ISCTE Business School
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VOLATILITY MODELING BASED ON GARCH-SKEWED-T-TYPE MODELS FOR CHINESE STOCK MARKET

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Abstract

As an emerging stock market with enormous potential, Chinese stock market has apparent volatility clustering appearance along with typical feature of leptokurtic, negative skewness and fat tail in its index yield series. The model based on traditional normal distribution often underestimate the risk, which would lead to profound loss for the investors and financial institution when the extreme events happened. VaR(Value at Risk), which measures risk as a certain value, is widely used in financial industry for its intuitive and concise characteristics. Since parameter method of the VaR calculation is the mostly implementation in practice, the choice of appropriate probability distribution function and variance can quite improve its accuracy. Therefore, the conditional variance is estimated by GARCH-type models and the assumption of normal distribution is replaced by skewed-t distribution. Compared with the common RiskMetrics based on normal distribution, the ARMA-GJR-GARCH-skewed-t model has better adaptability and precision for the VaR estimation of indices of Chinese stock markets.
Key words: Value at Risk, Volatility, GARCH

JEL Classification: C15, G17

Resumo

Como um mercado emergente de ações com enorme potencial, o mercado acionário chinês tem uma aparente aparência de agregação de volatilidade, juntamente com uma característica típica de leptocurtice, assimetria negativa e cauda gorda em sua série de índices de rendimento. O modelo baseado na distribuição normal tradicional freqüentemente subestima o risco, o que levaria a perdas profundas para os investidores e instituições financeiras quando os eventos extremos acontecessem. O VaR (Value at Risk), que mede o risco como um determinado valor, é amplamente utilizado no setor financeiro por suas características intuitivas e concisas. Como o método de parâmetro do cálculo do VaR é a maior parte da implementação na prática, a escolha da função de distribuição de probabilidade apropriada e da variância pode melhorar bastante sua precisão. Portanto, a variância condicional é estimada pelo modelo GARCH-types e a suposição de distribuição normal é substituída pela distribuição skewed-t. Comparado com o comum RiskMetrics baseado na distribuição normal e outros modelos do tipo GARCH-skewed-t, o modelo ARMA-GJR-GARCH-skewed-t tem melhor adaptabilidade e precisão para a estimativa de VaR de índices dos mercados de ações chineses.

Palavras-Chave: Valor em Risco, Volatilidade, GARCH

JEL Classificação: C15, G17

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Chapter 1: Introduction

1.1 Research background

The return of financial assets and their risks have always been a major concern for investors, financial institutions, and national authorities. In 1952, Markowitz who won the 1990 Nobel Prize in Economics by proposing the mean-variance model for risk measurement, first introduced the mathematical statistics method into the applied to study of portfolio selection. However, the normal distribution of the traditional finance hypothesis is constantly under attack in specific financial practices since in reality the returns of financial assets have the properties of negative skewness, leptokurtic and fat tail, which could not be fully described by the two parameters, mean and variance. And the variance is not always constant as the it was assumed in the normal distribution. As one of the stylized facts of time series, the volatility clustering, first documented as Mandelbrot (1963), indicates that the volatility has the gathering tendency as the large changes would be followed by large changes and the small one tends to be followed by small changes. In the believe that volatility is dependent upon past realizations of the asset process and related volatility process, Engle(1982) proposed the ARCH model and a few years later Bollerslev(1986) extended it into the GARCH model by build the ARMA model for the error variance. As more exogenous regressors were added into the GARCH model, the standard GARCH model generated into lots of derivative models such as the Absolute Value GARCH (AV-GARCH) model of Taylor (1986) and Schwert (1990), the exponential GARCH (E-GARCH) model by Nelson & Cao (1991), Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model with the asymmetry factor γ by Glosten, Jagannathan and Runkle (1993), the Threshold GARCH (T-GARCH) model of Zakoian (1994) and so on. Meanwhile, the normal distribution assumption applied to the GARCH was found out not enough to explain the negative skewness, leptokurtic and fat tail. Various distributions have been implemented to describe the dataset such as student-t distribution, generalized error distribution, normal inverse gaussian distribution, Johnson’s SU distribution and so on.

As the common characteristics of emerging markets, the government policy impacts heavily in the evolution of Chinese stock market. Unlike the efficient market of the developed countries, the development of the policy-dependent stock market relies heavily on the intervention of national policies instead of the market regulation. The stock returns react differently to the "good news"(positive shock) and "bad news"(negative shock), which leads to the asymmetry of yield distribution. Furthermore, the property of return distribution such as negative skewness, leptokurtic and fat tail acts against the normal distribution assumption applied to traditional financial theory of market asset pricing, portfolio management and risk management.

According to " China Household Finance Survey"(released on May 13, 2012 by the institution China Household Finance Survey, consist of Southwestern University Of Finance And Economics and The People's Bank Of China, nearly 6% of new investors in China's stock market are illiterate, 25% have only received primary education, and about one-third have only completed junior high school education. Overall, Chinese stock investors have a low level of education even without financial basics. The irrational investors, nonstandard market order and the incomplete competitive regulation buildup the irrationality of the Chinese stock market and makes the of the stock price more sensitive to the released information and end up with high volatility.

Figure 1-1 Curve of the close price points of Index SSE from year 1990 to 2019

Source: Sina Finance

Figure 1-2 Curve of the close price points of Index SHE from year 1991 to 2019

Source: Sina Finance

Figure 1-1 and 1-2 give the evolution curve of the close price for Index SSE and SZSE from its starting date until now, which illustrates the obvious volatility clustering. Take the recent 2015-2016 Chinese stock market crash for example, due to the collateral damage of the 2009 global financial crisis, the Chinese company have been waddling down their debt. The economy growth used to benefit from the “real estate” has been in the slump since 2014. In order to get rid of the predicament, Chinese government attempted to prosper the Chinese stock markets evolution by affirmative action policy such as encouraging the company to raise finance by equity financing meanwhile decrease the high leverage rate. High leveraged investment (the leverage rate raised from 2.5~3 times to 5.5~6 times by the China Securities Regulatory Commission(CSRC)) approaches like securities margin trading and structured fund were available for individuals even without deep pockets. Good news and promising future have mobilized the investment passion of irrational investors, which leads to the fast-forwarding of market bubbles due to the overheating of virtual market. Under the circumstance, the Chinese government began to strengthen the regulation by increasing the stamp duty and adjusting the leveraged financing costs while the CSRC inspected off-site funding resource. Since the risk hedging of Chinese stock index futures or option is not available for individuals, large number of highly leveraged accounts were forced to close due to the free-fall. The insufficiency of risk control and management took its toll on investors especially individuals with enormous stock market value vaporized, which evoke the implementation of robust risk management method such Value at Risk.

1.2 Relevant theoretical literature review

As an implied assumption in the GARCH model, the positive shocks and the negative shock share the same weight (the absolute values are equal) as for their impact to the conditional variance. The terminology “leverage effect” was proposed by Black (1976) to describe the significantly different effects on the stock price volatility by the positive shocks (actual rate of return is greater than expected rate of return) and the negative shocks (real rate of return is less than expected rate of return). As he observed, when the leverage effect exists, the volatility of the stock price increases due to the appearance of negative shocks and decreases with the appearance of positive shocks. From an economic point of view, Christie's (1982) explained that the impact of negative information(price decline) would reduce the ratio of shareholders' equity relative to debt as a result to increase the leverage rate, which expose the company stock into more risk and more volatile. The empirical researches also testify the exist of leverage effect in several countries’ market. The asymmetry impact was found in Dan Mai, Norway, Sweden and Finland by Booth(1997), in Greece by Koutmos(1993), in American by Cheung and Ng(1992), in Canada, France and Japan by Koutmos(1992), and in England by Yeh(2000), where the negative shocks impact more than the positive shocks.

With the development of Chinese stock market, the GARCH-type models become very popular in the implementation for the estimation of various Chinses indices. In early research, the practical evidence in Chinese stock markets found by the Chinese scholars seems to be contrary to the results found in the developed countries. Lu and Xu(2004) found the positive shocks impact more on the volatility based on the estimation of model Index SSE(from year 1990-2003) and SZSE(from year 1991-2003) modeled by E-GARCH and the reason is ascribed to

(1) the policy-dependent stock market, where both the government and public favor the stock price to rise;

(2) the irrational investment attitude of Chinese individual investor compared with the rational foreign investor most consist of institutional investors;

(3) the unavailability information for the individual Chinese investors with deficiency of company disclosure.

(4) the lack of short-selling method makes the investors benefit only from the price rise.

As more tests have been performed in these major indices with large volume among investors with the updating data, most research of Chinese stock market shows similar appearance as the findings in western country.

Lin(2018) performed the test of modelling and forecasting the stock market volatility of SSE Composite Index(daily closing prices over the period extending from July 26, 2013 to July 28, 2017) using GARCH models. To better describe the significant properties of SSE Composite Index as time-varying, clustering, leptokurtosis and significant ARCH and GARCH effects, he compared the fitting and forecast result of GARCH (1, 1) (symmetric), TARCH (1, 1) and EGARCH (1, 1) (asymmetric), and found the EGARCH (1, 1) outperforms the others. The advice he finally gave is to strengthen the country’s system construction, reduce excessive government intervention and advocate rational investment philosophy for Chinsese investors.

The SSE50 Index also been tested by Li and Zhang (2017) performed the unit root test, serial correlation test and (G) ARCH model method to analyze the effectiveness of the stock index and futures market. The dataset they chose is the high-frequency data of every five minutes since the early days of CSI 500 and SSE50 on April 16, 2015 to June 16, 2015 and they found the weak-form efficiency appeared in the Chinses stock index and future market. They also suggested the financial reform and structural reform by supply-side to be taken by Chinese government.

The industrial Index is also taken into consideration by the Chinese scholars. As the industry which attracts most of investment but has lower down its growth recently, the Index of real estate was studied by Zhou(2018) to understand the volatility and risks of China's real estate stock market. By selecting the daily closing prices from January 5,2010 to June 30, 2017 as sample data, he studied the ARCH effect on this industry index by modeling the volatility of the time serials date through GARCH and TARCH models, where he found the ARCH effect is aggregative and persistent. Moreover, the asymmetry and leverage effect also exist since the investors are more sensitive toward negative news. Their reactions to sell the stock while the price is decreasing would cause the stock price to fluctuate.

As a newly emerged section in Chinese stock market with huge potential and government support, the Index CHINEXT, considered as the IPO platform for the high-tech enterprise, also attracts attentions of researches' interest and its VaR based on GARCH model was estimated by Ren, Dan and Liang(2015) and Song and Chen(2018) with T and GED distribution against the normal distribution.

The Chinese scholars also tried to apply different GARCH-type models to get better analysis performance. Based on the GJR-GARCH, GJR-M-GARCH, E-GARCH and E-GARCH-M model, He and Sun(2003) found negative shocks impact more to the volatility of Index SSE, SZSE and SHE in the period from year 1993 to 2002 and they concluded the GJR-GARCH model has better performance.

Different factors like hot money, government policy, behavior of investors and so on that may have an influence toward Chinese stock market have been put special attention by some Chinese scholars.

Based on the nonlinear Granger causality test and a new GARCH-class model originated on the mixed data sampling regression (GARCH–MIDAS), Wei, Yu, Liu and Cao(2018) investigated the influence between hot money on the return and volatility of the Chinese stock market. In their empirical results, there is neither linear nor nonlinear causality exists between the growth rate of hot money and the return of Chinese stock market, which finally indicates that the hot money should not be a value driven factor. However, considering hot money shows a significant positive impact on the long-term volatility of the Chinese stock market, and relevance between the long-term volatility caused by hot money and the total volatility of the Chinese stock market is time-variant, they concluded that the huge volatilities in the Chinese stock market are not always elicited by international speculation capital flow. Their suggestion toward Chinese authorities is to pay further attention to more systemic reforms in the completion of transaction rules to effectively regulate the Chinese stock market.

The government's policy is another important influence factor in Chinese stock market. For instance, the bailout policies for the stock disaster in 1994 and in 2015 for Shanghai-Hong Kong Stock Connect have significant impact on the Chinese stock market over time. Wang, Tsai and Li(2017) implemented the family of GARCH models to evaluate the structural changes in risks with the execution of a series of policies. Their empirical result indicates that substantial volatility is in the wake of certain policies, which is a little bit off their primary target to improve or stabilize the stock market. Among plenty of policies, macro-control policies and transaction cost adjustments, which are refer to the double-edged sword, should be used with caution.

Investors' attitude also has great impact on the stock market. From the perspective of industry life cycle, by establishing GARCH, TARCH and EGARCH model, Du and Xie (2018) evaluate the influence of variations in QFII(Qualified Foreign Institutional Investor) shareholding tendency on the stock returns volatility of 19 industries in Chinese. Among their empirical results, firstly, in terms of the significant asymmetry effect appears in the returns' volatility of A-shares of listed companies in the construction, manufacturing and real estate industries, the E-GARCH model can better

reveal the impact of the stock returns volatility caused by the differences of QFII's shareholding. Secondly, instead of participating in the industries that already been developed and slow down to the recession period, QFII is more likely to hold those stocks of the growing industries with high potential such as finance or real estate for a good while.

Value of Risk method based on the GARCH-type models was also implemented in the Chinese market. Chen and Yu(2002) estimated the VaR based on GARCH model with t distribution and GED distribution while the VaR based the extended GARCH model such EGARCH and PARCH was estimated by Gong, Chen and Yang(2005)

But as a measure of risk, VaR is more frequently used in the estimation of China's stock index futures and spot markets in recent years.

Zhou and Li(2016) make the empirically investigation by apply a VAR-GARCH model with SSAEPD margins to the daily data of China's stock index futures and spot markets to find out the correlation between price discovery and information transmission. By implementing the VAR-DCC-GARCH model, Yue, Liu and Shan(2015) discovered the co-movement between Chinese nonferrous metal prices and global nonferrous metal prices represented by the nonferrous metal prices from London Metal Exchange (LME). In Zhao(2010)'s research, the VAR and multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models can be applied to the empirical analysis of the dynamic relationship between Renminbi (RMB) real effective exchange rate and stock price by using monthly data from January 1991 to June 2009.

1.3 Research idea and structure

With the self-improvement and the more completed regulation of Chinese stock market, the performance and characteristics also changed with the evolution. Therefore, we'd like to estimate the resent status of Chinese stock market through the GARCH-type(E-GARCH and GJR-GARCH) model with the skewed-t in order to better capture the volatility affected by leverage effect and the negative skewness, leptokurtic and fat tail feature of the dataset. Meanwhile, the VaR is also calculated and compare with the traditional RiskMetrics model for the practical application in reality. To give a general analysis of Chinese stock market, not only the most estimated Index SSE, SHE, SZSE and CHINEXT, the industrial representative Index SSEII, SSECI, SSEREI and SSEUI were also included in the estimation to see if our method can adapt to the whole market.

The dissertation is divided into 5 chapters:

Chapter 1 is the introduction part where we demonstrate the research background, relevant literature review and our research method and structure.

Chapter 2 is the methodology part where we present the academic theory and mathematic method for our model settings and the following tests.

Chapter 3 is the empirical evidence where we implemented the GARCH-type models in the dataset.

Chapter 4 consists of the VaR forecasting results showed by graph along with the backtesting method.

Chapter 5 is the conclusion to summarize the whole dissertation and research perspective.

Chapter 2: Technical method

The chapter 2 technical is organized in two parts. In the first part, we introduce the academic theory and mathematic method for our model settings. In the second part, we are going to elaborate several tests implemented in the model in details.

2.1 Introduction of model setting

At first, we are going to give the specific introduction of GARCH-type models including standard GARCH model, E-GARCH model and GARCH model and their detailed mathematical method. Secondly, we interpret how to calculate the value of risk based on the RiskMetrics(matched group) and GARCH-type models. Finally, we also introduce the skew-t distribution implemented in the GARCH-type model to fit better with the data of the leptokurtic and heavy tail.

2.1.1 Introduction of GARCH-type model

Denote x_t (t = 1, …, T) as a set of time series of financial returns. We can divide it the 'features' into three parts:

(1) Conditional mean which contains all the information about the location of the distribution.

(2) Conditional variance which contains a measure of the dispersion of the distribution.

(3) Shape parameters (including skewness and kurtosis which determines the conditional distribution shape.

In traditional economic models, the variance of the disturbance is assumed to be constant. However, many time-series data show both staged violent fluctuations and phased relative stability. In this case, it is not appropriate to assume that the variance is constant (the same variance). One way to predict variance is to introduce an independent variable to estimate volatility.

ARMA model:

Peter Whittle (1951) firstly described the general ARMA model in his thesis, which is used to provide a parsimonious description of a (weakly) stationary stochastic process in terms of two polynomials as for the statistical analysis of time series. The autoregressive–moving-average (ARMA) model is a hybrid model of the autoregression (AR) model and the moving average (MA) model.

The AR(p) model is written as followed with p order autoregressive terms:

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t . \quad \textbf{(1)}$$

The MA(q) model is written as followed with q order moving terms:

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \quad \textbf{(2)}$$

Then he ARMA (p, q) model consists of the AR(p) and MA(q) models with p autoregressive terms and q moving-average terms:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} . \quad \textbf{(3)}$$

Peter Whittle (1951) describe the general ARMA model was described through the mathematical analysis (Laurent series and Fourier analysis) and statistical inference. But the ARMA models were actually popularized by George E. P. Box and Jenkins (1970) in a book, where they expounded an iterative method for choosing and estimating the model. To be clarify, estimated by Hannan & Deistler (1988), the (Box–Jenkins) method was useful for low-order polynomials (of degree three or less).

ARCH(q) model:

The ARCH model is combined with the mean equation and the conditional variance equation, where the conditional variance σ2 of the error term would be influenced by the variables from the previous time:

$$y_t = x_t' b + \epsilon_t \quad \textbf{(4)}$$

$$\sigma_t^2 = \alpha_0 + \alpha_0 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad \textbf{(5)}$$

with the constrains of $\alpha_0 > 0$ and $\alpha_i \geq 0$, i>0.

Engle (1982) pointed out that even though the least squares method can produce unbiased estimates, it is more efficient to use the maximum likelihood estimation method to estimate the variables.

GARCH(p,q)

Bollerslev(1986) extended the original ARCH model into the generalized ARCH model by assuming that the error variance can be model by the ARMA model, so that the lagged values can be more elastic in the structure.

$$y_t = x_t'b + \varepsilon_t \quad \textbf{(6)}$$

$$\epsilon_t \mid \psi_{t-1} \sim \mathcal{N}(0, \sigma_t^2) \quad \textbf{(7)}$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \quad \textbf{(8)}$$

A crucial feature of the observed behavior of financial data is that whether the GARCH model can capture the volatility clustering, which may be quantified in the persistence parameter $\hat{P}$. It can be calculated as

$$\hat{P} = \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j . \quad \textbf{(9)}$$

E-GARCH model

Nelson & Cao (1991) extended the GARCH model into the exponential generalized autoregressive conditional heteroskedastic (EGARCH) model through the assumption that asset volatility is negative correlated with the return on assets. In other words, the volatility tends to decline when the return is positive as the asset prices rise. On the contrary, the volatility tends to increase when the return is negative as the asset prices fall.

$$\log_e\left(\sigma_t^2\right) = \left(\omega + \sum_{j=1}^{m} \zeta_j \upsilon_{jt}\right) + \sum_{j=1}^{q} \left(\alpha_j z_{t-j} + \gamma_j \left(\left|z_{t-j}\right| - E\left|z_{t-j}\right|\right)\right) + \sum_{j=1}^{p} \beta_j \log_e\left(\sigma_{t-j}^2\right) \quad \textbf{(10)}$$

where the coefficient α_j capturing the sign effect, γ_j the size of effect (impact of a magnitude of a shock (size) /arch effect / spillover effect) and β the persistence of past volatility (past volatility explains for the current volatility). The parameter Z_t allows the sign and the magnitude to have separate effects on the volatility while it can be a standard normal variable or come from a generalized error distribution. If α_j is statistically significant from zero, it indicates there exists asymmetric effect (bad news and good news of the same size have different impacts). For example, if α_j is negative, leverage effect: bad news has more impact than the good news of the same size.

The persistence $\hat{P}$ is given by,

$$\hat{P} = \sum_{j=1}^{p} \beta_j. \quad \textbf{(11)}$$

GJR-GARCH model

The asymmetric effects of positive and negative shocks in the ARCH process are taken into consideration by Glosten, Jagannathan and Runkle (1993) through the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) via the use of the indicator function I,

$$\sigma_t^2 = \left(\omega + \sum_{j=1}^{m} \zeta_j \upsilon_{jt} \right) + \sum_{j=1}^{q} \left(\alpha_j \varepsilon_{t-j}^2 + \gamma_j I_{t-j} \varepsilon_{t-j}^2 \right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad \textbf{(12)}$$

$$I_{t-j} = \begin{cases} 1 & if\ \varepsilon_{t-j} < 0 \\ 0 & otherwise \end{cases} \quad \textbf{(13)}$$

where γ_j now represents the "leverage" term. The positive γ_j indicates the exist of lever effect and the magnitude is the absolute value of γ_j. And the persistence formula also changed due to the presence of the indicator function and it crucially depends on the asymmetry of the conditional distribution used,

$$\hat{P} = \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{p} \beta_j + \sum_{j=1}^{q} \gamma_j \kappa \quad \textbf{(14)}$$

where κ is the expected value of the standardized residuals z_t below zero (effectively the probability of being below zero),

$$\kappa = E\left[I_{t-j} z_{t-j}^2 \right] = \int_{-\infty}^{0} f\left(z, 0, 1, \ldots\right) dz \quad \textbf{(15)}$$

the f function stands for the standardized conditional density with any additional skew and shape parameters. For instance, under the scenario of symmetric distributions, we have the κ simply equal to 0.5.

2.1.2 The data distribution of the model setting

The traditional GARCH model that based on the assumption of normal distribution is not sufficient to characterize the and heavy tail feature of Chinese stock market considering all the indices with the kurtosis above 5 and obvious negative skewness. Therefore, the distribution that is more capable to capture the asymmetry and peakness of data should be found and implemented to the GARCH model. Here we introduce the skew-t distribution.

2.1.2.1 The normal distribution

As a matched group, the RiskMatrics model we chose to give a comparison of Value at risk is the iGARCH model(1,1) based on the normal distribution. The Normal Distribution known as a spherical distribution is described completely by the first moments(mean) and the second moments(variance).

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \textbf{(16)}$$

2.1.2.2 The skewed-t Distribution

Bollerslev (1987) is the first person to apply the student-t distribution in the GARCH model for fitting the standardized innovations as an alternative to the normal distribution. Its probability density formula is described completely by a shape parameter υ,

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \quad \textbf{(17)}$$

where υ is the number of degrees of freedom and Γ is the gamma function. By adding the shape parameter υ, the Student distribution can fit the data with leptokurtic since the excess kurtosis will be equal to 6/(υ − 4) for ν > 4. But it cannot deal with the negative skewness since it has zero skewness when υ is bigger than 3.

Originated from the original student t distribution, Hansen (1994) proposed the skewed-t distribution, then it was extended by Fernandez and Steel (1998). By adding the skewness parameter based on the student-t distribution, the skewed-t distribution can better describe the asymmetric and fat tail features of financial asset serials. Define:

$$z_t = \left(\varepsilon_t - m\right)/s \quad \textbf{(18)}$$

$$f(\varepsilon|\xi) = \left[2/\left(\xi+1/\xi\right)\right]\cdot\left[g\left(\varepsilon/\xi\right)\mathrm{I}_{[0,\infty]}\left(\varepsilon\right)+g\left(\varepsilon\xi\right)\mathrm{I}_{[-\infty,0]}\left(\varepsilon\right)\right] \quad \textbf{(19)}$$

$$g(z|\nu) = \left[\Gamma\left(\left(\nu+1\right)/2\right)/\left(\sqrt{\pi\left(\nu-2\right)}\Gamma\left(\nu/2\right)\right)\right]\cdot\left[1+z^2/\left(\nu-2\right)\right]^{-(\nu+1)/2} \quad \textbf{(20)}$$

If the sequence z_t fulfill the condition(1)、(2)、(3) and has the mean of 0 and variance of 1, it can be considered follow a standard skewed-t distribution, write z_t~SKST(0,1,ξ,ν), so the probability density function of z_t is

$$f(z_t|\xi,\nu)=\frac{2}{\xi+1/\xi}s\left\{g\left[\xi\left(sz_t+m\right)|\nu\right]\mathrm{I}_{[-\infty,0]}\left(z_t+\frac{m}{s}\right)+g\left[\frac{sz_t+m}{\xi}|\nu\right]\mathrm{I}_{[0,\infty]}\left(z_t+\frac{m}{s}\right)\right\}$$

(21)

where ξ is the asymmetric parameter, so as the skewness parameter; υ (>2) is the degree of freedom, so as the tail parameter. At the meantime, when $z_t \geq -m/s$, $d_t = 1$, when $z_t < -m/s$, $d_t = -1$, the standard variance and mean function of the skewed-t distribution is defined respectively as follows:

$$s=\sqrt{g^2+g^{-2}-m^2-1} \quad \textbf{(22)}$$

$$m=\Gamma\left(\left(\nu-1\right)/2\right)\cdot\sqrt{\pi\left(\nu-2\right)}\left(\Gamma\left(\nu/2\right)\sqrt{\pi}\right)^{-1}\left(g-g^{-1}\right) \quad \textbf{(23)}$$

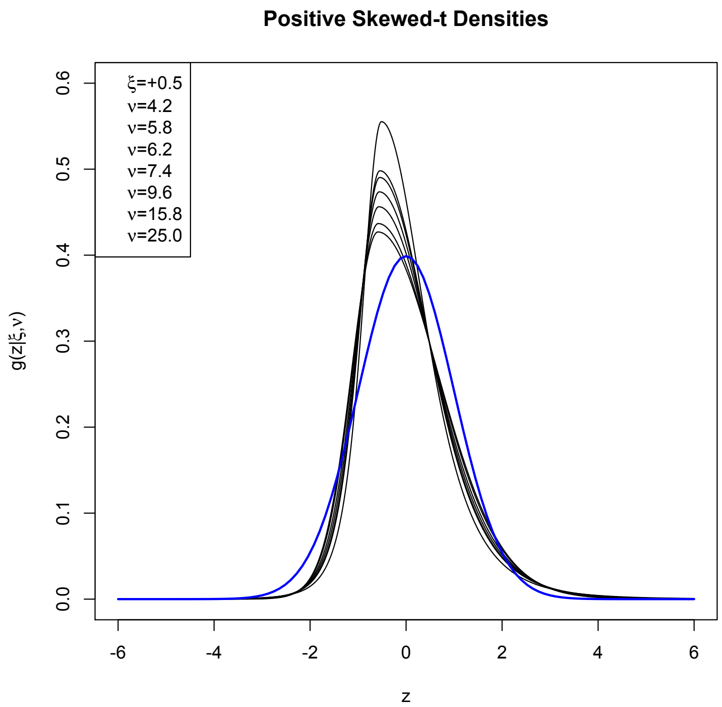

Figure 2-1 Positive skewed-t distribution density curve compared with normal distribution curve

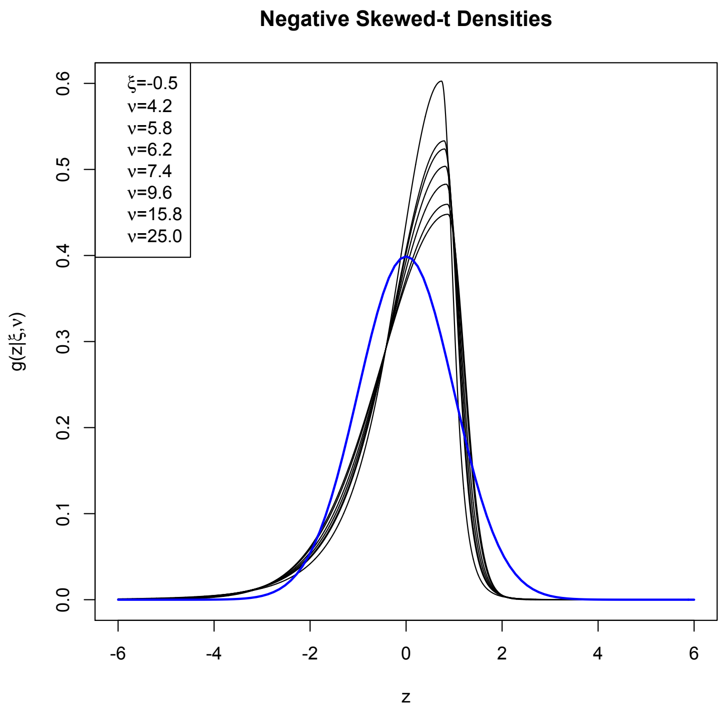

Figure 2-2 Negative skewed-t distribution density curve compared with normal distribution curve

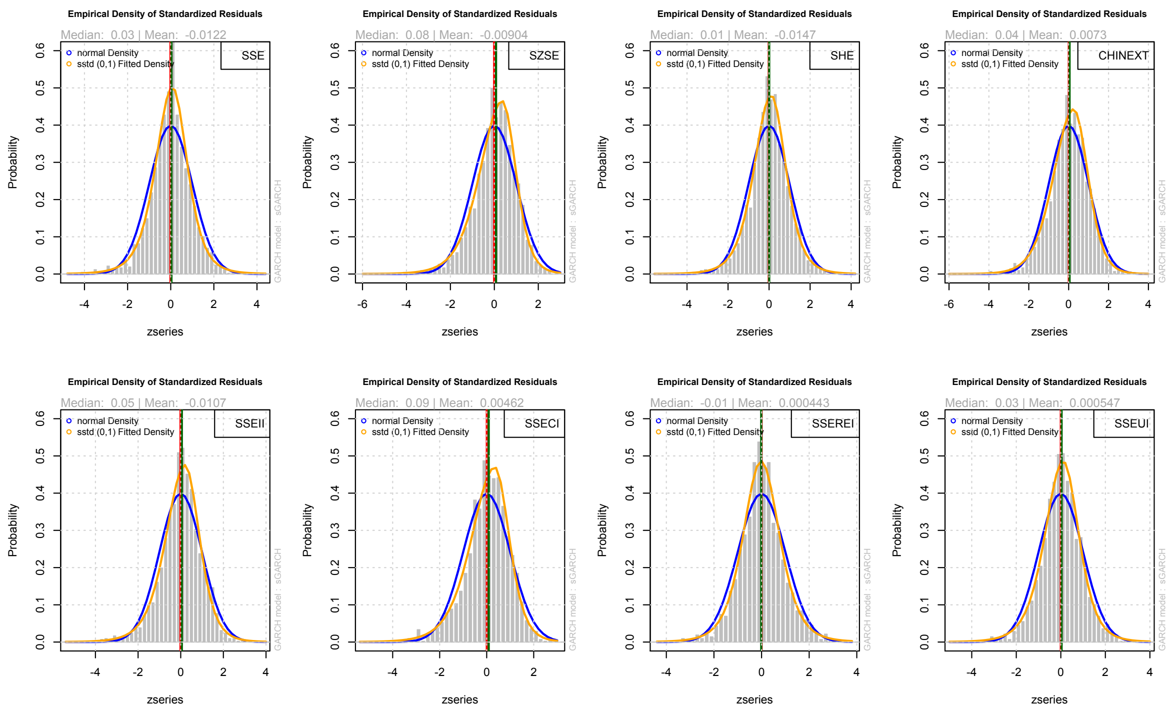

Figure 2-3 skewed-t distribution density curve of eight indices compared with their normal distribution curve

The figure 2-1 shows the comparison between the normal distribution density function curve and the probability density function graph with the skewness parameter ξ of +0.5 and the tail parameter ν of 4.2, 5.8, 6.2, 7.4, 9.6, 15.8, 25.0, respectively, and the comparison with the normal distribution density function. The figure 2-2 shows the comparison between the normal distribution density function curve and the probability density function graph with the skewness parameter ξ of -0.5 and the tail parameter ν of 4.2, 5.8, 6.2, 7.4, 9.6, 15.8, 25.0, respectively, and the comparison with the normal distribution density function.

As can be seen from figure 2-3, the histogram of the standardized residuals of the eight index returns of the observed period year 2010 to 2018 was presented, along with the fitted normal distribution density curve and skewed-t density curve. It's quite distinct that the skewed-t density curve (the orange one) can fit the dataset much better than the normal distribution density curve (the blue one) for both situations, the clear negative skewed situation of SZSE, CHINEXT, SSEII, SSECI and the leptokurtic situation of all indices.

2.2 Introduction of value at risk (VaR)

Value at risk (VaR) as a highly effective risk quantification technology developed internationally is a measurement of the risk. While in a stated duration, given the hypothetic market conditions, it can estimate how much a profile might lose (with a given probability).

The VaR of X at the confidence level α ∈ (0,1) is the smallest number y so that the probability that Y : = - X does not exceed y is at least 1-α, so to speak, mathematically, $VaR_\alpha(X)$ is the (1-α)-quantile of Y. As the most generalized VaR definition is presented as follow:

$$VaR_\alpha\left(X\right)=\inf\left\{x\in\mathbb{R}:F_X\left(x\right)>\alpha\right\}=F_Y^{-1}\left(1-\alpha\right) \quad \textbf{(24)}$$

2.2.1 VaR of RiskMetrics model

In order to make a comparison between the VaR estimated by GARCH-type models and the traditional method, RiskMetrics model is chosen as a benchmark. RiskMetrics (released in October 1994. by J.P. Morgan) is a methodology that contains techniques and data sets used to calculate the value at risk (VaR) of a portfolio of investments.

Unlike the skew-t distribution we used in GARCH type models, which fits well for the actual situation of Chinese stock market, the RiskMetrics methodology for calculating the VaR assumes that the asset returns follow a normal distribution. JP Morgan set a value for the decay factor, being 0.94 for daily and 0.99 for monthly holding periods. As we implement the decay factor 0.94 in the IGARCH(1,1)model of the daily stock returns under normal distribution, the Riskmetrics model is showed as follow:

$$\begin{cases} \varepsilon_t = z_t \sigma_t , \\ \sigma_t^2 = 0.06\varepsilon_{t-1}^2 + 0.94\sigma_{t-1}^2 . \end{cases} \quad \textbf{(25)}$$

where z_t follows i.i.d.(standard Gaussian) normal distribution $N(0,1)$

2.2.1 VaR of GARCH-type model

The traditional parametric method to calculated VaR is defined with the mean μ and the standard deviation σ as follow:

$$VaR(X) = \mu + \sigma N^{-1}(X) \quad \textbf{(26)}$$

where $N^{-1}(.)$ is the inverse of the cumulative normal distribution at a given confidence level X.

As for the GARCH-type model with skewed-t distribution, the standard deviation σ (unconditional variance) would be replaced with conditional variance calculated by the model. And the N^{-1} is also replaced with the critical value on the skewed-t distribution of the selected probability.

2.3 Introduction of the tests implemented in the model analysis

2.3.1 Introduction of the tests implemented in the model estimation

Autoregressive conditional heteroskedasticity test (ARCH test)

Under a dynamic conditional variance process, even an uncorrelated time series can be serially dependent, this time series that exhibits conditional heteroscedasticity (also called autocorrelation in the squared series), is said to have autoregressive conditional heteroscedastic (ARCH) effects. Engle (1982) proposed a Lagrange multiplier test (called ARCH test for short) to assess the significance of ARCH effects.

$$H_a : e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_m e_{t-m}^2 + u_t \quad \textbf{(27)}$$

$$H_0 : \alpha_0 = \alpha_1 = \ldots = \alpha_m = 0 \quad \textbf{(28)}$$

If we don't reject the null hypothesis, the parameter α_m is not significant, which left us with only the white noise u_t, and the alternative hypothesis is the existence of autocorrelation in the squared residuals, the test statistic is the usually the F statistic for the regression on the squared residuals that follows a $\chi 2$ distribution with m degrees of freedom. A large critical value indicates rejection of the null hypothesis in favor of the alternative.

In the R package "rugarch" we used, the ARCH-LM test have been replaced ARCH-LM statistics of Fisher and Gallagher (2012) since the advance version uses a weighted portmanteau test for testing the null hypothesis, which explains the distribution of the statistics of the values from the estimated models better.

Ljung–Box test (LB test)

The Ljung–Box test (also known as Ljung–Box Q test,) named after G.M. Ljung and G.E.P. Box(1978) is based on a series of lag orders to determine whether the relevance or randomness exists among the population sequence.

The null hypothesis of the Ljung–Box test is that the original data is independent, so to speak, the overall correlation coefficient is 0, and the observed correlations are only caused by the random sampling errors while the alternative hypothesis is that the data are not independently distributed(for instance: they existence of serial correlation).

The test statistic Q is calculated by the formula as follow:

$$Q = n\left(n+2\right)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \quad \textbf{(29)}$$

At the chosen significance level α, when $Q > \chi^2_{1-\alpha,h}$ (the critical region) we reject the hypothesis of randomness. The Ljung–Box test is commonly used in autoregressive integrated moving average (ARIMA) modeling and specifically applied to the residuals of a fitted ARIMA model to test if the autocorrelation exists in the residuals of the ARIMA model. In the R package "rugarch" we used, the the LB Q-statistics have been replaced with the Weighted Ljung-Box of Fisher and Gallagher (2012)

Sign Bias test

The Sign Bias Test, proposed by Engle and Ng (1993), can test the presence of leverage effects in the standardized residuals by regressing the squared standardized residuals on lagged negative and positive shocks. If the leverage effect isn't eliminated by the fitted model, there may exist possible misspecification of the GARCH model. The regression process is displayed as follows:

$$\hat{z}_t^2 = c_0 + c_1 I_{\hat{\varepsilon}_{t-1}<0} + c_2 I_{\hat{\varepsilon}_{t-1}<0}\hat{\varepsilon}_{t-1} + c_3 I_{\hat{\varepsilon}_{t-1}\geq 0}\hat{\varepsilon}_{t-1} + u_t \tag{30}$$

where I is the indicator function and $\hat{\varepsilon}_t$ is the estimated residuals from the GARCH process. The null hypotheses are H_0: $c_i = 0$ (for i = 1, 2, 3), and that jointly H_0: $c_1 = c_2 = c_3 = 0$. It can also be interpreted as the t test of coefficient significance and the significance test of regression equation in corresponding. If we don't reject null hypothesis, it indicates that the asymmetric effects still exist in our model which means the leverage effects wasn't eliminated by the fitted model.

2.3.2. Value at Risk backtesting method

The BackTesting method is implemented in the VaR value we estimated in the model to see whether the predicted performance is good or not.

Proportion of failures (POF) test

Proportion of failures (POF) test proposed by Kupiec (1995) considers the actual loss over VaR as a failure and the actual loss below VaR as a success. Then the observation of failure or success can be regarded as a series of independent Bernoulli tests and a likelihood ratio can be used to test whether the probability of exceptions is synchronized with the probability p implied by the VaR confidence level. The statistical formula of POF test is

$$LR_{POF} = -2\log\left(\frac{(1-p)^{N-x}p^x}{\left(1-\frac{x}{N}\right)^{N-x}\left(\frac{x}{N}\right)^x}\right) \tag{31}$$

where x is the number of failures, N the number of observations and p=1 – VaR level. This formula is asymptotically distributed as a chi-square variable with 1 degree of freedom. The null hypothesis is that the accuracy of the test model is equivalent to the failure rate that is equal to a specific probability. The VaR model would be considered to fail the test if this likelihood ratio exceeds a critical value which depends on the test confidence level.

The null hypothesis of the unconditional coverage (Kupiec) showed in the rugarch package is correct exceedances.

Christoffersen's Interval Forecast Tests

Interval forecasts evaluation was proposed by Christoffersen (1998) to measure whether the probability of observing an exception on a particular day depends on whether an exception occurred. Unlike the method presented in the POF test of unconditional probability of observing an exception, Christoffersen's test only measures the dependency between consecutive days.

Here is the statistical formula to test the independence in Christoffersen's interval forecast test:

$$LR_{CCI} = -2\log\left(\frac{\left(1-\pi\right)^{n00+n10}\pi^{n01+n11}}{\left(1-\pi_0\right)^{n00}\pi_0^{n01}\left(1-\pi_1\right)^{n10}\pi_1^{n11}}\right) \quad \textbf{(32)}$$

Table of Denotation Factors
n00 = Number of periods with no failures followed by a period with no failures.
n10 = Number of periods with failures followed by a period with no failures.
n01 = Number of periods with no failures followed by a period with failures.
n11 = Number of periods with failures followed by a period with failures.
π0 — Probability of having a failure on period t
given that no failure occurred on period t − 1 = n01 / (n00 + n01)
π1 — Probability of having a failure on period t,
given that a failure occurred on period t − 1 = n11 / (n10 + n11)
π — Probability of having a failure on period t = (n01 + n11 / (n00 + n01 + n10 + n11)

Figure 2-3 Table of Denotation Factors

The explanation was given in the Figure2-1. This formula is asymptotically distributed as a chi-square variable with 1 degree of freedom.

Conditional Coverage (Christoffersen) showed in the rugarch package is combined of the frequency POF test (LR_{POF}) and Interval Forecast Tests (LR_{CCI}) : $LR_{CC} = LR_{POF} + LR_{CCI}$ while its null hypothesis is correct exceedances and independence of failures.

Chapter 3: Data analysis and empirical research

Chapter 3 will be combined with two parts: (1) data analysis including data selection of the Chinese stock market and the following analysis of time serials data; (2) empirical evidence for the GARCH-type models implemented in the indices of Chinese stock market

3.1 Data analysis

3.1.1 The data selection.

The national representative indices for China are the Shanghai Composite Index (SSE, code: 000001.SS) and Shenzhen Composite Index (SZSE, code: 399106.SZ) which tells a whole story of the Chinese stock market. The Shenzhen Component Index (SHE, code: 399001.SZ) and the Growth Enterprises Market Index, also named as CHINEXT PRICE INDEX in Google Finance, (CHINEXT, code 399006.SZ) stands for the selected company listed in China with high volatility and return. Three industry indices are presented as SSE Industrial Index (SSEII, code: 000004.SS) SSE Commercial Index (SSECI, code: 000005.SS), SSE Real Estate Index (SSEREI, code: 000006.SS) and SSE Utilities Index (SSEUI, 000007.SS) to give an overall perspective for the Chinese industry development.

The different characteristics and details of these indices are described as follow:

The Shanghai Composite Index is a statistical indicator of the overall trend of listed stocks on the Shanghai Stock Exchange. The sample shares are all listed stocks, including A shares and B shares, reflecting the changes in the prices of listed stocks on the Shanghai Stock Exchange and its base period is since September 19, 1990 and the base point is 100 points.

The Shenzhen Composite Index is a statistical indicator of the overall trend of listed stocks on the Shenzhen Stock Exchange. The sample shares are all listed stocks, including A shares and B shares, reflecting the changes in the prices of listed stocks on the Shenzhen Stock Exchange and its base period is since April 3, 1991 and the base point is 100 points.

Shenzhen Component Index (hereinafter referred to as Shenzhen Stock Exchange) is the main stock index of Shenzhen Stock Exchange. It is based on a certain standard selected 40 representative listed companies as constituent stocks, the constituent stocks of free float as the number of shares, the use of Paasche method of weighting the stock price index. And its base period is since July 4, 1994 and the base point is 1000 points.

Growth Enterprises Market Board(English name: CHINEXT), is the Second-board Market as the complement to the Main-Board Market, making its debut on October 30, 2009. Being the same role in China as NASDAQ in America, it provides financing channels and growth space for companies as entrepreneurial enterprises, small and medium-sized enterprises and high-tech industrial enterprises that cannot be listed on the main board. Since most companies listed on the Growth Enterprises Market are engaged in high-tech business with high growth potential along with the government incentive policy, compared with other Chinese stock sectors, it expresses the characteristics of low threshold, high yield return and huge volatility.

Under the Shanghai Industry Classification Index, the listed companies on the Shanghai Stock Exchange are divided into five major categories according to their respective industries. The Industrial Index, Business Index, Real Estate Index and the Utilities Index reflects the economic situation of the industry sector and the overall changes in its share price respectively. And the listed company number is 985,139, 25 and 123 for each sector.

In order to get a general perspective of Chinese stock market, the sample starting point has been chosen as the early as the date when data for all indices were available and until the latest date. Since the Index CHINEXT was available after October 30, 2009 and we prefer not to use the data from the preliminary phase with obvious booming tendency. So the chosen data would cover the period from July 2010 to December 2018 including 2083 trading days. The closing price observed in the t and t+1 period is labeled as P_t and P_{t+1}, respectively. The continuous compound interest stock yield sequence R_t is computed as:

$$R_t = 100 \times \ln \frac{P_t}{P_{t-1}} = 100 \times \ln\left(P_t - P_{t-1}\right) \quad \textbf{(33)}$$

where the data source is NTES Finance

3.1.2. The basic data characteristics

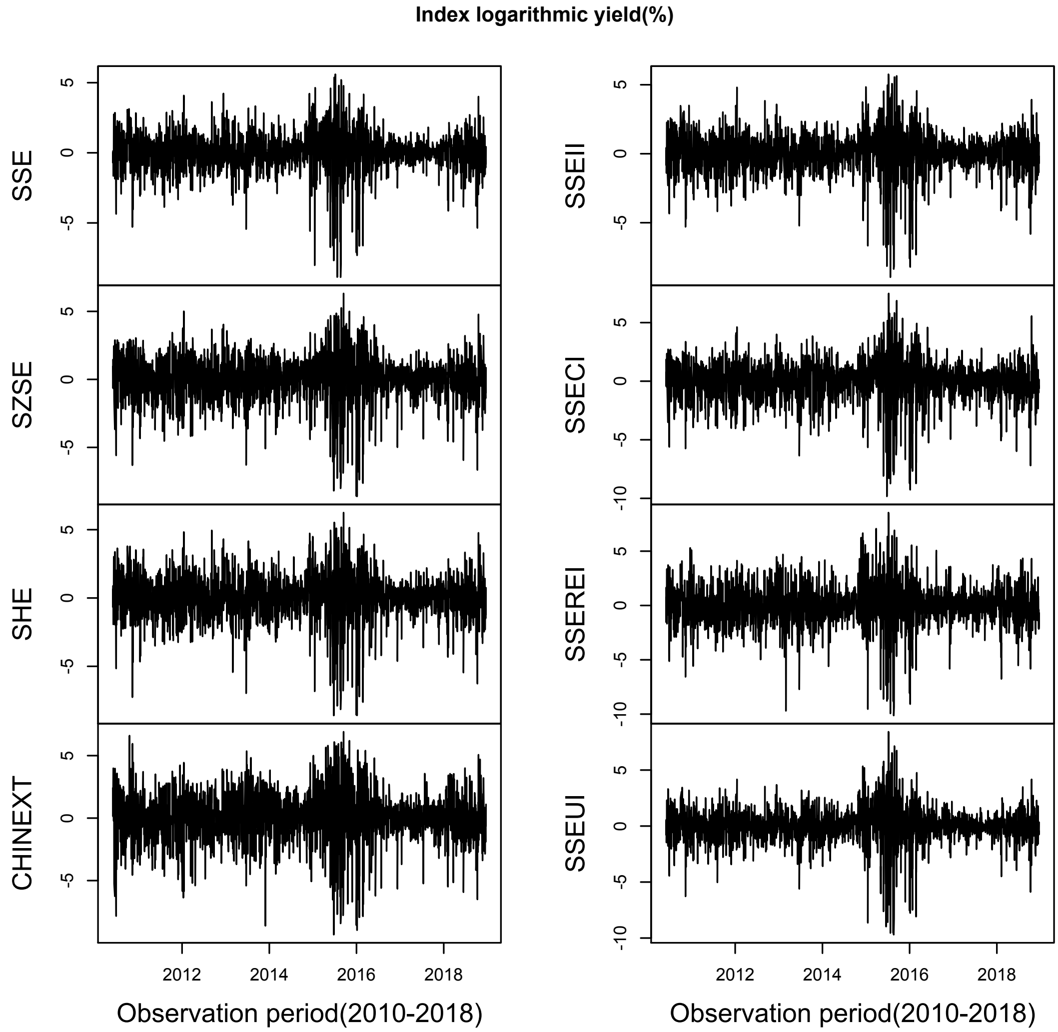

Figure 3-1 The logarithmic yield of the eight indices.

From Figure 3-1, we can observe that there are some terms of correlation between the logarithmic returns of the eight indices, which indicates they have a similar trend of evolution. Another apparent appearance is that high-volatility events exhibit the tendency to gather in time, support the volatility clustering statement, one of the stylized facts revealed by Rama (2001). The 'ARCH (Autoregressive conditional heteroskedasticity) effect' show by the eight indices point out that ARCH model can be used to evaluate the evolution. Obviously, the two indices, Shenzhen Component Index (SHE) and the Growth Enterprises Market Index (CHINEXT), which comprise the selected emerging companies, did show greater volatility than the rest of indices. Overall, all the indices show tremendous volatility over 2015-2016, relevant with the stock market crash mentioned in the background.

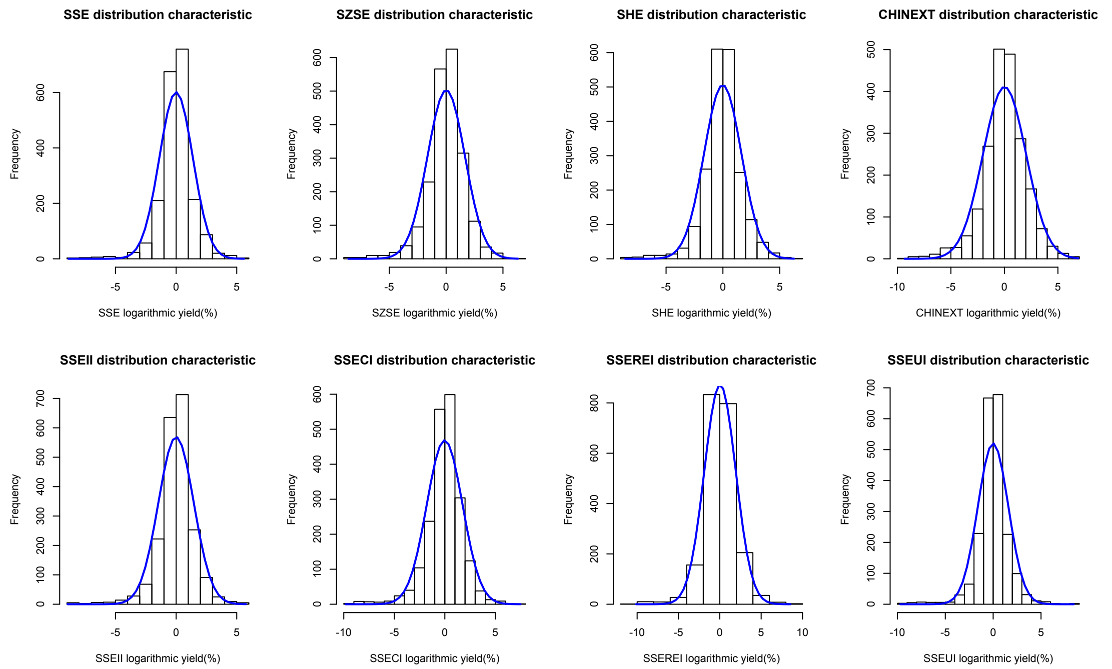

Figure 3-2 The histogram of eight indices' logarithm returns along with the normal distribution density curve

Figure 3-2 is the histogram of eight indices' logarithm return compared with the normal distribution density curve, which give us an intuitively point of view that the all the indices except SSE Real Estate Index have clearly characteristics of left skew and leptokurtosis.

Some data description and empirical evidence of Chinese stock markets will be revealed through previous methods as long as the goodness-of-fit test for its normality.

Table 3-1 Descriptive analysis of the eight indices' logarithm returns

Index	N	Skewness	Kurtosis	JB_TestX^2	JB_Test.P	L_Test.D	L_Test.P	SW_Test.W	SW_Test.P	KS_Test.D	KS_Test.P
SSE	2083	-0.9574	9.2268	3683.4	< 2.2e-16	0.0933	< 2.2e-16	0.9124	4.54E-33	0.0465	2.44E-04
SZSE	2083	-0.8487	6.3291	1211.9	< 2.2e-17	0.0801	< 2.2e-17	0.9474	1.18E-26	0.0777	2.34E-11
SHE	2083	-0.7036	6.7386	1385.0	< 2.2e-18	0.0768	< 2.2e-18	0.9436	1.71E-27	0.0705	2.09E-09
CHINEXT	2083	-0.5217	5.0902	473.7	< 2.2e-19	0.0622	< 2.2e-19	0.9687	6.81E-21	0.1320	< 2.2e-16
SSEII	2083	-0.9482	8.3695	2814.5	< 2.2e-20	0.0890	< 2.2e-20	0.9231	2.26E-31	0.0445	5.18E-04
SSECI	2083	-0.9485	7.3022	1918.8	< 2.2e-21	0.0821	< 2.2e-21	0.9341	2.08E-29	0.0850	1.65E-13
SSEREI	2083	-0.4954	6.9804	1460.3	< 2.2e-22	0.0692	< 2.2e-22	0.9451	3.49E-27	0.1001	< 2.2e-16
SSEUI	2083	-0.8233	10.1402	4660.1	< 2.2e-23	0.1000	< 2.2e-23	0.8962	2.25E-35	0.0535	1.36E-05

Table 3.1 shows some statistical characteristics of the eight indices. In terms of third-order moments parameter skewness, it can be seen that the skewness of the eight indices is negative, which means the eight indices have obvious asymmetry and all the indices show different degrees of left-bias. In terms of the fourth-order moments parameter kurtosis, it can be seen that the all the eight indices have the significant magnitude of the kurtosis larger than 3, which shows a more serious problem of excess kurtosis, that is, there is a problem of peaks and thick

tails compared with the normal distribution. This is consistent with the results observed directly in the log yield graph in Figure 3-1.

Several statistical tests such as Jarque-Bera test, Kolmogorov-Smirnov test, Lilliefors test and Shapiro-Wilk test have been implemented to see whether the dataset obeys the normality distribution. Since all the null hypothesis of normal distribution has been rejected, it can be deduced from the test result, that the assumption of the normal distribution of the logarithm return of the eight indices should be abandoned.

3.1.3 Data analysis of time series dataset

3.1.3.1 Unit root test

The most important and primary assumption for time series is the stationarity, which indicates the probability laws that govern its behavior of the process doesn't change over time. In mathematics and statistics, a stationary process or, to be more specific, a strict stationary process is described as a stochastic process whose unconditional joint probability distribution wouldn't change with the time. Since the conditions of strong stationary are too difficult to satisfy both in theory and in reality, instead, the time series is only required to fulfill the week stationarity with two assumptions: (1) the mean function is a constant over time; (2) the covariance function is only related to time interval. The non-stationary data can be converted to static through the difference method. The ADF (Enhanced Dickey-Fuller) test is the common test implemented in the dataset with large samples with the null hypothesis of defined as the existence of a unit root. The estimated time series data is stationarity is we reject the null hypothesis.

Table 3-2 The result of ADF test (unit root test)

Index	ADF test	1% level	5% level	10% level	P value
SSE	-32.5136	-3.4333	-2.8627	-2.5674	0.01
SZSE	-31.6061	-3.4333	-2.8627	-2.5674	0.01
SHE	-32.0922	-3.4333	-2.8627	-2.5674	0.01
CHINEXT	-32.5915	-3.4333	-2.8627	-2.5674	0.01
SSEII	-32.4256	-3.4333	-2.8627	-2.5674	0.01
SSECI	-30.9341	-3.4333	-2.8627	-2.5674	0.01
SSEREI	-32.5625	-3.4333	-2.8627	-2.5674	0.01
SSEUI	-32.3130	-3.4333	-2.8627	-2.5674	0.01

Table 3-2 is the ADF test result of the logarithm return of the eight indices. The t values of ADF test of all the indices are smaller than the t value in the 1%, 5% and 10% level confidence level. The null hypothesis of unit root exist is rejected at all the 1%, 5% and 10% level, which implies that the time serials of the eight indices is stationary.

3.1.3.2 ACF and PACF test

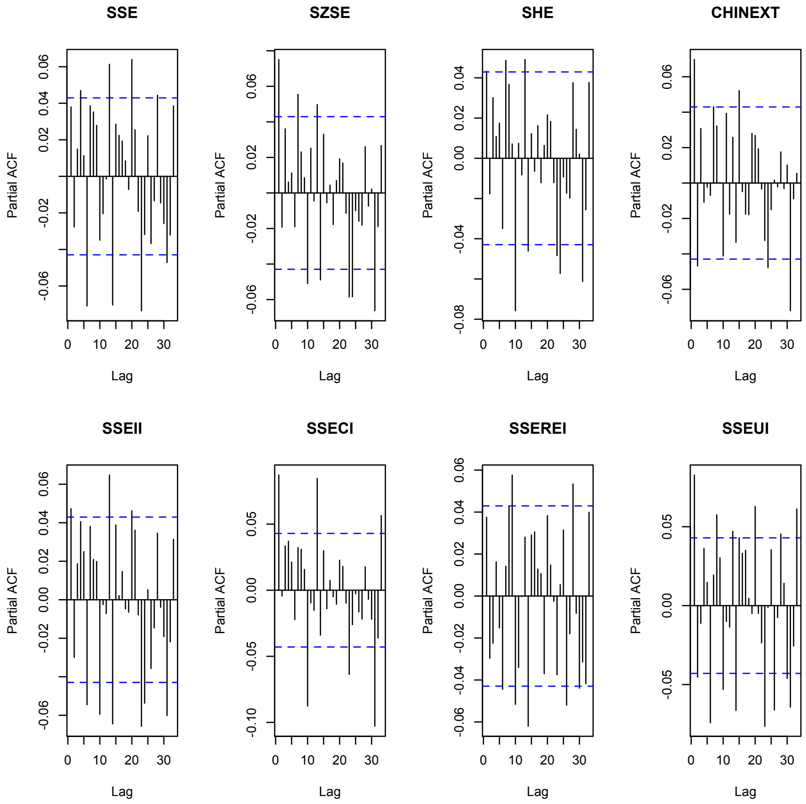

Figure 3-3 Graph of ACF test (Autocorrelation function test)

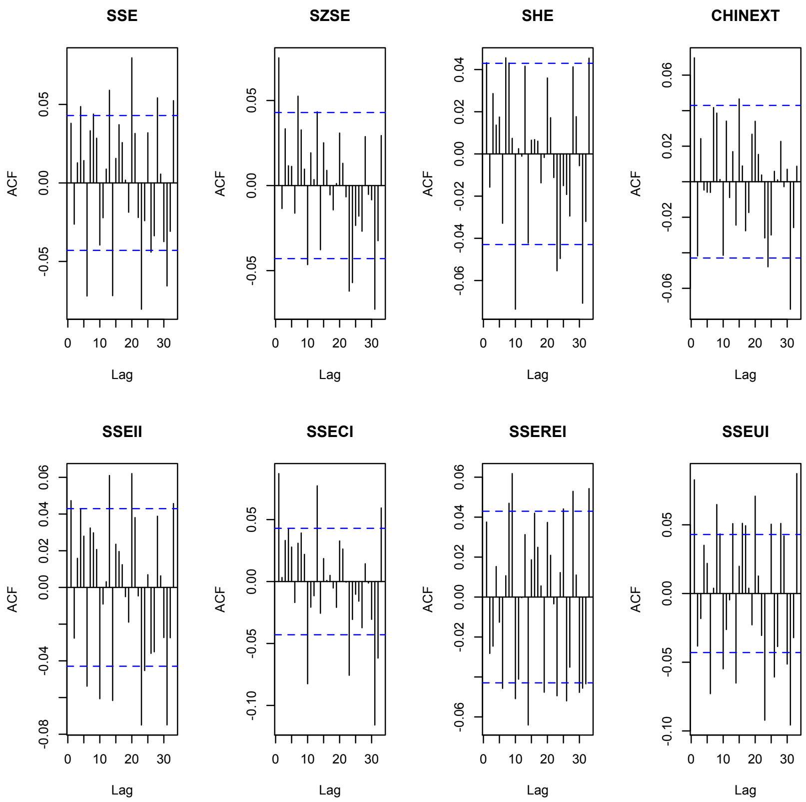

Figure 3-4 Graph of PACF test (Partial autocorrelation function test)

Figure 3-3 and 3-4 provide us the ACF and PACF test results by graph. The traditional method to determine the order of ARMA model through the phenomenon as tails off gradually or cuts off after q lags cannot be implement in our data due to the huge oscillation in the lags. To make it simple, the most suitable ARMA model would be chosen within the 5 order according to the parameter significance and AIC/BIC criterion.

3.1.3.3 Autoregressive conditional heteroscedasticity (ARCH) test for ARMA model

Before we estimate the error terms with the GARCH model, we need to build the ARMA model for the log return of eight indices. The ARMA modeling steps are as follow:

(1) Estimating the input time serials data to determine whether it is a stationary random process. If it is stable, go directly to step 2; if it is not stable, data processing such as difference should be performed, and then go to step 2 after processing.

(2) Model identification and ordering determination would be done based on the autocorrelation and partial autocorrelation function test of the data, combined with the AIC or BIC criteria.

(3) Then we estimate the parameters of the model and see if the parameters are statistically significant. If not, we would adjust the model.

After the suitable ARMA model was chosen for the data, the following estimation of GARCH model can be proceeded.

Table 3-3 The ARCH test for the residuals of ARMA(q,p) model of the logarithm return of the eight indices.

Index	model	LM(1)	p value	LM(30)	p value	LM(600)	p value
SSE	ARMA(3,2)	88.53	< 2.2e-16	425.77	< 2.2e-16	690.45	0.006
SZSE	ARMA(1,3)	81.25	< 2.2e-17	415.08	< 2.2e-17	690.50	0.006
SHE	ARMA(3,3)	73.85	< 2.2e-18	399.72	< 2.2e-18	663.86	0.036
CHINEXT	ARMA(1,1)	111.59	< 2.2e-19	380.01	< 2.2e-19	652.36	0.068
SSEII	ARMA(3,3)	122.47	< 2.2e-20	455.78	< 2.2e-20	695.29	0.004
SSECI	ARMA(2,3)	122.62	< 2.2e-21	550.50	< 2.2e-21	751.70	2.30E-05
SSEREI	ARMA(3,3)	76.36	< 2.2e-22	308.78	< 2.2e-22	598.60	0.508
SSEUI	ARMA(3,3)	171.70	< 2.2e-23	571.13	< 2.2e-23	768.02	3.76E-06

Footnote: LM(p) is the chi-squire value of the Lagrange Multiplier (LM) test for autoregressive conditional heteroscedasticity (ARCH) with p lag.

Lagrange multiplier test for conditional heteroscedasticity of is proposed by Engle (1982) The null hypothesis is no ARCH effects. From the test result, it's clear that the ARCH effects are obvious significant in high order lags of the model residuals until 600 lags, which indicates us to build the GARCH model for the estimation of disturbance terms.

3.2 Empirical research

The first step in building the GARCH-type model is to choose the suitable order for the model estimation. For most of situation, GARCH-type (1,1) can meet the requirements for dataset. If there are too many orders, the model will be unstable. Considering we use the ugarchspec function in package 'rugarch' of the statistical software R, the higher order choice would also cause the arima estimation model to have possible convergence problem as "optim gave code = 1". Therefore, the order of our GARCH-type model will be chosen in 3 based on the significance of parameters and the minimum AIC or BIC criteria. Finally, we would have four GARCH-type for every index estimation. The RiskMetrics model with iGARCH(1,1) is for matched group and we are going to implement the fit diagnostics through several tests such as the LB test, the ARCH test for residuals and the Sign Bias test for leverage effects in the standard GARCH, E-GARCH and GJR-GARCH model of the logarithm returns of eight indices of Chinese stock market.

3.2.1 GARCH-type model estimation process

The second step is to fit the chosen GARCH-type model into the empirical data as the logarithm returns of the eight indices. The ugarchroll function in package 'rugarch' provides cushy job for the fitting process. What we should do is to choose the suitable parameter for the 'ugarchroll' function:(1) "n.start" (2) "refit.window" (3) "solver"(3) "refit.window" (4) "refit.every"

(1) "n.start": considering the total sample size is 2083, we set the start data point at the 400^{th}, which gives us the forecast length of 1683.

(2) "refit.window": we choose the "recursive" as the "refit.window" since the choice of "moving" for the "refit.window" would lead to the failure of VaR calculation.

(3) “solver”: we choose the “hybrid” for the “solver” parameter because it offers an ergodic process by trying all the solvers including “solnp”, “nlminb”, “gosolnp” and “nloptr” by order, which give us the best chance to fit the model into the dataset.

(4) “refit.every”: one important parameter is the “refit.every”, which determines the model re-estimation frequency. If the large number is chosen like the number above 25(nearly one month), the model will adjust itself to the dataset too slowly, which means it can’t catch the short-time volatility. If the small number is chosen like 1 or 2, the enormous R program calculation would be demand. The comparison of different size of “refit.every” parameter was given as follow. We take the logarithm return of SSE index as the demonstration example to fit the GARCH-type model and show the VaR curve at 1% alpha level. It’s visually perceive that with “refit.every” parameter larger than 25, the “ugrachroll” function would have the problem to depict the rigorous shape of the VaR curve than the smaller parameter, especially for the E-GARCH and GJR-GARCH model with external regressors. Considering both the requirement of accuracy and efficiency, we set the “refit.every” parameter to 10 for the “ugarchroll” function.

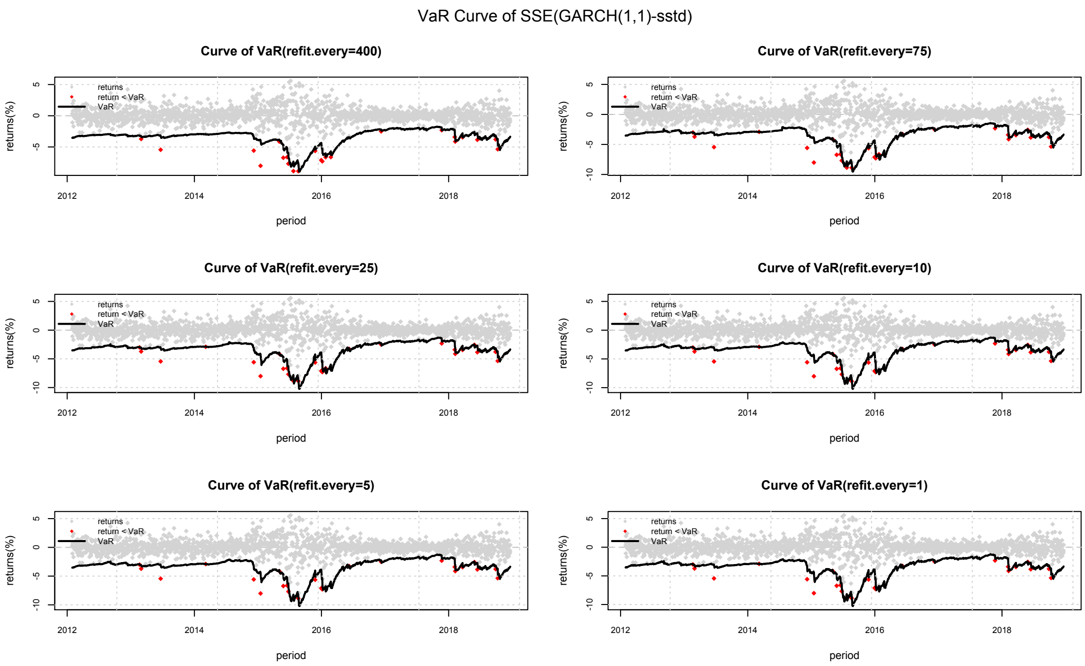

Figure 3-5 VaR Curve of SSE (GARCH(1,1)-sstd) with different size of “refit.every” parameter

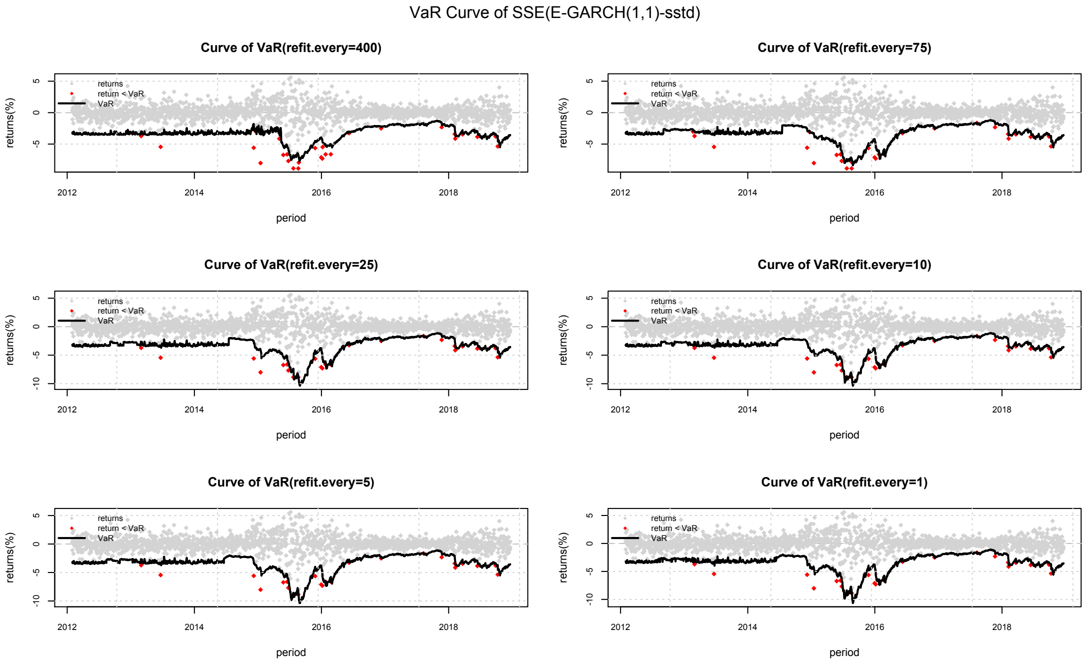

Figure 3-4 VaR Curve of SSE (E-GARCH(1,1)-sstd) with different size of "refit.every" parameter

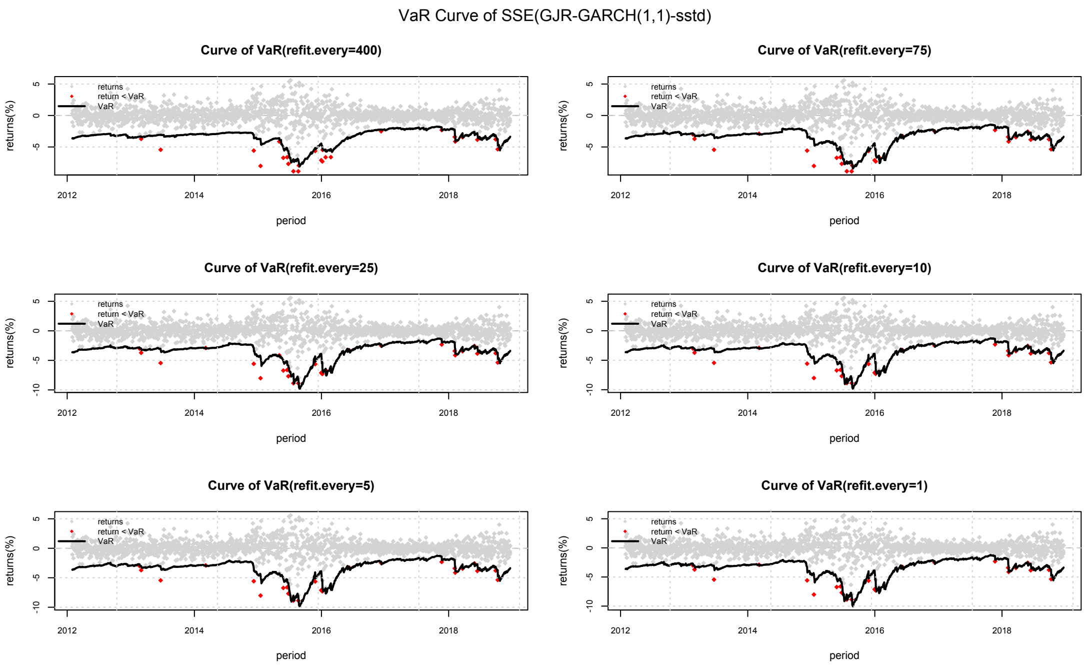

Figure 3-4 VaR Curve of SSE (GJR-GARCH(1,1)-sstd) with different size of "refit.every" parameter

GARCH fitting is performed after the order is determined. The LB test and the ARCH test are performed to check whether the residual is in accordance with the hypothetical white noise test. Generally, the residual term of the 10th order is guaranteed to be tested, that is, the p value is greater than 0.1, and then the GARCH model can be said. effective.

3.2.2 Parameter Estimation and Model Comparison

Table 3-4 Parameter estimation of GARCH-skewed-t model of the logarithm returns of eight indices

Model: GARCH-skewed-t							
Index	**mu**	**omega**	**alpha1**	**beta1**	**skew**	**shape**	$\hat{P}$
SSE	0.0165	0.0073	0.0533	0.9457	0.9502	4.6021	0.9990
	(0.4468)	(0.0288)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SZSE	0.0085	0.0151	0.058	0.9381	0.7775	7.0076	0.9961
	(0.7685)	(0.0594)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SHE	0.0005	0.0182	0.0523	0.9424	0.9359	5.4037	0.9947
	(0.9866)	(0.0245)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
CHINEXT	-0.0262	0.016	0.056	0.9417	0.8484	9.1145	0.9977
	(0.4572)	0.0845	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSEII	0.0124	0.0131	0.0549	0.9394	0.873	5.7889	0.9943
	(0.6079)	(0.0167)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSECI	-0.0382	0.0309	0.0592	0.9314	0.787	6.5446	0.9906
	(0.2248)	(0.0133)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSEREI	0.0363	0.0532	0.071	0.9168	1.0144	5.1322	0.9878
	(0.2836)	(0.005)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSEUI	-0.0095	0.0121	0.0586	0.9382	0.9322	5.1719	0.9968
	(0.6971)	(0.0284)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	

Footnote: the value in the bracket gives the p-value of estimated parameter in the fitted model.

Table 3-5 Parameter estimation of E-GARCH-skewed-t model of the logarithm returns of eight indices

Model: E-GARCH(1,1)-skewed-t								
Index	**mu**	**omega**	**alpha1**	**beta1**	**gamma1**	**skew**	**shape**	$\hat{P}$
SSE	0.0167	0.0029	-0.0128	0.9935	0.1249	0.9484	4.6316	0.9935
	(0.3879)	(0.1905)	(0.2507)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SZSE	-0.0024	0.0064	-0.0337	0.9926	0.1254	0.7666	7.2502	0.9926
	(0.9308)	(0.0075)	(0.0003)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SHE	-0.0087	0.0083	-0.0374	0.9897	0.1257	0.9328	5.3793	0.9897
	(0.76)	(0.001)	(0.0013)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
CHINEXT	-0.0228	0.0069	-0.0180	0.9944	0.1181	0.8519	9.1682	0.9944
	(0.5153)	(0.0005)	(0.0646)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSEII	0.0115	0.0040	-0.0170	0.9918	0.1237	0.8696	5.8126	0.9918
	(0.3596)	(0.0631)	(0.1013)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSECI	-0.0433	0.0097	-0.0142	0.9903	0.1271	0.7846	6.3744	0.9903
	(0.169)	(0.0000)	(0.1235)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
SSEREI	0.0450	0.0159	0.0134	0.9863	0.1471	1.0138	5.2575	0.9863
	(0.1734)	(0.0000)	0.2993	(0.0000)	(0.0262)	(0.0000)	(0.0000)	
SSEUI	-0.0096	0.0038	-0.0040	0.9935	0.1358	0.9290	5.2320	0.9935
	(0.6891)	(0.0992)	(0.7104)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	

Footnote: the value in the bracket gives the p-value of estimated parameter in the fitted model.

Table 3-6 Parameter estimation of ARMA-GJR-GARCH-skewed-t model of the logarithm returns of eight indices

Model: ARMA-GJR-GARCH-skewed-t												
Index	**mu**	**ar1**	**ar2**	**ma1**	**ma2**	**omega**	**alpha1**	**beta1**	**gamma1**	**skew**	**shape**	$\hat{P}$
SSE	0.0160					0.0075	0.0512	0.9452	0.0053	0.9505	4.5698	0.9990
	(0.4638)					(0.0311)	(0.0000)	(0.0000)	(0.7097)	(0.0000)	(0.0000)	
SZSE	0.1869	0.2399	0.7622	-0.1828	-0.8009	0.0232	0.0261	0.9348	0.0572	0.7837	6.3738	0.9871
	(0.3021)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0204)	(0.0162)	(0.0000)	(0.0011)	(0.0000)	(0.0000)	
SHE	0.0160					0.0075	0.0512	0.9452	0.0053	0.9505	4.5698	0.9918
	(0.8304)					(0.0198)	(0.0001)	(0.0000)	(0.0684)	(0.0000)	(0.0000)	
CHINEXT	-0.0264					0.0161	0.0558	0.9416	0.0006	0.8484	9.1199	0.9976
	(0.4582)					(0.0854)	(0.0000)	(0.0000)	(0.9654)	(0.0000)	(0.0000)	
SSEII	0.0106					0.0139	0.0504	0.9383	0.0094	0.8729	5.7599	0.9932
	(0.662)					(0.017)	(0.0000)	(0.0000)	(0.514)	(0.0000)	(0.0000)	
SSECI	0.1585	0.1911	0.8115	-0.1509	-0.8333	0.0415	0.0340	0.9312	0.0376	0.7875	6.0932	0.9823
	(0.5443)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0037)	(0.0072)	(0.0000)	(0.0205)	(0.0000)	(0.0000)	
SSEREI	0.0441					0.0462	0.0827	0.9215	-0.0282	1.0156	5.1712	0.9901
	(0.1968)					(0.0111)	(0.0000)	(0.0000)	(0.1172)	(0.0000)	(0.0000)	
SSEUI	0.0081					0.0114	0.0618	0.9396	-0.0084	0.9319	5.1846	0.9974
	(0.7412)					(0.0353)	(0.0000)	(0.0000)	(0.5453)	(0.0000)	(0.0000)	

Footnote: the value in the bracket gives the p-value of estimated parameter in the fitted model.

Table 3-4, 3-5 and 3-6 gives the result of estimated parameters for the fitted model. Only SZSE and SSECI index was fitted with ARMA (2,2)-GJR-GARCH (1,1)-skewed-t model since it can perform better parameter significance and the rest of models were all fitted with model order (1,1) since we would like to simplify the calculation process in the R software while keep the model stable.

The matched group RiskMatrics model was set to the fix parameter with alpha1 of 0.06 and beta1 of 0.94, therefore further estimation of the coefficient is skipped for this model.

A universal problem exists in all the three models as the parameter mu (stands for the constant term) is always not statistically significant (p-value larger than 5%). Since removing the interception parameter mu would make any difference for the significance of other parameters and it doesn't have the statistical interpretation, we choose to keep the constant term mu parameter even that it's not statistically significant.

Another parameter omega (as the intercept of the conditional variance model) is sometimes insignificant, but it should be kept in the model for reasons. If we force omega=0 and get the sum of parameter alpha and beta smaller than 1 (according to the design of the estimation procedure, restricts the parameters to a stationary region by defiding alpha+beta<1 in the GARCH model), it implies the conditional variance is decreasing over time in the model, which is generally undesirable.

As for the GARCH(1,1)-skewed-t model showed in the table 3-4, except for the interception mu, the rest of the parameters are statistically significant in 5% level. As for the parameter interpretation, α is often interpreted as the short run persistence where β denotes the long run persistence. The GARCH(1,1)-skewed-t models of the eight indices are stationary and the past volatility can explain for the current volatility since the persistence parameter $\hat{P}$ (sum of α and β) is smaller than 1.

As for the E-GARCH model showed in the table 3-5, except for the interception mu, the parameter omega and alpha1 are not always significant. The E-GARCH(1,1)-skewed-t models of the eight indices are stationary and the past volatility can explain for the current volatility since the persistence parameter $\hat{P}$ (value of β) is smaller than 1. The interception omega is insignificant with index SSE, SSEII and SSEUI while the parameter alpha1 is insignificant with index SSE, SSEII, SSECI, SSEREI and SSEUI. The parameter gamma1 is basically significant for all the indices except the index SSEREI. The negative alpha1 and positive gamma1 parameter of index SZSE, SHE, CHINEXT tells that the leverage effect does exist, which means the bad news has more impact than the good news of the same size.

As for the GJR-GARCH model showed in the table 3-6, besides the interception mu, the parameter gamma1 is basically insignificant with most indices including SSE, SHE, CHINEXT, SSEII, SSEREI and SSEUI. And the model of index CHINEXT also has the omega parameter insignificance. The GARCH(1,1)-skewed-t models of the eight indices are stationary and the past volatility can explain for the current volatility since the persistence parameter $\hat{P}$ is smaller than 1. But the parameter insignificant situation can be improved by adding the ARMA model to the mean. The index SZSE and SSECI with ARMA (2,2)-GJR-GARCH (1,1)-skewed-t model has all the parameters significant except the mu. The positive gamma indicates the leverage effect exists in the index SZSE and SSECI with the magnitude of 0.0572 and 0.0376 respectively.

Table 3-7 Model fitness analysis

Index	SSE	SZSE	SHE	CHINEXT	SSEII	SSECI	SSEREI	SSEUI
Model				**RiskMetrics**				
LB test (LB_5)	3.375	8.443	3.551	9.547	3.540	3.375	1.822	5.179
P-value	0.3427	**0.0229°**	0.3155	**0.012°**	0.3173	0.3427	0.6609	0.1394
ARCH test(Lag 3)	3.420	4.938	3.319	1.358	12.850	3.420	0.070	0.877
P-value	0.0644	**0.0263°**	0.0685	0.2440	**0.0003°**	0.0644	0.7910	0.3490
ARCH test(Lag 7)	5.552	5.903	4.815	2.428	14.470	5.552	1.147	3.488
P-value	0.1744	0.1479	0.2439	0.6274	**0.0016°**	0.1744	0.8885	0.4264
Sign Bias test	0.278	2.788	1.194	1.303	1.297	0.278	0.810	**0.013°**
P-value	0.7813	**0.0054°**	0.2326	0.1926	0.1947	0.7813	0.4179	0.9898
AIC	3.1474	3.6345	3.6097	4.0455	3.2827	3.7340	3.9061	3.3517
Model				**GARCH-skewed-t**				
LB test (LB_5)	3.364	8.439	3.598	9.443	3.553	9.333	1.649	5.272
P-value	0.3447	**0.0229°**	0.3086	**0.0128°**	0.3153	**0.0136°**	0.7032	0.1328
ARCH test(Lag 3)	5.179	4.734	4.455	1.312	14.660	9.284	0.089	1.052
P-value	**0.0229°**	**0.0296°**	**0.0348°**	0.2521	**0.0001°**	**0.0023°**	0.7658	0.3050
ARCH test(Lag 7)	7.125	5.819	5.637	2.327	16.170	11.156	0.983	3.616
P-value	0.0816	0.1539	0.1676	0.6484	**0.0006°**	**0.0099°**	0.9163	0.4054
Sign Bias test	0.416	2.705	1.124	1.231	1.190	2.022	0.811	**0.024°**
P-value	0.6773	**0.0069°**	0.2613	0.2185	0.2340	**0.0433°**	0.4177	0.9812
AIC	3.0686	3.5416	3.5426	4.0011	3.2135	3.6482	3.8522	3.2830
Model				**E-GARCH-skewed-t**				
LB test (LB_5)	3.033	7.534	3.396	9.025	3.034	9.091	1.445	4.492
P-value	0.4010	**0.0384°**	0.3395	**0.0163°**	0.4008	0.0157	0.7535	0.1988
ARCH test(Lag 3)	7.431	4.194	3.854	1.393	21.260	13.170	0.022	1.816
P-value	**0.0064°**	**0.0406°**	**0.0496°**	0.2379	**0.0000°**	**0.0003°**	0.8828	0.1778
ARCH test(Lag 7)	9.333	5.454	4.612	2.056	22.670	15.060	1.006	4.224
P-value	**0.0262°**	0.1825	0.2667	0.7056	**0.0000°**	0.0011	0.9126	0.3155
Sign Bias test	0.516	2.771	0.994	1.163	1.105	1.976	0.879	0.041
P-value	0.6061	**0.0056°**	0.3201	0.2450	0.2694	0.0483	0.3796	0.9674
AIC	3.0655	3.5303	3.5356	3.9959	3.2116	3.6474	3.8518	3.2817
Model				**GJR-GARCH-skewed-t**				
LB test (LB_5)	3.423	3.754	3.710	9.453	3.558	5.695	1.528	5.168
P-value	0.3352	0.9997	0.2926	**0.0127°**	0.3145	0.9811	0.7329	0.1403
ARCH test(Lag 3)	4.982	2.169	3.010	1.297	14.180	8.591	0.017	1.326
P-value	**0.0256°**	0.1408	0.0827	0.2547	**0.0002°**	**0.0034°**	0.8965	0.2496
ARCH test(Lag 7)	6.902	3.937	4.279	2.321	15.710	10.004	1.074	3.963
P-value	0.0912	0.3557	0.3082	0.6496	**0.0008°**	**0.0184°**	0.9012	0.3520
Sign Bias test	0.418	1.961	1.087	1.230	1.235	2.502	0.852	0.060
P-value	0.6756	0.0500	0.2770	0.2187	0.2170	**0.0124°**	0.3946	0.9524
AIC	3.0694	3.5333	3.5417	4.0020	3.2142	3.6410	3.8520	3.2838

Footnote: the P-value smaller than 0.05 is marked with the superscript of a full-stop sign and highlighted.

Table 3-7 summarizes several model fitness analysis results including the Ljung–Box test with order of 5, ARCH test of lag 3 and 7, sign bias test and the AIC value. If we don't reject LB test with the null hypothesis of no serial correlation, it indicates the residuals of the data is white noise, which means the model is well-fitted and already withdrawn all the information of the data. If we don't reject LB test with null hypothesis of no ARCH effect, it also indicates the fitted model can explain the data well. If we don't reject the sign bias test with the null hypothesis of existence of leverage effect, it indicates our model can well explain the skew leverage in the data.

But the out-performance seems terrible as all the three fitted models reject the null hypothesis more or less. During our 32 analysis results, only 10 models don't reject all the test, as 4 with the RiskMetrics model (matched group)(Index SSE, SHE, SSECI, and SSEREI), 1 with the GARCH-skewed-t model(Index SSEREI), 2 with the E-GARCH-skewed-t model(Index SSEREI and SSRUI) and 4 with the GJR-GARCH-skewed-t model(Index SZSE, SHE, SSEREI and SSEUI). The reason of the bad fitted result of the GARCH-type model for the two Index SSEII and SSECI may be the original data character since itself they stand for the Chinese industrial and commercial industry, less liquidity and inefficiency market.

Except the result of the models based on the dataset Index SSEII and SSECI, overall, we can conclude that the GJR-GARCH-skewed-t model perform better than other model in Chinese stock market. By adding the parameter for explaining the leverage effect, the E-GARCH and GJR-GARCH model trend to have the less AIC value between the estimated models.

3.3 VaR forecasting and backtesting

Chapter 4 is composed of the VaR forecast at 99% confidential interval according to the market risk requirement in Basel II and the corresponding VaR backtesting. The VaR prediction result would be demonstrate in pictorial format and the tests result would be collected in the table 4-1.

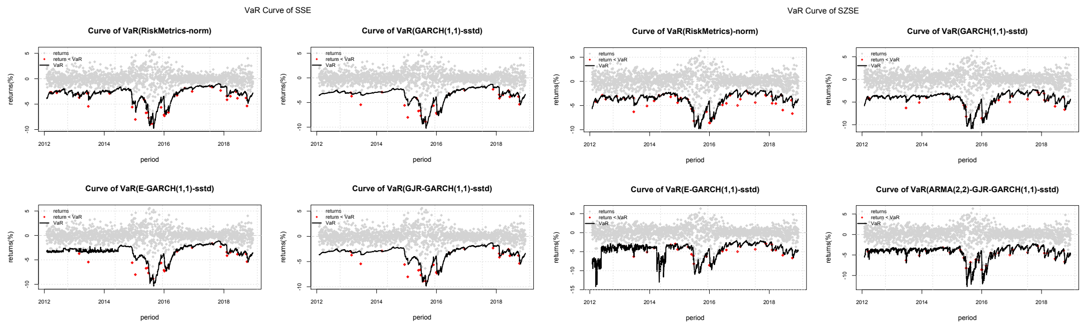

Figure 4-1 and 4-2 Value at Risk curve of index SSE/SZSE with comparison of the index's logarithm return

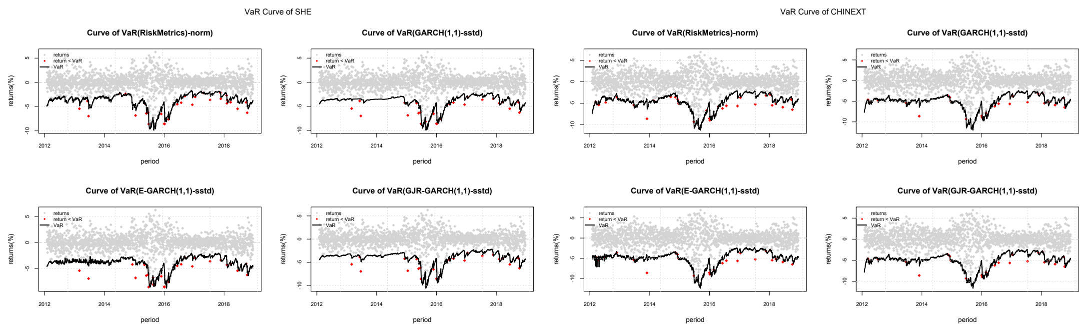

Figure 4-3 and 4-4 Value at Risk curve of index SHE/CHINEXT with comparison of the index's logarithm return

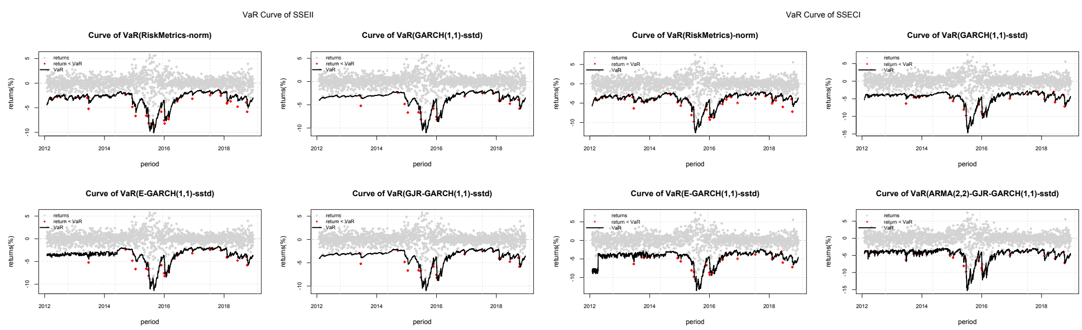

Figure 4-5 and 4-6 Value at Risk curve of index SSEII/SSECI with comparison of the index's logarithm return

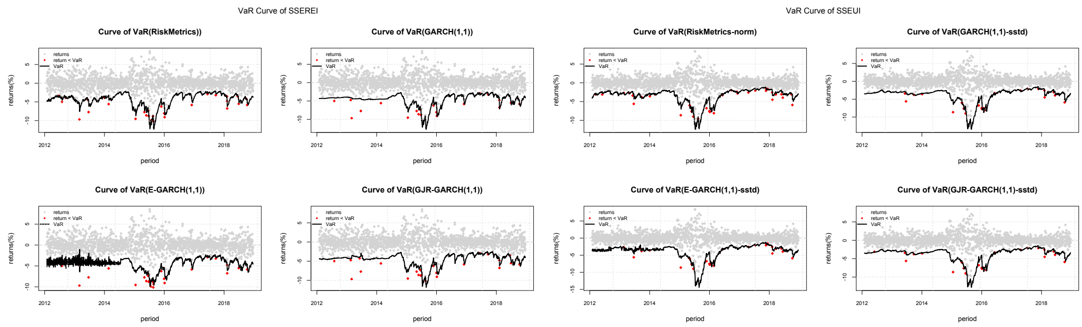

Figure 4-7 and 4-8 Value at Risk curve of index SSEREI/SSEUI with comparison of the index's logarithm return

In the visual perspective, the GARCH-type models with skewed-t distribution do show a better capacity for capturing the strong volatility than the traditional RiskMetrics method with normal distribution. By adding the leverage factor, the E-GARCH and GJR-GARCH model appear more adaptability and flexibility since it's VaR curve fitting gives more precision to the original data, which meet our requirement for not underestimation or overestimation for the VaR value. Due to the convergence problem happened from time to time, the E-GARCH model came up with the accuracy issue in the early stage of the forecast, which makes the GJR-GARCH model to be the model with best adaptability and presision.

Table 4-1 VaR backtesting results

Index	SSE	SZSE	SHE	CHINEXT	SSEII	SSECI	SSEREI	SSEUI
model	RiskMetrics(Backtest Length:1683 in 1% alpha level)							
Expected Exceed	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8
Actual VaR Exceed	36	43	36	30	40	40	29	36
Actual VaR Exceed in %	2.1%	2.6%	2.1%	1.8%	2.4%	2.4%	1.7%	2.1%
Kupiec LR.uc	16.627	28.744	16.627	8.446	23.241	23.241	7.309	16.627
P-value	**0°**	**0°**	**0°**	**0.004°**	**0°**	**0°**	**0.007°**	**0°**
Christoffersen LR.cc	18.202	31.001	18.202	9.536	25.190	25.190	7.713	18.202
P-value	**0°**	**0°**	**0°**	**0.008°**	**0°**	**0°**	0.021	**0°**
model	GARCH-skewed-t(Backtest Length:1683 in 1% alpha level)							
Expected Exceed	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8
Actual VaR Exceed	26	27	25	25	22	26	20	24
Actual VaR Exceed in %	1.5%	1.6%	1.5%	1.5%	1.3%	1.5%	1.2%	1.4%
Kupiec LR.uc	4.327	5.247	3.486	3.486	1.463	4.327	0.569	2.726
P-value	0.038	0.022	0.062	0.062	0.226	0.038	0.451	0.099
Christoffersen LR.cc	5.144	6.128	4.24	4.24	2.046	5.144	1.05	3.42
P-value	0.076	0.047	0.12	0.12	0.36	0.076	0.592	0.181
model	E-GARCH-skewed-t(Backtest Length:1683 in 1% alpha level)							
Expected Exceed	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8
Actual VaR Exceed	24	28	27	25	24	31	27	27
Actual VaR Exceed in %	1.4%	1.7%	1.6%	1.5%	1.4%	1.8%	1.6%	1.6%
Kupiec LR.uc	2.726	6.241	5.247	3.486	2.726	9.652	5.247	5.247
P-value	0.099	0.012	0.022	0.062	0.099	**0.002°**	0.022	0.022
Christoffersen LR.cc	3.42	7.189	6.128	4.24	3.42	9.926	5.81	6.128
P-value	0.181	0.027	0.047	0.12	0.181	**0.007°**	0.055	0.047
model	GJR-GARCH-skewed-t(Backtest Length:1683 in 1% alpha level)							
Expected Exceed	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8
Actual VaR Exceed	25	28	24	25	22	26	24	25
Actual VaR Exceed in %	1.5%	1.7%	1.4%	1.5%	1.3%	1.5%	1.4%	1.5%
Kupiec LR.uc	3.486	6.241	2.726	3.486	1.463	4.327	2.726	3.486
P-value	0.062	0.012	0.099	0.062	0.226	0.038	0.099	0.062
Christoffersen LR.cc	4.24	7.189	3.42	4.24	2.046	5.144	3.42	4.24
P-value	0.12	0.027	0.181	0.12	0.36	0.076	0.181	0.12

Footnote: the P-value smaller than 0.05 is marked with the superscript of a full-stop sign and highlighted.

Table 4-1 synthesizes the results of VaR backtesting including the unconditional coverage test (Kupiec LR.uc) and the conditional coverage test results (ChristoffersenLR.cc) at 1% alphal level. The number and percentage of actual VaR exceedance is also presented. The null hypothesis of the unconditional test is correct exceedances while the conditional coverage test estimates the independence of failure additionally. Critical value of the two tests is 6.635 for LR.uc test and 9.21 for LR.cc test. If the statistic value is smaller than the critical value, we don't reject the null hyphothesis.

The RiskMetrics method came up with the worst VaR forecast performance since it has the most actual exceedance among all the models and rejects both the unconditional test and conditional coverage test. Compared with the RiskMetrics, the GARCH-type models all perform well since they all accept the null hypothesis except for the specific dataset SSECI as we talked in the parameter estimation part.

In general, the GJR-GARCH-skewed-t performs averagely and sometimes better than other GARCH-type models.

Chapter 4: Conclusion and research perspectives

As an emerging stock market with enormous potential, Chinese stock market has apparent volatility clustering appearance along with typical feature of leptokurtic, negative skewness and fat tail in its index yield series.

As the result of model fitting and VaR estimation showed in the RiskMetrics, the model based on traditional normal distribution often underestimate the risk, which would lead to profound loss for the investors and financial institution when the extreme events happened.

Considering the model significant perspective, the leverage effect was testified in the Index SZSE and SSECI by the ARMA-GJR-GARCH-skewed-t model and in the Index SZSE, SHE and CHINEXT by the E-GARCH-skewed-t model. The negative shocks would impact more in the volatility of the Index with more liquidity in Chinese stock market. While in the industrial sector where most leading enterprises were stated owned, the leverage effect parameter is not significant. The stock market under Socialist society with the control of Chinese government shows more marketization features similar as the free market of western Capitalist society. Without directly government intervention, the market regulation starts takes part in the stock price evolution, which could be estimated by typical economic theory. With the popularization of education, the culture level and mantel strength also improved. According to the report released by the Panorama data service platform called Mobdata recently, until 2018, over half of the Chinese stock market is averaged or beyond bachelor’s degree, which indicates their investment behavior trend to more rational and traceable.

As for the VaR forecasting with 99% confidential interval, among the RiskMetris model and other GARCH-type model, the ARMA-GJR-GARCH-skewed-t model has better adaptability and precision for the estimation of indices of Chinese stock markets.

In the research perspective, due to the model limitation, the ARCH effect still exists in the residuals of GARCH-type models, even with all the parameters significant, which indicates the long-memory model such as fractionally integrated generalized autoregressive conditional heteroskedasticity(FIGARCH) model may work our better.

As for the dataset distribution, even the skewed-t distribution performs better than the normal distribution, sometimes it still fails to capture the excessive kurtosis and skewness. The high-order moment specific for the kurtosis, skewness should be included in the model building and contribute to the VaR estimation.

Bibliography

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. 1999. Coherent measures of risk. *Mathematical finance,* 9(3): 203-228.

Bhargava, A. 1986. On the theory of testing for unit roots in observed time series. *The Review of Economic Studies,* 53 (3): 369–384.

Black, F. 1976. Stuedies of stock price volatility changes.

Bollerslev, T. 1987. A conditionally heteroskedastic time series model for speculative prices and rates of return. *The review of economics and statistics,* 542-547.

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics,* 31 (3): 307–327. doi:10.1016/0304-4076(86)90063-1.

Chen, S.-D., & Yu, S.-D. 2002. The Analysis of China's Stock Market Based on VaR Method of GARCH Model. *Jilin University Journal Social Sciences Edition,* 4: 11-17.

Christie, A. A. 1982. The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of Financial Economics,* 10: 407–432.

Christoffersen, P. F. 1998. *Evaluating interval forecasts. International economic review,* 1: 841-862.

Cont, R. 2001. Empirical properties of asset returns: stylized facts and statistical issues. 223-236.

Dickey, D. A., & Fuller, W. A. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association,* 74 (366a): 427–431.

Ding, R. 2015. Analysis and Enlightenment of 2015 China's Stock Crash. *Business*, 27: 179-179.

Du, S. and Xie, P., 2018, June. An Analysis of the Influence of QFII Shareholding Tendency on Stock Returns Volatility: from the Perspective of Industry Life Cycle. *In 2018 3rd International Conference on Humanities Science, Management and Education Technology (HSMET 2018).* Atlantis Press.

Engle, R. F. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica,*50(4): 987–1007. JSTOR 1912773.

Follmer, H., & Schied, A. 2004. *Stochastic Finance: An Introduction in Discrete Time Walter de Gruyter.* Berlin: New York.

Gan, L., Yin, Z., Jia, N., Xu, S., Ma, S., & Zheng, L. 2013. Data you need to know about China: Research Report of China Household Finance Survey 2012. *Springer Science & Business Media*.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance,* 48(5): 1779-1801.

Gong, R., Chen, Z.-C., & Yang, D.-R. 2005. To Evaluate VaR of China Stock Marketing Comparatively by Using GARCH Family Model and Comment. *The journal of Quantitative & Technical Economics,* 22.7: 67-81.

Hannan, E. J., & Deistler, M. 1988. *The statistical theory of linear systems* (Vol. 70). SIAM.

Hannan, E. J. 2009. *Multiple time series* (Vol. 38). John Wiley & Sons.

He, X.-Q., & Sun, Q.-Y. 2003 The leverage effect of China's stock market and balance between risk and return. *NANFANGJINGJI,* 9: 62-65.

Jorion, P. 2000. Value at risk: The new benchmark for managing market risk.

Kupiec, P. 1995. Techniques for verifying the accuracy of risk measurement models.

Ljung, G. M., & Box, G. E. 1978. On a measure of lack of fit in time series models. *Biometrika,* 65(2): 297-303.

Li, X. and Zhang, W., 2017. Research on the Efficiency of Chinese Stock Index Future Market Based on the Test of GARCH Model. *Management & Engineering*, (28), pp.37-42.

Lin, Z., 2018. Modelling and forecasting the stock market volatility of SSE Composite Index using GARCH models. *Future Generation Computer Systems*, 79, pp.960-972.

Lu, R., & Xu, L. 2004. Asymmetric Effects of Policy Information on China's Stock Markets. *CHINA ECONOMIC QUARTERLY-BEIJING,* 3: 319-330.

Mandelbrot, B. B. The Variation of Certain Speculative Prices. *The Journal of Business 36,* No. 4: 394-419.

Mobdata. 2018. 2018 Guidance of Chinese Shareholders.

Morgan, J. P. 1994. *Introduction to riskmetrics.* New York: JP Morgan.

Nelson, D. B. 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica,* 59 (2): 347–370. JSTOR 2938260.

Engle, R. F., & Ng, V. K. 1993. Measuring and testing the impact of news on volatility. *The journal of finance,* 48(5): 1749-1778.

Ren, J.-Q., Dan, X.-T., & Liang, C. 2015. Comparative Analysis of China's Main Board and Growth Enterprise Market Risk-Based on GARCH-VAR Method. *Business,* 3: 128-132.

Song, Y.-H., & Chen, C. 2016. Risk Measurement of Chinese Growth Enterprise Market Based on GARCH Model. *Journal of Shenyang University of Technology(Social Science Edition),* 3: 57-63.

Fisher, T. J., & Gallagher, C. M. 2012. New weighted portmanteau statistics for time series goodness of fit testing. *Journal of the American Statistical Association,* 107(498): 777-787.

Wang, Y.C., Tsai, J.J. and Li, Q., 2017. Policy impact on the Chinese stock market: From the 1994 bailout policies to the 2015 Shanghai-Hong Kong stock connect. *International Journal of Financial Studies*, 5(1), p.4.

Wei, Y., Yu, Q., Liu, J. and Cao, Y., 2018. Hot money and China's stock market volatility: Further evidence using the GARCH–MIDAS model. *Physica A: Statistical Mechanics and its Applications*, 492, pp.923-930.

Whittle, P. 1951. *Hypothesis Testing in Time Series Analysis.* Almquist and Wicksell.

Whittle, P. 1963. *Prediction and Regulation.* English Universities Press. ISBN 0-8166-1147-5.

Yue, Y.D., Liu, D.C. and Shan, X.U., 2015. Price linkage between Chinese and international nonferrous metals commodity markets based on VAR-DCC-GARCH models. *Transactions of Nonferrous Metals Society of China*, 25(3), pp.1020-1026.

Zhao, H., 2010. Dynamic relationship between exchange rate and stock price: Evidence from China. *Research in International Business and Finance*, 24(2), pp.103-112.

Zhou, L., 2018, June. Research on the Volatility of China's Real Estate Stock Market. *In 2018 2nd International Conference on Management, Education and Social Science (ICMESS 2018)*. Atlantis Press.

Zhou, W. and Li, L., 2016. *Price Discovery and Information Transmission in Stock Index Futures and Spot Markets: Evidence from China based on a VAR-GARCH Model with SSAEPD Margins*. Available at SSRN 2797893.