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DEALING WITH INFLUENTIAL OBSERVATIONS IN ACCOUNTING EMPIRICAL RESEARCH

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Resumo

Esta dissertação tem por objectivo o estudo das observações influentes bem como o seu tratamento no modelo clássico de regressão linear. Na aplicação do modelo de regressão linear as observações têm diferentes pesos, pelo que a sua importância e influência podem induzir a resultados enganadores nos estudos empíricos se não forem tratadas de uma forma correcta. Para detectar esse tipo de observações é necessário recorrer a um conjunto de medidas de diagnóstico para que depois se possa proceder ao respectivo tratamento (geralmente a exclusão).

Assim, esta investigação tem por objectivo a análise de vários artigos publicados na área da contabilidade e cujo tratamento estatístico das observações influentes não está conforme as sugestões dos manuais de econometria podendo levar a conclusões distorcidas pela forma incorrecta como se lida com tais observações.

Deste modo, esta investigação é composta por três partes. No enquadramento teórico será referido o significado das observações influentes, a sua importância e a metodologia na sua identificação; numa segunda parte será feita uma análise de vários estudos empíricos na área da contabilidade com o intuito de identificar a metodologia geralmente utilizada na detecção de tais observações; e, finalmente, numa terceira fase pretendemos realizar um estudo empírico que consiste em tratar tecnicamente, segundo a forma sugerida pelos manuais de econometria, as observações influentes e comparar os resultados da estimação do modelo de regressão com aqueles que resultariam se fossem considerados os critérios que tradicionalmente são adoptados para identificar as observações influentes em *empirical accounting*.

Palavras-chave: Observações influentes, modelo de regressão linear, contabilidade, critérios de exclusão.

JEL Classification System: C51 – Econometric Modelling: Model Construction and Estimation; M41 – Accounting.

Abstract

The main objective of this dissertation is the study of influential observations and their treatment in the linear regression model. When the linear regression model is applied, the observations have different influence in the estimation results and their importance and influence will induce to wrong results if the empirical studies are not correctly treated. To detect these observations (influential observations) is indispensable to apply diagnostic measures and then proceed to the respective treatment (generally their exclusion).

Thus, the purpose of this investigation is to analyse some accounting published articles whose statistic treatment is not the more technically appropriate accordingly the econometric books, inducing to distorted results because of the incorrect form that these authors deal with that observations.

Therefore, this investigation is composed by three parts. Firstly, it will be done a theoretical framework of what are influential observations, their importance and the methodology that should be used in their identification; then, it will be analysed the methodology used to detect influential observations by various published accounting empirical studies; and, our final objective is to perform an empirical study that consists in treat technically and correctly the influential observations and compare the results of the regression model estimation with the results that we would obtained if were considered the traditional criteria adopted to identify the influential observations in empirical accounting.

Keywords: Influential observations, linear regression model, accounting, exclusion criteria.

JEL Classification System: C51 – Econometric Modelling: Model Construction and Estimation; M41 – Accounting.

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The important thing is never stop questioning.

Curiosity has its own reason for existing.

Albert Einstein

US (German-born) physicist (1879 – 1955)

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Executive Summary

The multiple linear regression model is one of the most used econometric frameworks in different scientific areas such as physical science, biology, business, humanities, technology, mathematics, sociology and accounting. However, in the empirical studies the rules suggested by the econometric books are not always correctly applied inducing misleading results and conclusions. Dealing with influential observations is one of these problems and it is the main purpose of this dissertation.

The sample observations have a different weight and impact in the estimation results, which increases the importance of analysing their impact on the model. The observations whose value has major impact and influence are named by **influential observations**.

The influential observations can be classified in three categories:

1. "Outliers" – designation for the dependent variable's observations whose value is very unusual according to the bulk of the variable observations;
2. "High leverage points" – designation for the explanatory variables whose value is very unusual according to the bulk of the variables observations;
3. "Both" – both dependent and explanatory variables observations whose value is very unusual when comparing with the other observations of the sample.

To detect the influential observations some methods can be used. The high leverage points are detected using methods based on the diagonal elements of the hat matrix and the outliers are identified by methods based on residuals.

The outlier's diagnostics are mainly based on ordinary, standardized and studentized residuals.

The ordinary residuals are given by the difference between the observed and estimated values for the dependent variable. Nevertheless, unfortunately the ordinary residuals are not always comparable due to the scale effect, so we have to standardize them. Furthermore these residuals are only a good substitute for the errors if the lines of the X matrix are homogeneous and the non-diagonal elements of the H matrix are sufficiently lower.

The standardized residuals (or internally studentized residuals) are obtained by the division of the ordinary residuals and their standard error. Notwithstanding, the standardization can produce negative effects because the residual value can seems lower than in reality. If the estimated model does not describe properly an observation, the residuals sum of square can be inflated by it high residual value which will induce into a lower standardized residual. This is the reason why, in outliers identification, the studentized residuals are generally used.

The application of the studentized residuals allows us to realize the influence of the residual in the estimated model. The residual impact can be analyzed by the exclusion of the observation associated (*i*th observation) and using the result mean error quadratic as a variability measure. The studentized residuals, also designated as *jackknife residuals* or *externally studentized residuals*, are given by the division of the ordinary residuals and their standard error calculated without the *i*th observation. This is a method that gives results more correct but is not the most efficient to be performed because it needs to be fitted repeatedly without the observation left out in turn.

The standardized residuals are a better substitute for errors than the ordinary residuals. However, for several authors, the studentized residuals are even more reliable.

The diagnostics performed on the H matrix are based in distance measures as the Mahalanobis distance or in scalar influence measures as the Atkinson measure, DFBETAS, DFFITS and Cook distance or based in other methods as the covariance ratio or Cook-Weisberg measure.

For a better understanding, the hat matrix has the designation of “hat” because it is the matrix that establishes the relation between the observed and estimated values of the dependent variable.

As the influential observations are detected we need to know how they should be treated. These observations should just be eliminated if they are truly anomalous. On the otherwise, we should keep them in the sample.

In case of eliminating the observations, the results of the model will improve but we take the risk of having disturbed results which will limit the capability of generalization of the sample conclusions for the respective population.

According to Hocking (1983), the studentized residuals (SR>2; Belsley *et al.*, 1980) and elements of the diagonal of the hat matrix ($h_{ii} < 2\frac{k}{n}$; Hoaglin and Welsch, 1978) are reasonable methods to detect suspect cases of influential observations. So we adopt these statistics to identify the outliers and high-leverage points in our sample.

Although, in the accounting studies, to avoid the influential observations distortions, the authors exclude the observations that they think that would be influential (generally they exclude the extremes of the sample) with the justification of having large samples and so the results would remain unchanged.

To analyse the impact of influential observations in empirical accounting studies, we have a sample of 24,644 observations per Company per year of 6,453 European listed companies from 14 countries, for which Worldsope Database data was available for all the variables (accounting standards, price, book value of equity per share and net income per share) for the period between 2000 and 2005.

In order to compare the procedures used to exclude influential observations we have followed three steps:

1) The observations are excluded accordingly the authors' criteria;
2) We apply the econometrics suggestion;
3) We estimate the Ohlson regression model based in each one of the resulting data sets and we compare the estimation results.

We choose three authors with different methods each to compare with our results.

The results obtained were completely different, the variables have different impact on the price according to the different authors, some authors exclude more observations that should be excluded and others exclude less, for some authors the variable NIPS has a negative coefficient estimate in some countries but for the other authors has not, the variables in some countries are not statistically significant but for other authors are.

Therefore, we notice that the criteria used by the authors were not the more correct according to the econometrics suggestions and conduced to misleading results.

The investigators do not give the deserved importance to the study of influential observations when developing their studies. However this is an extremely important matter that could significantly influence the empirical studies.

1. Introduction

The use of a regression model is very helpful to undertake many studies, and this model has allowed us to extract and understand crucial relations in a data set. However, the model observations have a different importance and influence. Observations with a greater influence are generally named by **influential observations** and they are very important to fit into the econometric issues analysis because they can have a significant impact on the estimation results from ordinary least squares.

The ordinary least squares method is the result of the minimization of the residual sum of squares. The estimation of parameters can has a tendency to reflect unusual cases and consequently a small number of observations can have a substantial impact on the estimation results.

Research by Chatterjee and Hadi (1988) has provided evidence that three types of influential observations can exist; outliers, where the dependent variable assumes a very different value from the others, **high-leverage** point in the **X** space, where the respective explanatory variables take on very different values from the others or we can also have **both** types of observations.

Due to the importance and influence of these observations, if they are not treated carefully incorrect results can be reached in the empirical studies. Therefore, for a good study it is necessary to detect these influential observations and develop their handling (generally excluding them).

The main purpose of this dissertation is to analyze how influential observations have an impact on the results of empirical studies. For this, we will consider various published accounting studies, where the authors do not correctly handle the influential observations and see the impact on the results of the same studies if they had been handled correctly.

But an important question emerges: how we should handle these observations? Should they be eliminated? When the influential observations are detected, they are normally excluded because they reduce the sum of the square residuals and the R^2 increase, which apparently improves the estimation results. Although we are eliminating observations that are part of our study, so we can be limiting the veracity of our study.

This investigation is composed of three parts. Firstly, we will see, from the perspective of several authors, the theory of what the influential observations are, their importance and the methodology which should be used to identify them. In the second part we will explore several empirical accounting studies to identify what type of methodology they use to detect and treat these observations. Finally, we will develop an empirical study, which consists of handling these observations correctly to compare the results of our estimated regression model with the results if we use the criteria traditionally adopted to identify influential observations in *empirical accounting*.

2. The influential observations in the OLS Method

When we perform a study applying the OLS method, the quality of our data and therefore of our investigation depends on the detection of influential points, outliers and high-leverages, which can cause many problems in a regression analysis and consequently distortions in the conclusion of the study. This is the **key point** of our investigation.

In this chapter we will understand the concept of influential observations, outliers and high-leverage points. Afterwards, we will study the origin of the hat matrix and its properties because it is the basis for our investigation. Then we will observe two types of diagnostics which are very useful to identify influential points: diagnostics based on residuals and diagnostics based on the hat matrix diagonal elements. Therefore, we will become aware of solutions for data handling treatment and finally, we will recognize the steps for the construction of a regression model and detection of influential observations.

2.1 The concept of influential observations

There are several definitions of influential observations which all point to observations that can considerably influence OLS estimation results.

Besley *et al.* (1980: 11) states that *…an **influential observation** is one which, either individually or together with several other observations has a demonstrably larger impact on the calculated values of various estimates (coefficients, standard errors, t-values, etc.)….*

According to Chatterjee and Haji (1988), they are observations that have a stronger influence on the model than the rest of observations. **Influential observations**, can be described as outliers, high-leverage points or both. The great difference between the outliers and the high-leverage points is the type of variables that assume values which are very distant from the bulk. In the case of outliers it is the dependent variable and in the case of high-leverage points they are the explanatory variables. We can also have cases where both, explanatory and dependent variables, are very distant from the other points of a database, which are therefore a third type of influential observations.

Rousseeuw and Leroy (2003) also state that the response variable is not the only one that can be outlying, the explanatory variable can do so as well, and in this case we are dealing with leverage points.

In the same scope, Barnett and Lewis (1995: 7) refer to outliers as *an observation (or subset of observations) which appears to be inconsistent with the remainder of that set of data*.

Based on these definitions we conclude that influential observations[1] are points that have a demonstrably large influence on estimating the linear model parameters because they have a significant distance from the rest of the data.

As several diagnostic measures are based on the hat matrix, we will discuss the hat matrix properties.

See appendix 1 for a description of the notations that are used in the dissertation.

2.2 Hat matrix

A regression analysis is used when we are looking for a functional relationship between the dependent variable (*Y*) and a set of explanatory variables (*Xs*). The general linear model is expressed by the following matrices equation:

$$y=X\beta+\varepsilon \qquad (2.2.1)$$

where β is the vector of unknown parameters that establish the relationship between the dependent variable and the explanatory variables.

The least squares estimator of β is given by:

$$\hat{\beta}=(X'X)^{-1}X'y \qquad (2.2.2)$$

If the Gauss-Markov conditions hold, the $k \times 1$ vector $\hat{\beta}$ has the following properties:

a. $E(\hat{\beta})=\beta$; (2.2.3a)

[1]The influential observations can be caused by "*keypunch errors, misplaced decimal points, recording or transmission errors, exceptional phenomena such as earthquakes or strikes, or members of a different population slipping into the sample*" (Rousseeuw and Leroy, 1987: vii)

b. $\mathrm{Var}(\hat{\beta}) = \sigma^2(X'X)^{-1}$ (2.2.3b)

$\hat{\beta}$ is the best linear unbiased estimator (**BLUE**) for β, i.e., it is the estimator with the smallest variance. Assuming the errors normality,

$$\hat{\beta} \sim N_k(\beta,\ \sigma^2(X'X)^{-1}) \quad (2.2.3c)$$

The matrix **H** is referred to as the hat matrix because it establishes a relationship between the observed and estimated values of the dependent variable.

$$\hat{\mathbf{y}} = X\hat{\beta} = X(X'X)^{-1}X'\mathbf{y} = H\mathbf{y} \quad (2.2.4)$$

Where,

$$H = X(X'X)^{-1}X' \quad (2.2.5)$$

The $n \times 1$ vector of fitted values has the following properties:

(a) $E(\hat{\mathbf{y}}) = X\beta$; (2.2.6a)

(b) $\mathrm{Var}(\hat{\mathbf{y}}) = \mathrm{Var}(H\mathbf{y}) = H\mathrm{Var}(\mathbf{y})H = H\sigma^2 IH = \sigma^2 H$; (2.2.6b)

(c) $\hat{\mathbf{y}} \sim N_n(X\beta,\ \sigma^2 H)$; (2.2.6c)

The values of $\hat{\mathbf{y}}_i$ can be written by the *i*th row elements of the hat matrix:

$$\hat{\mathbf{y}}_i = \sum_{j=1}^{n} h_{ij} y_j = h_{ii} y_{i+} \sum_{j \neq i}^{n} h_{ij} y_{j,} \qquad i=1,2,\dots,n. \quad (2.2.7)$$

From which the result is:

$$\frac{\partial \hat{y}_i}{\partial y_i} = h_{ii}, \qquad i=1,2,\dots,n \quad (2.2.8)$$

The estimated value results from a linear combination of the values observed and the weights are Hat elements matrix.

If each element of the principal diagonal, $h_{ii,}$ is close to 1 the other elements of the *i* line should be closer to 0 and the estimated value is mostly determined by y_i. As so, the *i*th observation should be an influential observation.

The Hat matrix has the following **properties**:

1. The *ij*th element of H matrix is:

$$h_{ij} = x_i'(\mathrm{X}'\mathrm{X})^{-1}x_j = x_j'(\mathrm{X}'\mathrm{X})^{-1}x_i = h_{ji} \qquad i, j = 1, 2, \ldots, n \tag{2.2.9}$$

2. The *ii*th elements (elements of the diagonal matrix, in which its sum is considered to be trace) of H matrix are:

$$h_{ii} = x_i'(\mathrm{X}'\mathrm{X})^{-1}x_i \qquad i = 1, 2, \ldots, n \tag{2.2.10}$$

3. The sum of the *ii*th elements of the matrix (trace) is *k:*

$$\sum_{i=1}^{n} h_{ii} = k \tag{2.2.11}$$

4. The H sum square elements is *k*:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} h_{ij}^2 = k \tag{2.2.12}$$

5. The H matrix is not affected by the nonsingular explanatory variables transformations. If **T** is a $k \times k$ nonsingular matrix and Z = XT, we will have:

$$\mathrm{H}_Z = \mathrm{Z}(\mathrm{Z}'\mathrm{Z})^{-1}\mathrm{Z}' = \mathrm{XT}(\mathrm{T}'\mathrm{X}'\mathrm{XT})^{-1}\mathrm{T}'\mathrm{X}' = \mathrm{H} \tag{2.2.13}$$

6. The Hat matrix is idempotent ($H = HH$) and symmetric ($H = H'$). Its rank and trace are the same:

$$\text{Rank(H)=Trace(H)=}tr\left[X(X'X)^{-1}X'\right] = tr\left(I_k\right) = k \tag{2.2.14}$$

The H matrix is idempotent because $H^2 = X(X'X)^{-1}X'X(X'X)^{-1}X' = IH = H$

7. $HX = X, (I-H)X = 0, HH = H^2 = H$ and $H(I-H) = 0$.

8. If X contains a constant column, subsequently

   a. $h_{ii} \geq \frac{1}{n}, \qquad i = 1, 2, \ldots, n$

b. $h_{ii} \leq \frac{1}{r}$, *r* is the number of X lines identical to x_i.

c. With both aforementioned properties and HJ = J, where J is an n × 1 vector we will have:

$$\sum_{i=1}^{n} h_{ij} = \sum_{j=1}^{n} h_{ij} = 1 \tag{2.2.15}$$

9. The principal diagonal elements of the H centered matrix give the distance between each case and the explanatory variables mean.

$$h_{ii} = (x_i - \overline{x})'(x_c' x)^{-1}(x_i - \overline{x}) \tag{2.2.16}$$

$\left(x_i - \overline{x}\right)$ represents a vector of *k* explanatory variables centered to the i*th* observation.

2.3 Diagnostics based on residuals

As has been mentioned before, the observations have a different influence on the general linear regression model. Some of them have a greater influence when compared to the rest of the observations. Residuals are very important for the regression diagnostic as they usually combine the diagnostic measures of influent observations and without studying them, the analysis is not complete.

As stated by Belsley *et al*. (1980) the residuals are a good form of detecting and solving problems like heteroscedasticity and autocorrelation, they are also very useful for testing the normality of the disturbance term.

The n × 1 vector of ordinary residuals is given by the difference between the observed and estimated values for the dependent variable:

$$e = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - H\mathbf{y} = (I - H)\mathbf{y} \tag{2.3.1}$$

Nevertheless, to measure the error (ε_i) is not especially easy because it cannot be observed and they have to be estimated. Although, each *i*th residual (e_i) can be an estimate for its error (ε_i). Through the (2.3.1) equation we can conclude the following:

$$
\begin{aligned}
e &= (\mathrm{I}-\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}')\mathbf{y} \\
&= (\mathrm{I}-\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}')(\mathrm{X}\beta+\varepsilon) \\
&= \mathrm{X}\beta-\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}'\mathrm{X}\beta+(\mathrm{I}-\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}')\varepsilon \\
&= (\mathrm{I}-\mathrm{X}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{X}')\varepsilon \\
&= (\mathrm{I}-\mathrm{H})\varepsilon
\end{aligned} \tag{2.3.2}
$$

The error has the following properties:

1. $E(\varepsilon)=0$ (2.3.3)

2. $\mathrm{var}(\varepsilon)=\sigma^2 I$ (2.3.4)

3. $\varepsilon \sim N(0,\sigma^2 I)$ (2.3.5)

In additional to (2.3.2) we conclude that:

$$\mathrm{Var(e)}=(I-H)\mathrm{Var}(\varepsilon)(I-H)=(I-H)\sigma^2 I(I-H)=\sigma^2(I-H)$$

Then,

$$\mathrm{Var}(e_i)=\sigma^2(1-h_{ii})$$

And consequently,

$$e_i=(1-h_{ii})\varepsilon_i-\sum_{j\neq i}^{n}h_{ij}\varepsilon_j \tag{2.3.6}$$

If the errors have a normal distribution, then the ordinary residuals will also have a normal distribution. Each e_i is a linear combination of all random errors ε_i.

$$e \sim N(0,\sigma^2(I-H)) \tag{2.3.7}$$

Therefore, as we can conclude, fitted values at distant points (h_{ii}) will have a relatively large variance ($\mathrm{var}(\hat{y}_i)=\sigma^2 h_{ii}$) and consequently the correspondent residual will have a relatively small variance ($\mathrm{var}(e_i)=\sigma^2(1-h_{ii})$).

The n × 1 vector of ordinary residuals has the following properties:

a. $$\overline{\mathrm{e}} = \frac{\sum_{i=1}^{n} e_i}{n} = 0 \qquad (2.3.8)$$

b. $$S^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-k} = \frac{RSS}{n-k} \qquad (2.3.9)$$

c. $$\mathrm{E(e)} = \mathrm{E[(I-H)\varepsilon]} = \mathrm{(I-H)E(\varepsilon)} = 0, \qquad (2.3.10)$$

d. $$\mathrm{Var(e)} = \sigma^2 \mathrm{(I-H)} \qquad (2.3.11)$$

e. $$\mathrm{e} \sim N_n(0, \sigma^2 \mathrm{(I-H)}) \qquad (2.3.12)$$

f. $$\frac{\mathrm{e'e}}{\sigma^2} \sim \chi^2_{(n-k)} \qquad (2.3.13)$$

where $\mathrm{e'e}$ is the residual sum of squares and $\chi^2_{(n-k)}$ denotes a χ^2 distribution with $n - k$ degrees of freedom.

g. $$\mathrm{Var}(e_i) = \sigma^2 (1 - h_{ii}) \qquad (2.3.14)$$

h. $$\mathrm{Cov}(e_i, e_j) = -\sigma^2 h_{ij} \qquad (2.3.15)$$

In conclusion, if $h_{ii} = 0$, the e_i and ε_i variances will be the same; if not, the e_i variance will always be lower than ε_i variance.

Two conditions must be considered for the residuals to be a reasonable substitute for ε. The first is that the X rows are homogeneous and the second is that the off-diagonal elements of H must be sufficiently small (Chatterjee and Haji, 1988).

Therefore, if the conditions mentioned before are valid, the ordinary residuals will have similar characteristics to errors, which mean linear independence, null average, constant variance, and normal distribution. Despite the ordinary residuals being very useful, they generally have different characteristics from the errors and we cannot assume that the ordinary residuals are an appropriate estimate for the errors, as we will see.

As we know, there are few hypotheses of the ordinary residuals variance being constant because the residuals depend on the principal diagonal of the H matrix, which are dependent on the explanatory variables' values, so the principal elements of H matrix diagonal probably assume values very distant from each other.

The *i*th residual does not have a common variance. It actually has a smaller variance than ε_i because of the factor $(1-h_{ii})$, if h_{ii} is large, the *i*th residual will be small. This is referred to as leverage and the *i*th point has high leverage. So, we can conclude that the ordinary residuals cannot be the best reasonable substitute.

In order for the ordinary residuals to become comparable, they must be placed on the same scale, which means converting them into standardized residuals, so we divide each residual by its estimated standard error.

The **standardized residuals** or **internally studentized residuals**:

$$r_i = \frac{e_i}{\sqrt{s^2(1-h_{ii})}} \tag{2.3.16}$$

s^2 is the residual mean square (MSE) and h_{ii} is the leverage for observation *i*. An observation with a high r_i value should be an influential observation.

The standardized residuals have the following properties:

a. $$\mathrm{E}(r_i) = 0 \tag{2.3.17}$$

b. $$\mathrm{Var}(r_i) = 1 \tag{2.3.18}$$

c. $$\mathrm{Cor}(r_i, r_j) = -\frac{h_{ij}}{\sqrt{(1-h_{ii})(1-h_{jj})}} \qquad i \neq j \tag{2.3.19}$$

d. $$\frac{r_i^2}{n-k} \sim Beta\left(\frac{1}{2}, \frac{n-k-1}{2}\right) \tag{2.3.20}$$

If the random factor ε follows the normal distribution, the standardized residuals will approximately follow the Student's *t* distribution.

Now the residuals are comparable, their variance is constant and through the residuals with a higher value we can identify the influential observations, although their value can seem lower than in reality because a peculiarly large error produces an unreasonably high value of s^2 or a residual with a low value which can be associated to an influential observation. Therefore, we have to study the **externally studentized residuals**, ***jackknife residuals*** or ***deletion residuals***.

The objective behind this alternative method is that influential observations can influence the predicted value when calculated with the residual at the *i*th observation, so we remove the observation which we suspect has a large influence on the model, and then calculate the mean square error without the *i*th observation, so we have $(n-1)-k = n-k-1$ degrees of freedom.

The difference between the actual dependent variable and the dependent variable without the *i*th observation should give us a more realistic vision of the quality that the model has at predicting a response for *i*th observation. Without the *i*th observation we can study the effect that occurs in the model. The studentized residual is calculated through the following expression:

$$t_i = \frac{y_i - \hat{y}_{i(i)}}{s_{d_i}} \tag{2.3.21}$$

An observation with a high t_i value might be an influential observation and the difference between y_i and the value estimated with *n*-1 observations is the deleted residual (d_i).

$$t_i = r_i\sqrt{\frac{s^2}{s_i^2}} = \frac{e_i}{\sqrt{s_i^2(1-h_{ii})}} = r_i\sqrt{\frac{n-k-1}{n-k-r_i^2}} \tag{2.3.22}$$

Where s_i^2 is the quadratic mean error acquired by fitting the model without the *i*th observation (the estimate of σ^2 with the *i*th observation deleted).

If X_i characteristic is equal to *k*, each externally studentized residual has *t*-Student distribution with n-k-1 degrees of freedom and if these are superior to 30, the distribution becomes normal.

$$t_i = \frac{e_i}{\sqrt{s_i^2(1-h_{ii})}} \sim t(n-k-1) \tag{2.3.23}$$

These types of residual studies ensure the influence of individual points on the mean quadratic error of prediction. If we have influential observations, the externally studentized residual will be greater than 10. Nevertheless, in cases that we have high-leverage points the residuals do not give any indication (Meloun *et al*, 2002).

Notwithstanding, we need to pay attention to two points of this method. The first is that the observed value deleted needs to be compared with other extreme order statistics from *t* distribution. The second is that there is an effect of under-estimation of σ^2 from deletion of the largest value of t_i.

So, after this analysis an important question is the following: which type of residual should be used for testing the errors in the linear regression model? We have already seen that the ordinary residuals are not the most appropriate and for the most statisticians, the externally studentized residuals are the most appropriate for diagnostic purposes. Although, one big disadvantage is that for the externally studentized residuals, the model needs to be fitted repeatedly without observations being left out consecutively so we need to look at their relationship between standardized residuals.

Other important point is that although the studentized residuals are very good for expressing how well a point is explained by the model, this method does not demonstrate how great the effect is on the fitted coefficients of omitting a point.

A latest note is that the standardized residuals are good only to identify heteroscedasticity and the studentized residuals to identify outliers (Meloun *et al.*, 2002).

To test the applicability of studentized residuals, it is important to understand perfectly the effects of eliminating influential observations.

2.4 The Consequences of eliminating observations

The relationship between the $\mathrm{X}'\mathrm{X}$ and $\mathrm{X}_i'\mathrm{X}_i$ matrixes, where $\mathrm{X}_i'\mathrm{X}_i$ excludes the *i*th observation is given by the following expression:

$$\mathrm{X}_i'\mathrm{X}_i = \mathrm{X}'\mathrm{X}[\mathrm{I} - (\mathrm{X}'\mathrm{X})^{-1}\mathrm{x}_i\mathrm{x}_i'] \qquad (2.4.1)$$

With this relationship between both matrixes, we can deduce the expressions for the more relevant measures.

Estimators for β

Due to,

$$\mathrm{X}_i'\mathrm{y}_i = \mathrm{X}'\mathrm{y} + \mathrm{x}_i\mathrm{y}_i \qquad (2.4.2)$$

the β estimator excluding the *i*th observation is:

$$\hat{\beta}_i = (\mathrm{X}_i'\mathrm{X}_i)^{-1}\mathrm{X}_i'\mathrm{y}_i = \hat{\beta} - \frac{e_i}{1-h_{ii}}(\mathrm{X}'\mathrm{X})^{-1}\mathrm{x}_{i,} \qquad {\scriptstyle i=1,2,\ldots,n} \qquad (2.4.3)$$

$\hat{\beta}_i$ is a $k \times 1$ vector and its elements are OLS estimators for β when the *i*th observation is excluded and e_i is the ordinary residual of the regression with all observations. If e_i increases or $1-h_{ii}$ decreases, or both, the *i*th observation may have a bigger influence in some elements of $\hat{\beta}$.

Ordinary residuals

In the case of the ordinary residuals we have:

$$e_j - e_{ji} = x'_j(\hat{\beta}_i - \hat{\beta}) = -\frac{e_i}{1-h_{ii}} h_{ij} \tag{2.4.4}$$

If $j = i$ then:

$$e_{ij} = \frac{e_i}{1-h_{ii}} \tag{2.4.5}$$

The estimated model without the *i*th observation will not be correctly predicted if that observation is an influential observation because it will always be different from the remaining observations.

Estimated values of y

Through the equation (2.4.3) we can also conclude that:

$$\hat{Y} - \hat{Y}_i = \frac{e_i}{1-h_{ii}} X(X'X)^{-1} x_i \tag{2.4.5}$$

And,

$$\hat{y}_{i(i)} = y_i - r_i \frac{h_{ii}}{1-h_{ii}} \tag{2.4.6}$$

If we want to study the impact of the elimination of the *i*th observation we have to calculate the vector $\hat{y} - \hat{y}_i$ squared:

$$(\hat{y} - \hat{y}_i)'(\hat{y} - \hat{y}_i) = h_{ii}\left(\frac{e_i}{1-h_{ii}}\right)^2 \tag{2.4.7}$$

Estimator for the errors variance

When we exclude the *i*th observation from the residual sum of squares we will have:

$$RSS_i = \sum_{j=1}^{n} e_{ji}^2 - e_{ii}^2 = \sum_{j=1}^{n} e_j^2 - \frac{e_i^2}{1-h_{ii}} \tag{2.4.8}$$

On one hand, the elimination of an outlier will reduce the residual sum of squares but on the other hand, we will have a reduction of one observation, thus we will have *n*-1 observations.

$$s_i^2 = \frac{n-k}{n-k-1}s^2 - (\frac{1}{n-k-1})\frac{e_i^2}{1-h_{ii}} = \frac{s^2}{n-k-1}(n-k-r_i^2) \qquad (2.4.9)$$

Hat matrix elements

The elements of the H matrix and the $X'X$ determinant are the following, respectively:

$$h_{jk(i)} = x_j'(X_i'X_i)^{-1}x_k = h_{jk} + \frac{1}{1-h_{ii}}h_{ij}h_{ik} \qquad (2.4.10)$$

$X'X$ **determinant**

$$|X_i'X_i| = (1-h_{ii})|X'X| \qquad (2.4.11)$$

2.5 Diagnostic based on the elements of H matrix

An influential observation has an effect on the estimates of the linear model parameters, which means that does not follow the general linear model tendency of the data and pulls the estimated results towards itself to minimize the distance between its actual response and the predicted response. Therefore, the influence of an observation is measured by the sensitivity of the model parameter estimated to the values of that observation, although cases with high leverage usually have more influence on the estimation of parameters.

It is very important to check the leverage h_{ii}, which measures the extremity of an observation and for each observation it is important to examine if it is huge because an observation with high leverage has the potential to influence the estimation results. The leverage looks at the extremity of the whole explanatory variable as one, we cannot have a high value for any explanatory variable analyzed separately but can have a high value when they are all considered together. So the residuals may give some light to whether influential observations exist, but a measure of the influence of the observations can be more reasonable.

The identification of outliers is usually performed by the residuals methods and for the high leverage points' identification it is more usual to perform methods based on the diagonal elements of the H matrix.

Diagnostic plots are very useful because they separate influential points into outliers and high-leverage points; the diagonal of the hat matrix indicates only the leverages and the residuals only outliers while the remaining diagnostics indicate both mutually.

2.5.1 Studentized Residuals

Generally, if there are no outliers in the regression model, the standardized and studentized residuals will have similar behaviour. If the *i*th observation is an outlier s_i^2 is lower than s^2, and as a result t_i will be bigger than r_i.

We know that one observation is an influential observation when $|t_i| > 2.0$ (indicates that the residual is not normal), although we have to pay attention to the residuals' distribution and if the sample dimension is reasonable.

As referred by Belsley *et al.* (1980) on one hand, the studentized residuals are a good form of examining the residuals' information because they have the same variance and they are without difficulty linked to the *t*-distribution, but on the other hand, many influential data points can have small studentized residuals. For this reason we need to study other measures to identify influential observations.

2.5.2 X space distance measures

To understand the leverage effect that was mentioned before we will start by studying the ordinary residual variance:

$$\mathrm{Var}(e_i) = \sigma^2(1 - h_{ii}) \tag{2.5.2.1}$$

As we have analyzed if h_{ii} is high the e_i variance will be low, which means that the observation pulls the model adjustment for itself; this phenomenon is termed **leverage effect**. If the X matrix column characteristic is equal to *k*, we have:

$$\sum_{i=1}^{k} h_{ii} = \mathrm{tr(H)=tr[X(X'X)^{-1}X'=tr[X'X(X'X)^{-1}] = tr(I)} = k \qquad (2.5.2.2)$$

So we need to have a factor that helps us defining when the h_{ii} value is large to attract our attention.

$$\frac{n-p}{p-1}\frac{\left[h_{ii}-(1/n)\right]}{1-h_{ii}} \sim F_{p-1,n-p} \qquad (2.5.2.3)$$

Thus if we want to be acquainted with the leverage effect through this expression we can, it is given by k/n.

The authors Hoaglin and Welsch (1978) define as high-leverage point the observations which:

$$h_{ii} > 2\frac{k}{n} \qquad (2.5.2.4)$$

Become aware of that when $h_{ii} = 1$, then $\hat{y}_i = y_i$ and $e_i = 0$.

We can also measure the distance between each explanatory variable and the average of all explanatory variables, known as **Mahalanobis distance,** this measure is very important for many disciplines because it gives us an estimation of distances between populations:

$$DM_i = [(\mathrm{x}_i - \mu)'\sum{}^{-1}(\mathrm{x}_i - \mu)]^{\frac{1}{2}} \qquad (2.5.2.5)$$

This measure follows a chi-square distribution with $k-1$ degrees of freedom and μ is the mean vector of the variables that we are studying and $\sum$ is the variance-covariance matrix: $\sum = \sum_{i=1}^{g}\frac{(n_i-1)S_i}{n}$, if they are unknown we use their estimators.

One important advantage of this measure is that the variable unit has no influence on the distance because each variable is standardized to a mean of zero and variance of one.

The distribution of $\hat{M}_{ik}$ *is positively skewed and* $Var(M_{ik})$ *is a quadratic function of* M_{ik} (Bedrick, 2005: 962). So, as Bedrick (2005) states, it could be expected that the normal approximation to $\hat{M}_{ik}$ is not adequate for small sample inferences. And for that problem this author suggests a square root transformation of $\hat{M}_{ik}$ because it tends to decrease the skew and moderate the dependence of the variance on $\hat{M}_{ik}$. The author also proposes a large sample distribution of $\hat{M}_{ik}^{5}$ which is normal with variance $Var\left(\hat{M}_{ik}^{5}\right)=\frac{.25Var\left(\hat{M}_{ik}\right)}{M_{ik}}$ that is linear in M_{ik}.

This method also determines the leverage effect, so we have to exclude row x_i' of the mean and of the X matrix variance, and we will have:

$$DM_i=\frac{n(n-2)}{n-1}\frac{h_{ii}-1/n}{1-h_{ii}}, \qquad i=1,2,...,n \tag{2.5.2.6}$$

As much as the distance measure is higher, the *i*th observation will be more influent. From this measure it is possible to conclude if an observation is unique for one or for all variables considered. It could happen that an observation is not unique for one single variable but become unique when we combine all the variables.

2.5.3 Diagnostics based on scalar influence measures

These measures are very important for the study because they communicate the influence of a specified point on all parameters.

Atkinson measure

This measure emphases the sensibility of a distance measure to the observations with high leverage.

$$A_i=\left|t_i\right|\times\sqrt{\frac{n-k}{k}\times\frac{h_{ii}}{1-h_{ii}}} \tag{2.5.3.1}$$

As we can see, as much as the studentized residual and the h_{ii} are bigger, the probability of the *i*th observation be an influent observation increases.

DFBETAS

The $\hat{\beta}_j$ can also be a good estimator to evaluate the observation influence, we can measure calculating the difference between $\hat{\beta}_j$ (that has *n* observations) and $\hat{\beta}_{j(i)}$ (that has $n-1$ observations) where we exclude the *i*th observation of the $\hat{\beta}_j$ estimation. This difference is influenced by the models variables we need to have in account a standard error:

$$DFBETAS_{ji} = \frac{\hat{\beta}_j - \hat{\beta}_{j(i)}}{s_{\hat{\beta}_{j(i)}}} = \frac{\hat{\beta}_j - \hat{\beta}_{j(i)}}{s_i\sqrt{a_{jj}}} = \frac{c_{ji}e_i}{s_i\sqrt{a_{jj}}(1-h_{ii})} = \frac{c_{ji}t_i}{\sqrt{a_{jj}(1-h_{ii})}} \quad (2.5.3.2)$$

Where a_{jj} represents the jth element of the principal diagonal of $(X'X)^{-1}$ matrix and c_{ji} represents the jith element of $(X'X)^{-1}X'$ matrix. The *DFBETAS* values grow when the studentized residual and/or the leverage effect also increases. Usually, *i*th is an influent observation in the estimate for the parameter β_j when the *DFBETAS* > 2. However, according the authors Besley *et al.* (1980) the influence of an observation decreases when the sample dimension increases so they suggest:

$$\left|DFBETAS_{ji}\right| > \frac{2}{\sqrt{n}} \quad (2.5.3.3)$$

<u>DFFITS</u>

Also designed as Welsch-Kuh distance, this measure enables us to evaluate the influence if the *i*th observation is removed on the prediction of the dependent variable. DFFITS is the difference between $\hat{y}_i$ that has *n* observations and $\hat{y}_{i(i)}$ where we exclude the *i*th observation of the $\hat{y}_i$ estimation and has $n-1$ observations. In this measure we must also consider the standard-error of $\hat{y}_{i(i)}$:

$$DFFITS_i = \frac{\hat{y}_i - \hat{y}_{i(i)}}{s_{\hat{y}_{i(i)}}} = \frac{\hat{y}_i - \hat{y}_{i(i)}}{s_i\sqrt{h_{ii}}} = \frac{\sqrt{h_{ii}}e_i}{s_i(1-h_{ii})} = \sqrt{\frac{h_{ii}}{1-h_{ii}}}t_i \quad (2.5.3.4)$$

As much as the h_{ii} and/or t_i are higher, the *i*th observation will have more influence on the *Y* prediction. According Belsey *et al* (1980) the *i*th observation has an important influence when:

$$\left|DFFITS_i\right| > 2\sqrt{\frac{k}{n}} \quad (2.5.3.5)$$

Although for Chatterjee and Haji (1998) this is when:

$$\left|DFFITS_i\right| > 2\sqrt{\frac{k}{n-k}} \quad (2.5.3.6)$$

<u>Cook distance</u>

The Cook distance (D_i) expresses the variance of the linear model parameters when a particular observation is left out of the data set, so it evaluates not only the impact of the *i*th observation on the β_j estimative but also on all parameters of β at the same time:

$$D_i = \frac{(\hat{\beta}-\hat{\beta}_i)'X'X(\hat{\beta}-\hat{\beta}_i)}{k \times s^2} = \frac{(\hat{y}_i-\hat{y})'(\hat{y}_i-\hat{y})}{k \times s^2} = \frac{r_i^2}{k} \times \frac{h_{ii}}{1-h_{ii}} \quad (2.5.3.7)$$

While the D_i value increases the *i*th observation, it becomes more influent, therefore when the *i*th point does not affect β significantly, the D_i value is low. A high value of D_i can be caused by an outlier or an extreme observation (h_{ii} = 1), the higher the cook distance, the more influential is the observation. Cook (1977) proposes an $F_{k,n-k}$ (α) distribution to identify the influent observations through the comparison of D_i and the 90 quintile (α=0.90). Weisberg (1985) suggests α=0.50 for samples with a reasonable dimension, if $D_i > 1$ the *i*th observation is influent.

We cannot conclude that a small D_i value means that there are no outliers, this author gives us an example that illustrates this fact well. When we have two outliers that share the same data values; the value of the Cook distance may be small because this measure considers the result on the parameters of removing the observations individually from the data set.

So this method can be very helpful for discovering influential observations when there is just a single outlier but can be unsuccessful if there is more than one.

2.5.4 Other diagnostic measures

<u>Covariance ratio</u>

We can study the influent observations through the determinants of variance-covariance matrix of the model estimators. Belsley *et al.* (1980) show that:

$$\det[X_i' X_i] = (1 - h_{ii}) \det(X'X) \qquad (2.5.4.1)$$

Then:

$$\frac{\det[X_i' X_i]^{-1}}{\det(X'X)^{-1}} = \frac{1}{1 - h_{ii}} \qquad (2.5.4.2)$$

The difference between $\det[X_i' X_i]^{-1}$ and $\det(X'X)^{-1}$ is the exclusion of the *i*th observation, and it is important because it indicates if the two matrices are close or if the *i*th observation does not affect the sensibility of the covariance matrix. However these analyses are only provided from the information of the X matrix.

In accordance with Belsey *et al* (1980), excluding the *i*th observation and $s_i^2\left(X_i'X_i\right)^{-1}$ and including all observations $s^2\left(X'X\right)^{-1}$, according to Belsley *et al*, 1980:

$$CR_i = \frac{\det[s_i^2(X_i'X_i)^{-1}]}{\det[s^2(X'X)^{-1}]} = \left(\frac{n-k-r_i^2}{n-k-1}\right)^k \frac{1}{1-h_{ii}} \tag{2.5.4.3}$$

Where *det* represents the determinant matrix. If s_i^2 is too different from s^2 this ratio will be significantly different from 1. These authors say *i*th is an influent observation when:

<u>Cook-Weisberg measure</u>

$$\left|CR_i - 1\right| > 3\times\frac{k}{n} \tag{2.5.4.4}$$

This measure represents a general diagnostic and evaluates the difference between the maximum of the logarithm of the likelihood function when all points are used and when excluding the *i*th observation:

$$LD_i = 2[L(\hat{\beta}) - L(\hat{\beta}_i)] \sim \chi^2_{k+1} \tag{2.5.4.5}$$

The observations are influential, for certain α level, when:

$$LD_i > \chi^2_{k+1}\left(1-\alpha\right) \tag{2.5.4.6}$$

Where $\chi^2_{k+1}\left(1-\alpha\right)$ is the quantile of χ^2 distribution.

With this measure it is possible to perform an analysis of the *i*th influence on the estimative parameters and on the errors variance estimate or on both. Through $LD_i(\hat{\beta})$ we can observe the individual observations' influence on the estimative for the parameters:

$$LD_i(\hat{\beta}) = n\times\ln\left[\frac{d_i\times h_{ii}}{1-h_{ii}}+1\right] \tag{2.5.4.7}$$

Where,

$$d_i = \frac{r_i^2}{n-k} \tag{2.5.4.8}$$

The individual observations' influence on the mean square error can be evaluate by the $LD_i(s^2)$ measure:

$$LD_i(s^2) = n\times\ln\left(\frac{n}{n-1}\right) + n\times\ln\left(1-d_i\right) + \frac{d_i\left(n-1\right)}{1-d_i} - 1 \tag{2.5.4.9}$$

And to evaluate each observation on $\hat{\beta}$ and s^2 together:

$$LD_i\left(\hat{\beta}, s^2\right) = n \times \ln\left(\frac{n}{n-1}\right) + n \times \ln\left(1-d_1\right) + \frac{d_i\left(n-1\right)}{\left(1-d_i\right)\left(1-h_{ii}\right)} - 1 \qquad (2.5.4.10)$$

After this analysis an important question that deserves our attention and is proposed by Martin and Kumar (2005) is the detection of outliers and influential observations with heteroscedasticity-corrected models. The heteroscedasticity corrections may inflate some errors' variance and consequently mask the signal that those observations are outliers. As previously mentioned, the outliers are identified by large values of studentized residuals and the heteroscedasticity corrections allocate a larger root mean square error to many observations that are far from the regression surface which will result in the masking of the signal that they are outliers. In the presence of this problem, analysts have to examine the ranking of the absolute value of the residuals to identify outliers.

In the case of influential observations, the DFFITS and the Cook distance are not enough for good analysis when they are heteroscedasticity-corrected. In this case we have to use the Cook-Weisberg measure. Analysts should observe the ranking of the likely displacements for observations that are linked with smaller error variances for a sign that observations are influential.

Peña (2005) proposes another type of statistic to analyse the influential observations which is very simple to compute and with an intuitive interpretation that should be seen as a complement of traditional analyses. This type of analysis allows us to discover other characteristics in the data such as clusters of high-leverage outliers.

The author suggests that instead of analysing the effects on the parameters, forecasts or likelihood function where we exclude an influential point we should look at how each point is influenced by the others in the sample, so for specific observation in the sample we evaluate the predicted change when another point in the sample is deleted. Therefore, we examine how each point is influenced by the rest of the data.

This new statistic is defined at the *i*th observation as the squared norm of the standardized vector, S_i:

$$S_i = \frac{1}{ks^2 h_{ii}} \sum_{j=1}^{n} \frac{h_{ji}^2 e_j^2}{\left(1-h_{jj}\right)^2} \qquad (2.5.4.11)$$

This statistic has three properties:

a. If the sample does not have outliers or high-leverage observations the expected value of the statistic will be approximately 1/k.

b. The statistic S_i has a normal distribution when the sample has a large size with many predictors.

c. The sensitivity statistic is discriminated between the outliers and the good points when the sample has a group of similar outliers with high leverage.

2.5.5 Graphical residuals analysis

We also can analyse residuals graphically and the authors Meloun *et al.* (2004) refer to five types of plots that can be used to detect influential observations.

Predicted residuals graph

On this graph the x-axis has the predicted residuals and the y-axis has the ordinary residuals. When a point has some distance from the line $y = x$ it means it is a high-leverage point. The outliers are situated on the line $y = x$ but far from its central pattern.

Williams graph

This graph has the diagonal elements of a hat matrix on the x-axis and the jack-knife residuals on the y-axis. Two border lines are drawn. One is for outliers where t is a t-student distribution with $n-k-1$ degrees of freedom.

Pregibon graph

This graph also has the diagonal elements of a hat matrix on the x-axis but has the square of normalized residuals on the y-axis. Two lines are formed $y = -x + 2(k+1)/n$ and $y = -x + 3(k+1)/n$ a point above the upper line is strongly influential and a point between the two lines is influential. The influential point can be an outlier or a high-leverage point or both.

McCulloh and Meeter graph

The x-axis has $\ln\left[H_{ii}/\left(k\left(1-H_{ii}\right)\right)\right]$ and y-axis has the logarithm of the square standardized residuals. The boundary line for high-leverage points is defined as $\mathrm{x}=\ln\left[2/\left(n-k\right)\times\left(t_{0.95}^{2}\left(n-h\right)\right)\right]$.

Gray's L-R graph

In this graph the x-axis has the diagonal elements of a hat matrix and on the y-axis the squared normalized residuals. In this graph all points are under the hypotenuse of a triangle with a 90º angle in the origin of the two axes. In this graph the contours of the critical influence are plotted and the positions of individual points are compared with them.

Graphically, we can detect the influential observations more easily, although for a large sample it is not very useful.

2.5.6 Treatment of influential observations

Now that we are conscious of the influential observation problems and how to detect them, we have another problem which is how to solve this problem.

Roddam *et al* (2002) propose two solutions:

1. Check if the values were correctly entered into the computer. We can also investigate whether the variables were correctly measured at the first attempt but in this case we have to keep the original measurement. Although if we notice that a transcription error or we are able to correct a measurement, we can substitute it with the correct value and re-analyse.
2. If the quality of the outlying observations appears fine or impossible to check and we are sure that the observation is a mistake, we can delete and re-analyse without the observation. However, we need to be careful and we must give a good justification explaining why we delete and not delete just because the model fit is better without the observation.

The easiest way of solving the outlier problem is deleting the observation but this is not very advisable and is only necessary when the error correction is not available. The influential observations should be treated carefully because the information can be very useful for the sample.

As a conclusion, regression diagnostics is a method that enables researchers to analyse the data more effectively and efficiently, so the principal objective of this method is to make researchers aware of some data points that can distort the estimation process and does not allow us to understand the real relationship between the dependent and independent variables (Chatterjee and Wiseman, 2001).

As evidenced when the influential observations are detected, they are normally excluded because it reduces the sum of the residuals square and the R^2 increases, which apparently improve the data fit. But is this right?

The authors Barnett and Lewis (1995) defend that is not necessary to have an extreme decision, eliminate the risk of losing important information or keep it in the sample with the risk of contamination. We can use some methods of inference, which minimize the influence of any outliers.

These observations should only be eliminated if they are truly anomalous, otherwise, we should keep them because if they are part of the population being studied and if they are eliminated, the results of the model will be improved but we run the risk of having disturbed results and limiting the capacity to generalize of the sample conclusions and the respective population.

Meloun *et al*. (2002) state that there are 4 steps to construct a regression model and detect influential observations. The first step consists of the examination of the scatter plot of individual variables and all possible pair combinations; here we can identify the influential observations that cause multicollinearity. The next step is to estimate the parameters and statistical characteristics like the *t*-student test for individual parameters, the correlation and calculation of R coefficient through the ordinary least-squares method (OLS). At this step we also verify the quality of the model through the means quadratic error of prediction and Akaike information criterion.

The following step is very important because it is the detection of influential observations. If we detect outliers then we have to decide whether or not to eliminate them from the data set or not. In the case of elimination we have to repeat the analyses of data. With the verification of the condition for the least squares method and the results of regression diagnostics we can construct a more accurate model.

Therefore, we need to proceed and ensure the data, model and method quality.

As was referred to above, the easiest way to solve the influential observations problem is through their elimination. This solution on one hand reduces both the ordinary least squares and the residual sum squares and on the other hand increases the R^2 value. Although we should keep all observations, the elimination should only be used when we are sure that the observation is really anomalous. The elimination of wrong observations can produce incorrect conclusions, so we need to be careful.

For a better understanding the authors Belsley *et al*. (1980) show us one application that is a good guide for our empirical application. The first thing to do is analyse the **validity of the model and the coefficients**. The model is valid if the probability associated to the F test is lower than 0.05. The coefficients are statistically significant if the probability associated to the *t* test is lower than 0.05.

Then we need to study the **residuals**. First we should see if the residuals have a normal distribution (through the Kolmogorov-Smirnov test). Afterwards, we see if there are outliers, we just need to observe if the value of the studentized residual is bigger than 2.

Once the outliers are detected, we will analyse the **leverage** and the **hat matrix diagonals**. Chatterjee & Hadi (1988) state $h_{ii} = \frac{1}{n} + \tilde{h}_{ii}$ where $\tilde{h}_{ii}$ represents the principal diagonal elements of the centred matrix H_c, we can conclude that one observation has high leverage when:

$$\tilde{h}_{ii} > 2 \times \frac{k}{n} - \frac{1}{n} = \frac{2k-1}{n} \tag{2.5.6.1}$$

Then, we study the measures that were mentioned previously like: coefficient sensitivity (DFBETAS), covariance matrix sensitivity (COVRATIO), DFFITS, and distance measures like Mahalanobis and Cook distances.

Finally, Belsley *et al.* (1980) suggest a complementary diagnostic for the suspicious observations that bring together the information which has been individually considered. We determine the interquartile range (IQ) for each series and indicate as extreme those values that exceed $\frac{7}{2}IQ$.

3. Diagnostic methods applied in published studies to handling influential observations

The objective of this section is to explore how the influential observations are handled by some authors in their studies. As we have already noted, the procedure that we use to minimize the impact of these observations will influence the results of our study and sometimes the methods used are not the most appropriate.

Arce and Mora (2002) investigated the value relevance of alternative accounting measures (earnings and book value) created in different European accounting systems. These authors focused their attention on the eight European countries with the most important stock markets: Belgium, France, Germany, Italy, the Netherlands, Switzerland, Spain and the United Kingdom. In the construction of their sample they included only firms with disclosing positive sales revenue, and excluded financial companies, property firms and investment trusts and deleted all firms/years with missing values for one or more variables. In order to control the extreme values, these authors removed observations in **the top and bottom percentiles** of the variables.

The **Kothari and Shanken (2003)** paper about the evidence of the economic determinants of the time-series variation in coefficient mapping of financial information in prices, provides some restrictions in order to avoid extreme values. These authors selected a random sample of 500 firms each year from 1967 to 2000. With a large sample the authors **exclude** firms with: negative book value of equity, **share prices lower than $2 and higher than $200**, **earnings below $10 per share and earnings above $20 per share**. Also, in this study, when they analyse the influence of growth of future earnings and future stock returns in price, and to mitigate the influence of outliers they establish that for future earnings growth greater than 50% per year it is set equal to 0.50 and less than 50% is set equal to −0.50.

Notwithstanding, they admit that this procedure can exclude observations that are not influential.

Based on the Olshon model, the authors **King and Langli (1998)** studied the relationship between accounting numbers and firm market values in three different countries (Germany, Norway and the United Kingdom) with different accounting practices. To construct the sample they used the following exclusions: financial firms,

firms with negative book value and **observations with the largest and smallest 1%** of observations ROE and price earnings.

Brown *et al.* (1999) studied that the presence of scale effects in regression levels increase R^2, and this effect increases in the scale factor coefficient of variation. To define the sample, the authors excluded the observations: i) with negative book value; ii) in the **top and bottom of one-half percent** of firms in terms of price of earnings per share (EPS), book value price per share (VPS) and non-recurring items such as a fraction of net income before non-recurring items, in each year; iii) that had studentized residuals with an absolute value greater than 4.

Collins *et al.* (1997) investigated the regular modifications in the value-relevance of earnings and book values over time. To construct their sample and in order to avoid influential observations they deleted the observations that were in the **top and bottom one half percent** of either earnings to price or book value to market value, removed the observations that were in the one half percent of firms with the most extreme values of one-time items as a percentage of income and deleted the points identified as outliers using the studentized residuals method (observations that have more than four standard deviations from zero).

Dechow *et al.* (1998) investigated different approaches that researchers use when examining the association between accounting information and long-window stock returns. To reduce the influence of influential observations, the authors deleted the **most extreme one percent** of observations.

The objective of the **Francis and Schipper (1999)** paper is to discuss and test some empirical implications of the claim that financial statements have lost their relevance over time. For better results and to avoid the influence of extreme values, the authors provide the following restrictions: i) delete the **extreme one percent** of each variable; ii) remove the observations in which the studentized residual is greater than 3; iii) exclude the observations in which Cook's D statistic is greater than 1.

The authors **Ali and Hwang (1999)** studied the relations between measures of the value relevance of financial accounting data and several country-specified factors suggested in prior research. In order to concretize this study, the authors used data of manufacturing firms from 16 non-U.S. countries during the period 1986-1995. To keep the sample away from extreme values, the authors made some exclusions: i) $\left|\Delta E_{it} / P_{it-1}\right| \leq 1$; ii) $\left|\Delta CFO_{it} / P_{it-1}\right| \leq 1$; iii) $0 < \mathrm{P}_{it} / \mathrm{BV}_{it} \leq 5$; iv) $\left|\mathrm{E}_{it} / \mathrm{BV}_{it}\right| \leq 0.5$, where ***i*** is a firm subscript, ***t*** is a year subscript, **BV** is book value, **P** is stock price, **E** is earnings before extraordinary items and **CFO** is cash flow from operations.

Barth *et al.* (1998) explored the roles of equity book value and net income as a function of financial health. To construct their regression summary statistics and eliminate the influential observations the authors only used the DFFITS statistic determined by Besley *et al.* (1980).

Based on data simulated and using Ohlson's valuation model, the authors **Barth and Clinch (2005)** studied the effects on inferences of 4 scale effects (additive and multiplicative omitted scale factors, scale-varying coefficients and heteroscedasticity). These authors designed their data by randomly selecting 500 compustat firms. To obtain data with an additive-omitted correlated scale effect, they determine the dividend value after **removing one percent from the top and bottom** of dividend payout and capital ratios.

As we can observe, there are some arbitrary decisions on influential observations deletion. The criteria used by the authors are not consistent with our review of the literature; in general the authors do not use a scientific base to exclude these observations. As we can conclude, the authors commonly exclude the extreme one percent of the variables used in their studies to avoid the extreme values, although we can have observations that are in the extreme one percent and not be an influential observation and on the other hand we can have an observation that is not in the extreme one percent and being an influential observation.

In the next chapter we will conclude about the differences that we can achieve by using the methods suggested in the econometric literature.

4. Empirical application and critical issues

In this section, we intend to conclude about the differences that an investigation can have if a different analysis of influential observations is made.

As we know, it is important detect, study and handle the influential observations. We have already seen that many authors do not perform the procedures suggested by the econometric handbooks and eliminate observations that cannot be influential. Our objective is to demonstrate that a different approach to deal with influential observations can induce us to different conclusions.

In our study we will focus on key variables of the theoretical accounting valuation model developed by Ohlson (1995).

According to our sample and considering the Ohlson regression model:

$$PRICE_{it} = \beta_1 + \beta_2 BVE + \beta_3 NIPS_{it} + \varepsilon_{it} \tag{4.1}$$

Where:

$PRICE_{\mathrm{it}}$ is the share price of the company (i) at the balance sheet date (t);

BVE_{it} is the book value equity;

$NIPS_{it}$ is the net income price share.

To study the impact of influential observations in empirical accounting studies, we used a sample of 24,644 year observations of 6,453 European listed companies from Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, The Netherlands, Portugal, Spain, Sweden and the United Kingdom, for which Worldsope Database data was available for all the variables (accounting standards, price, book value of equity per share and net income per share) for the period between 2000 and 2005. (See appendix 2)

In order to compare the procedures used to exclude influential observations we have followed three steps:

1) The observations are excluded according to the author's criteria;

2) We apply the econometric suggestions;

3) We estimate the Ohlson regression model based on each of the resulting data set and compare the estimation results.

Notwithstanding, we have to be aware that some exclusions applied by the authors have to be made because it is illogical to keep them in the study. These observations are not excluded to avoid influential observations but to make the investigation more objective and realistic, therefore we will also exclude them from our study, for example in the Ohlson' model, the observations with BVE<0 should be excluded from the sample.

We have chosen four papers from the last section to compare with our analyses:

	Authors	*Method performed*
Authors 1	Francis and Schipper (1999)	Extreme1%; SR>3; Cook'sD>1
Authors 2	Arce and Mora (2000)	Extreme 1%
Authors 3	Barth, Beaver and Landsman (1998)	|DFFIT|>3
Authors 4	Econometric suggestion	|SR|>2; Lev>2*mean(Lev)

Note: SP – share prices; E – earnings; SR – studentized residual.

The criteria for the exclusion of influential observations of these authors are better explained in the previous section.

In the sample of our method we exclude the observations with BVE<0. According to Hocking (1983), the studentized residuals (SR>2; Belsley *et al*, 1980) and elements of the diagonal of the hat matrix ($h_{ii} < 2\frac{k}{n}$; Hoaglin and Welsch; 1978) are reasonable methods to detect suspect cases of influential observations. So we will adopt these statistics to eliminate the outliers and high-leverage points.

Due to the problems of heteroscedasticity commonly found in empirical accounting when the Ohlson model is applied, we also correct the OLS coefficients standard errors according to the White procedure. The standard errors obtained for the coefficients are heteroscedastic consistent.

<u>Results of the Ohlson model in the UK</u>

The sample of this country is represented by 5,785 observations. According to the econometric method suggestions, exist 26 influential observations in the sample. Compared with the other methods performed by the accounting authors, differences of the results are visible. In the method used by authors 1 and 2, there are 227 and 186, respectively, and in the third method there are fewer influential observations, only 7, than in our method. Therefore, some of the accounting authors are excluding observations that are not influential and others are keeping them in the sample and consequently distorting the results.

In the econometric method, the model is globally valid and 59.9% of the variance of the price is explained by the book value and earnings variation. This is the method with the higher adjusted R^2 value. In accordance with the second and third method, the adjusted R^2 values are 31.2% and 36.5%, respectively. The first method has a result closer, 46.2%, to the econometric method but have also a difference of 13%.

The variation of the BVEPS influences the variation of the price by an average 1.158 units. The others authors have also different results but the authors that have results more distant are authors 3. According to all methods, the BVEPS is statistically significant. When we apply the White procedure (see appendix 4) the results remains unchanged.

The variable net income share price has very dissimilar results. In our method for each variation of one point, the price varies in average 0.214 units in the opposite direction. According to authors 2 and 3, the NIPS also varies in the opposite direction of the price but for authors 1, the NIPS tends to vary in the same way.

Finally, according to the econometrics suggestion, NIPS variables are statistically relevant because the significance of the t-test is lower than 0.05. Author 1 does not have the same result; for this author, NIPS is not statistically significant. Despite performing the White procedure, the variable NIPS is no longer significant for authors 4 (see appendix 4).

In conclusion, the three methods that we consider to exclude the influential observations lead us to different conclusions. In general, our method expresses that the variables for this model are adequate and influence the price. In the results of the other authors these conclusions are not so evident.

Results of the Ohlson model in France

The results of France (appendix 5) show that our method is the most explained by the variables. In the econometric suggestions, the variables (BVEPS and NIPS) explain 83.5% of the model against the 69.3% of authors 1, 57.4% of authors 2 and 70,5% of authors 3. The econometric method also presents the higher F-test value.

For all authors the model is globally valid (significance of F-Test is lower than 0.05) and the variables (BVEPS and NIPS) are statistically relevant (significance of T-test for all variables is lower than 0.05).

According to the econometric suggestion method, it is expected a variation of 1.048 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant. The method of authors 3 assumes the higher influence in the price (1.067 units).

Observing the performance of NIPS we see that in the econometric suggestions method the NIPS influence is lower in the behaviour of price than methods 1 and 2 but higher than method 3.

For all authors, both variables tend to vary in the same direction.

In this sample, 79 of the observations are influent, and as observed in the analysis of the UK, according to author 4, method that exerts less influent observations, they are kept in the sample and for the other two authors there are more influential observations. Authors 1 and 2 identified 193 and 146 influential observations, respectively.

Applying the White procedures we conclude that all methods and the respective variables continue to be statistically significant (see appendix 6).

In France, the results of the method used by authors 4 are clearly more similar to the econometric suggestion method. Notwithstanding, these authors do not have a scientific base so the results could not be correct.

<u>Results of the Ohlson model in Italy</u>

Analysing the results of Italy using the econometric suggestions, we notice that the BVEPS and NIPS explain 68% of price variation. Compared with the other methods we observe the following discrepancies: in authors 2 and 3 methods the variables only explain 55.9% and 50.4% of the model respectively. The method that is closer to ours is authors 1, in which the explanatory variables explain 69.3% of the dependent variable.

According to the results of all the authors, the model is globally valid (significance of F-Test is lower than 0.05) as the variable BVEPS is statistically relevant (significance of T-test is lower than 0.05).

The BVEPS significantly influences price behaviour in the econometric suggestions. It is the expected a variation of 1.142 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant. In the results of author 2, the variation of BVEPS has a greater influence, 1.573 points; although the T-test presents a higher value in our method which means that the result of our model has a major confidence.

The NIPS is the variable where the results diverge more. According to the econometric suggestions and authors 3 this variable is not statistically significant. However, according to authors 1 and 2 the variable is significantly relevant.

There are 40 influential observations in this sample. Method 3 still eliminates fewer observations than the other methods. The method that has a bigger approximation to the econometric method is that of author 1 (see appendix 7).

Considering the effects of heterocesdasticity (appendix 8), we conclude that the model continues globally valid for all authors. The variable BVEPS remains statistically relevant in all methods. The significant of NIPS (t-test) increases in all methods and in method 2 the variables are no longer statistically relevant.

Results of the Ohlson model in Austria

The method according the econometric suggestions has the higher value of $\bar{R}^2$, which means that the variable BVEPS and NIPS explain more the variations of the model (72.8%). In the method of authors 3, the variables only explain 54.6%.

The significance of the F-test is lower than 0.05 in all methods, so the model is globally valid for all authors, but in econometrics suggestion method, the F-test assumes the bigger value, which increases the confidence of our method.

Both variables in all methods are statistically significant. The BVEPS is more influential in method 1, the variation of the BVEPS influences the variation of the price by an average 0.527 units. According to the econometric suggestions, the price only varies 0.186 units although, the value of the T-test is higher in our method than in method 1.

Comparing the econometric suggestions to the other authors, it is visible that the other methods are raising the results. Our method has the lowest result (0.753) and for example, for author 1, the NIPS is 2 times more than it should be if the econometrics suggestions were applied. For all methods both estimated coefficient have positive sign so we can conclude that both variables tend to vary in the same direction.

According to the econometric methodology, 3 observations are influential and should be excluded. However, authors 1, 2 and 3 delete 22, 15 and 10 observations, respectively (appendix 9).

Attending to the White procedures (appendix 10), the significance of t-test for both variables increases in all methods including in our method for NIPS but remains lower than 0.05, keeping the variables statistically significant.

Results of the Ohlson model in Belgium

In Belgium, the econometric suggestions method also presents the higher $\bar{R}^2$ value**.** 90.4 % of the variation in price is explained by the variation of BVEPS and NIPS. The result of author 1 is very close to ours, where the variables explain 90.2% of model behaviour; nevertheless, the results of the other authors are very distant from ours.

The significance of the F-test is lower than 0.05 for all authors, therefore, the model is globally valid for all authors.

In the results of all authors, the significance of the t-test, in both variables, is also lower than 0.05, which means that the variables are statistically significant, even when we correct the OLS standard errors. Notwithstanding, the variables have a very different influence in the model.

According to the econometric suggestions, it is expected a variation of 0.611 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant. Authors 1, 2 and 3 have a more similar result (0.660, 0.650, and 0.561).

In the econometric suggestions, the variation of the NIPS influences the variation of the price by an average 2.580 units in the same direction. For authors 1 and 3 it varies 3.249 and 2.992, respectively. Although, the results of author 2 demonstrate that it is the expected variation of 5.124 units on price (dependent variable) per unitary variation on NIPS, assuming that all the rest remains constant. This result is extremely distant from the others.

As we can see, applying non-scientific methods induces us to different results. The author who has the most similar results is author 1. Authors 3 are more distant from the econometrics suggestion.

Finally, it is also important to mention that by applying the econometric suggestion method, we realized that there are 21 influential observations, but in methods 1, 2 and 3 are excluded 41, 27 and 28 observations (appendix 11).

According to the White procedures, the results remain unchanged (appendix 12).

Results of the Ohlson model in Denmark

The results of this country clearly demonstrate that applying non-scientific methods can lead to misleading results. For example, author 1 excludes 40 when there are only 11 influential observations.

According to the econometric suggestions, 97.9% of the variation of the price is explained by the book value equity and earnings. This presents the higher value. Using method 2, the $\bar{R}^2$ value decreases to 48.2%.

The significance of the F-test is lower than 0.05 in all methods, so the model is considered globally valid for all. The value of the F-test also presents large differences in all methods. Our method presents the higher value and the method of authors 2 presents the lower value.

In accordance with the econometric method, the BVEPS and NIPS are statistically significant variables. The BVEPS presents a value of 0.814; methods 1 and 3 have higher values but are closer to ours, and the values of method 2 are very distant. It is also important to mention that for author 2 the BVEPS variable is not statically significant.

On the other hand, NIPS is statically significant for all authors. In the econometric suggestions method, the variation of the NIPS influences the variation of the price by an average 5.622 unit in the same direction as we can observe in appendix 13, the results of other authors are also very different from ours. The result of author 2 is a great deal higher and the results of the other authors are significantly lower.

Considering the White procedures (appendix 14), the only significant rise in NIPS (t-test) is using author 2, but it is still lower than 0.05, so the variable continues to be statistically significant.

Results of the Ohlson model in Finland

Once again the econometrics method assumes the highest adjusted R^2 value. Applying the criteria of author 3, in this sample, only 1 observation should be excluded, when there are 42 influential observations (see appendix 15).

Both the significance of F-test and T-test are lower than 0.05 for all methods, which means that the model is globally valid and the variables are statistically significant, even when we apply the White procedures (appendix 16).

The BVEPS presents the same $\bar{R}^2$ results because econometrics suggestions method has the highest value for β_{BVEPS} (0.878 units) and the results of the other methods do not diverge significantly. The values are between 0.600 and 0.788.

According to the econometric suggestions it is the expected a variation of 2.162 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant. The results of β_{NIPS} are not so regular. The values of the other methods are all higher, the authors 1, 2 and 3 presents a value of 4.404, 5.177 and 3.449, respectively.

Results of the Ohlson model in Germany

In the German sample, there are 65 influential observations. Authors 1, 2 and 3 exclude 190, 131 and 28 observations respectively.

The model is globally valid and both variables are statistically significant for all authors. Nevertheless, the values of the results are very dissimilar.

When we correct the OLS standard errors the NIPS is no longer statistically relevant, so in Germany there is no relation between the NIPS and price. However, this is only visible in the econometric method because for the other methods the correctness of the OLS standard errors maintains the results (appendix 18).

In accordance with the econometric suggestions, 63.5% of the model is explained by the variables' behaviour. The results of authors 1 and 2 are not very distant but the results of author 3 are.

The econometric method demonstrates that the influence of the BVEPS in the price is lower than 1 unit and in the other methods is higher than 1 unit. Notwithstanding, the t-test of our method assumes the highest value (appendix 17).

Results of the Ohlson model in Greece

As we can observe in appendix 19, the NIPS has a major influence on the price rather than the BVEPS. The results of our method demonstrate that 47.3% of the variation in price is explained by the variation of BVEPS and NIPS. The method of authors 2 expresses that the dependent variables only explain 31% of the variations in price.

The model is globally valid in all methods. The variables are also statistically significant.

With regard to BVEPS, the results of all authors display higher values than our method. Author 3 has a closer result and authors 1 and 2, a more distant result.

Concerning NIPS, it is the expected variation of 4.366 units on price (dependent variable) per unitary variation on NIPS, assuming that all the rest remains constant. The result of authors 1 and 2 are, also, more distant showing a result of 6.756 and 6.928, respectively.

Regarding the number of observations excluded, we observe that according to the method of author 3, only two observations are excluded. In this sample, our method is not the method that excludes fewer observations, which means that author 3 certainly maintains observations in the sample that are influential and consequently induce wrong results (appendix 19).

Applying the White procedures in this sample (appendix 20), we verify that in the econometric suggestions the BVEPS variable is no longer statistically significant because the significance of t-test is higher than 0.05. Despite BVEPS still being statistically significant in the results of authors 3 the significance of BVEPS (t-test) increases.

Results of the Ohlson model in Ireland

In the econometric suggestions method, 79.5% of the price variation is explained by the model, assuming the higher value. In the other methods this value decreases; the result of author 1 is the nearest to ours (74%). For authors 2 and 3 the variables explain 68.8% and 68.4% of the price variation, respectively.

For all methods, the model is globally valid and both variables are statistically significant.

In relation to the BVEPS, it is the expected variation of 1.623 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant. The results according to methods of authors 1 and 2 are farther from ours, presenting a result lower than 1.

Regarding NIPS, the variation of the NIPS influences the variation of the price by an average 0.864 unit in the same direction. The results of authors 1 and 2 are extremely distant from ours.

As the sign of the estimated coefficient in all methods is positive, we can conclude that both variables tend to vary in the same direction.

Surprisingly in this case, our method is the method that eliminates more observations, which means that the other methods are keeping influential observations, namely the method used by author 2 that only excludes 2 observations (appendix 21).

Regarding to the White procedures, there are no different conclusions. In spite of the significance of the NIPS (t-test) increase in author 3 and in our method they are still lower than 0.05, so the variable continues to be statistically significant (appendix 22).

<u>Results of the Ohlson model in The Netherlands</u>

According to the results of our method, 72.8% of the model is explained by the variation of the explanatory variables. Both variables are statitiscally significant, therefore the model is globally valid (significance of t-test is lower than 0.05).

Regarding the econometrics method, the variation of the BVEPS influences the variation of the price by an average 0.903 units in the same direction. Concerning NIPS, the variation of this variable influences the variation of the price by an average 0.628 units in the opposite direction in the sample 16 observations are influential and must be excluded (see appendix 23).

When we observe the results of the other authors it follows that the results are distant from the correct results.

The results of authors 1 demonstrate that 83.1% of the variation in price is explained by the variation of BVEPS and NIPS. Regarding to author 3 the β_{BVEPS} is 1.245 against

0.903 of our method. To the contrary of our method, the results of all authors demonstrate that the NIPS positively influences the price and according to author 3, the variable NIPS is not statically significant.

In spite of only having 16 influential observations, authors 1, 2 and 3 exclude 60, 35 and 7 observations, respectively.

When we correct the OLS standard errors according to the White procedures, the t-test decision remains unchanged for all authors (see appendix 24).

Results of the Ohlson model in Portugal

In Portugal, the econometric suggestions method presents the highest adjusted R^2, 71.6% of price variation is explained by the dependent variables. The methods of the other authors display completely different results: 48.9%, 23% and 34.2% for authors 1, 2 and 3, respectively.

It is the expected variation of 0.756 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant; the author 3 result is 0.755; 0.710 for author 1 and 0,769 for author 2. Our method differentiates greatly from the other authors presenting the higher value of t-test.

The model is globally valid according to all methods. Also the variables' BVEPS and NIPS are statistically significant for all. These results are maintained even when the White procedure is applied (appendix 26).

Concerning the econometrics suggestions method, the variation of the NIPS influences the variation of the price by an average 0.862 units in the same direction. Observing the results of authors 1 and 2, we are led to very different conclusions. For these the price varies 1.508 and 1.472 points, respectively.

There are 14 influential observations, however author 4 eliminates only 1 observation (appendix 25). As we can see the results are completely unskewed.

Results of the Ohlson model in Spain

Regarding the econometric suggestions, the variation BVEPS and NIPS explains 73.1% of price variation. The model is globally valid and both variables are statistically significant. For each variation of one point in BVEPS the price varies 0.707 points. A variation of one unit in the NIPS causes a variation of 4.596 units in the price. This sample is constituted by 886 observations and 17 are influential so must be excluded.

Author 1 has similar $\bar{R}^2$ but the other authors have a very different result, for example, for author 3 only 53.2% of the price is explained by the dependent variables considered in the model.

For all authors the model is globally valid and both variables are statistically significant.

However, the t-test of BVEPS has the higher value in our method. And the results of β_{BVEPS} are very dissimilar. Concerning author 3, the variation of the BVEPS influences the variation of the price by an average 1.017 units in the same direction. Regarding author 1, the β_{BVEPS} presents a value of 0.510 units.

The variable NIPS also has very differing results. In accordance with author 3, the price only varies 2.564 units with the variation of one unit of NIPS in the same direction, assuming that all the rest remains and according to author 1, the price varies 6.781 units.

As in the other samples, the authors that excludes more observations is author 1, this author excludes 39 observations. On the contrary, the author 3 only excludes 7 observations.

Applying the White procedures, the significance of BVEPS (t-test) increases in authors 2 and 3, although it is still under 0.05 which means that the variable continues to be statistically significant. The significance of NIPS (t-test) also increases for author 3 but the variable is no longer statistically significant.

Results of the Ohlson model in Sweden

The results of Sweden demonstrate that the variation of the BVEPS and the NIPS explain 52.5% of the Ohlson model variation.

Both variables, BVEPS and NIPS, are statistically significant and consequently, as the results exhibit, the model is globally valid.

The BVEPS positively influences the price and the NIPS negatively. A variation of 1 unit of the BVEPS causes a variation of 1.174 units in the price assuming that all the rest remains constant. A variation of 1 point in the NIPS causes a variation of 0.149 points in the price but in the opposite direction, assuming that all the rest remains constant.

The Swedish sample is composed of 1782 observations, of which 13 were excluded, according the econometric suggestions.

Observing the results of the other authors, we see that the conclusions are very different from the econometric suggestions method.

For authors 1, the dependent variables explain 62.9% of price variation and for authors 2 it only explains 48.6%.

Observing all authors' results, the model is globally valid. As such the BVEPS is statistically significant. For author 2, the variable NIPS is not statistically significant.

Examining the β_{BVEPS}, it is evident that the methods used by the other authors lead to very different results. The econometric suggestions method presents the lowest value. For author 2 the price varies by 1.391 points.

Observing the behaviour of the variable NIPS, the conclusions between the authors are more accentuated. For author 2, the variable influences the price negatively as in the econometric suggestions method, although it is the expected variation of 0.248 units on price (dependent variable) per unitary variation on BVEPS, assuming that all the rest remains constant and in the econometric suggestions method only it is expected a variation of 0.149 units. The results of the other authors are even more distant because for author 1 the variable NIPS positively influences the behaviour of the price. And, finally, for author 3 this variable is not statistically significant.

Authors 1, 2 and 3 exclude 94, 60 and 13 observations, respectively.

Applying the White procedures, the BVEPS continues to have the same results. However, the results of the variable NIPS experience some changes. For author 3 and the econometric suggestions method this variable is no longer statistically significant.

5. Conclusion

A correct study of influential observations is crucial to perform a good study. The application of a non-scientific method can generate incorrect results and consequently incorrect conclusions of the investigation.

Regarding our method, we have reached the conclusions presented below about the Ohlson model.

On average, the BVEPS and NIPS influence 72% of the price variation, so these variables have a significant impact on price.

The price is more influenced by the BVEPS and NIPS in Denmark, Belgium and France by 97.9%, 90.4% and 83.5%, respectively.

The countries where the variables have a lower impact in the price are Greece, Sweden and the UK. Notwithstanding that in these countries the value of the adjusted R^2 is 47.3%, 52.5% and 59.9%, respectively, which we can consider also has a great impact.

In Ireland the BVEPS has a bigger influence on the price than in the other countries and in Austria has the lowest influence. Nevertheless, the BVEPS value does not show a significant difference between the countries.

Regarding NIPS, we observe that in Germany, the Netherlands, Sweden and the UK this variable tends to negatively influence the price. However, in Germany and Sweden the corresponding estimated coefficient is not statistically significant. The NIPS has a higher influence on the price in Denmark and this variable varies between -0.628 and 5.622.

The country that contains more influential is France, followed by Germany and the country with fewer observations is Austria, followed by Sweden, but we have to note the lower sample size in Austria.

Comparing our method that results from econometric suggestions with the other exclusion methods, we can perfectly observe the different results obtained from the same sample.

In all methods and for all countries, the significance of the F-test is always lower than 0.05, which means that at least one of the variables influences the variation of the price significantly.

Mostly, the adjusted R-square presents a higher value according to the econometrics suggestion.

The method that has a result closer to ours is the method used by authors 1 with 68.6%. For these authors the countries with a large impact of Olshon's model are Belgium, Denmark and the Netherlands and with a lower impact, the UK, Greece and Finland. The Netherlands and Finland are not on our list of the countries with a higher and lower influence, respectively.

Observing the results of BVEPS we conclude that they are also distorted. For authors 1, the UK is where this variable presents a lower value, for author 2 it is Denmark and for author 3 it is Austria, as in the econometric suggestions methods but with a higher value than in the econometric suggestions method. This variable assumes a higher expression in Germany for all authors.

In general, NIPS is the variable that is more times not statistically significant and when we apply the White procedure, the number of variables that are statistically significant doubles. Despite this, the results of author 1 demonstrate that there are no statistically significant variables.

According to the econometric suggestions method, in the Netherlands the variable NIPS is statistically significant for the model and negatively influences the price, for the rest of the authors this does not happen; the behaviour of the variable positively influences the price. The same thing happens with the UK, although only for authors 1 the variable influences positively the price.

Finally, we observe that author 1 excludes more observations, 1,127 observations; author 2 and author 3 exclude 826 and 183 respectively. Therefore, author 1 excludes many observations that are not influential, like authors 2 but to a lesser degree. Author 3 maintains in the sample influential observations.

We cannot conclude about the authors that have results more similar to ours because it depends on the subject because some authors have results more similar in NIPS, others in BVEPS. But one thing is definitely known, the results of the authors are different from ours that are based on econometric suggestions to exclude influential observations.

The objective of this dissertation was completed. We conclude that without an econometric study of the observations, the results of the investigations can be complete different and conduce to misleading conclusions.

In this dissertation have only been studied three methods used in empirical accounting studies by three different authors. It is important, for a future perspective, more analysis of other methods that are used by other authors. It could be also important to extend this study to other accounting themes that not only Ohlson regression model, to analyse how the estimated models are being modelling.

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Appendix

Appendix Index

Appendix 1 - The notation and respective meaning.

Notation	***Meaning***
$y, \hat{y}$	$n \times 1$ vector of dependent variable, where the elements are observed and estimated values, respectively.
$y_i, \hat{y}_i$	$n \times 1$ vector of dependent variable, where the elements are observed and estimated values, respectively, excluding the *ith* observation.
X	$n \times k$ $(n > k)$ matrix of predictors (explanatory variables) possibly including a constant predictor; so the number of explanatory variables in the model is $p = k - 1$
$\beta, \hat{\beta}$	$k \times 1$ vector of unknown coefficients (parameters) to be estimated and estimated, respectively.
ε, e	$n \times 1$ vector of random disturbances and residuals, respectively.
$X_{i,}$ X_j	The notation $_{(i)}$ or $_{(j)}$ is used to indicate the omission of the *ith* observation or *jth* variable, respectively.
σ^2, s^2	Error variance and the respective estimator.
s_i^2	Estimator of σ^2 when excluded de *ith* line of **X** and **y**.

Appendix 2 – Companies per Country included in the sample.

Country	*N.º of Companies*	*N.º of Company year observations*	*%*
Austria	130	527	2%
Belgium	167	754	3%
Denmark	215	1.043	4%
Finland	156	795	3%
France	970	4.263	17%
Germany	985	3.836	16%
Greece	348	1.578	6%
Ireland	95	387	2%
Italy	336	1.581	6%
Netherlands	231	1.062	4%
Portugal	79	348	1%
Spain	190	886	4%
Sweden	385	1.795	7%
UK	2.166	5.785	23%
Total	**6.453**	**24.640**	**100%**

Appendix 3 - Results of Ohlson model in the UK.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.462	0.312	0.365	0.599
F-test	2389.403	1267.817	1660.488	4300.951
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	1.736	3.630	15.02330	4.446
Constant(t-test)	23.271	6.189	1.938	9.686
β_{BVEPS}	0.923	1.274	1.487	1.158
BVEPS(t-test)	69.014	47.557	56.153	87.428
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.024	-1.746	-0.330	-0.214
NIPS (t-test)	0.434	-15.890	-5.815	-11.606
Significance of NIPS (t-test)	0.664	0.000	0.000	0.000
Number of observations	5558	5599	5778	5759
Number of observations excluded	227	186	7	26

Appendix 4 - Results of Ohlson model in the UK according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.462	0.312	0.365	0.599
F-test	2389.403	1267.817	1660.488	4300.951
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	1.736	3.630	15.023	4.446
Constant(t-test)	19.980	3.656	0.396	4.432
β_{BVEPS}	0.923	1.274	1.487	1.158
BVEPS(t-test)	31.496	12.170	5.912	11.163
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.024	-1.746	-0.330	-0.214
NIPS (t-test)	0.171	-2.346	-1.290	-2.622
Significance of NIPS (t-test)	0.864	0.019	0.197	0.009
Number of observations	5558	5599	5778	5759
Number of observations excluded	227	186	7	26

Appendix 5 - Results of Ohlson model in France.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.693	0.574	0.705	0.835
F-test	4583.160	2772.261	5045.483	10570.359
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	23.639	43.340	47.912	30.859
Constant(t-test)	27.791	11.518	11.635	14.317
β_{BVEPS}	0.705	0.981	1.067	1.048
BVEPS(t-test)	61.376	54.057	85.150	125.533
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	2.324	2.036	0.866	1.286
NIPS (t-test)	24.565	12.915	8.504	17.242
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	4070	4117	4223	4184
Number of observations excluded	193	146	40	79

Appendix 6 - Results of Ohlson model in France according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.693	0.574	0.705	0.835
F-test	4583.160	2772.261	5045.483	10570.36
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	23.639	43.340	47.912	30.859
Constant(t-test)	20.882	8.090	11.206	9.982
β_{BVEPS}	0.705	0.981	1.067	1.048
BVEPS(t-test)	22.674	11.996	27.331	30.144
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	2.324	2.036	0.866	1.286
NIPS (t-test)	10.108	3.085	2.412	5.887
Significance of NIPS (t-test)	0.000	0.002	0.016	0.000
Number of observations	4070	4117	4223	4184
Number of observations excluded	193	146	40	79

Appendix 7 - Results of Ohlson model in Italy.

	Authors 1	***Authors 2***	***Authors 3***	***Econometrics suggestions***
$\overline{R}^2$ ($Adjusted\ R^2$)	0.693	0.559	0.504	0.680
F-test	1694.562	968.674	803.684	1637.658
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	4.209	6.601	8.690	4.346
Constant(t-test)	9.428	4.830	5.897	15.368
β_{BVEPS}	1.357	1.573	1.450	1.142
BVEPS(t-test)	55.570	42.358	39.270	55.891
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.790	0.714	0.253	0.065
NIPS (t-test)	5.089	2.986	1.306	0.627
Significance of NIPS (t-test)	0.000	0.003	0.192	0.531
Number of observations	1504	1526	1578	1541
Number of observations excluded	77	55	3	40

Appendix 8 – Results of Ohlson model in Italy according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.693	0.560	0.504	0.680
F-test	1694.562	968.674	803.684	1637.658
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	4.209	6.601	8.690	4.346
Constant(t-test)	1.338	1.069	1.613	2.105
β_{BVEPS}	1.357	1.573	1.450	1.142
BVEPS(t-test)	26.257	17.143	9.647	21.161
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.790	0.714	0.253	0.065
NIPS (t-test)	3.251	1.862	0.763	0.377
Significance of NIPS (t-test)	0.001	0.063	0.445	0.706
Number of observations	1504	1526	1578	1541
Number of observations excluded	77	55	3	40

Appendix 9 - Results of Ohlson model in Austria.

	Author 1	***Author 2***	***Author 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.703	0.560	0.546	0.728
F-test	598.140	325.957	311.908	701.822
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	20.603	34.871	34.999	39.927
Constant(t-test)	11.877	12.000	13.005	13.985
β_{BVEPS}	0.527	0.173	0.245	0.186
BVEPS(t-test)	15.786	9.744	8.326	16.561
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.736	3.391	1.869	0.753
NIPS (t-test)	5.704	11.532	5.374	5.267
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	505	512	517	524
Number of observations excuded	22	15	10	3

Appendix 10 - Results of Ohlson model in Austria according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.703	0.560	0.547	0.728
F-test	598.140	325.957	311.908	701.822
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	20.603	34.871	34.999	39.927
Constant(t-test)	11.895	12.759	9.610	14.754
β_{BVEPS}	0.527	0.173	0.245	0.186
BVEPS(t-test)	7.107	2.995	2.181	6.039
Significance of BVEPS(t-test)	0.000	0.003	0.030	0.000
β_{NIPS}	1.736	3.391	1.869	0.753
NIPS (t-test)	2.226	2.648	1.965	2.446
Significance of NIPS (t-test)	0.026	0.008	0.050	0.015
Number of observations	505	512	517	524
Number of observations excluded	22	15	10	3

Appendix 11 - Results of Ohlson model in Belgium.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.902	0.871	0.782	0.904
F-test	3262.152	2450.204	1300.608	3447.090
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	69.082	157.285	81.197	91.692
Constant (t-test)	3.576	1.168	5.956	5.366
β_{BVEPS}	0.660	0.650	0.561	0.611
BVEPS(t-test)	40.877	27.038	23.856	34.588
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	3.249	5.124	2.992	2.580
NIPS (t-test)	14.466	18.783	13.694	14.189
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	713	727	726	733
Number of observations excluded	41	27	28	21

Appendix 12 - Results of Ohlson model in Belgium according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.902	0.871	0.782	0.904
F-test	3262.152	2450.204	1300.608	3447.090
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	69.082	157.285	81.197	91.692
Constant (t-test)	4.081	1.756	6.793	9.421
β_{BVEPS}	0.660	0.650	0.561	0.611
BVEPS(t-test)	7.035	6.053	5.138	10.144
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	3.249	5.124	2.992	2.580
NIPS (t-test)	3.164	3.159	3.602	4.245
Significance of NIPS (t-test)	0.002	0.002	0.000	0.000
Number of observations	713	727	726	733
Number of observations excluded	41	27	28	21

Appendix 13 - Results of Ohlson model in Denmark.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.889	0.482	0.718	0.979
F-test	4012.154	471.181	1281.184	23932.420
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	164.232	1111.845	150.941	283.599
Constant (t-test)	5.501	-1.691	4.768	-1.182
β_{BVEPS}	0.944	0.044	0.934	0.814
BVEPS(t-test)	47.499	0.403	29.754	41.852
Significance of BVEPS(t-test)	0.000	0.687	0.000	0.000
β_{NIPS}	1.766	18.254	2.133	5.622
NIPS (t-test)	8.447	21.751	10.000	50.201
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	1003	1010	1009	1032
Number of observations excluded	40	33	34	11

Appendix 14 - Results of Ohlson model in Denmark according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.889	0.482	0.718	0.979
F-test	4012.154	471.181	1281.184	23932.42
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	164.232	1111.845	150.941	283.599
Constant (t-test)	31.964	-1.319	4.304	-0.914
β_{BVEPS}	0.944	0.044	0.934	0.814
BVEPS(t-test)	12.758	0.076	14.446	9.905
Significance of BVEPS(t-test)	0.000	0.939	0.000	0.000
β_{NIPS}	1.766	18.254	2.133	5.622
NIPS (t-test)	3.845	2.044	5.556	10.701
Significance of NIPS (t-test)	0.000	0.041	0.000	0.000
Number of observations	1003	1010	1009	1032
Number of observations excluded	40	33	34	11

Appendix 15 - Results of Ohlson model in Finland.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.618	0.571	0.645	0.756
F-test	607.371	510.480	721.370	1167.796
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	3.869	4.730	4.975	3.424
Constant (t-test)	11.333	9.594	10.069	11.375
β_{BVEPS}	0.610	0.600	0.788	0.878
BVEPS(t-test)	14.244	11.522	19.022	29.386
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	4.404	5.177	3.449	2.162
NIPS (t-test)	15.534	15.236	12.572	10.974
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	752	766	794	753
Number of observations excluded	43	29	1	42

Appendix 16 - Results of Ohlson model in Finland according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.618	0.571	0.645	0.756
F-test	607.371	510.480	721.370	1167.796
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	3.869	4.730	4.975	3.424
Constant (t-test)	12.527	10.631	10.523	11.358
β_{BVEPS}	0.610	0.600	0.788	0.878
BVEPS(t-test)	10.374	10.034	5.756	13.595
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	4.404	5.177	3.449	2.162
NIPS (t-test)	10.024	9.043	9.676	5.054
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	752	766	794	753
Number of observations excluded	43	29	1	42

Appendix 17 - Results of Ohlson model in Germany.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.685	0.598	0.457	0.635
F-test	3971.655	2761.640	1600.503	3280.758
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	54.515	107.216	145.105	106.612
Constant (t-test)	2.931	-1.249	6.307	18.159
β_{BVEPS}	1.977	2.634	2.010	0.810
BVEPS(t-test)	73.743	54.126	51.927	78.797
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.988	4.973	0.779	-0.237
NIPS (t-test)	12.670	17.963	7.569	-10.140
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	3646	3705	3808	3771
Number of observations excluded	190	131	28	65

Appendix 18 - Results of Ohlson model in Germany according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.685	0.599	0.457	0.635
F-test	3971.655	2761.640	1600.503	3280.758
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	54.515	107.216	145.105	106.612
Constant (t-test)	2.343	-1.005	4.424	10.622
β_{BVEPS}	1.977	2.634	0.779	0.810
BVEPS(t-test)	20.185	14.413	2.636	6.742
Significance of BVEPS(t-test)	0.000	0.000	0.008	0.000
β_{NIPS}	1.988	4.973	2.010	-0.237
NIPS (t-test)	4.480	5.751	10.503	-1.420
Significance of NIPS (t-test)	0.000	0.000	0.000	0.156
Number of observations	3646	3705	3808	3771
Number of observations excluded	190	131	28	65

Appendix 19 - Results of Ohlson model in Greece.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.521	0.310	0.503	0.473
F-test	822.157	343.930	798.867	686.586
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	2.847	4.497	4.918	2.921
Constant (t-test)	12.379	9.417	18.756	28.672
β_{BVEPS}	0.687	0.673	0.386	0.256
BVEPS(t-test)	16.006	10.020	8.830	8.459
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	6.756	6.928	5.058	4.366
NIPS (t-test)	28.002	18.450	18.741	21.697
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	1510	1527	1576	1526
Number of observations excluded	68	51	2	52

Appendix 20 - Results of Ohlson model in Greece according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.521	0.310	0.503	0.473
F-test	822.157	343.930	798.867	686.586
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	2.847	4.497	4.918	2.921
Constant (t-test)	1.458	8.142	7.534	9.384
β_{BVEPS}	0.687	0.673	0.386	0.256
BVEPS(t-test)	10.555	8.854	2.170	1.877
Significance of BVEPS(t-test)	0.000	0.000	0.030	0.061
β_{NIPS}	6.756	6.928	5.058	4.366
NIPS (t-test)	13.620	12.778	6.804	8.531
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	1510	1527	1576	1526
Number of observations excluded	68	51	2	52

Appendix 21 - Results of Ohlson model in Ireland.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.740	0.688	0.684	0.795
F-test	526.330	410.393	416.044	706.882
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	2.246	2.533	3.085	1.961
Constant (t-test)	5.996	5.655	3.044	2.984
β_{BVEPS}	0.862	0.931	1.487	1.623
BVEPS(t-test)	12.825	12.345	23.156	31.821
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	4.655	4.143	1.229	0.864
NIPS (t-test)	11.796	9.413	5.439	5.796
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	369	372	385	366
Number of observations excluded	18	15	2	21

Appendix 22 - Results of Ohlson model in Ireland according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.740	0.688	0.684	0.795
F-test	526.330	410.393	416.044	706.882
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	2.246	2.533	3.085	1.961
Constant (t-test)	7.032	6.709	2.408	3.896
β_{BVEPS}	0.862	0.931	1.487	1.623
BVEPS(t-test)	6.470	6.490	7.717	17.741
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	4.655	4.143	1.229	0.864
NIPS (t-test)	6.030	4.664	2.031	2.683
Significance of NIPS (t-test)	0.000	0.000	0.043	0.008
Number of observations	370	372	385	366
Number of observations excluded	18	15	2	21

Appendix 23 - Results of Ohlson model in Netherland.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.831	0.743	0.740	0.728
F-test	2453.699	1487.187	1502.498	1400.668
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	7.804	10.370	19.959	11.542
Constant (t-test)	18.041	15.790	2.805	15.976
β_{BVEPS}	0.813	0.842	1.245	0.903
BVEPS(t-test)	58.216	46.665	54.807	49.887
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.240	1.040	0.012	-0.628
NIPS (t-test)	10.210	7.525	0.141	-14.470
Significance of NIPS (t-test)	0.000	0.000	0.888	0.000
Number of observations	1002	1027	1055	1046
Number of observations excluded	60	35	7	16

Appendix 24 - Results of Ohlson model in Netherland according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.831	0.743	0.740	0.728
F-test	2453.699	1487.187	1502.498	1400.688
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	7.804	10.370	19.959	11.542
Constant (t-test)	16.235	16.563	0.980	15.111
β_{BVEPS}	0.813	0.842	1.245	0.903
BVEPS(t-test)	31.624	31.572	8.169	32.780
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.240	1.040	0.012	-0.628
NIPS (t-test)	6.987	3.796	0.029	-3.076
Significance of NIPS (t-test)	0.000	0.000	0.977	0.002
Number of observations	1002	1027	1055	1046
Number of observations excluded	60	35	7	16

Appendix 25 - Results of Ohlson model in Portugal.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.489	0.230	0.342	0.716
F-test	160.132	51.327	90.809	420.590
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	4.044	7.656	7.713	3.394
Constant (t-test)	6.022	4.090	5.060	7.395
β_{BVEPS}	0.710	0.769	0.755	0.756
BVEPS(t-test)	12.574	7.231	11.044	24.717
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.508	1.472	0.910	0.862
NIPS (t-test)	5.797	3.014	2.478	5.174
Significance of NIPS (t-test)	0.000	0.003	0.014	0.000
Number of observations	333	338	347	334
Number of observations excluded	15	10	1	14

Appendix 26 - Results of Ohlson model in Portugal according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.489	0.230	0.342	0.716
F-test	160.132	51.327	90.809	420.590
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	4.044	7.656	7.713	3.394
Constant (t-test)	6.533	5.011	6.384	12.023
β_{BVEPS}	0.710	0.769	0.755	0.756
BVEPS(t-test)	7.811	7.358	15.209	21.658
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	1.508	1.472	0.910	0.862
NIPS (t-test)	3.481	2.393	2.045	2.388
Significance of NIPS (t-test)	0.001	0.017	0.042	0.018
Number of observations	333	338	347	334
Number of observations excluded	15	10	1	14

Appendix 27 - Results of Ohlson model in Spain.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ (*Adjusted* R^2)	0.748	0.532	0.537	0.731
F-test	1257.376	485.252	509.358	1180.953
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	5.969	10.485	10.789	6.322
Constant (t-test)	10.472	5.058	6.917	11.774
β_{BVEPS}	0.510	0.943	1.017	0.707
BVEPS(t-test)	12.314	13.445	16.464	18.190
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	6.781	4.407	2.564	4.596
NIPS (t-test)	21.966	8.352	6.340	17.802
Significance of NIPS (t-test)	0.000	0.000	0.000	0.000
Number of observations	847	853	879	869
Number of observations excluded	39	33	7	17

Appendix 28 - Results of Ohlson model in Spain according the White procedures.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.748	0.533	0.459	0.731
F-test	1257.376	485.252	375.825	1180.953
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	5.969	10.485	14.203	6.322
Constant (t-test)	8.249	4.586	2.371	10.229
β_{BVEPS}	0.510	4.407	1.010	0.707
BVEPS(t-test)	5.705	2.687	2.839	9.711
Significance of BVEPS(t-test)	0.000	0.007	0.005	0.000
β_{NIPS}	6.781	0.943	1.624	4.596
NIPS (t-test)	11.242	3.607	1.310	8.335
Significance of NIPS (t-test)	0.000	0.000	0.190	0.000
Number of observations	847	853	886	869
Number of observations excluded	39	33	7	17

Appendix 29 - Results of Ohlson model in Sweden.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.629	0.486	0.540	0.525
F-test	1439.769	821.122	1048.231	986.540
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	27.977	42.430	45.856	45.226
Constant (t-test)	16.911	11.064	13.030	15.028
β_{BVEPS}	1.214	1.391	1.266	1.174
BVEPS(t-test)	52.070	40.369	45.499	43.679
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.251	-0.053	-0.248	-0.149
NIPS (t-test)	4.709	-0.719	-6.201	-4.124
Significance of NIPS (t-test)	0.000	0.472	0.000	0.000
Number of observations	1701	1735	1782	1782
Number of observations excluded	94	60	13	13

Appendix 30 - Results of Ohlson model in Sweden according the White procedure.

	Authors 1	***Authors 2***	***Authors 3***	***Econometric suggestions***
$\bar{R}^2$ ($Adjusted\ R^2$)	0.629	0.486	0.540	0.525
F-test	1439.769	821.122	1048.231	986.540
Significance of F test	0.000	0.000	0.000	0.000
St error of estimate	27.977	42.430	45.856	45.226
Constant (t-test)	16.994	10.095	8.371	11.105
β_{BVEPS}	1.214	1.391	1.266	1.174
BVEPS(t-test)	31.242	19.213	14.651	16.826
Significance of BVEPS(t-test)	0.000	0.000	0.000	0.000
β_{NIPS}	0.251	-0.053	-0.248	-0.149
NIPS (t-test)	2.714	-0.227	-1.871	-1.034
Significance of NIPS (t-test)	0.007	0.820	0.062	0.301
Number of observations	1701	1735	1782	1782
Number of observations excluded	94	60	13	13