Efficient Frequency-Domain Detection for Massive MIMO Systems

Lorenzo Cabral[(1)], Daniel Fernandes[(1)], Francisco Cercas[(1,2)], Rui Dinis[(2,3)]
[(1)] ISCTE-IUL, Av. das Forças Armadas, Lisbon, Portugal
[(2)] Instituto de Telecomunicações, Av. Rovisco Pais, 1, Lisbon, Portugal
[(3)] FCT-UNL, Campus de Caparica, Caparica, Portugal

***Abstract*—Reduced-complexity implementations are critical for massive MIMO (Multiple Input, Multiple Output) systems. In this paper we consider the uplink of broadband massive MIMO systems employing SC-FDE (Single-Carrier with Frequency-Domain Equalization) schemes, where multiple users transmit to a single base station with a large number of antennas. We propose low-complexity frequency-domain detection schemes that allow excellent performance, but do not require matrix inversions.**

***Index Terms*—massive MIMO, 5G, SC-FDE, IB-DFE, ZF, MRC, EGC**

I. Introduction

Future 5G systems are expected to provide a huge increase in the user bit rate and overall system spectral efficiency when compared with current systems [1], [2]. MIMO (Multiple Input, Multiple Output) techniques allow multiple data in the same physical channel, which leads to significant gains in the system's spectral efficiency (in theory, upper-bonded by the minimum between the number of transmit and receive antennas, although in practical implementations it can be lower due to correlations between antennas) [3], [4].

Massive MIMO systems push the MIMO concept even further, involving tens or even hundreds of antennas, and allowing huge capacity gains. Therefore, massive MIMO schemes are expected to be central elements of the future 5G systems. However, the implementation complexity can be prohibitively high, namely in terms of signal processing requirements. Therefore, massive MIMO cannot be regarded simply as a scaled version of conventional MIMO schemes. In fact, it is desirable to have massive MIMO schemes with low complexity implementations [5], [6].

The multipath propagation effects set additional difficulties for broadband systems, since the channel becomes severely time-dispersive. SC-FDE (Single-Carrier with Frequency-Domain Equalization) is recognized as a promising technique for the uplink transmission since the frequency-domain receiver implementation makes it suitable for severely time-dispersive channels and the transmitted signals have much lower envelope fluctuations than OFDM (Orthogonal Frequency Division Multiplexing) signals, allowing an efficient power amplification [7], [8]. Iterative frequency-domain receivers such as the IB-DFE (Iterative Block Decision-Feedback Equalizer) [9], [10], [11] allow further performance improvements, which can be close to the MFB (Matched Filter Bound) [12]. These techniques were already shown to allow excellent performance in MIMO systems [11]. However, the complexity associated to conventional MIMO FDE receivers in general and IB-DFE receivers in particular can be very high for large MIMO systems. This is mainly due to the need to invert large matrices.

In this paper we consider the uplink transmission of wireless systems employing SC-FDE techniques and massive MIMO schemes. We consider multiple mobile terminals transmitting simultaneously to a single base station with a large number of antennas. We propose iterative frequency-domain receivers that do not require matrix inversions.

This paper is organized as follows: We start in Section II with the description of the adopted system, following to Section III where each subsection presents one of the receiver designs, beginning with Zero Forcing and ending with the EGC-based Iterative Receiver. The performance results are presented in Section IV and section V concludes this paper.

Throughout this paper we employ the following notation: matrices are denoted by upper-case, bold, non-italic letters and $\mathbb{E}(x)$ denotes the expectation of x. The transpose and Hermitian (conjugated transpose) of the matrix $\mathbf{X}$ are $\mathbf{X}^T$ and $\mathbf{X}^H$, respectively. The $N \times N$ identity matrix is $\mathbf{I}_N$.

II. System characterization

In this work we consider an uplink scenario where a Base Station (BS) equipped with R receive antennas is receiving the signals from T Mobile Terminals (MTs), as illustrated in Fig. 1 . Without loss of generality, we assume that each MT has a single antenna (the generalization for the case with multiple-antenna transmitters is straightforward) and the number of receive antennas is much higher than the number of transmit antennas (i.e., $R >> T$). Since $R >> 1$ and $T > 1$ this can be regarded as a massive MIMO scenario, at least at the receiver side.

An SC-FDE scheme is employed and we assume perfect synchronization and perfect channel estimation at the receiver side. No channel information is required at the transmitters, but we assume that the blocks transmitted by each MT arrive perfectly aligned at the BS (in practice, this means some kind of time advance mechanism is required by the MTs, although residual timing errors can be absorbed by the cyclic prefix). The transmitted block associated to the t^{th} transmitter (i.e., the t^{th} MT, $t = 1, 2, ..., T$) is $\{x_{n,t}; n = 0, 1, ..., N-1\}$, with N denoting the block size, common to all MTs, and $x_{n,t}$ is

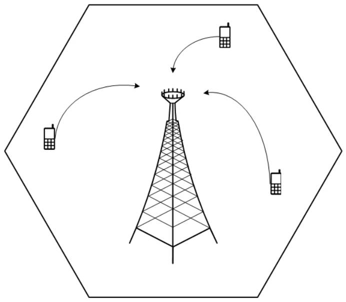

Fig. 1. Adopted uplink scenario.

selected from a given constellation. In this paper we consider QPSK modulation and Gray mapping. The corresponding frequency-domain block, i.e., its size-N DFT (Discrete Fourier Transform) is $\{X_{k,t}; k = 0, 1, ..., N-1\}$.

As with other prefix-assisted block transmission techniques, a cyclic prefix longer than the maximum overall channel impulse response is appended to each block before being transmitted trough a MIMO multipath channel. The received signal at the r^{th} receive antenna, $r = 1, 2, ..., R$, is sampled, the cyclic prefix is removed and passed to the frequency-domain by a DFT operation leading to the block $\{Y_{k,r}; k = 0, 1, ..., N-1\}$. In matrix format, the signal associated to the k^{th} subcarrier is

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{N}_k, \tag{1}$$

where $\mathbf{Y}_k$ is a size-R column vector with r^{th} element given by $Y_{k,r}$, $\mathbf{H}_k$ is the $R \times T$ channel matrix associated to the k^{th} subcarrier, $\mathbf{X}_k$ is a size-T column vector with t^{th} element given by $X_{k,t}$ and $\mathbf{N}_k$ denotes the channel noise, assumed white and Gaussian, with one-sided PSD (Power Spectral Density) N_0 and uncorrelated for different subcarriers and different antennas, i.e., $\mathbb{E}[\mathbf{N}_k \mathbf{N}_k^H] = N_0 \mathbf{I}_R$.

III. Receivers Design

Along this section we will present some receivers we can use with massive MIMO schemes. Two techniques that use matrix inversion including IB-DFE are first presented. As we know an IB-DFE receiver doesn't require the channel decoder output at the feedback loop which allows one to obtain high performance [13]. The use of matrix inversion make these techniques very complex to use with massive MIMO, consequently we explain two other techniques, that don't use matrix inversion. These four techniques are explained in detail in the following subsections.

A. Zero Forcing

The structure of the receiver that we are going to study is shown in Fig. 2. The signal received is then sampled at the receiver and the samples contained by the Cyclic Prefix (CP) are eliminated, leading to the time-domain the samples $\{y_n^{(r)}; n = 0, ..., N-1\}$. Then a size-N DFT results the corresponding frequency-domain block $\{\mathbf{Y}_k^{(r)}$; k=0, 1,...,N-1$\}$, with $\mathbf{Y}_k^{(r)}$ given by [14]

$$\mathbf{Y}_k^{(r)} = \left[\mathbf{Y}_k^{(1)}, ..., \mathbf{Y}_k^{(R)}\right]^T = \mathbf{S}_k \mathbf{H}_k + \mathbf{N}_k, \tag{2}$$

With $\mathbf{H}_k$ denoting the $\mathbf{R} \times \mathbf{T}$ channel matrix for k^{th} $(k = 0, 1, ..., N-1)$ frequency with *(r,t)*th element $\mathbf{H}_k^{(r,t)}$, $\mathbf{S}_k = \left[\mathbf{S}_k^{(1)}, .., \mathbf{S}_k^{(T)}\right]^T$ and $\mathbf{N}_k$ representing the frequency-domain term of the channel noise.The corresponding equalized samples are given by

$$\tilde{\mathbf{S}}_k = \mathbf{F}_k^T \mathbf{Y}_k \tag{3}$$

It can be demonstrated that $\left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \times \mathbf{H} = \mathbf{I}$. This is the well-known Zero Forcing (ZF) criterion [15]. Therefore, after the equalizer and for a linear ZF-based receiver, the data symbols can be obtained from the IDFT of the block $\{\tilde{\mathbf{S}}_k^{(l)}; k = 0, 1, ..., N-1\}$, where

$$\tilde{\mathbf{S}}_k = \left[\tilde{\mathbf{S}}_k^{(1)}, .., \tilde{\mathbf{S}}_k^{(R)}\right]^T = \left(\mathbf{H}_k^H \mathbf{H}_k\right)^{-1} \mathbf{H}_k \times \mathbf{Y}_k \tag{4}$$

Under the ZF criterion, the channel is fully inverted, resulting in a perfect equalized channel after the FDE. In the presence of channel noise, this inversion causes significant enhancement of the channel noise at subchannels with local deep notches, resulting in a higher Signal-to-Noise Ratio (SNR) reduction. However, in the absence of channel noise, this perfect inversion leads to exact values of the samples and avoids the last situation (see details in [16]).

B. IB-DFE Receivers

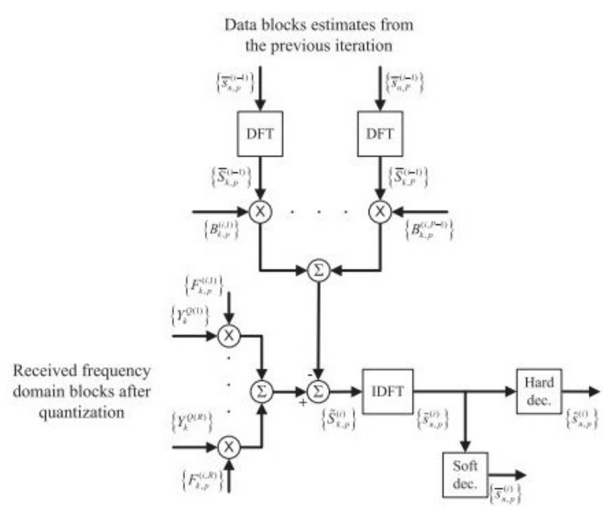

Fig. 2. IB-DFE receiver structure with N_{Rx}-branch space diversity.

The need to solve a system of *R* equations for every frequency of each user and each, which includes a FFT/IFFT pair, at each iteration, severely conditionates the complexity of these receivers. For the i^{th} iteration, the estimated symbols associated with the p^{th} MT $\{\hat{s}_{n,p}; n = 0, 1, ..., N-1\}$ are the hard decisions at the output of the time-domain detector $\{\tilde{s}_{n,p}; n = 0, 1, ..., N-1\} = IDFT\{\tilde{S}_{k,p}; k = 0, 1, ..., N-1\}$, where $\tilde{S}_{k,p}$ is given by:

$$\tilde{\mathbf{S}}_{k,p} = \mathbf{F}_{k,p}^{T}\mathbf{Y}_{k}^{Q} - \mathbf{B}_{k,p}^{T}\bar{\mathbf{S}}_{k,p}; \tag{5}$$

with $\mathbf{F}_{k,p}^{T} = \left[F_{k,p}^{(1)}, ..., F_{k,p}^{(R)}\right]^{T}$ and $\mathbf{B}_{k,p}^{T} = \left[B_{k,p}^{(1)}, ..., B_{k,p}^{(P)}\right]^{T}$ represents the feedforward and feedback coefficients, respectively. Vector $\bar{\mathbf{S}}_{k,p}$ denotes the average values conditioned at output detector for user p of a given iteration and it can be calculated as in [16], [17].

The receiver is characterized by $\mathbf{F}_{k,p}$ and $\mathbf{B}_{k,p}$ (k=0,1,...,N-1) coefficients, for a given iteration and detection of the p^{th} MT. These coefficients are selected in order to minimize the MSE, given by:

$$\Theta_{k,p} = \mathbb{E}\left[|\tilde{S}_{k,p} - S_{k,p}|^{2}\right] = \mathbb{E}\left[|\mathbf{F}_{k,p}^{T}\mathbf{Y}_{k}^{Q} - \mathbf{B}_{k,p}^{T}\bar{\mathbf{S}}_{k,p} - S_{k,p}|^{2}\right]$$

as can be consulted in [18]. It can be demonstrated that the optimum values of $\mathbf{F}_{k,p}$ and $\mathbf{B}_{k,p}$ are [17]:

$$\mathbf{F} = \kappa\Lambda\mathbf{H}^{H}\mathbf{e}_{p}, \tag{6}$$

and

$$\mathbf{B} = \alpha\mathbf{H}\mathbf{F} - \mathbf{e}_{p}, \tag{7}$$

with

$$\Lambda = \left(\mathbf{H}^{H}\left(\mathbf{I}_{P} - \mathbf{P}^{2}\right)\mathbf{H} + \mathbf{R}_{N^{Tot}}\mathbf{R}_{S}^{-1}|\alpha|^{-2}\right)^{-1}, \tag{8}$$

where κ is selected so that $\gamma_{p} = 1$, in order to obtain a normalized FDE with $\mathbb{E}\left[\tilde{s}_{n,p}\right] = s_{n,p}$. $\mathbf{R}_{S} = \mathbb{E}\left[\mathbf{S}^{*}\mathbf{S}^{T}\right] = 2\sigma_{S}^{2}\mathbf{I}_{P}$ and $\mathbf{R}_{N^{Tot}} = \mathbb{E}\left[\mathbf{N}^{\mathbf{Tot}^{*}}\mathbf{N}^{\mathbf{Tot}^{T}}\right] = |\alpha|^{2}\mathbf{R}_{N} + \mathbf{R}_{D}$, corresponding to the correlation matrices of $\mathbf{S}$ and $\mathbf{N}^{\mathbf{Tot}}$, respectively. $\mathbf{R}_{N} = 2\sigma_{N}^{2}\mathbf{I}_{R}$ and $\mathbf{R}_{D} = 2diag\left(\sigma_{D}^{(1)^{2}}, \sigma_{D}^{(2)^{2}}, ...\sigma_{D}^{(R)^{2}}\right)$ are the correlation matrices of the channel and quantization noise, respectively. σ_{S}^{2} and σ_{N}^{2} represent the symbol's variance and noise's variance, respectively with $\mathbf{P} = diag(\rho_{1}, ..., \rho_{P})$, where ρ_{p} denotes the correlation coefficients and represents a measure of the estimates reliability associated with the i^{th} iteration. It is given by:

$$\rho_{p} = \frac{\mathbb{E}\left[\hat{s}_{n,p}s_{n,p}^{*}\right]}{\mathbb{E}\left[|s_{n,p}|^{2}\right]} \tag{9}$$

and can be calculated as in [16].

C. MRC-based Iterative Receiver

Massive MIMO usually needs high dimension matrices that need to be inverted, which presents a heavy computational burden. To overcome this situation and to develop a lower complexity receiver we perform the Maximal-Ratio Combining (MRC) of the signals associated with each receiver antenna. This is a technique to combine the signals from multiple diversity branches. MRC performs the synchronization of the receiver signals and each one is multiplied by a weight factor proportional to the signal amplitude, thus offering the optimum value of SNR. The motivation behind this approach is that, $\mathbf{H}^{H}\mathbf{H} \approx k\mathbf{I}$, where $\mathbf{I}$ is an identity matrix and k a constant. For massive MIMO systems with $R >> 1$ and low correlation between the channels in receiver and transmitter antennas, the elements out of the main diagonal of the matrix, that is,

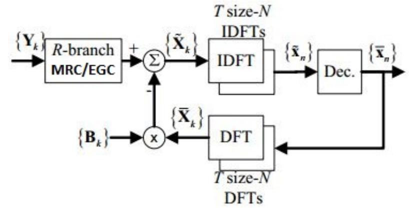

Fig. 3. IB-DFE receiver employed MRC/EGC.

$$\mathbf{F}_{k}^{H}\mathbf{H}_{k} \tag{10}$$

are much lower than those in the diagonal, where the element $(i,i')^{th}$ of the $\mathbf{F}$ matrix is $[\mathbf{F}]_{i,i'} = [\mathbf{H}]_{i,i'}$ and $\mathbf{H}_{k}$ denotes the $\mathbf{R}\times\mathbf{T}$ channel matrix for k^{th} frequency [19]. Notwithstanding, by employing a frequency-domain receiver with MRC for each frequency, based in $\mathbf{F}_{k}^{H}\mathbf{Y}_{k}$, the residual interference levels remains substantial, specially for moderate values of $\mathbf{R}/\mathbf{T}$. Therefore and in order to reduce this interference, we propose the interactive receiver depicted in Fig. 3, where

$$\bar{\mathbf{S}}_{k} = \Psi\mathbf{F}_{k}^{H}\mathbf{Y}_{k} - \mathbf{B}_{k}\overline{\mathbf{X}}_{k}, \tag{11}$$

with Ψ denoting the diagonal matrix where the $(t,t')^{th}$ element is given by $(\sum_{k=0}^{N-1}\sum_{r=1}^{R}|\mathbf{H}_{k}^{(r,t)}|^{2})^{-1}$. This parameter normalization is appropriate to guarantee that the overall frequency-response of the "channel plus receiver" for each MT has average 1 [9], [11]. The matrix $\mathbf{B}_{k}$ is used to reduce the residual ISI and inter-user interference. Clearly, their optimum values are

$$\mathbf{B}_{k} = \Psi\mathbf{F}_{k}^{H}\mathbf{H}_{k} - \mathbf{I} \tag{12}$$

This cancellation of the interference is made by $\overline{\mathbf{S}}_{k} = [\overline{\mathbf{S}}_{0}...\overline{\mathbf{S}}_{N-1}]$, where $\overline{\mathbf{S}}_{k}$ denotes the frequency-domain average values conditioned to the FDE's output at each preceding iteration, which can be computed as referred in [16]. For the first iteration we have not information about the transmitted symbols $\overline{\mathbf{S}}_{k} = 0$, which means our receiver can be regarded as the simple frequency-domain MRC of the signals associated to different receiver antennas. For the next iteration the average values, conditioned to the receiver output at preceding iteration, will be used to mitigate the residual ISI (Inter-Symbol Interference) and the inter-user interference. In general, for a moderate to high value of SNR (signal-to-noise ratio), the average values conditioned to the receiver output approach the transmitted signals as we increase the number of the iterations, which means that the cancellation of the interference made by $\mathbf{B}_{k}$ becomes more efficient and the performance improves. Furthermore, if the average values conditioned to the receiver output can be regarded as *"soft decisions"* [16], the error propagation effect in our receiver will be significantly reduced.

D. EGC-based Iterative Receiver

Within the context of the receivers that don't need matrix inversion operations, we performed the Equal Gain Combining (EGC) of the signals associated to the different receiver antennas. In EGC-based receiver, each signal branch is weighted with the same factor, regardless of the signal amplitude. Moreover, this is simpler to implement than MRC since no controller amplifiers/attenuators and channel estimation are needed. The motivation for this technique lies on the fact that, for a massive MIMO system with $R >> 1$, it can be demonstrated that $\exp\left(\mathbf{j} + \arg\left(\mathbf{H}^H\right)\right) \times \mathbf{H} \approx k\mathbf{I}$ with k designating a constant and $\mathbf{I}$ a identity matrix. As for MRC, an EGC-based receiver the elements out of the main diagonal of the matrix in $\mathbf{F}_k^H \mathbf{H}_k$ are much lower than the ones in the diagonal, so the matrix $\mathbf{F}$ becomes:

$$[\mathbf{F}]_{i,i'} = \exp\left(\mathbf{j} \arg\left([\mathbf{H}]_{i,i'}\right)\right) \tag{13}$$

Once again, we aim to cancel the interference's in the elements out of the main diagonal. For this reason, we implement the iterative EGC receiver depicted in Fig. 3, where the $\bar{\mathbf{S}}_k$ samples are given by (11) and the new $\mathbf{B}_k$ by (12). The interference's cancellation is made by the $\overline{\mathbf{S}}$ coefficients.

IV. Results

This section presents a set of performance results concerning the receiver design proposed in this paper. We consider the uplink of a massive MIMO system with $T = 4$ single-antenna transmitters unless otherwise stated, each one employing an SC-FDE modulation and a receiver with R antennas. The blocks have $N = 256$ data symbols, each one selected from a QSPK constellation, plus an appropriate CP. The channel has 100 slots, symbol-spaced, equal-power multipath components. Similar conclusions could be drawn for other rich multipath propagation conditions. We consider uncorrelated Ryeligh fading for different multipath components and different links between transmit and receive antennas. We assume perfect synchronization and channel estimation. For the sake of comparison, we also plot the MFB, which can be regarded as a lower bound on the optimum performance [16].

Let us start by comparing the conventional ZF, MRC and EGC schemes with the ideal IB-DFE receiver. As expected, the performance of the ZF receiver is acceptable since we are considering $R > T$, which reduces the probability of the inverting matrix to be ill conditioned. This would not be the case if $T = R$, especially for a small-to-moderate number of antennas. IB-DFE allows an additional gain, leading to a performance very close to the MFB just after a few iterations. In fact, 4 iterations are enough for the convergence of the IB-DFE receiver. However, the low-complexity techniques that do not require matrix inversions (EGC and MRC) have very poor performance, with high irreducible error floors. These error floors decrease as we increase the ratio R/T, but even for $R = 32 = 8T$, receive antennas we still have irreducible error floors in the vicinity of 10^{-2}.

Let us consider now our iterative FDE receivers where the first iteration is based on the MRC and EGC. Fig. 5 and 6 show the corresponding BER performance with different number of iterations. Clearly, our receivers can have excellent performance, even with $R = 4T = 16$. In fact, the performance approaches that of to the IB-DFE receiver after just 4 iterations.

It should be pointed out that our iterative receivers do not require matrix inversions and, in spite of that, they can have such a good performance when $R >> T$. Therefore, it would be interesting to evaluate their performance for smaller values of R/T or smaller values of R for a given values of T. The simulation results of this study are shown in Fig. 7. From this figure, we can observe that our EGC or MRC receiver only an option for R larger than T, with $R \geq 2T$, at least. Even in this case we can have irreducible errors above 10^{-4} for the ECG-based receiver and above 10^{-3} for the MRC-based receiver.

If we compare the performance we can see that the MRC

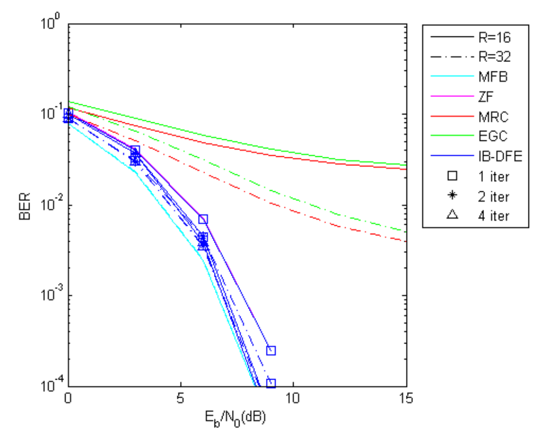

Fig. 4. BER performance for $T = 4$ and $R = 16$ or 32 receive antennas and an IB-DFE receiver, as well as conventional ZF, EGC and MRC frequency-domain receivers.

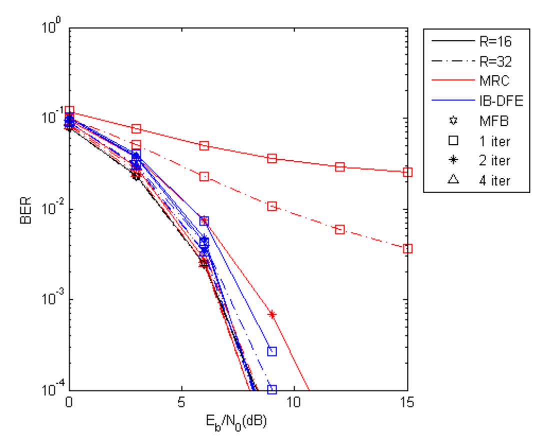

Fig. 5. BER values for $T = 4$ and $R = 16$ or 32 receiver antennas for the iterative receiver based on the MRC, as well as the IB-DFE receiver.

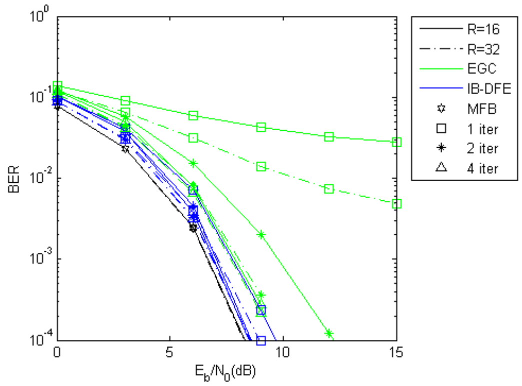

Fig. 6. BER values for $T = 4$ and $R = 16$ or 32 receiver antennas for the iterative receiver based on the EGC, as well as the IB-DFE receiver.

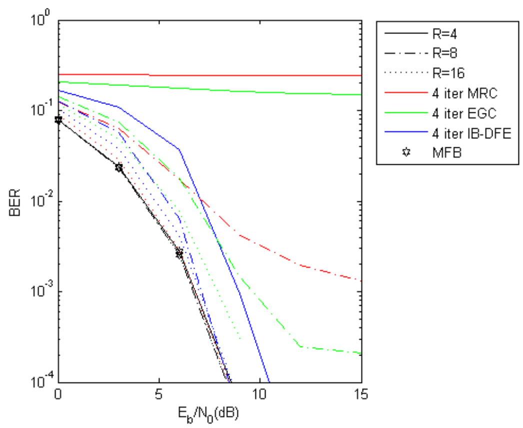

Fig. 7. BER values for $T = 4$ and different values of R, with the EGC-based and MRC-based receivers with 4 iterations.

presents a higher performance. This fact is shown in Fig. 7. The simulated results shown in Fig. 5 and 6 show that the EGC approaches the IB-DFE performance while the MRC approaches the MFB.

V. Conclusion

In this paper we considered the uplink of massive MIMO systems employing SC-FDE schemes, where multiple users transmit to a single base station with a large number of antennas. We proposed low-complexity frequency-domain detection schemes based on the MRC or EGC that do not require matrix inversions, while achieving an excellent performance, provided that the number of receiver antennas is at least twice as the number of transmitter ones.

The presented results exhibit an excellent performance at the 4^{th} iteration for the MRC and EGC techniques. In fact, we can observe that the 4^{th} iteration of the MRC technique already presents a performance very close to the MFB. Moreover, we have shown that this level of performance is achievable without the complexity associated with the inversion of large matrices. Therefore we have demonstrated that massive MIMO can be easily implemented, so it is a promising techniques to be used in future 5G systems.

Acknowledgment

This work was partially supported by Fundação para a Ciência e Tecnologia (FCT) under the project PEstOE/EEI/LA0008/2013.

References

[1] J. G. Andrews, *What will 5G be?*, IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065-1082, 2014.

[2] Rehman Talukdar and Mridul Saikia, *Evolution and Innovation in 5G Cellular Communication System and Beyond: A Study*, ArXiv e-prints, 2014

[3] G. J. Foschini, *Layered space-time architecture for wireless communication in a fading enviroment when using multi-element antennas*, Bell Labs Tech. J., vol. 1, no. 2, pp. 41-59, 1996.

[4] G. Raleigh and J. Cioffi, *Spatio-Temporal Coding for Wireless Communication*, IEEE. Trans. on Communications, vol. 46, pp. 357-366, 1998.

[5] E. G. Larsson, O. Edfors, F. Tufvesson and T. L. Marzetta, *Massive MIMO for next generation wireless systems*, IEEE Communications Magazine, vol. 52, no. 2, pp. 186-195, 2 2014.

[6] F. Rusek, D. Persson, Buon Kiong Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, *Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays*, IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40-60, 1 2013.

[7] A. Gusmão,R. Dinis, R. Conceição, N. Esteves, *Comparison of two modulation choices for broadband wireless communications*, Proc VTC'00-Spring, vol. 2, pp. 1300-1305, Tokyo, Japan, May 2000

[8] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar and B. Eidson, *Frequency domain equalization for single-carrier broadband wireless systems*, IEEE Communications Magazine, vol. 40, no. 4, pp. 58-66, 2002.

[9] N. Benvenuto and S. Tomasin, *Block iterative dfe for single carrier modulation* , Electron. Lett., vol. 39, no. 19, pp. 1144-1145, Sep.2002.

[10] N. Benvenuto, R. Dinis, D. Falconer, and S. Tomasin,*Single carrier modulation with nonlinear frequency domain equalization: An idea whose time has come-again*, Proceedings of the IEEE, vol. 98, no. 1, pp. 69-96, January, 2010.

[11] R. Dinis, R. Kalbasi, D.Falconer and A. Banihashemi, *Iterative Layered Space-Time Receivers for Single-Carrier Transmission over Severe Time Dispersive Channels*, IEEE Comm. Letters, vol. 8, no. 9, pp. 579-581, Sep. 2004.

[12] F. Amaral, R. Dinis, P. Montezuma, *Approaching the MFB with Block Transmission Techniques*, European Trans. on Telecommunications, vol. 23, no. 1, pp. 76-86, February, 2012.

[13] A. Gusmão, P. Torres, R. Dinis, N. Esteves, *A turbo FDE technique for reduced-CP SC-based block transmission systems*. IEEE Trans. Commun. vol. 55, no. 1, pp. 16-20, 2007.

[14] Fábio José Pinto da Silva, *Frequency-Domain Receiver Design for Doubly-Selective Channel*, July 2015.

[15] J. Proakis, *Digital Communications*, 4th Edition, McGraw-Hill, 2001.

[16] P. Silva, R. Dinis, *Frequency-Domain Multiuser Detection for CDMA Systems*, River Publishers, Aalborg, 2012.

[17] F. Ribeiro, R. Dinis, F. Cercas, A. Silva, *Receiver Design for the Uplink of Base Station Cooperation Systems employing SC-FDE Modulation*, EURASIP Journal on Wireless Communications and Networking, vol. 2015, no. 1, pp. 1-17, January, 2015.

[18] F. Ribeiro, R. Dinis, F. Cercas, A. Silva, *Analytical performance evaluation of Base Station cooperation systems using SC-FDE modulations with iterative receivers*, Globecom Workshops (GC Wkshps) 2012 IEEE , pp. 637641), Anaheim, USA, 2012

[19] P. Montezuma and R. Dinis, *Iterative Receiver Based on the EGC for Massive MIMO Schemes using SC-FDE Modulations*. Electronics letters vol. 52, no. 11, pp. 972-974, January 2016.