ISCTE Business School
Instituto Universitário de Lisboa

STRUCTURAL CREDIT RISK MODELS: ANALYSIS OF LISTED COMPANIES IN PORTUGAL

Inês Pereira Santos

Dissertation submitted as partial requirement for the conferral of

Master in Finance

Supervisor:

Prof. José Carlos Dias, Professor Associado com Agregação,
ISCTE Business School, Department of Finance

September 2018

ISCTE Business School
University Institute of Lisbon

STRUCTURAL CREDIT RISK MODELS: ANALYSIS OF LISTED COMPANIES IN PORTUGAL

Inês Pereira Santos

Abstract

Under the trend of financial globalization and considering the significant problems experienced by companies and banks during the Global Financial Crisis, the interest in credit risk measurement and management has increased substantially during the last decade.

This study empirically investigates the structural credit risk approach, initiated with the seminal studies of Black and Scholes (1973) and Merton (1974), which regard corporate securities as contingent claims on a firm's underlying assets. Throughout this study, we analyse and implement three structural credit risk models – the original Merton model (1974) and two commercial extensions of this model, the KMV model and the CreditGrades model – in order to evaluate the default probabilities of 8 listed Portuguese companies – EDP, Galp, Jerónimo Martins, Sonae, Nos, Cofina, Media Capital and Teixeira Duarte – during the years 2013 to 2017.

The obtained results suggest that the annual default probabilities determined by the three structural credit risk models are considerably different, taking into consideration the several refinements made by the KMV and CreditGrades models to the original Merton model. These two commercial extensions attempt to produce a more realistic output, that better reflects real-world default dynamics. In fact, the discrepancy of results produced by the 3 structural models intensifies, as the default probability determined by the structural approach, for a given company, also increases. Moreover, among the 3 structural credit risk models considered in this study, CreditGrades is considered the most reliable model as well as the one that displays the highest default probabilities, followed by the KMV model.

JEL Classification: G33, C53

Keywords: Credit risk, Default probability, Structural models, Portuguese listed compaies

Resumo

Tendo em conta a tendência de globalização financeira e as recentes dificuldades económicas experienciadas, quer por empresas quer por bancos, durante a Grande Crise Financeira, tem-se verificado, durante a última década, um crescente interesse na mensuração e gestão do risco de crédito.

Assim, este estudo tem como objetivo investigar a abordagem estrutural de risco de crédito, iniciada com os estudos de Black and Scholes (1973) e Merton (1974), que trata os títulos corporativos como reivindicações contingentes sobre os ativos subjacentes de uma empresa. Ao longo deste estudo são analisados e implementados três modelos estruturais – o modelo original de Merton (1974) e duas extensões comerciais deste modelo, o modelo KMV e o modelo CreditGrades – de modo a avaliar as probabilidades de incumprimento de 8 empresas portuguesas cotadas – a EDP, Galp, Jerónimo Martins, Sonae, Nos, Cofina, Media Capital e Teixeira Duarte – ao longo dos anos de 2013 até 2017.

Os resultados obtidos sugerem que as probabilidades anuais de incumprimento, determinadas pelos três modelos estruturais de risco de crédito, são consideravelmente diferentes, tendo em conta os diversos refinamentos realizados pelos modelos KMV e CreditGrades, ao modelo original de Merton. Estas duas extensões do modelo de Merton tentam produzir um resultado mais credível e realista, isto é, que melhor reflita as dinâmicas do mundo financeiro. De facto, é possível observar que a discrepância de resultados apresentados pelos 3 modelos aumenta, à medida que a probabilidade de incumprimento determinada pela abordagem estrutural também aumenta. Para além disso, de entre os 3 modelos estruturais de risco de crédito abordados neste estudo, o modelo de CreditGrades é considerado o mais credível, sendo também o que apresenta maiores probabilidades de incumprimento. Seguindo-se posteriormente, o modelo KMV.

Acknowledgements

This project is the last step to conclude the master's degree and I would like to express my sincere gratitude to all of those who have directly or indirectly contributed to this realization.

Firstly, I would like to gratefully thank my supervisor, Prof. José Carlos Dias, for his guidance and valuable time. His continuous support, motivation, useful comments and help throughout this long and challenging journey, were very important to successfully complete this work.

To my family, a big thank you! Especially to my parents, for all the patience, constant encouragement and love. For given me the good references and all the support to finish one more step in my academic life.

Finally, I would like to thank to all my friends and colleagues from work, that supported me throughout this journey. Their friendship, tolerance, comprehension, help and advices, were all very important and meaningful for me.

Table of Contents

Abstract I

Resumo II

Acknowledgements III

List of figures iii

List of tables iv

List of abbreviations v

1. Introduction 1

2. Review of literature 3

2.1. Default and credit risk 3

2.2. Default risk models 4

3. Structural credit risk models 8

3.1. The Merton model 8

3.1.1. Corporate liabilities as contingent claims 8

3.1.2. Model assumptions 10

3.1.3. Model setup 11

3.1.4. The implied credit spread of risky debt 12

3.1.5. Probability of default 13

3.2. The Moody's KMV approach 14

3.2.1. Measuring default probability 14

3.2.2. Estimate asset value and volatility 16

3.2.3. Calculate the distance-to-default 18

3.2.3.1. Non-financial firms 19

3.2.3.2. Financial firms 20

3.2.4. Compute the probability of default 20

3.2.5. The term structure of default risk 22

3.3. The CreditGrades model 24
3.3.1. The Model description 24
3.3.2. Survival probabilities 26
3.3.2.1. Approximated survival probability 26
3.3.2.2. Exact survival probability 27
3.3.3. Calibrating model parameters 28
4. Data and methodology 30
4.1. Data description 30
4.2. Methodology 32
4.2.1. Merton model 32
4.2.2. KMV model 33
4.2.3. CreditGrades model 35
5. Empirical results 37
5.1. The calculations and results 37
5.2. Sensitive analysis 42
5.3. Firm analysis 43
5.3.1. PSI-20 Index companies 43
5.3.2. Other listed companies 45
6. Conclusion 48
7. Annexes 55

List of figures

Figure 1.: Frequency distribution of a firm's asset value at the horizon of time H and probability of default, Crosbie and Bohn (2003).. 15

Figure 2.: From DD to PD: empirical mapping vs. the normal distributional assumption, Sun et al. (2012). 21

Figure 3.: Default risk over multiple time periods, Sun et al. (2012)............................. 23

Figure 4.: Model description, Finger et al. (2002). 26

List of tables

Table 1:. Companies profile 31
Table 2:. The results for structural models 38
Table 3:. Mean of the probabilities of default 39
Table 4:. Sensitive analysis 42
Table 5:. PSI-20 Index weighting 43
Table 6:. Default results: PSI-20 companies 44
Table 7:. Default results: other listed companies 45

List of abbreviations

ATM	At-the-Money
BS	Black and Scholes
BSM	Black-Scholes-Merton
CAPM	Capital Asset Pricing Model
CDS	Credit Default Swaps
CEDF	Cumulative Expected Default Frequency
DD	Distance-to-Default
DPT	Default Point
EDF	Expected Default Frequency
EU	European Union
FEDF	Forward Expected Default Frequency
IMF	International Monetary Fund
LTD	Long-term Debt
PSI	Portuguese Stock Index
PD	Probability of Default
STD	Short-term Debt
VK	Vasicek-Kealhofer
ZCB	Zero-Coupon Bond

1. Introduction

Default risk measurement and management have gained increasing prominence over the years and become one of the most important topics in finance today. The growth of off-balance-sheet derivatives, the declining and volatile collateral values and the increased number of bankruptcies, particularly during the financial crisis of 2007-2009, all have contributed to this increased importance.

In fact, the financial crisis of 2007-2009, which has also struck heavily the Portuguese economy, gave rise to an unprecedented number of bankruptcies in financial and non-financial institutions, many of which having had to be bailed-out with taxpayer's money.

Moreover, the new regulatory requirements, such as the Basel Accord, which incentives financial institutions to quantitatively measure and manage the risks associated to its corporate debt portfolios, also induce the interest in this financial subject.

In this sense, structural credit risk models appear as an integrated approach to measuring default risk, with potential for widening the scope and flexibility of risk management systems.

The structural approach is rooted in the seminal works of Black and Scholes (1973) and Merton (1974), which regard corporate securities as contingent claims on a firm's underlying assets. Afterward, many refinements and extensions to the original Merton (1974) model have been made and applied.

Then, the main purpose of this thesis is to apply the structural credit risk approach to evaluate the default probability of listed non-financial companies. In this setting, the analysis of three models – the original Merton (1974), and two extensions of this model, the KMV model and the CreditGrades model – is proposed and presented according to its temporal evolution.

These models are applied to eight listed Portuguese non-financial companies – EDP, Galp, Jerónimo Martins, Sonae, Nos, Cofina, Media Capital and Teixeira Duarte – covering utilities, oil, retail, media and construction sectors, from 2013 to 2017. The purpose of this study is to further analyze and compare the annual default measures from the three structural models.

From an academic point of view, this thesis aims to contribute to the state-of-art research in structural credit risk models, based on an analysis of the existing literature review and comparison of the models under study. From an empirical point of view, it aims to provide relevant information about the credit risk situation of the underlying companies, which were selected taking into consideration their dimension, importance, and influence in the Portuguese economy.

The structure of the thesis is divided as follows. The introduction chapter explains the background of the subject at hand.

Sections two and three present a general discussion about default risk, default risk models and the importance of credit risk modeling. It also provides a detailed description of structural credit risk approach, in particularly the theoretical background of the three models addressed in this study and previous empirical findings.

Section four describes the data and the methodology used in the implementation of the models and examines the parameters setting. The empirical results and comments are discussed in chapter five. Finally, chapter six concludes the thesis and gives directions for further research.

2. Review of literature

2.1. Default and credit risk

The risk of default can be defined, according to Crosbie and Bohn (2003), as the uncertainty surrounding a firm's ability to accomplish its debts and obligations in due time. As stated by Crosbie and Bohn (2003), default is considered a deceptively rare event and although a typical firm has about 2% probability of default in any year, there are considerable variations in default probabilities across firms.

Therefore, in order to compensate the lenders for that uncertainty and, knowing that loss suffered by a lender in the event of default is generally significant, firms are usually required to pay a spread over the default-free interest rate, that is directly proportional to the firm's probability of default.

Default risk measurement and management have then become one of the most important research topics in finance today, since the only way to distinguish the firms that will default from those that will not, it is to make probabilistic assessments of the likelihood of default.

In fact, default risk can be effectively managed in a portfolio, as well as other high cost rare events. The portfolio management of default risk requires, besides the default probability and loss given default, the measurement of default correlations, which stands for the degree in which the default risks of the several counterparties and borrowers are related in the portfolio. As pointed out by Crosbie and Bohn (2003), the basic elements of credit risk can be grouped into the standalone risk (where the asset is considered in isolation) and portfolio risk (where the asset is held as one of a number of assets in a portfolio) as follows:

- **Standalone Risk**:
    - Default probability: which is the probability of the borrower or counterparty fail in servicing its obligations;
    - Loss given default: the amount of the loss incurred in the event of default;
    - Migration risk: which is the value impact and probability of changes in default probability.
- **Portfolio Risk:**

- Exposure: the proportion of the portfolio that is exposed to the default risk of each counterparty or borrower;
- Default correlations: the degree to which the default risks of the counterparties and borrowers are related in the portfolio.

Being each of these items critical to the management of credit portfolios, default probability is possibly the most important and difficult to determine. Actually, default risk is one of the main risks faced by the banking industry and so a variety of theoretical models attempting to measure the probability of default have been recently produced and explored in the financial world.

2.2. Default risk models

During the last few years, under the trend of financial globalization and continuous innovation, default risk has become the key challenge of risk management and received considerable attention in both financial industry and academia.

The increased number of insolvencies at an international level over the past decade, followed by the recent Global Financial Crisis and instability of national monetary systems, have even increased the need for rigorous and accurate measures in the calculation of capital requirements and credit risks.

As stated by Laajimi (2012), the increasing interest in the credit risk modelling is also due to two main reasons. Firstly, according to the purposes set out in the Capital Accord 2006, known as Basel II, large banks can determine their capital requirements using an internal assessment of the probability of default of counterparties instead of the more constraining standardized model. Second, the huge increase of the securitization of loans and the development of new off-balance-sheet derivatives demanded more developed credit analysis methods.

Although the complexity of reality might not be perfectly described in a model, modelling provides useful guidance and assessments of the likelihood of default. Hence, in order to prevent future financial disasters and promote the stability and harmonious development of the global economy, a variety of theoretical default risk models have been recently produced and investigated in the academic literature.

There are two conceptually different approaches to assess the default risk: the structural approach and the reduced-form approach.

The structural approach is rooted in the pioneering work of Merton (1974), that uses the principles of option pricing framework of Black and Scholes (1973). This approach regards corporate securities as contingent claims on a firm's underlying assets, where the firm's asset value is assumed to follow a geometric Brownian motion and its capital structure is simple composed by a zero-coupon debt and common equity. This structural approach yields formulas for the value of both equity and risky corporate bond, as well as the default probability of the firm.
All credit risk models based on Merton's (1974) approach are known as structural models since the default risk is tied to the firm's value process and its capital structure.

The alternative approach to credit risk is the reduced-form approach, originally introduced by Jarrow and Turnbull (1992). This approach uses a default intensity, which is specified exogenously, to reflect the default probabilities. The default intensity is not correlated to any firm-specific variable and is the same for all bonds in the relevant credit risk class. In this case, the default event is unexpected since it can occur without any correlation with the firm value.

Both structural and reduced-form approaches are widely applied in credit risk analysis and they both have their own advantages and disadvantages. The main difference between the two approaches is that the structural model approach suggests that credit spread dynamics are a function of management decisions concerning the firm's capital structure.

Reduced-form models offer mathematical tractability and have achieved reasonable success at valuing defaultable debt, as reported for example in the studies of Duffee (1999) and Bakshi et al. (2001). However, since they do not attempt to explain credit spreads through a firm capital structure theory, reduced-form models are then considered poorer in their implications.

The structural credit risk approach is conceptually elegant and attractive on theoretical grounds, since it links the valuation of debt to the financial condition of the firm. Furthermore, structural models can also deal with yields and default probabilities at the same time, and for each firm, it derives an individual default probability.

However, the original Black-Scholes-Merton (1974) model relies on some shortcomings and unrealistic assumptions, including, for example, the requirements of a simple capital structure, where there is only one class of debt outstanding in the form of a zero-coupon bond, the interest rates are constants and the default can only occur at the maturity of the debt.

As a result, in order to overcome such weaknesses and make structural credit risk models to represent the corporate debt and the default process in a more realistic way, many refinements and extensions to the original Merton (1974) model have been made.

The extensions to the Black-Scholes-Merton (1974) model have gone into two main directions, the barrier models that began with Black and Cox (1976) and the compound option approach of Geske (1977).

In the Black and Cox (1976) model, the default event is defined as a barrier option, so that it can occur any time, before and up to maturity of the debt. Therefore, the firm defaults when the asset value falls below a specified barrier, which may be determined exogenous or endogenously within the model. In the original Black and Cox (1976) model it is used an exponential barrier and recovery is a fixed portion of the discounted face value of the debt. Later on, the model was also extended and some modifications were given by Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), Leland (1994) and Leland and Toft (1996).

The other class of structural credit risk models is the compound option approach, which was originally proposed by Geske (1977). It was noticed that when there are multiple cash flows, in the form of coupon payments, the equity is a compound call option (a call option on a call option) on the firm's assets and then each cash flow is a strike price for the compound option. In the compound option framework, the default event is endogenously determined at every cash flow, so there is a default probability associated with each cash flow over time. As a result, a complete term-structure of default probabilities may be generated by this model.

According to the studies of Jones et al. (1983), structural credit risk models do not perform so well in generating credit spreads and in terms of pricing risky debt, however they have been successful in using the market information to determine default probabilities and predict corporate default. Structural models offer a link between capital structure and asset valuation theory and are considered central to credit risk management practice and financial theory.

A practical implementation of the original BSM structural model, widely used as a benchmark in academic studies as well as in industry, is the KMV model. As described by Crosbie and Bohn (2003), multiple classes of liabilities are modelled, and the default point is determined as a linear combination of short-term and long-term debt. The KMV model uses the market information to produce two outputs, the Distance to Default (DD) and Expected Default Frequency (EDF), which provide forward-looking estimates of the default risk.

In 2002, it was jointly developed by Deutsche Bank, Goldman Sachs, J.P. Morgan, and the RiskMetrics Group, another alternative practical implementation of the standard structural model - the CreditGrades model. This model was initially proposed by Finger et al. (2002) and it belongs to the class of structural credit risk models, since it is inspired on the first passage time structural model of Black and Cox (1976). Its main difference from the classic structural approach is that the default barrier is assumed to be random, so that the default event can occur before maturity T (if the value of company assets hits the default barrier).

Due to its theoretical attractiveness, this dissertation examines the family of structural credit risk models and applies its methodology to a selected group of Portuguese listed companies, which are evaluated according to its probabilities of default. This study incorporates an empirical analysis of the original Merton (1974) model and compares its predictive accuracy against two representative theoretical extensions of this model: the KMV model and CreditGrades model.

3. Structural credit risk models

3.1. The Merton model

The literature on structural credit risk models was initiated by Merton (1974), who applies the Black and Scholes (1973) option pricing model to value corporate liabilities.

In practice, since corporate liabilities can be regarded as combinations of simple option contracts, option pricing models can also be used to price the several elements of the firm's capital structure. The application of the contingent claims analysis in corporate finance problems began when it was first recognized that the payoff structures of simple call and put option strategies are identical to the structures of risky pure discount debt and the equity of a levered firm.

Therefore, option pricing models can be used to price corporate securities, since the structure of corporate liabilities is viewed as contingent claims on the value of the firm.

3.1.1. Corporate liabilities as contingent claims

The Merton (1974) model is considered the simplest equilibrium model to price corporate debt since it makes some simplifying assumptions about the firm’s capital structure and bankruptcy procedure. The model assumes a simple capital structure, whereas the firm has only two classes of claims: a single class of debt, compared to a pure discount bond or zero-coupon bond (ZCB) with no coupon payments, and equity, which is composed by common stock. Hence,

$$V_t = E_t + D_t, \tag{1.1}$$

with V_t , E_t and D_t being, respectively, the asset value, the equity value and the debt value. The firm promises to pay the face value of the debt for the ZCB, a total amount of X, at the maturity date T of the firm's debt. If at the maturity date T, the face value of the debt (X) cannot be paid, the firm declares bankruptcy and the bondholders will take over the firm’s assets while the shareholders will receive nothing. The bankruptcy event is then interpreted as a transfer of the firm ownership from the stockholders to the debtholders.

As pointed out by Black and Scholes (1973) and Merton (1973), the firm’s equity can be viewed as a European call option on the firm’s value. Consequently, the equityholders may be viewed as having an option to buy back the firm by paying, to the debtholders, an exercise price equal to the face value of the firm's debt (X) at its maturity date T.

The put-call parity relationship can be applied to illustrate the claims of equityholders and bondholders, where it is considered that the payoffs of the firm's value, V_t, plus a put option written on it, P_t, are equivalent to the payoffs from a default-free zero coupon bond, X_t, plus a call option on the risky asset which represents the equity of a levered firm ($C_t \equiv E_t$):

$$V_t + P_t = X_t + E_t \tag{1.2}$$

$$\Leftrightarrow V_t = E_t + (X_t - P_t). \tag{1.3}$$

The value of the firm can then be divided into two claims. A higher-risk claim, which represents the shareholders' equity and it is equivalent to a call option on the firm's value with an exercise price equal to the face value of debt X and maturity date T, and a lower-risk claim, which represents the risky corporate debt and it is equivalent to a default-free debt minus a European put option with an exercise price equal to the debt's promised principal X and maturity date T (equal to the maturity of the corresponding risky debt).

According to equation (1.1), the risky debt can be defined as:

$$D_t = X_t - P_t. \tag{1.4}$$

Moreover, using the put-call parity relationship, notice that the bondholders' claim is equivalent to having the ownership of the entire firm and a short position in a European call option on the value of the firm, that is:

$$D_t = V_t - C_t = V_t - E_t. \tag{1.5}$$

The equityholder’s and bondholder’s payoff at maturity date are, respectively, defined as:

$$E_T = max(V_T - X, 0) = (V_T - X)^+ \tag{1.6}$$

and

$$D_T = min(V_T, X) = X - max(X - V_T, 0) = X - (X - V_T)^+. \tag{1.7}$$

If, at the maturity date T of the firm's debt:

- $\mathbf{V_T > X}$**:** Equityholders will pay, to bondholders, the face value of the bond and equity will have a positive value;
    - The debt will be paid off: $D_T = X$;

    - Equityholders will retain the excess value: $V_T - X$.
- **$V_T < X$:** Equity is worthless, then shareholders file for bankruptcy and bondholders will retain a value smaller than the promised payment X:
    - Equity is worthless: $E_t = 0$;
    - Debtholders will keep a value: $D_T = V_T < X$.

In fact, it is possible to notice that stockholders have protection against downside risk, in case of the depreciation of the firm's value below X, and have a right to the appreciation in the firm's value above X. The limited liability nature of equity explains this asymmetry between downside risk and upside potential.

3.1.2. Model assumptions

In order to valuing corporate liabilities as contingent claims using the pricing model of Merton (1974), the following assumptions are required:

1. There are no transaction costs or taxes, and all investors have free access to all available information.
2. Assets are perfectly divisible and trading takes place continuously in time with no restrictions on short selling of all assets.
3. There are sufficient investors in the market place with comparable wealth levels so that each investor can buy and sell as much of an asset as he wants, at the market price.
4. There is no limit for borrowing or lending, at the same rate of interest.
5. The Proposition I of Modigliani and Miller (1958), which states that, in the absence of corporate income taxes and other market imperfections, the market value of a firm is unaffected by its capital structure, is assumed to be valid.
6. There are no bankruptcy costs, and it occurs when the market value of the firm's assets is lower than the face value at the maturing debt.
7. The term structure is flat and known with certainty, that is, the price of a risk-free discount bond that promises to pay 1 dollar at time T is given by:

$$P(r, t, T) = e^{-r(T-t)}, \tag{1.8}$$

   where r is the instantaneous riskless rate of interest.

8. The dynamics for the value of the firm through time, are assumed to follow a geometric Brownian motion:

$$dV_t = \mu\,(V_t - \bar{P})\mathrm{dt} + \sigma V_t \mathrm{d}W_t^P, \tag{1.9}$$

where V is the value of the firm's assets, $\bar{P}$ is the net value of cash distributions and $\mathrm{d}W_t^P$ is a standard Gauss-Wiener process under P. The variables μ and σ^2 are assumed constants and represent respectively the instantaneous expected rate and variance of the firm's return per unit of time. The yield curve is flat at a constant risk-free rate r.

3.1.3. Model setup

Under the geometric Brownian motion assumption and considering a simple capital structure (where debt is composed by a zero-coupon bond and equity by common stock), the equity value of a levered firm at time-t is given by:

$$E_t = E_t^Q\left[e^{-r(T-t)}(V_T - X)^+\right] = V_t\,N(d_1) - X\,e^{-r(T-t)}\,N(d_2), \tag{1.10}$$

in which d_1 and d_2 are respectively given by:

$$d_1 = \frac{\ln\left(\frac{V_t}{X}\right) + \left(r + 0.5\,\sigma^2\right)(\mathrm{T-t})}{\sigma\sqrt{\mathrm{T-t}}} \tag{1.10a}$$

$$d_2 = \frac{\ln\left(\frac{V_t}{X}\right) + \left(r - 0.5\,\sigma^2\right)(\mathrm{T-t})}{\sigma\sqrt{\mathrm{T-t}}} = d_1 - \sigma\sqrt{T-t}\,, \tag{1.10b}$$

where:

- E_t: Equity's market value;
- X: Liabilities book value;
- V_t: Asset's market value;
- $T - t = \tau$: Maturity;
- r: Risk-free interest rate;
- $N(d_i)$ stands for the standard Gaussian cumulative distribution function.

Therefore, issuing bonds is identical to the shareholders selling to the bondholders all the firm's assets for the value of the issue plus a call option to repurchase that assets from the bondholders, with an exercise price equal to the face value of the corporate bonds.

Recalling that the value of the debt is the value of the firm minus the value of equity ($D_t = V_t - E_t$) and using the Merton (1974) pricing model as defined in equation (1.10) for valuing equity, the value of a risky discount corporate bond at time-t is given by:

$$D_t = V_t \, N(-d_1) + X \, e^{-r(T-t)} \, N(d_2). \tag{1.11}$$

The value of risky debt can also be presented as the price of a riskless bond (with a face value equal to the face value of the corporate debt) minus the price of a BS European put option, P_t, written on the value of the firm:

$$D_t = Xe^{-r(T-t)} - P_t = X_t - P_t. \tag{1.12}$$

Since the value of the firm is the sum of equity and debt values, it follows that:

$$V_t = E_t + Xe^{-r(T-t)} - P_t. \tag{1.13}$$

This last equation represents the put-call parity, which is equivalent to say that:

$$Assets = Equity + (PV(face\ value\ of\ debt) - \text{"Risk Premium"})$$

$$= Equity + Debt. \tag{1.14}$$

However, some firms fail to service their debt obligations and consequently default. Since prior to default, it is difficult to distinguish the firms that will default from those that will not, firms are required to pay a spread over the default-free interest rate. This spread is obviously an increasing function of the firm default's probability in order to compensate the lenders for the risk.

3.1.4. The implied credit spread of risky debt

When trying to model credit spreads it is usual to consider yields, since corporate bonds commonly have cash flow streams similar to those of treasury bonds. Considering that the yield to maturity on the risky corporate debt, provided by a firm that does not default, is $y(t,T)$, then:

$$X = D_t e^{y(t,T)(T-t)}. \tag{1.15}$$

Therefore, the yield at date t of a bond with maturity at time T is given by:

$$y(t,T) = \frac{1}{T-t} \ln \frac{X}{D_t}. \tag{1.16}$$

The difference between the yield on a defaultable bond and the yield of a corresponding treasury bond is called as the yield spread or credit spread and is denoted as:

$$s(t,T) = y(t,T) - r = \frac{1}{T-t} \ln\frac{X}{D_t} - r. \tag{1.17}$$

An alternative way to measure the riskiness of a bond is by its instantaneous return's standard deviation. While the credit spread measures the promised risk premium over the outstanding bond life, the bond's standard deviation measures the risk over the next instant of time relative to the risk of the firm, which is defined as:

$$\sigma_D = \frac{\partial D}{\partial V}\frac{V}{D}\sigma_V = V_t N(-d_1)\frac{V}{D}\sigma_V \equiv \eta_D \sigma_V, \tag{1.18}$$

where σ_D and σ_V are respectively the instantaneous standard deviation of the return on the bond and on the firm, and η_D measures the relative riskiness of the bond in terms of the riskiness of the firm at a given time, and it is given by:

$$\eta_D = N(-d_1)\frac{V}{D} = \frac{VN(-d_1)}{VN(-d_1)+Xe^{-r(T-t)}N(d_2)} = \frac{1}{1+\frac{X}{V}e^{-r(T-t)}\frac{N(d_2)}{N(-d_1)}}. \tag{1.19}$$

3.1.5. Probability of default

The probability of default of a given firm is the probability that the value of the assets of the firm will be less than the book value of the liabilities of the firm, which is equivalent to say that:

$$PD(t,T) = Prob(V_t \le X|V_t) = Prob(\ln(V_t) \le \ln(X)\,|V_t). \tag{1.20}$$

The real-world distance to default (DD), under the Merton (1974) model, can be defined as:

$$DD = \frac{\ln\left(\frac{V_t}{X}\right)+\left(\mu-\frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_V\sqrt{T-t}}. \tag{1.21}$$

Finally, under the Merton (1974) model, the risk-neutral probability of default is simply given by:

$$PD(t,T) = Prob(V_t \le X|V_t) = N(-DD) = N(-d_2) = 1 - N(d_2). \tag{1.22}$$

3.2. The Moody's KMV approach

The practical implementation of the Merton (1974) model has received considerable attention in recent years.

Oldrich Vasicek and Stephen Kealhofer, the founders of the KMV Corporation – a firm specialized in credit risk analysis that was acquired in April 2002 by Moody's to form the Moody's KMV – have extended the BSM (1973, 1974) framework to produce the Vasicek-Kealhofer (VK) model – a model of default probabilities.

As stated by Crosbie and Bohn (2003), this model assumes that the firm's equity is a perpetual option, with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point (DPT), the firm is assumed to default.

According to Sun et al. (2012), the KMV model is an extensive modification of the original Merton's approach, since it incorporates more realistic assumptions and empirical observations that better reflect real-world default dynamics. This is the big advantage of this model: it provides both up-to-date view of a firm's value and a timely warning of changes in credit risk.

The KMV model best applies to publicly traded companies for which the value of equity is market determined, however this model can also be modified and applied to firms without publicly traded equity.

3.2.1. Measuring default probability

The main output of the KMV model is the Expected Default Frequency (EDF) credit measure, which is the default probability of a given obligor, during the forthcoming year, or years.

The determination of the EDF value requires as inputs, the equity prices and certain items from financial statements. The probability of default is then computed as a function of the firm´s capital structure, the volatility of the asset returns and the current asset value. The EDF is specific for each firm and can be mapped into any rating system to derive the equivalent rating of the obligor.

According to Crosbie and Bohn (2003), there are three main elements for determining the default probability of a firm:

- **The market value of the firm's assets:** the present value of the future free cash flows produced by the assets of the firm discounted back at the appropriate discount rate. This incorporates relevant information about the firm's industry and measures the firm's prospects.
- **Asset Risk**: the uncertainty or risk of the asset value, which measures the firm's business and industry risk. Since the value of the firm's assets is an estimate, it is uncertain and, thus, it should always be understood in the context of the asset risk or firm's business.
- **Leverage**: the extent of the firm's contractual liabilities. A pertinent measure of the firm's leverage is the book value of liabilities relative to the market value of assets, since that represents the amount the firm must repay.

The firm´s probability of default increases as the current market value of the firm's assets decreases, the volatility of the firm's assets increases or the amount of liabilities increases. The firm defaults when the market value of the assets is insufficient to repay the liabilities, which happens when the market value of the firm's assets falls below the default point.

Therefore, the firm's default probability is the probability that the asset value will fall below the default point, represented by the Black area (EDF value) below the default point in Figure 2.

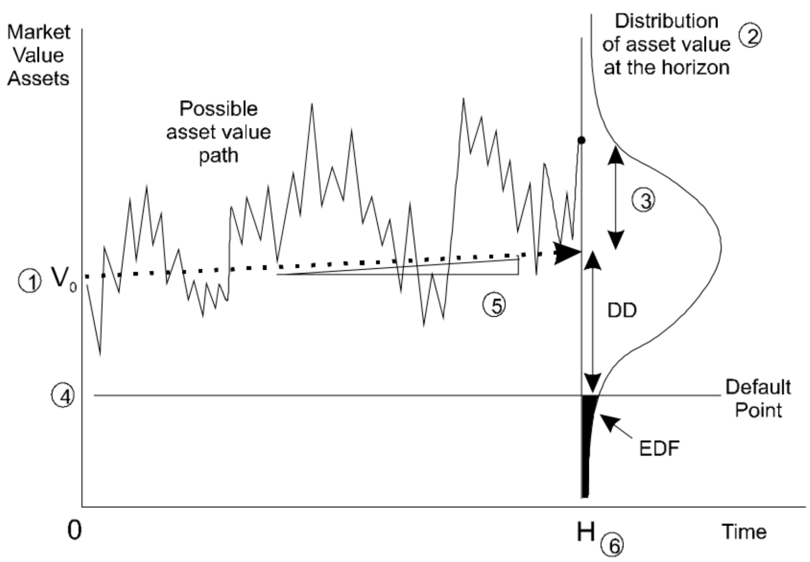


Figure 1.: Frequency distribution of a firm's asset value at the horizon of time H and probability of default, Crosbie and Bohn (2003)

Crosbie and Bohn (2003) found that in general many firms do not default when their asset value reaches the book value of their total liabilities. Usually, the default point lies somewhere between total liabilities and current liabilities.

As pointed out in the KMV model, the default probability of a firm is essentially determined through the following three steps:

1. **Estimate asset value and asset volatility of the firm**: from the book value of liabilities and market value and volatility of equity;
2. **Calculate the distance-to-default (DD):** from the book value of liabilities and the asset value and asset volatility (estimated in the first step);
3. **Calculate the default probability**: from the DD and the rate of default

3.2.2. Estimate asset value and volatility

Determining the firm's asset value and their volatility would be straightforward if all liabilities of the firm were traded and marked-to-market every day. As stated by Crouhy et al. (2000), the firm's asset value would be simply the sum of the market values of the firm's liabilities (equity + debt), and the volatility of the asset return could be basically derived from the historical time series of the reconstituted assets value.

However, in practice, not all firm's debt is traded and only the price of equity is directly observable for most firms, so that it is not possible to directly observe the market value of the firm. Therefore, alternative approaches may be used in order to perform and implement the Moody's KMV model, being two of them the non-linear system of equations approach and the iterative approach.

According to Crosbie and Bohn (2003), whenever the market price of equity is available, the market value of the firm and volatility of assets can be determined directly using the BSM option pricing model, which identifies equity as a call option on the underlying assets of the firm, with a strike price equal to the debt's value and the same debt's maturity, defined by:

$$E_t = V_t\, N(d_1) - X\, e^{-r\tau}\, N(d_2), \tag{2.1}$$

in which d_1 and d_2 are given by equations (1.10a) and (1.10b), respectively.

Furthermore, the volatility of the firm's assets (σ_V) is directly related with the volatility of equity (σ_E) through the following equation:

$$\sigma_E = \sigma_V \, N(d_1) \, \frac{V_t}{E_t}. \quad (2.2)$$

Hence, these two equations can be used to determine the time-t value of assets (V_t) and its volatility (σ_V), since the market value of equity is observable, and the equity volatility can be estimated.

This non-linear system of equations approach was used, for instance, by Jones et al. (1984) and Ogden (1987) for estimating the asset volatility. However, since d_1 and d_2 depend both on the two unknown values (asset value V_t and volatility σ_V), the solution of this system of equations is non-trivial and a numerical solution should be performed using a routine based on the Newton-Raphson algorithm (e.g in Excel or Matlab).

Nevertheless, this approach may not provide reasonable results, since in practice the market leverage changes too much for equation (2.2) to capture this market leverage dynamics and the models may bias the default probabilities in the wrong directions.

Thus, the calculation of the default probabilities through the non-linear system of equations approach may not provide a good discriminatory power in most of the cases.

Due to the limitations pointed to the previous approach, it is commonly preferable to use an iterative method to calculate the asset volatility σ_V, such as the one proposed by Crosbie and Bohn (2003) and Vassalou and Xing (2004). This alternative approach of getting asset value and volatility is relatively recent and it has proven to be very useful for predicting default's probabilities.

Firstly, the iterative approach uses an initial guess of the volatility to determine the asset value and to de-lever equity returns. As suggested by Vassalou and Xing (2004), one can use daily data from the past 12 months (e.g. 252 trading days) to obtain an estimate of the historical volatility of equity σ_E, which is then used as an initial value for the estimation of σ_V. Another alternative, suggested in Löffler and Posch (2011, Chapter 2), is to create a vector of asset prices V_{t-a}, for a = 0, 1, ..., 252, which is set as the sum of the market value of equity E_{t-a} (typically considered as the market capitalization) and the book value of liabilities X_{t-a} (typically considered the debt in one year plus half the long-term debt). Afterwards, the initial value for the estimation of σ_V is settled as the standard deviation of the log asset returns calculated with the V_{t-a} vector. These two approaches are considered equivalent, for practical purposes.

By rearranging the Black and Scholes (1973) formula, the asset value of the firm can be defined as:

$$V_t = \frac{E_t + X\, e^{-r(T-t)}\, N(d_2)}{N(d_1)}. \tag{2.3}$$

Using formula (2.3) for each trading day, the next step is to compute the asset value V_{t-a}, using E_{t-a} (as the market value of equity) and X_{t-a} (as the book value of the firm's liabilities) of each day t−a, with a maturity equal to T.

In this manner, the daily values for V_{t-a} are obtained, and it is now possible to compute the standard deviation of this new V_{t-a}vector, which will be then used as the value of σ_V for the next iteration. This procedure is repeated until the values of σ_V from two consecutive iterations converge.

After a few iterations, when σ_V value is obtained, the asset value (V_t) can be easily found through equation (2.3).

Then, the drift rate μ can be obtained by calculating the mean of the log asset returns of the final V_{t-a} vector, or alternatively it can be derived using the estimated asset values V_{t-a} and the CAPM.

3.2.3. Calculate the distance-to-default

The Distance-to-Default is the intermediate phase implemented in the KMV model, before computing the default probabilities.

According to Crosbie and Bohn (2003), the firm is assumed to default when the value of its assets hits the default point (DPT), which represents the amount of a firm's liabilities due at a given time horizon, that if not paid in time according to their contractual terms, would cause the firm’s default.

In the basic structural model of Merton (1974), since it is assumed a simple capital structure where equity is composed by common stock and debt is composed by a ZCB, the determination of the default point is straightforward. It is the face value of that ZCB with a maturity equal to the default horizon and, thus the potential default event can occur only at the maturity date of the ZCB when $V_T < X$. However, in practice, the potential default events are not limited to a specific maturity date, since the liabilities of the firms are usually comprised of multiple classes of debt with several maturities. Thus, due to its simplicity, the default probability model of Merton (1974) would not produce an optimal estimate of the default point for default prediction purposes.

Hence, in order to produce a more realistic model, the Moody's KMV model extends the original structural model of Merton (1974) by allowing a more realistic capital structure. In the current Moody's KMV model, multiple classes of liabilities are modeled, such as: short-term and long-term liabilities, preferred and common equity and convertible debt.

In their studies of defaults, Crosbie and Bohn (2003) have found that while some firms certainly default when their asset value reaches the book value of their total liabilities, or even before, many other firms continue to trade and service their debts.

In fact, the estimation of the default points is not an easy **t**ask, since there is no uniform template to describe all firm's liability structures. To overcome these problems, Moody's KMV employ separate algorithms for determining the default points of non-financial firms and financial firms.

3.2.3.1. Non-financial firms

Using a sample of several hundred of firms, Moody's KMV observed that, in case of non-financial firms, the default point – the asset value at which the firm will default – generally lies somewhere between total liabilities and or short-term debt.

Therefore, for a one-year time horizon, the critical threshold for default (DPT) is set at 100% of short-term liabilities (STD) plus 50% of long-term liabilities (LTD), that is:

$$DPT = STD + 0.5LTD. \qquad (2.4)$$

This tries to capture the idea that soon, the short-term debt requires a repayment of the principle while the long-term debt requires only the coupon payments.

The distance-to-default (DD) is then defined by:

$$DD = \frac{V_t - D^*}{\sigma_V V_t}. \qquad (2.5)$$

KMV measures the DD as the number of standard deviations the asset value is away from default and combines three key credit issues: the market value of the firm's assets, the industry and business risk, and the firm's leverage.

In the current Moody's KMV model, the distance-to-default is determined as follows:

$$DD = \frac{\ln V_t + (\mu - 0.5\,\sigma_V{}^2)\tau - \text{Payouts} - \ln D^*}{\sigma_V \sqrt{\tau}}. \tag{2.6}$$

In this equation, μ represents the asset's expected growth rate and the payouts represent the asset drainage through cash flows until T (and common dividends and debt coupons). As in general, μ is not easy to estimate, an alternative is to use a unique μ per industry or sector which would be easier to estimate.

3.2.3.2. Financial firms

According to Sun et al. (2012), in the case of financial firms, it is difficult to distinguish the long-term from the short-term liabilities. Therefore, Moody's KMV defines the critical threshold for default as a percent of total adjusted liabilities, depending on the subsector (e.g. investment banks, commercial banks, and nonbank financial institutions).

The critical threshold for default is then defined by:

$$DPT = x\% * Total\ Debt. \tag{2.7}$$

However, when analyzing default's probabilities for horizons longer than one year, Moody's KMV calibrate the default points to match project debt maturities, in order that obligations with longer maturities have higher weights in the calculation of the default point. This calibration is important since as time passes, long-term debt will become short-term liabilities as well as the expected growth rate of asset values will have an increasing impact on the long-term DD of the firm.

3.2.4. Compute the probability of default

Finally, the last phase of the Moody's KMV approach consists on mapping the DD to the actual default probabilities, recognized as Expected Default Frequencies. The EDF measures the default probability within a given period.

In the simple structural credit risk model, the DD follows a normal distribution since the Brownian motion assumption is used to model the dynamics of asset values. However, the predictions of this model depart significant from the actual default event. Instead, Moody's KMV obtained the relationship between distance-to-default and default probability, from a large sample of data on historical default and bankruptcy frequencies. As stated in Sun et al. (2012, pg. 15):

"The EDF model constructs the DD-to-PD mapping based on the empirical relationship (i.e., the relationship evidenced by historical data) between DDs and observed default rates."

Based on their database, which includes over 250,000 company-years of data and over 4,700 events of default or bankruptcy, it was generated a frequency table relating the likelihood of default to several levels of distance-to-default.

Moody's KMV has examined the relationship between distance-to-default and frequency of default for industry, size and time, among other effects, and has found that the relationship is constant across all of these variables.

In figure 2.2, it is plotted a stylized version of the DD-to-EDF mapping (the green line), along with the DD-to-PD mapping (the orange line) implied by a normal distribution.

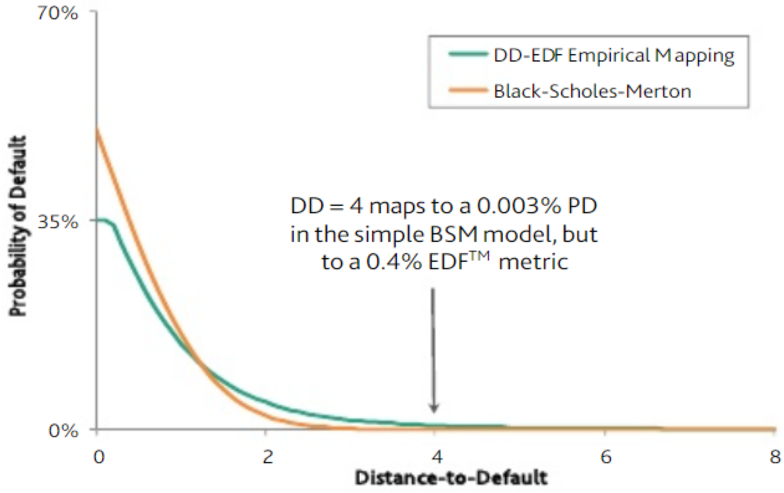


Figure 2.: From DD to PD: empirical mapping vs. the normal distributional assumption, Sun et al. (2012).

The empirical mapping of the DD-to-PD mapping provides a more realistic estimation of the default probability when a firm's net worth is close to zero, and moreover it accommodates the existence of "jumps-to-default", that may occur when the value of the firm assets experiences a sharp decline or a sudden collapse of their business environment.

3.2.5. The term structure of default risk

The Expected Default Frequencies model estimates the EDF for each firm, not only for a one-year horizon, but also a term structure of EDF measures at horizons of up to ten years.

According to Sun et al. (2012), the EDF term structure can be expressed in several ways, similar to the term structure of interest rates. The construction of EDF term structure starts with the cumulative t-year EDF ($CEDF_t$). The $CEDF_t$ represents the probability of default any time before and up to the end of year t.

Thus, the survival probability through year t can be determined as follows:

$$\text{Survival probability} = 1 - CEDF_t. \quad (2.8)$$

The annualized survival probability between time 0 and t, assuming a constant survival probability from a given year to the following one, is given by:

$$\text{Survival probability}_t = (1 - CEDF_t)^{\frac{1}{t}}. \quad (2.9)$$

Thus, the annualized t-year EDF is directly related to the $CEDF_t$ through the following equation:

$$EDF_t = 1 - (1 - CEDF_t)^{\frac{1}{t}}. \quad (2.10)$$

In the binomial tree of Figure 4, it is represented the timeline of defaults and the corresponding terms for default probabilities.

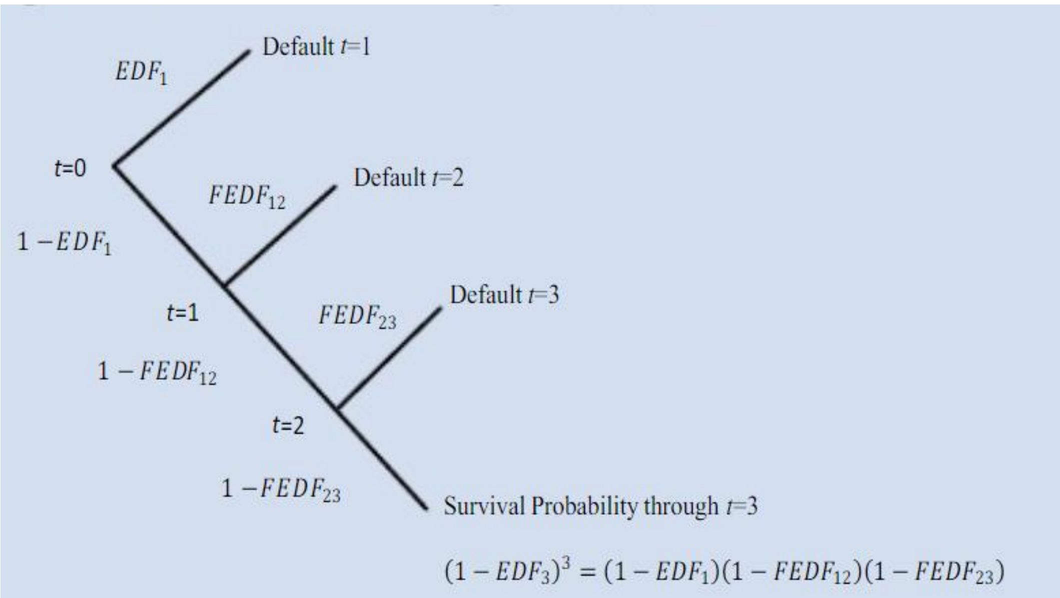


Figure 3.: Default risk over multiple time periods, Sun et al. (2012).

As stated by Sun et al. (2012), it is possible to recognize the similarity between the terminology used for the term structure of interest rate and the one used in this model. The forward EDF between time $t - 1$ and t ($FEDF_{t-1,t}$), can be defined as the default's probability between $t - 1$ and t conditional on survival until $t - 1$, such that the survival probability through time-t using this conditional information equates unconditional probability of survival implied by $CEDF_t$, that is:

$$(1 - CEDF_{t-1})(1 - FEDF_{t-1,t}) = 1 - CEDF_t. \tag{2.11}$$

For example, the firm's 2-year cumulative EDF ($CEDF_2$) can be determined by chaining together its first-year EDF and its forward EDF in year two (expected second-year EDF), as follows:

$$CEDF_2 = 1 - (1 - EDF_1)(1 - FEDF_{1,2}). \tag{2.12}$$

The annualized two-year EDF is then given by:

$$EDF_2 = 1 - \left[(1 - EDF_1)(1 - FEDF_{1,2})\right]^{\frac{1}{2}}. \tag{2.13}$$

3.3. The CreditGrades model

Another extension and practical implementation of Merton (1974) and Black and Scholes (1973) model, is the CreditGrades model proposed by Finger et al. (2002). This structural credit risk model was jointly developed by Goldman Sachs, J.P. Morgan, Deutsche Bank and the RiskMetrics Group, to create a benchmark model in the credit risk markets.

As a structural model, it also assumes that both debt and equity values of the firm can be viewed as options on the value of a firm's assets. However, its main difference from the classic structural approach is that the default barrier is assumed to be random, which allows the model to overcome the problem of low short-term spreads in the classical structural models, that have been heavily criticized.

According to Finger et al. (2002), the model employs approximations for the asset value, volatility and drift terms, which relate these quantities with the market observable quantities. Moreover, the CreditGrades model provides simple closed-form solutions that related the pricing of credit default swaps (CDS) to the equity price and equity volatility.

As stated by Byström (2005), the CreditGrades model is a simplified version of Merton (1974) model, in which the probability of default is only a function of asset volatility and leverage ratio.

3.3.1. The Model description

In the CreditGrades model, the dynamics for the firm's value are also assumed to follow a geometric Brownian motion:

$$dV_t = \mu V_t \mathrm{dt} + \sigma V_t \mathrm{d}W_t^P, \tag{3.1}$$

where V is the value of the firm's assets, μ is the expected continuously compounded expected rate of return on the firm's assets, σ is the volatility of the assets and W_t^P is a standard Brownian motion under P. To maintain a constant firm's leverage ratio over time, the model sets the drift rate μ to zero.

Several were the limitations pointed to the Merton (1974) model, being one of them to consider that asset value evolves by pure diffusion and the default barrier is fixed. This assumption produces unrealistic short-term credit spreads.

Hence, in order to overcome this limitation, Finger et al. (2002) introduces a random default barrier. The default barrier is defined as the amount of the firm's assets that remain in the case of default, and it is simply the recovery value that the debt holders receive. The recovery rate is different depending on the industry sector and the firm economic situation.

In the CreditGrades model, the default occurs at the first time that the asset price crosses the random default barrier B_t, which is given by:

$$B_t := \mathrm{DL}, \tag{3.2}$$

where D is the debt-per-share and L is the recovery rate. The recovery rate L is assumed to follow a lognormal distribution, with mean $\bar{L}$ and percentage standard deviation λ. The recovery rate is then given by:

$$L = \bar{L}e^{\lambda Z - \lambda^2/2}, \tag{3.3}$$

where λ and $L \in R^+$, and Z is a standard normal random variable, independent of the Brownian motion W_t^P. The value of Z is only revealed at the time of default (unknown at time t = 0). The expected value $\bar{L}$ and variance λ^2 of the random variable, are respectively defined as:

$$\bar{L} = \mathbb{E}[L] \tag{3.4}$$

and

$$\lambda^2 = \mathbb{V}[\ln(L)]. \tag{3.5}$$

Consequently, the random default barrier can be defined as:

$$B_t := \mathrm{D}\bar{L}e^{\lambda Z - \lambda^2/2}. \tag{3.6}$$

The CreditGrades model assumes a random recovery rate, and therefore a random default barrier, which allows to capture the uncertainty in the actual level of a firm's debt-per-share. According to Finger et al. (2002), with an uncertain recovery rate, the default barrier can be hit unexpectedly, resulting in a jump-like default event.

This is the main difference between this model and the classical structural default risk models that usually consider a constant default barrier (which is the face value of debt) or an only time-dependent default barrier.

3.3.2. Survival probabilities

Considering the initial asset value V_0, the firm will not default if:

$$V_0 e^{\sigma W_t - \sigma^2 t/2} > \mathrm{D}\bar{L}e^{\lambda Z - \lambda^2/2}. \tag{3.7}$$

Figure 5 represents a graphical description of the CreditGrades model.

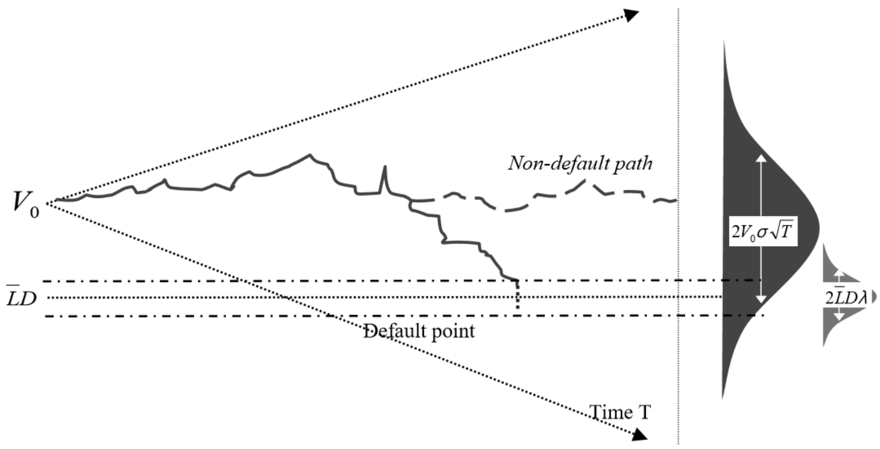


Figure 4.: Model description, Finger et al. (2002).

The survival probability of a firm can be determined through two alternative methods:

- Approximated Survival Probability;
- Exact Survival Probability.

3.3.2.1. Approximated survival probability

In the approximated survival probability, Finger et al. (2002) approximate the process X with a drifted-Brownian motion $\hat{X}$, with drift equal to - $\frac{\sigma^2}{2}$ and variance rate σ^2, which starts in the past at $-\Delta t = -\frac{\lambda^2}{\sigma^2}$ with $\hat{X}_{-\Delta t} = 0$. It is noticeable that for $t \geq 0$, the moments of $\hat{X}_t$ agree with the moments of X_t. Hence, with this approximation the uncertainty in the default barrier is replaced by the uncertainty in the level of the asset value at time 0. This approximation has minor impact, since it is the distance between the asset value and the default barrier that drives the model.

Using the distributions of Brownian motion for first hitting time, namely, for the following process $Y_t = a_t + bW_t$, with constant a and b, it is well-known that the succeeding formula is valid for every $y \leq 0$ (Musiela and Rutkowski (1998)):

$$\mathbb{P}(m_t^Y \geq y) = \mathbb{P}(Y_u \geq y, \forall\, 0 \leq u \leq t) = N\left(\frac{-y+\alpha_t}{b\sqrt{t}}\right) - e^{2ayb^{-2}}\, N\left(\frac{y+\alpha_t}{b\sqrt{t}}\right). \qquad (3.8)$$

Finger et al. (2002) applied this result to $\hat{X}$, by setting $a = -\frac{\lambda^2}{2}$, $b = \sigma$, $y = \ln\left(\frac{\bar{L}D}{V_0}\right) - \lambda^2$ and substitute t by $t + \frac{\lambda^2}{\sigma^2}$, in order to obtain a closed form formula for the survival probability up to time t, given by:

$$SP(t) = N\left(-\frac{\alpha_t}{2} + \frac{\ln(d)}{\alpha_t}\right) - dN\left(-\frac{\alpha_t}{2} - \frac{\ln(d)}{\alpha_t}\right), \qquad (3.9)$$

in which d and α_t^2 are defined as:

$$d = \frac{V_0 e^{\lambda^2}}{\bar{L}D} \qquad (3.9a)$$

and

$$\alpha_t^2 = \sigma^2 t + \lambda^2. \qquad (3.9b)$$

The survival probability given by equation (3.9) produces the counterintuitive result that there is a non-zero probability of default at t = 0, since it implicitly includes the possibility of default during the period $[-\Delta t, 0]$. Although this feature may be considered a technical problem of the modeling assumptions (specifically the lognormality of the default barrier), it aids in producing reasonable spreads for short (6 -month to 2-year) maturity instruments and in obtaining a simple formula for survival probability.

3.3.2.2. Exact survival probability

An alternative to the approximation of the process X with $\hat{X}$ (which does not contain the random variable Z), is to integrate out the random variable Z that follows a standard normal distribution.

A closed-form solution for determining the exact survival probability under the CreditGrades model was offered by Kiesel and Veraart (2008), to correct the initial formula given in Finger et al. (2002). The exact survival probability up to time t is then define as:

$$SP(t) = N_2\left(-\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{\alpha_t}{2} + \frac{\ln(d)}{\alpha_t}; \frac{\lambda}{\alpha_t}\right) - dN_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{\alpha_t}{2} - \frac{\ln(d)}{\alpha_t}; -\frac{\lambda}{\alpha_t}\right), \quad (3.10)$$

where d and α_t are given by equations (3.9a) and (3.9b), respectively, and N_2 is the cumulative bivariate normal distribution and it is given by:

$$N_2(a,b;\rho) = \int_{-\infty}^{a}\int_{-\infty}^{b} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\left(\frac{x^2-2\rho xy+y^2}{1-\rho^2}\right)\right) dxdy. \quad (3.11)$$

The CreditGrades model argues that for practical purposes, the numerical differences between the results given by the two approaches are marginal, however Kiesel and Veraart (2008) have demonstrated that in some circumstances the results may be significantly different, for example in the cases of highly leveraged firms.

To convert the survival probabilities of the CreditGrades model to a credit price, it is necessary to specify two additional parameters: the risk-free interest rate r and the recovery rate R on the underlying credit. Note that while $\bar{L}$ is the expected recovery averaged over all debt classes, R is the expected recovery on a specific class of a firm's debt. For an unsecured debt, the asset specific recovery R is generally lower than $\bar{L}$, as the secured debt will have a higher recovery.

3.3.3. Calibrating model parameters

In order to implement the CreditGrades model for determining the survival probability and the credit spread, there are several parameters that need to be calibrated.

Some variables are estimated from market data, such as the assets value of firm at the initial time t = 0, the volatility of the assets and the debt-per-share value.

The debt-per-share value is obtained from financial data of consolidated statements and it is computed by dividing the liabilities by the number of shares. In its turn, the number of shares used is simply the number of common shares plus the number of preferred shares and the financial debt is determined (for non-financial firms) as the sum of short-term and long-term borrowing, and one-half of the sum of other short-term and long-term liabilities. The inclusion of these two last items with a percentage factor of 50% attempts to correct for their inclusion of non-financial liabilities (e.g. deferred taxes or provisions).

Afterwards, the final debt is determined by reducing the financial debt by $\min[0.5 \times \text{financial debt}, \text{minority interest}]$, in order to adjust the effect of the liabilities of subsidiaries that are included in a consolidated balance sheet (even though the firm may own less than 100% of the subsidiary).

In this model, the distance-to default is determined from the Itô's lemma that relates the equity and asset volatilities through:

$$\sigma_E = \frac{\partial S_0}{\partial V_0}\left(\frac{V_0}{S_0}\right)\sigma_V. \tag{3.12}$$

The distance-to-default measure η, is defined as the number of annualized standard deviations separating the firm's current equity value from the default threshold, given by:

$$\eta = \frac{1}{\sigma_E}\ln\left(\frac{V_t}{\Lambda D}\right). \tag{3.13}$$

After analysing the behaviour of η near and far from the default threshold, Finger et al. (2002) stated that the initial value of the asset can be approximated by:

$$V_0 = S_0 + \bar{\Lambda}D. \tag{3.14}$$

Consequently, equation (3.12) can be rewritten and the asset volatility is set to:

$$\sigma_V = \sigma_E \frac{S_0}{S_0+\bar{\Lambda}D}. \tag{3.15}$$

This equation demonstrates that for a stable asset volatility, the equity volatility increases when the stock price decreases, and eventually reaches very high levels for a company at the brink of default.

Substituting equations (3.14) and (3.15) into equations (3.9a) and (3.9b), it is possible to conclude:

$$d = \frac{S_0+\bar{\Lambda}D}{\bar{\Lambda}D}e^{\lambda^2} \tag{3.16a}$$

and

$$\alpha_t^2 = \left(\sigma_E \frac{S_0}{S_0+\bar{\Lambda}D}\right)^2 t + \lambda^2. \tag{3.16b}$$

Consequently, these equations lead to a closed-form solution to determine the survival probability using equation (3.10).

The equity volatility, σ_E, can be estimated using a backward-looking volatility approach, from historical stock prices and the recovery rate $\bar{\Lambda}$ and its volatility λ can be based on statistics published by several rating agencies. The CreditGrades model proposes a recovery rate $\bar{\Lambda}$ = 0.50 and volatility λ = 0.30, for determining the survival probability.

4. Data and methodology

After a theoretical description of the structural approach in the previous sections, it follows a practical implementation of the models addressed in this study – the original Merton (1974) model and the two extensions, the KMV and CreditGrades model.

This section gives a descriptive overview of the sample and describes how the required data was collected. Further, it is discussed the methodology used for default probabilities calculations, according to each structural credit risk model.

4.1. Data description

In order to develop this study and apply the structural approach, we have chosen to assess the default probability of 8 Portuguese listed companies during the years 2013 to 2017. The main goal is to evaluate, not only the current default risk situation of the selected Portuguese companies, but also its evolution during the "post-financial crisis" period.

In fact, Portugal was one of the Eurozone member states most deeply affected by the financial crisis that hit Europe in 2008, with unemployment rising to 17% and the economy shrinking 4% in 2012. Due to the vulnerability of the Portuguese economy, financial markets began to become more apprehensive about the country's ability to fulfil its sovereign debt liabilities, so that in April 2011 Portugal was forced to follow Greece and Ireland in requesting external financial support. In May 2011, the International Monetary Fund (IMF) and the European Union (EU) approved a €78 billion bailout package. Along with the bailout, Portugal agreed to implement an economic adjustment program, which required to adopt austerity measures and implement several structural reforms.

The program, however, was controversial and painful for large fractions of the population, with high economic and social costs. In fact, Portugal experienced a deepened recession, as evidenced by record unemployment rates, lower nominal wages and the increased number of bankruptcies, caused by collapse in domestic demand.

However, as pointed out by the recent macroeconomic data, Portugal is recovering from the punishing debt crisis, and it has finally reached a turning point in its economic rehabilitation. A decisive moment in its recovery was for example the decision by Fitch Ratings to upgrade Portugal from junk status, in 2017 December.

Therefore, considering this recent evolution of the Portuguese economy, we have decided to choose 8 Portuguese companies, from different industries and dimensions, listed in the Portuguese Stock Index (PSI-all share) and assess them according to their probability of default, during the years 2013 to 2017.

In table 1, it is represented all the companies that were included in the analysis, a total of 8 companies, as well as the data regarding their respective supersector, number of outstanding shares and market capitalization.

Table 1:. Companies profile

Companies	***SuperSector***	***Index***	***Outstanding shares*** *31-12-2017*	***Market Capitalization*** *31-12-2017*
EDP	Utilities	PSI-20	3 656 537 715	10 485 911 563
Galp Energia-Nom	Oil & Gas	PSI-20	829 250 635	12 708 265 981
Jerónimo Martins	Retail	PSI-20	629 293 220	10 191 403 698
Sonae	Retail	PSI-20	2 000 000 000	2 252 000 000
Nos SGPS	Media	PSI-20	515 161 380	2 823 597 441
Cofina, SGPS	Media	PSI-all share	102 565 836	46 052 060
Media Capital	Media	PSI-all share	84 513 180	266 216 517
Teixeira Duarte	Construction & Materials	PSI-all share	420 000 000	93 660 000

As described in table 1, from the selected listed companies, 5 of them – EDP, Galp Energia, Jerónimo Martins, Sonae and Nos – belong to the Portuguese Stock Index PSI 20. The remaining companies – Cofina, Media Capital and Teixeira Duarte – do not fulfill the necessary PSI-20 Index requirements. We figured out that it could be interesting to apply the structural approach to companies that belong and do not belong to PSI-20 Index and compare the respective results.

The Portuguese Stock Index PSI 20 is a benchmark stock market index which tracks the performance of the maximal twenty companies[1] with the largest market capitalization.

[1] Nowadays, only 18 companies belong to the PSI-20 Index.

The eligible companies are required to fulfil the velocity threshold and minimum free float and they should in principle have a minimum Free Float market capitalization of € 100 million.

The financial data required to implement the structural approach addressed in this study has been collected from Bloomberg terminal, which provides real-time and historical financial market data. It includes all the trimestral and annual financial information (e.g. short and long-term liabilities, market capitalization, outstanding shares, minority interests, preferred equity...), addressed in the consolidated statements and balance sheet of each firm, as well as the daily closing prices of each selected firm and the Indices (PSI20 and PSI-ALL SHARE Index). The euro risk-free interest rates were attained in the European Central Bank website.

The implementation of the structural models and all calculations were performed in Excel with the support of the "Macro Iterate" from Löffler and Posch, the "Solver tool" and the "Bivar function of John C. Hull".

4.2. Methodology

4.2.1. Merton model

The Merton (1974) model is the simplest structural credit risk model to be implemented, in which it is assumed a simple firm's capital structure and bankruptcy procedure. The model's parameters and the firm's default probability are determined as follows.

Firstly, the debt value (X) is estimated from balance sheet data as the sum of all short term and long-term liabilities. The risk-free interest rate is obtained from prices of Treasury bonds and the equity value (E_t) and volatility (σ_E) are observed from the stock market. The volatility (σ_E) is obtained from the historical volatility and the equity value is determined by multiplying the number of shares outstanding by the stock price.

Afterwards, the value of the firm's assets (V_t) and their volatility (σ_V) is determined according to the non-linear system of equation approach. Firstly, it is assumed an initial value for firm's assets (V_t) and their volatility (σ_V). By doing so, it is possible to estimate the equity (E_t) and volatility (σ_E) values from the Black-Scholes formulas, using equations (4.1) and (4.2):

$$E_t = V_t \, N(d_1) - X \, e^{-rt} \, N(d_2) \tag{4.1}$$

and

$$\sigma_E = \sigma_V \, N(d_1) \, \frac{V_t}{E_t}, \tag{4.2}$$

in which d_1 and d_2 are given by equations (1.9a) and (1.9b), respectively.

Afterwards, these two equations are used to determine the time-t value of assets (V_t) and its volatility (σ_V). The solution of this system of equations is non-trivial, since it depends both on the two unknown values (asset value V_t and volatility σ_V) and a numerical solution is performed using a routine based on the Newton-Raphson algorithm in Excel. Finally, under the Merton (1974) model, the risk-neutral probability of default is given by:

$$PD(t, T) = N(-d_2) = 1 - N(d_2). \tag{4.3}$$

4.2.2. KMV model

The KMV model is an extension and practical implementation of the Merton (1974) model, since it incorporates more realistic assumptions and empirical observations that better reflect real-world default dynamics. One of the main advantages of this model is that it provides both up-to-date view of a firm's value and a timely warning of changes in credit risk.

The Expected Default Frequency (EDF) credit measure is the main output of the KMV model and it is determined through the following steps. Initially, we calculated the inputs of the model as follows:

- The market value of equity is determined daily by multiplying the number of outstanding shares by the stock quote at the end of each day;
- The book liabilities are defined as the debt in one year plus half the long-term debt and the daily value is assumed to be constant per quarter;
- The logarithm risk-free interest rate is determined daily from the country's risk-free interest rate, for each trading day.

Afterwards, the first step is to determine the value of the firm's assets (V_t) and its volatility (σ_V) according to the iterative approach. Firstly, the error tolerance is settled as 10E–10.

In the column "Iter k" it is created a vector of asset prices V_{t-a}, for a = 0, 1, ..., 252, which is settled as the sum of the market value of equity E_{t-a} and the book value of liabilities X_{t-a}. Afterwards, the initial value for the estimation of σ_V is settled as the standard deviation of the log asset returns calculated with the V_{t-a} vector. The column "Iter k+1" is defined using the formula (4.4) for each trading day of the past 12 months:

$$V_t = \frac{E_t + X\, e^{-r(T-t)}\, N(d_2)}{N(d_1)}. \tag{4.4}$$

Then, considering that the firm's debt maturity is one year and that it has relatively stable maturity structures (the firm issue new debt substitutes the debt that is going to be retired), we obtained the daily values for V_{t-a}, and a system of equations composed by 253 equations with 253 unknowns. The flowing step was to run the macro Iterate, so that the asset values in "Iter k+1" converge to those in column "Iter k", minimizing the sum of squared errors between these two columns. At that point, it is derived an estimate of the drift rate of asset returns μ, using the estimated asset values V_{t-a} and the CAPM.

The second step of the KMV model is to calculate the distance-to-default (DD). For a one-year time horizon, the default point (DPT) is simply the sum of 100% of short-term liabilities with 50% of long-term liabilities. Considering no payouts, the distance-to-default is then determined as follows by equation (4.5):

$$DD = \frac{\ln V_t + (\mu - 0.5\, \sigma_V{}^2)\tau - \text{Payouts} - \ln D^*}{\sigma_V \sqrt{\tau}}. \tag{4.5}$$

Finally, the EDF measure is obtained assuming a normal distribution, in which the default probability within a given period of time is given by:

$$EDF = N(-DD)^*. \tag{4.6}$$

Moreover, using the DD migration, it was also possible to estimate, for each firm, a term structure of EDF measures at a 5-year time horizon – the EDF Term Structure. The construction of EDF term structure starts with the cumulative t-year EDF ($CEDF_t$), which is determined, for each year, as follows:

$$CEDF_t = 1 - (1 - CEDF_{t-1})(1 - FEDF_{t-1,t}). \tag{4.7}$$

For example, the firm's 2-year cumulative EDF ($CEDF_2$) is given by:

$$CEDF_2 = 1 - (1 - EDF_1)(1 - FEDF_{1,2}). \tag{4.8}$$

The forward EDF ($FEDF_{t-1,t}$), previous calculated, is defined as the default's probability between $t-1$ and t conditional on survival until $t-1$.

Finally, the annualized t-year EDF is directly related to the $CEDF_t$ through the following equation:

$$EDF_t = 1 - (1 - CEDF_{t-1})(1 - FEDF_{t-1,t})^{\frac{1}{t}}. \quad (4.9)$$

For example, the annualized two-year EDF is then given by:

$$EDF_2 = 1 - \left[(1 - EDF_1)(1 - FEDF_{1,2})\right]^{\frac{1}{2}}. \quad (4.10)$$

4.2.3. CreditGrades model

The main difference from the classic structural approach is that in the CreditGrades model it is assumed a random default barrier, so that the default event can occur before the maturity date, if the value of company assets hits the default barrier.

Thus, in order to estimate the firm's default probabilities through the CreditGrades model, the required model parameters are determined through the follow assumptions and methodology. Firstly, the global recovery rate ($\overline{\Lambda}$) and volatility of the barrier (λ) are assumed to be constants and equal to 50% and 30%, respectively. Then, the debt-per-share value is obtained by dividing the liabilities by the number of shares:

- The total number of shares used is the sum of the number of common shares and the number of preferred shares;
  - The number of common shares is determined by dividing the firm's market capitalization by the current stock price;
  - The number of preferred shares is limited at half the number of common shares and is calculated by dividing the preferred equity by the market capitalization.
- The final debt (liabilities) is determined by reducing the financial debt by $\min[0.5 \times \text{financial debt}, \text{minority interest}]$;
  - The financial debt is determined as the sum of short-term and long-term borrowing, and one-half of the sum of other short-term and long-term

liabilities. Moreover, the "Accounts Payable" are not included in this calculation.

The information regarding the market capitalization, stock price and liabilities is obtained from financial data of consolidated statements. The historical volatility is estimated for the last 200 trading days from market data, and it is the annualized standard deviation of the daily returns during the last 200 trading days.

Afterwards, the initial asset value and its volatility are defined by equations (3.15) and (3.16), respectively, as well as the parameters d and α_t^2 are determined as settled by equations (3.17a) and (3.17b), respectively.

Finally, the survival probabilities of all firms are determined through the two alternative methods:

- In the Approximated Survival Probability, the firm survival probability is given by equation (4.8);

$$SP(t) = N\left(-\frac{\alpha_t}{2} + \frac{\ln(d)}{\alpha_t}\right) - dN\left(-\frac{\alpha_t}{2} - \frac{\ln(d)}{\alpha_t}\right). \quad (4.9)$$

- In the Exact Survival Probability, the survival probability of the firm follows a cumulative bivariate normal distribution and it is calculated by equation (4.10):

$$SP(t) = N_2\left(-\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{\alpha_t}{2} + \frac{\ln(d)}{\alpha_t}; \frac{\lambda}{\alpha_t}\right) - dN_2\left(\frac{\lambda}{2} + \frac{\ln(d)}{\lambda}, -\frac{\alpha_t}{2} - \frac{\ln(d)}{\alpha_t}; -\frac{\lambda}{\alpha_t}\right). \quad (4.10)$$

To conclude, the default probability is simple determined as:

$$\text{Default Probability} = 100\% - \text{Survival probability}.$$

5. Empirical results

This section addresses the results produced by each structural credit risk model under study, comparing its output, advantages and disadvantages, and credit measure predictive ability. Furthermore, it provides a sensitive analysis of the model's inputs parameters.

5.1. The calculations and results

After running the models, whose accounting and market information cover 5 years, from January 1, 2013 to December 31, 2017, the 1-year default probabilities were produced for each firm, during the time frame in analysis. The default probabilities, produced by the structural approach, take into account the firm's liabilities and may be viewed as forward-looking, since it uses current market information to provide an essential aid as a quantitative measure of solvency of the non-financial institution.

Commercial implementations, such as KMV and CreditGrades models, have refined the original Merton model in different ways. Each model incorporates more realistic assumptions and strives to produce an output that better reflects real-world default dynamics. These substantial modifications of Merton's original approach may then be used by market participants to evaluate potential investments.

The KMV model assumes that the firm´s equity is a perpetual option, and the default event occurs when the default point barrier is crossed for the first time. The CreditGrades is a more recent product, and in its implementation, the default barrier has a random component, which is a significant driver of short-term spreads. The default even occurs whenever the default threshold is crossed for the first time.

Again, the main goal of this study is to determine the default probabilities, of the selected Portuguese companies, according to the structural approach and compare the results produced by the original Merton model with its extensions.

Table 2 summarizes the results regarding the default probabilities determined by the three structural models, for the years 2013 until 2017. As it is possible to observe, through the KMV model we obtained two EDF credit measures: the default probabilities for a one-year horizon and a term structure of EDF measures for a five-year horizon. Furthermore, in the CreditGrades model, the firm's default probabilities were determined through the two alternative methods – the Approximated Survival Probability and the Exact Survival Probability. Detailed information about the default probability estimates is addressed in annexes 1 to 34.

Model		Non-Financial Institution	2013	2014	2015	2016	2017
Merton Model	Original Merton Model Approach	EDP	0.00000%	0.00007%	0.00047%	0.00178%	0.00000%
		Galp Energia-Nom	0.00000%	0.00000%	0.00817%	0.00001%	0.00000%
		Jerónimo Martins	0.00000%	0.00021%	0.00026%	0.00000%	0.00000%
		Sonae	0.00114%	0.00376%	0.00422%	0.01926%	0.00000%
		Nos SGPS	0.00002%	0.00001%	0.00000%	0.00000%	0.00000%
		Cofina, SGPS	0.03738%	0.18140%	0.00223%	0.03099%	0.09005%
		Media Capital	10.16541%	25.03066%	1.54354%	0.35860%	0.10149%
		Teixeira Duarte	4.05152%	1.30545%	0.29039%	6.19527%	2.90017%
KMV Model	KMV annual	EDP	0.00000%	0.00031%	0.00013%	0.00187%	0.00007%
		Galp Energia-Nom	0.00000%	0.00000%	0.00023%	0.00000%	0.00000%
		Jerónimo Martins	0.00000%	0.00042%	0.00003%	0.00000%	0.00000%
		Sonae	0.01039%	0.03128%	0.00660%	0.00472%	0.00000%
		Nos SGPS	0.00110%	0.00000%	0.00000%	0.00000%	0.00000%
		Cofina, SGPS	1.60015%	0.25748%	0.03819%	1.58207%	1.76670%
		Media Capital	7.77543%	33.69632%	0.22765%	0.22805%	0.02580%
		Teixeira Duarte	1.17834%	5.83069%	13.96145%	26.51577%	18.72683%
	EDF Term Structure	EDP	0.00000%	0.00015%	0.00015%	0.00058%	0.00048%
		Galp Energia-Nom	0.00000%	0.00000%	0.00008%	0.00006%	0.00005%
		Jerónimo Martins	0.00000%	0.00021%	0.00015%	0.00011%	0.00009%
		Sonae	0.01039%	0.02084%	0.01609%	0.01325%	0.01060%
		Nos SGPS	0.00110%	0.00055%	0.00037%	0.00027%	0.00022%
		Cofina, SGPS	1.60015%	0.93109%	0.63435%	0.87213%	1.05170%
		Media Capital	7.77543%	21.80263%	15.18653%	11.67152%	9.45631%
		Teixeira Duarte	1.17834%	3.53255%	7.14224%	12.41854%	13.71820%
CreditGrades Model	Approximate Probability of Default	EDP	3.45945%	1.96529%	1.47131%	1.97907%	1.04159%
		Galp Energia-Nom	0.00007%	0.00205%	0.00235%	0.00000%	0.00000%
		Jerónimo Martins	0.00000%	0.00014%	0.00000%	0.00000%	0.00000%
		Sonae	0.46493%	1.07699%	0.72608%	1.97907%	0.24159%
		Nos SGPS	0.00412%	0.00354%	0.00008%	0.00118%	0.00006%
		Cofina, SGPS	3.08169%	3.91801%	2.46340%	9.81802%	1.93745%
		Media Capital	7.59822%	23.00886%	0.02258%	0.81848%	0.12093%
		Teixeira Duarte	26.54874%	33.06555%	53.17765%	61.59463%	54.47726%
	Exact Probability of Default	EDP	2.67677%	1.70800%	1.30350%	1.74373%	0.88368%
		Galp Energia-Nom	0.00006%	0.00204%	0.00235%	0.00000%	0.00000%
		Jerónimo Martins	0.00000%	0.00014%	0.00000%	0.00000%	0.00000%
		Sonae	0.44914%	1.01520%	0.69652%	1.35371%	0.21993%
		Nos SGPS	0.00412%	0.00353%	0.00008%	0.00118%	0.00005%
		Cofina, SGPS	2.85808%	3.66682%	2.15471%	8.05142%	1.88608%
		Media Capital	7.54494%	23.00485%	0.02240%	0.81839%	0.12095%
		Teixeira Duarte	21.10132%	23.42303%	32.39565%	37.36353%	34.29311%

Table 2:. The results for structural models

According to the results described in table 2, the group of companies which not belong to the PSI-20 Index – Cofina, Media Capital and Teixeira Duarte – exhibit higher probabilities of default, when compared to the set of companies from PSI-20 – EDP, Galp, Jerómino Martins, Sonae and Nos. Within the latter, Galp, Jerómino Martins and Nos are identified as the companies with the lowest risk of default, displaying default probabilities around the zero percent, during the 5-year time period. On the other hand, Teixeira Duarte appears to be the company with the highest risk of default during the years of 2013 until 2017, displaying high and volatile default probabilities.

When analysing table 2, it is noticeable the differences in the results produced by Merton model and its extensions. Among the 3 structural models considered in this study, CreditGrades is the model that displays the highest values of default probabilities, followed by the KMV model. It is interesting to note that as long as the probability of default determined by the structural approach increases, the discrepancy in the results produced by the 3 structural models also intensifies.

This can also be recognised in table 3, which compares the average default probability of each structural model for three possible scenarios: when considering all companies under study, or only the group PSI-20 companies, or even just the group of companies that does not belong to PSI-20 Index.

Table 3:. Mean of the probabilities of default

<table>
<tr><th colspan="2">The Mean of Default Probabilities</th><th>All Companies</th><th>PSI-20 Companies</th><th>Other listed Companies</th></tr>
<tr><td colspan="2">Merton Model</td><td>1.30810%</td><td>0.00157%</td><td>3.48564%</td></tr>
<tr><td rowspan="2">KMV Model</td><td>KMV Annual</td><td>2.83670%</td><td>0.00229%</td><td>7.56073%</td></tr>
<tr><td>EDF Term Structure</td><td>2.72619%</td><td>0.00303%</td><td>7.26478%</td></tr>
<tr><td rowspan="2">CreditGrades Model</td><td>Approximate Probability of Default</td><td>7.40176%</td><td>0.57676%</td><td>18.77676%</td></tr>
<tr><td>Exact Probability of Default</td><td>5.26923%</td><td>0.48255%</td><td>13.24702%</td></tr>
</table>

In fact, CreditGrades is the model that, in the three scenarios, exhibits the highest mean of default probabilities, particularly under the Approximate Probability of Default method.

When considering only the group of companies from PSI-20, which exhibit low default risk, the mean of default probabilities determined by each structural model is considerable smaller and quite similar. Nevertheless, when considering all companies or only the group of companies that does not belong to PSI-20 Index, the same conclusion is no longer valid. The mean of default probabilities increases across all models, however in completely different proportions. For example, in the case of the group of companies that does not belong to PSI-20 Index, the mean of default probabilities produced by the CreditGrades model and the Merton model reaches a difference of 10 and 15 percentage points.

KMV and CreditGrades models are two extensive modifications of the basic Merton's approach, that aim to provide more accurate and trustworthy default predictions, since incorporate more realistic assumptions and empirical observations that better reflect real-world default dynamics.

The big advantage of the KMV model, when compared to the Merton's approach, is that it provides both up-to-date view of a firm's value and a timely warning of changes in credit risk. The probability of default is computed as a function of the firm´s capital structure, the volatility of the asset returns and the current asset value.

Another difference between the Merton and KMV model is the methodology used to determine the firm's asset value and their volatility. The Merton model follows the non-linear system of equations approach while the KMV model opts for the iterative approach. As already mentioned, the calculations of the default probabilities through the non-linear system of equations approach may not provide reasonable results and a good discriminatory power in most of the cases, since in practice this approach may not capture the market leverage dynamics and the models may bias the default probabilities in the wrong directions. The iterative approach is commonly preferable to use and it has proven to be very useful for predicting default's probabilities.

Within the KMV model, it was also applied the EDF Term Structure. This is a necessary ingredient for pricing, hedging and risk management of long term obligations. Particularly, portfolio models of credit risk require term structures of default probabilities as key inputs for the valuation of long-dated credit portfolios.

In this case, instead of calibrating a separate DD-to-EDF mapping for each time horizon, the EDF model employs a credit migration-based approach to build the EDF term structure. It applies the existing one-year DD-to-EDF mapping to the expected second-year DD level to produce the estimate of the second-year EDF. For example, as previous mentioned, the annualized two-year EDF is simply given by:

$$EDF_2 = 1 - \left[(1 - EDF_1)(1 - FEDF_{1,2})\right]^{\frac{1}{2}}. \tag{5.1}$$

The same methodology is applied to the following years. In this case, since the EDF of the previous year's travel over the next ones, the default probabilities results become more homogenous and the jumps are softened. Note that this is the case, for example, of Media Capital, in years of 2014 and 2015.

Among the structural models addressed in this study, CreditGrades is said to be the most reliable and accurate model. It focuses on a more complete capital structure, and then closest to the reality of the market. The input parameters are all observable in the market and the default threshold is volatile, which incorporates the uncertainty of the market when the liabilities of a firm change. According to table 2, CreditGrades is the model that exhibits the highest default probabilities values for almost all firms, over the 5 years in analysis.

Within the CreditGrades model, the results produced by the Approximate Probability of Default and Exact Probability of Default methods are identical, as expected, since the methodology applied in the implementation and calibration of the models is exactly the same. Only the last step (formula) to calculate the survival probability differs.

In the Approximate Probability of Default, the process X is approximated by a drifted-Brownian motion $\hat{X}$ such that the approximating process does not contain the random variable Z, which was considered for modeling the uncertainty in the default barrier. By doing so it is possible to apply the standard formula for the first passage time of a Brownian motion to calculate the survival probability. Alternatively, the Exact Probability of Default method integrates the random variable Z and still gets a closed form solution for the survival probability, which is expressed in terms of the most complex cumulative bivariate normal distribution function.

According to Finger et al. (2002), for practical purposes, the numerical differences between the survival probabilities given by the two approaches are marginal. However, Kiesel and Veraart (2008) found that there are circumstances in which these two formulas may yield significantly different results, as the particular case of highly leveraged companies such as banks. In fact, for companies that have a very low share-to-debt, the approximated survival probability is much lower than the exact one, which is the same as saying that the approximated default probability is much higher than the exact one.

In this study, the default probabilities given by the Approximate Probability of Default method are always equal or slightly higher than the ones observed in the Exact Probability of Default method. However, this difference becomes more noticeable for companies that have a higher probability of default, as it is the case of Teixeira Duarte. Teixeira Duarte is the company that, according to structural approach, has the highest probability of default and whose values determined by the approximated default probability are much higher than the exact one. This result is consistent with the Kiesel and Veraart (2008) founding, since Teixeira Duarte is a highly leveraged company.

Moreover, as it is possible to observe in tables of annexes 33 and 34, the two companies with the highest debt-to-equity and debt-to-capital ratios are Teixeira Duarte and Cofina, the companies that also display the highest discrepancy of results produced by the two methods. In fact, the debt-to-equity and debt-to-capital ratios are the most well known financial leverage ratios and are determined as the company's debt divided by its total equity and the company's debt divided by its total capital, respectively. All else being equal, the higher the debt-to-equity and debt-to-capital ratios, the riskier the company.

5.2. Sensitive analysis

Potential variations in the inputs of each model may lead to positive or negative effects in the default probability of a company. A sensitive analysis is then made, taking into consideration a positive variation of the inputs, ceteris paribus. Table 4 summarizes the several effects on default probability.

Table 4:. Sensitive analysis

Variables	*Merton Model*	*KMV Model*	*CreditGrades Model*
Market Capitalization	↘	↘	↘
Liabilities	↗	↗	↗
Asset Volatility	↗	↗	↗
Preferred Equity	-	-	↘
Minority Interest	-	-	↘

According to table 4, the default probability of a company will increase if, ceteris paribus, the amount of firm’s liabilities increases, or if the asset volatility increases. On the other hand, the default probability is a decreasing function of market capitalization.

The asset volatility refers to the amount of risk and uncertainty related to the size of changes in asset's value. A higher volatility means that the price of the security can change dramatically, over a short period of time, in either direction, which increase the instability of the business, and consequently the default probability of the company.

Similarly, an increase in the amount of liabilities of a firm, increases its financial leverage, which in turn intensifies the risk of possible transactions and the exposure to bankruptcy. On the other hand, as long as the market capitalization increases, that is the market value of the company's outstanding shares, the default probability of the company decreases.

Moreover, in CreditGrades model, the default probability of a firm is also a decreasing function of preferred equity and minotary interest. Preferred Equity is a class of ownership in a company that has a higher claim on its earnings and assets than common equity. In this case, the number of preferred shares increases the total number of shares so that, it reduces the amount of debt per share, and consequently the default probability. The minority interest, also known as non-controling interest, represents a percentage of ownership in a company by less than 50% of the outstanding shares, with a voting right. It includes part of the profit or loss and net assets of the subsidiary. Then, an increase in the minority interest of a company, will also reduce the amount of debt per share, and consequently its default probability.

5.3. Firm analysis

5.3.1. PSI-20 Index companies

From the selected companies, EDP, Galp Energia-Nom, Jerónimo Martins, Sonae and Nos belong to the Portuguese Stock Index PSI 20, which means that they are between the 20 Portuguese listed companies with the largest market capitalization, that fulfil the velocity threshold and minimum free float. Table 5 shows the weight of each selected company in the PSI-20 index at the date of 29 June 2018.

Table 5:. PSI-20 Index weighting

Companies	***% Weight in the Index***
EDP	13.09%
Galp Energia-Nom	12.98%
Jerónimo Martins	9.99%
Sonae	5.11%
Nos SGPS	7.87%

As would be expected, these are the companies that, according to the structural credit risk approach, exhibit the lowest probabilities of default. As it is possible to observe in table 6, among the five PSI-20 companies in analysis, EDP – a leader company in utilities sector in Portugal – is the company with the highest index weighting as well as the one that, during the time period in analysis, exhibits the highest risk of default.

Table 6:. Default results: PSI-20 companies

EDP	2013	2014	2015	2016	2017
Merton Model	0.00000%	0.00007%	0.00047%	0.00178%	0.00000%
KMV annual	0.00000%	0.00031%	0.00013%	0.00187%	0.00007%
Temporal EDF	0.00000%	0.00015%	0.00015%	0.00058%	0.00048%
Approximate	3.45945%	1.96529%	1.47131%	1.97907%	1.04159%
Exact	2.67677%	1.70800%	1.30350%	1.74373%	0.88368%
Galp Energia	2013	2014	2015	2016	2017
Merton Model	0.00000%	0.00000%	0.00817%	0.00001%	0.00000%
KMV annual	0.00000%	0.00000%	0.00023%	0.00000%	0.00000%
Temporal EDF	0.00000%	0.00000%	0.00008%	0.00006%	0.00005%
Approximate	0.00007%	0.00205%	0.00235%	0.00000%	0.00000%
Exact	0.00006%	0.00204%	0.00235%	0.00000%	0.00000%
Jerónimo Martins	2013	2014	2015	2016	2017
Merton Model	0.00000%	0.00021%	0.00026%	0.00000%	0.00000%
KMV annual	0.00000%	0.00042%	0.00003%	0.00000%	0.00000%
Temporal EDF	0.00000%	0.00021%	0.00015%	0.00011%	0.00009%
Approximate	0.00000%	0.00014%	0.00000%	0.00000%	0.00000%
Exact	0.00000%	0.00014%	0.00000%	0.00000%	0.00000%
Sonae	2013	2014	2015	2016	2017
Merton Model	0.00114%	0.00376%	0.00422%	0.01926%	0.00000%
KMV annual	0.01039%	0.03128%	0.00660%	0.00472%	0.00000%
Temporal EDF	0.01039%	0.02084%	0.01609%	0.01325%	0.01060%
Approximate	0.46493%	1.07699%	0.72608%	1.97907%	0.24159%
Exact	0.44914%	1.01520%	0.69652%	1.35371%	0.21993%
Nos	2013	2014	2015	2016	2017
Merton Model	0.00002%	0.00001%	0.00000%	0.00000%	0.00000%
KMV annual	0.00110%	0.00000%	0.00000%	0.00000%	0.00000%
Temporal EDF	0.00110%	0.00055%	0.00037%	0.00027%	0.00022%
Approximate	0.00412%	0.00354%	0.00008%	0.00118%	0.00006%
Exact	0.00412%	0.00353%	0.00008%	0.00118%	0.00005%

On the other hand, Jerónimo Martins – a Portuguese corporate group that operates in food distribution and specialized retail – is considered by all structure credit risk models, the company with the lowest probability of default. In fact, according to the Merton, KMV and CreditGrades models, the probability of default of this Portuguese company was approximately zero percent, for the years 2013 until 2017. Note that, in this case, CreditGrades is the model that displays the lowest probabilities of default, and in which the results given by the Approximate and Exact Probabilities of default are exactly the same, for all years. Sonae is a Portuguese company that also operates in the retail sector and displays low probabilities of default during the time period in analysis, however, when compared with Jerónimo Martins, the default risk determined by the structural approach is slightly higher.

Finally, according to the obtained results, Galp Energia, and Nos, which operate in oil and gas and media sectors, respectively, also display low probabilities of default around the zero percent.

5.3.2. Other listed companies

As previously discussed, the results obtained for the companies that do not belong to PSI-20 Index are completely different from those presented by the PSI-20 companies. Actually, the default probabilities determined for Cofina, Media Capital and Teixeira Duarte fluctuate wildly from year to year and achieve higher values, specifically under the CreditGrades model.

Table 7:. Default results: other listed companies

Cofina	2013	2014	2015	2016	2017
Merton Model	0.03738%	0.18140%	0.00223%	0.03099%	0.09005%
KMV annual	1.60015%	0.25748%	0.03819%	1.58207%	1.76670%
Temporal EDF	1.60015%	0.93109%	0.63435%	0.87213%	1.05170%
Approximate	3.08169%	3.91801%	2.46340%	9.81802%	1.93745%
Exact	2.85808%	3.66682%	2.15471%	8.05142%	1.88608%
Media Capital	2013	2014	2015	2016	2017
Merton Model	10.16541%	25.03066%	1.54354%	0.35860%	0.10149%
KMV annual	7.77543%	33.69632%	0.22765%	0.22805%	0.02580%
Temporal EDF	7.77543%	21.80263%	15.18653%	11.67152%	9.45631%
Approximate	7.59822%	23.00886%	0.02258%	0.81848%	0.12093%
Exact	7.54494%	23.00485%	0.02240%	0.81839%	0.12095%
Teixeira Duarte	2013	2014	2015	2016	2017
Merton Model	4.05152%	1.30545%	0.29039%	6.19527%	2.90017%
KMV annual	1.17834%	5.83069%	13.96145%	26.51577%	18.72683%
Temporal EDF	1.17834%	3.53255%	7.14224%	12.41854%	13.71820%
Approximate	26.54874%	33.06555%	53.17765%	61.59463%	54.47726%
Exact	21.10132%	23.42303%	32.39565%	37.36353%	34.29311%

When analysing the results reported in table 7, the differences between the KMV and CreditGrades models become clear, as well as the limitations pointed out to the Merton model. In most cases, the Merton model seems not to provide reasonable results and a good discriminatory power, when compared to other models.

It is possible to observe that for Cofina, a Portuguese media conglomerate owner of several newspapers (as Correio da Manhã, Record and Jornal de Negócios) and magazines (as TV Guia and Sábado), the results produced by the the KMV model are slightly higher than the Merton model. Nevertheless, only CreditGrades model could detect an unusual high default risk for Cofina, during the year of 2016, which is explained by the significant reduction of the Cofina's stock price during that year. Cofina's stock price decreased almost 50%, from 0,45 to 0,26 which consequently implies a proportional reduction of its market capitalization, from 45,6 to 26,7 million euros (annexe 18).

According to the Cofina's accounting report, there were two events that marked such stock price evolution: the announcement of the Group's performance during the year of 2015 and in the first quarter of 2016 (which exposed the decrease in net income and profit margin), as well as the announcement of a dividend payment. As previously analysed, when the market capitalization of a firm decreases, its probability of default increases. In this case, CreditGrades shows to be a highly sensitive model to this input variation.

Media Capital, just like Cofina, is a media company listed in the Portuguese Stock Index (PSI-ALL SHARE). It is the major media group in Portugal and owns several companies, as the television broadcaster – TVI – and a radio broadcaster that includes Radio Comercial, Cidade, M80, Vodafone FM and Smooth FM.

Nevertheless, it is interesting to note that the results displayed by Media Capital lead us to a different comparative approach of the models. In fact, for Media Capital, the default probabilities produced by all structural models are more homogenous and coincident, even when the default probabilities reach higher values. As showed in table 7, the results determined through the Approximate and Exact Default Probability methods, under the CreditGrades model, are practically the same and really close to the results presented by Merton and KMV models.

It is possible to observe that, during the time frame in analysis, there was a peak in the default risk displayed by Media Capital in the year of 2014, when the default probability of the company surpassed the 20%. Anyway, in 2013 the company already displayed a default risk of almost 10%.

However, in this case, that high default risk was perceived not only by the CreditGrades model, but also by the original Merton model and KMV model (which obtained even higher values of default risk than CreditGrades model). This risk can be mainly explained by the high equity volatility achieved in that year, but it is also justified by the low stock's prices and the high firm's liabilities (annexes 19, 20 and 21). In 2014, the firm's historical volatility reaches really high values, which had an immediate negative impact on its default risk, across all structural models, in that year.

After that and since 2015, Media Capital displayed low default probabilities, below 1%. When analysing the results (annexes 19, 20 and 21) we conclude that this change is due to 3 main reasons: a significant decline in the firm's volatility, a significant increase in the stock price and then market capitalization, and a decreased in the firm's liabilities.

Finally, as previously discussed, Teixeira Duarte is the company that, according to the structural approach, displayed the highest risk of default during the time frame in analysis. During the financial crisis of 2008, the construction was one of the sectors that mostly suffered and declined with the financial recession. Teixeira Duarte's group was one of the most affected with the abrupt stop of the construction and public works during Troika's years. Nevertheless, as shown in table 7, there is a consensus among the three structural models that classified 2016 as the worst year for Teixeira Duarte in terms of probability of default.

Although the KMV and CreditGrades models exhibit a coincident default risk behavior, i.e. the default's probabilities increased until 2016, where it reached the peak, and then in 2017 it declined somewhat, the probabilities displayed by the CreditGrades model are noticeably higher. It is interesting to note that this default risk behavior also coincides with the declining of stock price until 2016, and subsequent increase during 2017. Moreover, in this case, the default probabilities determined by Approximate Probability of Default method are considerably higher than the Exact one, and this difference increases as the default probability also increases.

When analysing the input parameters of the model (annexe 24), it is possible to conclude that the low stock price, and subsequent market capitalization, is the main explanation for this default risk variation, particularly under the GreditGrades and KMV models. In the Merton model, the volatility shows to be again the guiding thread of this variation, since the probability increases when the volatility increases, and vice versa. Nevertheless, the high default probabilities displayed by Teixeira Duarte, and its variations are also clearly justified by the firm's liabilities and high debt to equity ratio.

6. Conclusion

After the financial crisis of 2007-2009 and in a context of new regulatory requirements, such as the Basel Accord, default risk measurement and management has become an area of fast innovation and increasing interest from both academic and financial institutions.

In this sense, this study empirically investigates an integrated approach to measuring default risk: the structural credit risk models. The structural credit risk approach is rooted in the seminal paper of Merton (1974), which uses the principles of the option pricing framework of Black and Scholes (1973) to value corporate liabilities. In this setting, due to its theoretical attractiveness, we decided to analyse and investigate three structural credit risk models – the original Merton model (1974), and two extensions of this model, the KMV model and the CreditGrades model. These two commercial implementations have refined the original Merton model, incorporating more realistic assumptions which attempt to produce a more realistic output that better reflects real-world default dynamics.

Therefore, throughout this study, we determined the default probability of 8 selected Portuguese listed companies – EDP, Galp, Jerónimo Martins, Sonae, Nos, Cofina, Media Capital and Teixeira Duarte – covering utilities, oil, retail, media and construction sectors, during the years 2013 to 2017. After running the models, we could analyse and compare the annual default probabilities produced by the original Merton model and its two commercial extensions.

In this regard, our results suggest that the annual default probabilities determined by the three structural models are considerably different, taking into account the several modifications and refinements done, by the KMV and CreditGrades models, to the original Merton model.

Firstly, we have noticed that the discrepancy in the results produced by the 3 structural models intensifies, as long as the probability of default determined by the structural approach, for a given company, also increases. Secondly, among the 3 structural credit risk models considered in this study, CreditGrades is the model that displays the highest values of default probabilities, followed by the KMV model. Actually, among the 3 structural models, CreditGrades is said to be the most accurate and reliable, since it focuses on a more complete capital structure, and therefore closer to the reality of the market.

Thirdly, we could also observe that, as suggested by Kiesel and Veraart (2008), within the CreditGrades model, the results produced by the Approximate and Exact Probability of Default methods are identical; however, there are circumstances in which these two methods may yield significantly different results, as the case of highly leveraged companies. According to the results obtained in this study, the default probabilities given by the Approximate Probability of Default method are always equal or slightly higher than the default probabilities observed in the Exact Probability of Default method. This difference becomes more noticeable for companies that have a higher probability of default, as it is the case of Teixeira Duarte.

This study provides a relevant contribution to the finance literature, since it allows to better understand the structural approach to credit risk, specifically the models under analysis. Our investigation was based, not only, on an individual examination of each model, in terms of model setup, inputs parameters and sensitive analysis, but also on a practical implementation of each model, which allowed us to better perceive and compare the differences in the outputs produced by each model.

On the other hand, this study also provides relevant information about the credit risk situation, during the years 2013 to 2017, of the Portuguese companies under analysis which were selected taking into consideration their dimension, importance, and influence in the Portuguese economy. The 8 companies are listed in the Portuguese Stock Index (PSI-all share), however only 5 of them – EDP, Galp Energia, Jerónimo Martins, Sonae and Nos – belong to the Portuguese Stock Index PSI 20, while the remaining companies – Cofina, Media Capital and Teixeira Duarte – do not fulfill the necessary PSI-20 Index requirements.

In the results obtained from this study, the set of companies from PSI-20 exhibit lower probabilities of default, when compared to the group of companies which do not belong to the PSI-20 Index. Over the years 2013 to 2017, Teixeira Duarte appears to be the company with the highest default risk of default, displaying high and volatile default probabilities. On the other hand, Jerómino Martins, Galp and Nos are recognized as the companies with the lowest risk of default, during the 5-year period in analysis, displaying default probabilities around the zero percent.

However, some limitations have remained in this study. Firstly, some assumptions may be too strong, as the case of assuming that the assets of the firm follow a geometric

Brownian motion. Additionally, when applying the KMV model it was not possible to use the KMV's proprietary database that translates the Distance to Default of the firm to the Expected Default Frequency, since it was not available for public use. Thus, the EDF measure is obtained assuming a normal distribution, such as in the original Merton (1974) model. Moreover, in the CreditGrades model, it was not possible to use an ATM implied volatility to calculate the asset volatility, since that information is not available for the Portuguese companies under analysis. Thus, we had to use a historical volatility, estimated for the last 200 trading days.

Beyond the scope of this study, other relevant issues might require further research, for example, it could be of interest to create a model that would enable us to study and better understand the relationship between some company's financial indicators, macroeconomic indicators and the default probability determined by each structural model under analysis and evaluate the significance and correlations between them.

References

Arora, N., Bohn, J. R. & Zhu, F. 2005. ***Reduced form vs. structural models of credit risk: a case study of three models.*** San Francisco: Moody's KMV Company.

Allen, D. E. & Powell, R. J. 2011. Credit risk measurement methodologies. Paper presented in ***19th International Congress on modelling and simulation,*** Perth, Australia

Anderson, R. & Sundaresan S. 2000. A comparative study of structural models of corporate bond yields: An exploratory investigation. ***Journal of Banking & Finance,*** 24: 255-269.

Bakshi, G., Madan, D. & Zhang, F. 2001. ***Understanding the role of recovery in default risk models: Empirical comparisons and implied rates***, Working paper. University of Maryland.

Bharath, S. T. & Shumway, T. 2004. ***Forecasting default with the KMV-Merton model***, Working paper. University of Michigan

Black, F. & Cox, J. C. 1976. Valuing corporate securities: Some effects of bond indenture provisions. ***Journal of Finance,*** 31: 351–367.

Black, F. & Scholes, M. 1973. The Pricing of Options and Corporate Liabilities. ***Journal of Political Economy,*** 81: 637-654.

Byström, H. 2005. **Credit Default Swaps and Equity Prices**: The Itraxx CDS Index Market. Working Papers, Department of Economics, Lund University; No. 24. Department of Economics, Lund Universtiy.

Cao, C., Yu, F. & Zhong, Z. 2010. The Information Content of Option-Implied Volatility for Credit Default Swap Valuation. ***Journal of Financial Markets.*** 13: 321-343.

Collin-Dufresne, P. & Goldstein, R. S. 2001. Do credit spreads reflect stationary leverage ratios? ***Journal of Finance,*** 56: 1929–1957.

Cox, J. C. & Stephen A. R. 1976. The Valuation of Options for Alternative Stochastic Processes. ***Journal of Financial Economics,*** 3: 145-166.

Crosbie, P. J. & Bohn, J. R. 2003. ***Modeling Default Risk***, PhD dissertation. Technical Document, Moody's KMV.

Dias, J.C. 2017. ***Credit Risk: Lecture Notes.*** ISCTE-IUL Business School. Lisbon, Portugal

Duan, J. C., Gauthier, G. & Simonato, J. G. 2005. ***On the equivalence of the KMV and maximum likelihood methods for structural credit risk models.*** Working Paper. Montréal: Groupe d'études et de recherche en analyse des decisions.

Duffee, G. 1999. Estimating the price of default risk. ***The Review of Financial Studies***, 12: 197–226.

Duffie, D. & Singleton, K. J. 1999. Modeling term structures of defaultable bonds. ***The Review of Financial Studies,*** 12: 687–720.

Duffie, D. & Singleton, K. J. 2003. ***Credit Risk: Pricing, Measurement, and Management***. New Jersey: Princeton University Press, Princeton Series in Finance.

Finger, C. C., Filkelstein, V., Pan, G., Lardy, J. P., Ta, T. & Tierney, J. 2002. ***CreditGrades™, Technical Document***. New York: RiskMetrics Group, Inc.

Geske, R. 1977. The Valuation of Corporate Liabilities as Compound Options. ***Journal of Financial and Quantitative Analysis,*** 12: 541-552.

Geske, R. 1979. The Valuation of Compound Options. ***Journal of Financial Economics***, 7: 63-81.

Hull, J. C., Nelken, I. & White, A. D. 2004. Merton's Model, Credit Risk and Volatility Skew. ***Journal of Credit Risk***, 1: 3-27.

Iazzolino, G. & Fortino, A. 2012. Credit risk analysis and the KMV Black & Scholes model: a proposal of correction and an empirical analysis. ***Investment Management and Financial Innovations,*** 9: (2)

Jarrow, R. A. & Turnbull, S. M. 1995. Pricing derivatives on financial securities subject to credit risk. ***Journal of Finance***, 50: 53–86.

Jones, P., Mason, S. & Rosenfeld, E. 1983. Contingent claims analysis of corporate capital structures: An empirical investigation. ***Journal of Finance***, 39: 611–625.

Jones, E. P., Mason, S. P. & Rosenfeld, E. 1985. Contingent claims valuation of corporate liabilities: theory and empirical tests. In Friedman, B. M. (Eds.), ***Corporate Capital Structures in the United States***: 239-264. Chicago: University of Chicago Press

Kealhofer, S. & Bohn J. R. 1998. ***Portfolio Management of Default Risk***. San Francisco: KMV Corporation.

Kealhofer, S. 2003a. Quantifying Credit Risk I: Default Prediction. ***Financial Analysts Journal,*** 59: 30-44.

Kealhofer, S. 2003b. Quantifying Credit Risk II: Debt Valuation, ***Financial Analysts Journal***, 59: 78-92.

Kiesel, R. & Veraart, L. A. M. 2008. A Note on the Survival Probability in CreditGrades, ***Journal of Credit Risk***, 4: 65-74.

Klieštik, T. & Cúg, J. 2015. Comparison of Selected Models of Credit Risk. ***Procedia Economics and Finance,*** 23: 356 – 361

Laajimi, S. 2012. Structural Credit Risk Models: A Review. ***Insurance and Risk Management***, 80(1): 53-93

Leland, H. 1994. Corporate debt value, bond covenants, and optimal capital structure. ***Journal of Finance,*** 49: 1213–1252.

Leland, H. E. & Toft, K. B. 1996. Optimal capital structure, endogenous bankruptcy and the term structure of credit spreads. ***Journal of Finance,*** 51: 987–1019.

Löeffler, G. & Posch, P. N. 2011. **Credit Risk Modeling Using Excel and VBA**. United Kingdom: Wiley Finance

Longstaff, F. A. & Schwartz, E. S. 1995. A simple approach to valuing risky fixed and floating rate debt. ***Journal of Finance,*** 50(3): 89 – 819.

Merton, R. C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. ***The Journal of Finance,*** 29: 449-470.

Modigliani, F. & Miller, M. H. 1958. The Cost of Capital, Corporation Finance and the Theory of Investment. ***American Economic Review,*** 48: 261– 297.

Musiela, M. & Rutkowski, M. 1998. **Martingale Methods in Financial Modelling**. Springer-Verlag, Berlin.

Ogden, J. P. 1987. An Analysis of Yield Curve Notes. ***Journal of Finance,*** 42(1): 99 – 110.

Sun, Z., Munves D. & Hamilton, D. T. 2012. Public firm expected default frequency (EDFTM) credit measures: methodology, performance, and model extensions, ***Moody's Analytics Report.***

Vassalou, M. & Xing, Y. 2004. Default risk in equity returns, ***Journal of Finance*** 59: 831-868.

7. Annexes

Annexe 1: EDP – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	9 689.27	11 691.15	11 984.97	10 518.19	10 485.91
Equity volatility	σ_E	19.37%	23.94%	26.29%	27.70%	20.09%
Liabilities	L_t	31 121.34	30 903.87	30 415.47	30 347.37	28 594.79
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	5.92	4.90	4.50	4.20	5.82
Parâmetro	d_2	5.87	4.84	4.43	4.13	5.77
Equity value	E_t	9 689.24	11 663.16	11 941.32	10 518.16	10 485.88
Equity volatility	σ_E	19.37%	24.00%	26.29%	27.70%	20.09%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	40 781.31	42 595.03	42 477.96	41 124.74	39 293.23
Asset volatility	σ	4.60%	6.57%	7.39%	7.08%	5.36%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0000002%	0.0000662%	0.0004732%	0.0017815%	0.0000004%

Annexe 2: EDP – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	9 689.27	11 691.15	11 984.97	10 518.19	10 485.91
Equity volatility	σ_E	19.37%	23.94%	26.29%	27.70%	20.09%
Liabilities	L_t	20 125.35	19 433.14	19 255.64	18 956.26	17 427.42
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	29 795.69	31 141.89	31 317.32	29 723.56	28 042.88
Asset volatility	σ	6.55%	10.71%	10.71%	11.27%	10.09%
Asset drift rate	μ	1.56%	1.81%	2.26%	2.13%	1.55%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		5 461.63	5 021.02	4 596.15	3 497.09	2 395.83
Long-Term Debt		15 968.76	16 400.83	15 653.88	15 550.27	15 469.64
Other short term liabilities		3 667.74	2 941.39	3 499.67	4 068.05	3 864.23
Other long term liabilities		6 023.22	6 540.63	6 665.78	7 231.97	6 865.09
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	20 125.35	19 433.14	19 255.64	18 956.26	17 427.42
Distance to default	DD	6.19	4.52	4.70	4.12	4.82
Expected default frequency	EDF	0.0000000%	0.0003094%	0.0001299%	0.0018670%	0.0000727%

Annexe 3: EDP – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		5 461.63	5 021.02	4 596.15	3 497.09	1 448.13
Long term borrowings		15 968.76	16 400.83	15 653.88	15 550.27	15 469.64
Other short term liabilities		3 667.74	2 941.39	3 499.67	4 068.05	3 864.23
Other long term liabilities		6 023.22	6 540.63	6 665.78	7 231.97	6 865.09
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		3 082.81	3 287.68	3 451.72	4 330.09	3 934.32
Market Data		**2013**	**2014**	**2015**	**2016**	**2017**
Market capitalization		9 689.27	11 691.15	12 072.21	10 518.19	10 485.91
Stock price		2.67	3.22	3.32	2.89	2.89
# of common shares		3 628.94	3 633.05	3 635.11	3 634.48	3 634.63
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		3 628.94	3 633.05	3 635.11	3 634.48	3 634.63
Historical volatility (200D)		17.45%	22.57%	22.69%	23.72%	18.18%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.19%
Model parameters		**2013**	**2014**	**2015**	**2016**	**2017**
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	17.45%	22.57%	22.69%	23.72%	18.18%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		26 275.86	26 162.86	25 332.75	24 697.37	22 282.43
Debt		23 193.06	22 875.18	21 881.03	20 367.28	18 348.10
Debt-per-share	D	6.39	6.30	6.02	5.60	5.05
Calibration of the CreditGrades model		**2013**	**2014**	**2015**	**2016**	**2017**
Initial asset value	V_0	5.87	6.37	6.33	5.70	5.41
Asset volatility	σV	7.94%	11.41%	11.90%	12.05%	9.70%
d parameter	d	2.01	2.21	2.30	2.22	2.34
α parameter	α	0.31	0.32	0.32	0.32	0.32
Approximated survival probability		**2013**	**2014**	**2015**	**2016**	**2017**
Approximated survival probability	SP(0,t)	96.54055%	98.03471%	98.52869%	98.02093%	98.95841%
Aproximated probability of default	PD(0,t)	3.459455%	1.965290%	1.471313%	1.979068%	1.041594%
Exact survival probability		**2013**	**2014**	**2015**	**2016**	**2017**
Parameter a_1	a_1	2.17	2.50	2.63	2.51	2.69
Parameter b_1	b_1	2.09	2.31	2.42	2.31	2.55
parameter ρ_1	ρ1	0.97	0.93	0.93	0.93	0.95
Parameter a_2	a_2	2.47	2.80	2.93	2.81	2.99
Parameter b_2	b_2	-2.40	-2.63	-2.74	-2.63	-2.86
parameter ρ_2	ρ2	-0.97	-0.93	-0.93	-0.93	-0.95
Exact survival probability	SP(0,t)	97.32323%	98.29200%	98.69650%	98.25627%	99.11632%
Exact probability of default	PD(0,t)	2.676773%	1.708004%	1.303498%	1.743735%	0.883679%

Annexe 4: Galp – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	9 880.52	6 991.41	8 947.61	11 767.07	12 708.27
Equity volatility	σ_E	18.57%	24.29%	38.33%	30.36%	15.81%
Liabilities	L_t	7 301.62	6 790.70	6 604.91	5 895.74	6 268.00
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.01	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	8.07	5.81	3.99	5.51	10.49
Parâmetro	d_2	7.96	5.68	3.77	5.31	10.39
Equity value	E_t	9 880.52	6 991.41	8 947.61	11 767.06	12 708.26
Equity volatility	σ_E	18.57%	24.29%	38.33%	30.36%	15.81%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	17 175.27	13 788.26	15 578.80	17 713.16	19 022.85
Asset volatility	σ	10.68%	12.32%	22.01%	20.17%	10.56%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0000000%	0.0000007%	0.0081729%	0.0000054%	0.0000000%

Annexe 5: Galp – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	9 880.52	6 991.41	8 947.61	11 767.07	12 708.27
Equity volatility	σ_E	18.57%	24.29%	38.33%	30.36%	15.81%
Liabilities	L_t	5 065.93	4473.64	4318.41	4026.61	4344.00
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	14 941.68	11 469.10	13 283.23	15 828.07	17 084.56
Asset volatility	σ	12.60%	16.89%	24.89%	21.27%	11.80%
Asset drift rate	μ	3.08%	3.32%	4.88%	4.25%	2.76%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		2 022.27	1 315.29	1 295.01	1 272.65	1 440.00
Long-Term Debt		3 303.72	3 361.12	3 059.53	2 577.53	2 532.00
Other short term liabilities		807.96	841.29	736.90	884.84	980.00
Other long term liabilities		1 167.66	1 273.00	1 513.47	1 160.72	1 316.00
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	5 065.93	4 473.64	4 318.41	4 026.61	4 344.00
Distance to default	DD	8.77	5.69	4.59	6.53	11.78
Expected default frequency	EDF	0.0000000%	0.0000006%	0.0002254%	0.0000000%	0.0000000%

Annexe 6: Galp – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		2 022.27	1 315.29	1 295.01	1 272.65	1 440.00
Long term borrowings		3 303.72	3 361.12	3 059.53	2 577.53	2 532.00
Other short term liabilities		807.96	841.29	736.90	884.84	980.00
Other long term liabilities		1 167.66	1 273.00	1 513.47	1 160.72	1 316.00
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		1 254.89	1 420.18	1 416.05	1 562.94	1 461.00
Market Data		2013	2014	2015	2016	2017
Market capitalization		9 880.52	6 991.41	8 889.57	11 767.07	12 708.27
Stock price		11.92	8.43	10.72	14.19	15.33
# of common shares		829.25	829.25	829.25	829.25	829.25
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		829.25	829.25	829.25	829.25	829.25
Historical volatility (200D)		16.46%	22.78%	32.67%	22.37%	14.31%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	16.46%	22.78%	32.67%	22.37%	14.31%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		6 313.81	5 733.55	5 479.72	4 872.96	5 120.00
Debt		5 058.91	4 313.37	4 063.67	3 310.02	3 659.00
Debt-per-share	D	6.10	5.20	4.90	3.99	4.41
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	14.97	11.03	13.17	16.19	17.53
Asset volatility	σV	13.10%	17.41%	26.59%	19.61%	12.51%
d parameter	d	5.37	4.64	5.88	8.87	8.69
α parameter	α	0.33	0.35	0.40	0.36	0.33
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	99.99993%	99.99795%	99.99765%	100.00000%	100.00000%
Aproximated probability of default	PD(0,t)	0.000065%	0.002047%	0.002352%	0.000000%	0.000000%
Exact survival probability		2013	2014	2015	2016	2017
Parameter a_1	a_1	5.45	4.97	5.76	7.13	7.06
Parameter b_1	b_1	4.97	4.25	4.22	5.91	6.49
parameter ρ_1	$\rho 1$	0.92	0.86	0.75	0.84	0.92
Parameter a_2	a_2	5.75	5.27	6.06	7.43	7.36
Parameter b_2	b_2	-5.30	-4.60	-4.62	-6.27	-6.82
parameter ρ_2	$\rho 2$	-0.92	-0.86	-0.75	-0.84	-0.92
Exact survival probability	SP(0,t)	99.99994%	99.99796%	99.99765%	100.00000%	100.00000%
Exact probability of default	PD(0,t)	0.000065%	0.002042%	0.002354%	0.000000%	0.000000%

Annexe 7: Jerónimo Martins – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	8 933.19	5 238.00	7 597.77	9 263.12	10 191.40
Equity volatility	σ_E	27.72%	32.39%	35.38%	24.88%	18.49%
Liabilities	L_t	3 427.00	3 533.73	3 739.50	3 695.13	3 030.00
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	6.51	4.80	4.79	7.13	10.38
Parâmetro	d_2	6.30	4.60	4.55	6.95	10.24
Equity value	E_t	8 933.19	5 238.00	7 597.77	9 263.12	10 191.40
Equity volatility	σ_E	27.72%	32.39%	35.38%	24.88%	18.49%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	12 356.97	8 774.93	11 352.16	12 989.81	13 243.93
Asset volatility	σ	20.04%	19.33%	23.68%	17.74%	14.23%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0000000%	0.0002077%	0.0002626%	0.0000000%	0.0000000%

Annexe 8: Jerónimo Martins – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	8 933.19	5 238.00	7 597.77	9 263.12	10 191.40
Equity volatility	σ_E	27.72%	32.39%	35.38%	24.88%	18.49%
Liabilities	L_t	3 155.43	3 253.44	3 381.19	3 565.60	2 914.00
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	12 085.65	8 494.39	10 992.43	12 708.35	13 127.07
Asset volatility	σ	21.22%	21.67%	24.11%	18.08%	16.83%
Asset drift rate	μ	4.70%	2.89%	5.26%	3.76%	2.45%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		324.72	340.93	123.51	224.58	298.00
Long-Term Debt		369.07	373.88	534.42	114.83	232.00
Other short term liabilities		2 559.13	2 632.23	2 899.37	3 211.49	2 500.00
Other long term liabilities		174.08	186.70	182.20	144.23	0.00
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	3 155.43	3 253.44	3 381.19	3 565.60	2 914.00
Distance to default	DD	6.44	4.45	4.99	7.15	9.01
Expected default frequency	EDF	0.0000000%	0.0004213%	0.0000304%	0.0000000%	0.0000000%

Annexe 9: Jerónimo Martins – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		324.72	340.93	123.51	224.58	298.00
Long term borrowings		369.07	373.88	534.42	114.83	232.00
Other short term liabilities		2 559.13	2 632.23	2 899.37	3 211.49	2 500.00
Other long term liabilities		174.08	186.70	182.20	144.23	0.00
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		235.84	242.88	251.53	252.50	225.00
Market Data		2013	2014	2015	2016	2017
Market capitalization		8 933.19	5 238.00	10 191.40	9 263.12	7 538.07
Stock price		14.22	8.34	16.20	14.74	12.00
# of common shares		628.43	628.43	629.29	628.43	628.17
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		628.43	628.43	629.29	628.43	628.17
Historical volatility (200D)		24.29%	29.97%	29.51%	18.57%	15.70%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	24.29%	29.97%	29.51%	18.57%	15.70%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		2 060.40	2 124.27	2 198.71	2 017.27	1 780.00
Debt		1 824.56	1 881.39	1 947.19	1 764.77	1 555.00
Debt-per-share	D	2.90	2.99	3.09	2.81	2.48
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	15.67	9.83	17.74	16.14	13.24
Asset volatility	σV	22.04%	25.41%	26.94%	16.95%	14.24%
d parameter	d	11.81	7.19	12.55	12.58	11.70
α parameter	α	0.37	0.39	0.40	0.34	0.33
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	100.00000%	99.99986%	100.00000%	100.00000%	100.00000%
Aproximated probability of default	PD(0,t)	0.000000%	0.000139%	0.000000%	0.000000%	0.000000%
Exact survival probability		2013	2014	2015	2016	2017
Parameter a_1	a_1	8.08	6.42	8.28	8.29	8.05
Parameter b_1	b_1	6.45	4.82	6.07	7.18	7.24
parameter ρ_1	$\rho 1$	0.81	0.76	0.74	0.87	0.90
Parameter a_2	a_2	8.38	6.72	8.58	8.59	8.35
Parameter b_2	b_2	-6.82	-5.21	-6.48	-7.52	-7.57
parameter ρ_2	$\rho 2$	-0.81	-0.76	-0.74	-0.87	-0.90
Exact survival probability	SP(0,t)	100.00000%	99.99986%	100.00000%	100.00000%	100.00000%
Exact probability of default	PD(0,t)	0.000000%	0.000139%	0.000000%	0.000000%	0.000000%

Annexe 10: Sonae – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	2 098.00	2 048.00	2 112.00	1 748.00	2 252.00
Equity volatility	σ_E	29.36%	30.76%	31.51%	33.75%	22.31%
Liabilities	L_t	3 391.22	3 724.51	3 436.74	3 447.41	3 469.61
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	4.35	4.07	4.05	3.66	5.73
Parâmetro	d_2	4.24	3.96	3.93	3.55	5.65
Equity value	E_t	2 098.00	2 048.00	2 112.00	1 748.00	2 252.00
Equity volatility	σ_E	29.36%	30.76%	31.51%	33.75%	22.31%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	5 486.03	5 775.88	5 562.42	5 224.83	5 747.40
Asset volatility	σ	11.23%	10.91%	11.96%	11.29%	8.74%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0011374%	0.0037553%	0.0042171%	0.0192624%	0.0000008%

Annexe 11: Sonae – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	2 098.00	2 048.00	2 112.00	1 748.00	2 252.00
Equity volatility	σ_E	29.36%	30.76%	31.51%	33.75%	22.31%
Liabilities	L_t	2 775.63	3 183.53	2 723.02	2 756.20	2 777.19
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value A_t	A_t	4 870.99	5 234.33	4 845.85	4 515.73	5 049.84
Asset volatility σ	σ	15.89%	15.12%	15.67%	13.13%	9.53%
Asset drift rate μ	μ	3.97%	3.15%	3.48%	2.77%	2.65%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		1 532.64	2 147.71	1 519.85	1 564.72	1 527.99
Long-Term Debt		1 362.60	907.01	1 272.86	1 209.83	1 220.23
Other short term liabilities		450.19	494.84	489.45	500.28	556.78
Other long term liabilities		223.00	174.94	154.57	172.58	164.61
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	2 775.63	3 183.53	2 723.02	2 756.20	2 777.19
Distance to default	DD	3.71	3.42	3.82	3.90	6.50
Expected default frequency	EDF	0.0103913%	0.0312777%	0.0066043%	0.0047230%	0.0000000%

Annexe 12: Sonae – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		1 402.71	2 147.71	1 519.85	1 564.72	1 527.99
Long term borrowings		1 362.60	907.01	1 272.86	1 209.83	1 220.23
Other short term liabilities		450.19	494.84	489.45	500.28	556.78
Other long term liabilities		175.73	174.94	154.57	172.58	164.61
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		344.33	160.20	136.30	169.04	167.81
Market Data		2013	2014	2015	2016	2017
Market capitalization		2 098.00	2 048.00	2 096.00	1 748.00	2 252.00
Stock price		1.05	1.02	1.05	0.87	1.13
# of common shares		2 000.00	2 000.00	2 000.00	2 000.00	2 000.00
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		2 000.00	2 000.00	2 000.00	2 000.00	2 000.00
Historical volatility (200D)		26.99%	27.29%	27.80%	28.04%	18.53%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	26.99%	27.29%	27.80%	28.04%	18.53%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		3 078.27	3 389.61	3 114.73	3 110.98	3 108.92
Debt		2 733.94	3 229.41	2 978.42	2 941.94	2 941.11
Debt-per-share	D	1.37	1.61	1.49	1.47	1.47
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	1.73	1.83	1.79	1.61	1.86
Asset volatility	σV	16.34%	15.26%	16.25%	15.23%	11.21%
d parameter	d	2.77	2.48	2.63	2.39	2.77
α parameter	α	0.34	0.34	0.34	0.34	0.32
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	99.53507%	98.92301%	99.27392%	98.02093%	99.75841%
Aproximated probability of default	PD(0,t)	0.464926%	1.076990%	0.726084%	1.979068%	0.241592%
Exact survival probability		2013	2014	2015	2016	2017
Parameter a_1	a_1	3.25	2.88	3.08	2.76	3.25
Parameter b_1	b_1	2.82	2.53	2.67	2.43	3.02
parameter ρ_1	$\rho 1$	0.88	0.89	0.88	0.89	0.94
Parameter a_2	a_2	3.55	3.18	3.38	3.06	3.55
Parameter b_2	b_2	-3.16	-2.87	-3.01	-2.76	-3.34
parameter ρ_2	$\rho 2$	-0.88	-0.89	-0.88	-0.89	-0.94
Exact survival probability	SP(0,t)	99.55086%	98.98480%	99.30348%	98.64629%	99.78007%
Exact probability of default	PD(0,t)	0.449144%	1.015195%	0.696517%	1.353711%	0.219929%

Annexe 13: Nos – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	2 779.69	2 684.31	3 748.00	2 887.47	2 823.60
Equity volatility	σ_E	29.30%	27.93%	25.22%	27.12%	18.33%
Liabilities	L_t	1 869.66	1 895.80	1 912.97	1 929.54	1 880.70
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	5.29	5.47	6.57	5.70	8.37
Parâmetro	d_2	5.11	5.31	6.41	5.54	8.26
Equity value	E_t	2 779.69	2 684.31	3 748.00	2 887.46	2 823.60
Equity volatility	σ_E	29.30%	27.93%	25.22%	27.12%	18.33%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	4 647.59	4 581.83	5 668.59	4 833.49	4 718.28
Asset volatility	σ	17.52%	16.36%	16.68%	16.20%	10.97%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0000158%	0.0000056%	0.0000000%	0.0000016%	0.0000000%

Annexe 14: Nos – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	2 779.69	2 684.31	3 748.00	2 887.47	2 823.60
Equity volatility	σ_E	29.30%	27.93%	25.22%	27.12%	18.33%
Liabilities	L_t	1 315.94	1 498.89	1 337.60	1 345.20	1 316.85
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	4 094.40	4 184.56	5 090.93	4 233.91	4 150.24
Asset volatility	σ	26.43%	18.01%	18.13%	19.02%	12.48%
Asset drift rate	μ	2.17%	3.21%	3.49%	4.18%	3.28%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		558.25	892.69	533.12	498.29	197.30
Long-Term Debt		928.24	616.53	979.42	972.00	891.10
Other short term liabilities		203.98	209.29	229.11	262.56	555.70
Other long term liabilities		179.19	177.30	171.32	196.69	236.60
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	1 315.94	1 498.89	1 337.60	1 345.20	1 316.85
Distance to default	DD	4.24	5.79	7.48	6.15	9.40
Expected default frequency	EDF	0.0010959%	0.0000004%	0.0000000%	0.0000000%	0.0000000%

Annexe 15: Nos – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		558.25	892.69	533.12	498.29	197.30
Long term borrowings		925.04	607.78	979.42	972.00	891.10
Other short term liabilities		203.98	209.29	229.11	262.56	555.70
Other long term liabilities		182.39	186.04	171.32	196.69	236.60
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		9.62	9.82	9.43	9.04	9.33
Market Data		2013	2014	2015	2016	2017
Market capitalization		2 779.69	2 684.31	3 788.64	2 887.47	2 823.60
Stock price		5.40	5.24	7.25	5.64	5.48
# of common shares		514.76	512.66	522.86	512.14	515.16
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		514.76	512.66	522.86	512.14	515.16
Historical volatility (200D)		26.86%	24.51%	22.16%	22.41%	14.89%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	26.86%	24.51%	22.16%	22.41%	14.89%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		1 676.47	1 698.14	1 712.76	1 699.92	1 484.55
Debt		1 666.86	1 688.32	1 703.33	1 690.88	1 475.22
Debt-per-share	D	3.24	3.29	3.26	3.30	2.86
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	7.02	6.88	8.87	7.29	6.91
Asset volatility	σV	20.66%	18.65%	18.10%	17.33%	11.81%
d parameter	d	4.74	4.57	5.96	4.83	5.28
α parameter	α	0.36	0.35	0.35	0.35	0.32
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	99.995877%	99.996461%	99.999916%	99.998815%	99.999945%
Aproximated probability of default	PD(0,t)	0.004123%	0.003539%	0.000084%	0.001185%	0.000055%
Exact survival probability		2013	2014	2015	2016	2017
Parameter a_1	a_1	5.04	4.92	5.80	5.10	5.40
Parameter b_1	b_1	4.09	4.13	4.92	4.37	5.00
parameter ρ_1	$\rho 1$	0.82	0.85	0.86	0.87	0.93
Parameter a_2	a_2	5.34	5.22	6.10	5.40	5.70
Parameter b_2	b_2	-4.46	-4.48	-5.27	-4.72	-5.32
parameter ρ_2	$\rho 2$	-0.82	-0.85	-0.86	-0.87	-0.93
Exact survival probability	SP(0,t)	99.99588%	99.99647%	99.99992%	99.99882%	99.99995%
Exact probability of default	PD(0,t)	0.004123%	0.003534%	0.000084%	0.001183%	0.000055%

Annexe 16: Cofina – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	51.39	48.41	45.13	26.67	46.05
Equity volatility	σ_E	34.75%	39.93%	28.76%	32.58%	38.74%
Liabilities	L_t	119.15	114.18	106.68	95.71	82.48
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	3.48	3.03	4.17	3.49	3.26
Parâmetro	d_2	3.37	2.91	4.08	3.42	3.12
Equity value	E_t	51.39	48.41	45.13	26.67	46.05
Equity volatility	σ_E	34.75%	39.93%	28.76%	32.58%	38.74%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	170.42	162.68	152.24	123.19	129.15
Asset volatility	σ	10.48%	11.90%	8.52%	7.05%	13.82%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		0.0373808%	0.1813994%	0.0022321%	0.0309940%	0.0900539%

Annexe 17: Cofina – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	51.39	48.41	45.13	26.67	46.05
Equity volatility	σ_E	34.75%	39.93%	28.76%	32.58%	38.74%
Liabilities	L_t	85.63	84.41	75.67	75.22	71.21
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	136.81	132.88	121.10	102.35	117.69
Asset volatility	σ	21.60%	16.63%	14.23%	14.00%	22.36%
Asset drift rate	μ	1.79%	2.52%	1.90%	0.26%	-0.70%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		33.66	37.43	30.49	42.37	47.44
Long-Term Debt		58.08	49.32	49.54	33.19	16.74
Other short term liabilities		18.45	17.16	14.17	12.36	12.49
Other long term liabilities		8.96	10.32	12.49	7.79	5.81
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	85.63	84.41	75.67	75.22	71.21
Distance to default	DD	2.14	2.80	3.37	2.15	2.10
Expected default frequency	EDF	1.6001531%	0.2574810%	0.0381889%	1.5820730%	1.7667050%

Annexe 18: Cofina – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		31.64	37.43	30.49	42.37	47.44
Long term borrowings		58.08	49.32	49.54	33.19	16.74
Other short term liabilities		18.45	17.16	14.17	12.36	12.49
Other long term liabilities		8.96	10.27	12.49	7.79	5.81
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		0.77	0.09	0.00	0.00	0.00
Market Data		2013	2014	2015	2016	2017
Market capitalization		51.39	48.41	45.64	26.67	46.05
Stock price		0.50	0.47	0.45	0.26	0.45
# of common shares		102.57	102.57	102.57	102.57	102.57
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		102.57	102.57	102.57	102.57	102.57
Historical volatility (200D)		31.87%	35.16%	24.34%	30.32%	36.31%
Risk-free interest rate		0.10%	-0.09%	-0.40%	-0.84%	-0.73%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	31.87%	35.16%	24.34%	30.32%	36.31%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		103.42	100.46	93.35	85.63	73.33
Debt		102.65	100.37	93.35	85.63	73.33
Debt-per-share	D	1.00	0.98	0.91	0.83	0.71
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	1.00	0.96	0.90	0.68	0.81
Asset volatility	σV	15.94%	17.26%	12.03%	11.63%	20.21%
d parameter	d	2.19	2.15	2.16	1.78	2.47
α parameter	α	0.34	0.35	0.32	0.32	0.36
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	96.91831%	96.08199%	97.53660%	90.18198%	98.06255%
Aproximated probability of default	PD(0,t)	3.081692%	3.918006%	2.463400%	9.818024%	1.937451%
Exact survival probability		2013	2014	2015	2016	2017
Parameter a_1	a_1	2.46	2.40	2.42	1.76	2.86
Parameter b_1	b_1	2.14	2.04	2.23	1.62	2.32
parameter ρ_1	$\rho 1$	0.88	0.87	0.93	0.93	0.83
Parameter a_2	a_2	2.76	2.70	2.72	2.06	3.16
Parameter b_2	b_2	-2.48	-2.38	-2.55	-1.95	-2.68
parameter ρ_2	$\rho 2$	-0.88	-0.87	-0.93	-0.93	-0.83
Exact survival probability	SP(0,t)	97.14192%	96.33318%	97.84529%	91.94858%	98.11392%
Exact probability of default	PD(0,t)	2.858084%	3.666821%	2.154714%	8.051424%	1.886081%

Annexe 19: Media Capital – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	98.04	126.77	202.83	190.15	266.22
Equity volatility	σ_E	78.22%	107.92%	60.08%	49.86%	48.37%
Liabilities	L_t	206.09	197.16	191.97	176.14	159.80
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	1.54	1.17	2.47	2.95	3.39
Parâmetro	d_2	1.27	0.67	2.16	2.69	3.09
Equity value	E_t	98.03	126.77	202.83	190.15	266.22
Equity volatility	σ_E	78.22%	107.92%	60.08%	49.86%	48.37%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	301.51	312.42	395.26	367.75	427.19
Asset volatility	σ	27.10%	49.80%	31.04%	25.82%	30.15%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		10.1654145%	25.0306591%	1.5435434%	0.3586034%	0.1014918%

Annexe 20: Media Capital – Results of KMV Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	98.04	126.77	202.83	190.15	266.22
Equity volatility	σ_E	78.22%	107.92%	60.08%	49.86%	48.37%
Liabilities	L_t	163.26	141.18	131.67	133.28	125.71
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	259.73	252.19	335.00	324.54	392.86
Asset volatility	σ	29.24%	72.55%	31.30%	29.63%	30.61%
Asset drift rate	μ	-0.62%	-1.17%	0.32%	-0.57%	-2.98%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		55.87	36.49	24.38	62.01	47.72
Long-Term Debt		77.57	103.66	112.08	77.59	61.23
Other short term liabilities		64.58	48.71	46.99	28.41	43.89
Other long term liabilities		8.08	8.30	8.52	8.13	6.96
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	163.26	141.18	131.67	133.28	125.71
Distance to default	DD	1.42	0.42	2.84	2.84	3.47
Expected default frequency	EDF	7.7754282%	33.6963178%	0.2276502%	0.2280471%	0.0257995%

Annexe 21: Media Capital – Results of CreditGrades Model

Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		55.87	36.49	24.38	62.01	47.72
Long term borrowings		77.57	103.66	112.08	77.59	61.23
Other short term liabilities		64.58	48.71	46.99	28.41	43.89
Other long term liabilities		8.08	8.30	8.52	8.13	6.96
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		0.00	0.00	0.00	0.00	0.00
Market Data		2013	2014	2015	2016	2017
Market capitalization		98.04	126.77	202.83	190.15	266.22
Stock price		1.16	1.50	2.40	2.25	3.15
# of common shares		84.51	84.51	84.51	84.51	84.51
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		84.51	84.51	84.51	84.51	84.51
Historical volatility (200D)		59.64%	103.56%	24.34%	49.62%	48.27%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	59.64%	103.56%	24.34%	49.62%	48.27%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		169.76	168.65	164.21	157.87	134.38
Debt		169.76	168.65	164.21	157.87	134.38
Debt-per-share	D	2.01	2.00	1.94	1.87	1.59
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	2.16	2.50	3.37	3.18	3.95
Asset volatility	σV	31.96%	62.19%	17.32%	35.06%	38.55%
d parameter	d	2.36	2.74	3.80	3.73	5.43
α parameter	α	0.44	0.69	0.35	0.46	0.49
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	92.40178%	76.99114%	99.97742%	99.18152%	99.87907%
Aproximated probability of default	PD(0,t)	7.598216%	23.008856%	0.022579%	0.818478%	0.120930%
Exact survival probability		2013	2014	2015		2017
Parameter a_1	a_1	2.71	3.21	4.30	4.24	5.49
Parameter b_1	b_1	1.74	1.11	3.68	2.62	3.22
parameter ρ_1	$\rho 1$	0.68	0.43	0.87	0.65	0.61
Parameter a_2	a_2	3.01	3.51	4.60	4.54	5.79
Parameter b_2	b_2	-2.18	-1.80	-4.02	-3.08	-3.71
parameter ρ_2	$\rho 2$	-0.68	-0.43	-0.87	-0.65	-0.61
Exact survival probability	SP(0,t)	92.45506%	76.99515%	99.97760%	99.18161%	99.87905%
Exact probability of default	PD(0,t)	7.544941%	23.004848%	0.022403%	0.818394%	0.120953%

Annexe 22: Teixeira Duarte – Results of Merton Model

Inputs		2013	2014	2015	2016	2017
Equity value	E_t	373.80	298.62	142.38	78.12	93.66
Equity volatility	σ_E	57.60%	46.40%	37.07%	60.76%	51.91%
Liabilities	L_t	2 418.52	2 469.26	2 343.61	2 095.16	1 885.52
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Model Values from Black-Scholes formulae		2013	2014	2015	2016	2017
Parâmetro	d_1	1.82	2.28	2.78	1.56	1.92
Parâmetro	d_2	1.74	2.22	2.76	1.54	1.90
Equity value	E_t	373.80	298.62	142.38	78.12	93.66
Equity volatility	σ_E	57.60%	46.40%	37.07%	60.76%	51.91%
Estimates		2013	2014	2015	2016	2017
Asset value	Vt	2 786.96	2 769.56	2 495.29	2 189.88	1 992.66
Asset volatility	σ	8.00%	5.06%	2.12%	2.30%	2.51%
Probability of Default		2013	2014	2015	2016	2017
Prob(default)		4.0515202%	1.3054472%	0.2903937%	6.1952686%	2.9001725%

Annexe 23: Teixeira Duarte – Results of KMV Model

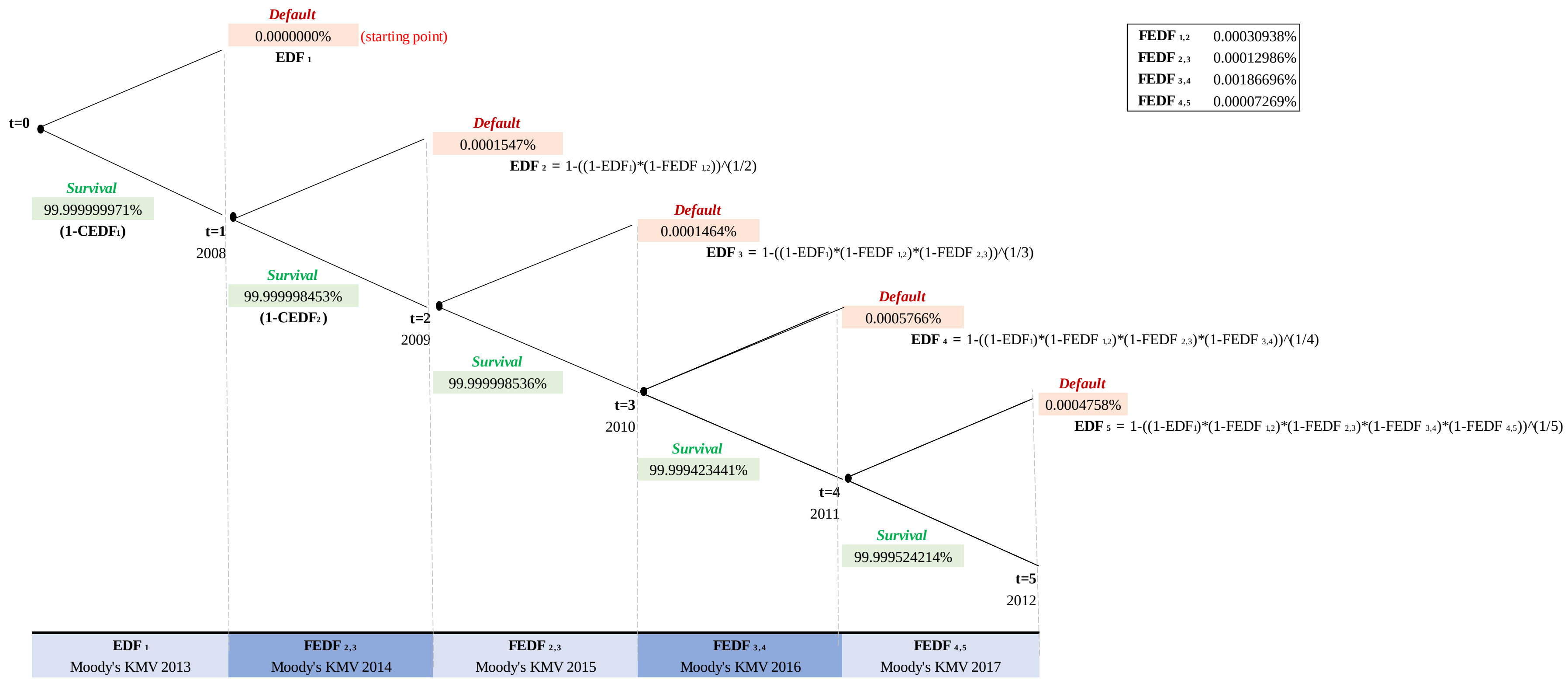
Inputs		2013	2014	2015	2016	2017
Equity value	E_t	373.80	298.62	142.38	190.15	266.22
Equity volatility	σ_E	57.60%	46.40%	37.07%	49.86%	48.37%
Liabilities	L_t	1 876.31	1 888.50	1 841.72	1 594.24	1 477.27
Risk free rate	r	0.09%	-0.09%	-0.40%	-0.85%	-0.74%
Horizon	T-t	1	1	1	1	1
Estimates		2013	2014	2015	2016	2017
Asset value	A_t	2 247.38	2 181.87	1 980.65	1 670.95	1 569.21
Asset volatility	σ	8.43%	10.03%	6.84%	6.23%	6.74%
Asset drift rate	μ	1.39%	1.80%	0.36%	-0.60%	0.18%
Balance Sheet Data		2013	2014	2015	2016	2017
Short term borrowings+Accounts Payable		870.26	930.88	947.41	792.09	459.06
Long-Term Debt		865.04	932.89	821.43	818.40	713.49
Other short term liabilities		463.85	376.86	392.43	301.24	609.97
Other long term liabilities		219.37	228.63	182.35	183.44	103.00
Default probability calculations		2013	2014	2015	2016	2017
Default barrier	PD*	1 876.31	1 888.50	1 841.72	1 594.24	1 477.27
Distance to default	DD	2.26	1.57	1.08	0.63	0.89
Expected default frequency	EDF	1.1783384%	5.8306869%	13.9614492%	26.5157739%	18.7268311%

Annexe 24: Teixeira Duarte – Results of CreditGrades Model

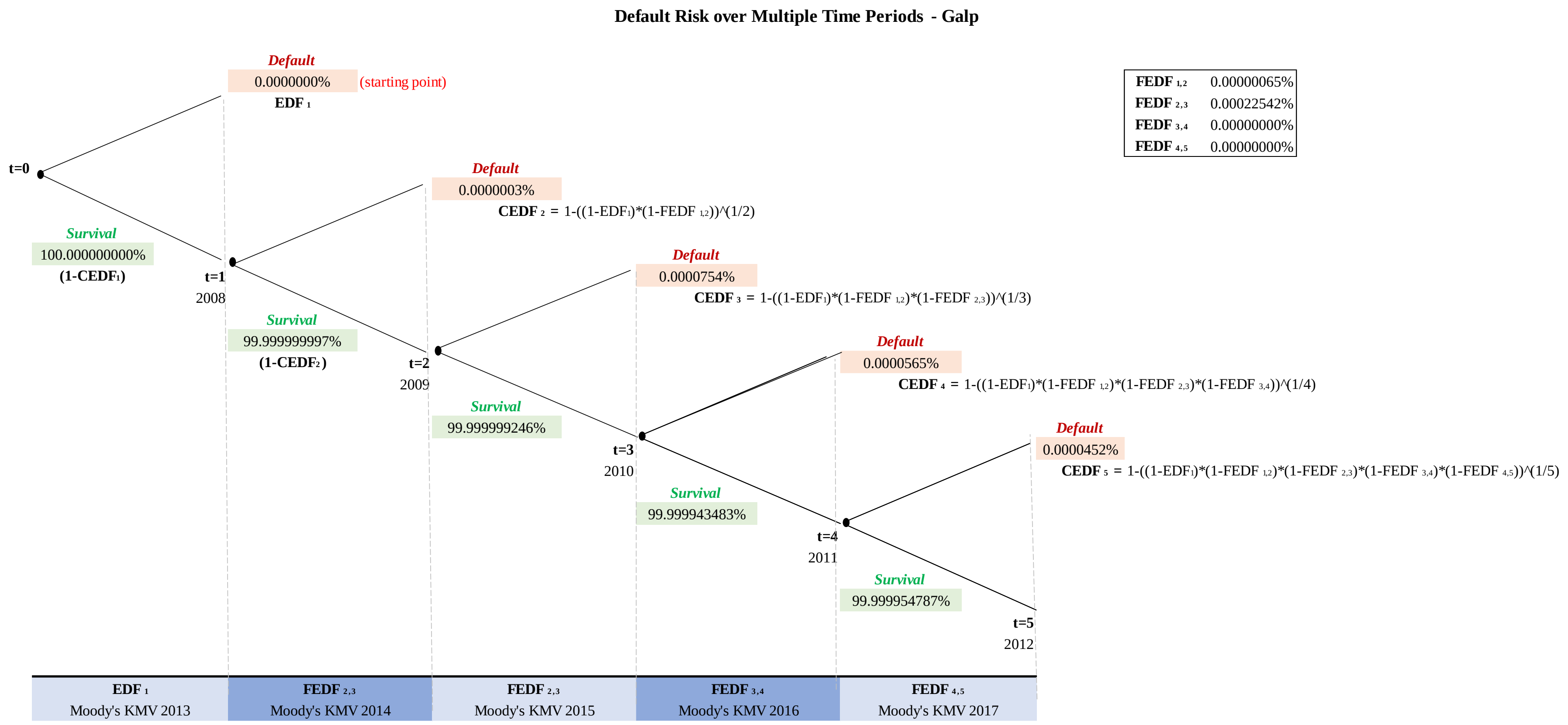
Balance sheet information		2013	2014	2015	2016	2017
Short term borrowings		870.26	930.88	947.41	792.09	459.06
Long term borrowings		865.04	932.89	821.43	818.40	713.49
Other short term liabilities		463.85	376.86	392.43	301.24	609.97
Other long term liabilities		219.37	228.63	182.35	183.44	103.00
Preferred equity		0.00	0.00	0.00	0.00	0.00
Minority interest		35.32	27.06	50.29	49.04	41.28
Market Data		2013	2014	2015	2016	2017
Market capitalization		373.80	298.62	131.88	78.12	93.66
Stock price		0.89	0.71	0.31	0.19	0.22
# of common shares		420.00	420.00	420.00	420.00	420.00
# of preferred shares		0.00	0.00	0.00	0.00	0.00
# of shares		420.00	420.00	420.00	420.00	420.00
Historical volatility (200D)		48.51%	39.42%	33.18%	48.35%	45.92%
Risk-free interest rate		0.09%	-0.09%	-0.40%	-0.83%	-0.74%
Model parameters		2013	2014	2015	2016	2017
Global recovery rate	$\bar{L}$	50.00%	50.00%	50.00%	50.00%	50.00%
Volatility of the barrier	l	30.00%	30.00%	30.00%	30.00%	30.00%
Asset specific recovery rate	R	48.51%	39.42%	33.18%	48.35%	45.92%
Time horizon (in years)	t	1.00	1.00	1.00	1.00	1.00
Financial debt		2 076.91	2 166.52	2 056.22	1 852.83	1 529.03
Debt		2 041.59	2 139.46	2 005.93	1 803.78	1 487.76
Debt-per-share	D	4.86	5.09	4.78	4.29	3.54
Calibration of the CreditGrades model		2013	2014	2015	2016	2017
Initial asset value	V_0	3.32	3.26	2.70	2.33	1.99
Asset volatility	σV	13.00%	8.60%	3.86%	3.85%	5.14%
d parameter	d	1.49	1.40	1.24	1.19	1.23
α parameter	α	0.33	0.31	0.30	0.30	0.30
Approximated survival probability		2013	2014	2015	2016	2017
Approximated survival probability	SP(0,t)	73.45126%	66.93445%	46.82235%	38.40537%	45.52274%
Aproximated probability of default	PD(0,t)	26.548743%	33.065553%	53.177655%	61.594632%	54.477260%
Exact survival probability		2013	2014	2015		2017
Parameter a_1	a_1	1.19	0.97	0.56	0.43	0.55
Parameter b_1	b_1	1.07	0.92	0.55	0.42	0.53
parameter ρ_1	$\rho 1$	0.92	0.96	0.99	0.99	0.99
Parameter a_2	a_2	1.49	1.27	0.86	0.73	0.85
Parameter b_2	b_2	-1.39	-1.23	-0.86	-0.72	-0.84
parameter ρ_2	$\rho 2$	-0.92	-0.96	-0.99	-0.99	-0.99
Exact survival probability	SP(0,t)	78.89868%	76.57697%	67.60435%	62.63647%	65.70689%
Exact probability of default	PD(0,t)	21.101324%	23.423025%	32.395654%	37.363533%	34.293112%

Annexe 25: EDP – EDF Term Structure

Default Risk over Multiple Time Periods - EDP

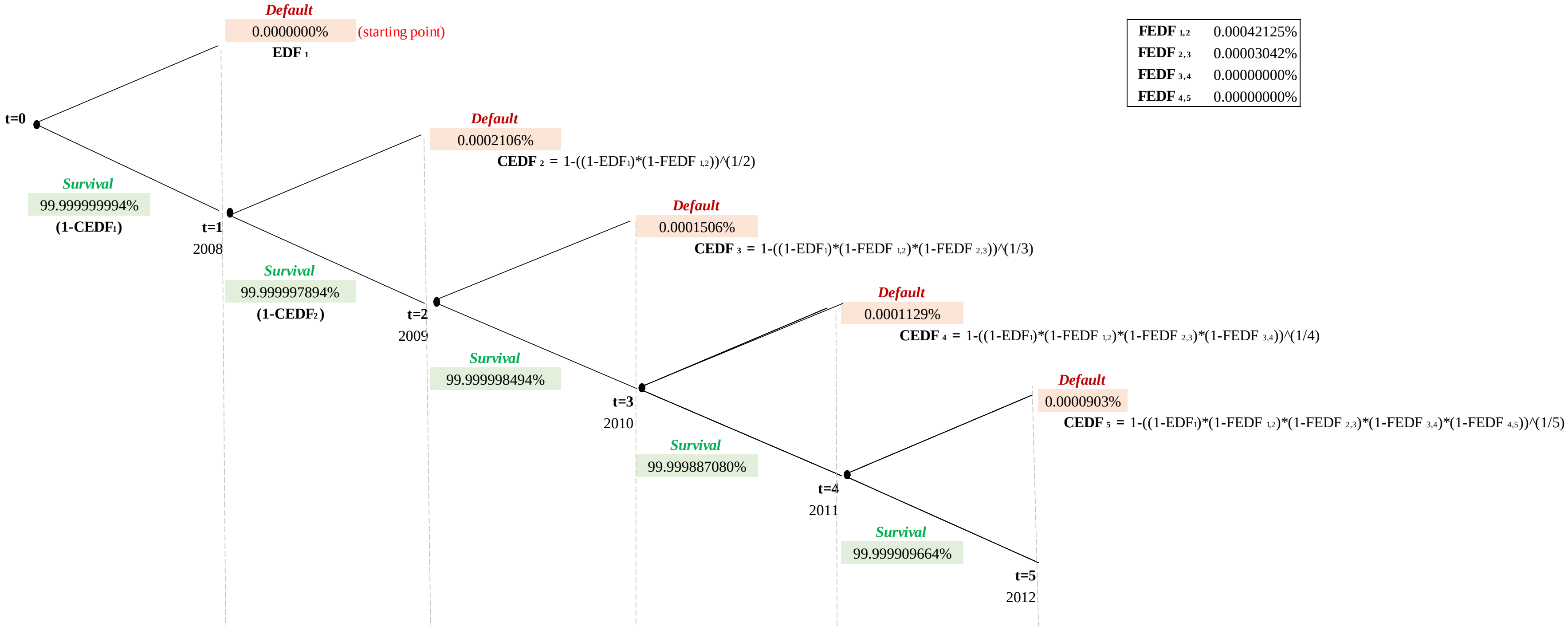


Annexe 26: Galp – EDF Term Structure



Annexe 27: Jerónimo Martins – EDF Term Structure

Default Risk over Multiple Time Periods - Jerónimo Martins

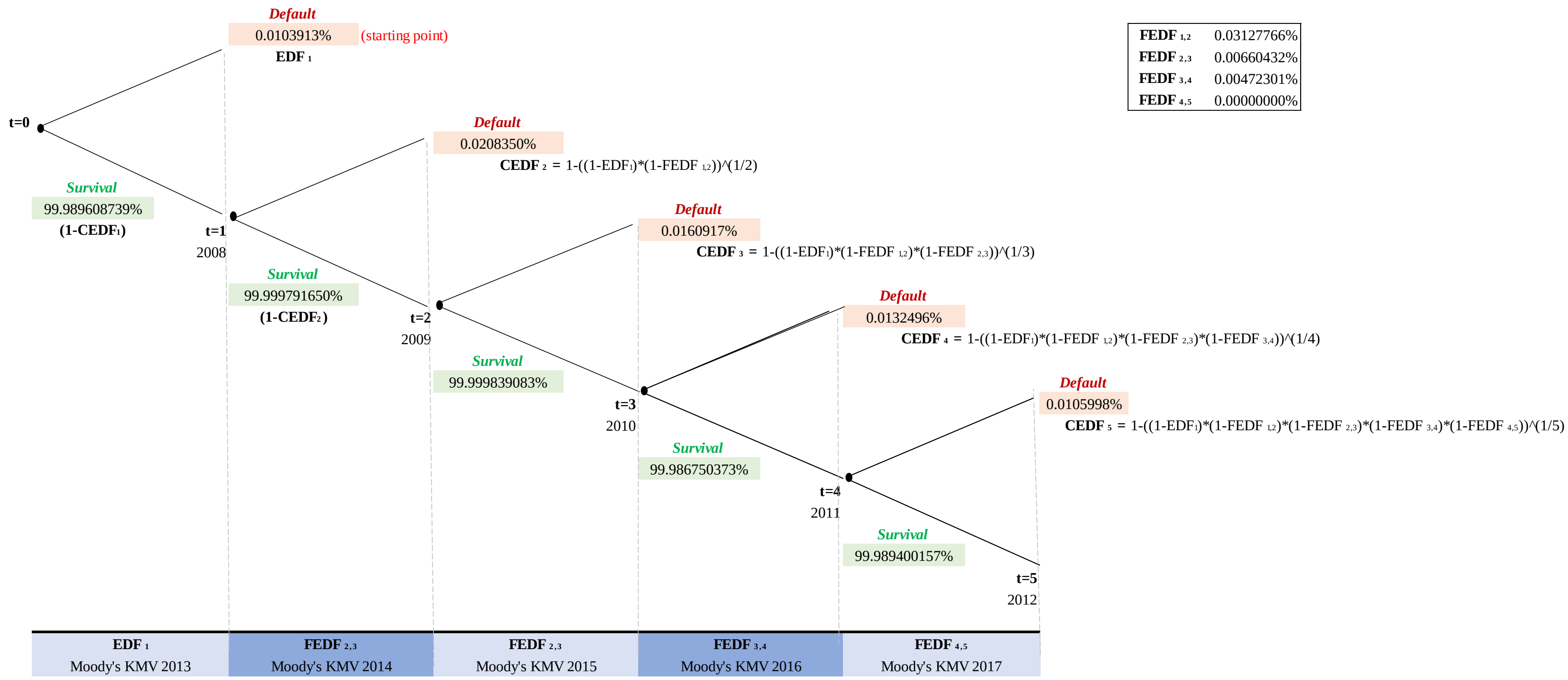


FEDF $_{1,2}$	0.00042125%
FEDF $_{2,3}$	0.00003042%
FEDF $_{3,4}$	0.00000000%
FEDF $_{4,5}$	0.00000000%

EDF $_1$	**FEDF $_{2,3}$**	**FEDF $_{2,3}$**	**FEDF $_{3,4}$**	**FEDF $_{4,5}$**
Moody's KMV 2013	Moody's KMV 2014	Moody's KMV 2015	Moody's KMV 2016	Moody's KMV 2017

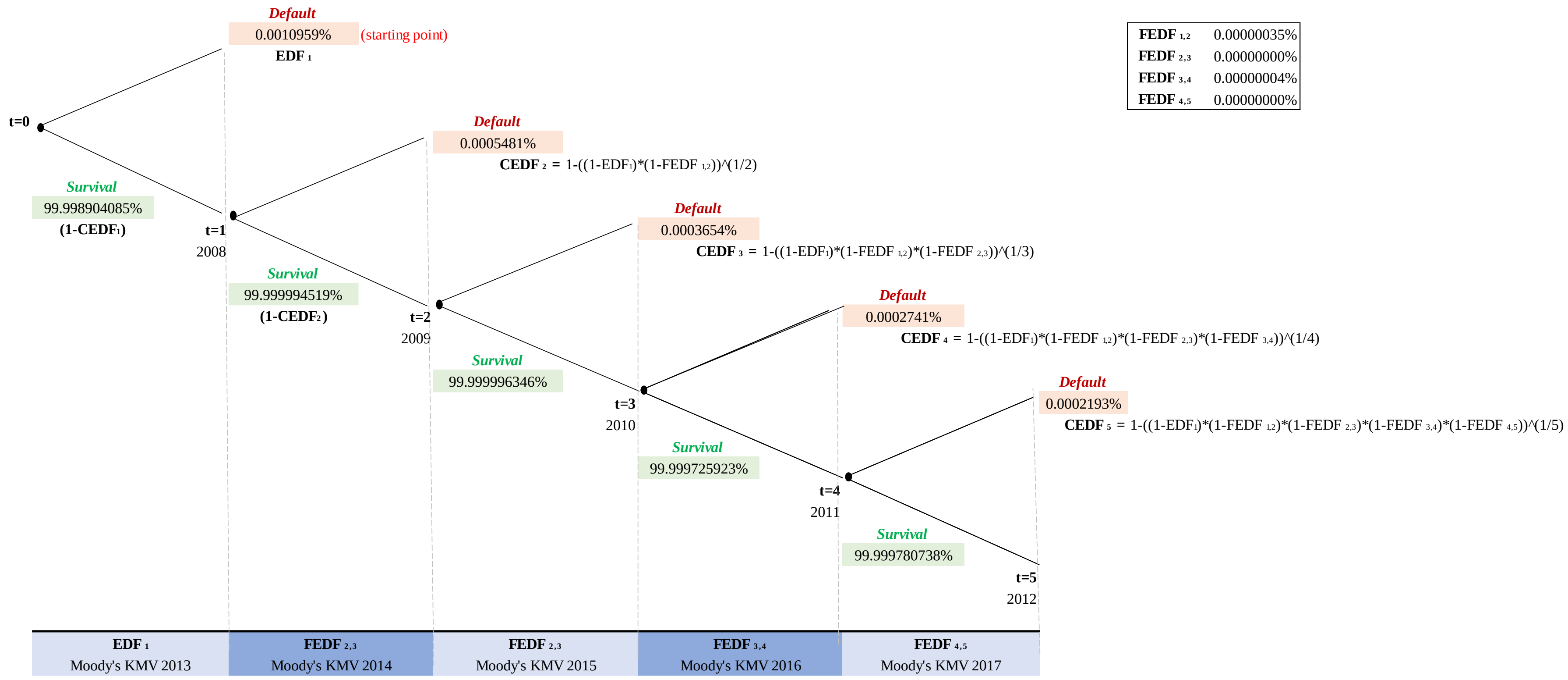
Annexe 28: Sonae – EDF Term Structure

Default Risk over Multiple Time Periods - Sonae



Annexe 29: Nos – EDF Term Structure

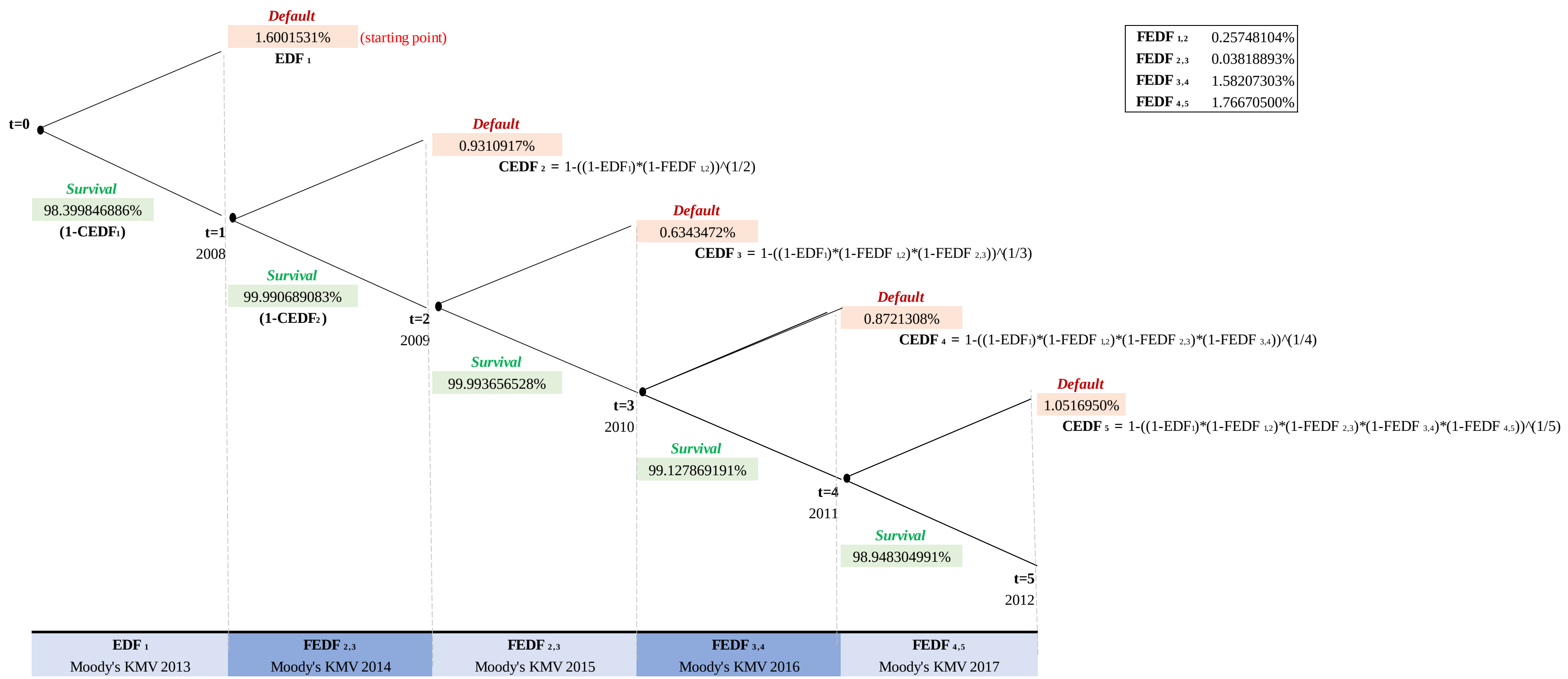
Default Risk over Multiple Time Periods - Nos



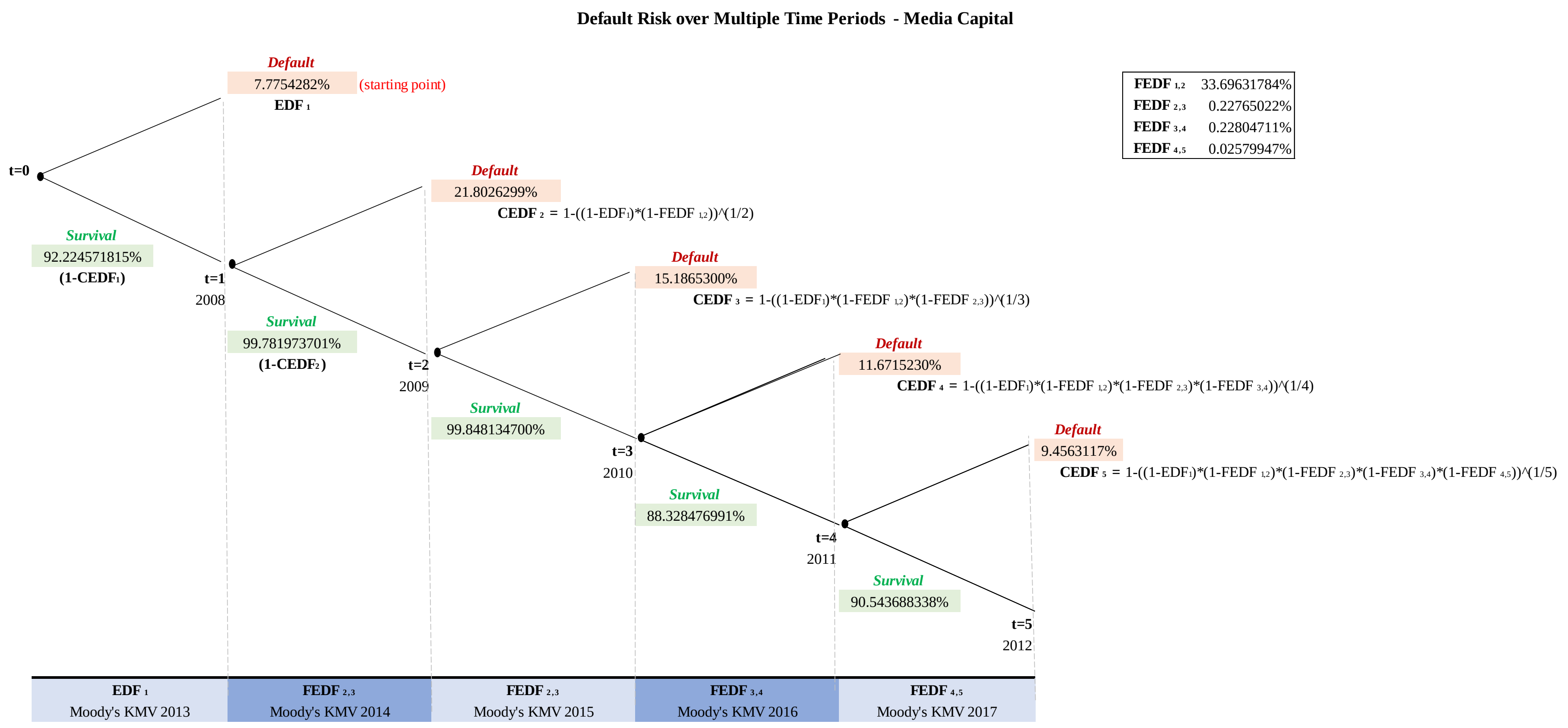
Annexe 30: Cofina – EDF Term Structure

Default Risk over Multiple Time Periods - Cofina

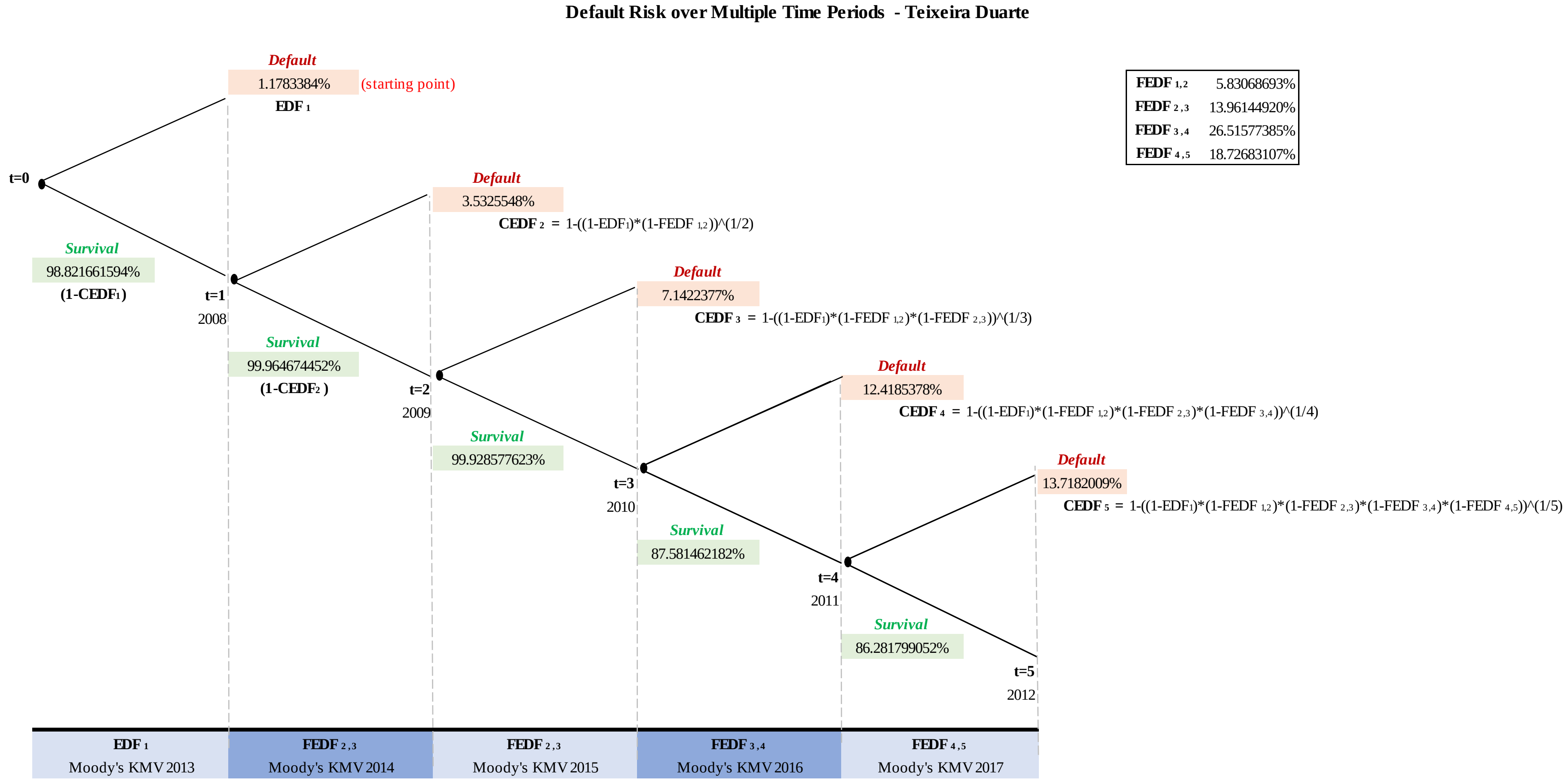
FEDF 1,2	0.25748104%
FEDF 2,3	0.03818893%
FEDF 3,4	1.58207303%
FEDF 4,5	1.76670500%



Annexe 31: Media Capital – EDF Term Structure



Annexe 32: Teixeira Duarte – EDF Term Structure



Annexe 33: Total Debt-to-Equity Ratio

Total Debt/Equity (in Millions of EUR)	Q1 2013	Q2 2013	Q3 2013	Q4 2013	Q1 2014	Q2 2014	Q3 2014	Q4 2014	Q1 2015	Q2 2015	Q3 2015	Q4 2015	Q1 2016	Q2 2016	Q3 2016	Q4 2016	Q1 2017	Q2 2017	Q3 2017	Q4 2017
EDP	168.738	175.018	177.867	174.878	160.918	163.174	171.237	169.588	159.373	158.289	161.159	158.978	152.329	143.732	139.402	131.233	129.160	143.690	124.371	125.500
Galp Energia-Nom	59.218	61.344	59.201	57.312	53.523	51.581	58.036	57.035	51.244	55.018	59.152	57.401	58.030	54.138	55.071	44.361	42.747	47.009	46.037	50.724
Jerónimo Martins	42.474	59.817	47.335	45.070	47.615	56.890	44.013	43.620	47.338	45.321	39.447	41.296	32.068	30.041	17.030	17.051	19.374	26.054	26.028	26.329
Sonae	149.055	144.378	90.462	83.470	116.257	109.452	107.560	101.107	109.101	95.832	84.802	88.382	87.121	91.269	81.375	76.147	83.963	83.647	72.870	69.697
Nos SGPS	437.505	386.224	106.987	107.683	99.605	109.386	106.387	105.651	103.601	117.364	110.150	103.315	96.480	121.291	115.614	113.636	94.834	107.084	95.687	100.184
Cofina, SGPS	580.785	606.759	551.153	462.553	475.936	396.013	356.689	382.665	344.195	371.985	314.044	299.368	283.624	257.129	247.299	256.865	245.734	312.270	229.177	176.491
Media Capital	90.916	100.622	103.931	86.212	91.247	90.063	103.129	88.013	106.245	92.653	97.601	86.488	82.419	85.671	90.247	71.488	71.295	82.355	84.681	67.559
Teixeira Duarte	424.225	514.935	531.876	434.760	381.515	389.724	370.860	345.827	331.504	307.052	332.908	303.914	375.786	391.647	392.519	334.776	346.217	351.668	350.174	248.732

Annexe 34:

Total Debt/Capital	Q1 2013	Q2 2013	Q3 2013	Q4 2013	Q1 2014	Q2 2014	Q3 2014	Q4 2014	Q1 2015	Q2 2015	Q3 2015	Q4 2015	Q1 2016	Q2 2016	Q3 2016	Q4 2016	Q1 2017	Q2 2017	Q3 2017	Q4 2017
EDP	62.789	63.639	64.011	63.620	61.674	62.002	63.132	62.906	61.445	61.284	61.709	61.387	60.369	58.971	58.229	56.754	56.362	58.964	55.431	55.654
Galp Energia-Nom	37.1931	38.02056	37.18623	36.43218	34.8633	34.02858	36.72332	36.32008	33.8816	35.49151	37.16701	36.46818	36.72079	35.12305	35.51335	30.729	29.94622	31.97687	31.52398	33.65353
Jerónimo Martins	29.812	37.428	32.127	31.068	32.256	36.261	30.562	30.372	32.129	31.187	28.288	29.226	24.282	23.101	14.552	14.567	16.230	20.669	20.653	20.842
Sonae	59.848	59.080	47.496	45.495	53.759	52.256	51.821	50.275	52.176	48.936	45.888	46.916	46.559	47.718	44.866	43.229	45.641	45.548	42.153	41.072
Nos SGPS	81.396	79.433	51.688	51.850	49.901	52.241	51.547	51.374	50.884	53.994	52.415	50.760	49.104	54.811	53.621	53.191	48.674	51.710	48.898	50.046
Cofina, SGPS	85.311	85.851	84.643	82.224	82.637	79.839	78.103	79.282	77.487	78.813	75.848	74.960	73.933	71.999	71.206	71.978	71.076	75.744	69.621	63.833
Media Capital	47.621	50.155	50.964	46.298	47.711	47.386	50.770	46.812	51.514	48.093	49.393	46.377	45.181	46.141	47.437	41.687	41.621	45.162	45.853	40.320
Teixeira Duarte	80.924	83.738	84.174	81.300	79.232	79.580	78.762	77.570	76.825	75.433	76.900	75.242	78.982	79.660	79.696	77.000	77.589	77.860	77.786	71.325