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WHY STANDARD RISK MODELS FAILED IN THE SUBPRIME CRISIS?

An approach based on Extreme Value Theory as a measure to quantify market risk of equity securities and portfolios

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Abstract

The assessment of risk is an important and complex task with which market regulators and financial institutions are faced, especially after the last subprime crisis. It is argued that since market data is endogenous to market behaviour, statistical analysis made in times of stability does not provide much guidance in times of crisis. It is well known that the use of Gaussian models to assess financial risk leads to an underestimation of risk. The reason is because these models are unable to capture some important facts such as heavy tails which indicate the presence of large fluctuations in returns.

This thesis provides an overview of the role of extreme value theory in risk management, as a method for modelling and measuring extreme risks. In this empirical study, the performance of different models in estimating value at risk and expected tail loss, using historical data, are compared. Daily returns of nine popular indices (PSI20, CAC40, DAX, Nikkei225, FTSE100, S&P500, Nasdaq, Dow Jones and Sensex) and seven stock market firms (Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs), during the period from 1999 to 2009, are modelled with empirical (or historical), Gaussian and generalized Pareto (peaks over threshold technique of extreme value theory). It is shown that the generalized Pareto distribution fits well to the extreme values using pre-crisis data. The results support the assumption of fat-tailed distributions of asset returns. As expected, the backtesting results show that extreme value theory, in both value at risk and expected tail loss estimation, outperform other models with normality assumption in all tests. Additionally, the results of the generalized Pareto distribution model are not significantly different from the empirical model. Further topics of interest, including software for extreme value theory to compute a tail risk measure, such as Matlab, are also presented.

Keywords: Value at risk, Expected Tail Loss, Extreme Value Theory, Generalized Pareto Distribution, Basel II

JEL classification: G01, G21, G24, G28, G32, G33

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Contents

List of Abbreviations

Apple	Apple Inc.
ARCH	Autoregressive Conditional Heteroskedasticity
BCP	Banco Comercial Português, S.A.
BES	Banco Espírito Santo, S.A.
BIS	Bank for International Settlements
CAC40	Cotation Assistée en Continu - French stock index
Committee	Basel Committee on Banking Supervision
CVAR	Conditional Value at Risk (the same meaning as ETL)
DAX	Deutscher Aktien Index - German stock index
DJ	Dow Jones Industrial Average - American stock index
ES	Expected Shortfall (the same meaning as ETL)
ETL	Expected Tail Loss
EVT	Extreme Value Theory
EWMA	Exponentially Weighted Moving Average
FTSE100	Financial Times Stock Exchange - English stock index
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GE	General Electric Company
GED	Generalized Error Distribution
GEV	Generalized Extreme Value
GPD	Generalized Pareto Distribution
GS	The Goldman Sachs Group, Inc.
HS	Historical Simulation
I.i.d	Independent and Identically Distributed
LB	Lehman Brothers Holdings Inc.
LEL-RM	Limited Expected Losses based Risk Management
LR	Likelihood ratio statistics
LRuc	Likelihood ratio statistics of The Unconditional Coverage Test
MS	Microsoft Corporation
Nasdaq	National Association of Securities Dealers Automated Quotations - American stock index
Nikkei225	Nihon Keizai Shimbun - Japanese stock index
POT	Peaks Over Threshold
PSI20	Portuguese Stock Índex
P&L	Profits and Losses
QQ-plot	Quantile-Quantile plot
Sensex	Bombay Stock Exchange Sensitive Index - Indian stock market index
S&P500	Standard & Poors - American stock index
USA	United States of America
VaR	Value at Risk

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1. Introduction

In recent years value at risk (VaR) has become a very popular measure of market risk and it has been adopted by central bank regulators as the major determinant of the capital requirements for banks in order to cover for potential losses arising from the market risks they are bearing. Recent directives issued by the Basel Committee have established VaR as the standard measure to quantify market risk. The solvency of banks is mainly important for the stability of the financial system. Central banks and the Basel Committee have a well-built concern in systemic risk, where insolvency in one sector of an economy can lead to a national crisis. The global recession following the stock market crash of 1987 prompted a revision of banking regulations as well as new minimum requirements owned by banks that were imposed in the G10 countries, and after adopted by the most of the countries in the world.

According to Jorion (2007), unforeseen adverse situations unaccounted for by existing models triggered huge losses, eventually ending in bankruptcies or almost bankruptcies[1]. The financial crisis that started in August 2007 is a case study for extreme risks and risk management practices. In recent years, the problem of extreme risks in financial markets has become topical following the crises in the Asian and Russian markets, and the unexpected big losses of investment banks such as Barings and Daiwa. The events prompted regulators to address the issue, and from the advent of the Basel Capital Accord of 1996[2] there has been a strong concern about quantifying market risk because banks were demanded to put up risk-adjusted capital as a buffer against likely shortfalls. The Amendment to the Basel Accord in 1996[3] and the broad lines maintained in the Basel Capital Accord of 2004[4] allowed financial institutions to employ their own internal market risk management models in order to determine capital requirements.

Unlike economic capital, when estimating the legal minimum required for the banks against its market risk exposures, the manager can use several risk models and risk metrics or simply apply the standardized rules that are set by the regulators. The banks can use an advanced risk model to estimate the market risk, validated by the regulator and provided that the risk management structure in the bank satisfies certain qualitative criteria. It can be one of the two broad types, either a scenario model or a VaR model. The scenario model is used by

[1] The last remarkable cases are Northern Rock, Bear Stearns, ANB Financial, First Integrity Bank, Roskilde Bank, IndyMac, First Heritage Bank, First National Bank of Nevada, IKB, Silver State, Fannie Mae, Freddie Mac, Lehman Brothers, AIG and Washington Mutual.

[2] See http://www.bis.org/publ/bcbs04a.pdf .

[3] See http://www.bis.org/publ/bcbs24a.pdf .

[4] See http://www.bis.org/publ/bcbs107.pdf .

smaller banks based on an aggregate maximum loss whereas major banks usually adopt the VaR model.

Since the financial crisis began in mid-2007, an important source of losses and the build up of leverage occurred in the trading book. A main contributing factor was that the current capital framework for market risk, based on the 1996 Amendment to the Capital Accord to incorporate market risks, does not capture some key risks. In response, the Basel Committee on Banking Supervision (the Committee) supplements the current VaR based trading book framework with an incremental risk capital charge, which includes default risk as well as migration risk, for unsecuritised credit products. An additional response to the crisis is the introduction of a "stressed VaR" requirement. Losses in most banks trading books during the financial crisis have been significantly higher than the minimum capital requirements under the former Pillar 1 market risk rules.

In June 2006, the Committee published a comprehensive version of the Basel II framework[5] which included the June 2004 Basel II framework, the elements of the 1988 Accord that were not revised during the Basel II process, the 1996 amendment to the Capital Accord to incorporate market risks and the July 2005 paper on the application of Basel II to trading activities and the treatment of double default effects. The Committee released consultative documents on the revisions to the Basel II market risk framework and the guidelines for computing capital for incremental risk in the trading book in July 2008[6] and more recently in July 2009[7]. The Committee has decided that the incremental risk capital charge should capture not only default risk but also migration risk. This decision is reflected in the proposed revisions to the Basel II market risk framework. Additional guidance on the incremental risk capital charge is provided in a separate document, the guidelines for computing capital for incremental risk in the trading book (referred to as "the Guidelines")[8].

According to the revised Basel II market risk framework, the precise number and composition of the stress scenarios to be applied will be determined by the Committee in consultation with the industry by March 2010. Furthermore, the Committee will evaluate a floor for the comprehensive risk capital charge which could be expressed as a percentage of the charge applicable under the standardised measurement method. This evaluation will be based on a quantitative impact study to be conducted in 2010. The improvements in the Basel II framework concerning internal VaR models in particular require banks to justify any

[5] See http://www.bis.org/publ/bcbs128.pdf .
[6] See http://www.bis.org/publ/bcbs140.pdf .
[7] See http://www.bis.org/publ/bcbs158.pdf .
[8] See http://www.bis.org/publ/bcbs159.pdf .

factors used in pricing which are left out in the calculation of VaR. They will also be required to use hypothetical backtesting at least for validation, to update market data at least monthly. To complement the incremental risk capital framework, the Committee extends the scope of the prudent valuation guidance to all positions subject to fair value accounting and make the language more consistent with the existing accounting guidance. The Committee has already conducted a preliminary analysis of the impact of an incremental risk capital charge where it included merely the default and migration risks, largely relying on the data collected from its quantitative impact study on incremental default risk in late 2007. It has collected additional data in 2009 to assess the impact of changes to the trading book capital framework. In the coming months, the Committee will review the calibration of the market risk framework in light of the results of this impact assessment. This review will include multipliers to the current and "stressed VaR" numbers. Banks are expected to comply with the revised requirements by December 31, 2010.

Under the revisions of the Basel II market risk framework proposed by the Basel Committee on Banking Supervision, VaR must be computed on a daily basis in a 99th percentile. In calculating VaR, an instantaneous price shock equivalent to a ten-day movement in prices is to be used, i.e., the minimum "holding period" will be ten trading days. The choice of historical observation period (sample period) for calculating VaR will be constrained to a minimum length of one year. Banks must update their data sets frequently by no less than once every month and reassess them whenever market prices are subject to material changes. No particular type of model is prescribed in the framework, however each model used need to capture all the material risks run by the bank. In this way, banks can use models based, for example, on variance-covariance matrices, historical simulations or Monte Carlo simulations. Banks can also recognise empirical correlations within broad risk categories (e.g. interest rates, exchange rates, equity prices and commodity prices, including related options volatilities in each risk factor category). In addition, banks must calculate the above mentioned "stressed VaR" measure. This measure is intended to replicate a VaR estimation that would be generated on the banks current portfolio if the relevant market factors were experiencing a period of stress and should therefore be based in the same conceptions than VaR, but with different calculations. Banks for International Settlements (BIS) did not prescribe any model to calculate this "stressed VaR" and banks can develop different techniques to translate this new addition[9]. The additional "stressed VaR"

[9] BIS gives an example, for many portfolios, a 12-month period relating to significant losses in 2007/2008 would adequately reflect a period of such stress.

requirement will also help to reduce the procyclicality of the minimum capital requirements for market risk.

VaR is formally defined as a quantile of the forecasted distribution of profits and losses (P&L) over a time span. The practical advantages of VaR methodology are largely counterbalanced by theoretical flaws[10] (see e.g. McNeil, Frey and Embrechts, 2005; Szegö, 2002 for a detailed review of VaR pitfalls), but, even so, VaR has become a regulatory exigency obliging financial institutions to obtain accurate and robust estimates in order to construct adequate capital structures.

The last years have been characterized by significant instabilities in the financial markets. With the latest market adversity started in the United States of America (USA) with the sub-prime mortgage crisis it is clear that there is a need for an approach that comes to terms with problems posed by extreme event estimation. Advances that have been made in VaR should not be lost with the probable (and well deserved) adoption of coherent risk measures into regulatory framework. This has led to numerous criticisms about the existing risk management systems and motivated the search for more appropriate methodologies able to cope with rare events that have heavy consequences. Concerning the extensive range of applications like risk management or regulatory requirements and considering that institutions can use their own approaches, the development of accurate techniques has become a topic of prime importance. While most methodologies could achieve that purpose for common everyday movements, they find themselves unable to account for unexpected events that take place in the crisis. It is well known that the use of Gaussian models to assess financial risk leads to an underestimation of risk. The reason is because these models are unable to capture some important facts such as heavy tails and volatility clustering which indicate the presence of large fluctuations in returns. By comparing the VaR and the Expected Tail Loss (ETL) calculated analytically and using simulations, but both approaches lead to almost the same result. Superior quality of VaR techniques can be employed to yield superior ETL forecasts. Academics and practitioners have extensively studied VaR to propose an unique risk management technique that generates accurate VaR estimations for long and short trading positions and for all types of financial assets. However, they have not yet succeeded as the testing frameworks of the proposals are still being developed. Numerous conditional volatility models that capture the main characteristics of asset returns (asymmetric and leptokurtic unconditional distribution of returns, power transformation and fractional integration of the

[10] VaR particularly appeals to non-technical audiences due to its conceptual simplicity.

conditional variance) under four distributional assumptions (normal, generalized error distribution (GED), Student-t, and skewed Student-t) have been estimated to find the best model for financial markets, long and short trading positions, and two confidence levels. By following this procedure, the risk manager can significantly reduce the number of competing models that accurately predict both the VaR and the ETL measures. ETL estimations can be significantly improved by using the knowledge obtained from advances in VaR estimation. This way, the VaR and the ETL should be regarded as partners, not rivals.

Further than traditional approaches, various alternative distributions have been proposed to describe fat-tail characteristics. One of the most popularity is based on the Extreme Value Theory (EVT). EVT has traditionally been used in fields like civil engineering, hydrology, meteorology and actuarial applications concerning loss severity distributions, recently being devoted to financial purposes. EVT provides a framework in which an estimate of anticipated forces could be made using historical data. Today, EVT is used in telecommunications, ocean wave modelling, thermodynamics of earthquakes, memory cell failure and many other fields. It is important to be aware of the limitations implied by the adoption of the EVT paradigm. EVT models are developed using asymptotic arguments, which should be kept in mind when applying them to finite samples. This extreme model provides a method to estimate VaR at high quantiles of the distribution, consequently focusing on extraordinary and unusual circumstances. This method focuses on the tails behaviour of distribution of returns. Instead of forcing a single distribution for the entire sample, it investigates only the tails of the return distributions, given that only tails are important for extreme values. Backtesting EVT representations found that EVT schemes could help financial institutions to avoid huge losses arising from market fluctuations. This simple exercise illustrates the advantages of EVT.

The empirical study examines the dynamics of extreme values of overnight returns before and during a financial crisis. It is shown that the generalized Pareto distribution (GPD) using the EVT fits well to extreme values of the exceedances distribution. The examination of tails (extreme values) provides answers to the extreme movements expected in financial markets and in assessing the financial fragility. In order to accomplish this task, a series of computational tools have been selected, such as Statistics Toolbox and Optimization Toolbox, an integrated environment for risk assessment developed in Matlab R2009a. This standard numerical or statistical software now provide functions or routines that can be used for EVT applications. Matlab has been designated because it provides a well-suited programming environment, where both numerical and interface design challenges can be met with a reduced development effort.

This thesis is structured as follows. In Section 2, the literature survey presents the definitions and reviews used in the empirical study. Section 3 delineates topics regarding the theoretical framework. Section 4 presents the empirical study that assesses the normal, the historical and the extreme values in risk management throughout the estimation of VaR and ETL. The empirical application is based on daily closings of the nine major developed market indices and seven stock market companies from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008. In particular, the EVT results are used to model the distributions underlying the risk measures by computing the estimations of the tail risk parameters. Section 5 states the concluding remarks and outlines some directions for further research.

2. Literature review

Baumol (1963) made the first attempt to estimate the risk that financial institutions face when he proposed a measure based on a standard deviation adjusted to a confidence level parameter that reflects the attitude towards risk. Since JP Morgan made available its RiskMetrics system on the Internet in 1994, the popularity of VaR and with it the debate among researchers about the validity of the underlying statistical assumptions increased. This is because VaR is essentially a point estimate of the tails of the empirical distribution. The assumed distribution for each market variable in Hull and White (1998) can be chosen in a variety of ways. One possibility is to select an appropriate standard distribution (e.g. a mixture of normals) and use maximum likelihood methods to find the best fit parameters. Another possibility is to smooth the historical distribution (e.g. using a kernel estimator). Using high frequency data others “stylized facts” of real-life returns have been studied namely: volatility clustering, long range dependence and aggregational Gaussianity. Many econometric models have been suggested to explain part of these asset return behavior and among then this study uses the Generalized autoregressive conditionally heteroscedastic model (GARCH). Other models have been suggested to capture this behaviour (Rydberg, 2000). The Hull and White approach provides one way of bridging the gap between the model building and historical simulation approaches. It shows how the model building approach can be modified to incorporate some of the attractive features of the historical simulation approach. Angelidis and Degiannakis (2005) suggest that *“(...) a risk manager must employ different volatility techniques in order to forecast accurately the VaR for long and short* *trading positions (...)"*, whereas Angelidis *et al.* (2004) considered that *"(...) the ARCH structure that produces the most accurate VaR forecasts is different for every portfolio (...)"*. Furthermore, Guermat and Harris (2002) applied an exponentially weighted likelihood model in three equity portfolios (US, UK, and Japan) and proved its superiority to the GARCH model under the normal and the Student-t distributions in terms of two backtesting measures (unconditional and conditional coverage). Moreover, Angelidis and Degiannakis (2004) studied the forecasting performance of various risk models to estimate the one-day-ahead realized volatility and the daily VaR. Regarding only on the VaR forecasts, they support that it was more important to model the fat tailed underlying distribution than the fractional integration of the volatility process. Similarly, Bams *et al.* (2005) argued that complex (simple) tail models often lead to overestimation (underestimation) of the VaR. On the one hand, Taleb (1997) and Hoppe (1999) argued that the underlying statistical assumptions are violated because they could not capture many features of the financial markets (e.g. intelligent agents). Under the same framework, many researchers (see for example Beder, 1995 and Angelidis *et al.* (2004)) showed that different risk management techniques produced different VaR forecasts and therefore, these risk estimates might be imprecise.

Bams and Wielhouwer (2000) drew similar conclusions, although sophisticated tail modelling results in better VaR estimates but with more uncertainty. They concluded that if the data generating process is close to be integrated, the use of the more general GARCH model introduces estimation error, which might result in the superiority of Exponentially Weighted Moving Average (EWMA). Guermat and Harris (2002) found that EWMA-based VaR forecasts are excessively volatile and unnecessarily high, when returns do not have conditionally normal distribution but fat tails. According to Brooks and Persand (2003) relative performance of different models depends on the loss function used but GARCH models provide reasonably accurate VaR. Christoffersen, Hahn and Inoue (2001) demonstrated that different models (EWMA, GARCH, Implied Volatility) might be optimal for different probability levels. Berkowitz and O'Brien (2002) examined VaR models used by six leading US banks. Their results indicated that these models are in some cases highly inaccurate. Their results indicated that banks models have difficulty dealing with changes in volatility. Žiković (2007) found that widespread VaR model consistently underpredict the true levels of risk especially at higher confidence intervals and that semi-parametric models provide superior VaR forecasts in transitional economies.

The normal or Gaussian model is well accepted in Economics and Finance because of the central limit theorem and the simplicity of concepts. The portfolio selection method of

Markowitz (1952), Sharpe's (1964) market equilibrium model and Black and Scholes (1973) option pricing theory are examples of developments taking a parent normal model as granted. This state of the art collapsed with the widespread use of computers, which provided exuberant evidence that skewness and kurtosis of empirical data could not support a normal fit in many instances of modelling financial returns. The standard VaR measure presumes that asset returns are normally distributed, whereas it is widely documented that they really exhibit non-zero skewness and excess kurtosis and, hence, the VaR measure either underestimates or overestimates the true risk. On the other hand, even if VaR is useful for financial institutions to understand the risk they face, it is now widely believed that VaR is not the best risk measure.

Although VaR is useful for financial institutions to see the contours of the risks they face, a growing number of papers clearly show that VaR is not an adequate risk measure. As a result, more general complex measures of risk have been proposed. VaR suffers from various shortcomings pointed out in recent studies. For example, numerical instability and difficulties occur for non-normal loss distributions, especially in the presence of "fat tails" or/and empirical discreteness. Artzner *et al.* (1997, 1998 and 1999) used an axiomatic approach to the problem of defining a satisfactory risk measure. Their study defined attributes that any good risk measure should satisfy and called for risk measures that satisfy these axioms "coherent". Additionally, the study demonstrate that VaR is not necessarily sub-additive, i.e., the VaR of a portfolio may be greater than the sum of individual VaR and therefore, managing risk by using it may fail to automatically stimulate diversification. VaR can only be made sub-additive if an usually implausible assumption is imposed on returns being normally (or more generally, elliptically) distributed. Sub-additivity expresses the fact that a portfolio will risk an amount, which is at most the sum of the separate amounts risked by its sub-portfolios. Moreover, it does not indicate the size of the potential loss, given that this loss exceeds the VaR. Furthermore, VaR is not a coherent measure of risk in the sense of Delbaen (2002) and Arztner *et al.* (1997, 1998 and 1999), and it does not take into account the severity of an incurred adverse loss event. A simple alternative measure of risk with some significant advantages over VaR is conditional VaR, expected shortfall or expected tail loss, abbreviated CVaR, ES and ETL respectively. ETL became the most popular alternative to VaR and equals the expected value of the loss, quantify dangers beyond VaR and it is coherent. Moreover, it provides a numerical efficient and stable tool in optimization problems under uncertainty. Some recent studies presenting these advantages and further desirable properties were included in Acerbi *et al.* (2001), Acerbi and Tasche (2002), Artzner *et al.* (1997, 1998 and

1999), Delbaen (2002), Rockafellar and Uryasev (2002), Testuri and Uryasev (2000), Yamai and Yoshiba (2002a/b/c/d) and Inui and Kijima (2005). Yamai and Yoshiba (2005) compared the two measures - VaR and ETL - and argued that VaR is not reliable during market turmoil as it can mislead rational investors, whereas ETL can be a better choice in the overall. While VaR represents a maximum loss one expects at a determined confidence level during a given holding period, ETL is the loss one expects to suffer, provided that the loss is equal to or greater than VaR. The authors conclude that although ETL is a superior risk measure to VaR, it lacks the depth of the theoretical and empirical research that VaR measure has. Instead of fighting for supremacy VaR and ETL should be used together, combined, giving a better insight into the risks from taking a market position. Furthermore, ETL is a coherent risk measure and hence its utility in evaluating the risk models can be rewarding. Currently, however, most researchers judge the models only by calculating the average number of violations. Even though VaR theoretical flaws outweigh its practical advantages, it is a regulatory obligation. Banks have to calculate their VaR figures to construct adequate capital requirements. VaR is incapable of distinguishing between situations where losses in the tail are only a bit worse, and those where they are overwhelming. Nowadays, ETL is not approved by the regulators as a risk measure that can be used to calculate economic capital. The field of ETL estimation and model comparison is just beginning to develop and there is an obvious lack of empirical research. After all, VaR and ETL are inherently connected in the sense that from the VaR surface of the tail ETL figures can be easily calculated.

Furthermore, Basak and Shapiro (2001) suggested an alternative risk management procedure, namely Limited Expected Losses based Risk Management (LEL-RM), that focuses on the expected loss also when (and if) losses occur. They substantiated that the proposed procedure generates losses lower than what VaR based risk management techniques generate.

An alternative way is to use regime-switching models, the latter are able to capture the previous facts. The issue of VaR calculation under regime-switching has been considered by Billio and Pelizzon (2000) and Guidolin and Timmermann (2006).

According to Mandelbrot (1963), the behaviour of assets returns have been extensively studied. Using low frequency data, he confirmed that log returns present heavier tails than the Gaussian's, so he suggested the use of Pareto stable distributions. In risk assessment, new ways of dealing with evidence provided by extreme order statistics are at the basis of more sophisticated methodologies to avoid extreme losses (Embrechts *et al.*, 2002). The problem is then how to model the rare phenomena that lies outside the range of available observations. In such a situation it seems essential to rely on a well founded methodology.

Most of the financial concepts developed in the past decades rest upon the assumption that returns follow a normal distribution and this is the most well-known classical parametric approach in estimating VaR and ETL. However, empirical results from McNeil (1997), Da Silva and Mendez (2003) and Jondeau and Rockinger (2003), demonstrated that extreme events do not follow Gaussian paradigm. Many have viewed the EVT in finance such as Embrechts *et al.* (1999), Bensalah (2000), Bradley and Taqqu (2002) and Brodin and Klüppelberg (2006). To investigate the extreme events, McNeil (1997) applied a method using EVT for modelling extreme historical Danish major fire insurances losses. His study indicated the usefulness of EVT in estimating tail distribution of losses. Not only is the EVT approach a convenient framework for the separate treatment of the tails of a distribution, as it allows asymmetry as evidence in LeBaron and Samanta (2005). EVT recently has found more application in hydrology and climatology (De Haan, 1990; Smith, 1989). As its name suggests, this theory is concerned with the modelling of extreme events and in the last few years various authors (Beirlant and Teugels, 1992; Beirlant *et al.*, 1996; Embrechts and Klüppelberg, 1993) have noted that the theory is as relevant to the modelling of extreme losses as it is to the modelling of high river levels or temperatures. Obviously, the empirical returns, especially in the high frequency are characterized by heavier tails than a normal distribution. EVT provides a firm theoretical foundation on which we can build statistical models describing extreme events. In many fields of modern science, engineering and insurance, EVT is well established (Embrechts *et al.* 1999; Reiss and Thomas, 1997). Recently, numerous research studies have analyzed the extreme variations that financial markets are subject to, mostly because of currency crises, stock market crashes and large credit defaults. The tail behaviour of financial series has, among others, been discussed in Koedijk *et al.* (1990), Dacorogna *et al.* (1995), Loretan and Phillips (1994), Longin (1996), Danielsson and de Vries (2000), Kuan and Webber (1998), Straetmans (1998), McNeil (1999), Jondeau and Rockinger (2003), Rootzen and Klüppelberg (1999), Neftci (2000), McNeil and Frey (2000) and Gençay *et al.* (2003b). An interesting discussion about the potential of EVT in risk management is given in Diebold *et al.* (1998). These recommendations are the natural consequence of the general admission that heavy tailed models provide much better fit than the normal model. Gilli and Këllezi (2003) advocated the use of EVT due to its firm theoretical grounds to compute both VaR and ETL. Furthermore, Gilli and Këllezi (2006) tried to illustrate EVT by using both block maxima method and peaks over the threshold (POT) in modelling tail-related risk measures, VaR, ETL and return level. They found that EVT is useful in assessing the size of extreme events. In depth, POT proved to be superior as it better exploits the information in sampling. Gençay and Selçuk (2004) have reviewed VaR estimation in some emerging markets using various models including EVT. The study revealed that EVT-based model provides more accurate VaR especially in a higher quantile. In depth, the GPD model fits well with the tail of the return distribution. Harmantzis *et al.* (2006) and Marinelli *et al.* (2007) have presented how EVT performs in VaR and ETL estimation compared to the Gaussian and historical simulation models together with the other heavy-tailed approach, the Stable Paretian model. Their empirical study supported that fat-tailed models can predict risk more accurately than non-fat-tailed ones and there exists the benefit of EVT framework especially method using GPD. However, Basel II recommendations maintained some remains of the normal model in the computation of VaR. For the purposes of this thesis, the key result in EVT is the Pickands-Balkema-de Haan theorem (Balkema and de Haan, 1974; Pickands, 1975) which essentially says that, for a wide class of distributions, losses which exceed high enough thresholds follow the GPD. The concern in this thesis is fitting the GPD to data on exceedances of high thresholds. This modelling approach was developed in Davison (1984), Davison and Smith (1990a/b) and other papers by these authors.

3. Theoretical Framework

This section introduces the definitions of two risk measures namely, VaR and ETL and outlines the key concepts of theoretical framework used in the empirical study which are Gaussian and EVT.

3.1. Value at Risk (VaR)

VaR is generally defined as the maximum potential loss that a portfolio can suffer within a fixed confidence level during a holding period (Jorion, 2007). Mathematically, McNeil *et al.* (2005) define VaR, in absolute value, at $\alpha \in (0,1)$ confidence level $VaR_\alpha(X)$ as follows.

$$VaR_\alpha(X) = inf\{x \mid P[X > x] \leq 1-\alpha\} = inf\{x \mid F(x) \geq \alpha\}, \quad \text{(i)}$$

where X is the loss of a given market index, and $inf\{x \mid P[X > x] \leq 1-\alpha\}$ indicates the smallest number x such that the probability that the loss X exceed x is no larger than $(1 - \alpha)$. Generally, VaR is simply a α-quantile of the probability distribution $F(x)$.

$$VaR_\alpha(X) = F(x)^{-1}(1-\alpha) \quad \text{(ii)}$$

where $F(x)^{-1}$ is the so called quantile function defined as the inverse of the distribution function $F(x)$.

According to the Basel II Accord, the financial entities compute a one percent VaR over a ten-day holding period, based on an historical observation period of at least one year of daily data. Each bank must meet, on a daily basis, a capital requirement expressed as the sum of: *i)* The higher of its previous days VaR number measured according to the parameters specified in revised Basel II market risk framework (2009) and an average of the daily VaR measures on each of the preceding sixty business days multiplied by a multiplication factor plus *ii)* The higher of its latest available "stressed VaR" number and an average of the "stressed VaR" calculated according to the preceding sixty business days multiplied by a multiplication factor. The multiplication factors will be set by individual supervisory authorities on the basis of their assessment of the quality of the banks risk management system, subject to an absolute minimum of three. Banks will be required to add to these factors a "plus" directly related to the ex-post performance of the model, thereby introducing a built-in positive incentive to maintain the predictive quality of the model. The "multiplication factor" was introduced because the normal hypothesis for the profit and loss distribution is widely recognized as unrealistic.

3.2. Coherent risk measures and Expected Tail Loss (ETL)

Hoppe (1999) revealed that the underlying statistical assumptions are violated because they cannot capture many features of the financial markets such as intelligent agents. Artzner *et al.* (1997, 1999) have used an axiomatic approach to the problem of defining a satisfactory risk measure. They defined attributes that a good risk measure should satisfy, and call risk measures that satisfy these axioms "coherent". Clearly, there are several axioms that should be satisfied by a good risk metric. A coherent risk measure ρ assigns to each loss X a risk measure $\rho(X)$ such that the following axioms are satisfied (Artzner *et al.*, 1999):

$$\rho(tX)=t\rho(X) \qquad \text{(homogeneity)} \tag{iii}$$

$$\rho(X)\leq \rho(Y), if\ \ X\ has\ a\ weak\ stochastic\ dominance\ over\ Y \qquad \text{(monotonicity)} \tag{iv}$$

$$\rho(X+n)=\rho(X)-n \qquad \text{(risk-free condition)} \tag{v}$$

$$\rho(X+Y)\leq \rho(X)+\rho(Y) \qquad \text{(sub-additivity)} \tag{vi}$$

for any number n and positive number t. These conditions guarantee that the risk function is convex, which in turn corresponds to risk aversion.

Homogeneity and monotonicity conditions are reasonable conditions to impose a priori, and together imply that the function $\rho(X)$ is convex. The risk-free condition means that the addition of a riskless asset to a portfolio will decrease its risk because it will increase the value of end-of-period portfolio. According to the last condition a risk measure is sub-additive if the measured risk of the sum of positions X and Y is less than or equal to the sum of the measured risks of the individual positions considered on their own. Furthermore, the risk measure need to aggregate risks in an intuitive way, accounting for the effects of diversification. The managers should ensure that the risk of a diversified portfolio is no greater than the corresponding weighted average of the risks of the constituents. Without sub-additivity there would be no incentive to hold portfolios and so could not be used for risk budgeting.

According to Artzner *et al.* (1999), generally, VaR is not a coherent risk measure because quantiles, unlike the variance operator, do not obey simple rules such as sub-additivity unless the returns have elliptical distribution. VaR can only be made sub-additive if an usually implausible assumption is imposed on returns being normally (or slightly more generally, elliptically) distributed because it behaves like the volatility of returns. Furthermore, if risks are not sub-additive, adding them together gives an underestimate of combined risks, and this makes the sum of risks effectively useless as a risk measure. If regulators use non-sub-additive risk measures to set capital requirements, a bank might be tempted to break itself up to reduce its regulatory capital requirements, because the sum of the capital requirements of the smaller units would be less than the capital requirement of the bank as a whole. This is maybe the most characterizing feature of a coherent risk measure and represents the concept of risk. The global risk of a portfolio will then be the sum of the risks of its parts only in the case when the latter can be triggered by concurrent events, namely if the sources of these risks may conspire to act altogether. In all other cases, the global risk of the portfolio will be strictly less than the sum of its partial risks thanks to risk diversification.

For a sub-additive measure, such as ETL is, portfolio diversification always leads to risk reduction, while for measures which violate this axiom, such as VaR, diversification may produce an increase in their value even when partial risks are triggered by mutually exclusive events. ETL can be defined as the expected value of the loss of the portfolio in the $100(1- \alpha)\%$ worst cases during a holding period (Artzner *et al.*, 1999).

ETL is closely related to VaR. It is known as the conditional expectation of loss given that the loss is beyond the VaR level. An intuitive expression to show that ETL can be interpreted as the expected loss that is incurred when VaR is exceeded (McNeil *et al.*, 2005).

(vii)

$$ETL_\alpha(X) = E\left[X \mid X \geq VaR_\alpha(X)\right]$$

3.3. Extreme Value Theory (EVT)

The most famous parametric approach for calculating VaR and ETL is based on the Gaussian assumption. It is assumed the independent identical distribution of standardized residual terms. On the other hand, EVT is used to model the risk of extreme, rare events (e.g., 1755 Lisbon, 1906 San Francisco or 2004 Aceh-Sumatra earthquakes). Critical questions related to the probability of a market crash or boom require an understanding of statistical behaviour expected in the tails. EVT allows us to measure a “tail index” that characterizes the density function in the tail of a distribution. Then is simulated a theoretical process that captures the extreme features of the empirical data and estimates the probability of extraordinary market movements. Embrechts, Klüppelberg and Mikosch (1997), Reiss and Thomas (1997) and Beirlant *et al.* (1996) provided a comprehensive source of the EVT for the finance and insurance literature. Danielsson and de Vries (1997), Embrechts (2000) and Gençay and Selçuk (2004) also provided references therein for EVT applications in finance. There are two well-known general approaches to model formulation: the block maxima or minima method stems from the behaviour of the *k* largest order statistics within a block for small values of *k* and POT roots in observations exceeding a high threshold.

The **theorem of Fisher and Tippet (1928) and Gnedenko (1943)** is the core of the EVT. The theory deals with the convergence of maxima (known as distribution of maxima or **block maxima method**). Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independent and identically distributed (i.i.d.) random variables from an unknown distribution function $F(x)$. Jenkinson (1955) and von Mises (1954) suggested the following one-parameter representation, with shape parameter *k*

$$H_k(x) = \begin{cases} e^{-(1+kx)^{-\frac{1}{k}}} & if\ k \neq 0 \\ e^{-e^{-x}} & if\ k = 0 \end{cases} \tag{viii}$$

EVT, even without exact knowledge of the distribution of the parent variable *X*, can derive certain limiting results of the distribution of maxima. As in general we do not know in advance the type of limiting distribution of the sample maxima, the generalized representation is particularly useful when maximum likelihood estimates have to be computed.

Denote the maximum of the first $m < n$ observations of *X* by $M_m = \max(X_1, \ldots, X_n)$. Given a sequence of $a_m > 0$ and b_m such that $(M_m - b_m)/a_m$, the sequence of normalized maxima converges in the following so-called generalized Extreme Value (GEV) distribution which uses a modelling technique known as the block maxima or minima method. This approach, divides an historical data set into a set of sub-intervals, or blocks, and the largest or smallest observation in each block is recorded and fitted to a GEV distribution. The cumulative function for the GEV distribution with location parameter μ, scale parameter σ, and shape parameter $k \neq 0$, is

$$f\left(x \mid k,\mu,\sigma\right) = e^{-\left(1+k\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{k}}}, \left(1+k\,\frac{(x-\mu)}{\sigma}\right) > 0 \tag{ix}$$

The probability density function is, consequently,

$$f\left(x \mid k,\mu,\sigma\right) = \left(\frac{1}{\sigma}\right)\cdot\left(1+k\,\frac{(x-\mu)}{\sigma}\right)^{-1-\frac{1}{k}}\cdot e^{-\left(1+k\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{k}}} \tag{x}$$

As the number of observations over which the maximum is taken tends towards infinity, the Fisher Tippet theorem summarizes three possible limiting extreme value distributions for the standardized maxima. When k is greater than zero the distribution is known as the Fréchet distribution and the fat-tail decay as a polynomial[11], meaning that $F(x)$ is leptokurtotic. The greater shape parameter means a more fat-tailed distribution. If k is less than zero, the distribution is known as the Weibull distribution, meaning that $F(x)$ is platokurtotic, and the tail decays with finite upper endpoint, such as the Beta. Finally, if k is equal to zero, it is the Gumbel distribution, meaning that $F(x)$ has normal kurtosis, and the tail can decay exponentially and have all finite moments, such as the normal, lognormal and gamma (Gumbel, 1958). Note the differences in the ranges of interest for the three extreme value distributions: Gumbel has no limit, Fréchet has a lower limit, while the reversed Weibull has an upper limit (see Figure I).

The three cases covered by the GEV distribution are often referred to as the Types I, II, and III. Each type corresponds to the limiting distribution of block maxima from a different class of underlying distributions. The GEV combines three simpler distributions into a single form, allowing a continuous range of possible shapes. The GEV distribution allows to "let the data decide" which distribution is appropriate. Among these three extreme value types of distributions the crucial point is to find the relevant distribution in modelling the behaviour of equity market returns. Since the concern is with stock market returns that are known to be fat tailed, then the choice cannot be a Gumbel distribution. Since returns are theoretically

[11] e.g. examples are the stable Paretian, Cauchy and Student-t distributions.

unbounded, the Weibull distribution is excluded. The focus will be on the Fréchet domain of attraction that encompasses numerous distributions ranging from the Student-t, Cauchy to the stable Paretian.

An alternative approach uses a modelling technique known as the **peak over threshold (POT) method or the distribution of exceedances** over a certain threshold. Suppose the following $X_1,\ldots,X_n$ be *n* observations and are all i.i.d. sequences of losses with distribution function $F_x(x)=P[X \le x]$ and the corresponding $Y_1,\ldots,Y_n$ are the excess over the threshold μ. The subject is to understand the distribution function *F* particularly on its lower tail. Firstly, it is described the distribution over a certain threshold μ using the GPD which is the main distributional model for excess over the threshold. The excess over threshold occurs when $X_i > \mu$. This approach sorts an historical data set, and fits the amount by which those observations exceed a specified threshold to a GPD. Like the exponential distribution, the GPD is often used to model the tails of another distribution. However, while the normal distribution might be a good model near its mode, it might not be a good fit to real data in the tails and a more complex model might be needed to describe the full range of the data. The GPD distribution allows a continuous range of possible shapes that includes both the exponential and Pareto distributions as special cases. Let denotes the distribution of excess values of *X* over threshold μ, which is called the conditional excess distribution function, is defined by

$$F_\mu(y)=P(X - \mu \le y \mid X > \mu)=\frac{F(x)-F(\mu)}{1-F(\mu)}, \quad 0 \le y \le x_F - \mu \qquad \text{(xi)}$$

where $y = x - \mu$ for $X > \mu$ is the excess over threshold and $x_t \le \infty$ is the right endpoint of *F*.

At this point EVT can prove to very helpful as it provides a powerful result about the conditional excess distribution function which is stated in the **theorem by Pickands (1975), Balkema and de Haan (1974)**. Following the theorem, for a sufficiently high threshold μ, the distribution function of the excess can be approximated by the generalized Pareto, i.e., the excess distribution $F_u(y)$ converges to the GPD ($G_{k,\sigma}(y)$) below as the threshold μ gets large,

$$F_\mu(y) \approx G_{k,\sigma}(y)=\begin{cases} 1-\left(1+k\dfrac{y}{\sigma}\right)^{-\frac{1}{k}} & if\ k \ne 0 \\ 1-e^{-\frac{y}{\sigma}} & if\ k=0 \end{cases}, \; y \in \begin{cases} [0,(x_F-\mu)] & if\ k \ge 0 \\ \left[0,-\dfrac{\sigma}{k}\right] & if\ k<0 \end{cases}, \qquad \text{(xii)}$$

where $\sigma > 0$, and the support is $y > 0$ when $k \ge 0$, and $0 \le y < \sigma/k$ when $k < 0$

In the sense of the above theorem, X_i with distribution of F assumes that the distribution of excesses (*y*) may be approximated by the GPD by estimating some scale parameter σ and tail index or shape parameter *k* as a function of a high threshold μ. The tail index *k* gives an indication of the heaviness of the tail, i.e., larger *k* means heavier tail. The parameters of the GPD can be estimated with various methods[12].

The cumulative function for the GPD with and threshold parameter μ, scale parameter σ, and shape parameter $k \neq 0$, is

$$f\left(y \mid k, \mu, \sigma\right) = 1 - \left(1 + k\frac{(y-\mu)}{\sigma}\right)^{-\frac{1}{k}} \quad \text{(xiii)}$$

The probability density function is, consequently,

$$f\left(y \mid k, \mu, \sigma\right) = \left(\frac{1}{\sigma}\right) \cdot \left(1 + k\frac{(y-\mu)}{\sigma}\right)^{-1-\frac{1}{k}} \quad \text{(xiv)}$$

The *k* parameter is known as the shape parameter and the case of most interest in finance is where *k* is superior to zero, which corresponds to the fat tails commonly founded in financial return data. If *k* and μ are equal to zero, the GPD is equivalent to the exponential distribution. If *k* is superior to zero and μ is equal to σ/k, the GPD is equivalent to the Pareto distribution. EVT tells us that the limiting distribution of extreme excess returns always has the same form. It is important because it allows us to estimate extreme probabilities and extreme quantiles, including VaR and ETL, without having to make strong assumptions about the full shape of the unknown parent distribution. For the security returns or high frequency foreign exchange returns, the estimates of *k* are usually less than *0,5* implying that the returns have finite variance (Longin, 1996; Dacorogna *et al.* 2001). For *k* greater than *-0,5,* which corresponds to heavy tails, Hosking and Wallis (1987) presents evidence that maximum likelihood regularity conditions are fulfilled and the maximum likelihood estimates are asymptotically normally distributed. Therefore, the approximate standard errors for the estimators of σ and *k* can be obtained through maximum likelihood estimation. The GPD is the limiting distribution of sample extremes (Embrechts, Klüppelberg and Mikosch, 1997). This distribution is analogous to the GEV. The two distributions differ in their definition of extremes. While the GEV is the limiting distribution of the extremes taken over *n* samples, the GPD defines extremes as all points above a certain threshold. Both distributions are

[12] The methods are the maximum likelihood estimation, the method of moments, the method of probability-weighted moments and the elemental percentile method. For detailed discussions about their use for fitting the GPD to data, see Hosking and Wallis (1987), Grimshaw (1993), Tajvidi (2003) and Castillo and Hadi (1997).

parametrized by the scale, location and shape parameters with the same interpretation in both cases. Jondeau and Rockinger (2003) fit a GPD to a range of emerging and developed market equity return series.

Additionally, if the i.i.d. condition fails, the EVT may still be an accurate approximation of the actual distribution function of maxima (Reiss and Thomas, 1997). Furthermore, a discussion on dependency, extremal index and its implications in practice can be found in Longin and Solnik (2001) and Embrechts *et al.* (1997).

Assuming a GPD function for the tail distribution, analytical expressions for VaR and ETL can be defined as a function of GPD parameters. Following McNeil (1999), the formula used to obtain VaR for a given probability α is,

$$\overline{\overline{VaR}}_{\alpha}(X) = \mu + \frac{\sigma}{k}\left[\left(\frac{n}{N_{\mu}}(1-\alpha)\right)^{-k} - 1\right] \tag{xv}$$

where n represents the number of observations and N_μ is the number of observations in the tail beyond the threshold μ. The associated ETL at $\overline{\overline{VaR}}_{\alpha}(X) > \mu$, can be calculated as,

$$\overline{\overline{ETL}}_{\alpha}(X) = \overline{\overline{VaR}}_{\alpha}(X) + E\left[X - \overline{\overline{VaR}}_{\alpha}(X) \mid X > \overline{\overline{VaR}}_{\alpha}(X)\right] = \frac{\overline{\overline{VaR}}_{\alpha}(X) + \sigma - k\mu}{1-k} \tag{xvi}$$

In effect, the first step is to estimate the parameters of extreme value distribution and then project the tail out beyond the data sample, thereby allows to estimate extreme risk measures and the probabilities associated with them.

4. Empirical study

As the financial system becomes more complex, the need for complicated statistical models to measure risk and to price assets becomes greater. Indeed, the credit crises, which started in the summer of 2007, showed that risk models are of somewhat lower quality than was generally believed. This does not suggest that statistical models should not be employed. On the contrary, they play a fundamental role in the internal risk management operations of financial institutions. According to Danielsson (2008) the main problem was the unrealistic expectations of what models can do. However, for practitioners, regulators, academics, and especially model designers, using models are very important not only for internal risk control but also for the assessment of systemic risk which is crucial for the regulation of financial institutions. Additionally, remembering the 1997 financial crisis in East Asia, where 75% drop in the Thai stock market contributed to a 554-point drop in the Dow Jones index, proven that VaR models have excessive dependency on history or unrealistic statistical assumptions. These crisis apparently moved events that have at least three things in common: they occur rarely, they are extreme in scope, and they are difficult to predict. The most crucial subject is to predict the likelihood and severity of a crash in financial markets and assess their probability and magnitude. Statisticians have applied a variety of techniques in their attempts to model rare events. These techniques frequently are based on EVT, a branch of statistics that analyzes events that deviate sharply from the norm, and copulas, which can be used to model the co-movement of dependent variables whose probability distributions are different from each other and might not be normal.

In this thesis the tail estimation of loss severity distributions assumes a particularly interest. In this situation it is essential to find a good statistical model for the largest observed historical losses. The benefits of this study are the better understanding and implication about the VaR and the ETL in fat-tailed environment, the efficiency in risk measurement prediction and the tail distribution of the financial returns.

4.1. Data and methods

This empirical study and its modelling is based on the EVT, a theory which until comparatively recently has found more application in hydrology and climatology (de Haan, 1990; Smith, 1989) than in finance. As its name suggests, this theory is concerned with the modelling of extreme events and in the last few years various authors (Beirlant and Teugels, 1992), Embrechts and Klüppelberg, 1993) have noted that the theory is as relevant to the modelling of extreme finance or insurance losses as it is to the modelling of high river levels or temperatures (following McNeil and Frey, 2000). For this study purposes, the key result in EVT is the Pickands-Balkema-de Haan theorem (Balkema and de Haan, 1974; Pickands, 1975) which essentially says that, for a wide class of distributions, losses which exceed high enough thresholds follow the GPD. The GPD can be fitted to data on excesses of high thresholds by a variety of methods including the maximum likelihood method. In this empirical study the maximum likelihood method has been chosen. Different methods can be used to estimate the parameters of the GPD. In this empirical study, the model will fit GPD to data on exceedances of high thresholds.

It will be presented how EVT framework performs under the select risk measures, such as VaR and ETL, on a data set. The data set is the daily closings of the nine major developed market indices and seven stock market companies from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008. This historical market data was downloaded from Yahoo[13]. There are 2.455 observations in the data set for the nine market indices and 2.226 observations in the data set for the seven stock market firms. Table I gives the list of the financial series considered in this empirical study. VaR and ETL estimation are made on daily basis and their calculations are based on the realized losses (left-tail) within given historical window size. The estimation is conducted at 99% confidence level.

The daily logarithmic returns (also called geometric, or continuously compounded, returns) are defined by $X_t = ln((P_t / P_t - 1)$, where P_t denotes the daily closing prices at day t. In the Figures II the historical prices and the empirical returns series are presented. This empirical study will exemplify the tail distribution estimation of a set of financial series of daily returns and use the results to quantify the market risk. This approach is compared to the Gaussian-based model and the historical simulation. Lastly, the predictive accuracy of the models is evaluated using backtesting procedure.

4.2. Empirical tests and estimated parameters

This empirical study presents a series of computational tools that can be used to calculate these different risk measures. The data, to analyse VaR and ETL Gaussian-based model, is tested in ECVaR software developed by Rho-Works[14]. The data and parameters to analyse the VaR and the ETL EVT based models are executed in a Matlab R2009a programming environment. Standard numerical or statistical software, like for example Matlab, now also provide functions or routines that can be used for EVT applications. Other software for extreme value analysis can be used such as Extreme Values In S-Plus (EVIS) developed by Alexander J. McNeil at the Swiss Federal Institute of Technology Zurich (ETH) and Xtremes developed by Rolf Reiss and Michael Thomas at the University of Siegen in Germany.

The implementation of the POT method involves the following steps: select the threshold μ, fit the GPD function to the exceedances over μ and then compute point and interval estimates for VaR and ETL. Modelling the exceedances over a given threshold provides estimations in high quantiles of the return distribution and the corresponding VaR and ETL, using the maximum likelihood estimation, which is one of the most common

[13] Available for free at http://finance.yahoo.com/ .
[14] Available for free at www.rhoworks.com/software/index.htm .

estimation procedures used in practice[15]. The greater computational complexity of the likelihood-based approach is nowadays no longer an obstacle for its use. Matlab has been chosen because it provides a well-suited programming environment, where both numerical and interface design challenges can be met with a reduced development effort. The tools integrated in Matlab permit the selection of several autoregressive models, such as GARCH. These tools also provide a graphical user interface. The estimation of the GPD parameters, such as k and σ, can be computed using Matlab software by fitting this distribution to the Nu excess losses given the data and the calculated loss exceeding threshold μ. VaR and ETL may be directly read in the plot or calculated from equations (xv) and (xvi) by replacing with our estimated parameters. The crucial step in estimating GPD parameters is the determination of the threshold μ. According to Pattarathammas and Mokkhavesa (2008) the choice of μ ultimately involves the trade-off between bias and variance. If the threshold is conservatively selected with few order statistics in the tail, then the tail estimate will be sensitive to outliers in the distribution and have a higher variance. On the other hand, to extend the tail more into the central part of the distribution it creates a more stable index but results in a biased value. This sensitive trade-off can be dealt in a variety of ways but there is no standard methodology of selecting the right threshold. However, in this empirical study the N_u is constant and calculated from Matlab (see Code IV, VI, IX and X and Table V) to be the 99th percentile of the GPD distribution, where n is the rolling window size which is equal to 2.455 and 2.226 observations, respectively, for the nine market indices and the seven stock market firms, or approximately ten years length. A tool that is very helpful for the selection of the threshold μ is the *sample mean excess plot.* These values are located at the beginning of a portion of the sample mean excess plot that is roughly linear. With this procedure, it actually fixes the number of index return data in the tail by using the largest one percent of the realized losses

[15] Maximum likelihood methods perform better when tails are thicker providing greater observations exceeding the threshold. This can be a severe constraint on effective estimation when studying relatively short histories of emerging markets. Additionally, it has the assumption about the distribution and dependence structure of the data that are used to calibrate the size of sub-samples and to estimate standard errors. Jansen and de Vries (1991) show that in the Fréchet domain of attraction that includes most distributions of financial returns, maximum likelihood methods are consistent but not the most efficient. An alternative to maximum likelihood estimation, a "nonparametric" school offers efficient estimators that rely on the largest order statistics of the parent distribution and only require that the data generating distribution be broadly well behaved. Nonparametric estimators have a long history in EVT, beginning with Hill's index first proposed in 1975 (Hill, 1975). The Hill index measures the average increase in the Pareto plot above the tail cut-off point and can be interpreted as the slope of the linear part of the Pareto quantile plot. The Hill index relies on the average distance between extreme observations and the tail cut-off point to extrapolate the behaviour of the tails into the broader part of the distribution. In the case that a Fréchet limit law applies, this index is a powerful measure of tail behaviour. The weakness of this index lies in the a priori need to determine the size of the tail.

as a threshold for historical rolling window. According to Pattarathammas and Mokkhavesa (2008) this effectively give us a random threshold at the $(N_u+1)^{th}$ order statistic.

McNeil and Frey (2000) proposed a two stage method that consisted in modelling the conditional distribution of asset returns against the current volatility and then fitting the GPD on the tails of residuals. On the other side, Danielsson and de Vries (2000) argued that for long time horizons an unconditional approach is better suited. Indeed, as Christoffersen and Diebold (2000) noticed, conditional volatility forecasting is not indicated for multiple day predictions. The above references can give a detailed discussion on these issues, including the i.i.d. assumptions. According to the above mentioned authors, the choice between conditional and unconditional approaches depends on the final use of the risk measures and the time horizon considered. For short time horizons of the order of several hours or days, and if an automatic updating of the parameters is feasible, a conditional approach may be indicated. For longer horizons, a non conditional approach might be justified by the fact that it provides stable estimates through time requiring less frequent updates.

Additionally, fitting the data in the tail is the main concern. The GPD was developed as a distribution that can model tails of a wide variety of distributions, based on theoretical arguments. One approach to distribution fitting is to use a non-parametric fit, such as the empirical cumulative distribution function, in regions where there are many observations, and to fit the GPD to the tail(s) of the data. In this study, to assess the GPD to tail data, functions in the Statistics Toolbox™ were used, for fitting this distribution by maximum likelihood (see Code V).

The tools for the examination of fat-tailness and asymptotic normality assumption in the data are the sample histogram, quantile-quantile (QQ) plot and the mean excess function. As other studies have also found, different methods of estimating the optimal tail size does not typically settle on a consensus (Lux, 1990). Extreme value analysis works with the right tail of the distribution that corresponds to return losses. To estimate the optimal size derived from a sub-sampling the use of the bootstrap technique is recommended (Danielsson and de Vries, 2000). This technique settles on a range of optimal tail sizes that vary across markets. A common remedy for that skewness is to estimate the parameter and its standard error on the log scale, where a normal approximation may be more reasonable. A QQ plot is a better way to assess normality than a histogram, because non-normality shows up as points that do not approximately follow a straight line. The optimal tail selection with QQ plot technique can evaluate whether the data have fat tails, displaying the quantiles of the sample data against those of a standard normal distribution. These techniques serve as a useful device to qualitatively compare the normality of the data without any other assumptions. QQ plot gives some idea about the underlying distribution of a sample. Specifically, the quantiles of an empirical distribution are plotted against the quantiles of a hypothesized distribution. If the sample comes from the hypothesized distribution or a linear transformation of the hypothesized distribution, the QQ plot should be linear. If there is a strong deviation from a straight line, then either the assumed shape parameter is wrong or the model selection is not accurate (see Code VII).

To quantify the precision of the estimates, the standard errors computed were used from the asymptotic covariance matrix of the maximum likelihood estimators. The function gplike computes, as its second output, a numerical approximation to that covariance matrix (see Code VI). The computation of these standard errors assumed that the Pareto model is correct and the simulation has enough data for the asymptotic approximation to the covariance matrix to hold.

Finally, it is important to backtest the results in order to examine the performance among various VaR and ETL methods within a given rolling window size. Entities that use models as a risk disclosure or risk management tool are facing growing pressure from internal and external parties such as senior management, regulators, auditors, investors, creditors, and credit rating agencies to provide estimates of the accuracy of the risk models being used. As the use of models extends from pure risk measurement to risk control in areas such as VaR-based stress testing and capital allocation, it is essential that the risk numbers provide accurate information, and that someone in the organization is accountable for producing the best possible risk estimates. In order to ensure the accuracy of the forecasted risk numbers, risk measurement models need regular backtests to analyze their accuracy and evaluate alternative models if the results are not entirely satisfactory. If a particular model does not perform its intended task properly it should be refined or replaced and the risk measurement process should continue. In its simplest form, the backtesting procedure consists of calculating the number or percentage of times that the actual portfolio returns fall outside the model estimation and comparing that number to the confidence level used. The setup for this test is the classic testing framework for a sequence of success and failures, also called Bernoulli trials. According to the 2006 revised framework from Basel II: international convergence of capital measurement and capital standards[16] it contains a detailed description of the backtests that supervisors will review and models that fail them will either be disallowed for use in

[16] See http://www.bis.org/publ/bcbs128.pdf .

regulatory capital calculations, or be subject to the highest multiplier value of four. Banking supervisors will only allow internal models to be used for regulatory capital calculation if they provide satisfactory results in backtests. The Basel Committee recommended a very simple type of backtest, which is based on a one percent daily VaR estimate and which covers a period of only 250 days. For purposes of the backtest, banks will compare daily end-of-day VaR estimates calibrated to a one-day, 99th percentile standard with the next day's hypothetical trading outcome[17] (see Directive 2006/49/EC of the European Parliament and the Council). Therefore, the expected number of exceedances is 2,5 and the standard error of the number of exceedances is $\sqrt{2{,}5 \times 99\%}$. The regulators are very conservative and Basel II will only consider that models having four exceptions or less as sufficiently accurate. These so called green zone models have a multiplier of three. If there are between five and nine exceptions, the model is yellow zone, which means it is admissible for regulatory capital calculations but the multiplier is increased from 0,4 to 0,85. Then the multiplier takes its maximum of value four, or the model is disallowed. Another coverage test is known as test of frequency of tail losses or Kupiec test. Kupiec's (1995) test attempts to determine whether the observed frequency of exceptions is consistent with the frequency of expected exceptions according to the model and chosen confidence interval. Under the null hypothesis that the model is "correct", the number of exceptions follows a binomial distribution. From Timotheos and Degiannakis (2006), the null and the alternative hypotheses should be,

$$\begin{aligned} H_0 &: N/T = 1-\alpha \\ H_1 &: N/T \neq 1-\alpha \end{aligned} \tag{xvii}$$

where α is the confidence level, N is number of exceptions and T is the sample size of the backtest, hence N/T is the proportion of excessive losses or violation ratio. The appropriate likelihood ratio unconditional[18] coverage test statistics is,

[17] The institution can monitor the accuracy and performance of its model by conducting a backtesting programme on both actual and hypothetical changes in the portfolio's value. Backtesting on hypothetical changes in the portfolio's value is based on a comparison between the portfolio's end‑of‑day value and, assuming unchanged positions, its value at the end of the subsequent day. Competent authorities may require institutions to perform backtesting on either hypothetical (using changes in portfolio value that would occur were end-of-day positions to remain unchanged), or actual trading (excluding fees, commissions, and net interest income) outcomes, or both.

[18] There are two general approaches in risk forecasting – either conditional on current market conditions or on the unconditional market environment. Both approaches have advantages and disadvantages. Thus, the choice of methodology is situation dependent. Furthermore, financial returns data have at least two stylized facts: fat tails and volatility dependence. For longer time horizons, an unconditional model is appropriate for the calculation of large loss forecasts. In many situations where the investment horizon is short, conditional volatility models may be preferable for risk forecasting. However, even if the time horizon is shorter, financial institutions often prefer unconditional risk forecast methods to avoid undesirable frequent changes in risk limits for traders and portfolio managers (Danielsson and de Vries, 2000).

$$LR_{uc} = -2\ln\left[(1-\alpha)^{T-N}\alpha^{N}\right] + 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] \qquad \text{(xviii)}$$

Automatically, the log likelihood ratio is chi-square distributed with one degree of freedom. This test can reject a model for both high and low failures. When Kupiec's test is bigger than the asymptotic chi-squared distribution, the null hypothesis that the model is accurate is rejected in the sense the total number of exceedances is close to the expected number.

4.3. Empirical tests and results

The main results for the tested data obtained for VaR and ETL are summarized in tables from II to VI and Figures from II to VIII. Table V displays the VaR and the ETL for different models and with different parametric fits. The rows are labelled according to the model used to compute the values: empirical (or historical) VaR and ETL, normal (or Gaussian) VaR and ETL and, finally, GPD VaR and ETL (with parameters calculated in Matlab).

The kurtosis of all the data sets is greater than three[19] indicating the evidence of fat-tailed. Regarding to the standard deviation, the firms Lehman Brothers, Apple, Goldman Sachs and Microsoft are most volatile. Lehman Brothers also exhibits the lowest and highest daily returns during the period from November 4, 1999 to September 12, 2009 following by Apple and Microsoft, respectively. The daily prices and the log returns of each country index and firm can be seen in Figures II (see Code I).

After computing the parameters k and σ that maximize the log-likelihood function for the sample defined by the observations exceeding the threshold μ, the results obtained for the left tail exceedances that the left tail is heavier than the right one. This can also be seen from the estimated value of the shape parameter which is positive in most of the cases, but higher in the left tail case, with the exceptions of CAC40, DAX, DJ, Apple and General Electric. Consider that if $k = 0$, $k > 0$ or $k < 0$, this indicates an exponentially decaying, power-decaying, or finite-tail distributions in the limit, respectively. The standard errors calculated indicate that the relative precision of the estimate for shape is quite a bit lower than sigma. VaR and ETL may be directly read in the plot or computed from equations (xv) and (xvi) where the parameters were replaced by the estimated values. As we can observe in Tables V and VI, with respect to the right tail, the left tail has a lower VaR but a higher ETL which

[19] Normal distribution has a kurtosis of three and skewness of zero. If the skewness is greater than three regardless to their mean or standard deviation, it represents the leptokurtic or fat-tailed distribution.

illustrates the importance to go beyond a simple VaR calculation. As described in section 4.2, it is possible to transform scale and location parameters to obtain a GPD model which fits the severity distribution itself in the tail area above the threshold.

VaR and ETL estimation for the empirical, normal and GPD are made on daily basis and their calculations are based on the realized losses (left-tail) within given historical window size, conducted at 99% confidence level. The results in Table V indicate that for general the left tail is heavier than the right one. Looking at estimated VaR and ETL, the firms Lehman Brothers, Apple, Goldman Sachs, Microsoft, General Electric and BCP are the most exposed to extreme losses, followed by and indices DJ, DAX, Sensex, CAC40 and Nikkei225. The less exposed firm is BES and the less exposed indices are S&P500, Nasdaq, FTSE100 and PSI20.

With this study the results suggest that within-sample risk measures are fairly sensitive to the model selected. For the probability level 99%, all models fits basically predict the same risk measures, except for the normal, which begins to exhibit its shortcomings and it underestimate both VaR and ETL (see Tables from V to VI and Figures from VII to VIII). Whether the asymptotic decay is algebraic or exponential is of some consequence. ETL is especially sensitive to this issue. The empirical and GPD models predict fairly consistent risk measures, particularly empirical ETL and GPD ETL. This fact results because the simplifying assumptions about market or portfolio behaviour used in VaR models that can only measure risk in normal market conditions (see Table VI). As we can see in (see Tables V and VI and Figures VII), when the market works with stable normal conditions, normal VaR and ETL can be precise, but when the market is characterized by significant instabilities, like the latest market adversity started in August 2007 from US sub-prime mortgage crisis, it is clear that there is a need for an approach that comes to terms with problems posed by extreme event estimation, such as the GPD or a non parametric model such as the empirical approach. Further, empirical VaR and ETL, in all cases, are larger than that predicted from using the parametric approach assuming normality. The confidence level and thus the quantile chosen for the VaR and ETL estimation had a great effect of extreme values in the asset's return distribution. This has the important implication that the existence of a fat-tailed return distribution implies that at high confidence levels the parametric-normal underestimates the exposure to market risk, with the difference likely to become larger for higher confidence levels chosen and fatter tails.

The bootstrap estimates for shape and sigma do not appear acceptably close to normality. The histogram of the bootstrap estimates for shape parameter appears to be asymmetric. A QQ plot for the estimates of sigma, on the unlogged scale, would confirm the skewness that is in the histogram. As a result, it would be more reasonable to construct a confidence interval for sigma by first computing one for log(sigma) under the assumption of normality and then do an exponentiation to transform that interval back to the original scale for sigma. A better characterization of fat-tailed distribution of each index returns can be seen in the histogram of the bootstrap (Figures IX and Code VII) and in QQ-plot (Figures X and Code VII). In this case, quantiles of empirical prices or index returns distributions are plotted against the standard normal quantiles. If the distribution of the returns is normal, the QQ-plot should close to linear along the 45-degree line. However, the empirical data shows that there exist the deviations from normality. In this case, the plot curve upward at the left and downward at the right which is the evidence of heavy tails distribution rather than normal distribution. As a result, it is sensible to estimate VaR and ETL using EVT technique through GPD.

Before using EVT to model the tails of the distribution of an individual index or stock market, the data must be i.i.d. Most financial return series exhibit some degree of autocorrelation and, more importantly, heteroskedasticity. A quick review of the data reveals that it is not i.i.d justify with the financial crisis that causes wild swings in the stock market. For example, the sample autocorrelation function (see Figure XI) of the returns associated with the selected Lehman Brothers stock market reveal some mild serial correlation. This tendency reflects a degree of heteroskedasticity in which today’s volatility is dependent on yesterday’s volatility. Unless the data is preconditioned or filtered, this dependence will undermine the value of EVT. To produce a series of i.i.d. observations, a GARCH model is needed to filter out serial dependence in the data (Figure XII and Code II). The step that involves a repeated application of GARCH filtration is one of the most important steps in the overall modelling approach. The GARCH model produces a series of i.i.d. observations that satisfy the requirements of EVT (see Figure XII). Once the data is filtered, the data must fit a probability distribution to model the daily movements. This empirical study never assumed that the data comes from a normal distribution or from any other simple parametric distribution. Rather, it is assumed a more flexible empirical distribution that will let the data speak for itself. A kernel density estimate works well for the interior of the distribution where most of the data is found, but it performs poorly when applied to the upper and lower tails. In this study it will be tested a reasonable model of the more extreme observations, large losses and large gains. Figures XIII and Code III provide the empirical cumulative distribution function for the Lehman Brothers stock market, with the kernel density estimate for the interior and the GPD estimate for the upper and lower tails. The underlying Matlab code uses the Statistics Toolbox function paretotails to automate the curve fit shown in Figure XIII.

Since a copula is a multivariate probability distribution whose individual variables are uniformly distributed, we can now use the univariate distributions that we just derived to transform the individual data of each index or stock market to the uniform scale, the form required to fit a copula (see Code VIII). Copulas have experienced a tremendous surge in popularity in recent years. They enable analysts to isolate the dependence structure of portfolios from the description of the individual variables, and offer a compelling alternative to the traditional assumption of jointly normal portfolio returns. By decoupling the univariate description of the individual variables from the multivariate description of the dependence structure, copulas offer significant theoretical and computational advantages over conventional risk management techniques.

To visually assess how good the fit is, the data was plotted in a scaled histogram of the tail data overlaid with the density function of the estimated GPD. The histogram was scaled so that the bar heights times their width sum to one (see Code V). To visually assess the GPD fit, the Matlab Statistics Toolbox function gpfit was used to plot the empirical cumulative distribution function curve to find the parameters for the GPD in the tails of the curve. Figures XIV 5 shows that the empirically generated cumulative distribution function curve matches quite well with the fitted GPD results. With the similarity of the curves providing a level of confidence in the results, the analysis for all indices and stock prices is repeated. The fitted density follows the shape of the data, and so the GPD model seems to be a good choice.

The six models of VaR and ETL estimation were backtested with historical series of indices and stock firms log returns starting from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2009. The window range size of 250 is placed between the 1st and the 250th data points, each model is estimated and the forecast is obtained for the 251st day. Next, the window is moved one period ahead to obtain the forecast of the 252nd day return with updated parameters from this new sample. Table VIII displays the result of the backtesting methodology by the Kupiec's unconditional coverage test as well as empirical statistical test (or Bernoulli test), with the parameters recommended by the Capital Accords (see Table VII). The rest of the columns are labelled according the model at which the risk measures are computed. The models tested in this study were namely: Gaussian, empirical and GPD. To show the validity and accuracy of overall period, the violations for each model are exhibited in Table VIII. As a decision rule, the interpretation of Bernoulli test and ratio Kupiec's test less than the losses effectively incurred, or the number of exceptions, cannot proof the null hypothesis (see equations (xvii) and (xviii)). The result in Table VIII presents that, in almost all models, violations are closest to the expected ratio at 99% confidence level. The table shows that VaR and ETL estimated under normal model are the less conservative comparing to GPD and empirical models. The VaR and ETL estimation based on empirical and GPD approach are very similar, especially when it is interesting to demonstrate the VaR and ETL performance on September 12, 2008 or "Lehman Brothers bankruptcy". The result shows that all of Gaussian-based models cannot capture the extreme loss occurred providing a result underestimated. The empirical ETL and the GPD ETL perform better than the Gaussian-based models. Although VaR estimation by empirical and GPD models are also rejected at 99% confidence level, performing less accurately against ETL estimation by empirical and GDP models. It can be inferred that the estimation under normality assumption tend to underestimate at a higher confidence level as its violations ratio are almost two times higher than the expected while the corresponding Kupiec's test is extremely rejected in almost all Gaussian VaR at 99% confidence level. Empirical ETL and GPD ETL perform well in all given confidence levels, unlike Gaussian models and VaR estimators. Unfortunately, the simulation cannot distinguish the estimation based on both empirical and GPD models as they perform insignificantly different in almost all cases. The GPD ETL and empirical ETL are the best overall choices. Between the two, GPD ETL is considered more appropriate for the equity markets because this model do not overestimate the expected losses. For example, if the risk manager is interested only in the higher confidence level and for short positions, he should use the GPD distribution. Any other model would generate inaccurate risk forecasts. To summarize, it is plausible to consider these models, which forecasts the market risk number accurately for equity positions with 99% confidence level. The risk manager can select any of these models, irrespective of the equity position, and satisfy the requirements of the Basel Committee.

5. Conclusions

The Basel Committee for Banking Supervision, in its Basel II proposals, has failed to address many of the key deficiencies of the global financial regulatory system and even created the potential for new sources of instability. The proposed regulations fail to consider the fact that risk is endogenous. Additionally, regulators always respond to crisis by retrenchment rules and increasing the minimum capital requirements but this fact exacerbated the problem because the only way out of the subprime crisis was to create liquidity. Basel Accord failed to control the systemic risk in financial market while the focus is on micromanaging the banks in their jurisdiction, and disregard on macro-financial decision making under uncertainty.

In risk management, the VaR methodology as a measure of market risk is popular with both financial institutions and regulators. Traditional VaR models tend to ignore extreme events and focus on modelling the entire empirical distribution of returns. By wrongly using the Central limit theorem it is often assumed that returns are normally or log-normally distributed, but little attention is paid to the distribution of the tails. Inference about the extreme tails is always uncertain, because of low number of observations and sensitivity to the values of individual extreme observations. The key to estimating the distribution of such events is the EVT theorem, which governs the distribution of extreme values, and shows how this distribution looks like asymptotically. In recent research is observed that statistical EVT has been successfully used for modelling stock and index prices log returns, since there is empirical evidence that all important samples exhibit heavy tail behaviour. However, the evidence for goodness-of-fit of an EVT model is thin and it is always important to complement with the empirical characteristics such as the VaR or the ETL. Besides, the classical normal model has very light tails, which clearly do not provide a good fit to the data. Bank supervisors increasingly rely on models as a key component in their activities and this carries with it the risk modelling danger throughout the financial system. The natural response to these limitations is for banks to implement severe stress tests to complement the results of their VaR analyses. Stress tests are exercises to determine the losses that might occur under unlikely but plausible circumstances. There has been a dramatic increase in the importance given to stress testing since the last financial crisis that started in August 2007. Indeed, many firms and regulators now regard stress tests as no less important than VaR methods for assessing the market risk exposure.

This thesis illustrated how EVT can be used to model tail related risk measures such as the VaR and the ETL. By combining Gaussian, historical and EVT, this empirical study illustrates an approach for modelling market risk and characterizing the tail behaviour during financial and economic crises. It was presented how EVT framework performed under the select risk measures, such as VaR and ETL, on a data set. EVT offers exciting possibilities to further our understanding of tail events. Using the daily closings of the nine major developed market indices and seven stock market companies from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2009 as an example, this study shows how Matlab, Statistics Toolbox, and Optimization Toolbox, enable the managers to apply this combined approach to evaluate a popular risk metric known as VaR. The study conclusion is that EVT can be useful for assessing the size of extreme events. In this application, the POT method proved that exploits the information in the data sample. For VaR and ETL estimations, the historical and the EVT methods are the most accurate for 99th confidence level, since the number of backtesting rejections is zero. According to the testing results, Gaussian method always underestimates VaR and ETL. There are few comparative studies of tail behaviour across markets in the literature. These results are based on fitting a GPD to the data return and then calculate the maximum likelihood estimation of the scale, location and shape parameters of the GPD distribution. Methods based around assumptions of normal distributions are likely to underestimate tail risk. Methods based on historical simulation can only provide very imprecise estimates of tail risk. EVT is the most scientific approach to predict the size of a rare event.

It is important that the models to be used in risk management should produce relatively stable quantile forecasts since adjusting the implemented capital frequently (daily) in light of the estimated VaR is costly to implement and regulate. In this respect, the GPD models provide robust tail estimates, and therefore more stable VaR projections in turbulent times. EVT method has solid foundations in the mathematical theory of the behaviour of extremes. However, even when we have abundant, good-quality data to work with and an accurate model, our parameter estimates are still subject to a standard error. Furthermore, inference is sensitive to small changes in the parameters and to the largest observed losses, when with the introduction of new extreme losses in the data set may cause severe impacts. Another aspect of data uncertainty is the familiar assumption of i.i.d.. In practice this assumption can be clustering, trends, seasonal effects and other kinds of dependencies. The tail risk is the result of the interaction among various factors. These include the tail index, the scale parameter, the tail probability, the confidence level and the dependence structure. To capture the information disregarded by VaR and ETL, it is essential to monitor diverse aspects of the profit/loss distribution, such as tail fatness and asymptotic dependence. These issues lead to a number of interesting statistical extensions to research. For example, it can be tested the dependence in the data under some form of volatility persistence applied to measures of VaR and ETL in each market specific tail.

Most studies of financial return distribution settle on the Fréchet domain of attraction. According to Longin (1996) who studied a long sample of a century of daily returns of the NYSE while the mean and variance definitely exist, third moments and higher such as skewness and kurtosis could be infinite. Dacorogna *et al.* (2001), da Silva *et al.* (2003) and Pattarathammas *et al.* (2008) found similar evidence for other developed and emerging market series. Basak and Shapiro (2001) argued that when investors use VaR for their risk management, their optimising behaviour may result in market positions that are subject to extreme loss because VaR provides misleading information regarding the distribution tail. Additionally, regarding Taleb (2007) “The Black Swan” described the existence and occurrence of high-impact, hard-to-predict and rare events that are beyond the realm of normal expectations. The "Black Swan Events (capitalized)" refers only to unexpected events of large magnitude and consequence and their dominant role in history. Taleb regards as undirected and unpredicted almost all major scientific discoveries, historical events, and artistic accomplishments as "black swans". This author gives the examples of the Internet, the personal computer, World War I, and the September 11, 2001 attacks. His claim was that almost all consequential events in history came from the unexpected, nevertheless humans later convinced themselves that these events are explainable in hindsight. A famous 1997 debate[20] between Nassim Taleb and Philippe Jorion set out some of the major points of contention. More recently, David Einhorn and Aaron Brown debated VaR in Global Association of Risk Professionals Review and they compared VaR to “*an airbag that works all the time, except when you have a car accident*”. New York Times reporter Joe Nocera wrote an extensive piece Risk Mismanagement on January 4, 2009 discussing the role VaR played in the financial crisis of 2007-2008[21].

A regulatory body may prefer a model overpredicting the risk since the institutions will allocate more capital for regulatory purposes. Institutions would prefer a method underpredicting the risk, since they have to allocate less capital for regulatory purposes, if they are using the estimation only to meet the regulatory requirements. For this reason, the implemented capital allocation ratio is increased by the regulatory bodies for those models that consistently underpredict the risk. Advances that have been made in VaR should not be lost with the adoption of coherent risk measures into regulatory framework. Superior quality of VaR techniques should yield superior ETL forecasts showing that VaR and ETL should be regarded as partners not rivals. The weak points of risk measurement models cannot be ignored and they will continually come back even when the model is switched. The focus of future research should be on improving both VaR and ETL estimation techniques as well as finding optimal combinations of VaR-ETL models. In addition, banks must calculate the “stressed VaR” or “stressed ETL” measures in order to stress the model parameters since these models have the unrealistic normality assumption. This additional “stressed VaR” or

[20] See http://www.derivativesstrategy.com/magazine/archive/1997/0497fea2.asp .
[21] See http://www.nytimes.com/2009/01/04/magazine/04risk-t.html .

"stressed ETL" will also help to reduce the procyclicality of regulation and the susceptibility of the financial system to systemic crises.

Additionally, the GPD models can also be implemented by the Basel Committee on Banking Supervision since they calculate robust tail estimations, and can be easily applied to measures of VaR and ETL in each market specific tail. GPD models are also more accurate and more appropriate for risk market projections, specially, in turbulent times. For example, The Committee could establish GPD ETL as the standard measure to quantify market risk, with a fixed threshold μ (instead of the fixed confidence level). According to this model, the financial entities could, for instance, compute a predetermined μ, over a ten-day holding period, based on an historical observation period of at least one year of daily data. The GPD ETL model will generate more accurate risk forecasts then the VaR or the ETL estimations with normal assumption (regarding the empirical results from this thesis). The practical advantages of GPD ETL methodology are largely superior then Gaussian methodology, and this can become a regulatory exigency obliging financial institutions to obtain accurate and robust estimates in order to construct adequate capital structures. With all the issues facing statistical modelling and finance being better understood, it is fundamental to increase the accuracy of the models and that is why the supervisors are increasingly advocating the use of improved models in assessing the risk of individual institutions and financial stability.

The solution to the supervisors to a problem like the subprime crisis is a Basel II more accurate, with reliable and complemented models. However, this regulatory risk assessment has a purely statistical basis and tend do neglect the subjacent financial product complexity. It is important to have in mind that risk models are mathematical models and the respective parameters need to be correct. This accomplish need not only a strict regulation and accurate models but supervisors that understand the products being traded in the markets, to have an idea of the magnitude and potential for systemic risk and endogenous risk with a pro-active compliance to act when necessary.

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Tables

Table I – Data analyzed

This table presents the symbol and the name of the data set tested in the empirical study. The dataset reports to i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and to ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays).

Symbol	Name	Start	End	Observations
PSI20	Portuguese Stock Index	06-Oct-1999	13-Jul-2009	2.455
CAC40	Cotation Assistée en Continu - French stock index			
DAX	Deutscher Aktien Index - German stock index			
Nikkei225	Nihon Keizai Shimbun - Japanese stock index			
FTSE100	Financial Times Stock Exchange - English stock index			
S&P500	Standard & Poors - American stock index			
Nasdaq	National Association of Securities Dealers Automated Quotations - American stock index			
DJ	Dow Jones Industrial Average - American stock index			
Sensex	Bombay Stock Exchange Sensitive Index - Indian stock market index			
Apple	Apple Inc.	4-Nov-1999	12-Sept-2008	2.226
MS	Microsoft Corporation			
LB	Lehman Brothers Holdings Inc.			
BES	Banco Espírito Santo, S.A.			
BCP	Banco Comercial Português, S.A.			
GE	General Electric Company			
GS	The Goldman Sachs Group, Inc.			

Source: Author

Tables II - Basic statistics

This table presents the basic statistics extracted from ECVaR software, such as daily/annual returns and daily/annual standard deviations, for seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).

Asset	Daily return | annual	Daily Std.Dev. | annual
SP500	-0,0157 % / -5,5761 %	0,0141 / 0,2691
DJ	-0,0098 % / -3,5008 %	0,0133 / 0,2534
Nasdaq	-0,0190 % / -6,6915 %	0,0195 / 0,3732
PSI20	-0,0234 % / -8,1706 %	0,0111 / 0,2117
CAC40	-0,0232 % / -8,1256 %	0,0159 / 0,3034
DAX	-0,0091 % / -3,2792 %	0,0169 / 0,3224
Sensex	0,0440 % / 17,4067 %	0,0180 / 0,3445
FTSE100	-0,0146 % / -5,1734 %	0,0135 / 0,2572
Nikkei225	-0,0284 % / -9,8517 %	0,0163 / 0,3114

This table presents the basic statistics extracted from ECVaR software, such as daily/annual returns and daily/annual standard deviations, for seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).

Asset	Daily return | annual	Daily Std.Dev. | annual
Lehman Bros	-0,1362 % / -39,1855 %	0,0519 / 0,9910
Apple	0,0259 % / 9,9279 %	0,0352 / 0,6717
Microsoft	-0,0539 % / -17,8688 %	0,0238 / 0,4538
Goldman Sachs	0,0327 % / 12,6702 %	0,0232 / 0,4431
GE	-0,0128 % / -4,5603 %	0,0181 / 0,3466
BES	-0,0518 % / -17,2253 %	0,0141 / 0,2688
BCP	-0,0686 % / -22,1518 %	0,0171 / 0,3274

Source: ECVaR

Tables III - Covariance Matrix

This table presents the covariance matrix extracted from ECVaR software for seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).

	SP500	DJ	Nasdaq	PSI20	CAC40	DAX	Sensex	FTSE100	Nikkei225
SP500	0,0001983	0,0001803	0,0002389	0,0000037	-0,0000071	-0,0000018	-0,0000108	0,0000343	-0,0000143
DJ	0,0001803	0,0001759	0,0002008	0,0000038	-0,0000078	0,0000007	-0,0000097	0,0000303	-0,0000128
Nasdaq	0,0002389	0,0002008	0,0003815	0,0000067	-0,0000018	-0,0000002	-0,0000071	0,0000383	-0,0000138
PSI20	0,0000037	0,0000038	0,0000067	0,0001227	0,0000472	-0,0000033	0,0000109	0,0000073	0,0000041
CAC40	-0,0000071	-0,0000078	-0,0000018	0,0000472	0,0002523	0,0000101	0,0000184	-0,0000042	-0,0000030
DAX	-0,0000018	0,0000007	-0,0000002	-0,0000033	0,0000101	0,0002848	0,0000060	-0,0000013	-0,0000021
Sensex	-0,0000108	-0,0000097	-0,0000071	0,0000109	0,0000184	0,0000060	0,0003252	0,0000048	0,0000028
FTSE100	0,0000343	0,0000303	0,0000383	0,0000073	-0,0000042	-0,0000013	0,0000048	0,0001813	-0,0000006
Nikkei225	-0,0000143	-0,0000128	-0,0000138	0,0000041	-0,0000030	-0,0000021	0,0000028	-0,0000006	0,0002657

This table presents the covariance matrix extracted from ECVaR software for seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).

	LB	Apple	Microsoft	GS	GE	BES	BCP
LB	0,0026905	0,0002155	0,0001353	-0,0000104	0,0001559	0,0000067	-0,0000122
Apple	0,0002155	0,0012361	0,0002407	0,0000273	0,0001963	0,0000031	-0,0000228
Microsoft	0,0001353	0,0002407	0,0005643	0,0000177	0,0001648	-0,0000063	0,0000005
GS	-0,0000104	0,0000273	0,0000177	0,0005379	0,0000108	-0,0000045	-0,0000005
GE	0,0001559	0,0001963	0,0001648	0,0000108	0,0003291	0,0000005	0,0000130
BES	0,0000067	0,0000031	-0,0000063	-0,0000045	0,0000005	0,0001979	0,0000091
BCP	-0,0000122	-0,0000228	0,0000005	-0,0000005	0,0000130	0,0000091	0,0002937

Source: ECVaR

Tables IV - Correlation matrix

This table presents the correlation matrix extracted from ECVaR software for seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).

	SP500	DJ	Nasdaq	PSI20	CAC40	DAX	Sensex	FTSE100	Nikkei225
SP500	1	0,9657592	0,8687463	0,0239265	-0,0317066	-0,0074449	-0,0426771	0,1807697	-0,0622411
DJ	0,9657592	1	0,7755985	0,0259543	-0,0372444	0,0032845	-0,0405574	0,1699810	-0,0594490
Nasdaq	0,8687463	0,7755985	1	0,0308857	-0,0058832	-0,0006801	-0,0202085	0,1458403	-0,0433528
PSI20	0,0239265	0,0259543	0,0308857	1	0,2684057	-0,0173957	0,0547316	0,0491710	0,0225367
CAC40	-0,0317066	-0,0372444	-0,0058832	0,2684057	1	0,0377219	0,0642478	-0,0198237	-0,0115991
DAX	-0,0074449	0,0032845	-0,0006801	-0,0173957	0,0377219	1	0,0197532	-0,0058454	-0,0077491
Sensex	-0,0426771	-0,0405574	-0,0202085	0,0547316	0,0642478	0,0197532	1	0,0198220	0,0093999
FTSE100	0,1807697	0,1699810	0,1458403	0,0491710	-0,0198237	-0,0058454	0,0198220	1	-0,0028705
Nikkei225	-0,0622411	-0,0594490	-0,0433528	0,0225367	-0,0115991	-0,0077491	0,0093999	-0,0028705	1

This table presents the correlation matrix extracted from ECVaR software for seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).

	LB	Apple	Microsoft	GS	GE	BES	BCP
LB	1	0,1182152	0,1098811	-0,0086096	0,1657204	0,0092344	-0,0137063
Apple	0,1182152	1	0,2882968	0,0335244	0,3079524	0,0062475	-0,0378095
Microsoft	0,1098811	0,2882968	1	0,0320992	0,3826815	-0,0189911	0,0012686
GS	-0,0086096	0,0335244	0,0320992	1	0,0257337	-0,0138068	-0,0012345
GE	0,1657204	0,3079524	0,3826815	0,0257337	1	0,0021126	0,0418507
BES	0,0092344	0,0062475	-0,0189911	-0,0138068	0,0021126	1	0,0375624
BCP	-0,0137063	-0,0378095	0,0012686	-0,0012345	0,0418507	0,0375624	1

Source: ECVaR

Table V – Descriptive statistics of daily returns for nine indices and seven stock market firms

Descriptive statistics of daily returns for nine popular indices (PSI20, CAC40, DAX, Nikkei225, FTSE100, S&P500, Nasdaq, Dow Jones and Sensex) and seven stock market firms (Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs) from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008. This results obtained for VaR and ETL were modelled with empirical (or historical), Gaussian and generalized Pareto (peaks over threshold technique of extreme value theory).

	[6-Oct-1999; 13-Jul-2009]									[4-Nov-1999; 12-Sept-2008]						
	PSI20	**CAC40**	**DAX**	**Nikkei225**	**FTSE100**	**S&P500**	**Nasdaq**	**DJ**	**Sensex**	**Apple**	**MS**	**LB**	**BES**	**BCP**	**GE**	**GS**
Number of Observations (n)	2455	2455	2455	2455	2455	2455	2455	2455	2455	2226	2226	2226	2226	2226	2226	2226
Number of Exceedances	25	25	25	25	25	25	25	25	25	22	22	22	22	22	22	22
Shape / Tail index (k)	0,37651	-0,00502	-0,03509	0,35164	0,03334	0,43245	0,12812	-0,02514	0,37065	* -0,64537	0,21663	0,70320	0,14236	0,06623	-0,07569	0,15136
Scale (σ)	0,00884	0,01860	0,01796	0,00997	0,01496	0,00866	0,01434	0,01918	0,00861	0,02891	0,01713	0,03862	0,01254	0,00992	0,01738	0,01924
Mean Exceedance (u)	-0,00065	-0,00086	-0,00076	-0,00086	-0,00067	-0,00071	-0,00063	-0,00095	-0,00021	-0,00077	-0,00133	-0,00325	-0,00098	-0,00131	-0,00080	-0,00055
Threshold (μ)	2,6%	4,2%	4,7%	4,0%	3,6%	4,0%	3,5%	5,5%	5,0%	8,6%	5,7%	8,7%	3,2%	5,1%	5,2%	6,6%
stdErr = k	0,30370	0,34760	0,24760	0,28040	0,26280	0,36320	0,24820	0,21840	0,26420	0,22630	0,28650	0,33030	0,28020	0,25720	0,20980	0,36600
stdErr = σ	0,00310	0,00750	0,00570	0,00340	0,00490	0,00350	0,00460	0,00570	0,00280	0,00810	0,00610	0,01400	0,00440	0,00330	0,00520	0,00810
returns (min)	-0,10384	-0,09477	-0,07434	-0,12111	-0,09272	-0,09451	-0,08200	-0,10183	-0,11815	-0,73125	-0,66016	-0,75274	-0,39290	-0,17096	-0,13684	-0,14072
returns (max)	0,09703	0,10624	0,10795	0,13244	0,09384	0,10947	0,10515	0,13260	0,15992	0,12829	0,17877	0,77704	0,09669	0,09677	0,11718	0,15077
Daily mean	-0,00023	-0,00023	-0,00009	-0,00028	-0,00015	-0,00016	-0,00010	-0,00019	0,00044	0,00026	-0,00054	-0,00136	-0,00052	-0,00069	-0,00013	0,00033
Daily volatility	0,01109	0,01587	0,01685	0,01633	0,01347	0,01408	0,01325	0,01949	0,01804	0,03902	0,02561	0,05232	0,01519	0,01722	0,01814	0,02309
Daily skewness	-0,31268	0,03499	0,08226	-0,30016	-0,10282	-0,09479	0,02465	0,10786	-0,16584	-7,02139	-7,80326	-3,66841	-11,37301	-0,61135	-0,10924	0,23299
Daily kurtosis	9,82445	5,02586	4,12428	6,35437	6,18686	7,67599	7,49791	3,96248	5,67634	122,51119	200,65090	114,74481	261,35075	9,21056	5,56506	4,24425
Significance Level (α)	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%	1,00%
Normal Percentil	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635	-2,32635
Standard Normal ETL	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521	-2,66521
Empirical VaR	3,48%	4,53%	5,06%	4,68%	4,02%	4,02%	3,81%	5,49%	5,07%	7,89%	6,39%	11,03%	3,14%	5,13%	4,64%	5,86%
Normal VaR	2,60%	3,72%	3,93%	3,83%	3,15%	3,29%	3,09%	4,55%	4,15%	9,05%	6,01%	12,31%	3,59%	4,07%	4,23%	5,34%
GPD VaR	2,67%	4,21%	4,74%	4,05%	3,59%	3,98%	3,51%	5,49%	4,98%	8,53%	5,72%	8,63%	3,15%	5,13%	5,13%	6,54%
Empirical ETL	4,60%	5,91%	5,95%	6,52%	5,36%	5,76%	5,31%	6,84%	6,65%	17,68%	11,45%	28,51%	7,75%	7,20%	6,74%	7,74%
Normal ETL	2,98%	4,25%	4,50%	4,38%	3,60%	3,77%	3,54%	5,21%	4,76%	10,37%	6,88%	14,08%	4,10%	4,66%	4,85%	6,12%
GPD ETL	4,09%	6,06%	6,48%	5,60%	5,14%	5,52%	5,16%	7,36%	6,36%	10,30%	7,90%	21,53%	4,61%	6,19%	6,75%	8,80%

* Warning: Maximum likelihood has converged to an estimate of K < -1/2. Confidence intervals and standard errors can not be computed reliably.

Source: Author

Tables VI - Descriptive assessment of empirical, normal and GPD models in VaR and ETL estimation

Descriptive statistics of daily returns for nine popular indices (PSI20, CAC40, DAX, Nikkei225, FTSE100, S&P500, Nasdaq, Dow Jones and Sensex) and seven stock market firms (Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs) from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008. This results obtained for VaR and ETL were modelled with empirical (or historical), Gaussian and generalized Pareto (peaks over threshold technique of extreme value theory). The empirical and GPD models predict fairly consistent risk measures, particularly empirical ETL and GPD ETL. This fact results because the simplifying assumptions about market or portfolio behaviour used in VaR models that can only measure risk in normal market conditions.

Δ GPD	**PSI20**	**CAC40**	**DAX**	**Nikkei225**	**FTSE100**	**S&P500**	**Nasdaq**	**DJ**	**Sensex**	**Apple**	**MS**	**LB**	**BES**	**BCP**	**GE**	**GS**	**∑**
								VaR									
Empirical	0,82%	0,31%	0,32%	0,63%	0,43%	0,04%	0,30%	0,01%	0,09%	-0,64%	0,67%	2,41%	0,00%	0,00%	-0,49%	-0,68%	**4,21%**
Normal	-0,06%	-0,50%	-0,81%	-0,23%	-0,44%	-0,69%	-0,42%	-0,94%	-0,83%	0,52%	0,29%	3,68%	0,44%	-1,05%	-0,90%	-1,20%	**-3,13%**
								ETL									
Empirical	0,51%	-0,16%	-0,53%	0,92%	0,22%	0,24%	0,15%	-0,52%	0,29%	7,39%	3,55%	6,98%	3,14%	1,01%	-0,01%	-1,07%	**22,12%**
Normal	-1,11%	-1,81%	-1,97%	-1,22%	-1,53%	-1,75%	-1,62%	-2,14%	-1,60%	0,08%	-1,02%	-7,45%	-0,51%	-1,53%	-1,90%	-2,68%	**-29,78%**

	PSI20	**CAC40**	**DAX**	**Nikkei225**	**FTSE100**	**S&P500**	**Nasdaq**	**DJ**	**Sensex**	**Apple**	**MS**	**LB**	**BES**	**BCP**	**GE**	**GS**	
GPD ETL	4,09%	6,06%	6,48%	5,60%	5,14%	5,52%	5,16%	7,36%	6,36%	10,30%	7,90%	21,53%	4,61%	6,19%	6,75%	8,80%	∑
Δ Normal VaR	-1,49%	-2,35%	-2,55%	-1,77%	-1,99%	-2,23%	-2,07%	-2,80%	-2,21%	-1,25%	-1,89%	-9,22%	-1,02%	-2,11%	-2,51%	-3,46%	**-40,93%**
Δ Normal ETL	-1,11%	-1,81%	-1,97%	-1,22%	-1,53%	-1,75%	-1,62%	-2,14%	-1,60%	0,08%	-1,02%	-7,45%	-0,51%	-1,53%	-1,90%	-2,68%	**-29,78%**

Source: Author

Tables VII - Supervisory Framework for the use of "backtesting" in conjunction with the internal models approach to market risk capital requirements

Model is accurate		
Exceptions (our of 250)	Coverage = 99%	
	exact	type 1
0	8.1 %	100.0 %
1	20.5 %	91.9 %
2	25.7 %	71.4 %
3	21.5 %	45.7 %
4	13.4 %	24.2 %
5	6.7 %	10.8 %
6	2.7 %	4.1 %
7	1.0 %	1.4 %
8	0.3 %	0.4 %
9	0.1 %	0.1 %
10	0.0 %	0.0 %
11	0.0 %	0.0 %
12	0.0 %	0.0 %
13	0.0 %	0.0 %
14	0.0 %	0.0 %
15	0.0 %	0.0 %

Model is inaccurate: Possible alternative levels of coverage								
Exceptions (our of 250)	Coverage = 98%		Coverage = 97%		Coverage = 96%		Coverage = 95%	
	exact	type 2	exact	type 2	exact	type 2	exact	type 2
0	0.6 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %
1	3.3 %	0.6 %	0.4 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %
2	8.3 %	3.9 %	1.5 %	0.4 %	0.2 %	0.0 %	0.0 %	0.0 %
3	14.0 %	12.2 %	3.8 %	1.9 %	0.7 %	0.2 %	0.1 %	0.0 %
4	17.7 %	26.2 %	7.2 %	5.7 %	1.8 %	0.9 %	0.3 %	0.1 %
5	17.7 %	43.9 %	10.9 %	12.8 %	3.6 %	2.7 %	0.9 %	0.5 %
6	14.8 %	61.6 %	13.8 %	23.7 %	6.2 %	6.3 %	1.8 %	1.3 %
7	10.5 %	76.4 %	14.9 %	37.5 %	9.0 %	12.5 %	3.4 %	3.1 %
8	6.5 %	86.9 %	14.0 %	52.4 %	11.3 %	21.5 %	5.4 %	6.5 %
9	3.6 %	93.4 %	11.6 %	66.3 %	12.7 %	32.8 %	7.6 %	11.9 %
10	1.8 %	97.0 %	8.6 %	77.9 %	12.8 %	45.5 %	9.6 %	19.5 %
11	0.8 %	98.7 %	5.8 %	86.6 %	11.6 %	58.3 %	11.1 %	29.1 %
12	0.3 %	99.5 %	3.6 %	92.4 %	9.6 %	69.9 %	11.6 %	40.2 %
13	0.1 %	99.8 %	2.0 %	96.0 %	7.3 %	79.5 %	11.2 %	51.8 %
14	0.0 %	99.9 %	1.1 %	98.0 %	5.2 %	86.9 %	10.0 %	62.9 %
15	0.0 %	100.0 %	0.5 %	99.1 %	3.4 %	92.1 %	8.2 %	72.9 %

Notes: The table reports both exact probabilities of obtaining a certain number of exceptions from a sample of 250 independent observations under several assumptions about the true level of coverage, as well as type 1 or type 2 error probabilities derived from these exact probabilities.

The left-hand portion of the table pertains to the case where the model is accurate and its true level of coverage is 99%. Thus, the probability of any given observation being an exception is 1% (100% - 99% = 1%). The column labelled "exact" reports the probability of obtaining exactly the number of exceptions shown under this assumption in a sample of 250 independent observations. The column labelled "type 1" reports the probability that using a given number of exceptions as the cut-off for rejecting a model will imply erroneous rejection of an accurate model using a sample of 250 independent observations. For example, if the cut-off level is set at five or more exceptions, the type 1 column reports the probability of falsely rejecting an accurate model with 250 independent observations is 10.8%.

The right-hand portion of the table pertains to models that are inaccurate. In particular, the table concentrates of four specific inaccurate models, namely models whose true levels of coverage are 98%, 97%, 96% and 95% respectively. For each inaccurate model, the "exact" column reports the probability of obtaining exactly the number of exceptions shown under this assumption in a sample of 250 independent observations. The columns labelled "type 2" report the probability that using a given number of exceptions as the cut-off for rejecting a model will imply erroneous acceptance of an inaccurate model with the assumed level of coverage using a sample of 250 independent observations. For example, if the cut-off level is set at five or more exceptions, the type 2 column for an assumed coverage level of 97% reports the probability of falsely accepting a model with only 97% coverage with 250 independent observations is 12.8%.

Zone	Number of exceptions	Increase in scaling factor	Cumulative probability
Green Zone	0	0.00	8.11%
	1	0.00	28.58%
	2	0.00	54.32%
	3	0.00	75.81%
	4	0.00	89.22%
Yellow Zone	5	0.40	95.88%
	6	0.50	98.63%
	7	0.65	99.60%
	8	0.75	99.89%
	9	0.85	99.97%
Red Zone	10 or more	1.00	99.99%

Notes: The table defines the green, yellow and red zones that supervisors will use to assess backtesting results in conjunction with the internal models approach to market risk capital requirements. The boundaries shown in the table are based on a sample of 250 observations. For other sample sizes, the yellow zone begins at the point where the cumulative probability equals or exceeds 95%, and the red zone begins at the point where the cumulative probability equals or exceeds 99.99%.

The cumulative probability is simply the probability of obtaining a given number or fewer exceptions in a sample of 250 observations when the true coverage level is 99%. For example, the cumulative probability shown for four exceptions is the probability of obtaining between zero and four exceptions.

Note that these cumulative probabilities and the type 1 error probabilities reported in Table 1 do not sum to one because the cumulative probability for a given number of exceptions includes the possibility of obtaining exactly that number of exceptions, as does the type 1 error probability. Thus, the sum of these two probabilities exceeds one by the amount of the probability of obtaining exactly that number of exceptions.

Source: BIS

Table VIII - Descriptive statistics of “backtesting” the six models of VaR and ETL estimation by Bernoulli test and Kupiec test

The six models of VaR and ETL estimation were backtested with historical series of indices and stock firms log returns starting from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2009. The backtesting methodology consists in the Kupiec’s unconditional coverage test as well as empirical statistical test (or Bernoulli), with the parameters recommended by the Capital Accords.

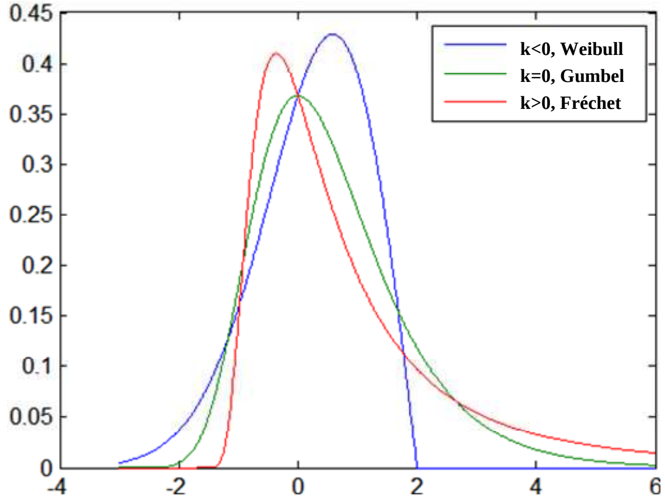
	PSI20	CAC40	DAX	Nikkei225	FTSE100	S&P500	Nasdaq	DJ	Sensex	Apple	MS	LB	BES	BCP	GE	GS
Number of Observations (n)	2.455	2.455	2.455	2.455	2.455	2.455	2.455	2.455	2.455	2.226	2.226	2.226	2.226	2.226	2.226	2.226
Significance Level	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%
Rolling Window Size	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250	250
Sample Size of the Backtest	2.205	2.205	2.205	2.205	2.205	2.205	2.205	2.205	2.205	1.976	1.976	1.976	1.976	1.976	1.976	1.976
Expected number of Exceptions	22	22	22	22	22	22	22	22	22	20	20	20	20	20	20	20
Cutoff Value (~N)	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326	2,326
Chi-squared Critical	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635	6,635
# Empirical VaR Events	**40**	**35**	**35**	**36**	**41**	**42**	**40**	**35**	**39**	**24**	**27**	**35**	**40**	**40**	**27**	**30**
BT	3,842	2,772	2,772	2,986	4,056	4,270	3,842	2,772	3,628	0,959	1,637	3,446	4,576	4,576	1,637	2,315
Bernoulli trails Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Reject	Reject	Reject	Accept	Accept
π_obs Empirical VaR Events	1,81%	1,59%	1,59%	1,63%	1,86%	1,90%	1,81%	1,59%	1,77%	1,21%	1,37%	1,77%	2,02%	2,02%	1,37%	1,52%
LR_{uc}	11,893	6,519	6,519	7,484	13,126	14,409	11,893	6,519	10,711	0,860	2,404	9,657	16,148	16,148	2,404	4,626
Kupiec Test	Reject	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject	Accept	Accept	Reject	Reject	Reject	Accept	Accept
# Normal VaR Events	**55**	**56**	**54**	**51**	**62**	**54**	**53**	**36**	**47**	**20**	**22**	**39**	**41**	**66**	**25**	**23**
BT	7,052	7,266	6,838	6,196	8,551	6,838	6,624	2,986	5,340	0,054	0,506	4,350	4,802	10,455	1,185	0,733
Bernoulli trails Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Reject	Reject	Reject	Accept	Accept
π_obs Normal VaR Events	2,49%	2,54%	2,45%	2,31%	2,81%	2,45%	2,40%	1,63%	2,13%	1,01%	1,11%	1,97%	2,07%	3,34%	1,27%	1,16%
LR_{uc}	35,142	37,019	33,302	28,014	49,030	33,302	31,501	7,484	21,529	0,003	0,247	14,742	17,604	67,813	1,295	0,510
Kupiec Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Reject	Reject	Reject	Accept	Accept
# GPD VaR Events	**49**	**34**	**35**	**37**	**43**	**37**	**39**	**29**	**32**	**21**	**20**	**15**	**22**	**27**	**15**	**10**
BT	5,768	2,558	2,772	3,200	4,484	3,200	3,628	1,488	2,130	0,280	0,054	-1,076	0,506	1,637	-1,076	-2,207
Bernoulli trails Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
π_obs GPD VaR Events	2,22%	1,54%	1,59%	1,68%	1,95%	1,68%	1,77%	1,32%	1,45%	1,06%	1,01%	0,76%	1,11%	1,37%	0,76%	0,51%
LR_{uc}	24,688	5,613	6,519	8,505	15,740	8,505	10,711	2,013	3,981	0,077	0,003	1,263	0,247	2,404	1,263	5,947
Kupiec Test	Reject	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
# Empirical ETL Events	**27**	**22**	**28**	**25**	**27**	**29**	**26**	**21**	**22**	**12**	**12**	**17**	**12**	**23**	**12**	**12**
BT	1,059	-0,011	1,273	0,631	1,059	1,488	0,845	-0,225	-0,011	-1,754	-1,754	-0,624	-1,754	0,733	-1,754	-1,754
Bernoulli trails Test	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
π_obs Empirical ETL Events	1,22%	1,00%	1,27%	1,13%	1,22%	1,32%	1,18%	0,95%	1,00%	0,61%	0,61%	0,86%	0,61%	1,16%	0,61%	0,61%
LR_{uc}	1,048	0,000	1,494	0,382	1,048	2,013	0,676	0,051	0,000	3,581	3,581	0,409	3,581	0,510	3,581	3,581
Kupiec Test	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
# Normal ETL Events	**44**	**43**	**38**	**38**	**45**	**40**	**38**	**29**	**25**	**21**	**18**	**35**	**30**	**51**	**18**	**11**
BT	4,698	4,484	3,414	3,414	4,912	3,842	3,414	1,488	0,631	0,280	-0,398	3,446	2,315	7,063	-0,398	-1,981
Bernoulli trails Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Accept	Accept	Reject	Accept	Reject	Accept	Accept
π_obs Normal ETL Events	2,00%	1,95%	1,72%	1,72%	2,04%	1,81%	1,72%	1,32%	1,13%	1,06%	0,91%	1,77%	1,52%	2,58%	0,91%	0,56%
LR_{uc}	17,119	15,740	9,582	9,582	18,544	11,893	9,582	2,013	0,382	0,077	0,163	9,657	4,626	34,734	0,163	4,672
Kupiec Test	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Accept	Accept	Accept	Reject	Accept	Reject	Accept	Accept
# GPD ETL Events	**23**	**13**	**12**	**23**	**24**	**25**	**27**	**23**	**26**	**10**	**11**	**10**	**10**	**20**	**10**	**10**
BT	0,203	-1,937	-2,151	0,203	0,417	0,631	1,059	0,203	0,845	-2,207	-1,981	-2,207	-2,207	0,054	-2,207	-2,207
Bernoulli trails Test	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
π_obs GPD ETL Events	1,04%	0,59%	0,54%	1,04%	1,09%	1,13%	1,22%	1,04%	1,18%	0,51%	0,56%	0,51%	0,51%	1,01%	0,51%	0,51%
LR_{uc}	0,041	4,400	5,544	0,041	0,169	0,382	1,048	0,041	0,676	5,947	4,672	5,947	5,947	0,003	5,947	5,947
Kupiec Test	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Source: Author

Figures

Figure I - Three possible limiting extreme value distributions for the standardized maxima

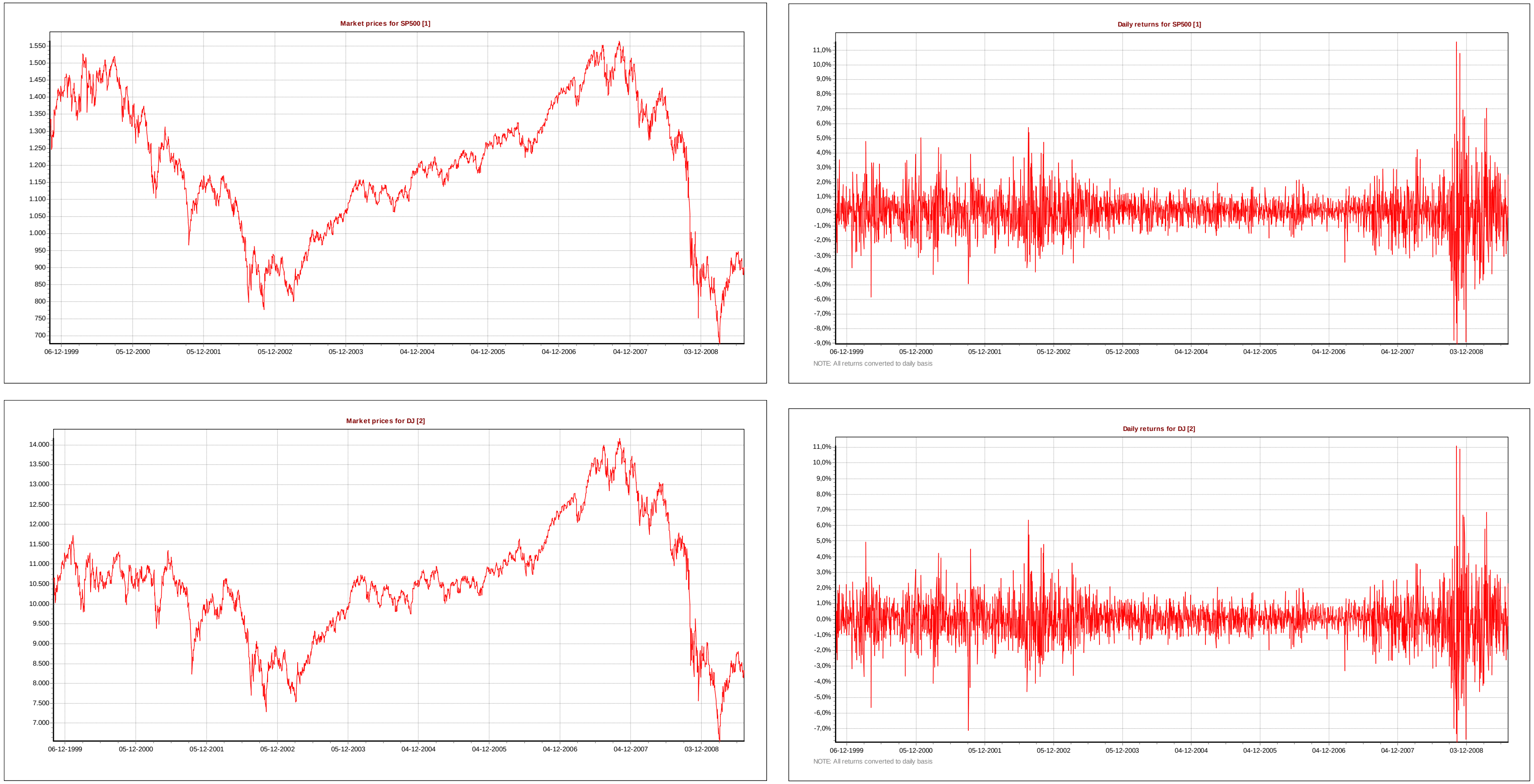
Fisher Tippet theorem summarizes three possible limiting extreme value distributions for the standardized maxima. The Generalized Extreme Value (GEV) distribution unites the type Gumbel, type Fréchet and type Weibull extreme value distributions into a single family, to allow a continuous range of possible shapes. It is parameterized with location and scale parameters, μ and σ, and a shape parameter, k. When $k < 0$, the GEV is equivalent to the Weibull extreme value. When $k > 0$, the GEV is equivalent to the Fréchet. In the limit as k approaches 0, the GEV becomes the type Gumbel. Notice that for $k < 0$ or $k > 0$, the density has zero probability above or below, respectively, the upper or lower bound $-(1/k)$.



Source: Matlab

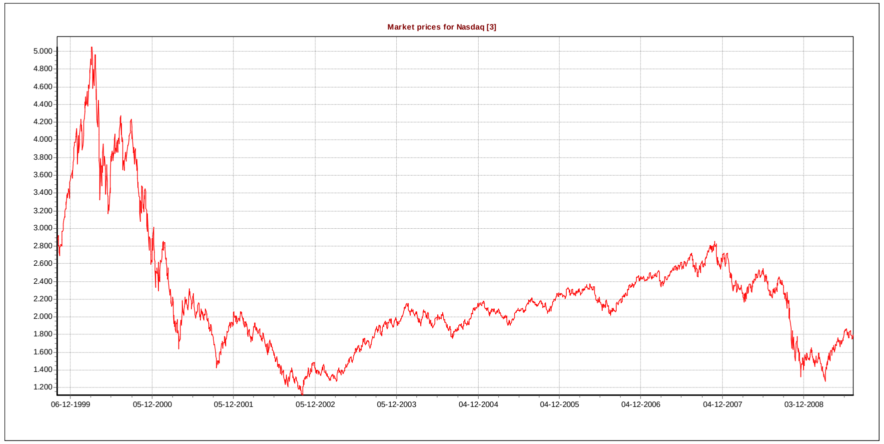
Figures II – The historical prices and index returns series

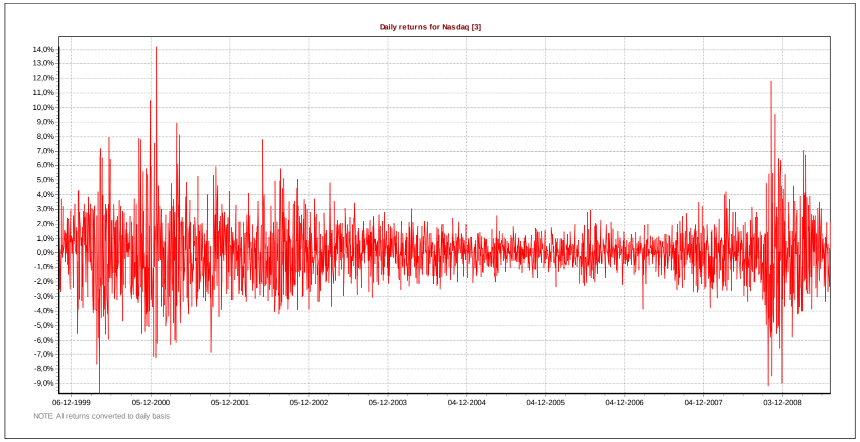
These figures report the descriptive statistics of the daily prices and the daily returns of S&P500 and Dow Jones. The total number of observations is 2.455 days ranging from October 6, 1999 to July 13, 2009 (excluding holidays).

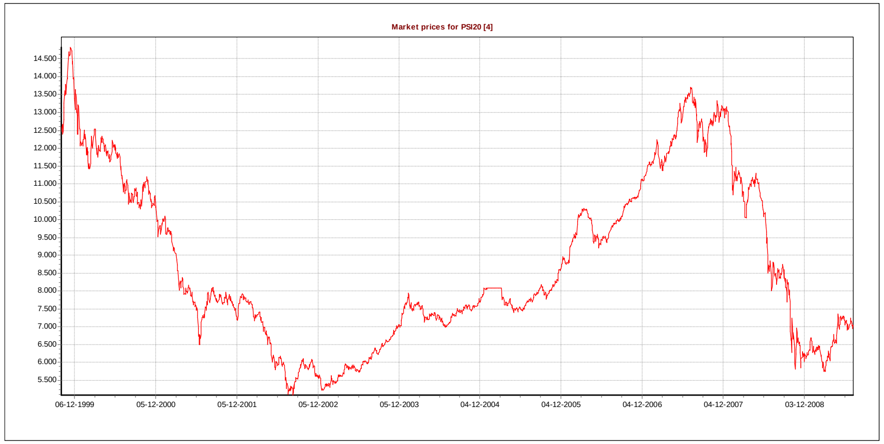


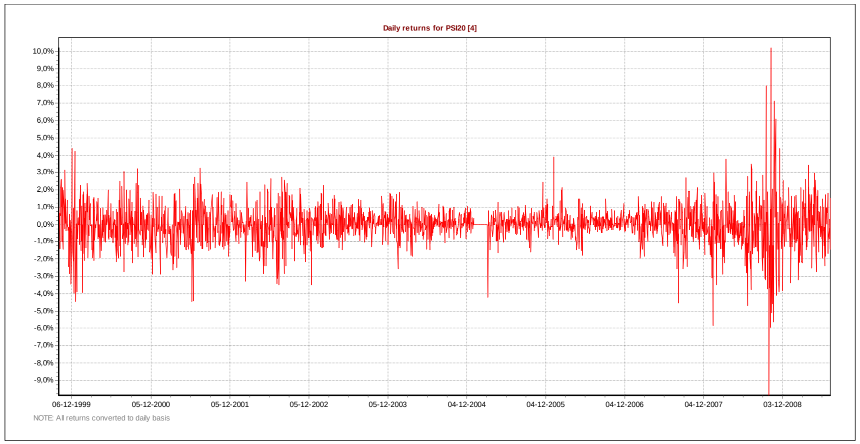
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of Nasdaq and PSI20. The total number of observations is 2.455 days ranging from October 6, 1999 to July 13, 2009 (excluding holidays).



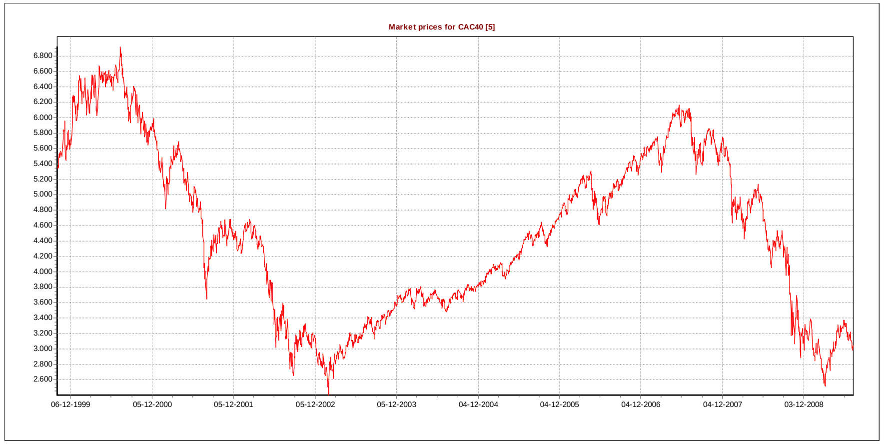


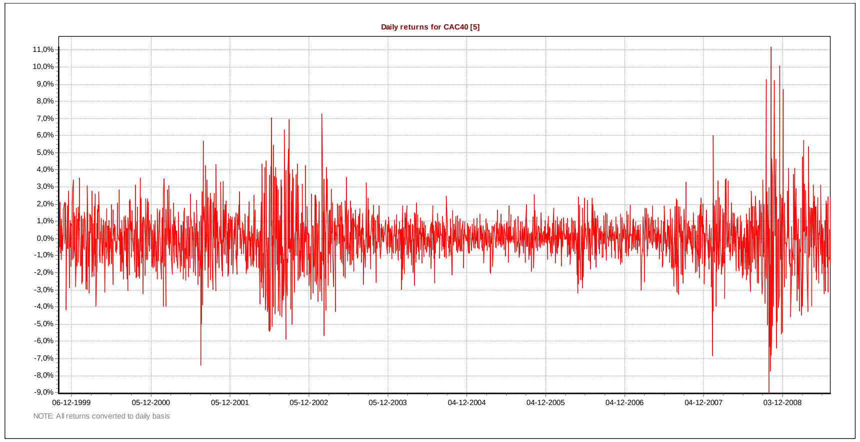


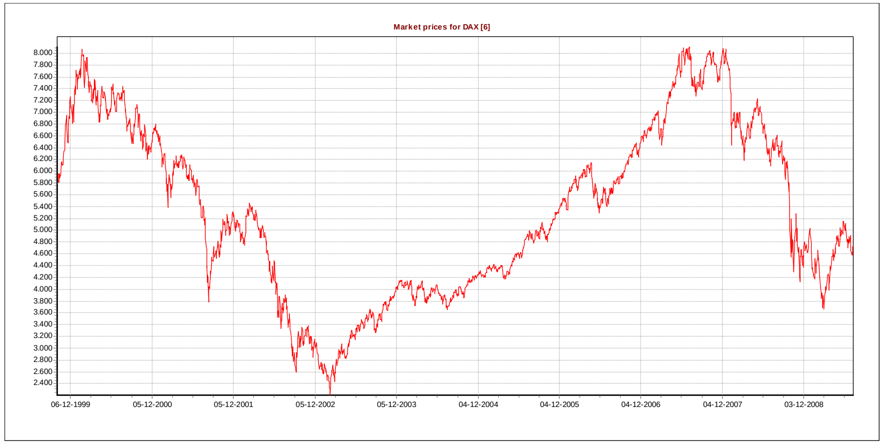


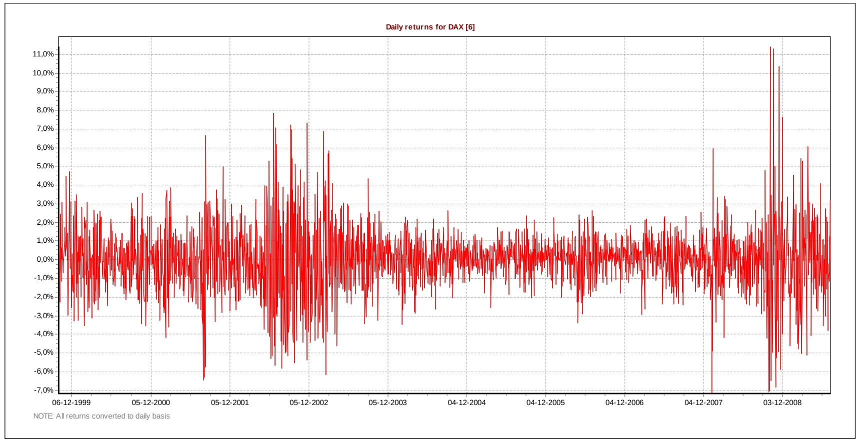
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of CAC40 and DAX. The total number of observations is 2.455 days ranging from October 6, 1999 to July 13, 2009 (excluding holidays).



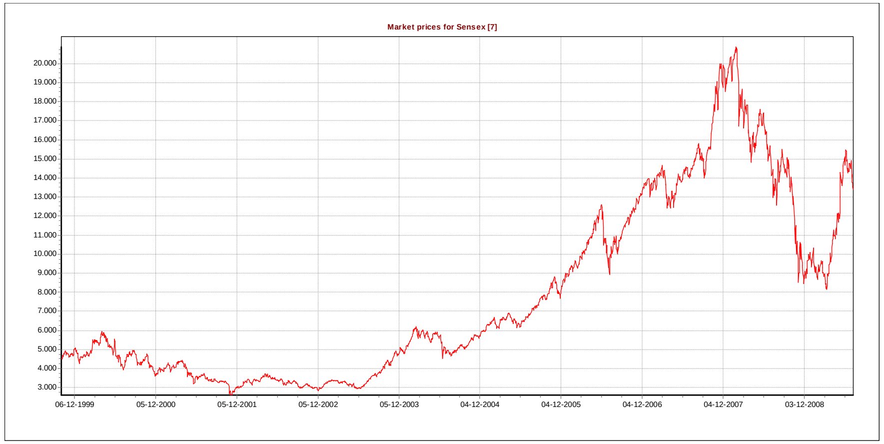


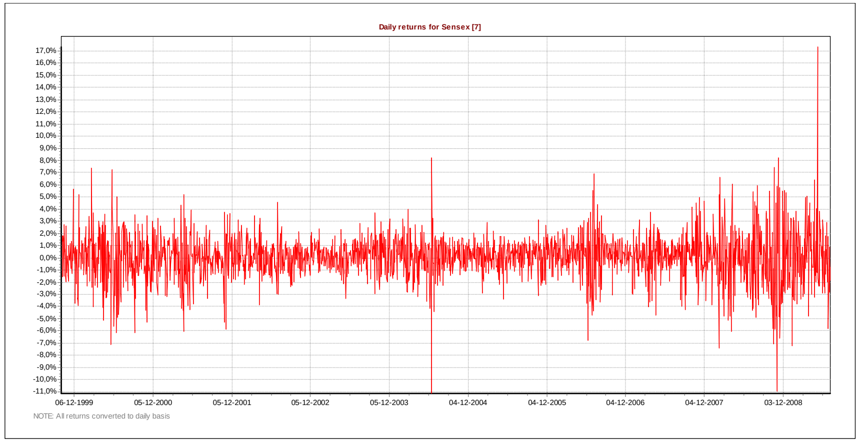


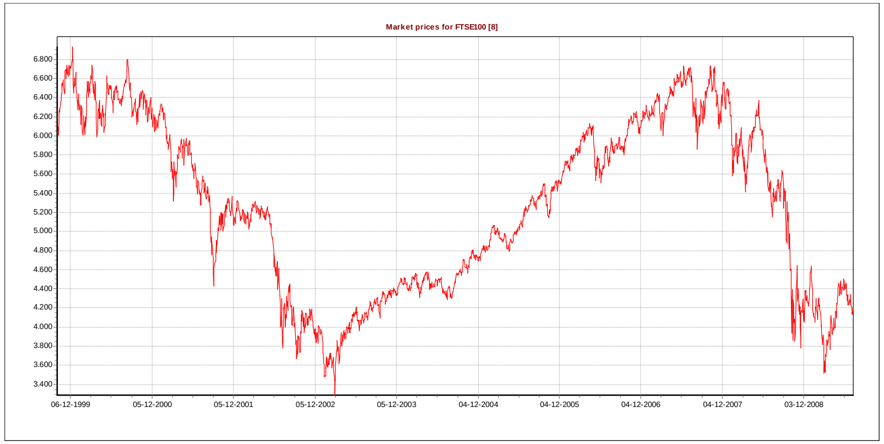


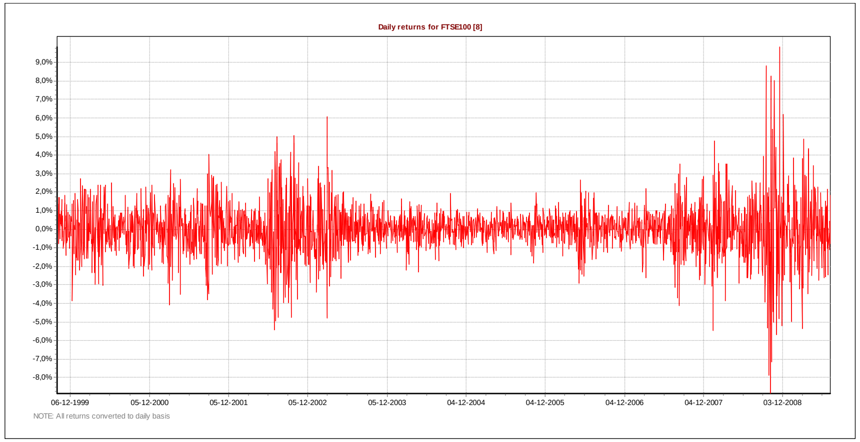
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of Sensex and FTSE100. The total number of observations is 2.455 days ranging from October 6, 1999 to July 13, 2009 (excluding holidays).



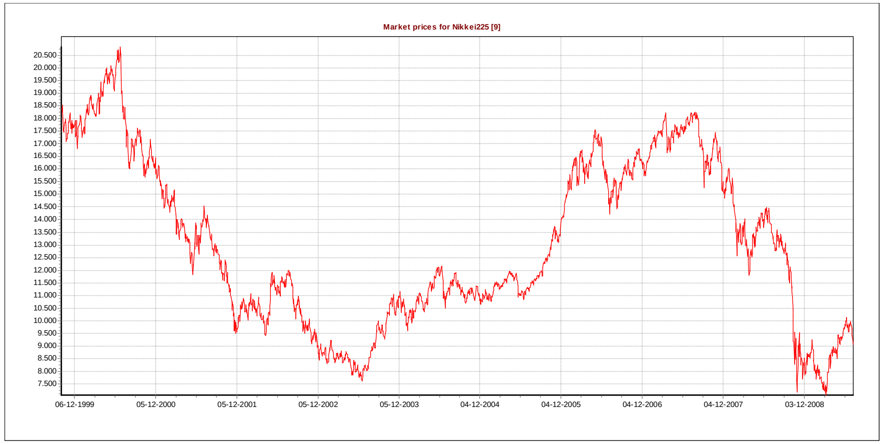


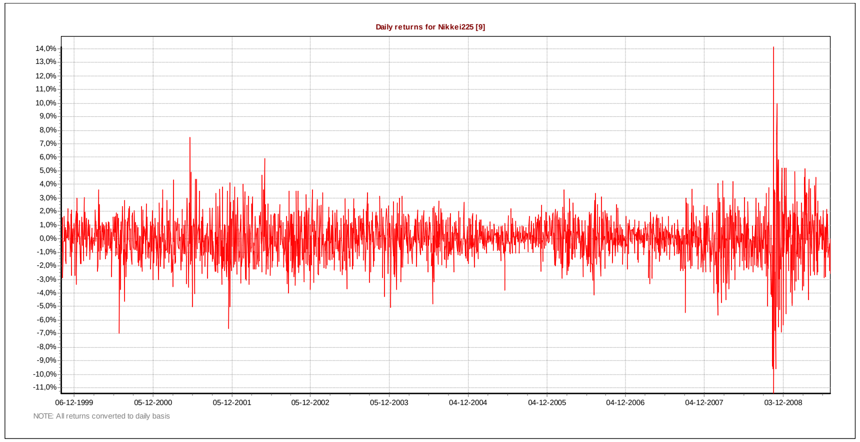


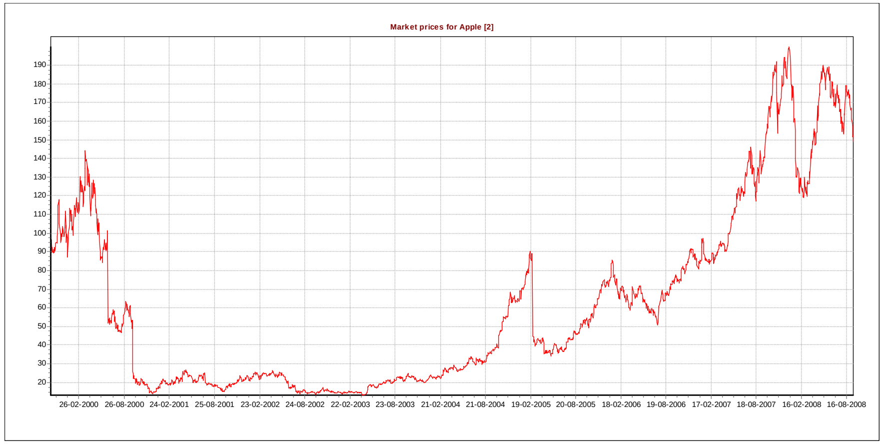


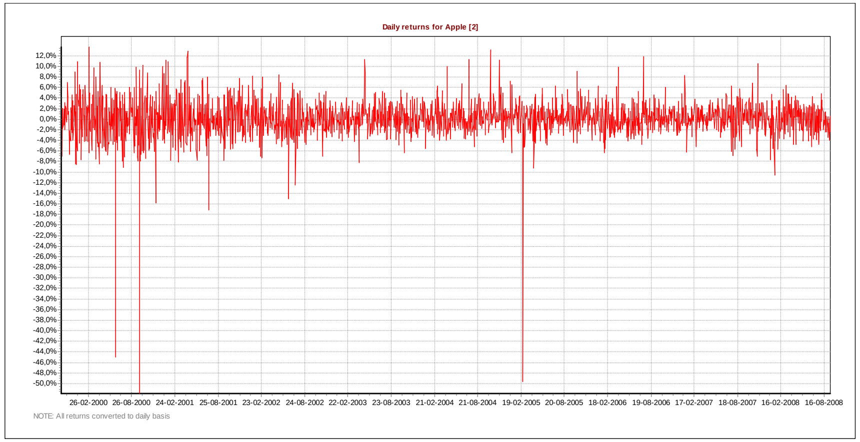
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of Nikkei225 and Apple. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays).



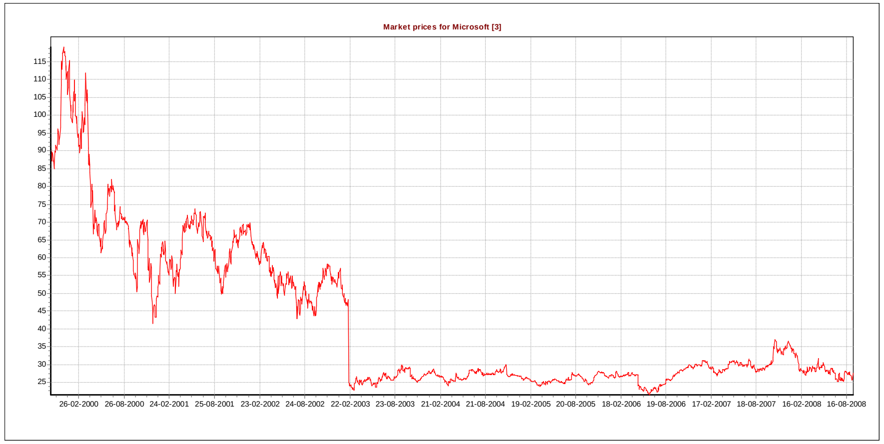


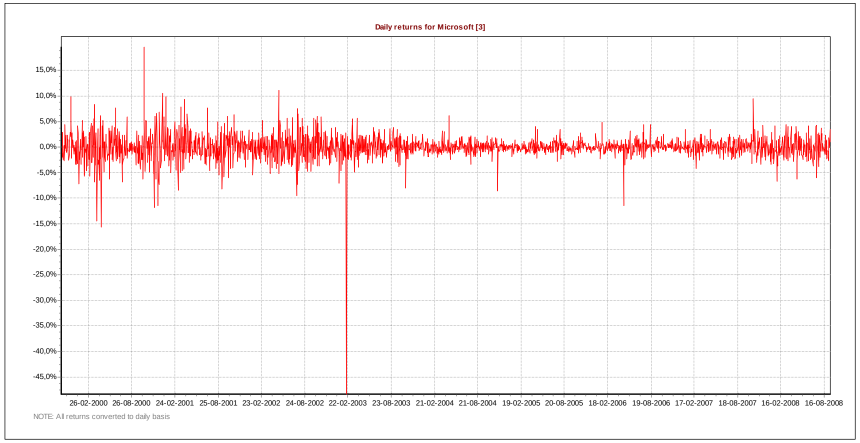


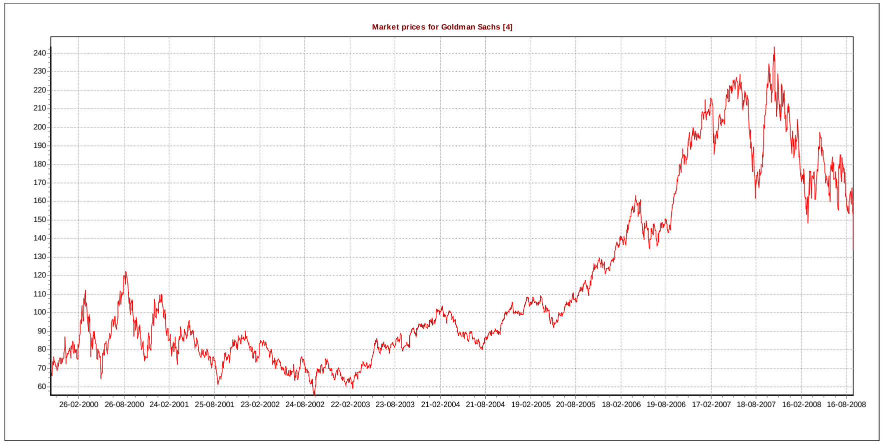


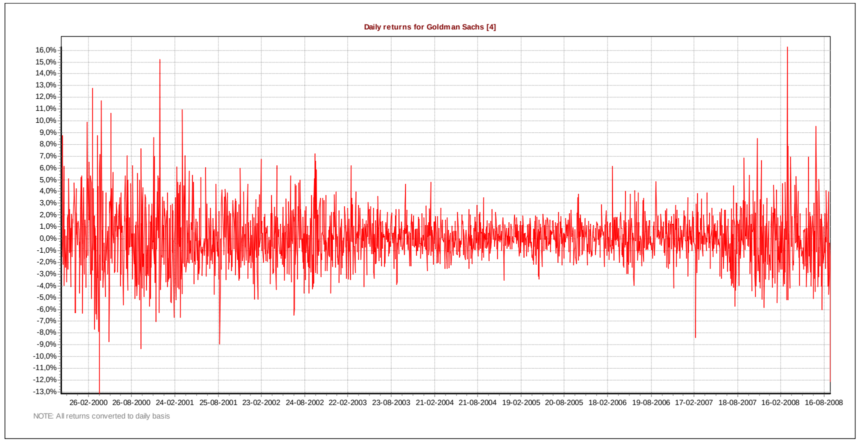
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of Microsoft and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).



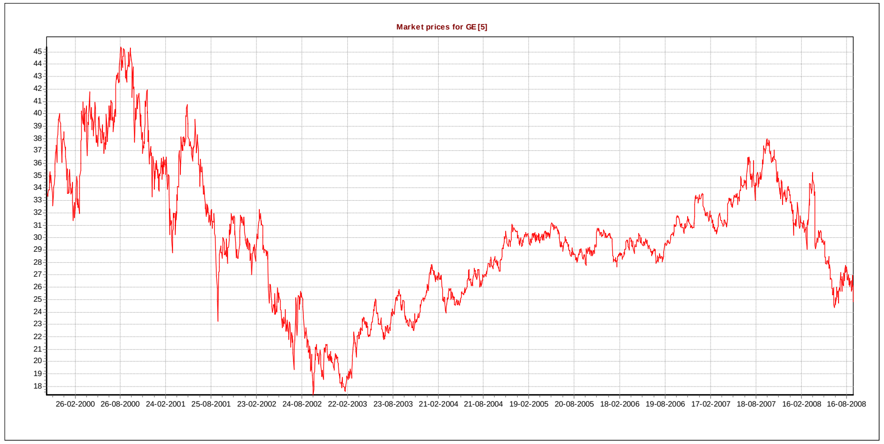


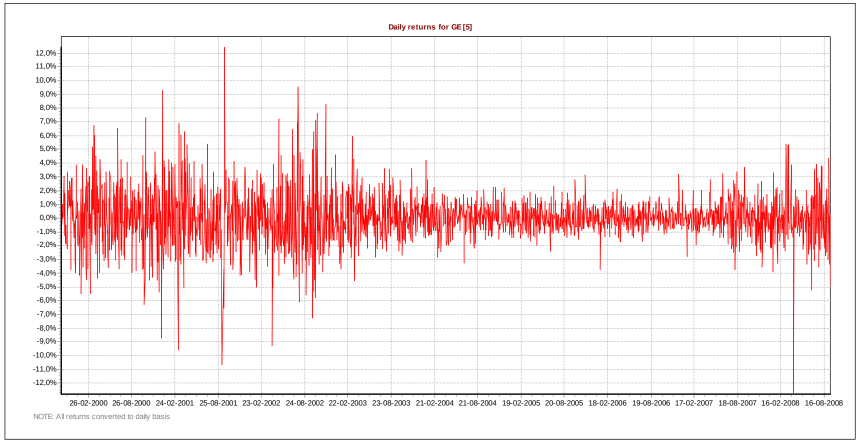


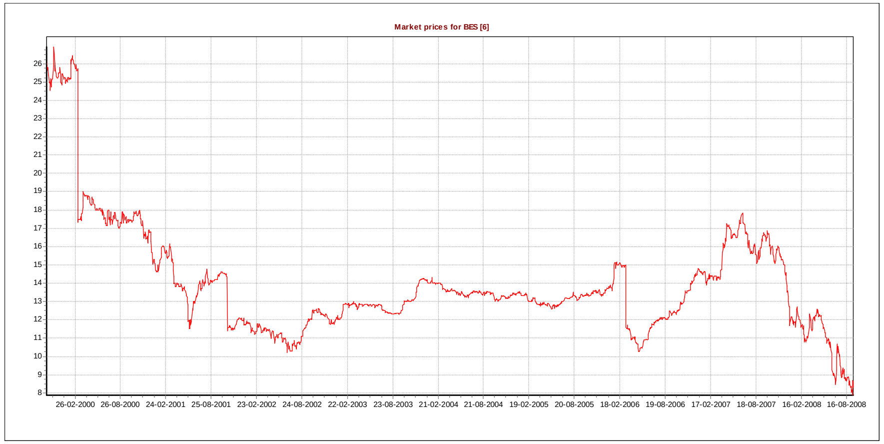


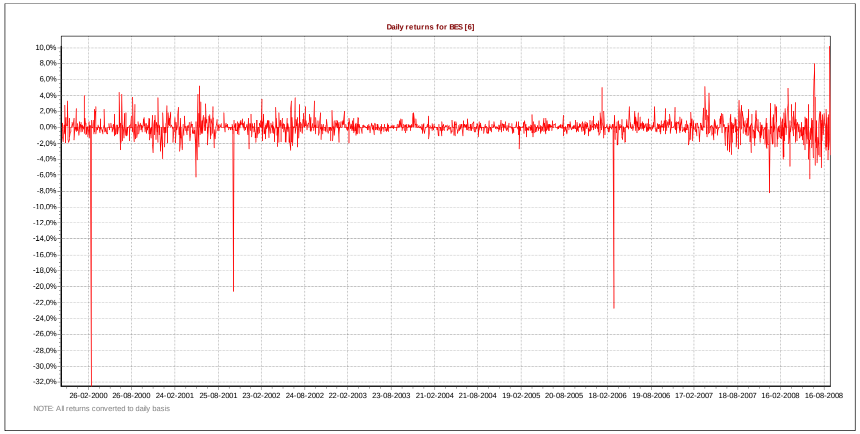
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of General Electric and BES. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).



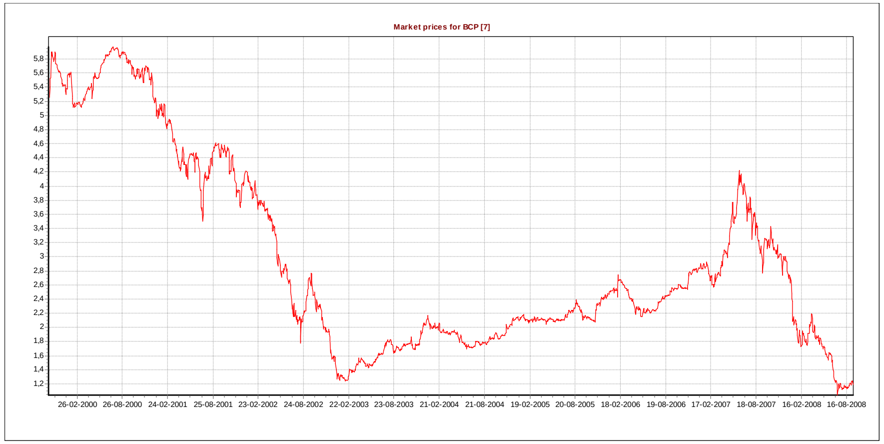


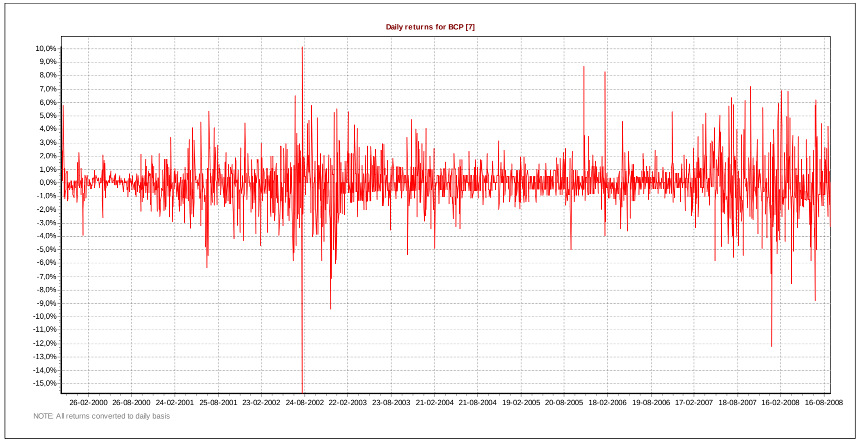


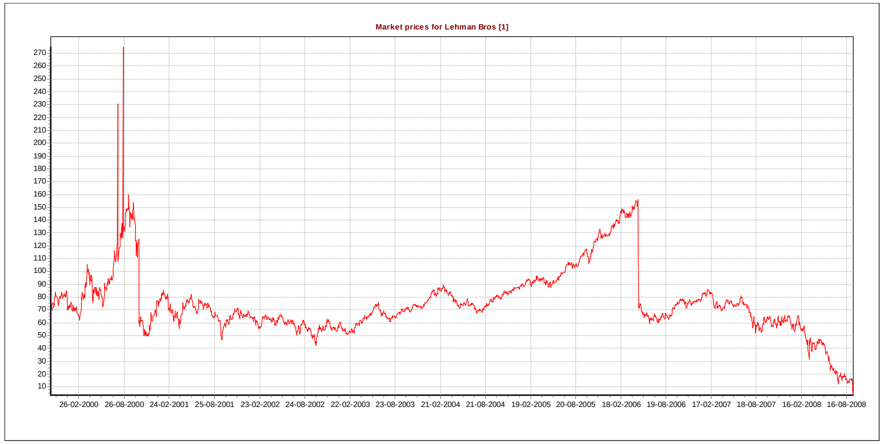


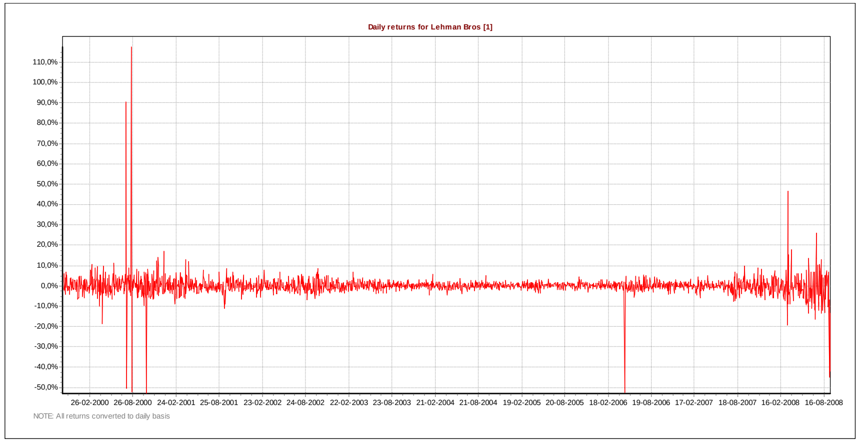
Source: ECVaR

These figures report the descriptive statistics of the daily prices and the daily returns of BCP and Lehman Brothers. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).





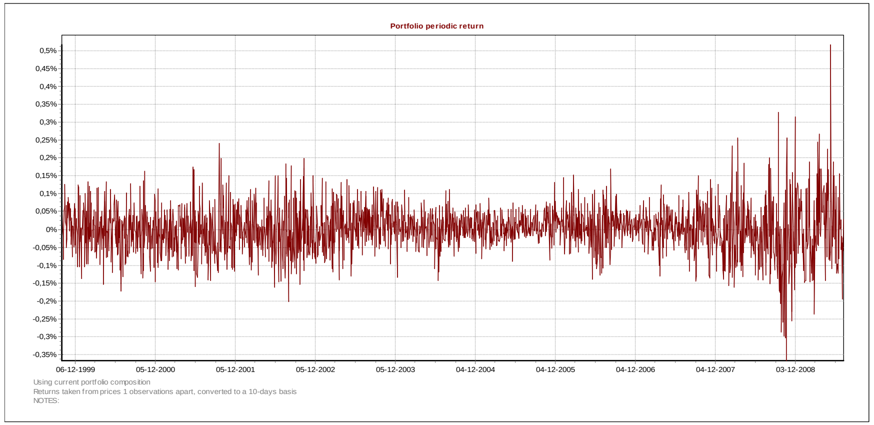


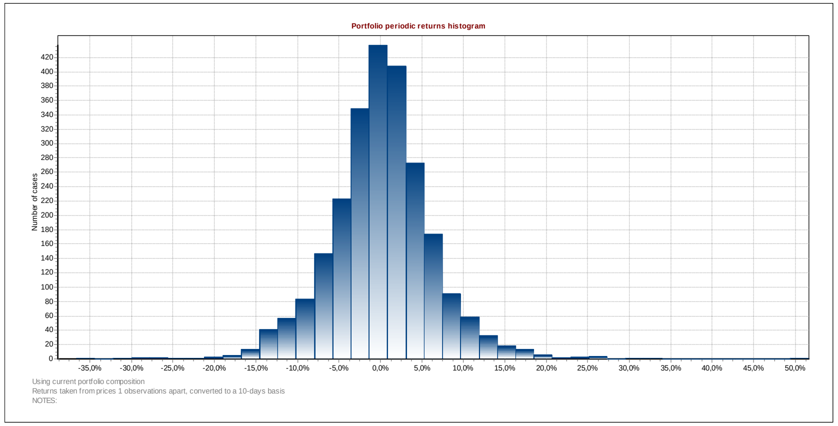


Source: ECVaR

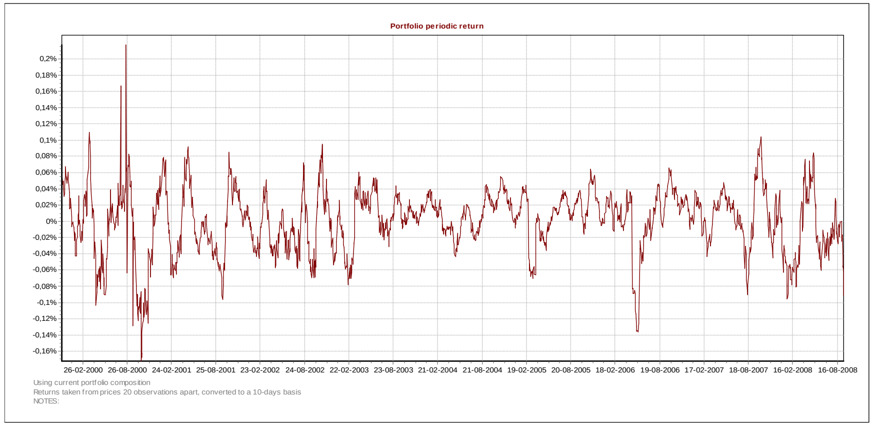
Figures III – Returns and respective histogram for the nine index portfolio (seven countries) and for the seven firms stock market portfolio

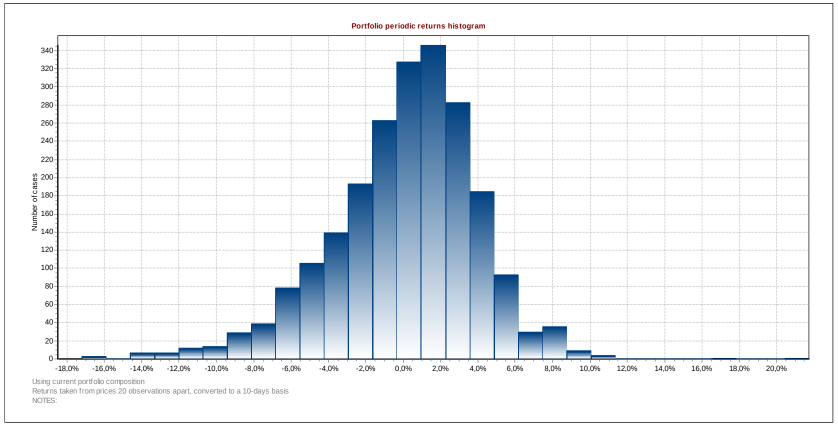
This figure presents the return and the respective histogram extracted from ECVaR software for the portfolio composed by seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).





This figure presents the return and the respective histogram extracted from ECVaR software for the portfolio composed by seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).

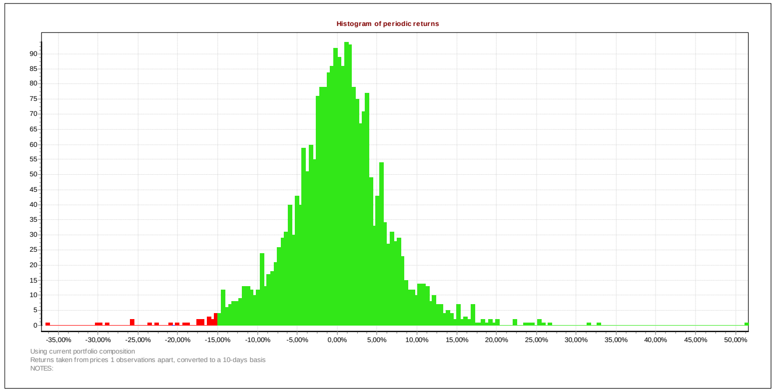




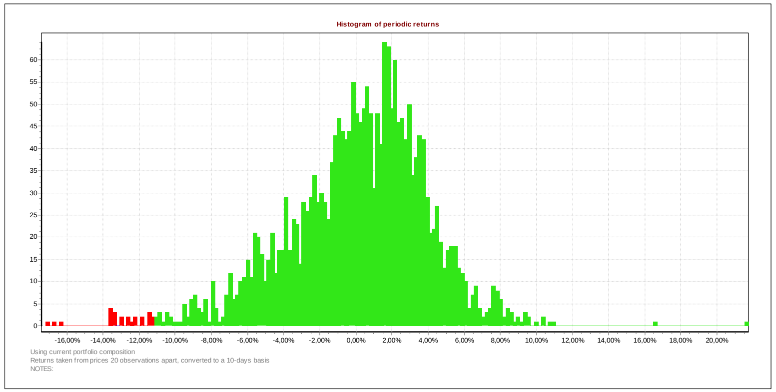
Source: ECVaR

Figures IV - Histograms for the nine index portfolio (seven countries) and for the seven firms stock market portfolio (VaR and ETL estimation)

This figure presents the histogram return extracted from ECVaR software for the portfolio composed by seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays). The portfolio is not likely to lose more than 15,301% of their global value after tens-days following 13-Jul-2009, with a 99% of confidence (VaR estimation). The portfolio is not likely to lose more than 20,610% of their global value after ten-days following 13-Jul-2009, with a 99% of confidence (ETL estimation).



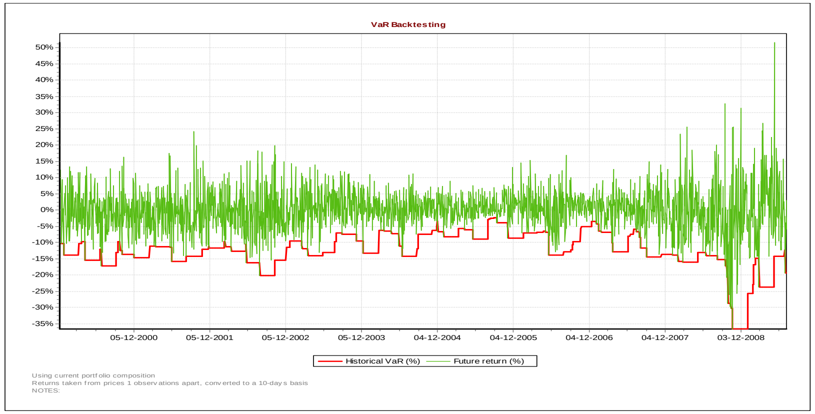
This figure presents the histogram return extracted from ECVaR software for the portfolio composed by seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays). The portfolio is not likely to lose more than 39,52% of their global value after ten-days following 12-Sep-2008, with a 99% of confidence (VaR estimation). The portfolio is not likely to lose more than 45,93% of their global value after ten-days following 13-Jul-2009, with a 99% of confidence (ETL estimation).

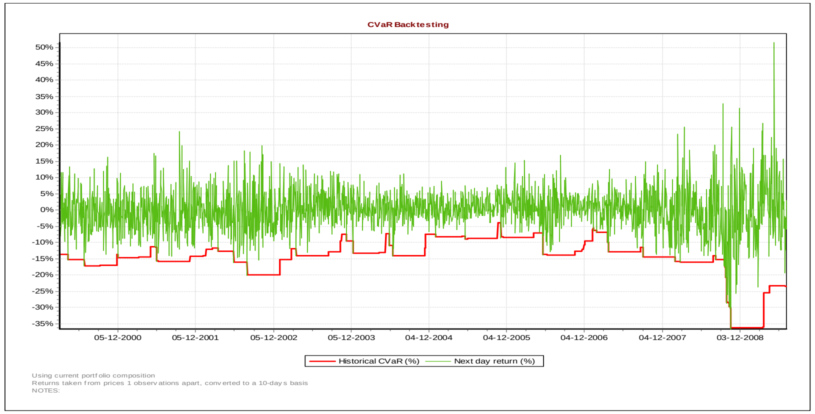


Source: ECVaR

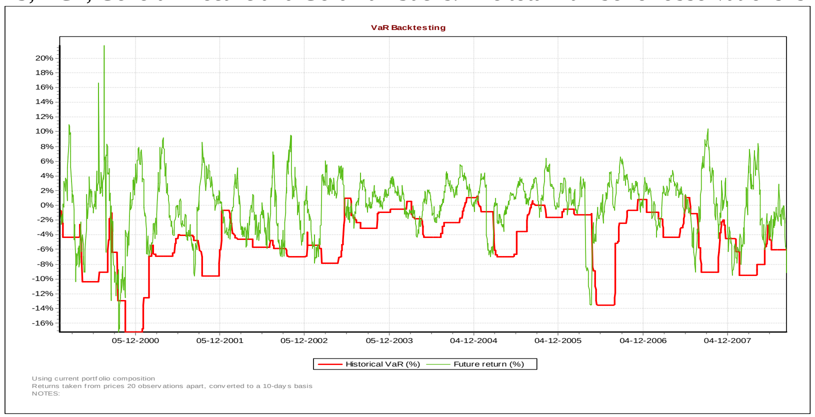
Figures V – Backtesting representation of VaR and ETL estimation for the nine index portfolio (seven countries) and for the seven firms stock market portfolio (VaR and ETL estimation)

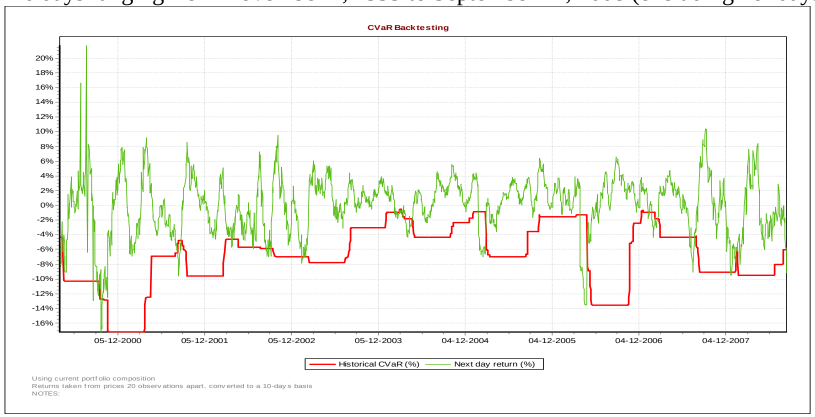
This figure represents the backtesting results for, namely, VaR and ETL estimation extracted from ECVaR software for the portfolio composed by seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).





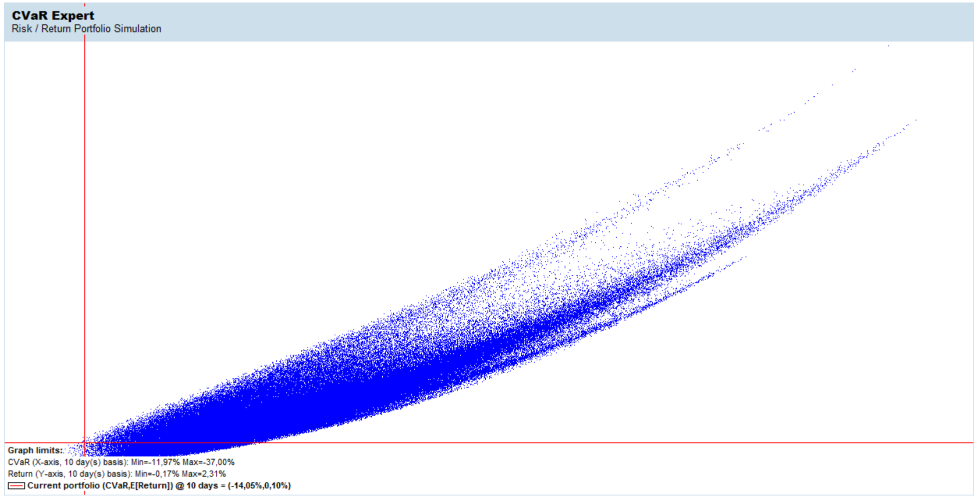
This figure represents the backtesting results for, namely, VaR and ETL estimation extracted from ECVaR software for the portfolio composed by seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).



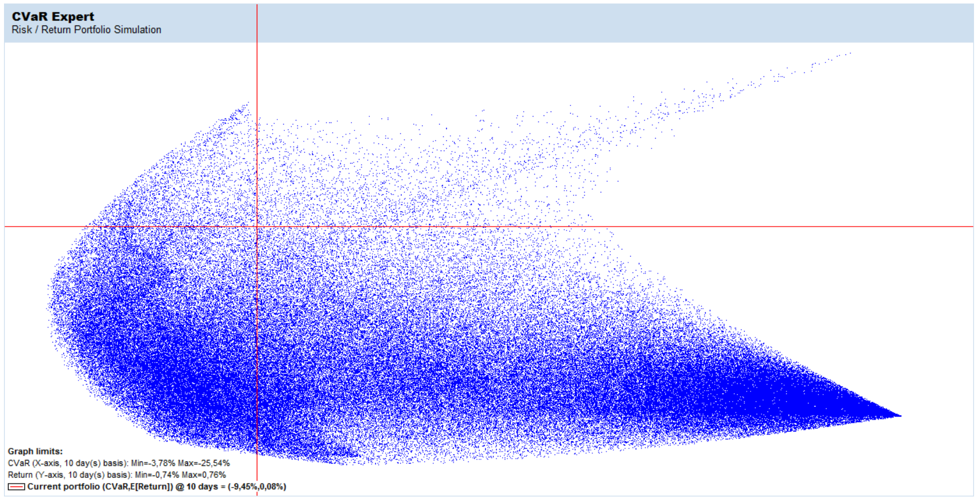


Source: ECVaR

Figures VI – Risk and return representation of the nine index portfolio (seven countries) and the seven firms stock market portfolio



This figure represents the risk versus return extracted from ECVaR software for the portfolio composed by seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India. The total number of observations is 2.455 days from October 6, 1999 to July 13, 2009 (excluding holidays).

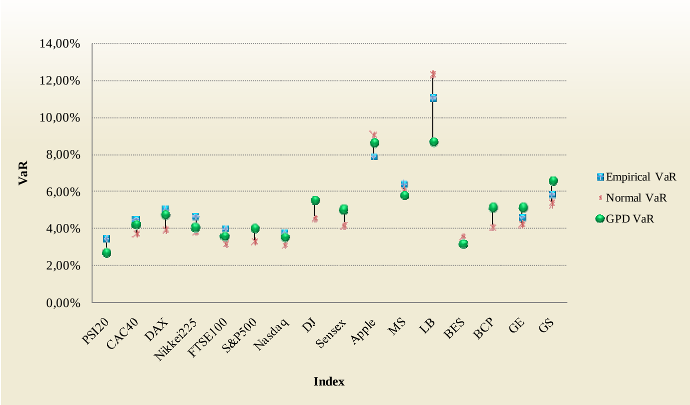


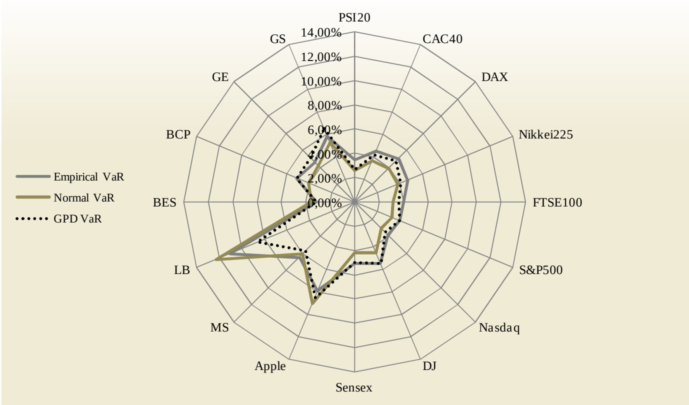
This figure represents the risk versus return extracted from ECVaR software for the portfolio composed by seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.226 days ranging from November 4, 1999 to September 12, 2008 (excluding holidays).

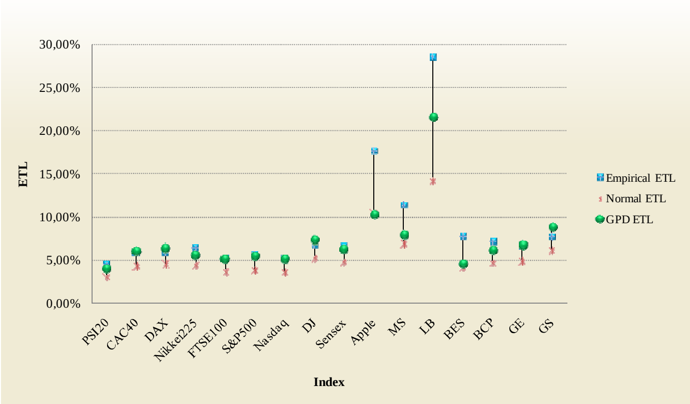
Source: ECVaR

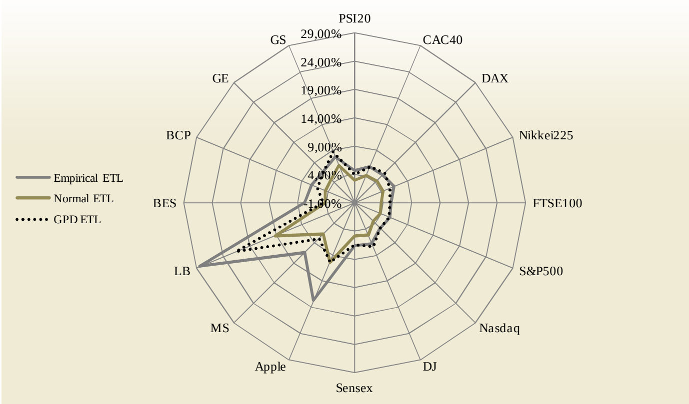
Figures VII – Visual comparison between empirical, normal and GPD models in VaR and ETL estimations

These figure represent the comparison results for, namely, VaR and ETL estimation by empirical, normal and GPD models of the portfolios composed by i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and by ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays). The empirical and GPD models predict fairly consistent risk measures, particularly empirical ETL and GPD ETL. This fact results because the simplifying assumptions about market or portfolio behaviour used in VaR models that can only measure risk in normal market conditions.





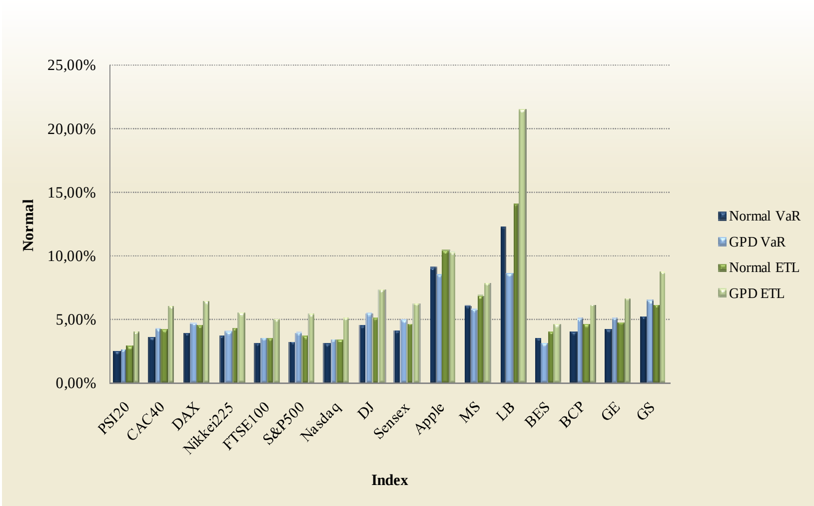




Source: Author

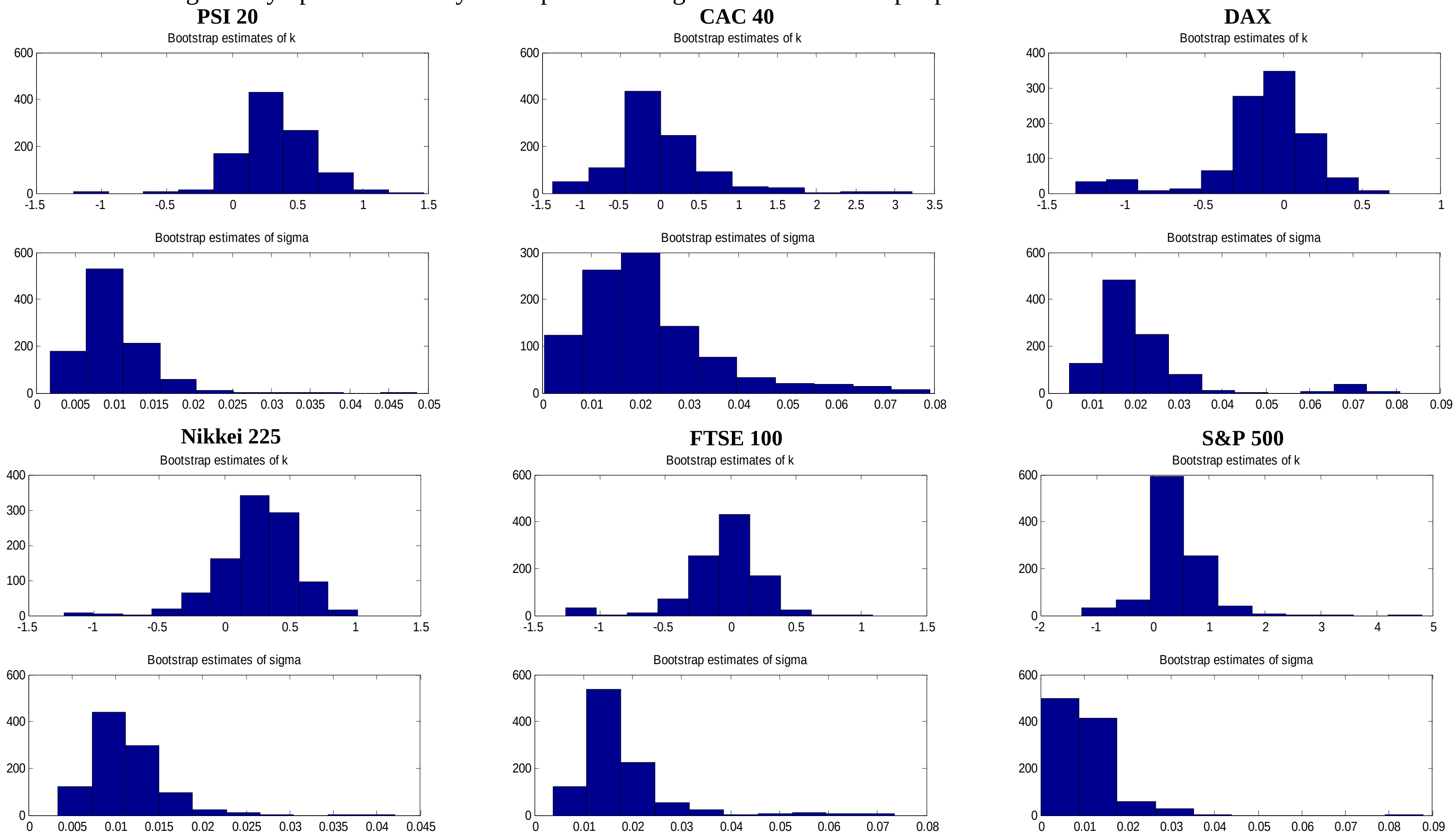
Figure VIII - Visual comparison between normal and GPD models in VaR and ETL estimations

This figure represents the comparison results for, namely, VaR and ETL estimation by normal and GDP models of the portfolios composed by i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and by ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays). The GPD models predict fairly consistent risk measures, particularly empirical ETL and GPD ETL. This fact results because the simplifying assumptions about market or portfolio behaviour used in VaR models that can only measure risk in normal market conditions.

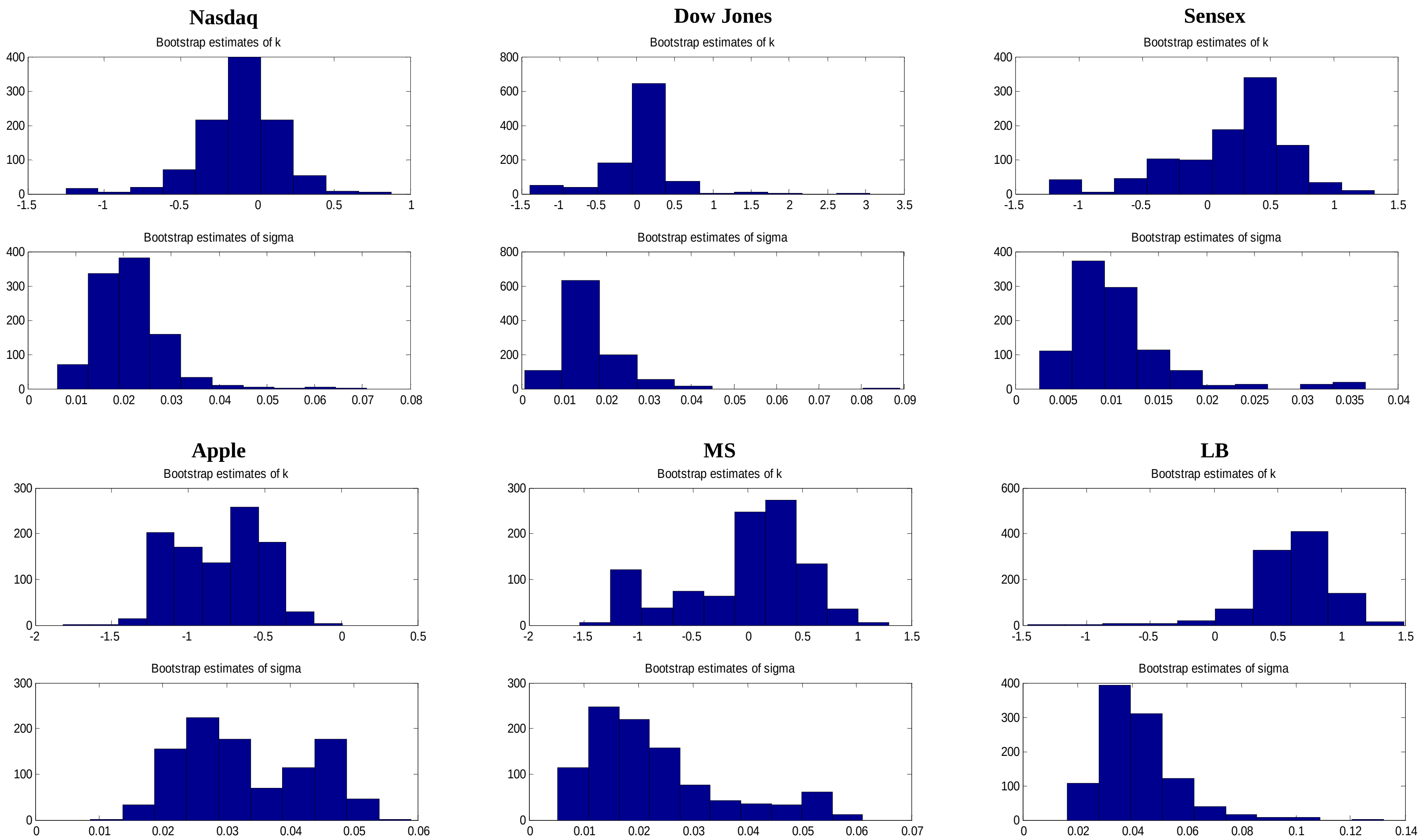


Source: Author

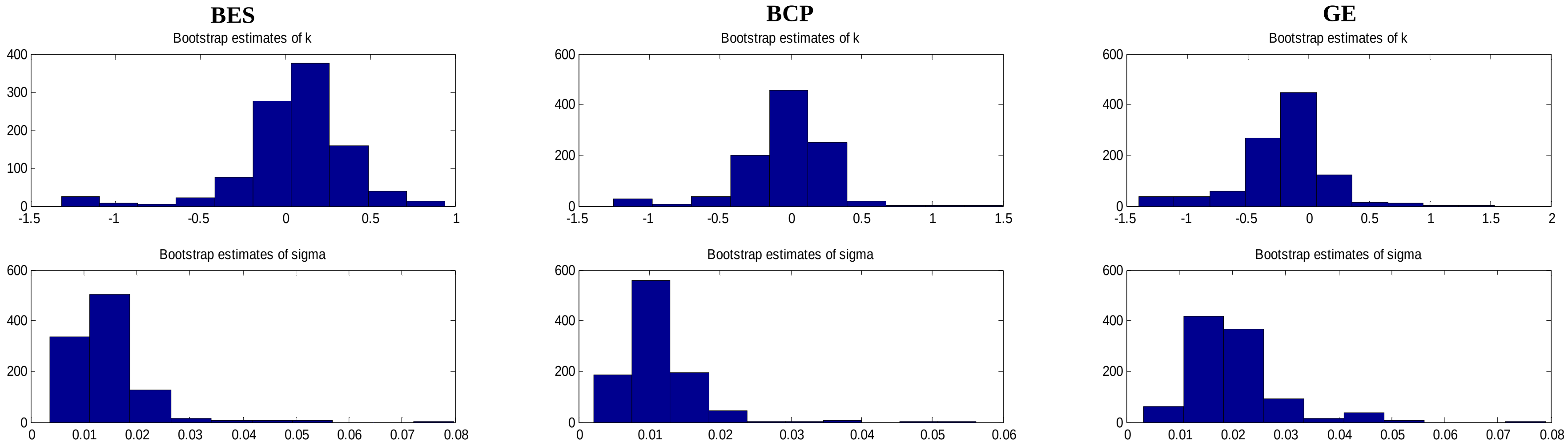
Figures IX - Checking the asymptotic normality assumption: Histograms of the bootstrap replicates

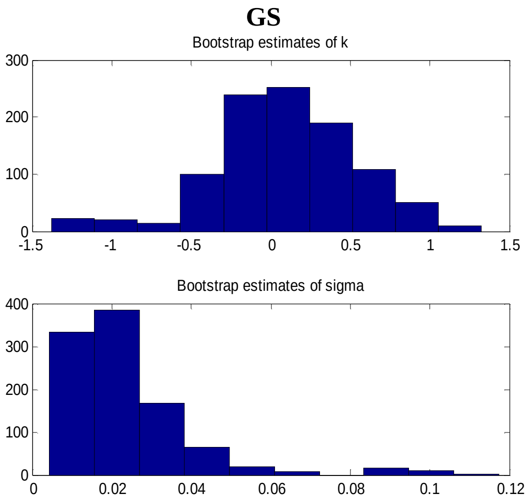


Source: Matlab



Source: Matlab

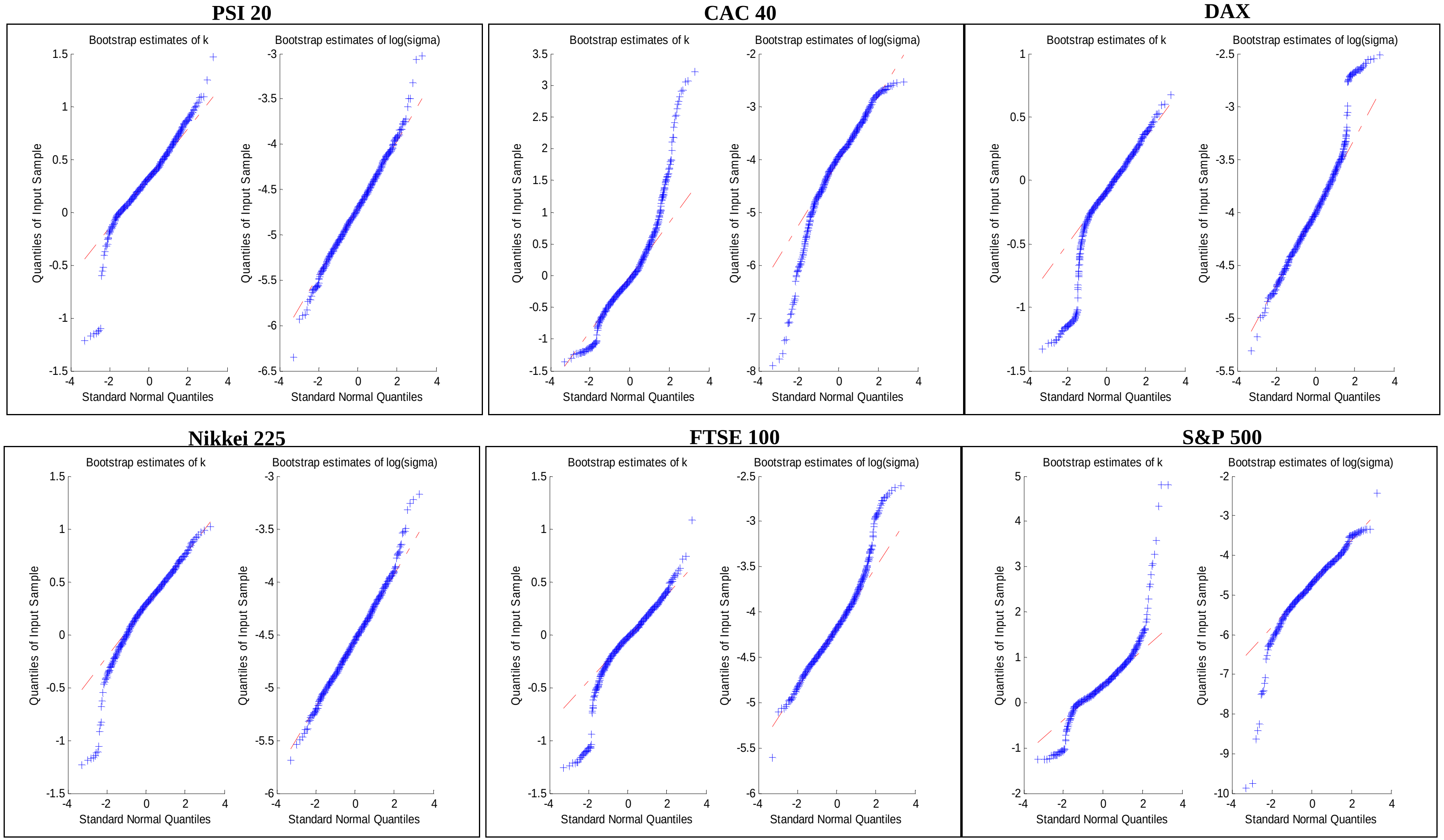




These figures represent the assessment of the asymptotic normality assumption using histograms of the bootstrap replicates. The dataset was tested to i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and to ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays). The histogram of the bootstrap estimates for shape parameter appears to be asymmetric.

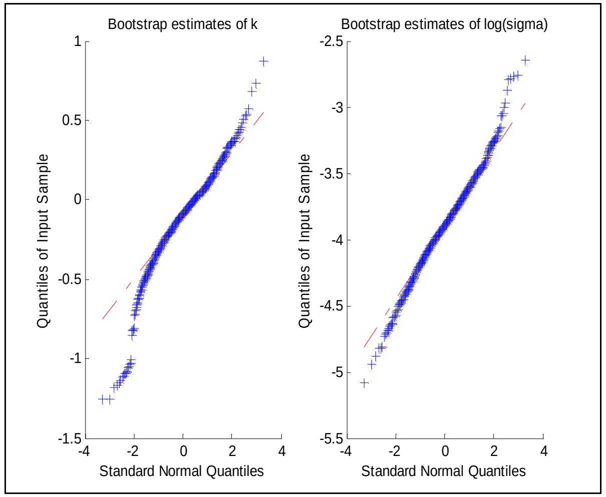
Source: Matlab

Figures X - Checking the asymptotic normality assumption: QQ plot

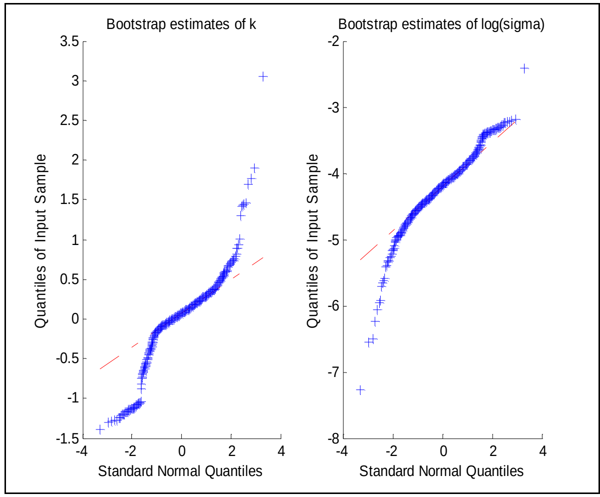


Source: Matlab

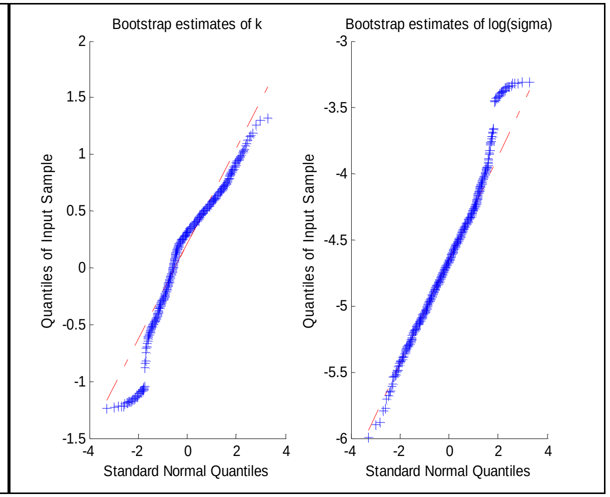
Nasdaq



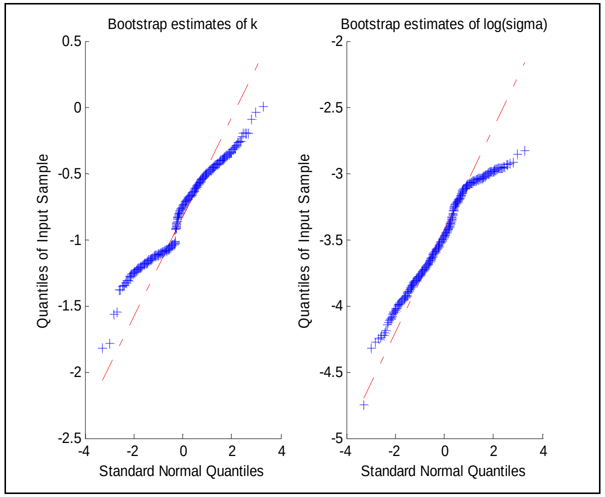
Dow Jones



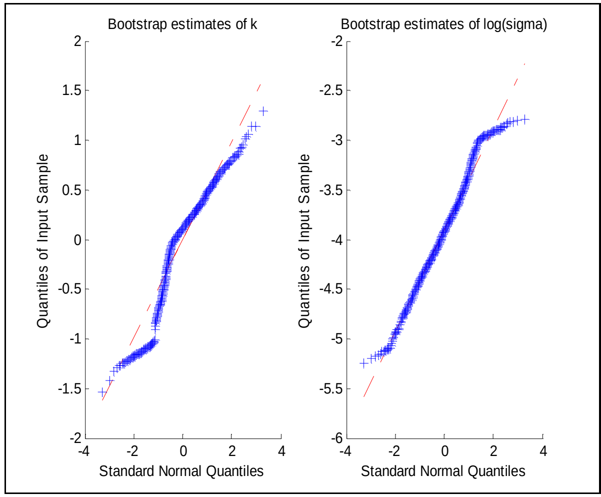
Sensex



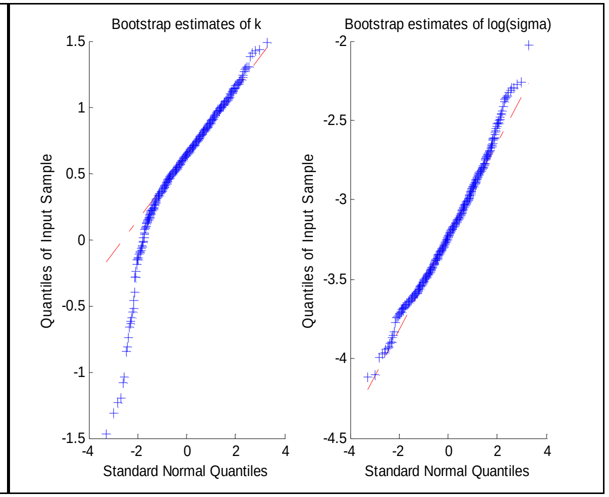
Apple



MS

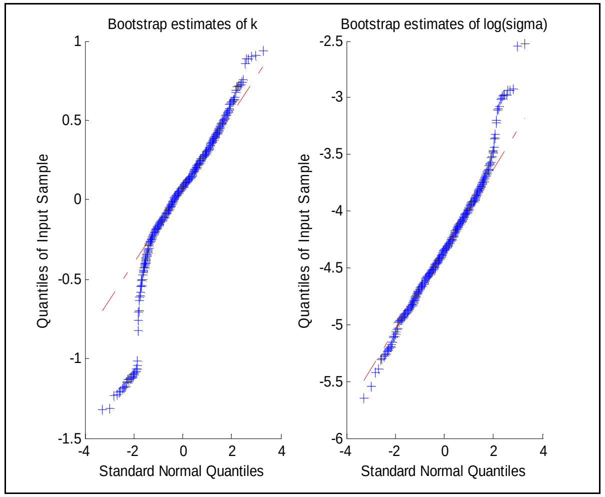


LB

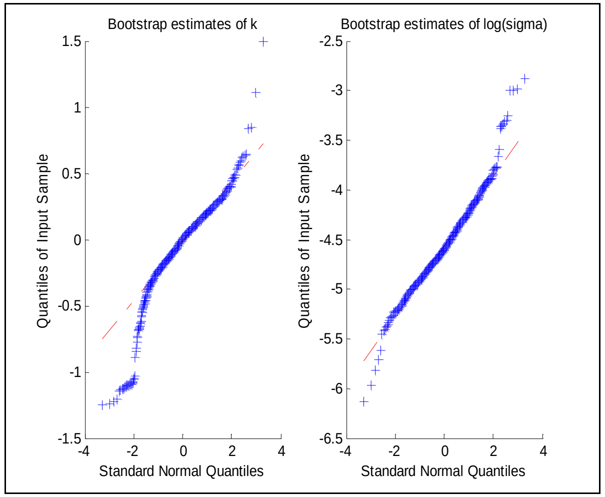


Source: Matlab

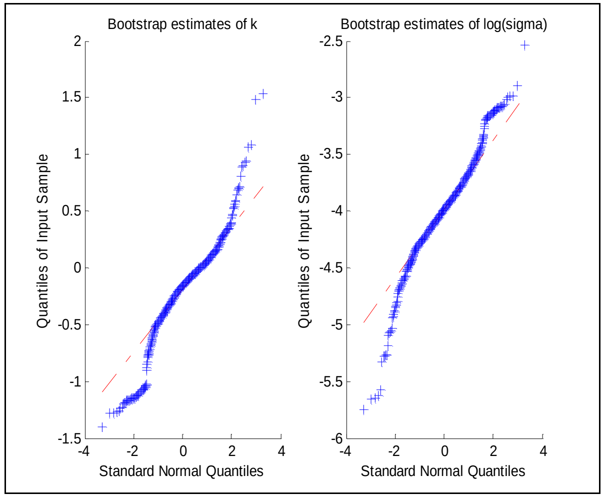
BES



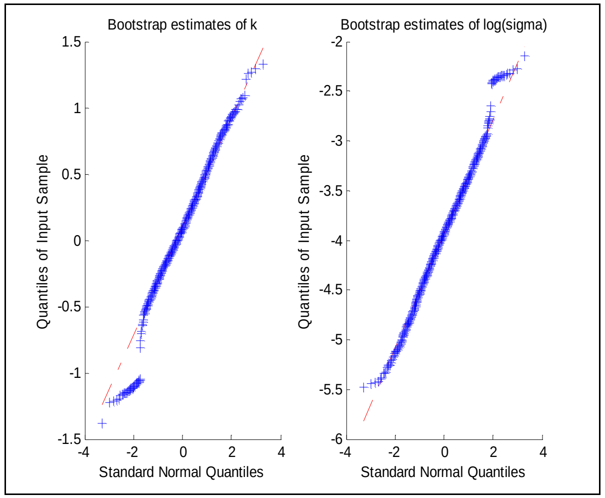
BCP



GE



GS

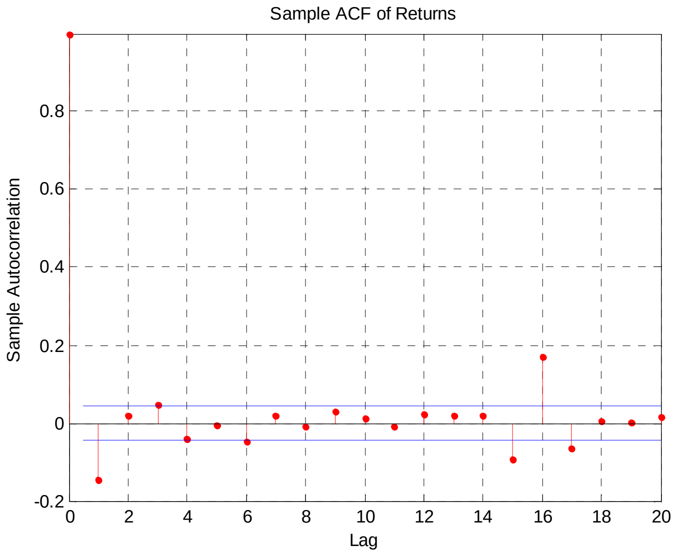


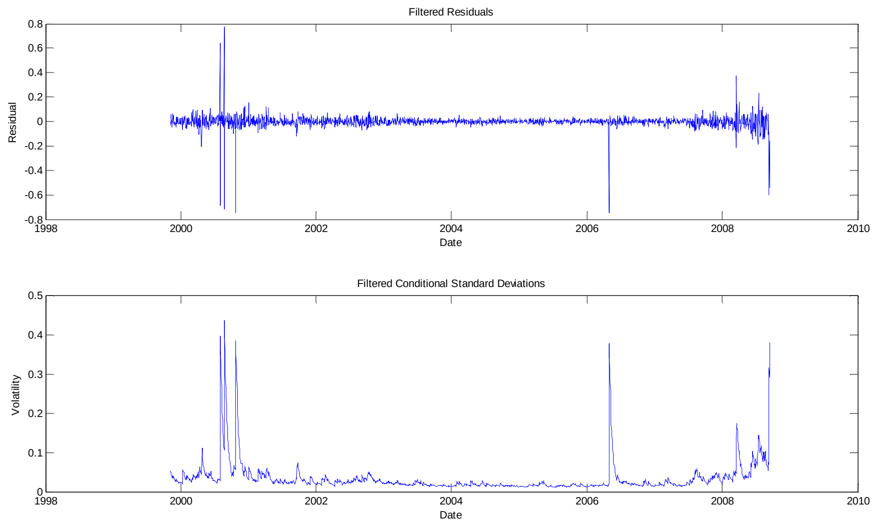
These figures represent the assessment of the asymptotic normality assumption using Q-Q plot. The dataset was tested to i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and to ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays). The QQ plot shows that points that do not approximately follow the straight line. In this way, it can be assumed that the returns have heavy tails rather than normal distribution.

Source: Matlab

Figures XI - Illustration of the result before filter the returns for each price and index using GARCH method (without i.i.d. assumption) – Example Lehman Brothers

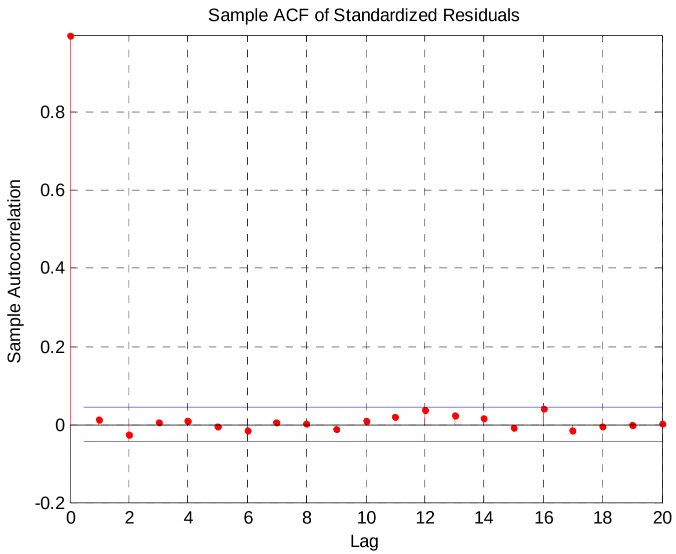
Before using EVT to model the tails of the distribution of an individual index or stock market, the data must be i.i.d. A quick review of the data reveals that it is not i.i.d justify with the financial crisis that causes wild swings in the stock market. For the Lehman Brothers stock market example, the sample autocorrelation function of the returns reveals some mild serial correlation. Unless the data is preconditioned or filtered, this dependence will undermine the value of EVT. To produce a series of i.i.d. observations, a GARCH model is needed to filter out serial dependence in the data.

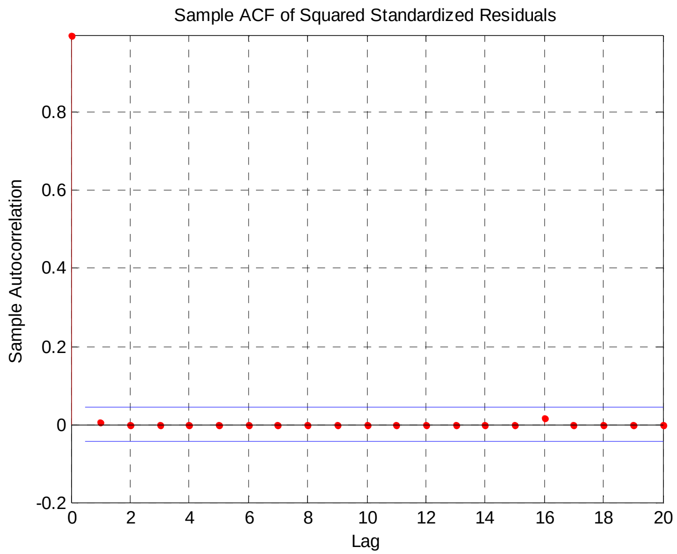




Figures XII – Illustration of the result after filter the returns for each price and index using GARCH method (i.i.d. assumption) – Example Lehman Brothers

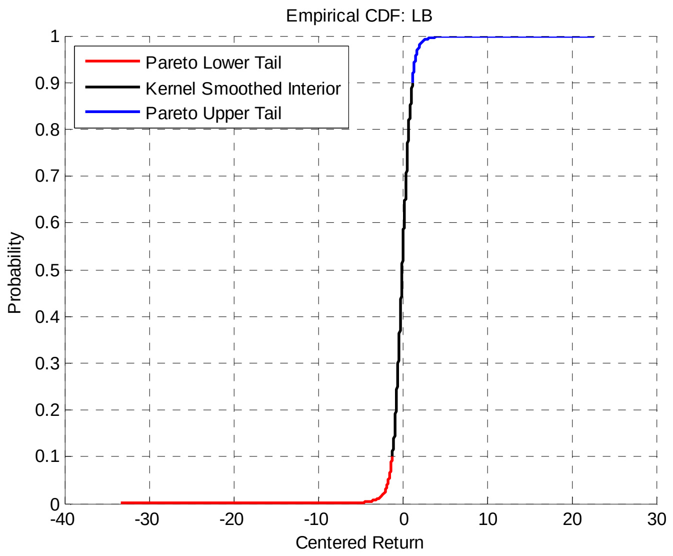
To produce a series of i.i.d. observations, a GARCH model is needed to filter out serial dependence in data. The step that involves a repeated application of GARCH filtration is one of the most important steps in the overall modelling approach. The GARCH model can produce a series of i.i.d. observations that satisfy the requirements of EVT. The sample ACF of the squared returns illustrates the degree of persistence in variance, and implies that GARCH modeling may significantly condition the data used in the subsequent tail estimation process.



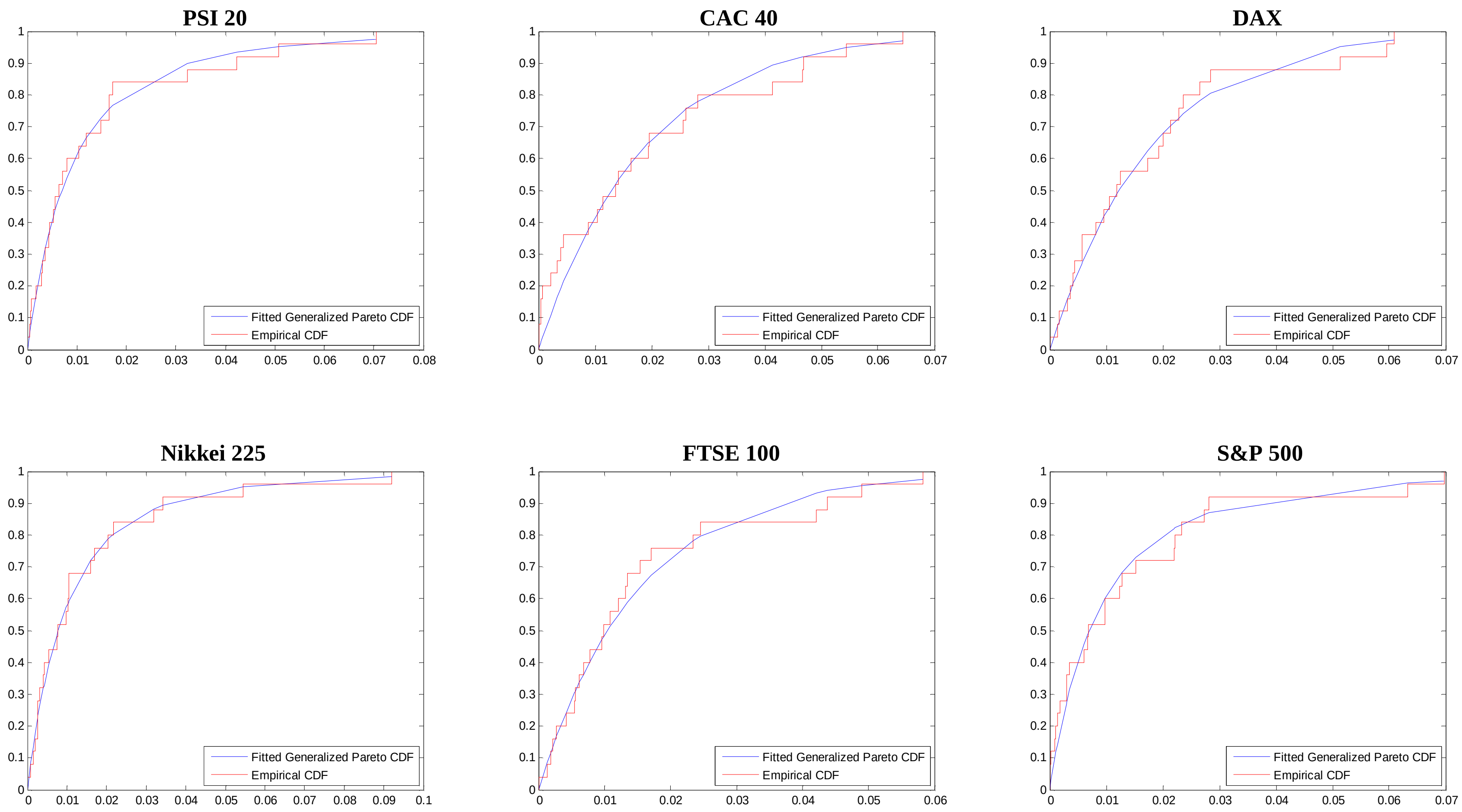


Figures XIII – Estimation of the semi-parametric cumulative distribution function – Example Lehman Brothers

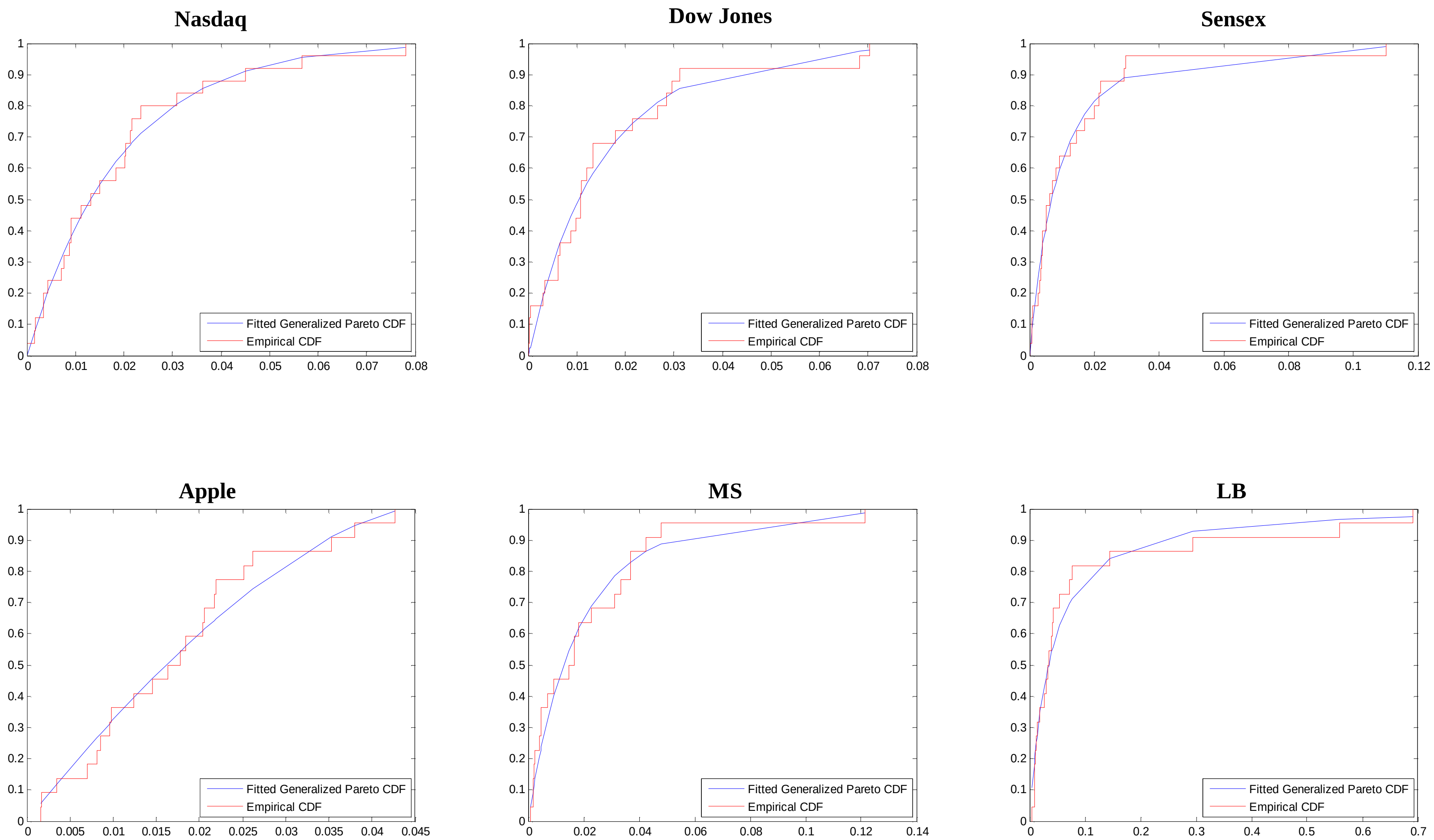
Once the data is filtered, the data must fit a probability distribution to model the daily movements. This figure represents the empirical cumulative distribution function for the Lehman Brothers stock market, with the kernel density estimate for the interior and the GPD estimate for the upper and lower tails. The underlying Matlab code uses the Statistics Toolbox function paretotails to automate the curve fit.



Figures XIV - Checking the Fit Visually

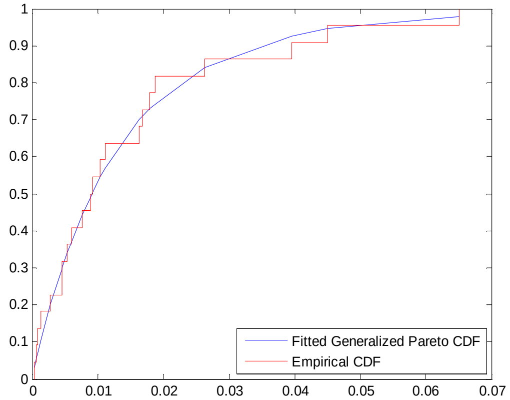


Source: Matlab

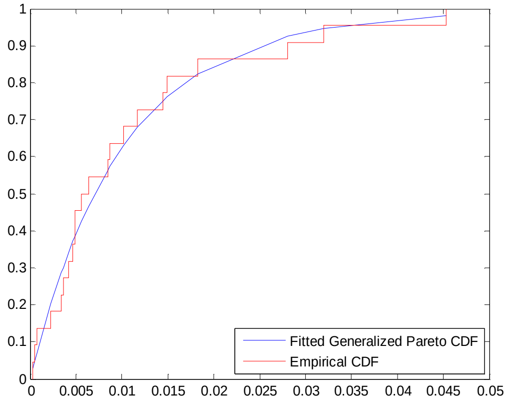


Source: Matlab

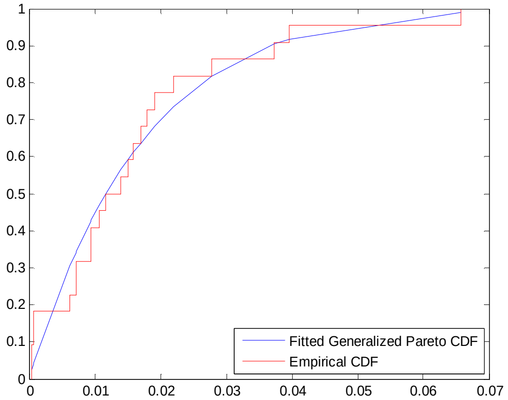
BES



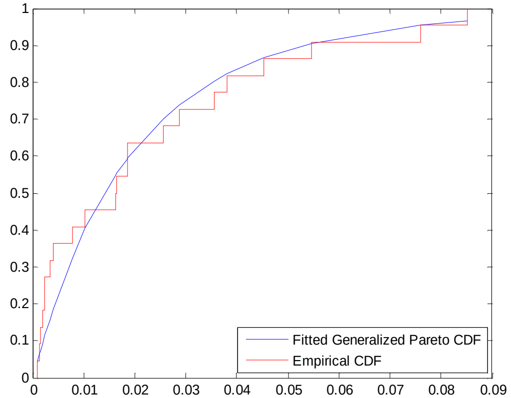
BCP



GE



GS



To visually assess the GPD fit, the Matlab Statistics Toolbox function gpfit was used to plot the empirical cumulative distribution function curve to find the parameters for the GPD in the tails of the curve. The dataset was tested to i) seven countries indices: Portugal, France, Germany, Japan, United Kingdom, United States of America and India; and to ii) seven firms: Apple, Microsoft, Lehman Brothers, BES, BCP, General Electric and Goldman Sachs. The total number of observations is 2.455 days and 2.226 days ranging from, respectively, October 6, 1999 to July 13, 2009 and November 4, 1999 to September 12, 2008 (excluding holidays). These figures show that the empirically generated cumulative distribution function curve matches quite well with the fitted GPD results. The fitted density follows the shape of the data, and so the GPD model seems to be a good choice.

Source: Matlab

Matlab and Excel Code

Matlab Code I - Prices and Returns

```
load INDEXDATA_INDICES   % Import daily index closings

countries = {'Portugal' 'France' 'Germany' 'Japan' 'UK' 'US' 'US_N' ...
'US_DJ' 'India'};
prices    = [IndexData.Portugal IndexData.France IndexData.Germany ...
IndexData.Japan IndexData.UK IndexData.US IndexData.US_N ...
IndexData.US_DJ IndexData.India];

figure
plot(IndexData.Dates, ret2price(price2ret(prices)))
datetick('x')
xlabel('Date')
ylabel('Index Value')
title ('Relative Daily Index Closings')
legend(countries, 'Location', 'NorthWest')

returns = price2ret(prices);  % Logarithmic returns
T       = size(returns,1);    % # of returns (i.e., historical sample size)

index = 1;  % 1 = Portugal, 2 = France, 3 = Germany, 4 = Japan, 5 = UK, 6 =
US_S&P 7 = US_N 8 = US_DJ 9 = India

figure
plot(IndexData.Dates(2:end), returns(:,index)), datetick('x')
xlabel('Date'), ylabel('Return'), title('Daily Logarithmic Returns')


load INDEXDATA_Firms  % Import daily prices closings

firms = {'LB' 'Apple' 'MS' 'BES' 'BCP' 'GE' 'GS'};
prices    = [IndexData.LB IndexData.Apple IndexData.BES ...
IndexData.BCP IndexData.GE IndexData.GS];

figure
plot(IndexData.Dates, ret2price(price2ret(prices)))
datetick('x')
xlabel('Date')
ylabel('Index Value')
title ('Relative Daily Index Closings')
legend(firms, 'Location', 'NorthWest')

returns = price2ret(prices);  % Logarithmic returns
T       = size(returns,1);    % # of returns (i.e., historical sample size)

index = 1;  % 1 = LB, 2 = Apple, 3 = MS, 4 = BES, 5 = BCP, 6 = GE 7 = GS

figure
plot(IndexData.Dates(2:end), returns(:,index)), datetick('x')
xlabel('Date'), ylabel('Return'), title('Daily Logarithmic Returns')
```

Matlab Code II - Filter the Returns for Each Price (GARCH)

```
figure
autocorr(returns(:,index))
title('Sample ACF of Returns')

nIndices = size(prices,2);     % # of indices

spec(1:nIndices) = garchset('Distribution' , 'T'  , 'Display', 'off', ...
                            'VarianceModel', 'GJR', 'P', 1, 'Q', 1, 'R',
1);

residuals = NaN(T, nIndices);  % preallocate storage
sigmas    = NaN(T, nIndices);

for i = 1:nIndices
    [spec(i)       , errors, LLF, ...
     residuals(:,i), sigmas(:,i)] = garchfit(spec(i), returns(:,i));
end

subplot(2,1,1)
plot(IndexData.Dates(2:end), residuals(:,index))
datetick('x')
xlabel('Date'), ylabel('Residual'), title ('Filtered Residuals')

subplot(2,1,2)
plot(IndexData.Dates(2:end), sigmas(:,index))
datetick('x')
xlabel('Date'), ylabel('Volatility')
title ('Filtered Conditional Standard Deviations')

residuals = residuals ./ sigmas;

figure
autocorr(residuals(:,index))
title('Sample ACF of Standardized Residuals')

figure
autocorr(residuals(:,index).^2)
title('Sample ACF of Squared Standardized Residuals')
```

Matlab Code III - Estimate the Semi-Parametric Cumulative Distributions Functions

```
tailFraction = 0.1;      % Decimal fraction of residuals allocated to each
tail

OBJ = cell(nIndices,1);  % Cell array of Pareto tail objects

for i = 1:nIndices
    OBJ{i} = paretotails(residuals(:,i), tailFraction, 1 - tailFraction,
'kernel');
end

figure, hold('on'), grid('on')

minProbability = OBJ{index}.cdf((min(residuals(:,index))));
maxProbability = OBJ{index}.cdf((max(residuals(:,index))));

pLowerTail = linspace(minProbability  , tailFraction    , 200); % sample
lower tail
pUpperTail = linspace(1 - tailFraction, maxProbability  , 200); % sample
upper tail
pInterior  = linspace(tailFraction    , 1 - tailFraction, 200); % sample
interior

plot(OBJ{index}.icdf(pLowerTail), pLowerTail, 'red'  , 'LineWidth', 2)
plot(OBJ{index}.icdf(pInterior) , pInterior , 'black', 'LineWidth', 2)
plot(OBJ{index}.icdf(pUpperTail), pUpperTail, 'blue' , 'LineWidth', 2)

xlabel('Centered Return'), ylabel('Probability')
title (['Empirical CDF: ' firms{index}])
legend({'Pareto Lower Tail' 'Kernel Smoothed Interior' ...
        'Pareto Upper Tail'}, 'Location', 'NorthWest')
```

Matlab Code IV - Estimating parameters

```
load INDEXDATA_Firms   % Import daily prices closings
firms = {'LB' 'Apple' 'MS' 'BES' 'BCP' 'GE' 'GS'};
prices    = [IndexData.LB IndexData.Apple IndexData.BES ...
IndexData.BCP IndexData.GE IndexData.GS];

load INDEXDATA_INDICES   % Import daily index closings
countries = {'Portugal' 'France' 'Germany' 'Japan' 'UK' 'US' 'US_N' ...
'US_DJ' 'India'};
prices    = [IndexData.Portugal IndexData.France IndexData.Germany ...
IndexData.Japan IndexData.UK IndexData.US IndexData.US_N ...
IndexData.US_DJ IndexData.India];

returns = price2ret(prices);  % Logarithmic returns
T       = size(returns,1);    % # of returns (i.e., historical sample size)

x = returns; % # for each one returns
q = quantile(x,.99);
y = x(x>q) - q;
n = numel(y)

paramEsts = gpfit(y);
kHat      = paramEsts(1)   % Tail index parameter
sigmaHat  = paramEsts(2)   % Scale parameter
```

Matlab Code V - Assess the GPD Fit

```
bins = 0:.1:1;
h = bar(bins,histc(y,bins)/(length(y)*.25),'histc');
set(h,'FaceColor',[.9 .9 .9]);
ygrid = linspace(0,1.1*max(y),n);
line(ygrid,gppdf(ygrid,kHat,sigmaHat));
xlim([0,1]); xlabel('Exceedance'); ylabel('Probability Density');

[F,yi] = ecdf(y);
plot(yi,gpcdf(yi,kHat,sigmaHat),'-');
hold on; stairs(yi,F,'r'); hold off;
legend('Fitted Generalized Pareto CDF','Empirical
CDF','location','southeast');
```

Matlab Code VI - Computing Standard Errors for the Parameter Estimates

```
[nll,acov] = gplike(paramEsts, y);
stdErr = sqrt(diag(acov))
```

Matlab Code VII - Checking the Asymptotic Normality Assumption

```
replEsts = bootstrp(1000,@gpfit,y);

subplot(2,1,1), hist(replEsts(:,1)); title('Bootstrap estimates of k');
subplot(2,1,2), hist(replEsts(:,2)); title('Bootstrap estimates of sigma');

subplot(1,2,1), qqplot(replEsts(:,1)); title('Bootstrap estimates of k');
subplot(1,2,2), qqplot(log(replEsts(:,2))); title('Bootstrap estimates of
log(sigma)');

[paramEsts,paramCI] = gpfit(y);

kHat
kCI  = paramCI(:,1)

sigmaHat
sigmaCI  = paramCI(:,2)
```

Matlab Code VIII - Calibrate the t Copula

```
U = zeros(size(residuals));

for i = 1:nIndices
    U(:,i) = OBJ{i}.cdf(residuals(:,i)); % transform margin to uniform
end

[R, DoF] = copulafit('t', U, 'Method', 'ApproximateML'); % fit the copula

options     = statset('Display', 'off', 'TolX', 1e-4);

corrcoef(returns)  % linear correlation matrix of daily returns

DoF                % scalar degrees of freedom parameter of the optimized t
copula
```

Excel Macros Code IX - Mean Exceedances

```
Function exmean(returns, threshold)
With Application
' .Volatile
x = .Transpose(.Transpose(returns))
t = .Transpose(.Transpose(threshold))
n = UBound(x, 1)
nex = 0
cumex = 0
For i = 1 To n
    xi = .Small(x, i)
    If xi <= t Then
        nex = nex + 1
        cumex = cumex + xi
    End If
Next i
exmean = cumex / nex
End With
End Function
```

Excel Macros Code X - Exceedances

```
Function excount(returns, threshold)
With Application
' .Volatile
x = .Transpose(.Transpose(returns))
t = .Transpose(.Transpose(threshold))
n = UBound(x, 1)
nex = 0
For i = 1 To n
    xi = .Small(x, i)
    If xi <= t Then
        nex = nex + 1
    End If
Next i
excount = nex
End With
End Function
```