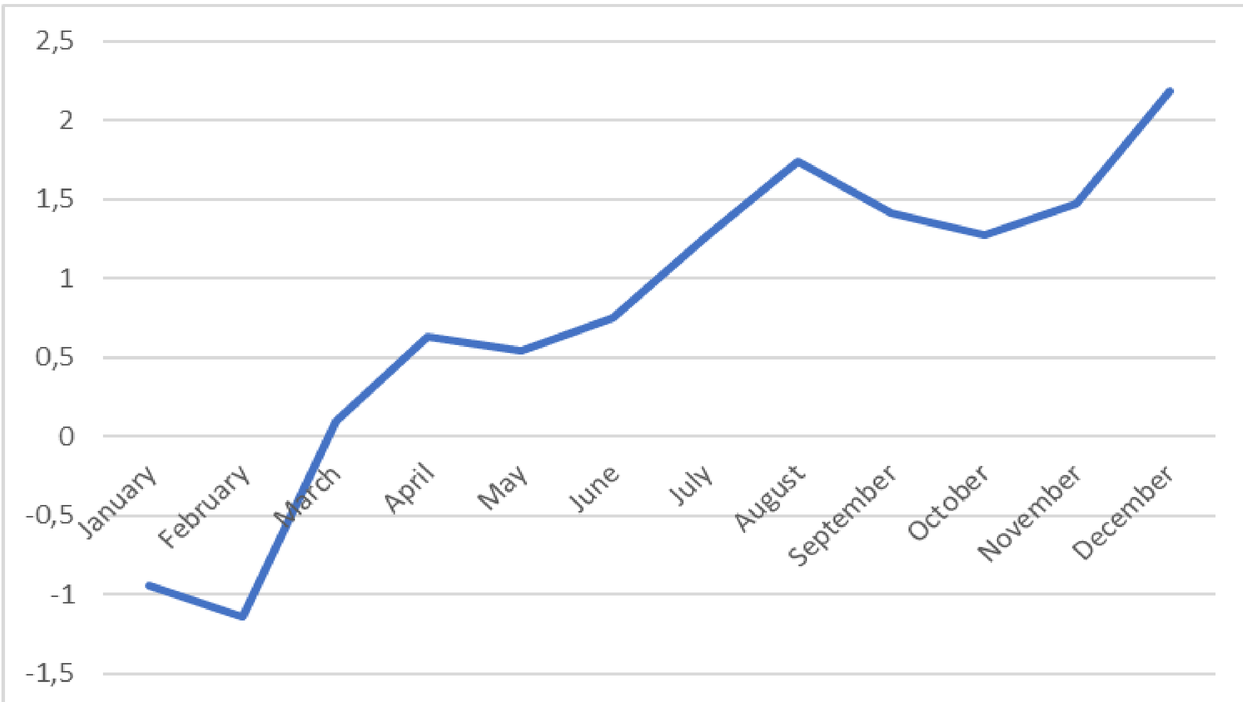

Figure 3.1. - Average of the Accumulated Returns per Day (S&P 500)

4. Results

In this chapter, the results obtained for the call options, put options and beta-zero straddle options are presented. The results are exposed on three different tables for each type of option, indicating the mean return, the confidence interval, the minimum and maximum returns and the standard deviation of the results. The t-test is also used to verify the null hypotheses that the mean return is zero. Therefore, the t-statistic and the corresponding p-values are reported also on the tables. For the call and put options the results are divided in 5 different groups, depending on their moneyness level, from 5% below the index value to 10% above.

4.1 Call Options

This table reports the mean returns of call options. The index is the S&P 500 and the sample period is from January 1st to December 31st of 2016. The returns are recorded in daily percentage and divided per moneyness level, from -5% to 10%.

Daily S&P 500 Call Option Returns					
Moneyness (%)	**-5 to -3**	**-3 to -1**	**-1 to 0**	**0 to 5**	**5 to 10**
Mean Return	-0.09909	0.09597	0.15736	0.19367	0.17587
95% Confidence	[-0.18969. -0.00849]	[0.025595. 0.121562]	[0.080976. 0.233752]	[0.169813. 0.217525]	[0.157667. 0.194082]
t-Statistic	(-2.14494)	(2.67499)	(4.04219)	(15.91575)	(18.94505)
P-value	0.01604	0.00377	2.8363E-05	8.48781E-56	2.94612E-73
Minimum	-0.83827	-0.96139	-0.99944	-0.99968	-0.86314
Maximum	9.85275	5.56901	3.23366	2.75476	1.57676

Standard Deviation	2.12808	1.64754	1.27760	0.80725	0.38846

Table 4.1. - Daily SPX Call Option Returns

By observing the results of the S&P 500 call options in Table 2, it is possible to verify that as argued by Bakshi, Madan and Panayotov (2010), the average returns are not positive for all the types of moneyness, this can be confirmed by the average returns obtained for call options 5% to 3% OTM as they are –0.09909% per day, or –2.97% per month. Contrary to what was expected and proposed in Proposition 1, the prediction that expected returns are always positive for S&P 500 call options is denied.

The remaining results are divided as: call options that are 3% to 1% OTM the average returns are 0.09597% per day, or 2.88% per month. ATM call options, between –1% of moneyness to 5%, have the tendency to earn average returns between 0.15736% and 0.19367% per day, or between 4.72% and 5.81% per month. ITM call options, between 5% and 10% of moneyness earn in average 0.17587% per day, or 5.28% per month.

Even though a sample of a one-year period does not reflect well the behaviour of the market, and in this case, of the call options returns, the results obtained are in accordance with the ones obtained by Bakshi, Madan and Panayotov (2010).

To simplify the visualization of how the average call options returns varied along the sample year, the following graph was created:

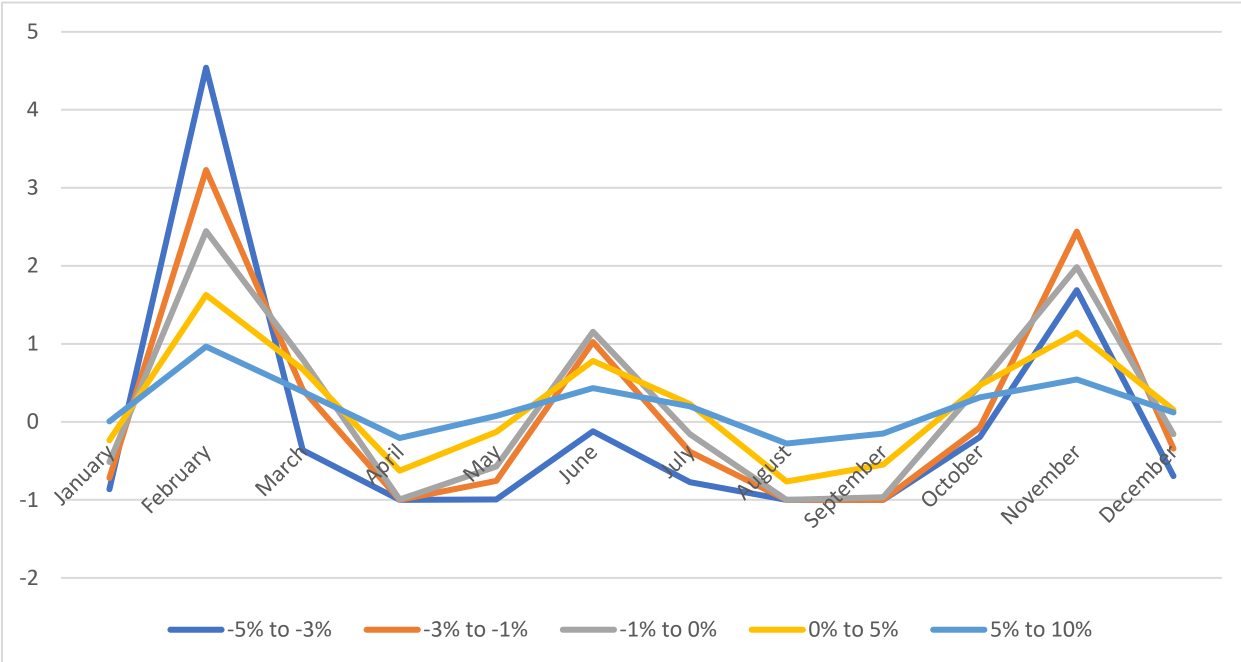

Figure 4.1. - S&P 500 Call Options Average Returns

Also, to facilitate the visualization of how the call options returns and the S&P 500 index returns behaved during 2016, the below graph shows the returns of both variables:

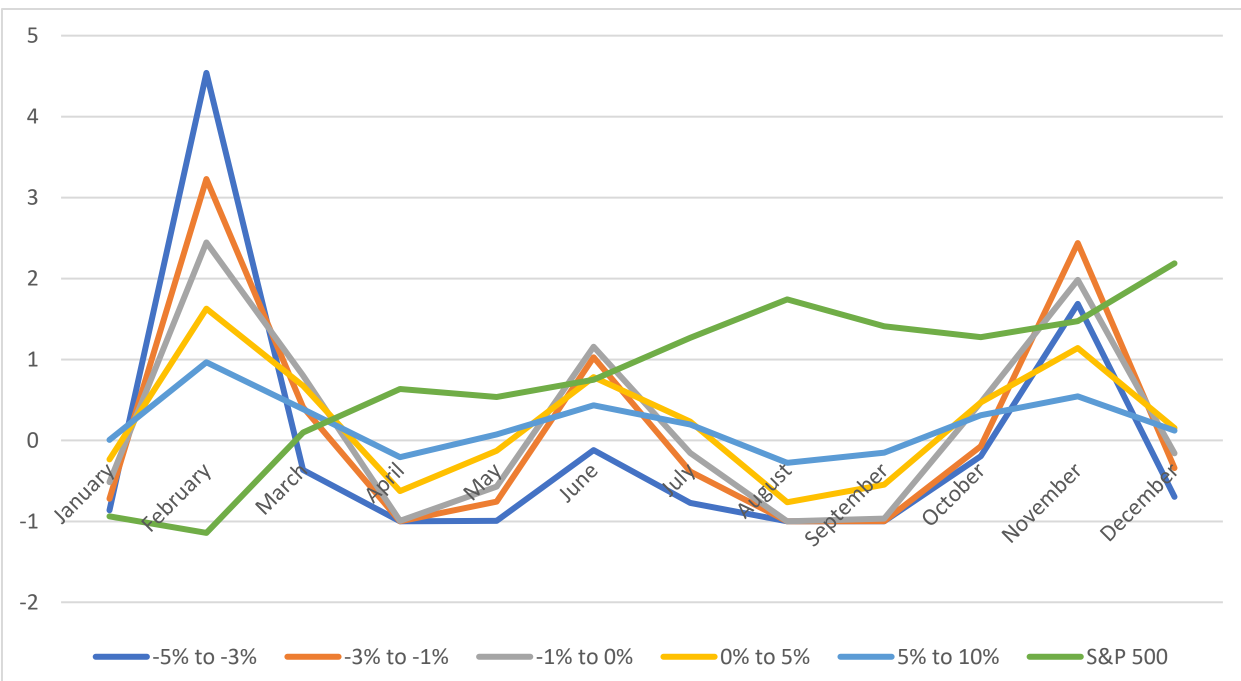

Figure 4.2. - S&P 500 Call Options Average Returns with the S&P 500 Index returns

4.2 Put Options

This table reports the mean returns of put options. The index is the S&P 500 and the sample period is from January 1st to December 31st of 2016. The returns are recorded in daily percentage and divided per moneyness level, from -5% to 10%.

Daily S&P 500 Put Option Returns					
Moneyness (%)	**-5 to -3**	**-3 to -1**	**-1 to 0**	**0 to 5**	**5 to 10**
Mean Return	-1.00000	-0.99893	-0.94538	-0.53587	-0.23396
95% Confidence	-	[-0.99971. -0.99813]	[-0.95449. -0.93626]	[-0.54876. -0.52298]	[-0.25403. -0.21388]
t-Statistic	-	(-2471.46847)	(-203.50583)	(-81.52243)	(-22.884756)
P-value	-	0.00000	0.00000	0.00000	3.05657E-84
Minimum	-1.00000	-0.98773	-0.99926	-0.99978	-0.93519
Maximum	-1.00000	-0.48106	-0.07091	0.38388	0.29164
Standard Deviation	0.00000	0.01865	0.15096	0.39637	0.25146

Table 4.2. - Daily SPX Put Option Returns

From table 3 it is possible to observe the results of the S&P 500 put options, that contrary to the results obtained on call options, put options results are in accordance with their respective proposition, i.e. Proposition 2. As proved by Coval and Shumway (2001), Bakshi, Madan and Panayotov (2010) and, also observed on this thesis, the mean return of S&P 500 put options is consistently negative and tend to decrease along the strike price, OTM put options present have lower returns than ATM put options.

The results are divided as: put options 5% to 3% OTM have returns of –1% per day, or –30% per month, put options that are 3% to 1% OTM present average returns of -0.99893% per day, or

-29.97% per month, ATM put options between –1% of moneyness and 5%, have the tendency to earn average returns between -0.94538% and -0.53587% per day, or between -28.36% and -16.08% per month and 5% to 10% ITM have average returns of -0.23396% per day, or -7.02% per month.

As observable, the mean returns vary between -1% and -0.23396% per day or -30% and -7.02% per month, consequently in accordance with Proposition 2.

The following graph provides a better visualization of the put options returns during 2016:

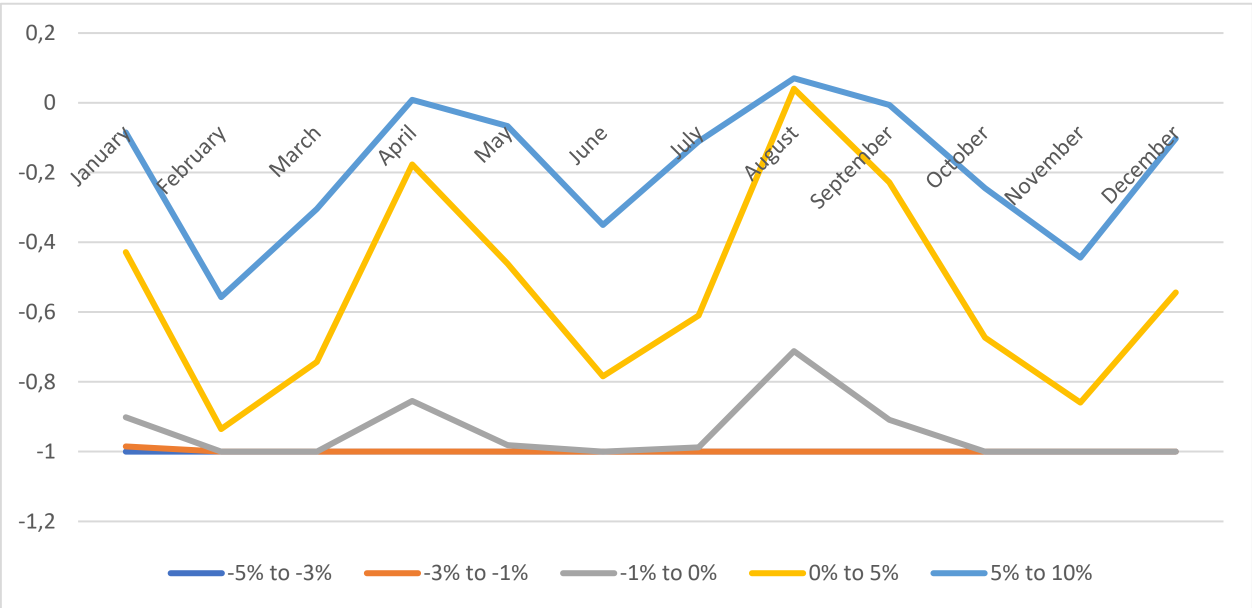

Figure 4.3. - S&P 500 Put Options Average Returns

Also, to facilitate the visualization of how the put options returns and the S&P 500 index returns behaved during 2016, the following graph shows the returns of both variables:

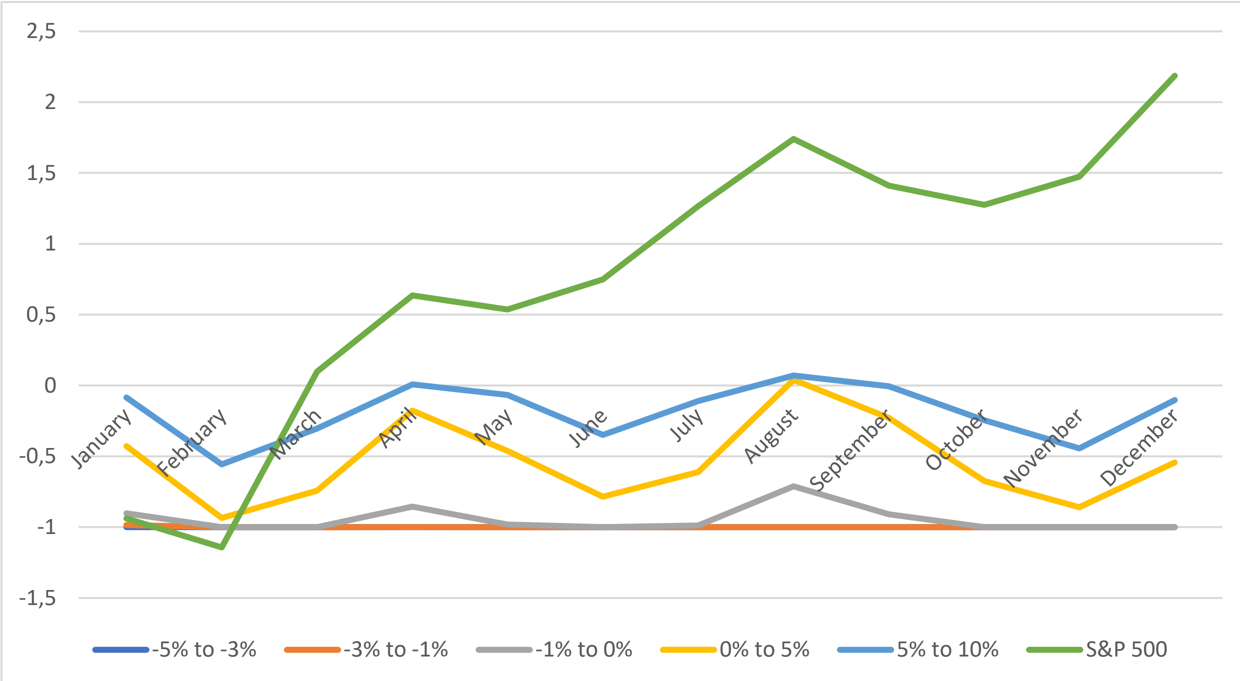

Figure 4.4. - S&P 500 Put Options Average Returns with the S&P 500 Index returns

It is easily observed the low returns obtained by put options when comparing with the S&P 500 index returns. Also, when Figure 4.4. and Figure 4.2. are compared, the difference of returns obtained by the different type of contracts is, also, remarkable.

4.3 Zero-Beta Straddle Options

One of the assumptions of CAPM is that it assumes options are redundant assets, thus in the following pages, straddle positions will be studied since they are not influenced by market changes. To form zero-beta straddle positions, it is necessary to combine a long position in a call option with a long position in a put option with the same strike price and time to expiration, so the obtained overall beta is zero. By performing this combination, the expected returns should be equal to the risk-free rate, at least, according to the Black-Scholes/CAPM. However, several authors such as Coval and Shumway (2001) and Goltz and Lai (2008) have reached opposing results.

Since zero-beta straddle options are not sensitive to market changes, their returns will also not be sensitive to market returns. However, they are sensitive to market volatility, making them perfect to study the effects of stochastic volatility[14].

According to Black and Scholes (1973), a call option beta can be computed as:

$$\boldsymbol{\beta_c = \frac{s}{C}\mathcal{N}[\frac{\log\left(\frac{s}{K}\right)+\left(r-\lambda+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}]\beta_s}\ , \tag{14}$$

where:

- s: Price of the underlying asset;
- C: Call option price;
- K: Option strike price;
- r: Short-term interest rate;
- σ: Underlying asset's volatility;
- t: Time to expiration;
- $\mathcal{N}\ [\,.\,]$: Cumulative normal distribution;
- β_s: Underlying asset's beta;
- λ: Dividend yield of the underlying asset.

The put option beta can be computed as:

$$\boldsymbol{\beta_p = \frac{s}{P}(\mathcal{N}\left[\frac{\log\left(\frac{s}{K}\right)+\left(r-\lambda+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right]-1)\beta_s}\ , \tag{15}$$

[14] Stochastic volatility specifies the fact that the volatility of asset prices is not constant.

where:

- P: Put option price.

As demonstrated in Coval and Shumway (2001), the zero-beta straddle options are created by solving the following equations:

$$\boldsymbol{r_v = \theta r_c + (1-\theta) r_p} \tag{16}$$

$$\boldsymbol{\theta \beta_c + (1 - \theta)\beta_p = 0 ,}$$

where:

- r_v : Straddle returns;
- θ : Fraction of the straddle's value in call options;
- β_c: Call option's beta;
- β_p: Put option's beta.

These equations are solved by the weight function:

$$\boldsymbol{\theta = \frac{-\beta_p}{\beta_c - \beta_p} .} \tag{17}$$

Substituting equation 17 into equation 16, the straddle returns can be calculated as:

$$\boldsymbol{r_v = \frac{-\beta_p}{\beta_c - \beta_p} r_c + \frac{\beta_c}{\beta_c - \beta_p} r_p ,} \tag{18}$$

where the call and put option's beta will be computed through the Black-Scholes beta shown in equations 15 and 16. Contrary to Coval and Shumway (2001) that use the assumption of put-call parity to form the weights for their straddles, nowadays put-call parity does not always hold.

Also, the study will only focus on options ATM, since the main objective is to verify whether the returns are equal to the risk-free rate or not. Therefore, the remaining options would not have any added value.

This table reports the mean returns of zero-beta straddle options created with ATM options. The index is the S&P 500 and the sample period is from January 1st to December 31st of 2016. The returns are recorded per day.

Daily S&P 500 Zero-Beta Straddle Option Returns (ATM)	
Mean Return	-0.35305
95% Confidence	[-0.41909. -0.287]
t-Statistic	(-10.53040)
P-value	7.27127E-22
Minimum	-0.99654
Maximum	0.88235
Standard Deviation	0.52047

Table 4.3. - Daily S&P 500 Zero-Beta Straddle Option Returns (ATM)

The results achieved provide the same conclusion that Coval and Shumway (2001) and Goltz and Lai (2008) arrived on their articles. The mean return obtained for zero-beta straddle option is -0.35305% per day, or -10.59% per month, and is highly statistically significant.

Following the Black-Scholes/CAPM assumption, the expected mean returns should be equal to the risk-free rate, however, the mean returns obtained in this thesis are negative, what goes in accordance with other authors conclusions. It suggests that other risks are being priced, since the market beta risk was neutralized by forming beta-zero straddle options.

Below can be found a graph that reflects how the mean returns varied during the one-year sample:

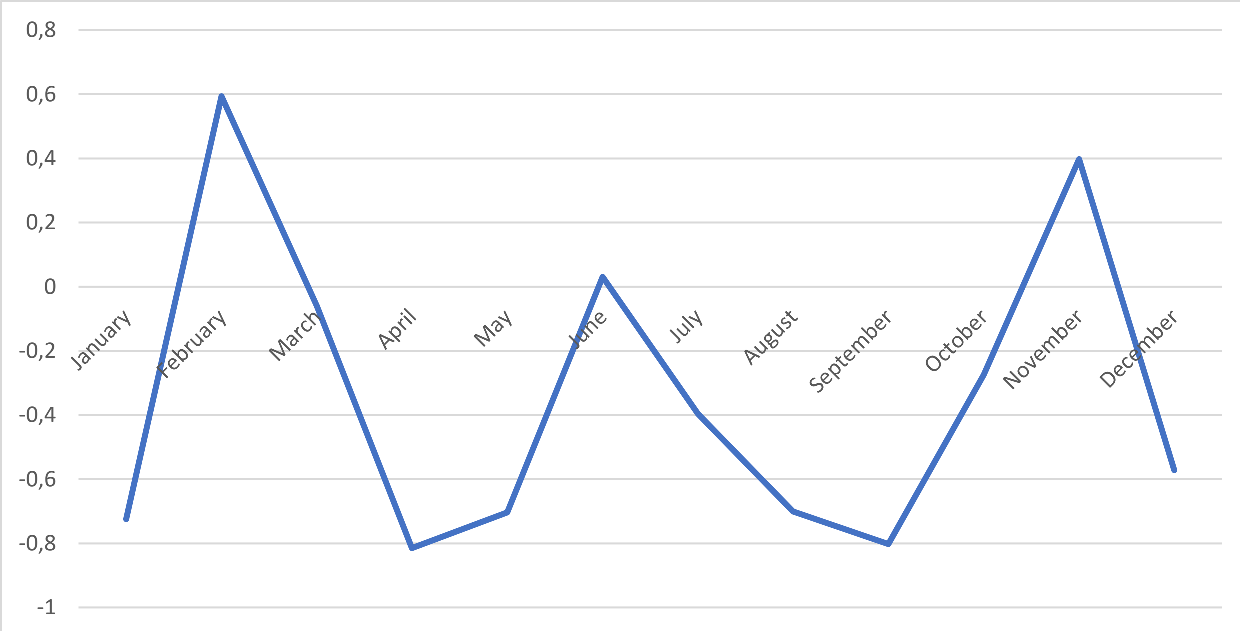

Figure 4.5 - S&P 500 Zero-Beta Straddle Option Returns

4.4 Answers to the Proposed Questions

After the elaboration of the above studies, several conclusions were possible to achieve and with them, the responses for the research question and for the sub-questions. Therefore, the questions that were proposed for this thesis are enumerated with their respective responses.

Since the sub-questions are used as support to the research question, first, it will be listed the answers obtained for them and, only after, the answers obtained for the research question.

Following the above introduction, the sub-questions are below:

"What is the real meaning of expected option returns?"

As the question suggests, expected option returns is the amount that the buyer of the option is expected to receive if he opts to exercise the option.

"Which type of option is expected to generate the highest returns?"

As indicated in the Black-Scholes/CAPM asset-pricing theory, call options are expected to always have positive long-run returns higher than those of their underlying securities. On the contrary, returns on put options are expected to be below the risk-free rate. Therefore, it is expected for call options to have higher returns than put options.

In addition to the above, the results obtained on this thesis also confirm that call options earn, indeed, higher returns than put options. To illustrate this fact, below can be found a table and a graph comparing both results.

Daily S&P 500 Call and Put Options Returns					
Moneyness (%)	**-5 to -3**	**-3 to -1**	**-1 to 0**	**0 to 5**	**5 to 10**
Mean Return (Call Option)	-0.09909	0.09597	0.15736	0.19367	0.17587
Mean Return (Put Option)	-1.00000	-0.99893	-0.94538	-0.53587	-0.23396

Table 4.4. - Daily S&P 500 Call and Put Options Returns

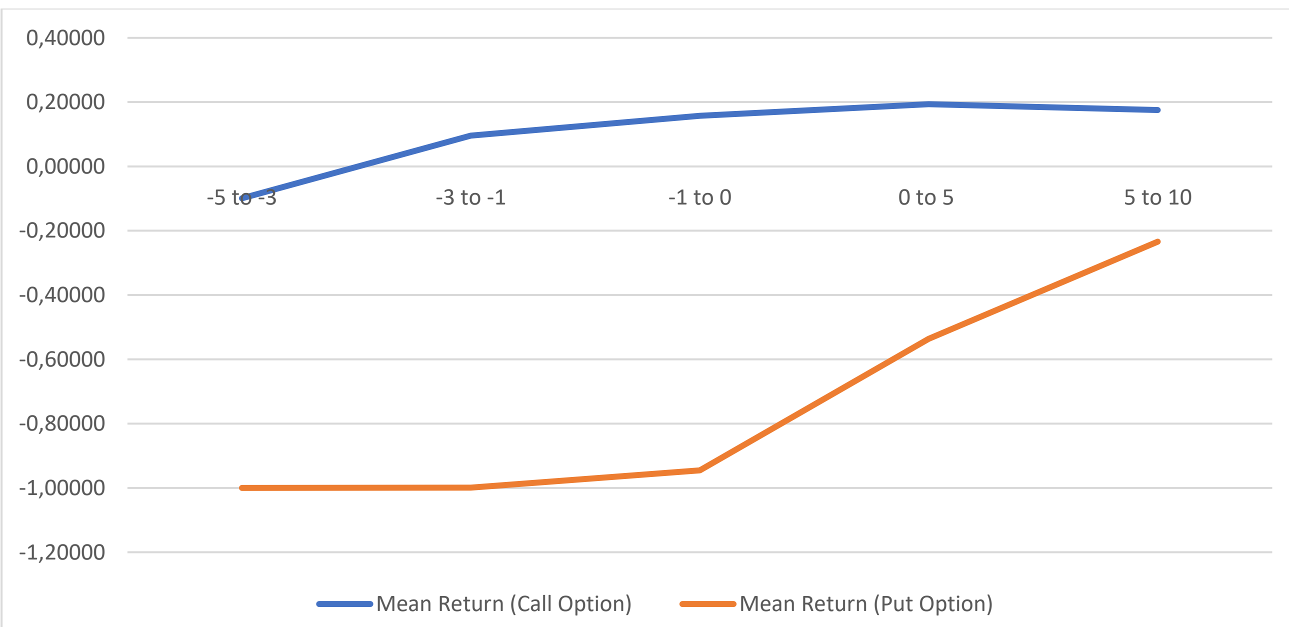

Figure 4.6 - Daily S&P 500 Call and Put Options Returns

"Do Out-of-the-money options have expected positive returns?"

Black-Scholes/CAPM asset-pricing theory claims that call option returns are positive and increase with the strike price. Thus, OTM call options are expected to have positive returns. Even though, the results obtained are not in agreement with the above affirmation, call options 1% to 3% OTM have positive returns, confirming that OTM options can have positive expected returns.
Put options, as indicated by the Black-Scholes/CAPM asset-pricing theory and confirmed by the obtained results, the returns expected are all negative.

"Do moneyness level influence the expected option returns?"

Since moneyness informs if the option contract is in-the-money (ITM), at-the-money (ATM) or out-of-the-money (OTM), it is logic to affirm that it will influence the expected returns. Also, the results found confirm that the returns tend to increase with the moneyness level. Even in the put options returns that are all negative, the returns are increasing with the moneyness level (Strike Price).

Following the exhibition of the answers obtained for the sub-questions, the answers gotten for the research question can be finally stated.

"What is the relation between expected option returns and the volatility of the underlying securities?"

First, volatility is a statistical measure that measures the risk of a security. Usually risk is directly proportional to volatility, meaning that the higher the volatility the riskier the security.
To study how volatility influences the expected option returns, zero-beta straddle options were created. By forming this position, the expected returns should be equal to the risk-free rate, since they are not sensitive to market changes, but are sensitive to market volatility.

The results obtained for this type of options, allow the conclusion that volatility may have influence on the options returns, since the returns should be equal to the risk-free rate but, on the contrary, the expected returns are negative. These results are settled by other authors too, as Coval and Shumway (2001) and Goltz and Lai (2008) that arrived at the same conclusions.

5. Conclusion

In summary, the results achieved on this thesis go in accordance with the ones obtained by other authors. This is also confirmed if compared with the scientific articles selected to form the chapter 2 of the Literature Review.

As observable by the results obtained on chapter 4.1 of the Call Options, Proposition 1 was not verified. Proposition 1 affirms the following:

"If the stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any call option on that security will have a positive expected return that is increasing in the strike price."

The results contradict the proposition, and with it, Coval and Shumway (2001) that have confirmed on their article that the above proposition was correct. On the other hand, the results are in accordance with Bakshi, Madan and Panayotov (2010) that reached similar results to the ones found on this thesis.

Different from the conclusions achieved for Proposition 1, on chapter 4.2 of the Put Options, Proposition 2 was confirmed. Proposition 2 affirms the following:

"If a stochastic discount factor is negatively correlated with the price of a given security over all ranges of the security price, any put option on that security will have an expected return below the risk-free rate that is increasing in the strike price."

In this case, the results are in accordance with the conclusions achieved by Coval and Shumway (2001) and Bakshi, Madan and Panayotov (2010) on their articles.

In the last tests performed, in order to verify how volatility influences the expected option returns, zero-beta straddle options were created. Taking in consideration their characteristics, changes that happen on the underlying index, S&P 500, will not have any influence on the expected returns, only changes on the market volatility will have influence on them. According to the Black-Scholes/CAPM asset-pricing theory the returns obtained from zero-beta straddle options should be equal to the risk-free rate. By performing this test, it is possible to answer to the research question: *"What is the relation between expected option returns and the volatility of the underlying securities?"*. Since the results obtained indicate that the expected returns are negative, it is possible to conclude that in addition to market risk there is another risk associated with option contracts pricing. In accordance with Coval and Shumway (2001), since the returns are fundamentally determined by innovations in market volatility, it implies that systematic stochastic volatility may be an important factor for pricing assets.

The main constraint present on this thesis is the fact that the data available to perform it is limited to only one year, between January and December of 2016. Nonetheless, the results obtained are in conformity with the results achieved by other authors.

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