Do Heterogeneous Expectations Constitute a Challenge for Policy Interaction?[☆]

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Abstract

Yes, indeed; at least for macroeconomic policy interaction. We examine a Neo-Classical economy and provide the conditions for policy arrangements to successfully stabilize the economy when agents have either rational or adaptive expectations. For a contemporaneous-data monetary policy rule, the monetarist solution is unique and stationary under a passive fiscal/active monetary policy regime if monetary policy appropriately incorporates expectational heterogeneity. In contrast, the active fiscal/passive monetary policy regime's fiscalist solution is prone to explosiveness due to empirically plausible expectational heterogeneity. Nevertheless, this can be a well-defined, rather orthodox equilibrium. For operational monetary policy rules, only the results for the fiscalist solution prevail. Moreover, our results are plausible from an adaptive learning viewpoint.

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1. INTRODUCTION

Modeling expectations in modern macroeconomics is dominated by the paradigm of homogeneous expectations. Even when a continuum of agents is assumed, routinely subjective expectations coincide with the average expectations as symmetry among agents is imposed. The prevalence of homogeneous expectations reaches far beyond the dominating *rational expectations hypothesis* (REH) into the literature on bounded rationality. One example is the standard adaptive learning approach as discussed in Evans and Honkapohja (2001).

However, recent empirical and experimental research provides compelling evidence undermining the homogeneous expectations hypothesis. Evidence in favor of the heterogeneous expectations hypothesis based on survey data can be found in Branch (2004) or Bovi (2013).[1] Cornea et al. (2013) present evidence based on aggregate time series, and Hommes (2011), Pfajfar and Žakelj (2013), as well as Assenza et al. (2013) document the pervasiveness of heterogeneous expectations in laboratory experiments. Hommes (2011) reviews this literature.

These findings have triggered a notable number of studies tackling the issue of how expectational heterogeneity may affect aggregate economic dynamics. Examples are the seminal work of Brock and Hommes (1997) on dynamic predictor selection, or, the contributions of Branch and Evans (2006), Branch and McGough (2009, 2010), Berardi (2009), Kurz et al. (2013), Massaro (2013), Gasteiger (2014), and Hommes and Lustenhouwer (2015). Nonetheless, the issue of fiscal and monetary *policy interaction*, so far, has only been examined under the homogeneous expectations hypothesis. This is somewhat surprising given the finding that not only fiscal and monetary policy interaction, but also the expectational set-up can have important consequences for aggregate economic dynamics

[1]Further evidence is provided in Carroll (2003), Mankiw et al. (2003), and Branch (2007).

in general and the determination of the price level in particular. Prominent examples for analyses under homogeneous expectations are Leeper (1991) and Evans and Honkapohja (2007). The core questions in this strand of the literature are whether fiscal variables affect the price level and what policy arrangements successfully stabilize the economy. The answer crucially depends on the policy regime in place and private sector behavior, including its expectations.[2] In fact, depending on the particular policy regime, typically a certain unique stationary *rational expectations equilibria* (REE) is possible and stabilizes the economy. One is routinely denoted the *fiscalist* solution, where price level determination depends on fiscal variables, whereas the other one is referred to as *monetarist* solution, in which the price level is independent of fiscal variables.[3]

Our primary contribution is to examine the determinacy properties of various fiscal and monetary policy regimes under heterogeneous expectations.[4] Thereby we address the issue of whether fiscal variables and private sector expectations can affect inflation, and, whether heterogeneous private sector expectations can constitute a new challenge for policy interaction with regard to stabilization policy. The key novelty is that we embed fiscal and monetary policy interaction à la Leeper (1991) into a heterogeneous expectations set-up à la Branch and McGough (2009).[5] Fiscal and monetary policy arrangements that successfully stabilize the

[2] Recently Cochrane (2011), Leeper and Zhou (2013), and Sims (2013) argued that fiscal policy has a crucial role in the determination of the price level and that current fiscal and monetary policy arrangements fail to account for this.

[3] Davig and Leeper (2006, 2011) empirically document the related fiscal and monetary policy regimes in post-war US data.

[4] As our heterogeneous expectations model can be written as an *associated rational expectations* model, see Branch and McGough (2004), we can analyze RE solutions. Thus, a situation in which there exists a unique stationary REE is referred to local *determinacy*. Moreover, local *indeterminacy* denotes the existence of multiple stationary REE. Finally, if no stationary REE exists, the economy is said to feature local *divergence* or *explosiveness*.

[5] Woodford (2013) recently forcefully illustrates the desirability of policy recommendations, which are robust across various reasonable expectational set-ups. However, he focuses on homogeneous expectations and abstracts from macro policy interaction.

economy by generating determinacy are of interest. Successful arrangements under heterogeneous expectations are of even higher relevance, as expectational heterogeneity can be an important source of economic instability for the monetarist solution in *New-Keynesian* models (see, e.g., Zhao, 2007; Branch and McGough, 2009; Massaro, 2013). In contrast, our results are derived in a model with an admittedly simpler production side, but we consider a much broader set of policy regimes, while nesting the policy regimes of the aforementioned studies. In consequence, our approach permits various new insights.

Thus, our work not only states a straightforward extension of the seminal contribution of Leeper (1991) on policy interaction under the REH, and the complementary analysis by Branch et al. (2008). It also contributes to a burgeoning strand of the literature, which considers macroeconomic policy interaction under different expectational set-ups and its implications for stabilization policy. In particular, see Evans and Honkapohja (2005, 2007) or Eusepi and Preston (2012) under homogeneous adaptive learning.[6] Our analysis extends this literature by putting forth a theory of fiscal and monetary policy interaction under heterogeneous rather than homogeneous expectations. This generates new restrictions on policy interaction, which are relevant for the design of stabilization policies.

In particular, we assume that agents either have *rational* (RE) or *adaptive expectations* (AE). One can interpret such a set-up as one of persistent heterogeneity. Evans and Honkapohja (2013) argue that this is a plausible assumption, even when agents may entertain various forecasting models.[7]

[6] The mentioned studies examine Leeper (1991)-type policy interaction in a system linearised around a deterministic steady-state. For global analyses of policy interaction we refer the reader to Evans et al. (2008) or Benhabib et al. (2014).

[7] Others have considered persistently heterogeneous expectations before. For instance Honkapohja and Mitra (2006) investigate monetary policy under coexistence of two types of forecasts arising from two different adaptive learning rules. Berardi's (2009) set-up implies

Despite the fact that such a modeling approach partly neglects the plurality of predictors that the afore-mentioned evidence suggests, it is appealing for at least three reasons. First, a common feature of the evidence is the presence of a relatively large share of agents with AE among agents with access to various predictors. Branch (2004, p.617) finds a share of agents with AE around 47%.[8] Also Bovi (2013) finds favorable evidence for persistent heterogeneity in expectations. Furthermore, the evidence discussed in Massaro (2013, p.687) suggests that a share of backward-looking agents in the range of 20% to 60% seems plausible. Second, this approach allows for analytical tractability, and third, the model nests the RE benchmark case. The latter, and limiting the analysis to a *Neo-Classical* economy, facilitates a direct comparison to the related literature on fiscal and monetary policy interaction (i.e., Leeper, 1991; Evans and Honkapohja, 2007).[9]

Assuming expectational heterogeneity in this way, introduces lagged inflation as a new state variable to the economy. This eventually changes the dynamic properties of the economy and the resulting policy implications. Actually, we show that different restrictions on RE solutions can emerge when we focus on the determinate cases. One involves inflation depending on fiscal variables, i.e., the fiscalist solution, whereas others do not, i.e., monetarist solutions.

Subsequently we examine the full set of REE and find that four different types of stationary solutions are possible. We relate the four types of solutions to different policy regimes and show under which conditions the shares of agents with RE and AE have a crucial role in determining economic outcomes. A key

persistent heterogeneity asymptotically. Branch and McGough (2009), Branch and McGough (2011), or Gasteiger (2014) simply assume persistent heterogeneity.

[8]This also includes the special case of *naïve* expectations. Branch (2004) also shows that these shares vary over time with different volatility regimes.

[9]Our analysis is extendable to a *New-Keynesian* model. This is the goal in a related paper. Eusepi and Preston (2012) provide a suitable framework under homogeneous expectations.

result is that whether or not the fiscalist solution is stationary, turns out to depend crucially on the share of agents with RE. Surprisingly, even in the non-stationary case, as long as monetary policy is passive, the equilibrium may be well-defined and exhibit 'orthodox' properties (see, McCallum, 2003, p.1172).

In contrast, non-explosiveness of the monetarist solution appears to be less vulnerable to the presence of heterogeneous expectations under a contemporaneous-data rule. This can be explained by the extent to which monetary policy incorporates heterogeneous private sector expectations. In fact, obeying a generalized version of the Taylor (1993)-principle that guarantees that, in response to a change in inflation, the real interest rate always moves more in the same direction than inflation itself, generates determinacy. In this sense, active monetary policy is no longer unconstrained, but constrained by expectational heterogeneity.

Following Branch et al. (2008), we assess the generality of our findings, by considering operational interest rate rules instead of Leeper's (1991) contemporaneous-data rule. It turns out, that our finding for the fiscalist solution is robust with regard to these alternative specifications of monetary policy, whereas the monetarist solution is no longer determinate under operational rules. This is a remarkable result, as most of the monetary literature builds on solutions of this type and develops predictions conditional on this solution being determinate.[10]

Finally, following Evans and Honkapohja (2007), we assess the plausibility of our findings from an adaptive learning viewpoint by replacing agents with RE by agents who behave like econometricians. They estimate the structural parameters by a least-squares (LS) regression model, base their forecasts on this model, and, repeat estimation as well as forecast updating whenever new data becomes available. A REE is plausible, when it is locally stable under such LS

[10]Kirsanova et al. (2009) denote this the 'current consensus assignment'.

learning, and it turns out that all our findings are indeed plausible.

The remainder of the paper is organized as follows. We present a simple *Neo-Classical* economy under heterogeneous expectations and the derivation of the aggregate equilibrium conditions from individual behavior in Section 2. Section 3 analyzes the dynamic properties of the model, presents our main results and discusses their policy implications. In Section 4 we present results for alternative monetary policy specifications and an assessment of the plausibility of our results from an adaptive learning viewpoint. Section 5 concludes.

2. THE MODEL

We develop our analysis in a heterogeneous expectations version of the model outlined in Evans and Honkapohja (2007). We consider infinitely many households and each individual household i of type ς has a utility function, which depends on real consumption in period s, $c_s^\varsigma(i)$, beginning of period real money balances, $\pi_s^{-1} m_{s-1}^\varsigma(i)$, where $m_s^\varsigma(i) = M_s^\varsigma(i)/P_s$, $M_s^\varsigma(i)$ denotes nominal money balances, and P_s is the aggregate price level. Thus, $\pi_s = P_s/P_{s-1}$ is the gross inflation rate. The household's maximization problem is given by

$$\max \quad E_t^\varsigma \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_s^\varsigma(i)^{(1-\sigma_1)}}{(1-\sigma_1)} + \mathcal{A} \frac{\left(\frac{m_{s-1}^\varsigma(i)}{\pi_s}\right)^{(1-\sigma_2)}}{(1-\sigma_2)} \right] \right\} \tag{1}$$

$$\text{s.t.} \quad c_s^\varsigma(i) + m_s^\varsigma(i) + b_s^\varsigma(i) + \tau_s = y + \frac{m_{s-1}^\varsigma(i)}{\pi_s} + R_{s-1} \frac{b_{s-1}^\varsigma(i)}{\pi_s}, \tag{2}$$

where (2) is the household's budget constraint. Moreover, $0 < \beta < 1$ is the discount factor, $\sigma_1 > 0$ and $\sigma_2 > 0$ are the elasticities of substitution, and $\mathcal{A}$ is a relative weight on real balances. $y > 0$ is a constant endowment and $b_s^\varsigma(i) = B_s^\varsigma(i)/P_s$ are end-of-period holdings of bonds in real terms, where $B_s^\varsigma(i)$ is the nominal end of period nominal government bond holdings. Next, τ_s are real lump-sum taxes, and R_{s-1} is the pre-determined gross nominal interest rate paid at the beginning of period s. Finally, the government is assumed to purchase and waste constant $g \geq 0$ in each period.[11]

The subjective expectations operator of a household of type ς is denoted $E_t^{\varsigma}\{\cdot\}$. We assume that all households are perfectly identical apart from the way they form expectations. In this regard, a household is considered to be of one of the two types $\varsigma \in \{1, 2\}$. Following the heterogeneous expectations set-up of Branch and McGough (2009), for any variable q_t we have

$$E_t^1 q_{t+1} = E_t q_{t+1}, \tag{3}$$

$$E_t^2 q_{t+1} = \iota E_t^2 q_t = \iota^2 q_{t-1}, \text{ and} \tag{4}$$

$$\widehat{E}_t q_{t+1} = \chi E_t q_{t+1} + (1-\chi)\iota^2 q_{t-1}. \tag{5}$$

Here χ is the share of agents of type $\varsigma = 1$ forming RE as in (3). Agents of type $\varsigma = 2$ form AE for unobserved and next period variables, and ι is the coefficient that these agents use to forecast variables based on the most recent observation according to (4). Aggregate expectations are given by (5). We restrict the coefficient to $\iota > 0$ and consider $\chi \in (0, 1]$.[12]

Appendix A shows that optimal behavior of households and market clearing conditions yield the aggregate *Fisher relation* and a money market equilibrium

[11] As there is no idiosyncratic income risk, we deny agents to trade state-contingent claims.

[12] See Branch and McGough (2009, p.1038) for more details on the subjective expectations operator. Agents of type $\varsigma = 1$ can be thought of *'really good forecasters'*.

condition in period t, expressed in deviations from steady-state, i.e.,

$$\tilde{R}_t^{-1} = \beta \widehat{E}_t\{\tilde{\pi}_{t+1}^{-1}\}, \quad \text{and} \tag{6}$$

$$\tilde{m}_t = \tilde{C}\widehat{E}_t\{\tilde{\pi}_{t+1}\}, \tag{7}$$

where transversality condition $\lim_{t\to\infty} \beta^t E_t^{\varsigma} c_t^{\varsigma}(i)^{-\sigma_1}(b_t^{\varsigma}(i) + m_t^{\varsigma}(i)) = 0$ holds.[13]

Next, the government budget constraint in real terms is given by

$$b_t + m_t + \tau_t = g + \frac{m_{t-1}}{\pi_t} + R_{t-1}\frac{b_{t-1}}{\pi_t}. \tag{8}$$

It basically states that government spending and interest payments on debt outstanding can be funded by issuing new debt, seigniorage, and taxes.

Following Leeper (1991), we assume two independent public authorities that interact with each other. First, there is a fiscal authority with tax rule

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t. \tag{9}$$

The rule implies that the authority responds to previous period real debt. ψ_t is the exogenous fiscal policy shock. Second, there is a central bank conducting monetary policy according to the contemporaneous-data interest rate rule

$$R_t = \alpha_0 + \alpha\pi_t + \theta_t. \tag{10}$$

Thus, this rule relates the central bank's policy instrument to its mandate of controlling inflation and captures monetary policy shocks, θ_t. Here θ_t and ψ_t are assumed to be exogenous *iid* mean zero random shocks. The feedback of policy

[13] $\tilde{q}_t$ represents the respective variable in deviation from steady-state, i.e., $\tilde{q}_t \equiv (q_t - q)$.

to the targeted variable is governed by the coefficients γ and α. These coefficients determine qualitatively different types of fiscal and monetary policies (see Leeper, 1991; Evans and Honkapohja, 2007).

DEFINITION 1. *If $|\beta^{-1} - \gamma| > 1$, the fiscal authority's policy is active(AF). In contrast, if $|\beta^{-1} - \gamma| < 1$, fiscal policy is considered to be passive (PF). The central bank's policy is active (AM) if $|\alpha\beta| > 1$ and passive (PM) if $|\alpha\beta| < 1$.*

This definition is based on the roots of the economic system considered, i.e., $\alpha\beta$ and $\beta^{-1} - \gamma$. As policy parameters α and γ enter these roots, the above definition divides the policy parameter space into regions where either none, one, or, both roots are (un-)stable. Therefore the dynamic properties of the system are fundamentally different in each region. The aforementioned authors explain that, for the empirically realistic case, $0 \leq \gamma < \beta^{-1}$, AF implies that under rule (9) the additional tax revenue generated from a small increase in the steady-state level of debt is lower than the increase in the related interest payments. For PF, the reverse is true. Moreover, $\alpha > \beta^{-1}$ implies a positive response of the real interest rate to an increase in steady-state inflation. Notice that this condition is often referred to as the Taylor (1993)-principle.[14]

According to Leeper (1991), in economic terms, it follows that a passive policy of either the central bank or the fiscal authority is constraint by private sector behavior, including its expectations, and the active policy of the other authority. The passive policy aims at balancing the inter-temporal budget constraint, either by means of generating inflation or sufficient tax revenue.

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[14]For instance, in the *New-Keynesian* benchmark model under the REH with rule (10), the principle is $\alpha > 1$.

3. DYNAMIC PROPERTIES UNDER POLICY INTERACTION

3.1. Main Results

The linearized economy (6)-(7), including the policy block (8) to (10) as well as the expectational set-up (3) to (5), can be expressed by a two-dimensional system (as in Evans and Honkapohja, 2007)

$$\tilde{\pi}_t = (\alpha\beta)^{-1}\chi E_t\tilde{\pi}_{t+1} + (\alpha\beta)^{-1}(1-\chi)\iota^2\tilde{\pi}_{t-1} - \alpha^{-1}\theta_t \tag{11}$$

$$\begin{aligned} 0 &= \tilde{b}_{t+1} + \varphi_1\chi E_t\tilde{\pi}_{t+1} + \varphi_1(1-\chi)\iota^2\tilde{\pi}_{t-1} + \varphi_2\tilde{\pi}_t \\ &\quad - (\beta^{-1}-\gamma)\tilde{b}_t + \psi_{t+1} + \varphi_3\theta_{t+1} + \varphi_4\theta_t, \quad \text{where} \end{aligned} \tag{12}$$

$$\varphi_1 = [\tilde{C}\beta\alpha + m\pi^{-2} + Rb\pi^{-2}], \varphi_2 = [-\pi^{-1}\tilde{C}\beta\alpha - \pi^{-1}b\alpha],$$

$$\varphi_3 = \tilde{C}\beta, \varphi_4 = [-\pi^{-1}\tilde{C}\beta - \pi^{-1}b].$$

Following their example, we abstract from the special cases $\alpha = 0$, $\alpha\beta = 1$, $\gamma\beta = 1$, and $\beta^{-1} - \gamma = 1$. Hereby we rule out the peculiar case of eigenvalues on the unit circle as well as a scenario of no policy feedback, or, a scenario where the government exactly pays all debt (including interest) off.

In Appendix B we define $y_t \equiv [\tilde{\pi}_t, \tilde{b}_t, \tilde{\pi}_{t-1}]'$ and show that $\lambda_1 \equiv (\beta^{-1} - \gamma)^{-1}$, $\lambda_2 \equiv \frac{(\alpha\beta)-\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$, and $\lambda_3 \equiv \frac{(\alpha\beta)+\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$, $\Theta \equiv (1-\chi)\iota^2$, are the eigenvalues of our economic system. The crucial difference between the economy in Evans and Honkapohja (2007) and the one herein is, that the latter involves the dynamics of one free and two predetermined variables in presence of heterogeneous expectations, i.e., $\chi < 1$. The additional state variable is $\tilde{\pi}_{t-1}$. This has important consequences for the question of when a REE is locally determinate.

Technically speaking, local determinacy requires that the number of eigenvalues inside (outside) the unit circle matches the number of free (predetermined) variables, which is one (two) in our case. If the number of eigenvalues inside the unit circle exceeds (is below) the number of free variables, then the economy is said to be locally explosive (indeterminate).[15]

Now we pursue one of our main goals, which is to relate qualitatively different economic dynamics to certain policy regimes. First, note that λ_1 is similar to the root related to fiscal policy in Definition 1 above. Furthermore, it is obvious that $|\lambda_1| > 1$ if $|(\beta^{-1} - \gamma)| < 1$ is the case. This corresponds to PF under homogeneous expectations and the reverse is true in case of AF. Second, inspection of λ_2 and λ_3 suggests to refine the notion of AM and PM as follows.

DEFINITION 2. *Monetary policy is passive under heterogeneous expectations (PMHE) if* $(\alpha\beta) < (\chi + \Theta)$. *Moreover, monetary policy is active under heterogeneous expectations (AMHE) if* $(\alpha\beta) > (\chi + \Theta)$.

This is a straightforward modification. In case of AMHE, we will find $|\lambda_2| < 1$ and $|\lambda_3| > 1$ and with PMHE it turns out that both $|\lambda_2|$ and $|\lambda_3|$ are either inside or outside the unit circle. Thus, the modification allows us to divide the policy parameter space in a way similar to Leeper (1991) and Evans and Honkapohja (2007). Likewise, as PMHE (AMHE) corresponds to PM (AM) for $\chi = 1$, the definition nests the natural RE benchmark case. Finally, as we argue below, the definition of AMHE can be regarded as a generalized Taylor (1993)-principle, $\alpha > \beta^{-1}(\chi + \Theta)$. Thus, a central bank that aims at satisfying this generalized principle, will have to explicitly incorporate private sector expectations into its policy decisions.[16] In the subsequent analysis, this turns out to be one of the main challenges for policy interaction constituted by heterogeneous expectations.

[15] Branch and McGough (2004) have shown that one can examine the determinacy properties of the economy herein by utilizing the standard techniques as outlined in Blanchard and Kahn (1980) or Klein (2000). Corresponding to their approach, (11)-(12) states the *associated RE* model and solutions to this model are also solutions to the heterogeneous expectations economy.

[16] For $\iota = 1$ the principle collapses to its homogeneous expectations counterpart $\alpha > \beta^{-1}$.

For the moment, let us focus on the stationary cases. In Appendix B, we argue that linear restrictions of the type

$$\tilde{\pi}_t = K_1\tilde{b}_t + K_2\theta_t + K_3\tilde{\pi}_{t-1} \tag{13}$$

emerge and yield a stationary solution. In particular we find that:

(i) In the case of AF/PMHE, $|\lambda_1| < 1$, and $|\lambda_2|, |\lambda_3| > 1$;

(ii) In the case of PF/AMHE, $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$ with $K_1 = 0$;

(iii) In the case of PF/PMHE, $|\lambda_1|, |\lambda_2| > 1$ and $|\lambda_3| < 1$ with $K_1 = 0$.

In the homogeneous RE version of this economy a PF/PM regime leads to indeterminacy and an AF/AM regime yields local divergence. Thus, we ask to what extent these findings carry over to the heterogeneous expectations version.

In order to do so, we examine the whole set of REE. We define $v_t \equiv [\theta_t, \psi_t]'$ and recast the economy (11)-(12) as

$$y_t = \mathbf{M}E_t y_{t+1} + \mathbf{N}y_{t-1} + \mathbf{P}v_t + \mathbf{R}v_{t-1}, \quad \text{where} \tag{14}$$

$$\mathbf{M} = \begin{pmatrix} (\alpha\beta)^{-1}\chi & 0 & 0 \\ -\varphi_1(\alpha\beta)^{-1}\chi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{N} = \begin{pmatrix} (\alpha\beta)^{-1}\Theta & 0 & 0 \\ -\varphi_1(\alpha\beta)^{-1}\Theta - \varphi_2 & \beta^{-1} - \gamma & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\mathbf{P} = \begin{pmatrix} -\alpha^{-1} & 0 \\ \varphi_1\alpha^{-1} - \varphi_3 & -1 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad \mathbf{R} = \begin{pmatrix} 0 & 0 \\ -\varphi_4 & 0 \\ 0 & 0 \end{pmatrix}. \tag{15}$$

We assume that REE follow

$$y_t = \mathbf{A} + \mathbf{B}y_{t-1} + \mathbf{C}v_t + \mathbf{D}v_{t-1}. \tag{16}$$

In consequence, the very same *undetermined coefficient* reasoning as in Evans and Honkapohja (2007, p.678) leads to the following proposition.

PROPOSITION 1. *One can verify that there exist four types of solutions:*

(I) *One solution is characterized by satisfying restriction* (i) *and matrix* $\mathbf{B} = (\kappa\chi)^{-1}\times$

$$\begin{pmatrix} -\beta\Theta\varphi_1 - \left(\alpha\beta^2 + (\beta\gamma-1)\chi\right)\varphi_2 & -\beta^{-1}\left[(\alpha(\beta\gamma-1)+\Theta)\beta^2 + (\beta\gamma-1)^2\chi\right] & 0 \\ \beta\left(\Theta\varphi_1^2 + \alpha\beta\varphi_2\varphi_1 + \chi\varphi_2^2\right) & \beta(\alpha(\beta\gamma-1)+\Theta)\varphi_1 + (\beta\gamma-1)\chi\varphi_2 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

where $\kappa \equiv (\beta\gamma-1)\varphi_1 - \beta\varphi_2$*.* $\mathbf{A} = 0$*, and* $\mathbf{C}$ *as well as* $\mathbf{D}$ *are also uniquely determined. In this case, the eigenvalues of matrix* $\mathbf{B}$ *are* $\{0, \chi^{-1}\Theta\lambda_2, \chi^{-1}\Theta \lambda_3\}$*. We denote this the fiscalist solution under heterogeneous expectations. In case of* $\chi = 1$ *this solution corresponds to the traditional fiscalist solution.*

(II) *A second solution satisfies restriction* (ii) *with matrices* $\mathbf{B} = \begin{pmatrix} \chi^{-1}\Theta\lambda_3 & 0 & 0 \\ -\chi^{-1}\Theta\lambda_3\varphi_1 - \varphi_2 & \lambda_1^{-1} & 0 \\ 1 & 0 & 0 \end{pmatrix}$*, and* $\mathbf{A} = 0$*. Moreover,* $\mathbf{C}$ *and* $\mathbf{D}$ *are uniquely determined. The eigenvalues of matrix* $\mathbf{B}$ *are* $\left\{0, \lambda_1^{-1}, \chi^{-1}\ \Theta\lambda_3\right\}$*. This can be denoted the monetarist solution under heterogeneous expectations. For* $\chi = 1$ *this solution is the traditional monetarist solution.*

(III) *A third solution, satisfying restriction* (iii)*, is possible and is characterized by matrices* $\mathbf{B} = \begin{pmatrix} \chi^{-1}\Theta\lambda_2 & 0 & 0 \\ -\chi^{-1}\Theta\lambda_2\varphi_1 - \varphi_2 & \lambda_1^{-1} & 0 \\ 1 & 0 & 0 \end{pmatrix}$*,* $\mathbf{A} = 0$*,* $\mathbf{C}$ *and* $\mathbf{D}$ *uniquely determined. The eigenvalues of matrix* $\mathbf{B}$ *are* $\left\{0, \lambda_1^{-1}, \chi^{-1}\Theta\lambda_2\right\}$*. Again, this solution states nothing but the monetarist solution.*

(IV) *Finally, there is a continuum of non-fundamental solutions characterized*

by matrices $\mathbf{B} = \begin{pmatrix} \chi^{-1}(\alpha\beta) & 0 & -\chi^{-1}\Theta \\ -\chi^{-1}(\alpha\beta)\varphi_1 - \varphi_2 & \chi^{-1}\Theta\varphi_1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, *and* $\mathbf{A} = 0$.
However there exist multiple solutions for $\mathbf{C}$ *and* $\mathbf{D}$.

Note that (IV) describes a situation, where there exist multiple stationary solution paths for inflation, indexed by their initial values or eventually sunspots for a given level of the nominal money supply. This in turn engenders multiple paths for real balance growth, see Leeper (1991).

Next, we restrict attention to the parameter space $\alpha > 0$, $\gamma \geq 0$, and $\beta^{-1} > \gamma \geq 0$. As argued above, the literature regards this as the empirical realistic case and for this case we derive our main results. As we prove in Appendix C, we can relate the solutions to certain policy regimes, which is our main goal.

PROPOSITION 2. *Assume the monetary policy rule (10). For the empirically realistic case it holds that:*

(I) In a PF/ AMHE regime determinacy prevails.

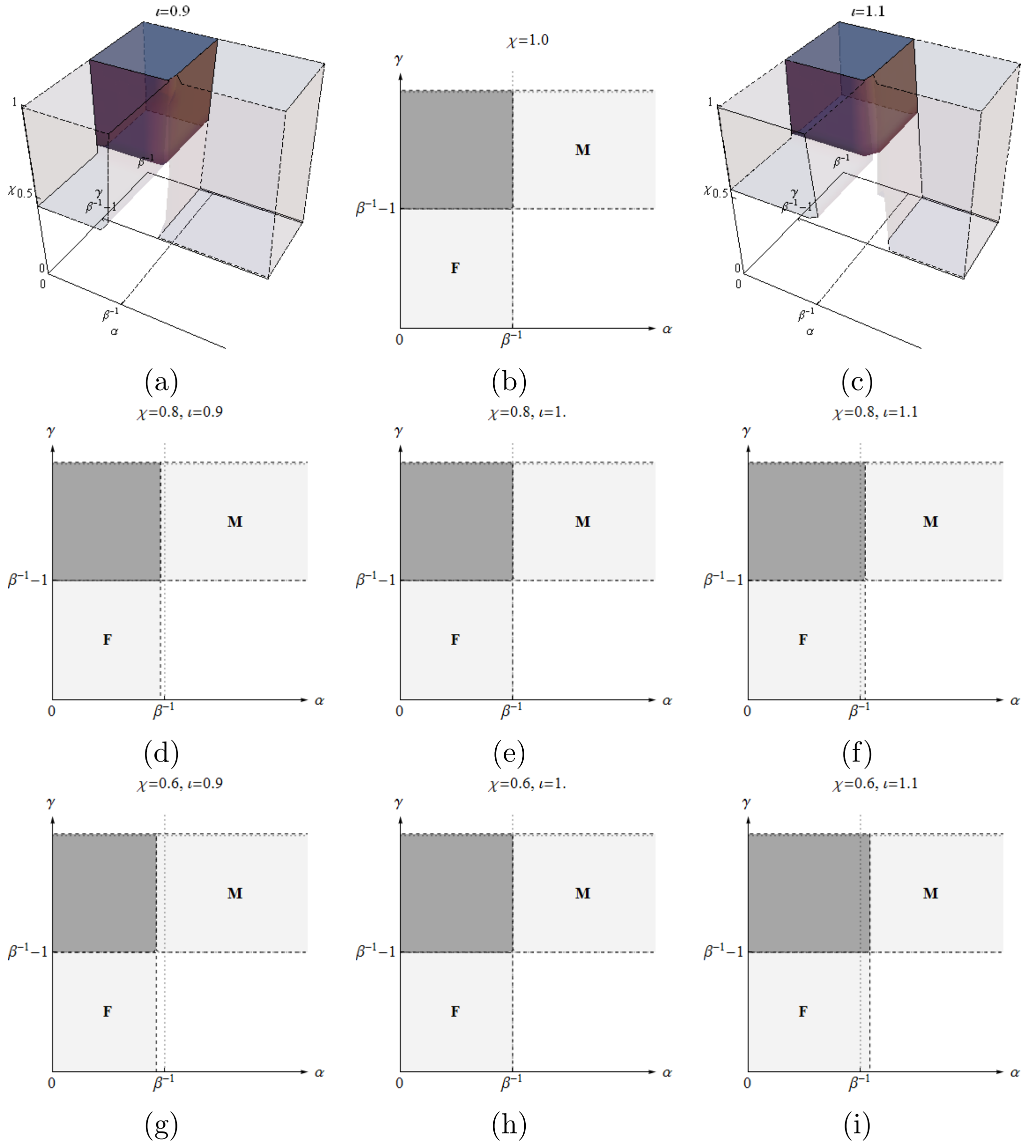
(II) A PF/PMHE regime results in local indeterminacy or divergence, depending on the share of agents with RE.

(III) An AF/AMHE regime yields local divergence.

(IV) Moreover, an AF/PMHE regime may lead to determinacy, if the share of agents with RE in the economy is sufficiently high. If this share is too low, the regime triggers local divergence.

Proposition 2 is our main result and has important policy implications. Therefore the remainder of this section illustrates our findings graphically, provides an intuitive explanation and discusses the implications in detail.

In Panels 1a and 1c below we numerically illustrate our findings of Propositions 1 and 2 in the α-γ-χ-space, i.e., the coefficients from the interest rate rule, the tax rule, and the share of agents forming RE respectively. The remaining panels in Figure 1 are illustrations in the α-γ-space.



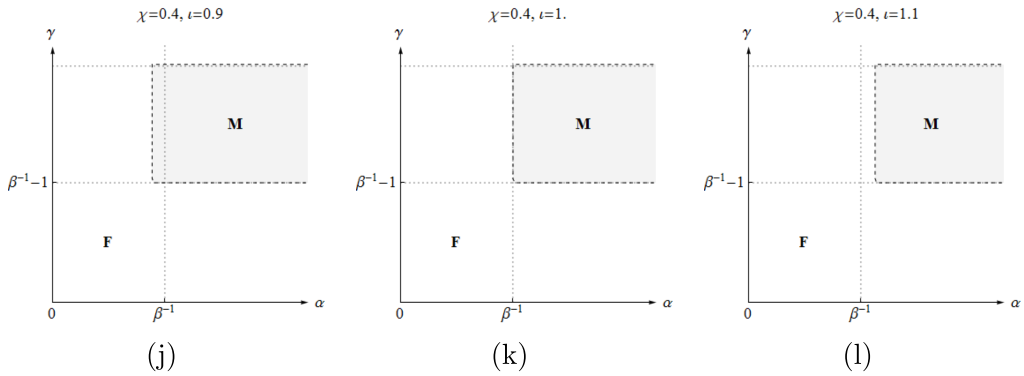


Figure 1: Regions of local determinacy (light grey), indeterminacy (dark grey), and explosiveness (remainder) in the empirical relevant space, i.e., $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$ for $\beta = 0.99$. **M** (**F**) is the monetarist (fiscalist) solution.

In Panels 1a to 1c the value $\chi = 1$ represents an illustration of the results obtained by Evans and Honkapohja (2007) for the homogeneous RE benchmark case. The additional implications of heterogeneous expectations for the dynamics of the economy become evident, once we consider the cases of $\chi < 1$. In particular, the region of approximately $\alpha \in [0, \beta^{-1}]$ and below $\chi \approx 0.5$. In this area of the parameter space the PF/PMHE regime, and more important, the AF/PMHE regime have fundamentally different dynamic properties as is known from homogeneous expectations benchmark, i.e., local explosiveness.[17]

Consider Panel 1a. Some intuition can be developed for the local explosiveness result by entertaining a scenario, where an unanticipated contractionary monetary policy shock hits the economy in steady-state. Given $\chi = 1$, and PMHE, the shock contemporaneously raises R_t. This triggers a *substitution effect*: agents substitute nominal money balances for nominal bond holdings, which means an expansion in nominal debt. However, the inter-temporal government budget constraint needs to be satisfied, i.e., current real government debt outstanding must

[17]Technically local divergence occurs, because a policy regime fails to ensure that $0 < |K_1|, |K_2|, |K_3| < 1$ in (13). Consequently, one or more of the coefficients are larger than one in modulus and the dynamics of π_t become explosive.

be backed by the future discounted sum of primary government surpluses and seigniorage. Given AF, this can only happen by a jump in P_t that lowers current real government debt outstanding, and in turn increases π_t. The more passive fiscal policy, the weaker this effect. In the subsequent periods, due to PMHE, the substitution effect dies out and variables return to their steady-state.

Expectational heterogeneity, $\chi < 1$, opens a *self-fulfilling channel*, which is active in the periods following the shock. It's interplay with the substitution effect can explain the local explosiveness. Specifically, the self-referential nature of the model will induce an upward revision of inflation expectations of agents with AE and yield to a further increase of π_t. Given the contemporaneous-data rule, R_t will again be raised. When the self-fulfilling channel quantitatively outweighs the dampening nature of the PMHE stance, the raise in R_t re-enforces the interplay of the two effects and triggers an explosive path of π_t. The quantitative importance of the self-fulfilling channel looms larger with decreasing χ, and thereby poses a restriction on policy interaction.

Variation of ι within the rather wide range[18] $\iota \in \{0.9, 1.0, 1.1\}$ in Figure 1 reveals further insights regarding the interplay between χ and ι. Consider the determinate cases in Panels 1d to 1l. One can observe that for $\iota < 1$, the determinate region in the α-γ-space of the monetarist (fiscalist) solution increases (decreases) with decreasing χ within the considered parameter space. The opposite is true for $\iota > 1$ and the regions remain constant for $\iota = 1$. This behavior of the determinacy regions is directly related to the definition of AMHE and PMHE from above and how χ and ι restrict α regarding the monetary policy stance. However, notice that, as long as agents with non-rational expectations

[18]Notice that the range of $\iota \in [0.9, 1.1]$ is rather large. If type $\varsigma = 2$ agents observe a 1% deviation of inflation in $t-1$, their forecast for the period $t+1$ deviation is in the range of $[0.81\%, 1.21\%]$.

have forecasts that are a function of past data, their share is more decisive for the possibility of a determinate outcome, not their particular functional form, e.g., whether agents with AE discount ($\iota < 1$) or extrapolate ($\iota > 1$) past observations. For example, for $\chi = 0.4$ the fiscalist solution is explosive for any ι.

3.2. Further Discussion

Our main results for rule (17) are summarized in the second column of Table 1. Contrasting them with the RE benchmark (first column of Table 1) reveals various economic implications. First and foremost, the PF/AMHE regime yields local determinacy under a contemporaneous-data rule. However, heterogeneous expectations impose an informational challenge on the central bank. Recall from Definition 2 that AMHE requires $(\alpha\beta) > (\chi + \Theta)$. Therefore the central bank needs to respond sufficiently strong to inflation, which entails to successfully track private sector expectations, i.e., parameters χ and ι. In the logic of Leeper (1991), it turns out that not only PF is constrained by AMHE and private sector behavior, but for $\chi < 1$ also the central bank is constrained by private sector expectations. However, the challenge of tracking private sector expectations can eventually be met by modern central banks.[19]

Also notice that for the homogeneous RE benchmark case AMHE means nothing but $\alpha > \beta^{-1}$. This is equivalent to AM and known as the Taylor (1993)-principle. It is fair to say, that the core of this prescription, i.e., more than one-for-one response of the nominal interest rate to deviations in inflation, is to affect the real interest rate. In particular, in response to positive (negative) inflation deviations, the real interest rate should increase (decrease) in order to lower (stimulate) aggregate demand, see, for instance, Orphanides and Williams

[19]In fact, central banks try to track expectations, e.g., the *Survey of Professional Forecasters*.

Table 1. Overview on Results

Monetary Policy Rule and Regime	Solution	Expectational Set-Up			
		$E_t^1 = E_t$		$E_t^1 = E_t^*$	
		$\chi = 1$	$\chi < 1$	$\chi = 1$	$\chi < 1$
(10)		Leeper (1991)	This Paper	Evans and Honkapohja (2007)	This Paper
PF/AMHE	M[a]	D[b]	D	E-stable	E-stable
PF/PMHE	∞	I	I or E	not E-stable	not E-stable
AF/PMHE	F	D	D or E	E-stable	E-stable
AF/AMHE	F or M	E	E	E-stable	E-stable
(17)		Branch et al. (2008)	This Paper	This Paper	
PF/AMHE	∞	E	E	E-stable	
PF/PMHE	∞	I	I or E	E-stable	
AF/PMHE	F	D	D or E	E-stable	
AF/AMHE	F	E	E	E-stable	
(18)		Branch et al. (2008)	This Paper	This Paper	
PF/AMHE	∞	I	I or E	E-stable	
PF/PMHE	∞	I	I or E	E-stable	
AF/PMHE	F	D	D or E	E-stable	
AF/AMHE	F	D	D or E	E-stable	

[a] M = monetarist, F = fiscalist, or, ∞ = continuum of non-fundamental solutions.
[b] D = determinate, I = indeterminate, or, E = explosive.

(2005b, p.499) or Taylor (1999). In this light, even when $\alpha > \beta^{-1}(\chi + \Theta)$ implies $1 > \alpha > \beta^{-1}(\chi + \Theta)$, policy is compliant with the Taylor (1993)-principle in the way that the nominal interest rate setting affects the real interest rate. Or, one can simply view $\alpha > \beta^{-1}(\chi + \Theta)$ as a generalized version of the principle, which has to hold in a world of heterogeneous expectations.[20]

The view that under deviations from REH an AM may imply a response different from the original Taylor (1993)-principle appears consistent with existing numerical results in the literature on monetary policy rules in heterogeneous expectations set-ups. For instance, Anufriev et al. (2013) conduct a non-linear

[20] A similar argument can be made for the *New-Keynesian* model in Branch and McGough (2009). In their model, the condition, for the nominal interest setting to affect the real interest in the desired way, is $\alpha_\pi + \lambda^{-1}[1 - \beta(\chi + \Theta)]\alpha_y > 1$. α_π and α_y are the coefficients of the monetary policy rule for inflation and output gap, and, λ is the sensitivity of inflation to changes in the output gap in the *New-Keynesian Phillips Curve*.

analysis in a model that is very similar to ours. The key differences are that they assume PF and dynamic predictor selection. They find that obeying $\alpha > \beta^{-1}$ is desirable as inflation is successfully stabilized, but does not guarantee convergence to the monetarist solution under the REH. Likewise, in linearized *New-Keynesian* models, e.g., Branch and McGough (2009) or Massaro (2013), $\alpha > 1$, may not generate determinate outcomes. Under the assumption of social learning with similar simple monetary policy rules Arifovic et al. (2013) find that the classic Taylor (1993)-principle, $\alpha > 1$, is not necessary for the convergence. Moreover, the simulations of De Grauwe (2010) suggest that given $\alpha > 1$, the larger α, the more successful is stabilization policy. Under homogeneous adaptive learning under optimal policy (see Orphanides and Williams, 2005a,b, 2007a,b) similar findings occur. These authors also find that responding to inflation expectations rather than realized inflation improves stabilization policy as well.

Second, our result for the AF/PMHE regime deserves special attention. Based on the homogeneous RE benchmark case, on may argue that, once fiscal policy switches from PF to AF, the central bank can bring about determinacy by switching from AM to PM. This argument acknowledges the fact that it is usually the central bank that is more flexible and faster in implementing policy changes. However, an AF/PMHE regime makes the economy prone to local divergence, if roughly the majority of agents has AE. This is in the empirical range that is documented by Branch (2004) or discussed in Massaro (2013), i.e., $\chi \in [0.4, 0.8]$. Thus, one can view this finding as a challenge to policy interaction. The policy rules considered do not allow for successfully stabilization policy for certain χ. Eventually, fiscal rules that account for private sector expectations may be able to safeguard the economy against explosive dynamics in inflation in this situation.

Be also aware that the divergence induced by agents with AE may be a well-defined equilibrium. In fact, P_t, M_t, and B_t diverge, but the transversality conditions may be satisfied along these paths. Thus, the non-stationary fiscalist solution herein is different from the one found by McCallum (2001, p.20ff.) under the REH and AM (i.e., constant money supply). In our case, the price level and nominal money balances, and, necessarily also π_t and ΔM_t[21], move together. McCallum (2003, p.1172) notices this 'orthodox' property of the fiscalist solution in the stationary case under homogeneous expectations, i.e., AF/PMHE with $\chi = 1$. Furthermore, Woodford (2003, p.1184) regards this policy arrangement as the 'primary case' that one should consider.

Third, the PF/PMHE, in theory, may be a more unpleasant regime than is known under homogeneous RE. In this case both the fiscalist and the monetarist solution, as part of the continuum of possible solutions, are stationary for the benchmark case $\chi = 1$. However, when the share of agents with AE becomes sufficiently high, this regime leads to divergence for the whole continuum of solutions. In fact, the dynamics of π_t and b_t become complex under this regime. However it is worthwhile that the non-stationary fiscalist solution again may have a rather *orthodox* behavior.

Fourth, our analysis confirms the finding of the homogeneous expectations literature on policy interaction that an AF/AMHE regime leads to local explosiveness. Thus, the expectational set-up does not affect the simple logic that the economy diverges if authorities ignore government solvency requirements.

Finally, Branch and McGough (2009) demonstrate in the very same expectational set-up as ours with a forward-looking interest rate rule that it is rather the weight on past data (discounting vs. extrapolating past data) than the share of agents with AE, which is crucial in engendering determinacy. In the presence of

[21] This can be verified in an analysis similar to McCallum and Nelson (2005).

purely AE, monetary policy can again implement the monetarist solution with more moderate feedback to inflation relative to the RE benchmark. In contrast, if AE are extrapolative, the opposite is true. Our results for the contemporaneous-data rule (10) are only to some extent consistent with the ones of Branch and McGough (2009). For the monetarist solution, we can confirm their finding. However, in case of the fiscalist solution, the effect of the weight of past data influences the size of the determinacy region exactly in the opposite direction. Moreover, the magnitude of ι is of secondary importance when χ is too small, as the fiscalist solution then becomes explosive for any $\iota > 0$.[22]

4. ROBUSTNESS

4.1. Implementability Concerns

Starting with McCallum (1999), many authors have questioned whether a rule like (10) may be operational or implementable. The key issue is that current period observations of aggregate variables are hardly available to policy makers. Subsequently Branch et al. (2008) argue that the well-known implementability concerns regarding rule (10) have to be addressed in the context of policy interaction by considering a backward-looking or a forward-looking rule, i.e.,

$$R_t = \alpha_0 + \alpha\pi_{t-1} + \theta_t, \quad \text{or} \tag{17}$$

$$R_t = \alpha_0 + \alpha\widehat{E}_t\pi_{t+1} + \theta_t.^{23} \tag{18}$$

[22] A related question to the above analysis is how a transitory monetary or fiscal policy shock propagates through this economy. This analysis can be found in an online appendix, which is available on the author's website: www.urleiwand.com. In short, we find persistent responses to transitory shocks, and, depending on the policy regime, dampening oscillations. These results are solely driven by expectational heterogeneity.

As we prove in Appendix D, our results for rule (17) are the following.

PROPOSITION 3. *Assume the monetary policy rule (17). For the empirically realistic case it holds that:*

(I) In a PF/ AMHE regime there is local divergence.

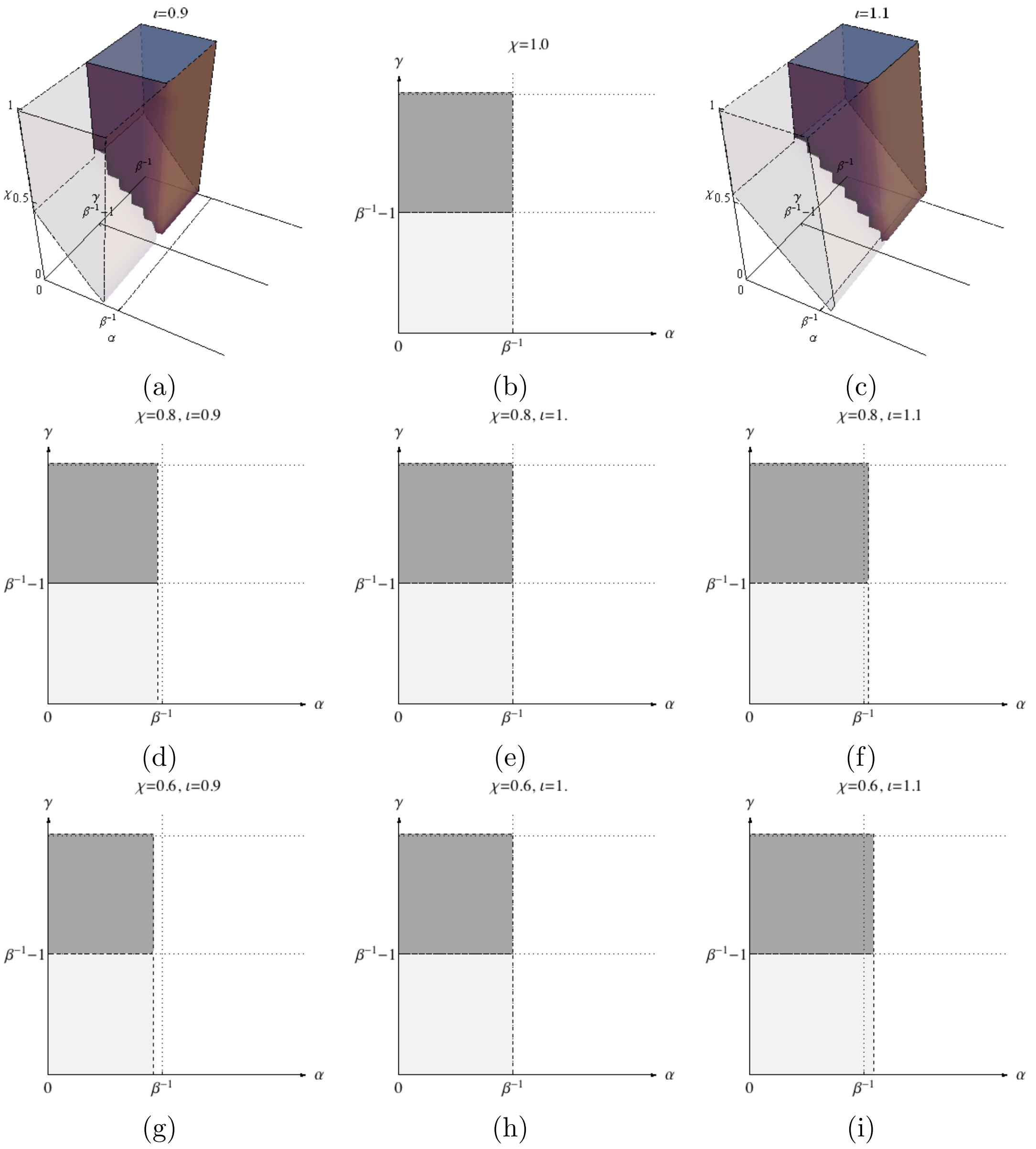
(II) A PF/PMHE regime results in local indeterminacy or divergence. The latter is true, if PMHE is overly passive, which depends on the share of agents with RE and monetary policy feedback α.

(III) An AF/AMHE regime yields local divergence.

(IV) An AF/PMHE regime may lead to determinacy, if PMHE is not too passive. Again, this depends on the share of agents with RE and monetary policy feedback α. If PMHE is overly passive, the regime triggers local divergence.

The panels in Figure 2 below provide a numerical exposition of our results in Propositions 3 in the α-γ-χ-space. Again, $\chi = 1$ is the RE benchmark (as in Branch et al., 2008). We observe that only PM can lead to stationary solutions and AF is a necessary condition for determinacy. In consequence, only the fiscalist solution can be determinate. Branch et al. (2008, p.1099)'s intuition for this result stems on the substitution effect described above: under a backward-looking rule, an unanticipated contractionary monetary policy shock unambiguously raises R_t, which induces substitution of nominal money balances for nominal bonds, which, as discussed above, creates inflation. In the subsequent period, as R_t responds actively to π_{t-1}, it fails to offset the shock, but reinforces the substitution effect and local divergence occurs.

[23]Interest rate rule (18) is a straightforward adaption of the rule $R_t = \alpha_0 + \alpha E_t \pi_{t+1} + \theta_t$ considered in Branch et al. (2008). The intention is to analyse a rule that is assumed to feed back to aggregate private sector expectations.


ι=0.9
χ=1.0
ι=1.1
χ=0.8, ι=0.9
χ=0.8, ι=1.
χ=0.8, ι=1.1
χ=0.6, ι=0.9
χ=0.6, ι=1.
χ=0.6, ι=1.1
γ
α
β⁻¹
β⁻¹−1
0
1
0.5
χ

(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

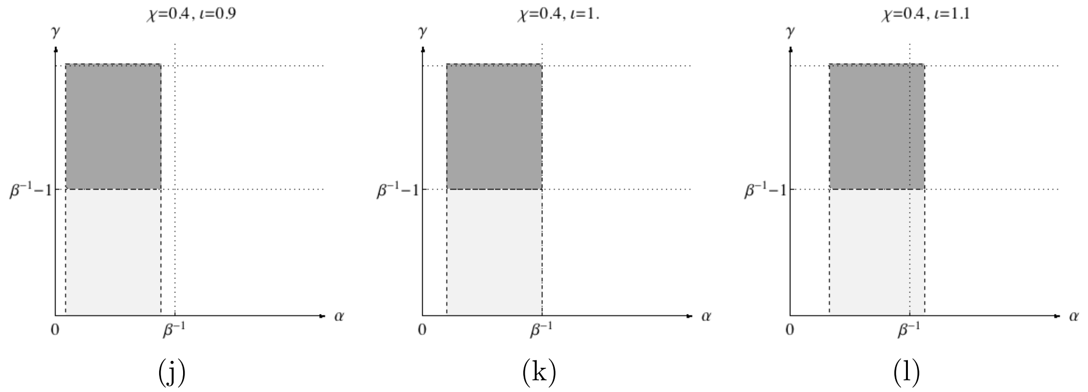


Figure 2: Regions of local determinacy (light grey), indeterminacy (dark grey), and explosiveness (remainder) for backward-looking monetary policy in the empirical relevant α-γ-χ-space, i.e., $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$, $\beta = 0.99$.

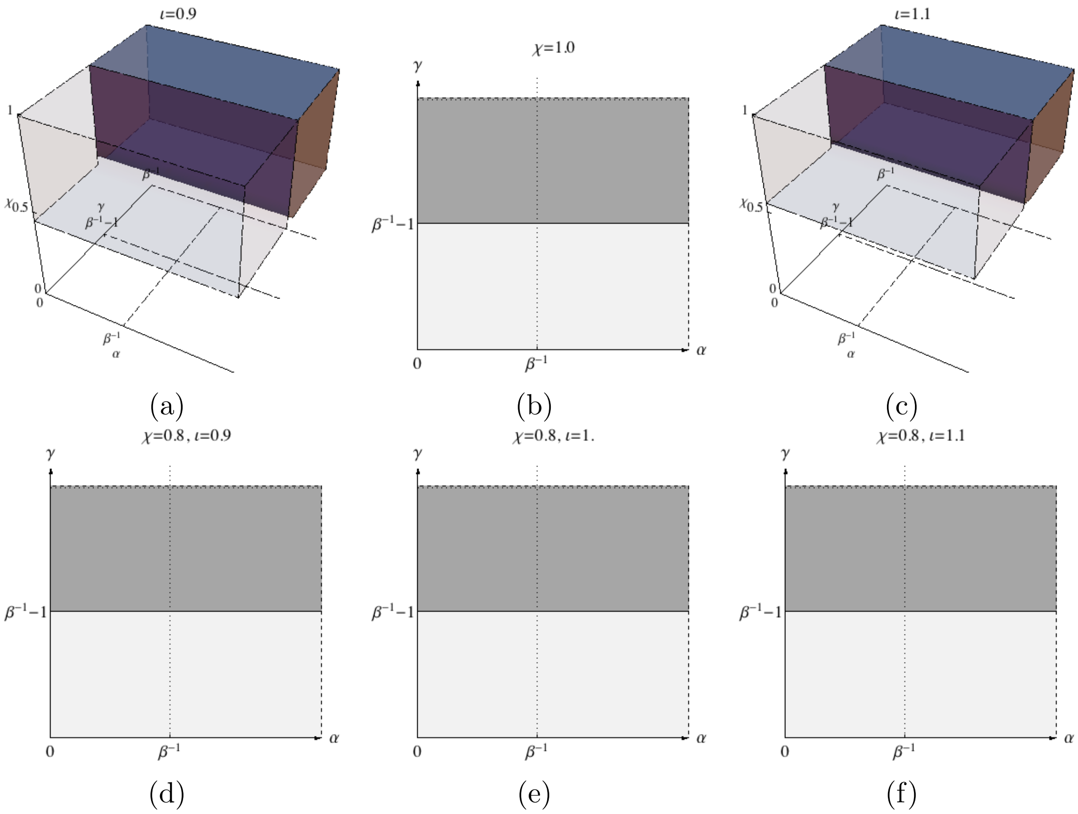
However, considering heterogeneous expectations provides additional insights. On the one side, these results partially extend the findings of Branch et al. (2008) for the AMHE stance to the case of heterogeneous expectations. On the other side, below $\chi \approx 0.5$ our results also give new insights regarding the PMHE stance. As one can see from Panels 2j to 2l, if policy is overly passive, i.e., low values of α, then such a policy triggers local divergence in both the AF/PMHE and the PM/PMHE regime. The intuitive explanation is again the interplay of the substitution effect and the self-fulfilling channel, as described above. In this sense, expectational heterogeneity restricts the central bank further constituting an additional challenge. A central bank aiming at a determinate outcome faces an upper and lower bound on α. Neither can policy be active, nor overly passive. Finally, the effect of a variation of ι is similar to our observations from above.

Next, we demonstrate in Appendix E the following results for rule (18).

PROPOSITION 4. *Assume the monetary policy rule (18). For the empirically realistic case it holds that exclusively under AF determinacy may prevail. The latter depends on the share of agents with RE.*

Figure 3 illustrates our results in Proposition 4 in the α-γ-χ-space. The RE benchmark ($\chi = 1$) confirms the findings of Branch et al. (2008). For this rule, there is no constraint on monetary policy. Nevertheless, once the χ decreases approximately below 0.5, the self-fulfilling channel again triggers local divergence.

The results above show that AF is a necessary condition for determinacy in the empirically realistic case for both, the backward- or forward-looking interest rate rule. However, in the latter case a sufficiently large share of agents with RE is necessary as well. For both rules, if policy interaction obtains a unique stationary REE, it is the fiscalist solution. A new challenge to policy interaction under the backward-looking rule, which emerges from heterogeneous expectations, is that monetary policy cannot be overly passive. So policy interaction needs to be designed more carefully in this case.



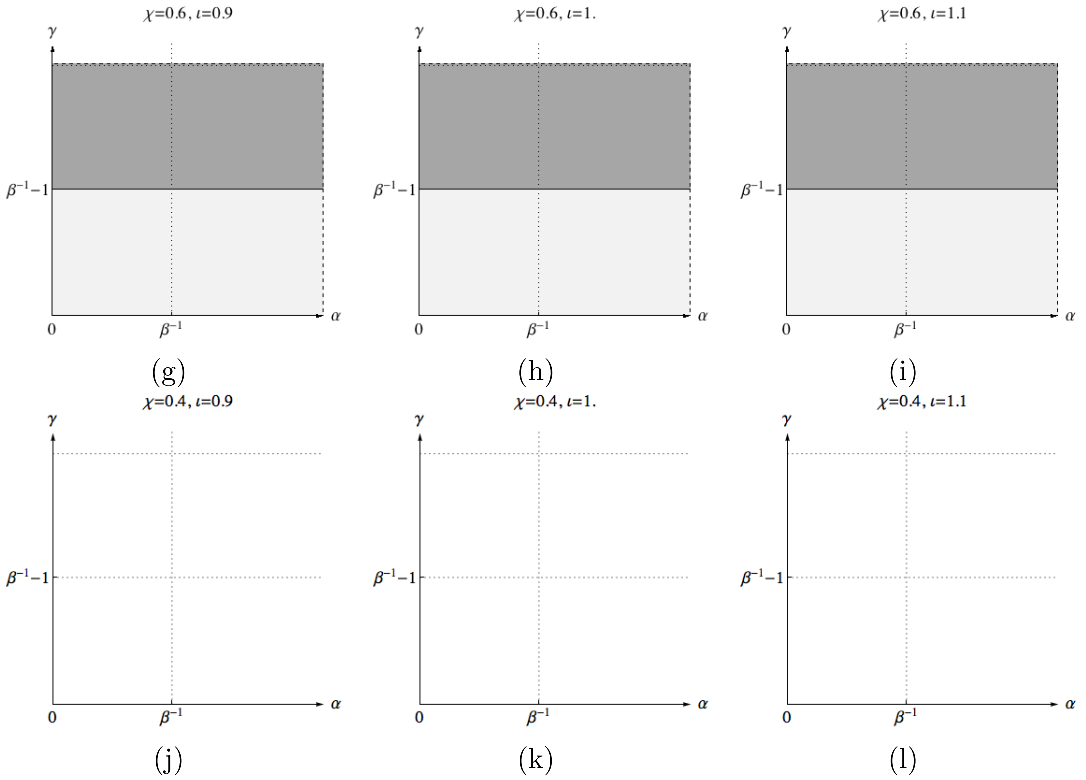


Figure 3: Regions of local determinacy (light grey), indeterminacy (dark grey), and explosiveness (remainder) for forward-looking monetary policy in the empirical relevant α-γ-χ-space, i.e., $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$, $\beta = 0.99$.

The finding regarding the backward-looking rule that monetary policy must be passive to achieve determinacy confirms the result of Branch et al. (2008) and contrasts the one of Schmitt-Grohé and Uribe (2007). Note that the latter study considers a production economy with physical capital and sticky prices. Especially nominal rigidities appear to have a crucial impact on the findings in the literature. For instance, compare our findings for the PF/AMHE regime to the ones of Bullard and Mitra (2002). They also examine the determinacy properties of rules (10), (17), and (18) in the *New-Keynesian* model under the REH for sort of a PF/AMHE regime, and in each case determinacy prevails.

Another interesting result states the fact that monetary policy plays no role in bringing about determinacy under the forward-looking rule, which extends the RE benchmark result of Branch et al. (2008) to the case of heterogeneous expectations. This result is also in line with Schmitt-Grohé and Uribe (2007). For AE with $\theta < 1$, our findings can also be related to Zhao (2007). The main finding therein is that under an interest rate rule with feedback to expected inflation, the monetarist solution can be implemented with weaker responses to inflation compared to the homogeneous RE benchmark. This is consistent with our finding for the contemporaneous-data rule, but contradicts our finding for the forward-looking rule, where AMHE causes local divergence. However, direct comparison is infeasible, as Zhao (2007) focuses on optimal feedback to π_t and does not provide conditions, under which policy fails to stabilize the economy.

In sum, one possible view on our results is that for a sufficiently large share of agents with RE, and AF/PMHE regime yields determinacy, independent of whether monetary policy is specified by a contemporaneous-data, backward- or forward-looking interest rate rule. Moreover, in the non-stationary case, the fiscalist solution may state a well-defined equilibrium with *orthodox* properties, as the divergence is triggered by expectational heterogeneity.

4.2. Plausibility from the Adaptive Learning Viewpoint

Evans and Honkapohja's (2007) analysis also addresses the concern of whether Leeper's (1991) findings regarding the monetarist and fiscalist solution under the REH are plausible from the adaptive learning viewpoint. Thus, it appears logical to assess our findings along the lines of Evans and Honkapohja (2007) and to consider the issue of stability of a solution under LS learning. Therefore, in this subsection, we assume that type $\varsigma = 1$ agents act like econometricians. The subjective period t forecast of any variable q_t is denoted $E_t^* q_{t+1}$. For given subjective expectations, this behavior generates a sequence of temporary equilibria.

All derivations in Appendix A remain valid under this assumption and for all three interest rate rules the economy can then be expressed as

$$y_t = \mathbf{M}E_t^* y_{t+1} + \mathbf{N}y_{t-1} + \mathbf{P}v_t + \mathbf{R}v_{t-1}, \tag{19}$$

and the agents consider (16) to be their *perceive law of motion* (PLM). As notation and analysis exactly follow Evans and Honkapohja (2007, p.679ff.) we will refrain from laying out the details, but instead state and discuss our results for the interest rate rules, (10), (17), and (18) in turn.

Contemporaneous-data rule. Recast the economy (11)-(12) to fit (19). In Appendix F we prove the following result. Our findings regarding E-stability are also contrasted with the literature in the third and forth column of Table 1.

PROPOSITION 5. *Assume the monetary policy rule (10). For the empirically realistic case, conditional on the REE of interest being stationary it holds that:*

(I) The monetarist solution is E-stable if

$$\chi + \Theta\lambda_2 < (\alpha\beta) \quad \wedge \quad \chi\lambda_1 + \Theta\lambda_2 < (\alpha\beta). \tag{20}$$

(II) The fiscalist solution is E-stable if

$$\frac{(\gamma + 1 - \beta^{-1})\chi}{(\alpha\beta)} < 0 \quad \wedge \tag{21}$$

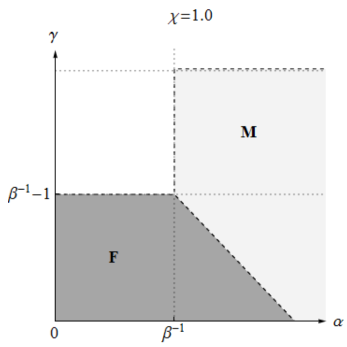
$$\frac{(\beta^{-1} - \gamma)\chi}{(\alpha\beta)} + \frac{\sqrt{\beta^2[\beta\varphi_2 + \beta\varphi_1(\beta^{-1} - \gamma)]^2[(\alpha\beta)^2 - 4\Theta\chi]}}{2\alpha\beta^2[\beta\varphi_2 + \beta\varphi_1(\beta^{-1} - \gamma)]} > \frac{1}{2} \quad \wedge \tag{22}$$

$$\frac{(\beta^{-1} - \gamma)\chi}{(\alpha\beta)} - \frac{\sqrt{\beta^2[\beta\varphi_2 + \beta\varphi_1(\beta^{-1} - \gamma)]^2[(\alpha\beta)^2 - 4\Theta\chi]}}{2\alpha\beta^2[\beta\varphi_2 + \beta\varphi_1(\beta^{-1} - \gamma)]} > \frac{1}{2} \tag{23}$$

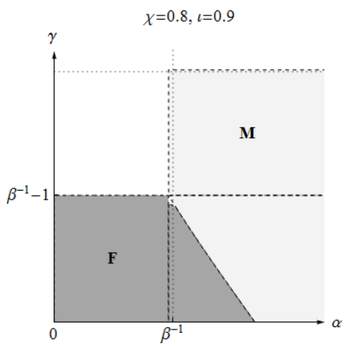
is true for the real parts of the left-hand side.

(III) None of the non-fundamental solutions is E-stable.

Note that for $\chi = 1$ conditions (20) to (23) reduce to the ones in Evans and Honkapohja (2007, p.680). Panels 4a to 4j indicate the E-stability regions for the monetarist and fiscalist solution respectively. Calibration of $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ is discussed in the online appendix. It is worthwhile that the regions cover not only the determinacy regions from Figure 1, but also show that local divergence (compare Panels 4h-4j to 1j-1l) is a plausible outcome under LS learning.



(a)



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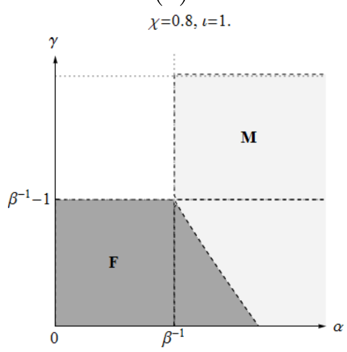
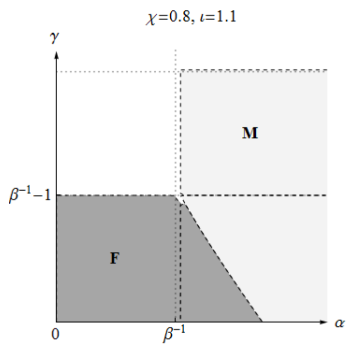


(c)



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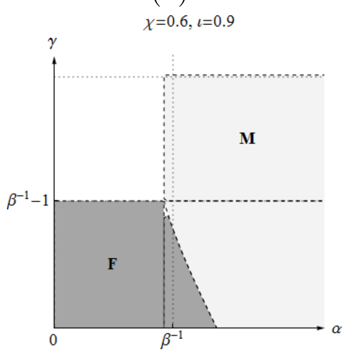
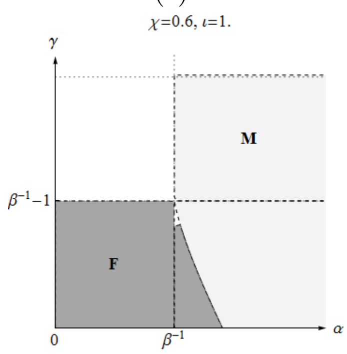
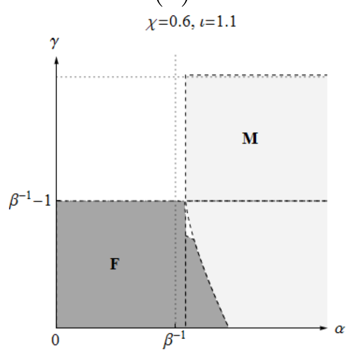


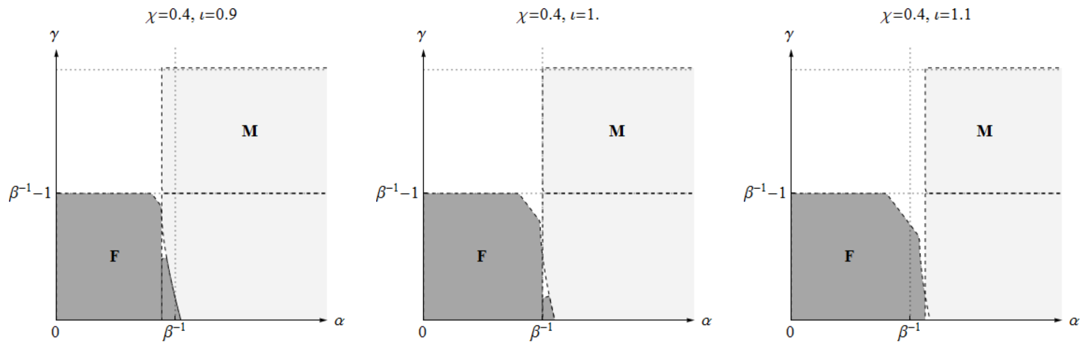
(e)



(f)



(g)



(h) (i) (j)

Figure 4: Regions of local E-stability for monetarist solution (light grey), fiscalist solution (dark grey), and E-instability (remainder) in the empirical relevant α-γ-space, i.e., $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$, for $\chi \in \{0.4, 0.6, 0.8, 1.0\}$, $\iota \in \{0.9, 1.0, 1.1\}$, and $\beta = 0.99$. **M** (**F**) is the monetarist (fiscalist) solution.

Backward-looking rule. Define $v_t \equiv [\theta_t, \psi_t, \eta_t]'$ and rewrite system (D.1)-(D.2) to fit (19). In Appendix G we demonstrate that the proposition below holds.

PROPOSITION 6. *Assume the monetary policy rule (17). For the empirically realistic case, given that the fiscalist solution is stationary, it is also E-stable.*

This is result may be anticipated, as in this particular case the model appears to be correctly specified as $\mathbf{M} = 0$.

Forward-looking rule. System (E.1)-(E.2) can be written in the form of (19). In Appendix H we demonstrate that, due to $\mathbf{M} = 0$, the proposition below holds.

PROPOSITION 7. *Assume the monetary policy rule (18). For the empirically realistic case, given that the fiscalist solution is stationary, it is also E-stable.*

The propositions above focus on the case where solutions are stationary. However, we find it a remarkable result that for all three monetary policy rules, the fiscalist solution in the AF/PMHE regime appears to be E-stable, even when it is explosive due to expectational heterogeneity. Thus, the prediction that the economy under the AF/PMHE eventually diverges from the steady-state due to expectational heterogeneity and that this may be a well-defined equilibrium, is also plausible from an adaptive learning viewpoint. Be aware that under this regime the fiscalist solution is not necessarily 'fragile' (see Evans and Honkapohja, 2007, p.681ff.) in the sense that it is E-stable in the neighbourhood of the steady-state, but asymptotically loses stability under LS learning. The latter is known to be the case under AF/AMHE for $\chi = 1$, but whether or not the non-stationary fiscalist solution under the AF/PMHE is fragile, ultimately needs to be addressed in a global analysis, which is left for future research.

5. CONCLUSIONS

In sum, this paper puts forth a *Neo-Classical* theory of fiscal and monetary policy interaction under heterogeneous expectations. The coexistence of agents with RE and AE gives rise to economic dynamics strikingly different from the homogeneous RE benchmark case.

For plausible assumptions on the parameter space, we show that the monetarist solution can be the unique stationary RE solution in a PF/AMHE regime under a contemporaneous-data interest rate rule. This is true, as the central bank obeys a generalized Taylor (1993)-principle by incorporating knowledge about the heterogeneous nature of private sector expectations. To this extent, even active policy becomes constrained by heterogeneous expectations.

Moreover, we find that an AF/AMHE regime leads to local divergence and a PF/PMHE regime results in local divergence as well, or opens the door to arbitrary large economic fluctuations associated with indeterminacy.

Furthermore, the fiscalist solution, where inflation depends on public debt, can be the unique stationary RE solution, given there is an AF/PMHE regime in place. Nevertheless, under this regime, ultimately the shares of agents with

RE and AE become decisive for stationarity. If the share of agents with RE goes below one half, a value within the empirically relevant range, the fiscalist solution becomes explosive. This stands in sharp contrast to our findings for the monetarist solution. More important, the non-stationary fiscalist solution may be a well-defined equilibrium implying *orthodox* behavior for macroeconomic aggregates.

Remarkably, once we consider more implementable interest rate rules, the fiscalist solution remains the sole possibly stationary solution. The central bank may still have to incorporate private sector expectations, even when it is pursuing a passive policy and active fiscal policy is a necessary condition for a determinate outcome. However, depending on the shares of agents with RE and AE, the fiscalist solution may become stationary. We also demonstrate that all our findings are plausible from an adaptive learning viewpoint.

Overall, these results suggest that heterogeneous private sector expectations constitute a novel challenge to current fiscal and monetary policy arrangements and their ability to successfully stabilize the economy.

We believe that the concern of persistent expectational heterogeneity and bounded rationality in general, and with regard to policy interaction in particular, is of high relevance for academics as well as policy makers. One can view the present paper as a generalized way of addressing this concern. Clearly, our modeling approach aims at analytical results. It is rather stylized, and might neglect important aspects. One exemplary issue is to address nominal rigidities and its implications for policy interaction under heterogeneous expectations. This issue is left to future research.

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A. MODEL DERIVATIONS

Consider the problem of individual household i. We define $W^{\varsigma}_{t+1}(i) \equiv m^{\varsigma}_t(i) + b^{\varsigma}_t(i)$ and $x^{\varsigma}_{t+1}(i) = m^{\varsigma}_t(i)$. Then the household's problem can be solved by the very same Lagrangian as in Evans and Honkapohja (2007), i.e.,

$$\mathcal{L} = E^{\varsigma}_t \Bigg\{ \sum_{t=0}^{\infty} \beta^t \left[(1-\sigma_1)^{-1} c^{\varsigma}_t(i)^{(1-\sigma_1)} + \mathcal{A}(1-\sigma_2)^{-1}(x^{\varsigma}_t(i)\pi_t^{-1})^{(1-\sigma_2)} \right]$$
$$-\beta^{t+1}\mu_{1,t+1}\left[W^{\varsigma}_{t+1}(i) - y + c_t(i) + \tau_t - x^{\varsigma}_t(i)\pi_t^{-1} - R_{t-1}\pi_t^{-1}(W^{\varsigma}_t(i) - x^{\varsigma}_t(i)) \right]$$
$$-\beta^{t+1}\mu_{2,t+1}\left[x^{\varsigma}_{t+1}(i) - m^{\varsigma}_t(i) \right] \Bigg\}. \tag{A.1}$$

This yields the first-order conditions

$$E^{\varsigma}_t\{c^{\varsigma}_t(i)^{-\sigma_1}\} - \beta E^{\varsigma}_t\{\mu_{1,t+1}\} = 0, \tag{A.2}$$

$$E^{\varsigma}_t\{\mu_{2,t+1}\} = 0, \tag{A.3}$$

$$\beta^{-1}R^{-1}_{t-1}E^{\varsigma}_t\{\mu_{1,t}\} = E^{\varsigma}_t\{\mu_{1,t+1}\pi_t^{-1}\}, \tag{A.4}$$

$$E^{\varsigma}_t\{\mu_{2,t}\} = \mathcal{A}E^{\varsigma}_t\{\pi_t^{-1}(x^{\varsigma}_t(i)\pi_t^{-1})^{-\sigma_2}\} + \beta E^{\varsigma}_t\{(\pi_t^{-1} - R_{t-1}\pi_t^{-1})\mu_{1,t+1}\}, \tag{A.5}$$

where we make use of *Assumption A3* (Branch and McGough, 2009, p.1038). Re-arranging terms within (A.5), plugging in (A.4), forwarding the resulting expression, using *Assumption A5* and combining it with (A.2)-(A.3) yields

$$0 = \mathcal{A}E^{\varsigma}_t\{\pi^{\sigma_2-1}_{t+1}m^{\varsigma}_t(i)^{-\sigma_2}\} + (R^{-1}_t - 1)\beta^{-1}E^{\varsigma}_t\{c^{\varsigma}_t(i)^{-\sigma_1}\}. \tag{A.6}$$

If every agent can observe his own period t choices of $c_t^\varsigma(i)$ and $m_t^\varsigma(i)$, and within-type expectations are identical, then in fact individual money demand is

$$0 = \mathcal{A} m_t^\varsigma(i)^{-\sigma_2} E_t^\varsigma\{\pi_{t+1}^{\sigma_2 - 1}\} + (R_t^{-1} - 1)\beta^{-1} c_t^\varsigma(i)^{-\sigma_1}. \tag{A.7}$$

We can use the very same assumption together with (A.2) and (A.4) to derive individual consumption demand

$$c_t^\varsigma(i)^{-\sigma_1} = \beta R_t E_t^\varsigma\{c_{t+1}^\varsigma(i)^{-\sigma_1}\pi_{t+1}^{-1}\}, \tag{A.8}$$

where R_t is set by the central bank and states publicly available information. Clearly, in the non-stochastic steady-state we have $R = \beta^{-1}\pi$. Next we linearize (A.8) at the non-stochastic steady-state. Variables are expressed as deviations from the steady-state, i.e., $\tilde{q}_t \equiv (q_t - q)$ for any variable q_t. Thus, we arrive at

$$\tilde{c}_t^\varsigma(i) = E_t^\varsigma\{\tilde{c}_{t+1}^\varsigma(i)\} - \sigma_1^{-1} c \left(R^{-1}\tilde{R}_t - \pi^{-1} E_t^\varsigma\{\tilde{\pi}_{t+1}\}\right). \tag{A.9}$$

Take into account that all individual agents of the same type will make similar decisions, i.e., $c_t^1(i) = c_t^1$ and $c_t^2(i) = c_t^2$. Therefore we aggregate as follows

$$c_t = \int_0^\chi c_t^1(i)di + \int_\chi^1 c_t^2(i)di = \int_0^\chi c_t^1 di + \int_\chi^1 c_t^2 di = \chi c_t^1 + (1-\chi)c_t^2. \tag{A.10}$$

Next, the agent knows the structure of the economy, so it is natural to assume that $E_t^\varsigma\{\tilde{c}_{t+1}^\varsigma(i)\} = (y - g)$. Together with (A.10) it follows that

$$\tilde{c}_t = (y - g) - \sigma_1^{-1} c \left(R^{-1}\tilde{R}_t - \pi^{-1}\widehat{E}_t\{\tilde{\pi}_{t+1}\}\right) \tag{A.11}$$

Imposing goods market clearing, $\tilde{c}_t = (y - g)$, and *Assumption A1* yields the Fisher relation

$$\tilde{R}_t^{-1} = \beta \widehat{E}_t\{\tilde{\pi}_{t+1}^{-1}\}. \tag{A.12}$$

Linearization of (A.7) and rearranging terms results in

$$\tilde{m}_t^{\varsigma}(i) = \sigma_2^{-1}(\sigma_2 - 1)m\pi^{-1}E_t^{\varsigma}\{\tilde{\pi}_{t+1}^{-1}\} - \sigma_2^{-1}m(R-1)^{-1}R^{-1}\tilde{R}_t + \sigma_2^{-1}\sigma_1 mc^{-1}\tilde{c}_t^{\varsigma}(i). \tag{A.13}$$

The reasoning for (A.10) above also applies to $m_t^{\varsigma}(i)$, thus

$$m_t = \int_0^{\chi} m_t^1(i)di + \int_{\chi}^1 m_t^2(i)di = \int_0^{\chi} m_t^1 di + \int_{\chi}^1 m_t^2 di = \chi m_t^1 + (1-\chi)m_t^2. \tag{A.14}$$

Aggregating (A.13) by the help of (A.14), imposing the Fisher relation, goods market clearing as well as the steady-state relationship $m = \hat{C}((1-\beta\pi^{-1})(\pi^{\sigma_2-1})^{-1})^{-1/\sigma_2}$, where $\hat{C} \equiv (\mathcal{A}\beta)^{1/\sigma_2}(y-g)^{\sigma_1/\sigma_2}$, leads to the money market equilibrium condition

$$\tilde{m}_t = \left[\left(\frac{-\hat{C}\beta}{\sigma_2}\right)(\pi-\beta)^{-(1+\sigma_2)/\sigma_2}\left(\frac{\sigma_2 - 1}{\sigma_2}\right)\hat{C}(\pi-\beta)^{-1/\sigma_2}\right]\widehat{E}_t\{\tilde{\pi}_{t+1}\} + const., \tag{A.15}$$

or, following Evans and Honkapohja (2007, p.688) and ignoring the constant, we can express (A.15) more compact as $\tilde{m}_t = \tilde{C}\widehat{E}_t\{\tilde{\pi}_{t+1}\}$.

B. DETERMINACY CONDITIONS AND LINEAR RESTRICTIONS

By defining $y_t \equiv [\tilde{\pi}_t, \tilde{b}_t, \tilde{\pi}_{t-1}]'$, the system can be rearranged as

$$y_t = \mathbf{J}y_{t+1} + \mathbf{F}_1\eta_{t+1} + \mathbf{F}_2\theta_{t+1} + \mathbf{F}_3\theta_t + \mathbf{F}_4\psi_{t+1}, \quad \text{where} \tag{B.1}$$

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & (\beta^{-1} - \gamma)^{-1} & \frac{((\alpha\beta)\varphi_1 + \varphi_2)}{(\beta^{-1} - \gamma)} \\ -\frac{\chi}{\Theta} & 0 & \frac{(\alpha\beta)}{\Theta} \end{pmatrix} \tag{B.2}$$

is the Jacobian of the system.[24] Note that $\eta_{t+1} = \tilde{\pi}_{t+1} - E_t\tilde{\pi}_{t+1}$ is a *martingale difference sequence* as we assume $E_t\eta_{t+1} = 0$. We also define $\Theta \equiv (1 - \chi)\iota^2$.

System (B.1), given $\breve{y}_t \equiv [x_t, z_t, x_{t-1}]'$, can be rewritten as

$$\breve{y}_t = \mathbf{\Lambda}\breve{y}_{t+1} + \mathbf{Q}^{-1}\left[\mathbf{F}_1\eta_{t+1} + \mathbf{F}_2\theta_{t+1} + \mathbf{F}_3\theta_t + \mathbf{F}_4\psi_{t+1}\right], \tag{B.3}$$

where (B.3) follows from diagonalizing matrix $\mathbf{J}$ in (B.1). Note that $E_t\tilde{\pi}_{t+1} = \tilde{\pi}_{t+1} - \eta_{t+1}$, $\mathbf{J} = (\mathbf{Q\Lambda Q}^{-1})$ is a decomposition of $\mathbf{J}$ into its eigenvalues and its right eigenvector, and $\breve{y}_{t+1} = \mathbf{Q}^{-1}[\tilde{\pi}_{t+1}, \tilde{b}_{t+1}, \tilde{\pi}_t]'$.

The important matrices in (B.3) are given by

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad \text{and} \tag{B.4}$$

[24] Note that information regarding any matrix not reported herein is irrelevant for the analysis and omitted for clarity of exposition. More information is available from the author.

$$\mathbf{Q}^{-1} = \begin{pmatrix} \frac{\beta(\beta\gamma-1)\chi(\alpha\beta\varphi_1+\varphi_2)}{(\alpha(\beta\gamma-1)+\Theta)\beta^2+(\beta\gamma-1)^2\chi} & 1 & \frac{\beta^2\Theta(\alpha\beta\varphi_1+\varphi_2)}{(\alpha(\beta\gamma-1)+\Theta)\beta^2+(\beta\gamma-1)^2\chi} \\ \frac{\chi}{\sqrt{\alpha^2\beta^2-4\Theta\chi}} & 0 & -\frac{\Theta}{\sqrt{\alpha^2\beta^2-4\Theta\chi}}\lambda_2 \\ -\frac{\chi}{\sqrt{\alpha^2\beta^2-4\Theta\chi}} & 0 & \frac{\Theta}{\sqrt{\alpha^2\beta^2-4\Theta\chi}}\lambda_3 \end{pmatrix}, \qquad \text{(B.5)}$$

where $\Theta \equiv (1-\chi)\iota^2$. $\lambda_1 \equiv (\beta^{-1}-\gamma)^{-1}$, $\lambda_2 \equiv \frac{(\alpha\beta)-\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$, and $\lambda_3 \equiv \frac{(\alpha\beta)+\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$ are the eigenvalues of $\mathbf{J}$.

Paralleling the analysis of Evans and Honkapohja (2007), from (B.3), and given $[C_1, C_2, C_3]' = -\mathbf{Q}^{-1}\mathbf{F}_3$ we can figure out three different cases. First, given an AF regime, $|(\beta^{-1}-\gamma)^{-1}| < 1$, stationarity of the solution requires that $E_t x_{t+1} = \lambda_1^{-1}(x_t + C_1\theta_t) = 0$ to rule out that $|E_t x_{t+s}| \to \infty$ as $s \to \infty$. This yields restriction (i) with coefficients

$$\begin{aligned} K_1 &= \left[\frac{\sqrt{(\alpha\beta)^2-4\Theta\chi}(\beta^{-1}-\gamma)(\lambda_1-\lambda_3)(\lambda_1-\lambda_2)}{\chi[(\alpha\beta)\varphi_1+\varphi_2](\lambda_3-\lambda_2)}\right], \\ K_2 &= \left[\frac{\sqrt{(\alpha\beta)^2-4\Theta\chi}(\lambda_1-\lambda_3)(\lambda_1-\lambda_2)}{\chi(\lambda_3-\lambda_2)}\right] \times \\ &\left[\frac{\beta}{[(\alpha\beta)-\lambda_1\Theta-(\beta^{-1}-\gamma)\chi]} - \frac{(\beta\varphi_1+\varphi_4)}{[(\alpha\beta)\varphi_1+\varphi_2]}\right], \quad \text{and} \quad K_3 = \frac{\Theta}{\chi}\lambda_1. \end{aligned}$$

Moreover, in the PF/AMHE regime, where $|(\alpha\beta)| > (\chi+\Theta)$ is true, stationarity of the solution requires that $E_t z_{t+1} = \lambda_2^{-1}(z_t + C_2\theta_t) = 0$ to rule out that $|E_t z_{t+s}| \to \infty$ as $s \to \infty$. Restriction (ii) follows with coefficients $K_1 = 0$, $K_2 = -\chi^{-1}\beta\lambda_2$, and $K_3 = \chi^{-1}\Theta\lambda_2$. Finally, in the PF/PMHE regime, where $|(\alpha\beta)| < (\chi+\Theta)$ is true, stationarity of the solution requires that $x_t = \lambda_3^{-1}(x_{t-1} + C_3\theta_t) = 0$ to rule out that $|x_{t+s}| \to \infty$ as $s \to \infty$. This leads to restriction (iii) with coefficients $K_1 = 0$, $K_2 = -\chi^{-1}\beta\lambda_3$, and $K_3 = \chi^{-1}\Theta\lambda_3$.

C. PROOF OF PROPOSITION 2

Proof. We consider the empirical relevant parameter space to be $\alpha > 0$, $\beta > 0$, and $\chi \in (0, 1]$. Following the arguments in Evans and Honkapohja (2007, p.681), we assume $\beta^{-1} > \gamma \geq 0$. The characteristic polynomial of $\mathbf{J}$ is given by

$$\begin{aligned}\mathcal{P}(\psi) = &-\psi^3 + \left[(\beta^{-1} - \gamma)^{-1} + \Theta^{-1}(\alpha\beta)\right]\psi^2 \\ &- \left[\Theta^{-1}((\alpha\beta)(\beta^{-1} - \gamma)^{-1} + \chi)\right]\psi + \Theta^{-1}\chi(\beta^{-1} - \gamma)^{-1},\end{aligned} \tag{C.1}$$

where it's roots coincide with the eigenvalues λ_1, λ_2, and λ_3. The assumptions on γ above imply that there is at least one real root, λ_1.

Moreover, *Descartes' rule of signs* suggests that there is a maximum of three positive real roots and zero negative real roots. Furthermore note that $\mathcal{P}(-\infty) \to +\infty$, $\mathcal{P}(-1) > 0$, $\mathcal{P}(0) > 0$, and $\mathcal{P}(\infty) \to -\infty$.

Next, with regard to λ_2 and λ_3, if $(\alpha\beta) > (\chi + \Theta)$, then $\mathcal{P}(1) < 0$, and either there is one real root or a pair of complex conjugates with the same modulus inside the unit circle. In case of $(\alpha\beta) < (\chi + \Theta)$, then $\mathcal{P}(1) > 0$, and there is no real root inside the unit circle. However, λ_2 and λ_3 may also form a pair of complex conjugates. In this case their identical modulus can be inside or outside the unit circle. In order to analyze the various possible cases, it is useful to calculate the discriminant of $\mathcal{P}(\psi)$, which is given by

$$\mathcal{D} = \frac{(\alpha^2\beta^2 - 4\Theta\chi)\left[\beta^2(\alpha(\beta\gamma - 1) + \Theta) + \chi(\beta\gamma - 1)^2\right]^2}{\Theta^4(\beta\gamma - 1)^4}. \tag{C.2}$$

According to Irving (2004, p.154), three cases are possible. First, if $\mathcal{D} > 0$, then $\mathcal{P}(\psi)$ has three distinct real roots. Second, if $\mathcal{D} < 0$, then $\mathcal{P}(\psi)$ has one real root and a pair of complex conjugates with identical modulus. We ignore the third case, where $\mathcal{D} = 0$ and $\mathcal{P}(\psi)$ has multiple real roots. One can verify that the sign of $\mathcal{D}$ depends on whether $(\alpha\beta)$ is larger or smaller than $\sqrt{4\chi\Theta}$. Furthermore, note that $(\chi + \Theta) \geq \sqrt{4\chi\Theta}$.

Now, in case of PF, i.e., $\gamma > \beta^{-1} - 1$, the root λ_1 is real and outside the unit circle. Likewise root λ_1 is real and inside the unit circle in case of AF, i.e., $\gamma < \beta^{-1} - 1$. Consequently, in a PF/AMHE regime it follows that $(\alpha\beta) > (\chi + \Theta) \geq \sqrt{4\chi\Theta}$ and there are three distinct real roots, $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$, which yield local determinacy. In contrast, under an AF/AMHE regime there is local divergence from the steady-state as this policy regime yields $|\lambda_1| < 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$.

Next, given a PF/PMHE regime, it is true that, when $(\chi + \Theta) > (\alpha\beta) > \sqrt{4\chi\Theta}$, there are three distinct real roots, $|\lambda_1| > 1$, $|\lambda_2| > 1$, and $|\lambda_3| > 1$ and this results in local indeterminacy. In case of $(\chi + \Theta) \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ there is a pair of complex conjugates, λ_2 and λ_3, with identical modulus. If $\lambda_2\lambda_3 = (\chi/\Theta) < 1$, then their identical modulus is inside the unit circle. If $\lambda_2\lambda_3 = (\chi/\Theta) > 1$, then it is outside the unit circle.

In sum, when $(\chi + \Theta) \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ is true, a PF/PMHE regime leads to local indeterminacy if $(\chi/\Theta) > 1$, as $|\lambda_1|, |\lambda_2|, |\lambda_3| > 1$. And, if $(\chi/\Theta) < 1$ there is local divergence from the steady-state as $|\lambda_1| > 1$, and $|\lambda_2|, |\lambda_3| < 1$.

Finally, for the AF/PMHE regime similar arguments apply. In case of $(\chi + \Theta) > (\alpha\beta) > \sqrt{4\chi\Theta}$, there are three distinct real roots, $|\lambda_1| < 1$, and $|\lambda_2|, |\lambda_3| > 1$ and local determinacy prevails. However, when $(\chi + \Theta) \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ is true, an AF/PMHE regime does only yield local determinacy if $\lambda_2\lambda_3 = (\chi/\Theta) > 1$, but results in local divergence if $\lambda_2\lambda_3 = (\chi/\Theta) < 1$ $\square$

D. PROOF OF PROPOSITION 3

Proof. Following the approach outlined in Subsection 3.1 above, the economy given by a linearized version of (6)-(7), including the policy block (8)-(9), and (17) as well as the expectational set-up (3) to (5) can be expressed by

$$\tilde{\pi}_t = \chi^{-1}\left[(\alpha\beta) - (1-\chi)\iota^2\right]\tilde{\pi}_{t-2} - \chi^{-1}\beta\theta_t + \eta_t \tag{D.1}$$

$$\begin{aligned} 0 &= \tilde{b}_{t+1} + \phi_1\chi E_t\tilde{\pi}_{t+1} + [\phi_1(1-\chi)\iota^2 + \phi_5]\tilde{\pi}_{t-1} + \phi_2\tilde{\pi}_t \\ &- (\beta^{-1}-\gamma)\tilde{b}_t + \psi_{t+1} + \phi_3\theta_{t+1} + \phi_4\theta_t, \end{aligned} \tag{D.2}$$

and the coefficients $\phi_1 = [m\pi^{-2} + Rb\pi^{-2}]$, $\phi_2 = [\tilde{C}\beta\alpha]$, $\phi_3 = \tilde{C}\beta$, $\phi_4 = [-\pi^{-1}\tilde{C}\beta - \pi^{-1}b]$, $\phi_5 = [-\pi^{-1}\tilde{C}\beta\alpha - \pi^{-1}b\alpha]$.

For $y_t \equiv [\tilde{\pi}_t, \tilde{b}_t, \tilde{\pi}_{t-1}, \tilde{\pi}_{t-2}]'$, this yields the following Jacobian

$$\mathbf{J}_{BW} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ \frac{\chi\phi_1}{(\beta^{-1}-\gamma)} & (\beta^{-1}-\gamma)^{-1} & \frac{\phi_2}{(\beta^{-1}-\gamma)} & \frac{(\Theta\phi_1+\phi_5)}{(\beta^{-1}-\gamma)} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\chi}{(\alpha\beta)-\Theta} & 0 \end{pmatrix}. \tag{D.3}$$

Next, a similar decomposition as in Appendix B above, $\mathbf{J}_{BW} = (\mathbf{Q}_{BW}\mathbf{\Lambda}_{BW}\mathbf{Q}_{BW}^{-1})$, yields $\mathbf{\Lambda}_{BW} = \text{diag}(\lambda_0, \lambda_1, \lambda_2, \lambda_3)$, where the eigenvalues of $\mathbf{J}_{BW}$ are now given by $\lambda_0 \equiv 0$, $\lambda_1 \equiv (\beta^{-1}-\gamma)^{-1}$, $\lambda_2 \equiv -\frac{\sqrt{\chi}}{\sqrt{(\alpha\beta)-\Theta}}$, and $\lambda_3 \equiv \frac{\sqrt{\chi}}{\sqrt{(\alpha\beta)-\Theta}}$.

Next, consider the characteristic polynomial of $\mathbf{J}_{BW}$ given by

$$\mathcal{P}_{BW}(\psi) = \psi^4 - (\beta^{-1}-\gamma)^{-1}\psi^3 - \frac{\chi}{[(\alpha\beta)-\Theta]}\psi^2 + \frac{\chi}{(\beta^{-1}-\gamma)[(\alpha\beta)-\Theta]}\psi, \tag{D.4}$$

where it's roots coincide with the eigenvalues λ_0, λ_1, λ_2, and λ_3. λ_0 is a real root, and, due to the assumptions from above, λ_1 is so too.

Moreover, *Descartes' rule of signs* suggests that there is a maximum of three positive real roots and one (zero) negative real root if $(\alpha\beta) > (<)\Theta$.

The discriminant of $\mathcal{P}_{BW}(\psi)$, can be computed as

$$\mathcal{D}_{BW} = \frac{4\chi^3\left[\beta^3(\Theta - (\alpha\beta)) + \beta(\beta\gamma - \Theta)^2\chi\right]^2}{(\beta\gamma - 1)^6((\alpha\beta) - \Theta)^5}. \tag{D.5}$$

If $\chi = 0$, then $\mathcal{D}_{BW} = 0$, and three of the four roots of $\mathcal{P}_{BW}(\psi)$ are equal to zero. Moreover, with assumptions $\beta \neq 0$ and $\chi \in (0,1]$, we can rule out $\mathcal{D}_{BW} = 0$. Next, as discussed in Irving (2004, p.167), if $\mathcal{D}_{BW} > 0$, then $\mathcal{P}_{BW}(\psi)$ has four distinct real roots. In contrast, for $\mathcal{D}_{BW} < 0$, $\mathcal{P}_{BW}(\psi)$ has two distinct real roots and a pair of complex conjugates. It can be shown that $\mathcal{D}_{BW} > 0$ if $(\alpha\beta) > \Theta$, and that $\mathcal{D}_{BW} < 0$ if $(\alpha\beta) < \Theta$.

In case of the PF/AMHE regime, $(\alpha\beta) > (\chi + \Theta) \geq \Theta$ there are four distinct real roots, $|\lambda_0| < 1$ $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| < 1$, which implies local divergence.

Next, it is straightforward that the AF/AMHE regime, has similar implications as there are four distinct real roots, $|\lambda_0|, |\lambda_1|, |\lambda_2|, |\lambda_3| < 1$.

For the PF/PMHE regime, one can verify that, if $(\chi + \Theta) > (\alpha\beta) > \Theta$, there exist four distinct real roots $|\lambda_0| < 1$, and $|\lambda_1|, |\lambda_2|, |\lambda_3| > 1$, which renders the economy locally indeterminate. When $(\chi + \Theta) \geq \Theta > (\alpha\beta)$, the roots λ_2 and λ_3 form a pair of complex conjugates with identical modulus $\lambda_2\lambda_3 = \chi/[\Theta - (\alpha\beta)]$. It follows that if $(\alpha\beta) > (\Theta - \chi)$, then $\lambda_2\lambda_3 > 1$, and if $(\alpha\beta) < (\Theta - \chi)$, then $\lambda_2\lambda_3 < 1$. Thus, when $(\chi + \Theta) > (\alpha\beta) > \Theta$, the PF/PMHE regime yields local indeterminacy if $(\alpha\beta) > (\Theta - \chi)$, as $|\lambda_0| < 1$ and $|\lambda_1|, |\lambda_2|, |\lambda_3| > 1$. However, if $(\alpha\beta) < (\Theta - \chi)$, then $|\lambda_0|, |\lambda_2|, |\lambda_3| < 1$ and $|\lambda_1| > 1$ and there is local divergence.

For the AF/PMHE regime an equivalent reasoning can be used. For $(\chi + \Theta) >$

$(\alpha\beta) > \Theta$ there are four distinct real roots and it follows that $|\lambda_0|, |\lambda_1| < 1$ and $|\lambda_2|, |\lambda_3| > 1$, which yields local determinacy. The same is true if $(\chi + \Theta) \geq \Theta > (\alpha\beta)$ and at the same time $(\alpha\beta) > (\Theta - \chi)$. But once $(\alpha\beta) < (\Theta - \chi)$ the result is $|\lambda_0|, |\lambda_1|, |\lambda_2|, |\lambda_3| < 1$, which implies local divergence. $\square$

E. PROOF OF PROPOSITION 4

Proof. Following the approach outlined in Appendix D while utilizing rule (18) we can derive the following dynamical system

$$\tilde{\pi}_t = -\chi^{-1}(1-\chi)\iota^2\tilde{\pi}_{t-2} + \chi^{-1}\frac{\beta}{[1-(\alpha\beta)]}\theta_{t-1} + \eta_t \tag{E.1}$$

$$\begin{aligned} 0 = {} & \tilde{b}_{t+1} + \chi[\phi_1 + \phi_7]E_t\tilde{\pi}_{t+1} + (1-\chi)\iota^2[\phi_1 + \phi_5]\tilde{\pi}_{t-1} + (1-\chi)\iota^2\phi_2\tilde{\pi}_t \\ & - (\beta^{-1} - \gamma)\tilde{b}_t + \chi\phi_6 E_t\tilde{\pi}_{t+2} + \psi_{t+1} + \phi_3\theta_{t+1} + \phi_4\theta_t, \end{aligned} \tag{E.2}$$

with coefficients $\phi_1 = [m\pi^{-2} + Rb\pi^{-2}]$, $\phi_2 = [\tilde{C}\beta\alpha]$, $\phi_3 = \tilde{C}\beta$, $\phi_4 = [-\pi^{-1}\tilde{C}\beta - \pi^{-1}b]$, $\phi_5 = [-\pi^{-1}\tilde{C}\beta\alpha - \pi^{-1}b\alpha]$, $\phi_6 = \phi_2$, $\phi_7 = \phi_5$.

This yields the following Jacobian

$$\mathbf{J}_{FW} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{\chi\phi_6}{(\beta^{-1}-\gamma)} & \frac{\chi(\phi_1+\phi_7)}{(\beta^{-1}-\gamma)} & (\beta^{-1}-\gamma)^{-1} & \frac{\phi_2}{(\beta^{-1}-\gamma)} & \frac{\Theta(\phi_1+\phi_5)}{(\beta^{-1}-\gamma)} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{\chi}{(\alpha\beta)-\Theta} & 0 \end{pmatrix}, \tag{E.3}$$

where $y_t \equiv [\tilde{\pi}_{t+1}, \tilde{\pi}_t, \tilde{b}_t, \tilde{\pi}_{t-1}, \tilde{\pi}_{t-2}]'$. Again, a similar decomposition as in Appendix B above, $\mathbf{J}_{FW} = (\mathbf{Q}_{FW}\mathbf{\Lambda}_{FW}\mathbf{Q}_{FW}^{-1})$, yields $\mathbf{\Lambda}_{FW} = \text{diag}(\lambda_0, \lambda_{00}, \lambda_1, \lambda_2, \lambda_3)$, where the eigenvalues of $\mathbf{J}_{FW}$ are now given by $\lambda_{00} = \lambda_0 \equiv 0$, $\lambda_1 \equiv (\beta^{-1} - \gamma)^{-1}$, $\lambda_2 \equiv -\frac{i\sqrt{\chi}}{\sqrt{\Theta}}$, and $\lambda_3 \equiv \frac{i\sqrt{\chi}}{\sqrt{\Theta}}$.

Next, consider the characteristic polynomial of $\mathbf{J}_{FW}$ is given by

$$\mathcal{P}_{FW}(\psi) = -\psi^5 + (\beta^{-1} - \gamma)^{-1}\psi^4 - \frac{\chi}{\Theta}\psi^3 + \frac{\chi}{\Theta}(\beta^{-1} - \gamma)^{-1}\psi^2, \tag{E.4}$$

where it's roots coincide with the eigenvalues λ_{00}, λ_0, λ_1, λ_2, and λ_3. λ_{00} and λ_0 are real roots, and, due to the assumptions from above, λ_1 is so too. Clearly λ_2 and λ_3 are a pair of complex conjugates with identical modulus $\lambda_2\lambda_3 = (\chi/\Theta)$. If $\lambda_2\lambda_3 = (\chi/\Theta) < 1$, then their identical modulus is inside the unit circle. If $\lambda_2\lambda_3 = (\chi/\Theta) > 1$, then it is outside the unit circle.

Descartes' rule of signs implies that there is a maximum of three positive real roots and zero negative real roots. Next, the discriminant of $\mathcal{P}_{FW}(\psi)$, can be computed as $\mathcal{D}_{FW} = 0$, which confirms the multiplicity, see Irving (2004, p.173).

Now, inspection of the eigenvalues makes clear that monetary policy does not affect the eigenvalues. In addition, under the PF regime, there are three distinct real roots, $|\lambda_{00}|, |\lambda_0| < 1$, and $|\lambda_1| > 1$. For sufficiently large χ, local indeterminacy follows as $\lambda_2\lambda_3 = (\chi/\Theta) > 1$, otherwise there is local divergence.

In contrast, the AF regime generates three distinct real roots, $|\lambda_{00}|, |\lambda_0|, |\lambda_1| < 1$, and $|\lambda_1| > 1$, and, for sufficiently large χ, local determinacy, as $\lambda_2\lambda_3 = (\chi/\Theta) > 1$. Otherwise there is local divergence from the steady-state. $\square$

F. PROOF OF PROPOSITION 5

Proof. As shown in Evans and Honkapohja (2007, p.689ff.), the E-stability conditions are given by

$$D_A T_A(\bar{A}, \bar{B}) = M(I + \bar{B}), \tag{F.1}$$

$$D_B T_B(\bar{B}) = \bar{B}' \otimes M + (I \otimes M\bar{B}), \tag{F.2}$$

$$D_C T_C(\bar{B}, \bar{C}, \bar{D}) = (I \otimes M\bar{B}), \tag{F.3}$$

$$D_D T_D(\bar{B}, \bar{D}) = (I \otimes M\bar{B}), \tag{F.4}$$

where $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ characterize the REE of interest. For a REE to be locally stable under LS learning, the real parts of all eigenvalues of matrices (F.1) to (F.4) have to be less than one.

We will restrict attention to the empirical realistic parameter space for the various stationary solutions from Proposition 1. Note that we rather sketch the proof and will not report matrices (F.1) to (F.4) for the individual cases due to space constraints. Mathematica routines for the details are available.

1. For the monetarist solution (III) from Proposition 1 the non-zero eigenvalues of matrices (F.1) to (F.4) are given by $\{\frac{\chi}{(\alpha\beta)} + \frac{\Theta\lambda_2}{(\alpha\beta)}, \frac{\Theta\lambda_2}{(\alpha\beta)}, \frac{2\Theta\lambda_2}{(\alpha\beta)}, \frac{\chi\lambda_1^{-1}}{(\alpha\beta)} + \frac{\Theta\lambda_2}{(\alpha\beta)}\}$. For AMHE it holds that $(\alpha\beta) > (\chi + \Theta) \geq \sqrt{4\Theta\chi}$ and therefore the solution is E-stable if the reported conditions are satisfied.
2. The continuum of non-fundamental solutions (VI) is not E-stable, as the non-zero eigenvalues of $D_C T_C$ and $D_D T_D$ are equal to unity. Alternatively, it can be shown that the real part of at least one eigenvalue of $D_B T_B$ is larger or equal to $3/2$.
3. For the fiscalist solution (I) the non-zero eigenvalues of matrices (F.1) to (F.4) are given by $\{1 - \frac{(\beta^{-1}-1-\gamma)\chi}{(\alpha\beta)}, 1 - \frac{(\beta^{-1}-\gamma)\chi}{(\alpha\beta)}\}$, and $\{\frac{3}{2} - \frac{(\beta^{-1}-\gamma)\chi}{(\alpha\beta)} - \frac{\sqrt{\beta^2[\beta\varphi_2+\beta\varphi_1(\beta^{-1}-\gamma)]^2[(\alpha\beta)^2-4\Theta\chi]}}{2\alpha\beta^2[\beta\varphi_2+\beta\varphi_1(\beta^{-1}-\gamma)]}, \frac{3}{2} - \frac{(\beta^{-1}-\gamma)\chi}{(\alpha\beta)} + \frac{\sqrt{\beta^2[\beta\varphi_2+\beta\varphi_1(\beta^{-1}-\gamma)]^2[(\alpha\beta)^2-4\Theta\chi]}}{2\alpha\beta^2[\beta\varphi_2+\beta\varphi_1(\beta^{-1}-\gamma)]}\}$. Thus, the solution can only be E-stable for the conditions given.

□

G. PROOF OF PROPOSITION 6

Proof. For the empirical realistic parameter space and monetary policy rule (17) it can be verified that for the fiscalist solution in Proposition 3, characterized by $\mathbf{M} = 0$, $\mathbf{B} = \{\{0,0,0\}, \{-\varphi_2, (\beta^{-1} - \gamma), -\frac{\alpha\beta\varphi_1 - \Theta\varphi_1 + \varphi_5\chi}{\chi}\}, \{1,0,0\}\}$, and some matrices $\bar{A}, \bar{C}, \bar{D}$, the eigenvalues of matrices (F.1) to (F.4) are all zero. $\square$

H. PROOF OF PROPOSITION 7

Proof. Under monetary policy rule (18) one can show for the empirical realistic parameter space that the fiscalist solution from Proposition 4, characterized by $\mathbf{M} = \{\{0,0,0\}, \{\chi\varphi_6, 0, 0\}, \{0,0,0\}\}$, $\mathbf{B} = \{\{0,0,-\chi^{-1}\Theta\}, \{0, (\beta^{-1} - \gamma), \Theta\left(\chi^{-1}\varphi_1 - \varphi_5 + \varphi_7\right)\}, \{0,0,0\}\}$, and some matrices $\bar{A}, \bar{C}, \bar{D}$, yields eigenvalues of matrices (F.1) to (F.4) all equal to zero. $\square$