ISCTE Business School
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IMPLIED VOLATILITY:
CAN WE IMPROVE VAR MODELS?

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"We must be careful not to confuse data
with the abstractions we use to analyze them."

William James

Resumo

Os modelos atuais de *value-at-risk* (VaR) são suportados por uma componente histórica de preços e de metodologias que estimam matrizes de covariância, respeitando meramente informação histórica, tais como: pesos igualmente ponderados, médias móveis exponenciais ponderadas, GARCH, etc. Todos estes métodos observam uma lacuna: são desenvolvidos com informação histórica. Nesse sentido, ninguém pode garantir que os mercados vão continuar a comportar-se da mesma maneira que antigamente.

Dado o exposto, aliado com *returns* históricas de ações, incorporamos volatilidades implícitas *at-the-money* derivadas de opções cotadas com prazos de vencimento reduzidos. As volatilidades implícitas irão ser utilizadas com vista ao refinamento das matrizes de covariância com uma medida *forward looking*. Criamos 8 modelos paramétricos (normal vs T-Student) e 7 históricos (sem ajustamentos vs ajustados à volatilidade resultado da decomposição de Cholesky) VaR que estimam o VaR diário para o nosso portefólio, o qual concentra 10 ações liquidadas cotadas nas bolsas norte americanas.

Através do *backtesting* dos nossos resultados, comparamos os modelos VaR que usam informação histórica e estimam a volatilidade, com os que incorporam ajustamentos da volatilidade implícita. Os resultados do *backtesting* produzem testes estatísticos com *p-values* mais elevados para modelos que usam volatilidade implícita. Esta melhoria, embora marginal, é mais preponderante nos resultados do teste BCP, o que sugere uma influência mais significativa das volatilidades implícitas na redução dos excedentes de *clustering*.

Por fim, desenvolvemos simulações nos pesos dos portefólios de modo a aferir se, em média, esta melhoria é de facto apenas marginal, mais significativa, ou se nem sequer existe. Depois de simular 500 portefólios diferentes, observamos que existe um aumento significativo dos *p-values* do teste BCP quando se utiliza as volatilidades implícitas para atualizar as matrizes de covariância. Por outras palavras, a volatilidade implícita provoca reduções, positivas no fundamental, nos excedentes de *clustering* do VaR.

Palavras chave: Volatilidade Implícita, Ações, Value-at-Risk, Backtesting.

Classificação JEL: C32, G17

Abstract

Current value-at-risk (VaR) models are based on historical prices and (co)variance estimation methods that relies purely on historical data: equally weighted, exponentially weighted moving average (EWMA), generalized autoregressive conditional heteroscedasticity (GARCH), etc. All these methods suffer from one main flaw: they are *backward looking*. In other words, no one can guarantee that markets will continue performing in the same manner as they did in the past.

Thus, along with historical stock returns, we incorporate at-the-money implied volatilities derived from listed options with nearest expiration. Implied volatilities will be used to refine our covariance matrices with this *forward looking* measure. We create eight parametric (normal vs. Student's t) and seven historical (no adjustment vs. volatility adjusted based on the Cholesky decomposition method) VaR models that estimate 1-day total VaR for our portfolio, which consists of 10 liquid stocks listed on US exchanges.

We backtest and compare VaR models that use historical prices and well known volatility estimation methods, with its peers that incorporate implied volatility adjustment. Backtest results show mostly marginal increase of statistical test p-values for models that use implied volatility. This marginal improvement is mainly with Berkowitz, Christoffersen and Pelletier (BCP) test results, which suggests that the main contribution of implied volatility lies in the reduction of VaR exceedance clustering.

Finally, we perform portfolio weights simulation to verify whether, on average, this improvement is indeed just marginal, more significant, or does not exist at all. After simulating 500 different portfolios, we conclude that there exist a significant increase in BCP test p-values when implied volatility is being used to update covariance matrices. In other words, implied volatility can indeed help us to reduce VaR exceedance clustering.

Keywords: Implied Volatility, Stocks, Value-at-Risk, Backtesting.

JEL Classification: C32, G17

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List of Abbreviations

AAPL – Apple Inc.

ACF – Auto-Correlation Function

AMZN – Amazon.com, Inc.

ATM – At-the-Money (Options)

BAC – Bank of America Corporation

BCP – Berkowitz, Christoffersen and Pelletier (Test)

CC – Conditional Coverage (Test)

DCC – Dynamic Conditional Correlation

EW – Equally Weighted

EWMA - Exponentially Weighted Moving Average

F – Ford Motor Company

GARCH - Generalized Autoregressive Conditional Heteroscedasticity

GE – General Electric Company

INTC – Intel Corporation

IV – Implied Volatility

KO – The Coca-Cola Company

LR - Likelihood Ratio (Test)

MSFT – Microsoft Corporation

OTC – Over-the-Counter (Market)

PFE – Pfizer Inc.

P&L - Profit and Loss

T – AT&T Inc.

UC – Unconditional Coverage (Test)

VaR – Value-at-Risk

1 Introduction

Volatility estimation of future asset returns is one of the most important topics in the financial world nowadays. There are several well-known methods used by practitioners and academics, such as: equally weighted, EWMA, GARCH (symmetric and asymmetric versions) and stochastic volatility models. It is a stylized fact, also supported by Alexander (2008), Dowd (2005), and other authors, that EWMA and GARCH models are robust, most stable and produce satisfactory results across different market regimes.

One of the most widely used tools for measuring financial risk within banks and investment firms is the value-at-risk (VaR) method. It represents the portfolio or single asset loss over specific time horizon, that we are $100(1-\alpha)\%$ sure will not be exceeded. The main input into VaR models (parametric and adjusted historical) is variance, more specifically, estimation of future variance of asset returns. In case we are dealing with multivariate time-series, we need to include covariances to capture dependencies between portfolio constituents. This dissertation will focus on equity VaR and usage of multivariate time series of asset returns. Thus, both variance and covariance estimation will be in the scope of this work.

All VaR models are based on historical prices and/or (co)variances estimated by methods that rely, directly or implicitly, on historical data. It means that they are highly dependent on market conditions that already occurred in the past. But nobody can guarantee that markets will continue to perform in the same manner. This raises the question if it is possible to find a better, forward-looking estimator of future realized volatility, which will in turn better anticipate changes in market behavior. We try to find a solution for this issue in the options market.

Option prices represents an additional source of data. From them we can extract information about expectation of future volatility of the underlying stocks. Time series of implied volatility was obtained for ten US stocks that constitutes our portfolio. Most liquid options are usually the ones with shortest maturity and with strikes closest to current stock price. Thus, we will focus our attention to implied volatility derived from options with nearest month expiration and that are at-the-money, and investigate if information embedded in these options can provide us input that is valuable enough to improve performance of VaR models.

First, using out-of-sample data for stocks in our portfolio, we construct lists of daily covariance matrices forecasts using well-known methods: equally weighted, EWMA, and multivariate dynamic conditional correlation (DCC) GARCH. We then extract correlations from them and use it to create new, *"hybrid"* type of covariance matrices that use *those* correlations, but also incorporate implied volatilities: Implied Vol/Equally Weighted, Implied Vol/EWMA and Implied Vol/DCC GARCH.

Subsequently, we assign equal weights to our stocks and thus create a typical portfolio that we shall follow through the rest of this work. We estimate t-distribution degrees of freedom for these portfolio returns, which is required for estimating parametric Student's t total VaR.

Two types of VaR models are used to estimate profit and loss (P&L) distribution of future stock and portfolio returns: parametric (normal and Student's t version) and historical (non-adjusted and volatility adjusted based on the Cholesky decomposition method). Parametric VaR models are used along with the following covariance matrices: Equally weighted, EWMA, Implied Vol/Equally Weighted, Implied Vol/EWMA. Historical VaR models (volatility adjusted based on the Cholesky decomposition method) are used with all the available covariance matrices: Equally Weighted, EWMA, multivariate DCC-GARCH, Implied Vol/Equally Weighted, Implied Vol/EWMA and Implied Vol/DCC-GARCH. Finally, historical VaR models (non-adjusted) do not assume any P&L distribution, thus do not require any covariance estimation. In total we have six parametric (three normal and three student-t) and nine historical (one non-adjusted, eight volatility adjusted based on the Cholesky decomposition method) VaR models.

Finally, using all 15 models and out-of-sample data, we estimate 1-day dynamic VaR (total for parametric) under three different significance levels. Thus, in total 45 different time-series of 1-day VaR for 10-years period are created, that should be faced with time-series of realized portfolio log-returns for the same period. Dynamic VaR implies that portfolio weights are not changed, thus we assume daily rebalancing in order to keep weights at the constant levels throughout entire time horizon.

To compare estimated out-of-sample VaR with actual portfolio log-returns, we perform backtesting and evaluate results using several statistical tests: Unconditional Coverage test (Kupiec, 1995), Conditional Coverage test (Christoffersen, 1998) and BCP (Berkowitz, Christoffersen and Pelletier, 2009) test at several different lags. Moreover, we make visual inspection of the graphs to detect some unusual paterns or VaR exceedance clustering. Backtesting and evaluation is performed for two reasons: firstly, to evaluate overall performance of our VaR models and detect the best ones among them, and secondly, more relevant for the topic of this dissertation, to directly compare the models that use pure historical data with the *hybrid* ones that incorporate implied volatility, and observe VaR exceedance clustering behaviour. As implied volatility is the *forward looking* measure, we expect that its inclusion could help us better anticipate the changes in current market conditions and ultimately lead to reduction of VaR exceedance clustering, which is the main cause of value-at-risk underestimation.

As expected, the improvement, although marginal, is reflected through the increase of BCP test p-values at all lags up to five. This indicates that incorporating implied volatility might indeed help us with the reduction of VaR exceedance clustering. On the other hand, we do not observe any improvements with UC and CC test p-values, mainly because most of the models that use purely historical data already have decent results and do not have problems with passing these two basic tests.

Since backtesting and statistical tests are performed based on one specific portfolio only, this raises an important question of VaR models robustness and stability. What will happen if some other portfolio weights are used? Are results going to be changed and will some VaR models perform better or worse than before?

To evaluate stability of VaR models, we perform portfolio weights simulation and generate five hundred portfolios, each with different seed and random weights assigned to its constituents. Then we analyze behavior of median p-values, number of null-hypothesis rejections and number of VaR exceedances, in order to check whether models are robust and stable enough.

The simulation results showed that the difference between classic and *hybrid* models is no longer marginal, but now more concrete. On average, we observe significant increase of p-values generated from BCP test at all lags up to five. This gives us stronger evidence that *hybrid* models perform better than the *backward looking* ones that use purely historical data. Thus, incorporation of implied volatility can indeed help us with reduction of VaR exceedance clustering. Again, UC and CC p-values are similar as all historical volatility adjusted models, both *hybrid* and regular ones, have a decent amount of VaR exceedances.

The structure of this work is the following: In Section 2, we review and challenge the current literature and papers related to this field. The Value-at-Risk concept is described briefly in the third section. Next, Section 4 is dedicated to the explanation of variables and sources for obtaining stocks and implied volatility data. Subsequently, in Section 5 we describe the covariance estimation methods and statistical tests used for backtesting evaluation. In the Section 6, we backtest and analyse one specific, equally weighted portfolio. Section 7 is used for portfolio weights simulation where we check the stability of results from the previous section. We make conclusion in the Section 8 and summarize our work. Finally, last section is dedicated for presenting the bibliography that was used.

2 Review of Literature

Volatility forecasting as topic of financial econometrics, has occupied academics and practitioners for years. The first autoregressive conditional heteroscedasticity (ARCH) model for estimating variance was introduced by Engle (1982) and has later been generalized (GARCH) by Bollerslev (1986). The main contribution of GARCH model is that it can capture conditional variance (variance changing over time) and volatility clustering behaviour.

Equities and commodities have a common property called *leverage effect*. It means that a volatility increase is more likely to happen after a huge negative return rather than after a positive return of the same magnitude. That was the main motive for Engle (1990) to propose asymmetric A-GARCH model, which was later evolved to GJR GARCH by Glosten et al. (1993). The major property of asymmetric GARCH model is its ability to properly capture this *leverage effect*. Now there exists several other asymmetric versions: E-GARCH, T-GARCH, P-GARCH.

Exponentially weighted moving average (EWMA) is also a very popular method for covariance estimation. Depending on smoothing constant λ, it puts more or less weight to recent observations, and influence of past observations on the estimator decay exponentially as we move back in time.

One special case of EWMA model with predefined parameter λ is RiskMetrics^{TM}, presented by JP Morgan (1996) in the RiskMetrics Technical Document. For estimating daily variances and covariances, the decay parameter equals to $\lambda = 0.94$ and for monthly estimation they propose $\lambda = 0.97$.

When it comes to option pricing, there is no document which made more scientific contribution than "*The pricing of options and corporate liabilities*" by Black and Scholes (1973). Black-Scholes pricing formula provides the framework for calculating European option prices based on several inputs: volatility, risk-free rate, strike, time to maturity and price of the underlying asset. When we "invert" this formula and use option prices from the market as an input, using numerical procedure we can get implied volatilities as an output. These volatilities represent the expectation of investment community, more specifically of option traders, about the future variance of the market returns.

However, the fact that the implied volatility is not constant with respect to the strike of the option from which the volatility is implied, as imposed by Black and Scholes (1973), indicates that there is some extra information embedded in option prices which is not captured by the Black-Scholes formula. There are several papers that used the implied volatility curve as an input to variance estimation, and tried to estimate future probability density functions of asset returns.

Most of the papers that are dealing with implied volatility, for example Bentes (2015) and Nishina et al. (2006), are interested in volatility forecasting *per se*. In other words, they are comparing volatility forecasts based on implied volatility, with actual, realized volatility, to see whether they can perform better than traditional volatility forecasting methods presented above. Nevertheless, there are some exceptions where implied volatility was tested for other purposes such as improving VaR models performance. For instance, Giot (2005) assesses the information content of volatility forecasts based on the VIX[1] and VXN[2] implied volatility indexes in a daily market risk evaluation framework. His empirical findings show that "*volatility forecasts based on the implied volatility indexes provide meaningful results when market risk must be quantified*".

On the other hand, Kim and Ryu (2015) tried to estimate VaR using implied volatility from the options on index of the KOSPI 200[3]. Their empirical results show that the model-free implied volatility VKOSPI (which is equivalent to US-based VIX) does not improve the performance of suggested VaR models. Yet, when they incorporated implied volatility derived from OTM options and combined it with GJR-GARCH model, they were able to improve the overall performance of VaR models.

When it comes to research of the option volatility smile, Xing et al. (2010) tried to predict future equity returns based on the steepness of the volatility skew. They concluded that stocks with the steepest slopes of their options' smiles underperform peer companies by 10.9% per year on a risk-adjusted basis. They also managed to prove that most in-

[1] The Chicago Board Options Exchange (CBOE) Volatility Index of S&P 500.
[2] The Chicago Board Options Exchange (CBOE) Volatility Index of Nasdaq 100.
[3] Korea Composite Stock Price Index consisting of 200 biggest companies.

formed traders with negative news are most likely to trade out-of-the-money put options and that these options have the highest information content embedded.

Feng, Zhang and Friesen (2015) came to a similar conclusion when they investigated the relationship between the option-implied volatility smile, stock returns and heterogeneous beliefs. They studied the implied volatility slope of both call and put options with different delta's and found that "*stocks with a steeper put slope earn lower future returns, while stocks with a steeper call slope earn higher future returns*".

There is a significant amount of academic work on dynamics of implied distributions, more specifically how those can be translated into implied probability density function of future asset returns. Breeden and Litzenberger (1978) showed that second order derivative of a put option with respect to its exercise price is equivalent to the current value of the risk neutral probability density of S_t, which represents an asset price at some future time t.

Parametrization of volatility surface was also in the scope of researchers and practitioners. This topic is very significant in the presence of low liquidity in the options market. Gatheral and Jacquier (2013) showed how to calibrate stochastic volatility inspired (SVI) parametrization in a way that guarantees the absence of static arbitrage. The no-arbitrage principle is important in the context of translating implied volatility surface into series of implied PDFs[4] to secure non-negative values for latter.

In our modest opinion, the main flow of these works is that they work exclusively with single stocks or indices, thus applicability in a real world situation other than univariate is questionable. It is very rare to see institutional investors with portfolio consisting of only one instrument or with portfolios that closely follows the weights from a specific market index.

Another major issue we find important is verification and stability of the proposed models. The vast majority of scientific papers that are dealing with implied volatility and VaR are using UC, CC and similar statistical tests that check exceedances up to first lag only. More advanced tests, like BCP that checks independence and exceedance clustering at higher lags, are not used in these papers. Moreover, results presented are characteristic for one specific index or stock, thus it is not sure whether these VaR models will behave in the same manner if some other index or shares are being used.

[4]Probability density functions.

Our dissertation will try to answer these questions and analyze applicability of implied volatility VaR models in a true multivariate environment, with ten different stocks. Moreover, BCP which is a more sophisticated statistical test, will be used to check exceedance clustering all the way up to lag five.

Instead of using only one specific portfolio, we check for robustness and stability of our VaR models by *shaking* shares' weights and simulating several hundred different portfolios to check if test results are consistent enough. We believe that this will provide enough evidence whether proposed VaR models could be applicable to the real portfolios with dozens or even hundreds of stocks.

3 Data

3.1 Stocks

Stock prices are pulled from Yahoo Finance using R package *quantmode* and function `getSymbols.yahoo()`. We use adjusted daily closing prices from January 2001 - June 2017, for the following stocks that are included in our portfolio:

- Apple [AAPL]
- Amazon [AMZN]
- Bank of America Corporation [BAC]
- Ford [F]
- General Electric [GE]
- Intel [INTC]
- Coca-Cola [KO]
- Microsoft [MSFT]
- Pfizer [PFE]
- AT&T [T]

After downloading stock prices in USD, we convert those to daily log returns for the purpose of estimating daily VaR in percentage points.

Adjusted closing prices are used since they are already corrected for all stock splits, dividends, and other important events that might impact log returns.

3.2 Implied Volatilities

Implied volatility data is obtained through Quandl, which is the main aggregator of financial databases. R package *Quandl* and function `Quandl.datatable()` are available for pulling database into R Studio.

The original provider of database is Quantcha, Inc., a Redmond, Washington (US) based company. Quantcha is a financial software and services company focused on stocks and options investors. Moreover, this company provides a suite of tools for searching, filtering, and analyzing stock market investments.[1]

The dataset contains historical (March 2002 – June 2017) daily implied volatilities for the ten stocks from our portfolio. Implied volatilities are derived based on *"inverted"* Black-Scholes formula where option and stock prices from the market are used as an input. There is no analytical solution so numerical procedure is required to generate IV values for different strikes.

This dataset is updated on a daily basis, and includes at-the-money (ATM) implied volatilities for calls, puts, and means, as well as skew steepness indicators. Implied volatilities are provided for constant future maturities of 10, 20, 30, 60, 90, 120, 150, 180, 270, 360, 720, and 1080 calendar days. The data is calculated using end of day (EOD) market data provided by the option exchanges.[2]

Since we are dealing with 1-day VaR, we extrapolate volatilities for constant maturities of 10 and 20 days to create time series of IVs with constant maturity of 1 day. As implied volatilities are annualized, daily volatility is calculated using the *"square root of time rule"* assuming that there are 252 trading days per each calendar year.

We decided to choose mean ATM implied volatility as the input to our VaR models since it is calculated as arithmetic mean of calls and puts ATM IVs, thus we do not have to make any assumptions and this should keep us free from model risk, at least regarding selection of implied volatility variables.

[1]Company description obtained via https://www.quandl.com/publishers/QUANTCHA, access date: 04 October 2018.

[2]Implied volatility data specification obtained via https://www.quandl.com/data/VOL-US-Equity-Historical-Option-Implied-Volatilities/documentation/introduction, access date: 04 October 2018.

4 Value-at-Risk (VaR)

As a broad definition, value-at-risk represents a portfolio loss that we are sure will not be exceeded, within certain confidence and specific time horizon. The main parameters of VaR are thus the significance level α (equivalent to 1-α confidence level), and the risk horizon h over which the VaR is estimated. Depending on the context and whether we need to comply with certain regulations or not, different alphas can be used. However, in this work we work exclusively with three different values of α: 0.01, 0.05 and 0.1. Put it the other way around, we are sure that our portfolio loss will not exceed the estimated VaR with 99%, 95% and 90% confidence, respectively.

A more formal definition of VaR states that the $100\alpha\%$ h-day Value-at-Risk at time t (usually omitted), $VaR_{ht,\alpha}$, is minus the α quantile of the h-day discounted P&L (or return) distribution. Throughout this work, we will work exclusively with log returns. Moreover, we deal with 1-day VaR, thus the discounting effect is negligible and therefore can be omitted without creating any bias in the results.

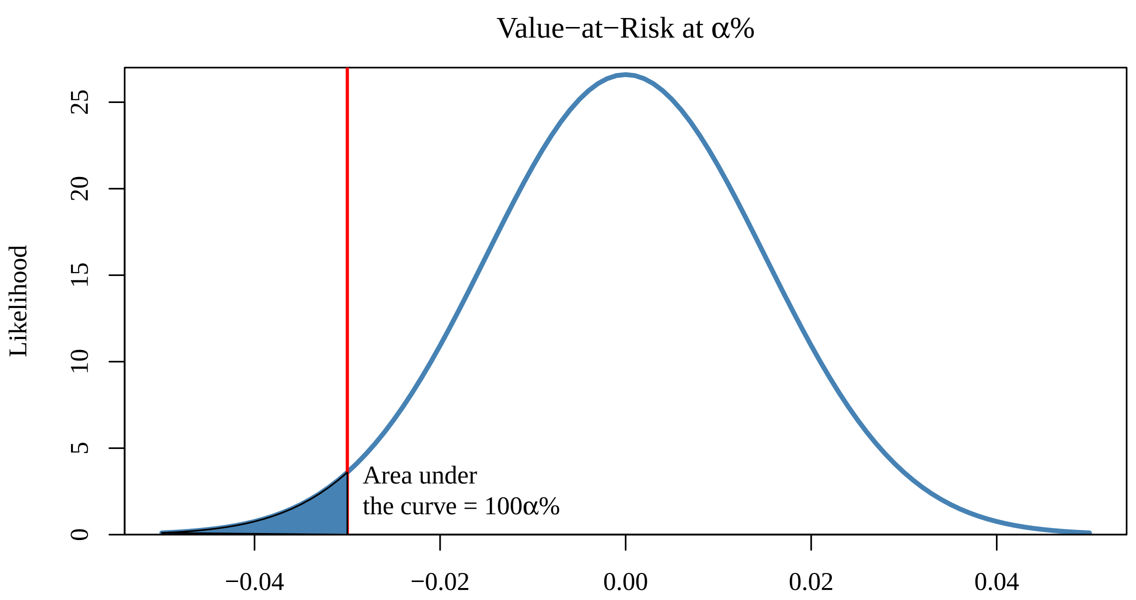

Figure 4.1: Value-at-Risk, PDF of Portfolio Returns

In the language of statistics, the value-at-risk estimate at significance level α for a time horizon h, at time t, indicates the portfolio or single asset loss over h-days at time t that is exceeded with probability α, such that:

$$P[r_{t+h} < -VaR_{t+h,\alpha}|I_t] = \alpha \tag{4.1}$$

where r_{t+h} is the return from time t to time $t+h$ and I_t represents information set at time t.

Alexander (2008) divides VaR models into three different categories according to their resolution method, in other words how they model the returns distribution:

- Parametric linear VaR;
- Historical simulation VaR;
- Monte-Carlo VaR.

Both parametric and Monte-Carlo VaR are directly dependent on variance and co-variance estimation. On the other hand, pure historical VaR is the only method which does not make any distributional assumptions as it simply represents the α -quantile of an empirical h-day discounted P&L or returns distribution.

However, volatility-adjusted historical VaR, introduced by Duffie and Pan (1997) and Hull and White (1998), also uses variances and covariances to refine historical returns, and make it more *actual* to the current regime of the market. Thus, the main prerequisite for any good VaR model is to produce solid forecasts of variances, and covariances in case of a multi-factor model.

Throughout this work, we treat each stock in our portfolio as a single risk factor. We have two reasons for this. Firstly, we wish to calculate total VaR and make predictions as accurate as possible. Secondly, to directly compare VaR models that use standard co-variance estimation methods with the ones that incorporate implied volatility adjustment. As implied volatility from exotic option on basket of stocks that matches our portfolio is obviously not available, but only from the single stocks that constitutes the portfolio, any other comparison would be inappropriate.

Moreover, to simplify calculations, we always report dynamic VaR. Thus, we assume that portfolio is constantly rebalanced to keep the portfolio weights constant.

4.1 Parametric VaR

Parametric value-at-risk assumes that we use some of the well known families of distributions to model our portfolio returns. Most commonly used are normal and Student's t distribution. These distributions in the context of VaR are fully explained by the mean and standard deviation of discounted portfolio returns, and, in case of Student's t, degrees of freedom. As we are interested in 1-day VaR, a reasonable assumption can be made that the expected daily portfolio excess return over the risk free rate is zero. Thus, the only thing that we require to locate the quantile of interest is the covariance matrix of stock returns, and the vector of portfolio weights.

4.1.1 Parametric - Normal VaR

We start by deriving formula for x_α, which represents the α-quantile return, i.e. the return such that $P(X < x_\alpha) = \alpha$.

Under the assumption that asset returns are normally distributed, thus $X_h \sim N(\mu_h, \sigma_h^2)$, where μ_h and σ_h are the estimates for the mean and standard deviation of the discounted asset returns, over the future period h. we apply the standard normal transformation:

$$P(X_h < x_{h,\alpha}) = P(\frac{X_h - \mu_h}{\sigma_h} < \frac{x_h - \mu_h}{\sigma_h}) = P(Z < \frac{x_h - \mu_h}{\sigma_h}) = \alpha,$$

where Z is a standard normal variable. Therefore:

$$\frac{x_{h,\alpha} - \mu_h}{\sigma_h} = \Phi^{-1}(\alpha), \tag{4.2}$$

where $\Phi^{-1}(\alpha)$ represents the standard normal α quantile. Thus, we can finally provide formula for the $100\alpha\%$ h-day parametric normal VaR, that is presented as the percentage of the single asset or portfolio value:

$$VaR_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h - \mu_h. \tag{4.3}$$

As mentioned earlier, we do not calculate VaR based on portfolio that is treated as univariate time-series, but we attribute it to the different risk factors. Therefore, equivalent formula for the parametric normal VaR in multivariate framework is the following:

$$VaR_{h,\alpha} = \Phi^{-1}(1-\alpha)\sqrt{\boldsymbol{w'}\boldsymbol{V_h}\boldsymbol{w}} - \boldsymbol{w'}\boldsymbol{\mu_h}, \tag{4.4}$$

where $\boldsymbol{w'}$ represents vector of portfolio weights, $\boldsymbol{V_h}$ is the covariance matrix of risk factors, in our case 10×10 matrix of 1-day stock returns, and $\boldsymbol{\mu_h}$ is the vector of average discounted stock returns. Obviously, the second part of this equation can be omitted, as we assume that vector of means of 1-day discounted returns is a zero vector.

4.1.2 Parametric - Student's t VaR

Very often, the assumption that returns of financial instruments follow a normal distribution is unrealistic. It is a stylized fact that empirical stock returns have more of a leptokurtic distribution, thus heavier tails and higher peak, when compared to the equivalent normal distribution with same variance. Thus, if we try to fit past returns with normal distribution but the actual returns exhibit excess kurtosis, it is very likely that we will underestimate VaR at low significance levels.

The exact boundary at which significance level Student's t VaR is higher than normal VaR depends on degrees of freedom, the main parameter that determines the tail shape of the t-distribution. This parameter can be obtained by fitting the empirical distribution using the maximum likelihood method. With the increase of degrees of freedom, the t-distribution converges to the normal one. Again, how many previous observations should be included in the rolling sample is arbitrary. We believe that 1-year represents a reasonable size, thus in this work we always estimate degrees of freedom based on the last 250 daily portfolio returns.

Value-at-Risk can be calculated as:

$$VaR_{h,\alpha,\nu} = \sqrt{\nu^{-1}(\nu-2)}T_{\nu}^{-1}(1-\alpha)\sigma_h - \mu_h, \tag{4.5}$$

where ν represents degrees of freedom and $T_{\nu}^{-1}(1-\alpha)$ is the quantile of the *regular* Student's t distribution. The adjustment $\sqrt{\nu^{-1}(\nu-2)}$ has to be made to obtain the quantile of the standardized Student's t distribution.

Another common property of empirical distributions of asset returns is that they are negatively skewed. It is worth noting that there exists non-central t-distribution which is able to *capture* this effect, however here we shall work with symmetric t-distribution only.

4.2 Historical VaR

4.2.1 Historical - No Adjustment

Contrary to the parametric method, historical VaR do not have to make any distributional assumptions about the future returns. As a general definition, the $100\alpha\%h$-day historical VaR is simply the α quantile of the empirical h-day discounted distribution of returns. Estimating historical VaR is therefore a pretty simple task. After choosing sample size (size of the rolling window in backtesting environment), we compute h-day returns for the assets that constitutes the portfolio. Empirical h-day portfolio's return distribution is created by keeping the portfolio weights constant. Finally, from this distribution of returns, we find the α quantile that we are interested in. Although simple historical simulation has its advantages, the main downside of this method is that market conditions change over time. Selection of sample size can highly affect our VaR estimation. Thus we need to find the way to make this sample of returns better reflect the current market conditions.

4.2.2 Historical - Volatility Adjusted

Volatility adjusted, or more formally, volatility weighted method, was proposed by Duffie and Pan (1997) and Hull and White (1998). This method assumes that we still assign the same weight to each observation, but we adjust the entire series of returns in a way that it matches the current volatility estimation. Thus, the entire sample size is adjusted to match the current volatility regime of the market.

When we are dealing with univariate time-series of returns, either as a single stock or portfolio treated as the single asset, this methodology is simple. After obtaining a series of volatility estimates (using EWMA, GARCH or other methods), we adjust the time-series of returns as following:

$$\hat{r_t} = \frac{\hat{\sigma_T}}{\hat{\sigma_t}} r_t \tag{4.6}$$

where $\hat{\sigma_T}$ represents the current volatility estimate, and $\hat{\sigma_t}$ is the past volatility estimate made at time t.

Cholesky Decomposition

Throughout this work, we treat each stock as the risk factor. As we wish to make the volatility adjustment at the risk factor level, thus for the vector of portfolio returns $\boldsymbol{X_t}$, we need to perform a Cholesky decomposition. This method allows us to adjust the volatility of all stocks in the portfolio simultaneously, while preserving their correlations. According to Miller (2014), when the covariance matrix satisfies certain minimum requirements, we can decompose it by rewriting it as the product of the lower triangular matrix, $\boldsymbol{L}$, and its transpose, $\boldsymbol{L'}$:

$$\boldsymbol{\Sigma = LL'}. \tag{4.7}$$

Now, the vector of stock returns generated at time t can be *updated* as follows:

$$\boldsymbol{\tilde{X}_t = X_t(L_T L_t^{-1})'}, \tag{4.8}$$

where $\boldsymbol{\Sigma_T = L_T L'_T}$ and $\boldsymbol{\Sigma_t = L_t L'_t}$.

This transformation changes our stock returns vector covariance matrix from $\boldsymbol{\Sigma_t}$ to $\boldsymbol{\Sigma_T}$, which was our initial intention.

In this work, the following covariance estimation methods will be used to refine historical returns: equally weighted, EWMA, GARCH and hybrid implied volatility, that will be presented in the next section. Although adjusting returns with covariances estimated using equally weighted method might sound like an oxymoron, it actually makes sense to perform it if the rolling windows for decomposition and covariance estimation are of a different size. With Cholesky decomposition, we always use 1000 observations in the rolling window but *only* last 250 for calculating historical covariances based on equally weighted method. This gives us longer time series, thus more observations from which to draw the quantile we are looking for, but makes it better reflect the current market conditions.

Even though *volatility adjusted* is the official name of this method, all historical volatility-adjusted VaR models here will be called 'Cholesky adjusted', to indicate that the adjustment was performed on a risk factor (individual stock) level.

5 Methodology

5.1 Covariance Estimation

The covariance matrices of the returns of stocks that constitutes our portfolio represent the key component of our VaR models. With parametric VaR, they are used to calculate the standard deviation of the entire portfolio. On the other hand, with historical, adjusted VaR by Cholesky decomposition, it is used to refine historical returns with most actual volatility while, at the same time, not disturbing correlations.

The covariance matrix $\mathbf{\Sigma}$ in our case is 10×10 matrix that has variances of stock returns on the main diagonal and covariances on places other then main diagonal. Thus, for each covariance matrix, we need to estimate 55 different values: 10 variances and 45 covariances (covariance of returns between stock a and b is identical to the covariance of returns between stock b and a). Therefore, the covariance matrix $\mathbf{\Sigma}$ can be easily decomposed as $\mathbf{\Sigma} = \boldsymbol{VRV}$ where $\boldsymbol{V}$ is 10×10 diagonal matrix with 10 different volatilities on its diagonal, zeros otherwise, and $\boldsymbol{R}$ represents 10×10 correlation matrix. Once the covariance matrix is estimated, we can use the portfolio weights vector $\boldsymbol{w}$ to calculate variance of our entire portfolio as $\boldsymbol{\sigma_p^2} = \boldsymbol{w'\Sigma w}$. Throughout this chapter, for the purpose of plotting and comparing the differences between different estimation methods, we will assume an equally weighted portfolio.

Several standard methods for covariance calculation are used: equally weighted, exponentially weighted moving average (EWMA), multivariate dynamic conditional correlation GARCH (DCC GARCH). Moreover, we introduce *hybrid* implied volatility covariance matrices to check their applicability in the VaR framework.

5.1.1 Equally Weighted

One of the simplest but still widely used methods is based on assigning the equal weights to all past observation within defined time frame. Then we use the rolling window of size T to estimate covariance matrix of i stock returns. The size of the rolling window is arbitrary and it needs to reconcile two opposite interests: larger T means more observations for estimating the value, thus more confidence in the estimation, but consequently makes forecasted values not properly reflecting the current market conditions. In our case, we use $T = 250$ which represents a rolling window size of one calendar year.

As (co)variances are calculated on a daily basis, it is assumed that discounted returns have an expectation of zero. This gives us the following formula for variance calculation:

$$\hat{\sigma}^2_{i,t} = T^{-1} \sum_{k=1}^{T} r^2_{i,t-k} \ . \tag{5.1}$$

Similarly, we can define the following formula for covariance estimation:

$$\hat{\sigma}^2_{ij,t} = T^{-1} \sum_{k=1}^{T} r_{i,t-k} r_{j,t-k} \ , \tag{5.2}$$

which represents the covariance estimate for two stock returns, i and j at time t, based on the previous T (250) daily returns. Once we have both variances and covariances, we can derive the following formula for the correlation between two stocks i and j:

$$\hat{\rho}_{ij,t} = \frac{\hat{\sigma}_{ij,t}}{\hat{\sigma}_{i,t} \hat{\sigma}_{j,t}} \ . \tag{5.3}$$

To plot portfolio standard deviation across time, we use an equally weighted portfolio. Then, for each day, based on portfolio weights vector $\boldsymbol{w}$ and covariance matrix $\boldsymbol{\Sigma}$, we can obtain time series of forecasted portfolio standard deviations using equally weighted method with rolling window size of 250 days. Thus, to construct the time series of portfolio volatility forecasts that starts on 1 January 2007, we use multivariate time series of stock returns that starts roughly on 1 January 2006.

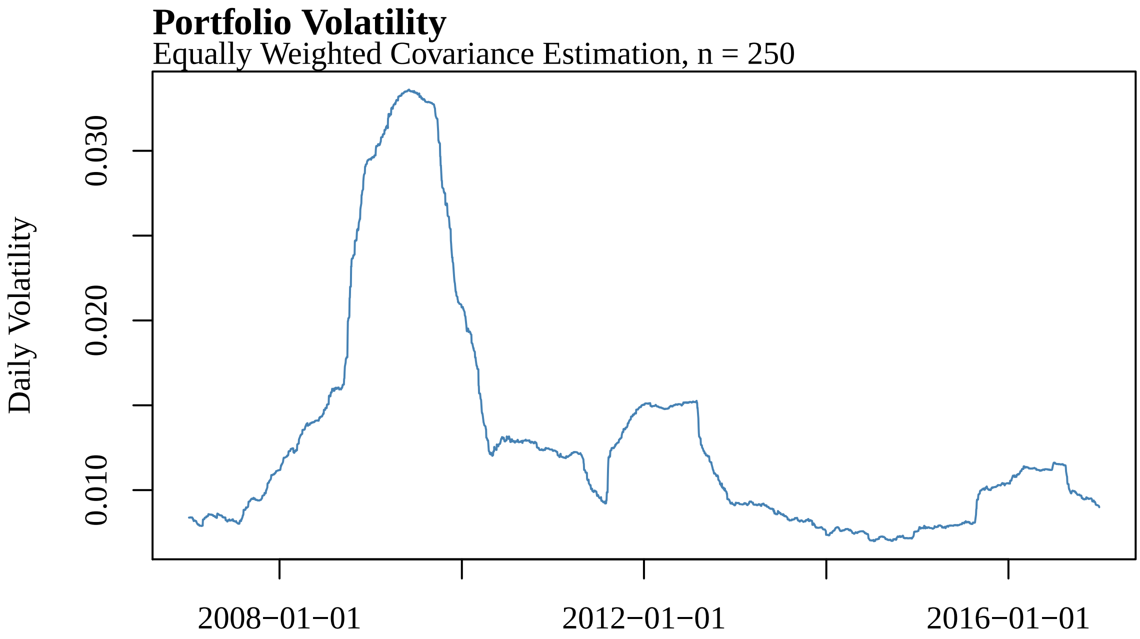

Figure 5.1: Portfolio Volatility Forecasting - Equally Weighted Method

5.1.2 EWMA - Exponentially Weighted Moving Average

As clearly seen from figure 5.1, the main issue with the equally weighted method is that it is not reactive enough and suffer from so called *ghost feature*. When the extreme observation drops out from the sample, it is no longer counted. Until then, it was treated the same way as it occurred last day or last month, as long as it is included in the rolling window.

As a logical alternative, exponentially weighted moving average method was proposed, where larger weights were assigned to more recent observations and vice versa. As mentioned by Dowd (2008, 129), "*this type of weighting scheme might be justified by claiming that volatility tends to change over time in a stable way, which is certainly more reasonable than assuming it to be constant*". Thus, EWMA is still considered unconditional volatility estimation method for the time-series of returns that are assumed to be i.i.d. and can be used with parametric methods.

To estimate future variance, we use:

$$\hat{\sigma}_t^2 = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} r_{t-i}^2 \tag{5.4}$$

likewise, for covariance:

$$\hat{\sigma}_{12,t} = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}r_{1,t-i}r_{2,t-i}. \tag{5.5}$$

When performing actual calculations, the above formulas are more useful in their recursive forms:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \tag{5.6}$$

as well as

$$\hat{\sigma}_{12,t} = (1-\lambda)r_{1,t-1}r_{2,t-1} + \lambda\hat{\sigma}_{12,t-1}. \tag{5.7}$$

Obviously, $0 < \lambda < 1$ represent the smoothing parameter and it is an arbitrary value. Higher λ gives less weight to recent observations and lower λ does the opposite, thus make it more reactive.

We use the value proposed in the RiskMetrics – Technical Document, which states that the optimal parameter is $\lambda = 0.94$, when we are dealing with daily returns. Obviously, the size of the rolling window does not play much of a role here as after some time observations become statistically insignificant. With $\lambda = 0.94$, the weight on the 100th observation is 0.21%, and on the 250th observation it is only 0.00002%. Nevertheless, make the initial estimation with 250 past observations and always use all available data. Thus, the window size gradually increases each day by one. Daily portfolio volatility estimated using EWMA method is plotted in the figure 5.2.

5.1.3 GARCH - Generalized Autoregressive Conditional Heteroscedasticity

Moving average models, both equally weighted and EWMA, consider that returns are independent and identically distributed (i.i.d.). Therefore, forecast of volatility is always equal to the current estimate, despite the time-horizon we are interested in. Obviously, this assumption is very unrealistic. Moreover, the EWMA method assumes that the λ parameter is constant. It implies that it is not responsive to current market conditions and stays the same whether market is turbulent or we have a calm period. That was the main motivation for inventing the new method for conditional covariance estimation.

The plain ARCH model was introduced by Engle (1982) and it was later generalized

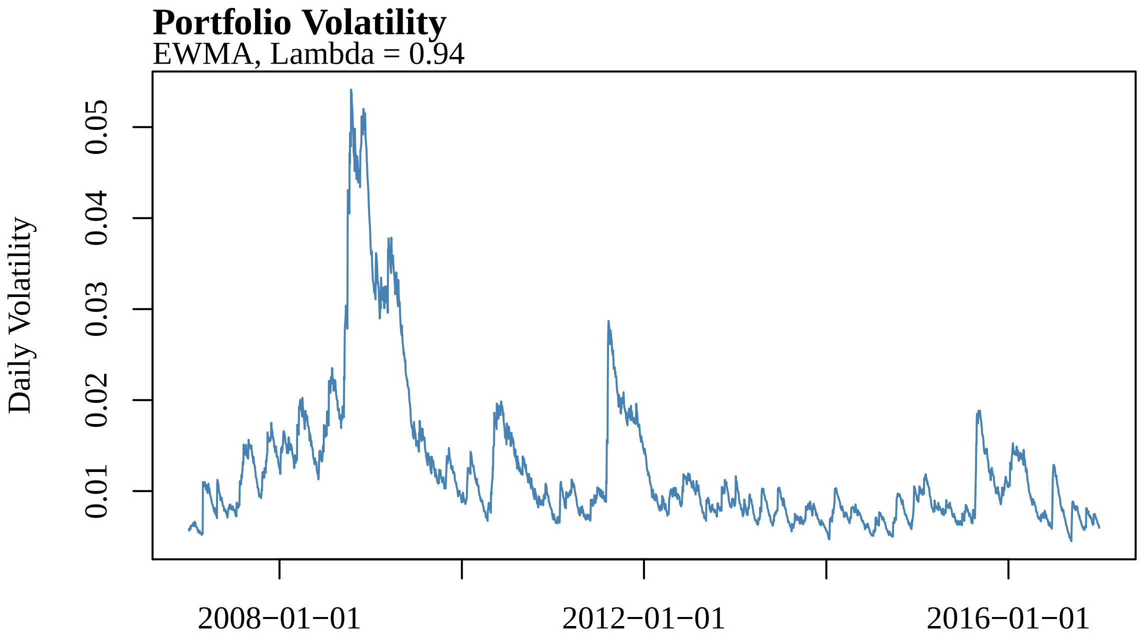

Figure 5.2: Portfolio Volatility Forecasting - EWMA

(GARCH) by Bollerslev (1986). After that, a lot of other sub-types were created.

As mentioned by Alexander (2008, a, 131), "*the volatility can be higher or lower than average over the short term but as the forecast horizon increases, the GARCH volatility forecasts converge to the long term volatility*". Moreover, GARCH models are able to capture volatility clustering.

The most basic GARCH (1,1) model for the conditional volatility can be presented in the following form:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{5.8}$$

where we use the following AR(1) process to model the returns:

$$r_t = \mu + \rho r_{t-1} + \epsilon_t, \epsilon_t \sim (0, \sigma_t^2) \tag{5.9}$$

under the constraints:

$$\omega > 0, \quad \alpha, \beta \geq 0, \quad \alpha + \beta < 1. \tag{5.10}$$

Finally, parameters $\mu, \rho, \omega, \alpha$ and β are estimated by maximizing the (log) likelihood function.

The basic GARCH (1,1) model implies that the response of the conditional variance to the negative market shocks is the same as the response to the positive market shocks.

However, in financial markets, there is a well known phenomenon called the *leverage effect*. It means that *bad news* cause larger volatility increase than *good news*. The best way to prove existence of the leverage effect is to observe the correlation between VIX and S&P500. When markets are performing well, volatility goes down and vice versa, strong bear market probably means that VIX is *skyrocketing*. Here we will deal with symmetric multivariate GARCH models only, however it should be mentioned that there are several asymmetric versions like A-GARCH, GJR-GARCH and E-GARCH, that are able to capture the *leverage effect*.

As GARCH forecasts are applied here with disaggregated historical VaR, we need to use multivariate GARCH models to obtain covariances. It seems that volatility clustering has its counterparty in the multivariate world, called the correlation clustering. It is a known fact that during period of crisis, correlations tend to increase and stay at high levels for long period of time. There are several multivariate GARCH methods proposed by Alexander (2008, a), according to the asset class of the instruments, we wish to estimate the covariance for:

- Constant correlation GARCH (CC-GARCH) and dynamic conditional correlation (DCC-GARCH) for covariance matrices of FX exchange rates and equity indices.
- Factor GARCH (F-GARCH) for estimating covariance matrices of stocks.
- Orthogonal GARCH (O-GARCH) for estimating covariance matrices of interest rates or any other like commodity futures.

F-GARCH is able to capture systematic risk only, thus idiosyncratic risk is not included. Even though F-GARCH was initially proposed for equities, here we use DCC-GARCH due to the fact that we are interested in total risk.

Since the constant conditional correlation is obviously a very unrealistic assumption, a new type of model called Dynamic Conditional Correlation (DCC) was introduced by Engle (2002) and Tse and Tsui (2002), which allows the correlation matrix to be time varying with the following dynamics:

$$\Sigma_t = \boldsymbol{V}_t \boldsymbol{R}_t \boldsymbol{V}_t \ , \tag{5.11}$$

where $\boldsymbol{V}_t$ is a diagonal matrix with conditional volatilities and $\boldsymbol{R}_t$ represents the time varying correlation matrix. To secure positive semi-definiteness of $\boldsymbol{R}_t$ Engle (2002) proposes a specific proxy process Q_t. Interested readers can check Engle (2002) for more details.

Again, for the purpose of plotting portfolio volatility, we assign equal portfolio weights for our ten stocks. The first 500 observations are used to estimate first set of parameters using maximum likelihood method. Then, after each 22 trading days (roughly one calendar month), the model is refitted and new set of parameters is obtained. Window size gradually increases until we reach 1000 observations. Afterwards, we keep rolling window size constant at this level. This methodology will be used again when we perform estimation of portfolio volatility, backtesting and simulation using specific vectors of portfolio weights.

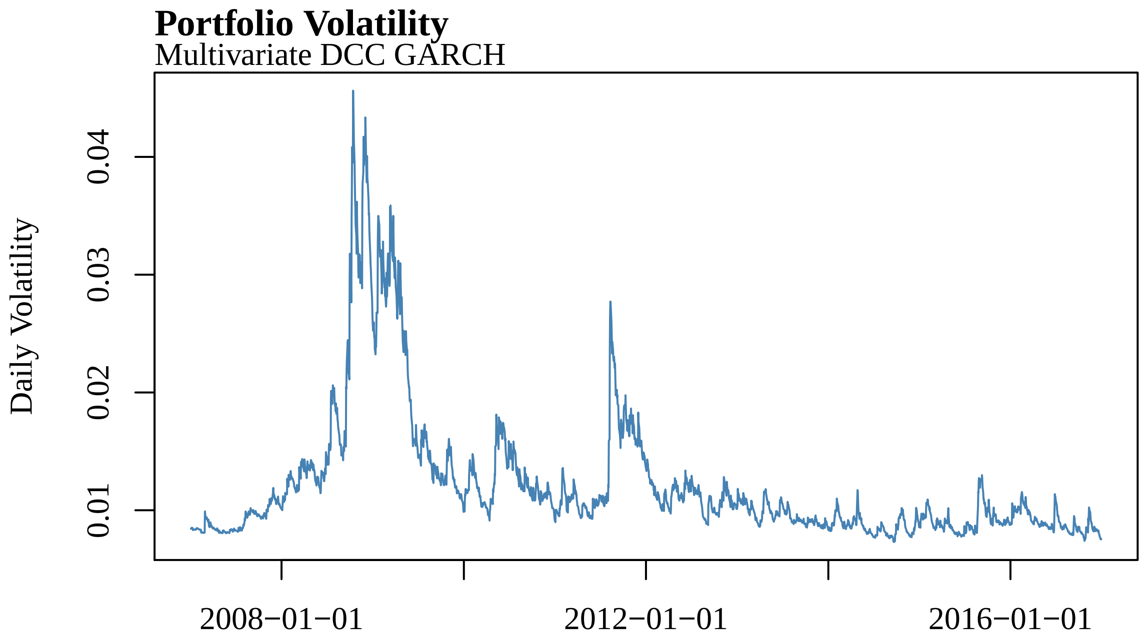

Figure 5.3: Portfolio Volatility Forecasting - DCC GARCH

5.1.4 Hybrid - Implied Volatility

So far, we spoke about implied volatility and implied variance only. However, since we are dealing with portfolio of stocks and total VaR, thus, more than one risk factor, we need to fill non-diagonal elements of covariance matrices. There exist, at least to our knowledge, only two cases when it is possible to derive *true* implied covariances or implied correlations from the option prices.

The first one is clearly with foreign exchange (FX) options market, which is liquid enough to get critical amount of data. In the review of Campa and Chang (1997) paper, Allen, Boudoukh, and Saunders (2004: 240) correctly concluded that *"given the implied volatilities of, for example, the USD/GBP, GBP/Euro, and Euro/USD, one could obtain the USD/GBP and USD/Euro implied volatility as well as the implied correlation between the two"*. Unfortunately, the scope of this work does not include FX VaR, thus this method is not helpful.

Second case is present when we are dealing with options on more than one stock, such as a spread, quanto, or basket. First, let us write the formula for the variance of the difference between two stocks, a and b:

$$\sigma^2_{a-b} = \sigma^2_a + \sigma^2_b - 2\rho_{a,b}\sigma_a\sigma_b \,. \tag{5.12}$$

Now we can rearrange the equation in order to get correlation on the left hand side:

$$\rho_{a,b} = \frac{\sigma^2_a + \sigma^2_b - 2\sigma^2_{a-b}}{2\sigma^2_a\sigma^2_b} \,. \tag{5.13}$$

This is the perfect formula for implied correlation between stocks a and b. Unfortunately, there is one main flaw with this approach. As indicated by Dowd (2005, 141), "*in this particular case, this (formula) means that we need options on a and on b, and also, more problematically, an option on the difference between a and b (e.g., a spread, quanto or diff option)*". All these options are exotic and, even if they exist, they are traded exclusively in the over-the-counter (OTC) markets. Therefore, they have very low liquidity and more important, it is not possible to obtain critical amount of data required for proper estimation and backtesting. Thus, we need to find alternative way of obtaining implied covariances.

As indicated at the beginning of this section, the covariance matrix $\mathbf{\Sigma}$ can be written as $\mathbf{\Sigma} = \boldsymbol{VRV}$, where $\boldsymbol{V}$ is a diagonal matrix with (implied) volatilities on its diagonal. Since implied volatilities for single stock options are already obtained, the critical component is the (implied) correlation matrix $\boldsymbol{R}$, which is obviously missing. Thus, we create three *hybrid* types of covariance matrices with implied volatility diagonal matrix $\boldsymbol{V}$ and correlation matrices $\boldsymbol{R}$ with (historical) correlations already calculated using well established methods: equally weighted, EWMA and DCC GARCH.

To make sure that covariance matrices are always estimated using out-of-sample data, implied volatilities are shifted one day ahead. For instance, to construct hybrid covariance

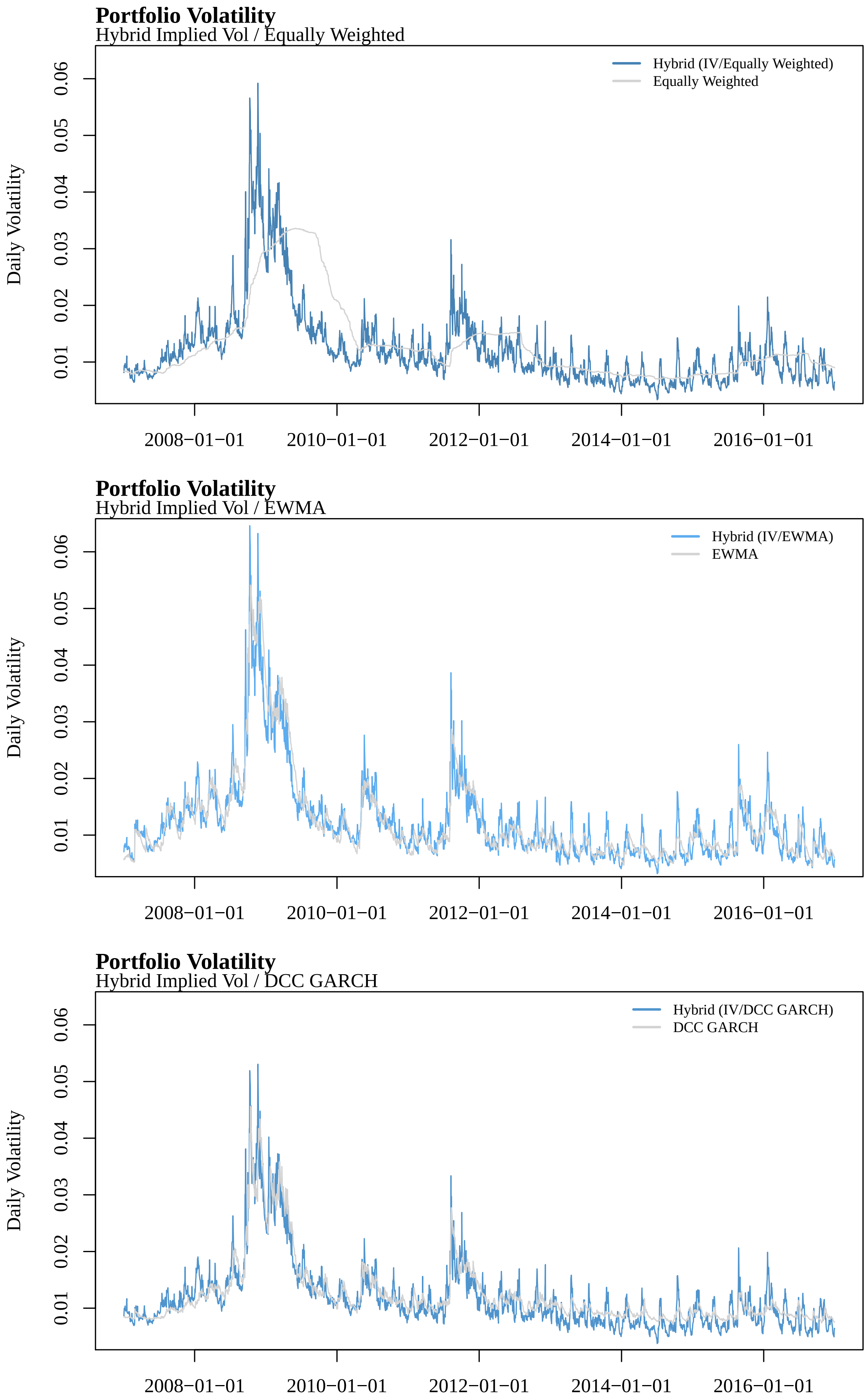

Figure 5.4: Portfolio Volatility Forecasting - Hybrid Covariance Matrices

matrix Σ estimate for June 15, we use correlation matrix based on data available up to (including) June 14, and diagonal matrix of implied volatilities obtained on June 14.

Ensuring that these *hybrid* matrices stay positive semi-definite is a concern that we need to tackle. The positive semi-definiteness property of the covariance matrix Σ is inherited from its *parental* correlation matrix $\boldsymbol{R}$. As long as this correlation matrix $\boldsymbol{R}$ is positive semi-definite, then the covariance matrix Σ is guaranteed to be positive semi-definite as well. As all correlation matrices are estimated from historical data, this is not more of a problem than if the covariance matrices were estimated from historical data alone.

Even though we are fully aware that these covariance matrices are *forward looking* in terms of volatilities, but still *backward looking* in terms of correlations, this is still our best guess about future volatility of the portfolio, when multiplied with portfolio weights. These *hybrid* matrices will be used to check whether the *forward looking* component $\boldsymbol{V}$ is strong enough to improve performance of our VaR models, mainly in terms of $\boldsymbol{p}$-values derived from BCP test, which checks for autocorrelation of VaR exceedances.

Here we plot portfolio volatility estimated using all three *hybrid* covariance matrices and compare it with the portfolio volatility using standard estimation methods that does not incorporate implied volatility. As it can be seen from the figure 5.4, it is indicative that the biggest difference is between portfolio volatility estimated using equally weighted (rolling window size 250 days) method and *hybrid* equally weighted/IV model. On the other hand, EWMA and DCC GARCH graphs are very similar to its *hybrid* counterparties that incorporate implied volatility.

Finally, in figure 5.5 we plot the differences between *hybrid* portfolio volatilities versus the *regular* ones. It is interesting to observe that portfolio volatility estimated using traditional methods that utilize purely historical data is, on average, higher than volatility estimated using our hybrid covariance matrices that leverage implied volatility from the options market. This opposes the stylized fact (Eraker, 2009) that implied volatility is, on average, higher than realized volatility, at least when comparing volatility index such as VIX with its counterparty, S&P500 index. Moreover, it means that VaR estimated using hybrid methods is, on average, lower than VaR estimated with other methods, *ceteris paribus*. Thus, less capital has to be devoted for the purpose of covering prospective losses.

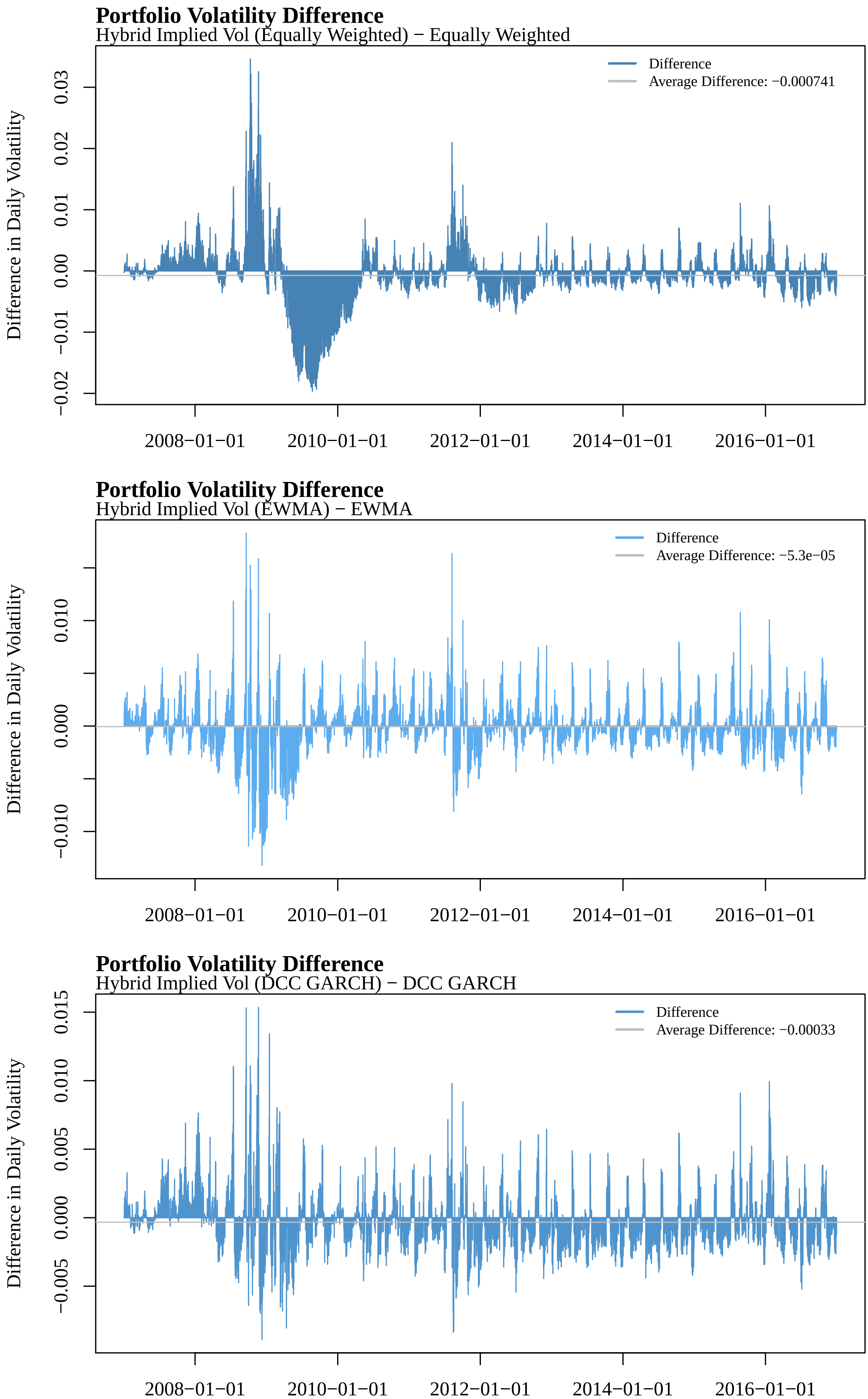

Figure 5.5: Portfolio Volatility - Difference (Hybrid vs. Regular Covariance Matrices)

5.2 Evaluating performance of VaR models

To compare the performance of VaR models, unconditional coverage (Kupiec LR), conditional coverage (Christoffersen), and BCP tests are used. The first two fall into likelihood ratio tests and the last one is Ljung-Box, which tests for autocorrelation. Statistical tests however cannot be performed on the actual VaR and returns series. Thus, a backtesting engine was created, which takes estimated VaR (using out-of-sample data) for each specific day, and compare it to the actual, realized portfolio return on that date. In case the portfolio return on specific day is lower than estimated VaR, we count it as an exceedance and value 1 is assigned, zero otherwise.

If the VaR model is well specified, the number of exceedances is expected to match the significance level. For instance $VaR_{1,5\%}$ is expected to have: $0.05 \times 252 \approx 12.6$ exceedances per calendar year. Large deviations from that number will reject the assumption that the model is correct.

Moreover, we also tend to avoid exceedence clustering. If VaR exceedances are clustered, statistical tests (like Kupiec LR) that are not considering autocorrelation might show good performance, while the actual risk might be underestimated. Good performing models are able to handle this issue as well.

Backtesting is performed based on a 10-year period (2007-2016) with daily frequency, thus 2518 observations (trading days) in total. We calculate p-values based on statistical tests for each calendar year and for the entire period as a whole. Each VaR model is backtested for three significance levels: 0.01, 0.05 and 0.1.

5.2.1 Unconditional Coverage (UC) Test

The unconditional coverage test introduced by Kupiec (1995) is a likelihood ratio test based on the number of exceedances where the indicator function is represented by:

$$I_{\alpha,t+1} = \begin{cases} 1, & \text{if } r_{t+1} < VaR_{1,\alpha,t} \\ 0, & \text{otherwise.} \end{cases} .$$

Now the null and alternative hypotheses are being defined as follows:

$$H_0 : E(X_{n,\alpha}) = n(1-\alpha)$$

$$H_1 : E(X_{n,\alpha}) \neq n(1-\alpha) ,$$

where $X_{n,\alpha}$ is a number of successes (each day when indicator function has a value of 0 and VaR is not exceeded).

The test statistic is calculated by:

$$LR_{uc} = \frac{\pi_{exp}^{n_1}(1-\pi_{exp})^{n_0}}{\pi_{obs}^{n_1}(1-\pi_{obs})^{n_0}} , \tag{5.14}$$

where π_{exp} is the expected proportion of exceedances, π_{obs} is the realized proportion of exceedances, n_1 is the observed number of exceedances and n_0 is the number of days when there was no exceedance.

The distribution of test statistic under the null hypothesis is: $-2ln(LR_{uc}) \sim \chi^2(1df)$.

If the null hypothesis is not rejected, it is assumed that the model is well specified. In other words, the expected number of exceedances matches the realized number of exceedances, under the specific significance level.

5.2.2 Conditional Coverage (CC) Test

This test proposed by Christoffersen (1998) summarises unconditional coverage and independence test into one unique test. It checks for exceedance rate and independence rate at the same time.

The null and alternative hypotheses are defined as follows:

$$H_o : \pi(exp) = \pi(obs) \text{ and } \pi(n_{11}) = \pi(n_{01}) \times \pi(n_{10})$$

$$H_1 : \pi(exp) \neq \pi(obs) \text{ or } \pi(n_{11}) \neq \pi(n_{01}) \times \pi(n_{10}) .$$

The test statistic is calculated by:

$$LR_{cc} = \frac{\pi_{exp}^{n_1}(1-\pi_{exp})^{n_0}}{\pi_{01}^{n_{01}}(1-\pi_{01})^{n_{00}}\pi_{11}^{n_{11}}(1-\pi_{11})^{n_{10}}} \; . \tag{5.15}$$

Under the null hypothesis, the test statistic is distributed: $-2ln(LR_{cc}) \sim \chi^2(2df)$.

If the null hypothesis is not rejected, it can be assumed that the expected number of exceedances matches the realized number of exceedances and that there is no first order autocorrelation of exceedances.

5.2.3 Berkowitz, Christoffersen and Pelletier (BCP) Test

The main downside of conditional coverage test is that it tests for the first order autocorrelation of exceedances only.

Thus, we use Berkowitz, Christoffersen and Pelletier (2009) test that checks for first K autocorrelations of exceedances. The test statistic is specified as per following:

$$BCP(K) = T(T+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{T-k} \; , \tag{5.16}$$

where:

- $\hat{\rho}_k^2 = Corr(I_{t,\alpha} - \alpha, I_{t+k} - \alpha)$ is the k_{th} lag sample autocorrelation of the series $I_{t,\alpha}$
- $I_{\alpha,t+1} = \begin{cases} 1, & \text{if } y_{t+1} < VaR_{1,\alpha,t} \\ 0, & \text{otherwise.} \end{cases}$ is the exceedance indicator
- K is the maximum autocorrelation lag considered in the test
- T is the sample size

If our model is well specified, autocorrelation coefficients should be statistically insignificant at all lags. Therefore, the null and alternative hypotheses are defined as follows:

$$H_o : \rho_k = 0, \forall k$$

$$H_1 : \rho_k \neq 0, \exists k \; .$$

Under the null hypothesis, the test statistic is distributed: $BCP(K) \sim \chi^2_K$.

K is the arbitrary parameter which should reconcile trade-off between detecting non-independence presence at higher-order lags on one hand, and the power of the test (less degrees of freedom in the distribution of the test statistic) on the other. In this work, we shall use values between 1 and 5, thus five different statistical tests and p-values will be generated.

6 Backtesting Results

VaR models are evaluated based on the portfolio that consists of ten stocks. Specific weights must be chosen in order to generate time series of portfolio returns, estimate degrees of freedom for Student's t-distribution (related to parametric VaR), perform VaR estimation, backtest the models, and finally evaluate results. As indicated earlier, we report dynamic VaR, thus it is assumed that portfolio weights are not changed over time and portfolio is constantly rebalanced to keep them at constant levels.

To avoid any bias in the results, we choose an equally weighted portfolio. Thus, each stock has been assigned a weight of 10%. Now that the vector of portfolio weights is selected, portfolio returns are calculated for the entire backtesting period and beyond, all the way from 2001 to 2016. As shown in **Section 4**, for historical adjusted VaR with rolling window of size 1,000 (trading days) and with at least 250 observations to have meaningful estimation of covariance matrices, we need 5 years of additional data that precedes the start of the backtesting period.

VaR is estimated based on eight parametric and seven historical models, and then compared with the actual portfolio returns. Evaluation results such as number of exceedances and p-values from statistical tests for three significance levels: **0.01, 0.05 and 0.1**, can be seen in the tables below.

It is worth mentioning that the results presented here treat the **period 2007 - 2016** as a whole. Thus, the period of 2008-2009 financial crisis is included, which obviously deteriorates the performance of most models. In fact, there are some models which *easily* pass statistical tests for each individual year, even the years of crisis, but fail when evaluated on the entire sample of 2518 trading days. For full evaluation, we therefore provide **Appendix A** with complete backtesting results for each specific year between 2007 and 2016, as well as graphs with marked exceedances.

Cells in the tables below are formatted conditionally, based on the p-values. In case the p-value is above 0.05, it is marked with green color. On the other hand, if the p-value is between 0.01 and 0.05, cell is marked with yellow. Finally, cells with p-values lower than 0.01 have a white background.

6.1 VaR Backtesting, Alpha = 1%

For significance level 0.01 and sample size 2518, expected number of VaR exceedances is **25**. Even though this is not the official statistical test, our starting point will always be to compare the actual number of exceedances with the expected. Obviously, all parametric normal VaR models, as well as Student's t with equally weighted covariance estimation method, fail to pass the most basic UC and CC tests, due to much greater number of VaR exceedances than expected.

On the other hand, three parametric models with t-distribution are able to pass UC and CC test, although with p-values higher than 0.01 but not than 0.05: IV/Equally Weighted, EWMA and IV/EWMA. Out of those three, the two models that incorporate implied volatility are able to pass BCP tests at all lags up to five, IV/EWMA does it with p-values significantly higher than 0.05.

Table 6.1: VaR Models Backtesting, Alpha = 0.01

Models	Exceeds	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Parametric VaR								
Normal-Eqw	58	0.000	0.000	0.557	0.000	0.000	0.000	0.000
Std t-EqW	46	0.000	0.001	0.197	0.000	0.000	0.000	0.000
Normal-IV/Eqw	47	0.000	0.000	0.222	0.001	0.004	0.011	0.022
Std t-IV/EqW	37	0.027	0.026	0.045	0.018	0.035	0.057	0.083
Normal-EWMA	53	0.000	0.000	0.068	0.004	0.006	0.015	0.019
Std t-EWMA	38	0.017	0.019	0.056	0.000	0.000	0.000	0.000
Normal-IV/EWMA	55	0.000	0.000	0.851	0.745	0.891	0.956	0.859
Std t-IV/EWMA	36	0.042	0.105	0.492	0.624	0.703	0.746	0.780
Historical VaR								
No Adjustment	41	0.004	0.006	0.097	0.000	0.000	0.000	0.000
EqW Adjusted	34	0.094	0.056	0.021	0.000	0.000	0.000	0.000
IV/EqW Adjusted	34	0.094	0.056	0.021	0.005	0.011	0.021	0.034
EWMA Adjusted	26	0.870	0.089	0.001	0.000	0.000	0.000	0.000
IV/EWMA Adjusted	34	0.094	0.056	0.021	0.005	0.011	0.021	0.034
GARCH Adjusted	30	0.349	0.097	0.005	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	31	0.261	0.089	0.008	0.001	0.002	0.000	0.000

When it comes to adjusted historical VaR, exceedances are **between 26 - 34** which indicates that models slightly underestimate VaR, but still perform better than the parametric ones, at least regarding the actual number of exceedances. Both UC and CC test

p-values are above 0.05 for all six volatility adjusted models.

Regarding BCP test p-values, results are not encouraging. Only IV/Equally Weighted and IV/EWMA have some p-values above 0.01, but still evidently lower than 0.05. Even though both models comes from the hybrid implied volatility family, this represents just a marginal improvement and we cannot make any meaningful conclusions.

As it can be seen in the **Appendix A** where a full evaluation of the results is presented, significance level of 1% is not so thankful for BCP test as a lot of NA values are generated. In cases where zero or only one exceedance is recorded in a specific year, the test statistic cannot be calculated. Additionally, estimating autocorrelation coefficients based on roughly 25 or 30 observations can be misleading. Thus, BCP results will be more meaningful when we switch to significance levels of 5% and 10%.

6.2 VaR Backtesting, Alpha = 5%

At significance level of 0.05, we expect to have **126** exceedances. Out of the parametric models, the ones which use normal distribution now perform better than the ones which use Student's t distribution to model the distribution of returns. When it comes to p-values from UC and CC tests, best results are provided by normal hybrid IV/EqW and hybrid IV/EWMA, although regular normal EWMA also pass the test with p-values over 0.05 and slightly higher number of exceedances.

Table 6.2: VaR Models Backtesting, Alpha = 0.05

Models	Exceeds	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Parametric VaR								
Normal-Eqw	145	0.088	0.037	0.038	0.000	0.000	0.000	0.000
Std t-EqW	176	0.000	0.000	0.008	0.000	0.000	0.000	0.000
Normal-IV/Eqw	139	0.238	0.482	0.796	0.000	0.001	0.000	0.001
Std t-IV/EqW	167	0.000	0.001	0.347	0.025	0.058	0.003	0.008
Normal-EWMA	145	0.088	0.155	0.387	0.004	0.013	0.000	0.000
Std t-EWMA	172	0.000	0.000	0.813	0.074	0.138	0.000	0.001
Normal-IV/EWMA	139	0.238	0.401	0.522	0.016	0.034	0.002	0.005
Std t-IV/EWMA	170	0.000	0.001	0.879	0.026	0.062	0.073	0.127
Historical VaR								
No Adjustment	155	0.010	0.008	0.060	0.000	0.000	0.000	0.000
EqW Adjusted	146	0.073	0.035	0.043	0.000	0.000	0.000	0.000
IV/EqW Adjusted	134	0.463	0.688	0.654	0.023	0.044	0.004	0.004
EWMA Adjusted	116	0.359	0.630	0.767	0.000	0.001	0.000	0.000
IV/EWMA Adjusted	132	0.580	0.858	0.975	0.018	0.024	0.004	0.003
GARCH Adjusted	138	0.276	0.254	0.186	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	142	0.149	0.272	0.456	0.248	0.257	0.004	0.005

Out of the models that are able to pass UC and CC test, the best performing one in terms of BCP test results is the normal IV/EWMA. However, it has p-values higher than 0.01, but not than 0.05. Moreover, these p-values are valid only up to the lag three. Thus, out of our parametric models, there does not exist one that is able to pass both UC and CC test, as well as BCP test, up to lag five.

Historical adjusted models again show slightly better performance comparing to the parametric ones, in terms of number of exceedances, which is now between **116 - 146**. When it comes to BCP test results, at least up to lag three, IV/GARCH model performs better comparing to other models. Moreover, two other hybrid IV models have p-values higher than 0.01, but still under the 0.05 threshold. Unfortunately, at lags higher than three, we do not have any model that is able to pass BCP test.

6.3 VaR Backtesting, Alpha = 10%

Finally, we check the performance of VaR models at 0.1 significance level. The expected number of exceedances, considering the number of observations in the sample, is **252**. Still, when it comes to parametric models, normal ones show better performance than Student's t, latter obviously underestimating VaR. The best performing parametric model is the normal hybrid IV/EWMA, which is able to pass UC, CC, and all BCP tests up to lag five, with p-values significantly higher than 0.05.

Table 6.3: VaR Models Backtesting, Alpha = 0.1

Models	Exceeds	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Parametric VaR								
Normal-Eqw	223	0.052	0.003	0.002	0.000	0.000	0.000	0.000
Std t-EqW	284	0.036	0.005	0.010	0.000	0.000	0.000	0.000
Normal-IV/Eqw	231	0.162	0.303	0.503	0.028	0.006	0.002	0.004
Std t-IV/EqW	290	0.013	0.044	0.756	0.016	0.002	0.000	0.000
Normal-EWMA	243	0.557	0.715	0.574	0.002	0.002	0.000	0.001
Std t-EWMA	303	0.001	0.004	0.618	0.007	0.004	0.001	0.001
Normal-IV/EWMA	229	0.125	0.307	0.968	0.225	0.354	0.244	0.342
Std t-IV/EWMA	301	0.001	0.006	0.722	0.043	0.033	0.014	0.028
Historical VaR								
No Adjustment	263	0.460	0.031	0.008	0.000	0.000	0.000	0.000
EqW Adjusted	249	0.852	0.032	0.006	0.000	0.000	0.000	0.000
IV/EqW Adjusted	252	0.989	0.349	0.134	0.006	0.002	0.000	0.001
EWMA Adjusted	231	0.162	0.343	0.667	0.004	0.001	0.000	0.001
IV/EWMA Adjusted	242	0.513	0.265	0.122	0.025	0.043	0.023	0.041
GARCH Adjusted	240	0.430	0.287	0.157	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	262	0.501	0.391	0.221	0.006	0.005	0.001	0.002

Once again, historical volatility-adjusted models show slightly better performance regarding number of exceedances, which is in the range of **231 - 263**. When it comes to

BCP test results, IV/EWMA is the only one which has p-values higher than 0.01 at all lags up to five. Unfortunately, except at lag one, these p-values are not exceeding 0.05.

The results presented here show that in certain cases hybrid models that incorporate implied volatility indeed show some improvement, when compared to its counterparties that use pure historical data. This improvement is mostly reflected in the marginal increase of the BCP test p-values, except in the case of hybrid Student's t IV/EWMA at 1% VaR, and hybrid normal IV/EWMA at 10% VaR, when this improvement is more obvious. On the other hand, when it comes to UC and CC p-values, there is no clear difference between the models, mainly because all volatility adjusted models, both standard and *hybrid* ones, are able to pass these tests without any problems.

This means that the main advantage of *hybrid* implied volatility models could be in the reduction of VaR exceedance clustering. As implied volatility represents the *forward looking* measure, exceedance clustering is the area where we indeed expect to observe the largest improvement.

However, we need to be aware that the results presented here are specific just for one single portfolio, with equal portfolio weights. Thus, we need to find the way to check stability of these results and robustness of our models in case that some other portfolio is used. Moreover, we are interested to verify whether the increase in BCP p-values is just marginal as seen here, a bit more significant, or does not exist at all.

That is the main motivation to proceed further with this research, and jump into the next section dedicated for portfolio weights simulation.

7 Portfolio Weights Simulation

So far we backtested and compared VaR models using one specific, equally weighted portfolio consisting of ten stocks, (AAPL, AMZN, BAC, F, GE, INTC, KO, MSFT, PFE, T). Therefore, VaR models evaluation results from the previous section such as the number of exceedances, or p-values generated from UC, CC and BCP tests, are valid for this specific vector of portfolio weights where each element is equal to 0.1.

Here, a different approach will be used to generate portfolio weights. First, we will use the `set.seed()` function to assign seed for random number generator, specific to each single simulation. Then, weights will be randomly *drawn* from the normal distribution with mean 0.1 and standard deviation 0.05. In order to secure that portfolio weights sum up to one, the weight on last stock w_{10} will be calculated as $1 - \sum_{k=1}^{9} w_k$.

7.1 Algorithm

In order to check robustness of created VaR models and stability of results, additional 500 portfolios are simulated by iterating seed from 1 to 500. The following algorithm is used to perform portfolio weights simulation and evaluation of VaR models robustness and stability:

1. Select subset of the VaR models that will be used for simulations.

2. Generate list of 500 vectors with length 10, each containing randomly generated portfolio weights. Weights are *drawn* from the normal distribution with mean 0.1 and standard deviation 0.05.

3. For each vector of portfolio weights, create time-series of portfolio returns with daily frequency, based on realized returns (2007-2016) of specific stocks that constitutes the portfolio. Thus, list of 500 different time-series of portfolio returns is

created.

4. For each time-series of portfolio returns, estimate 1-day VaR (2007-2016) using out-of-sample data, for three significance levels: 0.01, 0.05, 0.1.

5. Calculate summary statistics based on 500 simulations (median p-value for UC, CC, BCP, average number of exceedances, cumulative % simulations that fall below expected number of exceedances) for each significance level and model from the subset.

6. Select best performing model(s), evaluate its robustness and plot results.

Here, we plot the first 100 simulations to observe how portfolio weights are *shaked*, based on step 2 of the proposed algorithm. The exceedances below zero and above one, as well as white areas on the graph, means that some weights are negative. Thus, certain stocks are sometimes shorted and leverage is used to invest more funds in other stocks, however the total sum of weights always equals one.

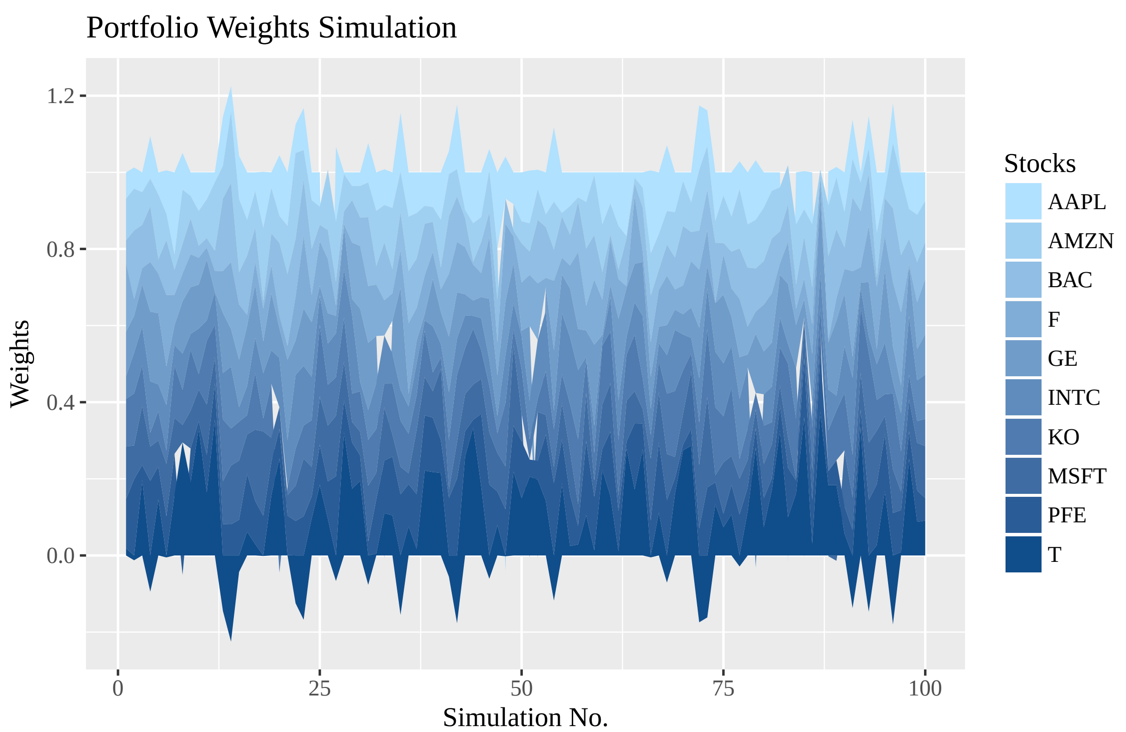

Figure 7.1: Portfolio Weights Simulation - First 100 Seeds

7.2 Simulation Results

As there was a lot of variation in the performance of parametric models (normal vs. Student's t) depending on VaR significance level, here we perform simulation based on the following six models that are free of any distributional assumptions:

- Historical (Cholesky Adjusted - Equally Weighted) VaR
- Historical (Cholesky Adjusted - Implied Vol / Equally Weighted) VaR
- Historical (Cholesky Adjusted - EWMA) VaR
- Historical (Cholesky Adjusted - Implied Vol / EWMA) VaR
- Historical (Cholesky Adjusted - DCC GARCH) VaR
- Historical (Cholesky Adjusted - Implied Vol / DCC GARCH) VaR

Similarly to the tabular comparison from the previous chapter, same values are reported but now as an agregated statistics: average number of exceedances and median p-values from UC, CC and BCP tests at lags one to five. Since we expect that number of exceedances has symmetrical distribution, mean value is reported. As for p-values from statistical tests, expected distribution is right skewed. Thus, more appropriate measure is the median since just a few outliers could easily *push* the mean above the threshold of 0.01 or 0.05. Moreover, we can easily observe whether at least at half of the simulations, our model is able to pass the statistical test.

Based on these summary statistical measures, we see that at 0.05 and 0.1 significance level, results slightly improved when compared to the ones from the previous section, where we analysed only one specific portfolio. Models that incorporate implied volatility to a certain extent perform better than the others without this *innovation*.

At 5% VaR, IV/Equally Weighted, IV/EWMA and IV/GARCH models passed all statistical tests on at least half of the simulations. Regular GARCH and Equally Weighted models passed BCP tests only up to the lag one and EWMA did not passed UC test, mainly due to the average number of exceedances lower than expected.

Regarding 10% VaR, again the best model is IV/EWMA, followed by IV/GARCH and IV/Equally Weighted. Other models are also able to pass UC and CC tests and BCP but only up to the lag one.

Table 7.1: Portfolio Weights Simulation - Summary Statistics

Models	Exceeds Mean	UC p-value Median	CC p-value Median	BCP1 p-value Median	BCP2 p-value Median	BCP3 p-value Median	BCP4 p-value Median	BCP5 p-value Median
VaR, Alpha = 0.01								
EqW Adjusted	15.7	0.027	0.012	0.001	0.000	0.000	0.000	0.000
IV/EqW Adjusted	12.0	0.001	0.000	0.000	0.000	0.000	0.000	0.000
EWMA Adjusted	11.5	0.001	0.000	0.000	0.000	0.000	0.000	0.000
IV/EWMA Adjusted	10.9	0.001	0.000	0.000	0.000	0.001	0.004	0.007
GARCH Adjusted	14.5	0.027	0.002	0.000	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	13.0	0.003	0.000	0.000	0.000	0.000	0.000	0.000
VaR, Alpha = 0.05								
EqW Adjusted	115.4	0.196	0.058	0.069	0.000	0.000	0.000	0.000
IV/EqW Adjusted	107.1	0.094	0.203	0.690	0.053	0.084	0.030	0.041
EWMA Adjusted	92.2	0.002	0.005	0.291	0.001	0.003	0.000	0.001
IV/EWMA Adjusted	99.6	0.014	0.049	0.702	0.247	0.388	0.199	0.189
GARCH Adjusted	107.9	0.105	0.082	0.161	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	113.5	0.175	0.331	0.651	0.132	0.177	0.047	0.055
VaR, Alpha = 0.1								
EqW Adjusted	249.3	0.321	0.018	0.008	0.000	0.000	0.000	0.000
IV/EqW Adjusted	253.9	0.207	0.152	0.217	0.016	0.009	0.003	0.005
EWMA Adjusted	229.8	0.125	0.159	0.356	0.006	0.003	0.001	0.001
IV/EWMA Adjusted	241.9	0.232	0.200	0.315	0.075	0.102	0.064	0.061
GARCH Adjusted	253.1	0.289	0.118	0.108	0.000	0.000	0.000	0.000
IV/GARCH Adjusted	262.7	0.142	0.155	0.259	0.025	0.014	0.006	0.009

Unfortunately, at 1% VaR, with at least half of the simulations, models did not pass the threshold. With CC and especially UC test, the reason is obvious. Models are too conservative as the mean of exceedances is *much* lower than expected 25. With median p-values from BCP tests, most of the values are zero or very close to it. Thus we cannot make proper comparison of the models.

Even though portfolio weights simulation for 5% and 10% VaR gives strong evidence that implied volatility models perform better than its peers, we should still be careful with the interpretation of the summary p-values of statistical tests, even we decided to use the median instead of the mean. To be on the safe side here, we decide to show cumulatively how many times, out of 500 simulations, our models are able to pass statistical tests at 0.01 significance. This will also help us to better anticipate the differences between the models for 1% VaR, as there were a lot of zero median p-values, which are obviously incomparable.

As the differences between models for 5% and 10% VaR are already interpreted, we shall focus our attention to number of null-hypothesis non-rejections at 1% VaR. Finally we have enough data to make proper comparison of the models. Regarding UC and CC test, highest passing rate has Equally Weighted model. When it comes to BCP, the best model is once again Implied Volatility / EWMA, as it is able to capture exceedances at lags higher than one, in roughly $40\% - 45\%$ of the total number of simulations.

Again, we use conditional formatting to emphasize the differences. Values above 400 represent more than 80% of the total number of simulations, and are thus marked with green. On the other hand, we use yellow color to present values between 250 and 400 which is the range of 50-80%. Finally, light yellow is used for those cases when the number of simulations that pass statistical test at 0.01 significance is between 100 and 250, which in percentage terms represents the range between 20% and 50%.

Table 7.2: Portfolio Weights Simulation - Non-Rejections of Null Hypothesis at 0.01 (out of 500)

Models	UC Non-Rejects	CC Non-Rejects	BCP1 Non-Rejects	BCP2 Non-Rejects	BCP3 Non-Rejects	BCP4 Non-Rejects	BCP5 Non-Rejects
VaR, Alpha = 0.01							
Equally Weighted	315	257	140	21	21	22	22
IV/Equally Weighted	182	88	101	98	107	110	113
EWMA	158	39	122	90	91	91	95
IV/EWMA	145	90	201	207	224	227	246
DCC GARCH	298	146	46	40	61	52	61
IV/DCC GARCH	213	106	90	90	99	100	108
VaR, Alpha = 0.05							
Equally Weighted	396	358	409	2	2	1	0
IV/Equally Weighted	341	368	499	391	417	346	378
EWMA	196	214	482	119	172	76	96
IV/EWMA	272	316	497	471	481	458	456
DCC GARCH	345	348	472	64	75	10	16
IV/DCC GARCH	381	399	499	452	470	362	389
VaR, Alpha = 0.1							
Equally Weighted	422	300	225	0	0	0	0
IV/Equally Weighted	397	406	497	296	230	135	184
EWMA	372	384	499	204	161	70	89
IV/EWMA	382	401	498	427	458	423	438
DCC GARCH	415	407	473	11	2	0	0
IV/DCC GARCH	372	385	499	349	277	187	235

One remark must be made here as there are some simulations that produce NA p-value for BCP tests at 0.01 significance. When this occur, such observation is removed from the sample and not counted. This approach might be conservative since NA value means that there is zero or only one exceedance, thus obviously there cannot be autocorrelation of exceedances. However, there are only a few such cases (out of 500) so counting NA as a *failure* will not create large negative bias in our results.

Even though in the previous section we mostly observed just the marginal increase of BCP test p-values, after performing several hundred simulations we can be confident that models which incorporate implied volatility adjustment are indeed better in avoiding VaR exceedance clustering.

The main problem of the methods which use historical volatility is that they are always somewhat slow to pick up changes in market conditions, since they are backward looking. On the other hand, implied volatility being forward looking, is expected to perform better in the area of exceedance clustering reduction, and this claim was backed by the simulation results.

When it comes to UC and CC test, most of the volatility adjusted models at 5% and 10% VaR do not have a problem with passing them. Moreover, there is no obvious difference between the hybrid models and the regular ones. At 1% significance, both types of models are conservative and overestimating VaR. Thus, we cannot claim that there exists any difference between the models, when the number of exceedances is considered as the criteria for comparison.

Again, we invite the reader to check complete simulation results that are provided in the **Appendix B**.

8 Conclusion

The main purpose of this work was to investigate whether the current covariance estimation and forecasting methods within Value-at-Risk framework could be improved, by incorporating implied volatility data obtained from the options market. If the implied volatility is indeed a true *forward looking* measure, then VaR exceedance clustering reduction is the area where it is most likely to get some improvement.

Each time we estimate variances and covariances, we need to decide how *reactive* our model should be. On one hand, slow reactive methods like equally weighted which assumes unconditional variance within defined window size, gives us smooth forecasts and we can extract some general knowledge about the underlying process. Unfortunately, during turbulent periods when we desperately need to have good forecasts, it shows poor performance because it cannot adapt to turbulent market conditions quickly enough.

On the other hand, more reactive models like EWMA and GARCH, of which the latter assumes conditional variance, are more closely following current regime of the market. By choosing parameters like λ for EWMA, we decide how reactive we wish our model to be. This parameter is obviously arbitrary. Even with GARCH models which use maximum likelihood to estimate optimal values for parameters, by choosing size of the rolling window and how often we refit the model, we implicitly determine how reactive it would be. Thus, we are always exposed to model risk. The nice thing about implied volatilities is that we do not need to make any assumptions in that sense. Implied volatility always represents some general market consensus about the *future* volatility of the underlying asset, until the options expiry date.

As this research is based on portfolio of ten (obviously) correlated stocks from the US market, vector of implied volatilities with these stocks as the underlyings was not enough. We had to find the way to fill non-diagonal elements of implied covariance matrices. True implied covariances are available only with FX options and exotic options with two (or more) stocks as an underlying. None of these two are applicable to this work. Thus, new *hybrid* type of covariance matrix was created, based on implied volatilities and historical correlations.

To summarize this part, we use three types of *traditional* covariance estimation methods: equally weighted, EWMA and DCC GARCH. Additionally, correlations were isolated from these well know methods and combined with implied volatilities, to create three new types of *hybrid* covariance matrices: IV/equally weighted, IV/EWMA and IV/DCC GARCH. Thus, in total we have six different covariance matrix estimation techniques.

Equally weighted, IV/equally weighted, EWMA and IV/EWMA were used with parametric (normal and Student's t) VaR to estimate volatility of the entire portfolio. Multivariate DCC GARCH and IV/DCC GARCH were added when it comes to refining historical returns using Cholesky decomposition method.

Short remark must be made whether implied volatility should be used or not with parametric VaR. Implied volatility was derived by *inverting* Black-Sholes formula which assumes constant volatility, obviously wrong due to the volatility smile effect. However, since we are always obtaining IV derived from ATM options, thus from one specific point on the volatility smile, we can consider implied volatility unconditional and use it with parametric VaR models, which is not the case with GARCH method.

As part of evaluation process, equal weights were assigned to stocks that constitutes the portfolio and time-series of portfolio returns was created based on the actual daily returns within 10-year period (2007-2016). Value-at-Risk was estimated using out-of-sample data with eight parametric (normal and Student's t) and seven historical (no adjustment and Cholesky decomposition adjusted) models and confronted with the actual time-series of realized portfolio returns.

We backtested and compared VaR models that use purely historical data and well known volatility estimation methods, with its peers that incorporate implied volatility adjustment. Statistical tests were performed on a 10-year period as a whole, but also separately for each year. As 1-day VaR is estimated, we use daily frequency and three significance levels: 0.01, 0.05 and 0.1.

Results showed mostly marginal improvement of BCP statistical test p-values for *hybrid* models that use implied volatility, usually at lags higher than one. This indicates that the main contribution of *hybrid* models, if proved significant, lies in the reduction of VaR exceedance clustering at higher lags, as it was our initial assumption.

At 1% VaR, the best performing model was the parametric Student's t IV/EWMA, which was able to pass BCP test at all lags with p-values significantly higher than 0.05. On the other hand, at 5% significance VaR, the model that showed the highest p-values was the historical volatility adjusted IV/GARCH, although only up to lag three. Finally, at 10% VaR, parametric normal IV/EWMA was superior comparing to others as it has BCP p-values higher than 0.05 at all lags up to five.

When it comes to UC and CC test p-values, we have not observed any meaningful difference between the models that use standard covariance estimation methods and the *hybrid* ones that incorporate implied volatility. The reason is mainly because most of the historical volatility adjusted models that use purely historical data already showed solid performance, thus there was not much space for improvement.

These statistical test results were generated based on one specific vector of equal portfolio weights. Thus, a simulation of portfolio weights was performed by generating 500 different time-series of portfolio returns. BCP test results significantly improved when compared to the previous section where only one portfolio was observed. On the other hand, UC and CC test results were mostly consistent with the equally weighted portfolio.

For VaR 5% and 10%, IV/EWMA was able to pass UC and CC tests at 0.01 significance at more than 50% of the total number of simulations. When it comes to BCP test results are much better, IV/EWMA model was able to pass tests (from lag 1 to lag 5) at roughly 90% of all simulations.

For VaR 1%, best results with UC and CC test are present with historical adjusted equally weighted method, mainly because other models are too conservative. However, when it comes to BCP test, IV/EWMA is again dominant when compared to others, with passing rate between 40% - 45% of the total number of simulations.

This gives us enough confidence that when it comes to VaR exceedance clustering, our *hybrid* models are indeed, on average, better than its counterparties that does not incorporate the implied volatility innovation. Moreover, we have not observed any significant change in p-values derived from UC and CC tests. This indicates that hybrid models are reducing exceedance clustering, but not at the expense of some other parameters, such as the total number of VaR exceedances.

A possible extension of this work could be to combine implied volatility with other types of multivariate GARCH covariance estimation methods such as orthogonal GARCH (O-GARCH) or factor GARCH (F-GARCH). Also, instead of using $\lambda = 0.94$ with EWMA method as proposed by RiskMetrics[1], one could use some other parameter value set for λ.

Moreover, this work uses ATM options with nearest expiration. It would be interesting to observe the results when implied volatility is *taken* from some other section of volatility surface, for instance from deep out-of-the money put options with longer expiration.

[1] 1996 RiskMetrics^{TM} Technical Document

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A Appendix: Backtesting Results

Backtesting results for the following models:

1. Equally Weighted (n=250), Parametric (Normal) Total VaR
2. Equally Weighted (n=250), Parametric (Student-t) Total VaR
3. Implied Vol / Equally Weighted (n=250), Parametric (Normal) Total VaR
4. Implied Vol / Equally Weighted (n=250), Parametric (Student-t) Total VaR
5. EWMA (Lambda = 0.94), Parametric (Normal) Total VaR
6. EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR
7. Implied Vol / EWMA (Lambda = 0.94), Parametric (Normal) Total VaR
8. Implied Vol / EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR
9. Historical (No Adjustment) VaR
10. Historical (Cholesky - Equally Weighted) VaR
11. Historical (Cholesky - Implied Vol / Equally Weighted) VaR
12. Historical (Cholesky - EWMA 0.94) VaR
13. Historical (Cholesky - Implied Vol / EWMA 0.94) VaR
14. Historical (Cholesky - DCC GARCH) VaR
15. Historical (Cholesky - Implied Vol / DCC GARCH) VaR

Backtest period: **01-01-2007 / 31-12-2016 (10-years)**

Significance levels: **0.01, 0.05, 0.10**

Portfolio Weights (Equally Weighted Portfolio):

AAPL	AMZN	BAC	F	GE	INTC	KO	MSFT	PFE	T
0.10000	**0.10000**	**0.10000**	**0.10000**	**0.10000**	**0.10000**	**0.10000**	**0.10000**	**0.10000**	**0.10000**

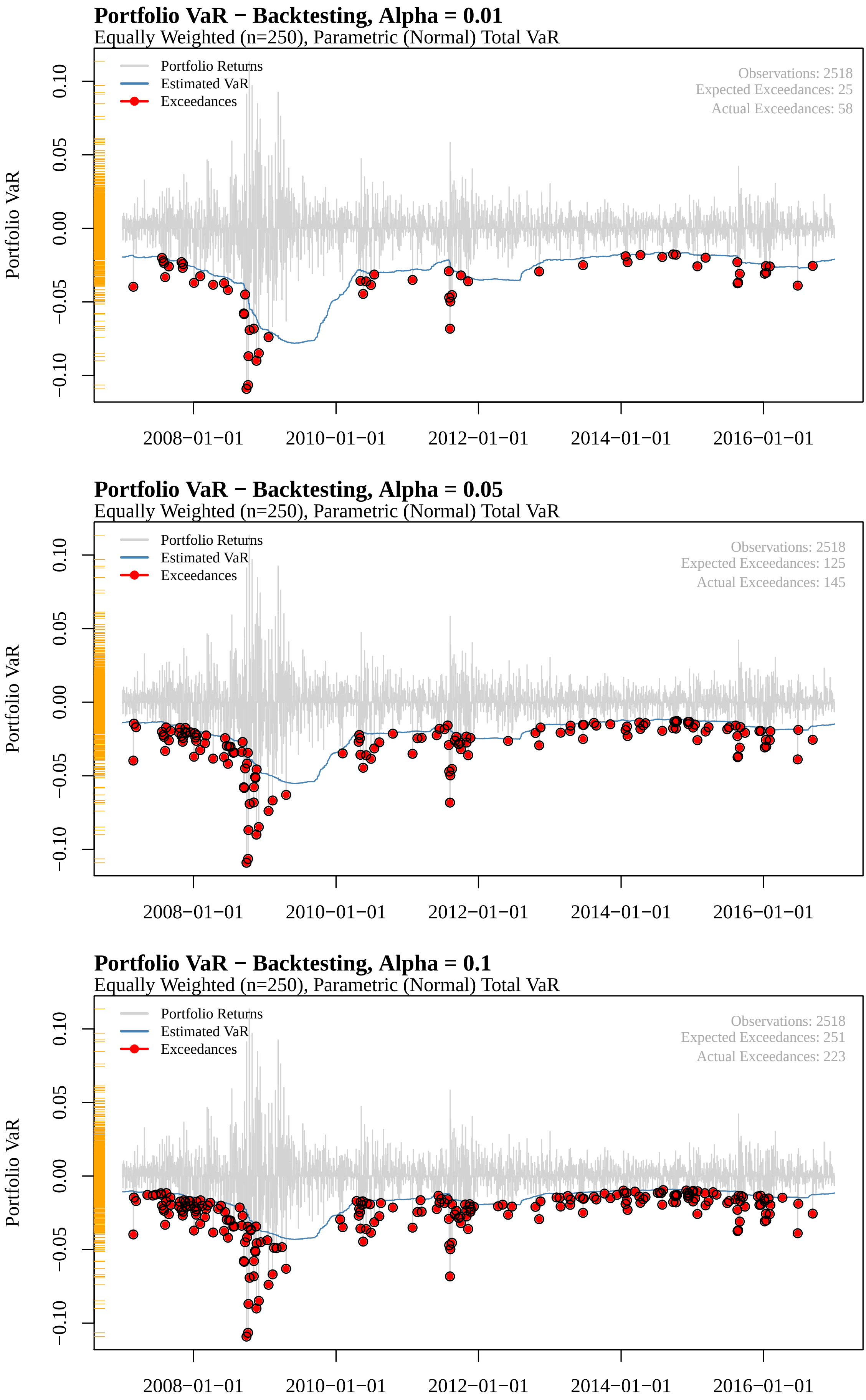


Figure A.1: Equally Weighted (n=250), Parametric (Normal) Total VaR

Table A.1: Equally Weighted (n=250), Parametric (Normal) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.001	0.004	0.552	0.387	0.329	0.011	0.013
2008	0.000	0.000	0.311	0.273	0.452	0.616	0.494
2009	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2010	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2011	0.006	0.017	0.599	0.000	0.000	0.000	0.000
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.061	0.150	0.696	0.857	0.927	0.961	0.978
2015	0.061	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.061	0.150	0.696	0.060	0.123	0.021	0.039
ALL	**0.000**	**0.000**	**0.557**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.081	0.046	0.190	0.157	0.028	0.026	0.047
2008	0.000	0.000	0.648	0.477	0.087	0.049	0.026
2009	0.001	0.004	0.847	0.963	0.990	0.997	0.999
2010	0.637	0.540	0.464	0.054	0.091	0.132	0.174
2011	0.014	0.035	0.394	0.485	0.278	0.162	0.109
2012	0.004	0.016	0.795	0.934	0.977	0.991	0.011
2013	0.274	0.330	0.213	0.004	0.005	0.011	0.020
2014	0.141	0.124	0.102	0.000	0.001	0.000	0.000
2015	0.345	0.414	0.296	0.579	0.778	0.704	0.825
2016	0.155	0.181	0.125	0.094	0.069	0.047	0.079
ALL	**0.088**	**0.037**	**0.038**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 10%							
2007	0.422	0.706	0.821	0.257	0.147	0.092	0.152
2008	0.000	0.000	0.749	0.947	0.781	0.846	0.921
2009	0.000	0.000	0.647	0.810	0.261	0.108	0.167
2010	0.175	0.355	0.610	0.058	0.053	0.094	0.078
2011	0.238	0.401	0.491	0.626	0.544	0.235	0.323
2012	0.000	0.000	0.645	0.808	0.887	0.930	0.491
2013	0.021	0.071	0.907	0.456	0.373	0.381	0.384
2014	0.326	0.416	0.354	0.067	0.052	0.000	0.000
2015	0.867	0.131	0.024	0.049	0.103	0.122	0.194
2016	0.002	0.002	0.039	0.014	0.005	0.001	0.002
ALL	**0.052**	**0.003**	**0.002**	**0.000**	**0.000**	**0.000**	**0.000**

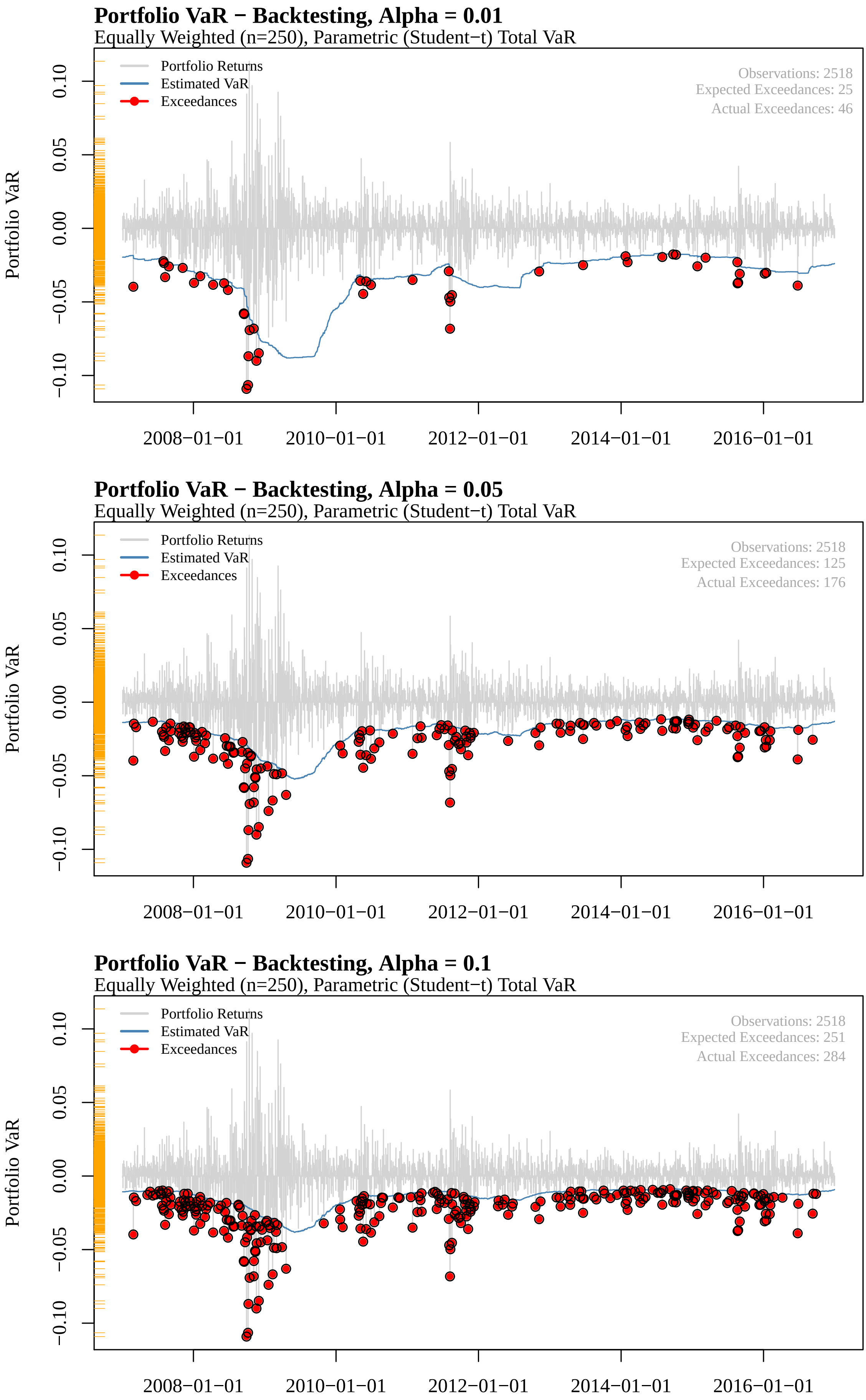


Figure A.2: Equally Weighted (n=250), Parametric (Student-t) Total VaR

Table A.2: Equally Weighted (n=250), Parametric (Student-t) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.060	0.148	0.695	0.857	0.124	0.024	0.044
2008	0.000	0.000	0.347	0.215	0.278	0.412	0.446
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.388	0.646	0.796	0.935	0.977	0.992	0.997
2011	0.061	0.150	0.696	0.000	0.000	0.000	0.000
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2015	0.061	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.000**	**0.001**	**0.197**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.003	0.012	0.823	0.452	0.030	0.033	0.058
2008	0.000	0.000	0.615	0.794	0.441	0.297	0.198
2009	0.078	0.174	0.647	0.810	0.261	0.108	0.167
2010	0.691	0.404	0.346	0.217	0.156	0.117	0.190
2011	0.000	0.000	0.686	0.852	0.471	0.268	0.385
2012	0.004	0.016	0.795	0.934	0.977	0.991	0.011
2013	0.908	0.919	0.672	0.209	0.108	0.143	0.175
2014	0.048	0.007	0.003	0.000	0.000	0.000	0.000
2015	0.226	0.355	0.393	0.691	0.862	0.739	0.850
2016	0.274	0.054	0.002	0.004	0.005	0.005	0.005
ALL	**0.000**	**0.000**	**0.008**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 10%							
2007	0.048	0.126	0.634	0.235	0.136	0.086	0.133
2008	0.000	0.000	0.709	0.933	0.684	0.799	0.867
2009	0.021	0.071	0.907	0.058	0.010	0.011	0.000
2010	0.867	0.692	0.371	0.194	0.124	0.084	0.021
2011	0.001	0.003	0.870	0.761	0.812	0.686	0.743
2012	0.000	0.001	0.506	0.491	0.493	0.581	0.572
2013	0.175	0.355	0.610	0.772	0.012	0.025	0.024
2014	0.078	0.152	0.406	0.279	0.359	0.016	0.032
2015	0.169	0.103	0.082	0.176	0.216	0.326	0.436
2016	0.021	0.036	0.188	0.021	0.022	0.019	0.037
ALL	**0.036**	**0.005**	**0.010**	**0.000**	**0.000**	**0.000**	**0.000**

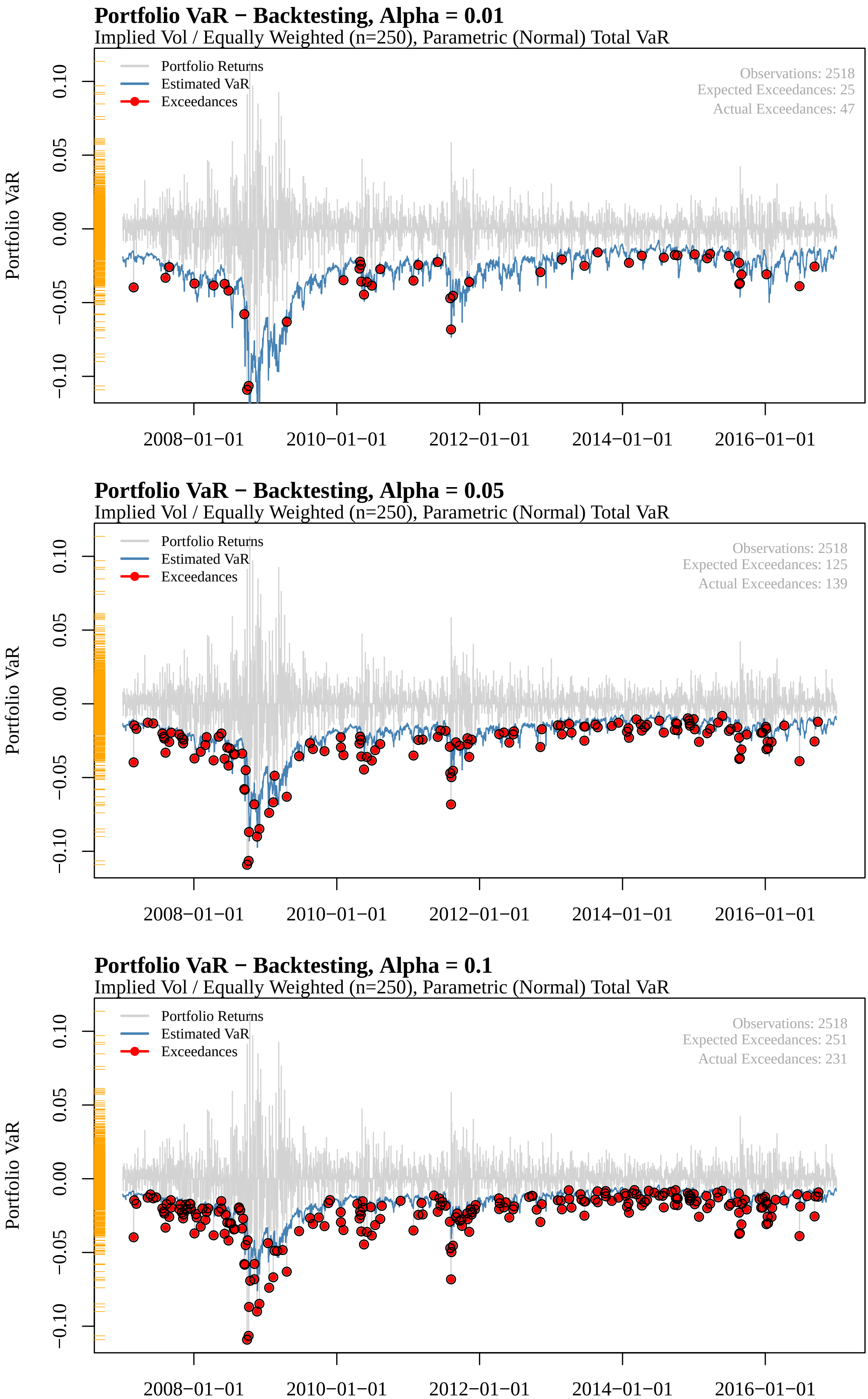


Figure A.3: Implied Vol / Equally Weighted (n=250), Parametric (Normal) Total VaR

Table A.3: IV / Equally Weighted (n=250), Parametric (Normal) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.763	0.921	0.847	0.963	0.990	0.997	0.999
2008	0.020	0.055	0.648	0.810	0.901	0.947	0.970
2009	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2010	0.001	0.005	0.553	0.007	0.010	0.011	0.012
2011	0.020	0.054	0.647	0.151	0.262	0.378	0.489
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.768	0.923	0.847	0.963	0.990	0.997	0.999
2014	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2015	0.006	0.001	0.000	0.000	0.001	0.003	0.006
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.000**	**0.000**	**0.222**	**0.001**	**0.004**	**0.011**	**0.022**
Alpha = 5%							
2007	0.491	0.303	0.309	0.593	0.460	0.390	0.531
2008	0.007	0.003	0.108	0.218	0.300	0.377	0.518
2009	0.155	0.280	0.599	0.757	0.840	0.524	0.624
2010	0.861	0.851	0.551	0.115	0.197	0.284	0.372
2011	0.226	0.475	0.879	0.175	0.182	0.233	0.221
2012	0.083	0.181	0.645	0.154	0.266	0.382	0.171
2013	0.861	0.851	0.551	0.702	0.196	0.253	0.304
2014	0.084	0.212	0.742	0.314	0.274	0.047	0.077
2015	0.345	0.414	0.296	0.579	0.540	0.519	0.664
2016	0.436	0.480	0.286	0.014	0.020	0.023	0.026
ALL	**0.238**	**0.482**	**0.796**	**0.000**	**0.001**	**0.000**	**0.001**
Alpha = 10%							
2007	0.851	0.425	0.244	0.344	0.206	0.255	0.348
2008	0.021	0.030	0.218	0.357	0.554	0.592	0.726
2009	0.011	0.017	0.346	0.409	0.603	0.750	0.851
2010	0.113	0.233	0.498	0.633	0.714	0.771	0.486
2011	0.435	0.719	0.827	0.580	0.609	0.740	0.739
2012	0.012	0.042	0.798	0.937	0.521	0.678	0.795
2013	0.175	0.181	0.156	0.134	0.024	0.024	0.023
2014	0.169	0.388	0.978	0.508	0.643	0.138	0.199
2015	0.966	0.562	0.254	0.461	0.617	0.613	0.611
2016	0.069	0.132	0.356	0.102	0.036	0.073	0.128
ALL	**0.162**	**0.303**	**0.503**	**0.028**	**0.006**	**0.002**	**0.004**

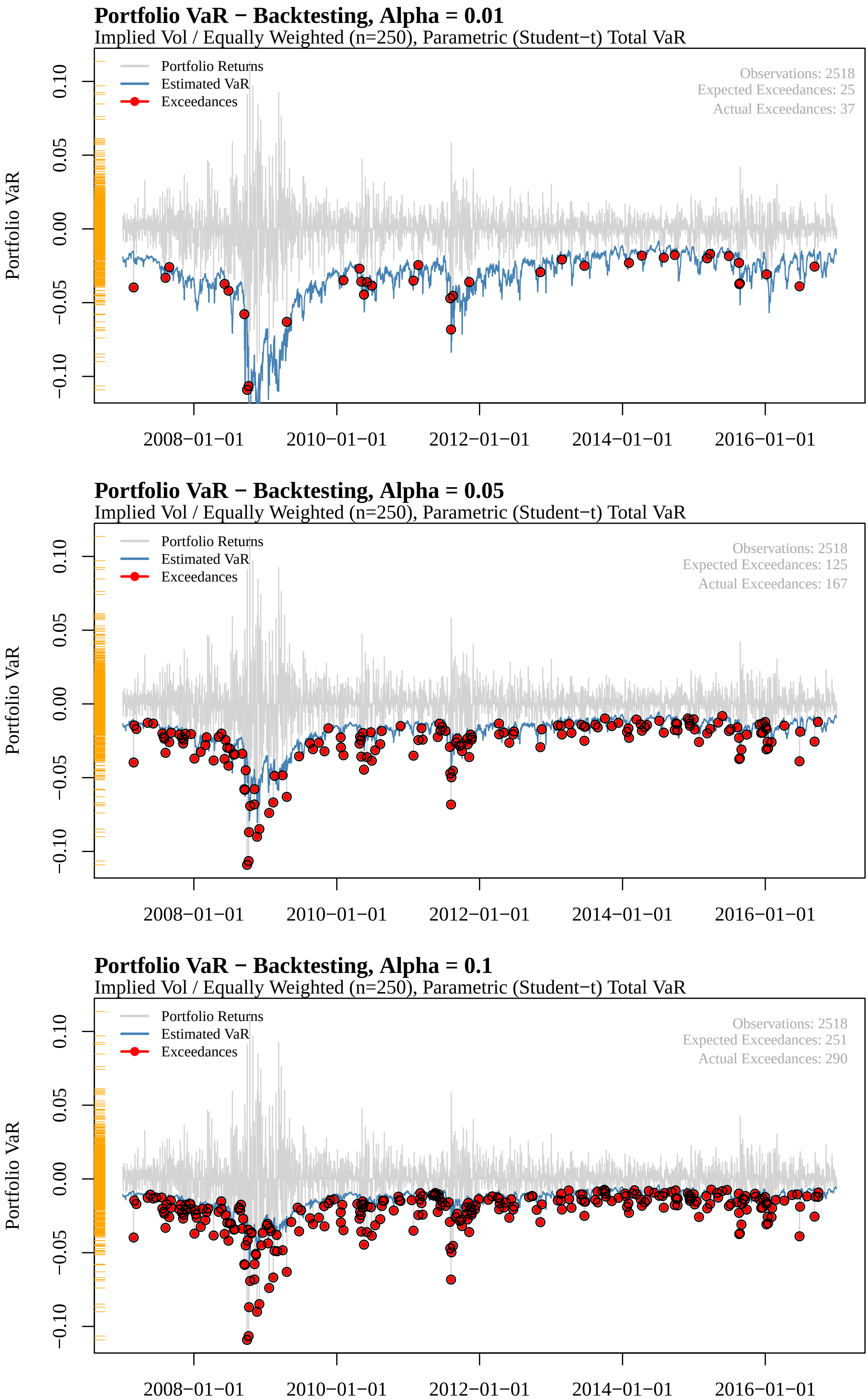


Figure A.4: Implied Vol / Equally Weighted (n=250), Parametric (Student-t) Total VaR

Table A.4: IV / Equally Weighted (n=250), Parametric (Student-t) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.763	0.921	0.847	0.963	0.990	0.997	0.999
2008	0.168	0.350	0.746	0.900	0.957	0.980	0.991
2009	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2010	0.061	0.150	0.696	0.857	0.927	0.961	0.978
2011	0.061	0.150	0.696	0.060	0.123	0.204	0.297
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2014	0.388	0.646	0.796	0.935	0.977	0.992	0.997
2015	0.061	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.027**	**0.026**	**0.045**	**0.018**	**0.035**	**0.057**	**0.083**
Alpha = 5%							
2007	0.220	0.136	0.245	0.502	0.554	0.593	0.727
2008	0.001	0.001	0.249	0.503	0.671	0.651	0.754
2009	0.637	0.540	0.464	0.584	0.654	0.692	0.731
2010	0.345	0.640	0.983	0.581	0.538	0.518	0.662
2011	0.001	0.003	0.834	0.659	0.783	0.764	0.834
2012	0.163	0.190	0.127	0.097	0.176	0.264	0.181
2013	0.691	0.895	0.791	0.933	0.512	0.521	0.527
2014	0.084	0.212	0.742	0.314	0.274	0.047	0.077
2015	0.084	0.092	0.137	0.319	0.297	0.438	0.571
2016	0.637	0.158	0.018	0.004	0.007	0.011	0.016
ALL	**0.000**	**0.001**	**0.347**	**0.025**	**0.058**	**0.003**	**0.008**
Alpha = 10%							
2007	0.693	0.339	0.205	0.174	0.147	0.210	0.281
2008	0.000	0.000	0.149	0.293	0.466	0.546	0.474
2009	0.800	0.562	0.341	0.557	0.747	0.727	0.080
2010	0.708	0.735	0.468	0.295	0.227	0.183	0.019
2011	0.000	0.001	0.900	0.940	0.979	0.660	0.776
2012	0.122	0.290	0.774	0.771	0.373	0.522	0.652
2013	0.640	0.381	0.148	0.124	0.029	0.044	0.061
2014	0.169	0.388	0.978	0.508	0.643	0.138	0.199
2015	0.238	0.491	0.863	0.701	0.858	0.924	0.870
2016	0.113	0.222	0.456	0.179	0.087	0.160	0.254
ALL	**0.013**	**0.044**	**0.756**	**0.016**	**0.002**	**0.000**	**0.000**

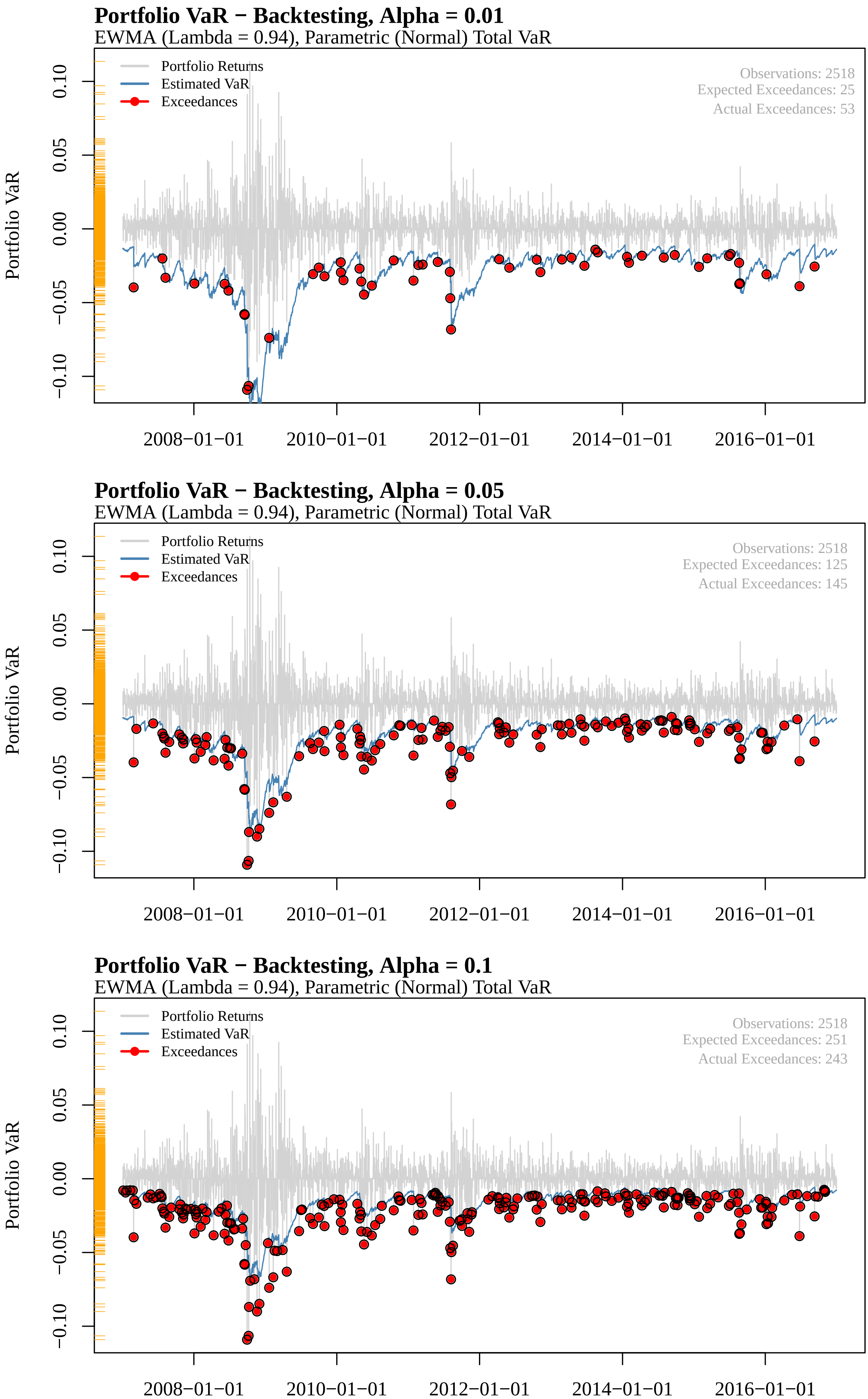


Figure A.5: EWMA (Lambda = 0.94), Parametric (Normal) Total VaR

Table A.5: EWMA (Lambda = 0.94), Parametric (Normal) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.763	0.921	0.847	0.963	0.990	0.997	0.999
2008	0.020	0.055	0.648	0.149	0.266	0.390	0.510
2009	0.388	0.646	0.796	0.935	0.977	0.992	0.997
2010	0.006	0.011	0.125	0.267	0.404	0.524	0.624
2011	0.020	0.054	0.647	0.000	0.000	0.000	0.000
2012	0.380	0.638	0.795	0.934	0.977	0.991	0.997
2013	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2014	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2015	0.020	0.002	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.000**	**0.000**	**0.068**	**0.004**	**0.006**	**0.015**	**0.019**
Alpha = 5%							
2007	0.873	0.539	0.422	0.609	0.720	0.259	0.344
2008	0.027	0.013	0.146	0.203	0.320	0.124	0.181
2009	0.274	0.393	0.553	0.386	0.519	0.621	0.700
2010	0.226	0.475	0.879	0.688	0.689	0.702	0.716
2011	0.141	0.084	0.217	0.124	0.125	0.014	0.015
2012	0.657	0.717	0.437	0.547	0.091	0.132	0.176
2013	0.691	0.895	0.791	0.616	0.372	0.399	0.417
2014	0.014	0.035	0.460	0.197	0.129	0.000	0.000
2015	0.908	0.364	0.086	0.203	0.278	0.339	0.446
2016	0.155	0.280	0.599	0.267	0.404	0.240	0.334
ALL	**0.088**	**0.155**	**0.387**	**0.004**	**0.013**	**0.000**	**0.000**
Alpha = 10%							
2007	0.422	0.016	0.037	0.104	0.208	0.223	0.273
2008	0.121	0.032	0.064	0.117	0.214	0.244	0.337
2009	0.113	0.071	0.217	0.372	0.560	0.709	0.587
2010	0.259	0.456	0.607	0.418	0.601	0.504	0.442
2011	0.169	0.341	0.599	0.761	0.292	0.408	0.550
2012	0.387	0.546	0.524	0.489	0.186	0.263	0.337
2013	0.175	0.355	0.610	0.772	0.115	0.191	0.162
2014	0.238	0.401	0.491	0.141	0.138	0.001	0.002
2015	0.365	0.424	0.302	0.322	0.459	0.332	0.424
2016	0.175	0.163	0.137	0.019	0.016	0.034	0.063
ALL	**0.557**	**0.715**	**0.574**	**0.002**	**0.002**	**0.000**	**0.001**

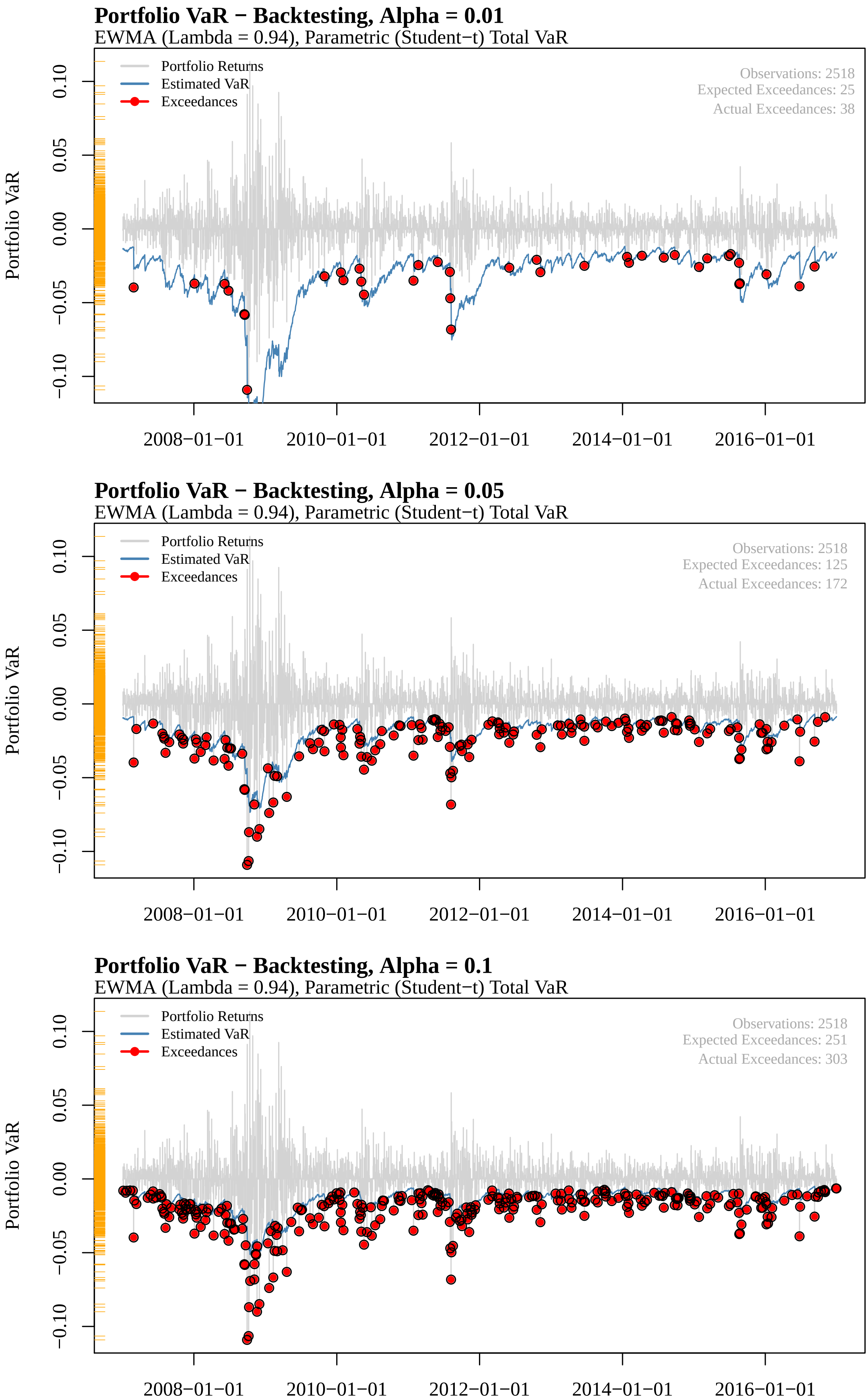


Figure A.6: EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR

Table A.6: EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.276	0.550	0.949	0.996	1.000	1.000	1.000
2008	0.062	0.152	0.696	0.059	0.124	0.209	0.308
2009	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2010	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2011	0.061	0.150	0.696	0.000	0.000	0.000	0.000
2012	0.758	0.919	0.846	0.963	0.990	0.997	0.999
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2015	0.020	0.002	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.017**	**0.019**	**0.056**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.873	0.539	0.422	0.609	0.720	0.259	0.344
2008	0.014	0.006	0.126	0.215	0.316	0.166	0.223
2009	0.691	0.404	0.346	0.619	0.795	0.896	0.846
2010	0.084	0.207	0.690	0.813	0.881	0.616	0.460
2011	0.000	0.001	0.937	0.993	0.587	0.695	0.731
2012	0.669	0.886	0.798	0.334	0.229	0.357	0.488
2013	0.500	0.791	0.907	0.987	0.664	0.619	0.589
2014	0.014	0.035	0.460	0.197	0.129	0.000	0.000
2015	0.691	0.423	0.141	0.320	0.382	0.425	0.555
2016	0.861	0.279	0.046	0.115	0.181	0.248	0.314
ALL	**0.000**	**0.000**	**0.813**	**0.074**	**0.138**	**0.000**	**0.001**
Alpha = 10%							
2007	0.048	0.081	0.315	0.512	0.627	0.710	0.684
2008	0.013	0.016	0.176	0.331	0.523	0.459	0.525
2009	0.708	0.376	0.234	0.488	0.630	0.729	0.798
2010	0.435	0.719	0.827	0.580	0.287	0.168	0.105
2011	0.000	0.001	0.687	0.823	0.525	0.286	0.382
2012	1.000	0.934	0.718	0.200	0.230	0.239	0.242
2013	0.966	0.609	0.284	0.116	0.059	0.071	0.081
2014	0.238	0.401	0.491	0.141	0.138	0.001	0.002
2015	0.966	0.242	0.064	0.160	0.209	0.246	0.275
2016	0.640	0.654	0.411	0.045	0.027	0.052	0.085
ALL	**0.001**	**0.004**	**0.618**	**0.007**	**0.004**	**0.001**	**0.001**

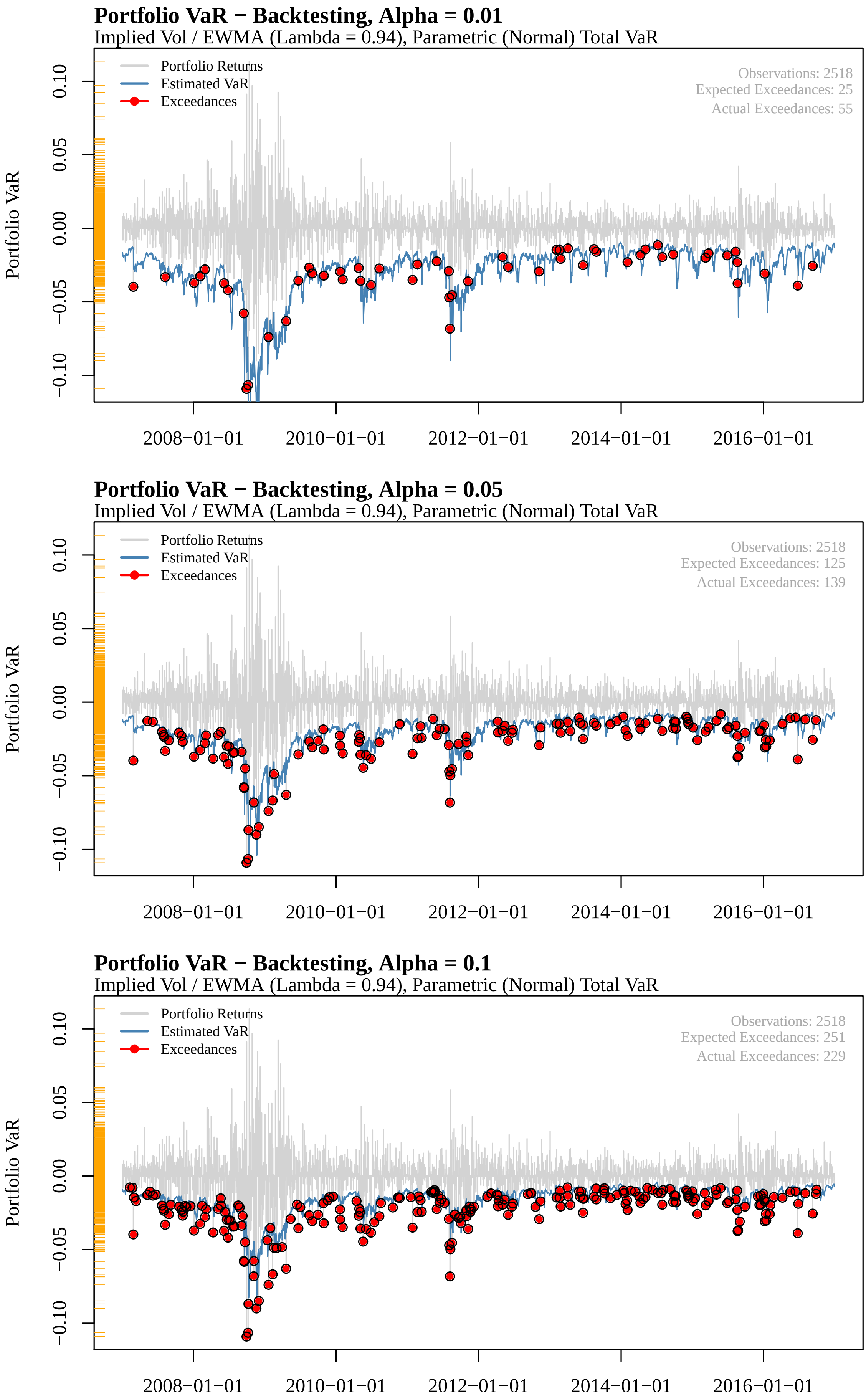


Figure A.7: Implied Vol / EWMA (Lambda = 0.94), Parametric (Normal) Total VaR

Table A.7: IV / EWMA (Lambda = 0.94), Parametric (Normal) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.737	0.930	0.898	0.984	0.997	0.999	1.000
2008	0.006	0.017	0.600	0.758	0.857	0.911	0.944
2009	0.061	0.150	0.696	0.857	0.927	0.961	0.978
2010	0.061	0.150	0.696	0.857	0.927	0.961	0.978
2011	0.006	0.017	0.599	0.001	0.004	0.003	0.006
2012	0.758	0.919	0.846	0.963	0.990	0.997	0.999
2013	0.020	0.054	0.647	0.810	0.261	0.377	0.488
2014	0.061	0.150	0.696	0.857	0.927	0.961	0.978
2015	0.061	0.051	0.020	0.061	0.124	0.205	0.298
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.000**	**0.000**	**0.851**	**0.745**	**0.891**	**0.956**	**0.859**
Alpha = 5%							
2007	0.647	0.543	0.463	0.582	0.639	0.136	0.180
2008	0.007	0.003	0.108	0.218	0.300	0.377	0.518
2009	0.436	0.488	0.508	0.487	0.596	0.577	0.647
2010	0.861	0.851	0.551	0.115	0.197	0.284	0.372
2011	0.226	0.475	0.879	0.175	0.182	0.080	0.083
2012	0.286	0.342	0.216	0.216	0.331	0.435	0.378
2013	0.908	0.919	0.672	0.622	0.272	0.321	0.359
2014	0.345	0.215	0.277	0.060	0.077	0.002	0.005
2015	0.345	0.414	0.296	0.579	0.540	0.519	0.664
2016	0.861	0.567	0.462	0.614	0.703	0.748	0.791
ALL	**0.238**	**0.401**	**0.522**	**0.016**	**0.034**	**0.002**	**0.005**
Alpha = 10%							
2007	0.506	0.096	0.124	0.306	0.500	0.548	0.691
2008	0.247	0.267	0.286	0.503	0.669	0.714	0.804
2009	0.259	0.093	0.167	0.336	0.482	0.601	0.722
2010	0.069	0.189	0.879	0.688	0.855	0.938	0.911
2011	0.116	0.273	0.713	0.916	0.586	0.739	0.836
2012	0.075	0.202	0.872	0.694	0.247	0.384	0.521
2013	0.365	0.175	0.063	0.031	0.016	0.029	0.023
2014	0.563	0.287	0.197	0.357	0.286	0.007	0.013
2015	0.640	0.341	0.130	0.227	0.397	0.478	0.543
2016	0.069	0.132	0.356	0.428	0.128	0.224	0.338
ALL	**0.125**	**0.307**	**0.968**	**0.225**	**0.354**	**0.244**	**0.342**

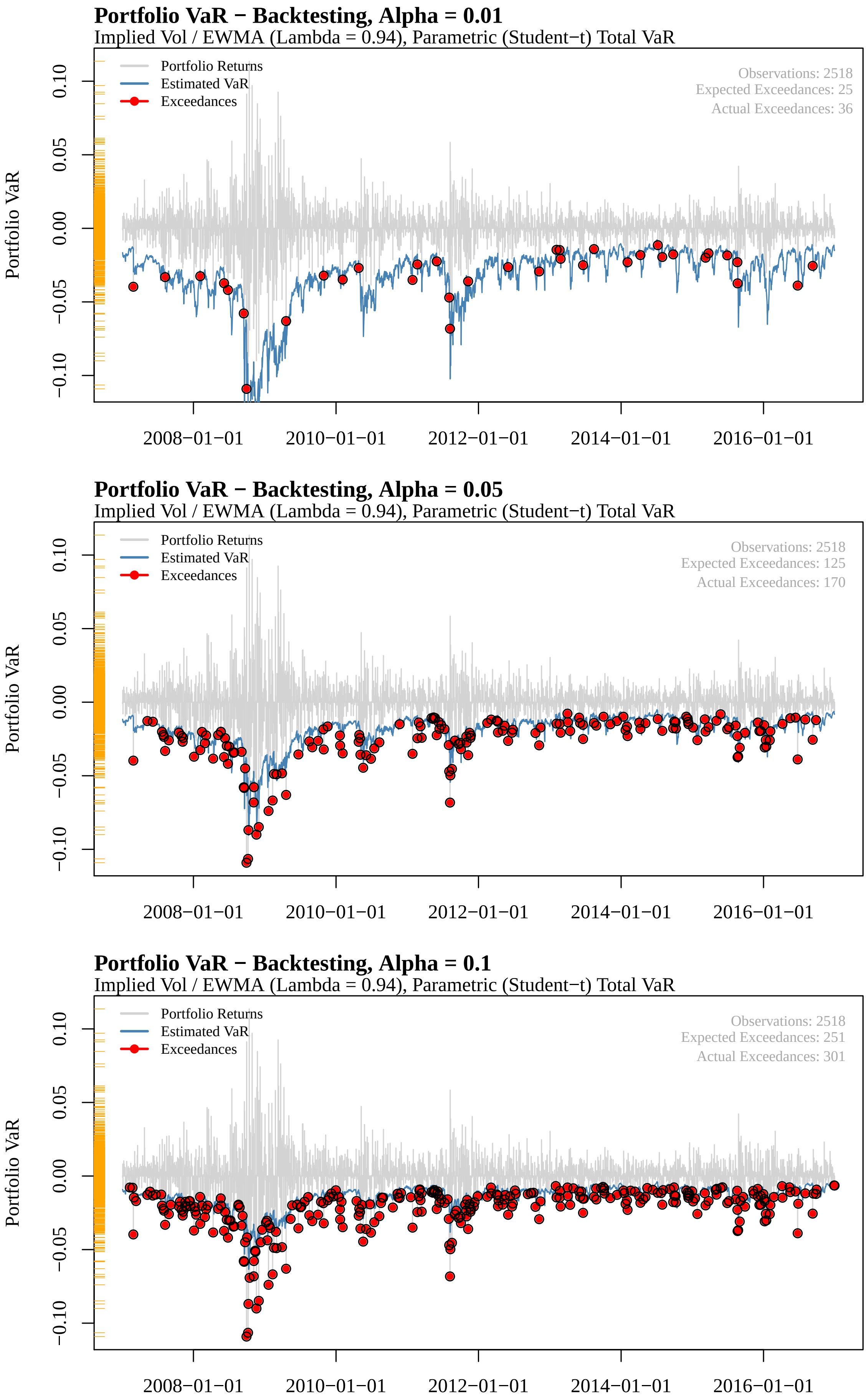


Figure A.8: Implied Vol / EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR

Table A.8: IV / EWMA (Lambda = 0.94), Parametric (Student-t) Total VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.737	0.930	0.898	0.984	0.997	0.999	1.000
2008	0.168	0.350	0.746	0.900	0.957	0.980	0.991
2009	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2010	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2011	0.061	0.150	0.696	0.060	0.123	0.204	0.297
2012	0.742	0.932	0.898	0.983	0.997	0.999	1.000
2013	0.166	0.346	0.746	0.899	0.031	0.062	0.105
2014	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2015	0.166	0.079	0.003	0.013	0.032	0.063	0.107
2016	0.733	0.928	0.898	0.984	0.997	0.999	1.000
ALL	**0.042**	**0.105**	**0.492**	**0.624**	**0.703**	**0.746**	**0.780**
Alpha = 5%							
2007	0.873	0.539	0.422	0.609	0.720	0.259	0.344
2008	0.001	0.001	0.249	0.503	0.671	0.805	0.876
2009	0.908	0.488	0.383	0.626	0.633	0.756	0.748
2010	0.908	0.919	0.672	0.209	0.347	0.481	0.601
2011	0.000	0.000	0.802	0.941	0.782	0.874	0.938
2012	0.885	0.918	0.678	0.214	0.355	0.400	0.520
2013	0.500	0.791	0.907	0.987	0.664	0.619	0.589
2014	0.226	0.475	0.879	0.175	0.182	0.011	0.017
2015	0.141	0.278	0.498	0.603	0.493	0.432	0.569
2016	0.908	0.516	0.421	0.642	0.254	0.358	0.422
ALL	**0.000**	**0.001**	**0.879**	**0.026**	**0.062**	**0.073**	**0.127**
Alpha = 10%							
2007	0.693	0.339	0.205	0.174	0.263	0.345	0.430
2008	0.001	0.001	0.135	0.271	0.453	0.603	0.629
2009	0.238	0.093	0.112	0.279	0.308	0.339	0.263
2010	0.966	0.506	0.291	0.537	0.500	0.074	0.124
2011	0.000	0.001	0.687	0.823	0.942	0.488	0.604
2012	0.670	0.909	0.924	0.800	0.481	0.518	0.544
2013	0.966	0.609	0.284	0.116	0.059	0.071	0.081
2014	0.326	0.592	0.778	0.300	0.358	0.030	0.056
2015	0.326	0.221	0.126	0.303	0.469	0.517	0.370
2016	0.493	0.776	0.845	0.003	0.005	0.012	0.023
ALL	**0.001**	**0.006**	**0.722**	**0.043**	**0.033**	**0.014**	**0.028**

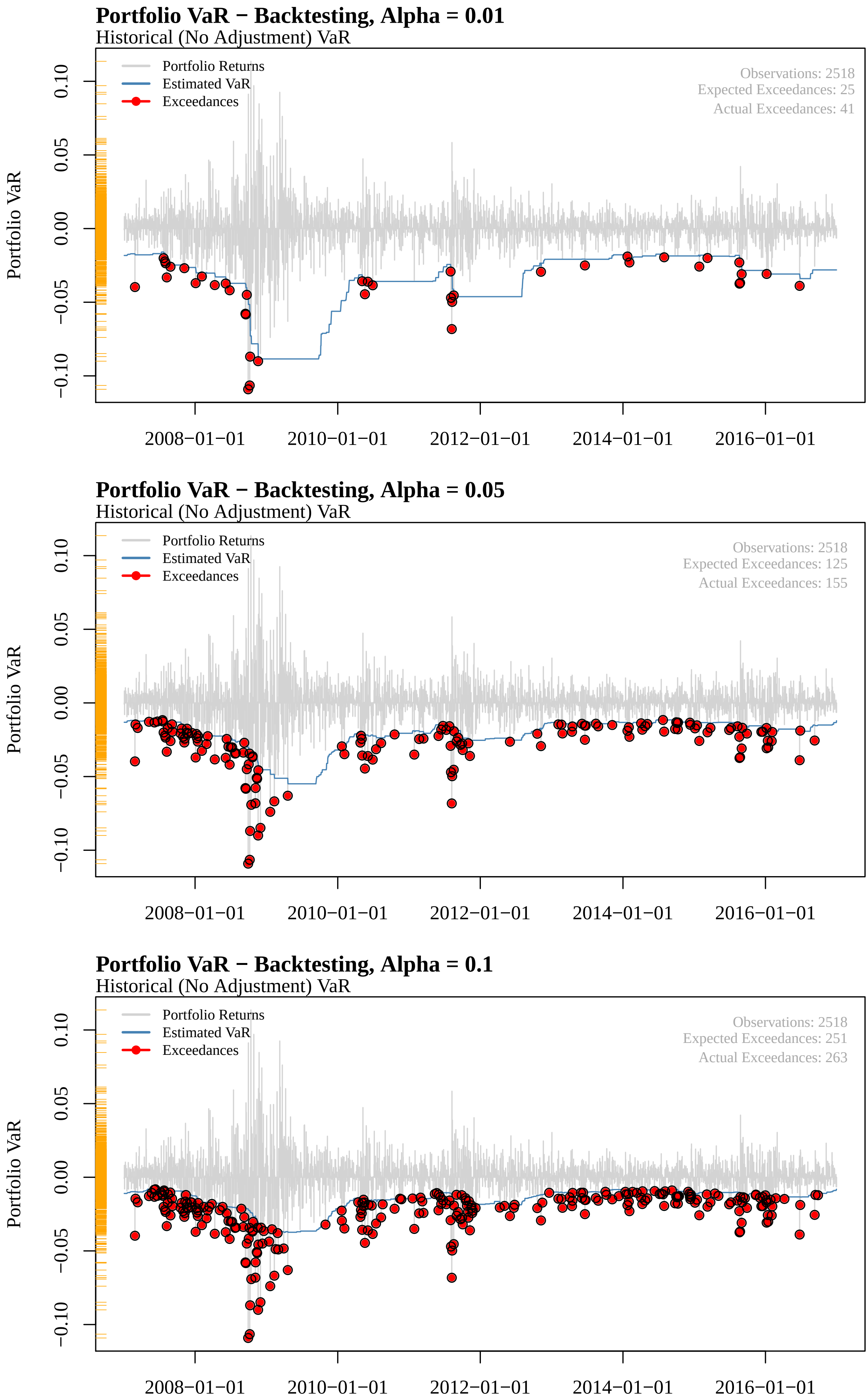


Figure A.9: Historical (No Adjustment)

Table A.9: Historical (No Adjustment) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.019	0.053	0.646	0.809	0.263	0.109	0.049
2008	0.000	0.000	0.424	0.098	0.166	0.228	0.077
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.388	0.646	0.796	0.935	0.977	0.992	0.997
2011	0.166	0.346	0.746	0.000	0.000	0.000	0.000
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.768	0.923	0.847	0.963	0.990	0.997	0.999
2015	0.061	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.733	0.928	0.898	0.984	0.997	1.000	1.000
ALL	**0.004**	**0.006**	**0.097**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.001	0.000	0.077	0.010	0.016	0.022	0.040
2008	0.000	0.000	0.827	0.718	0.246	0.107	0.045
2009	0.001	0.004	0.847	0.963	0.990	0.997	0.999
2010	0.861	0.539	0.423	0.099	0.174	0.256	0.341
2011	0.014	0.035	0.394	0.485	0.278	0.162	0.109
2012	0.001	0.004	0.846	0.963	0.990	0.997	0.999
2013	0.861	0.851	0.551	0.115	0.040	0.062	0.086
2014	0.345	0.414	0.296	0.337	0.536	0.015	0.031
2015	0.345	0.414	0.296	0.579	0.778	0.704	0.825
2016	0.274	0.054	0.002	0.004	0.005	0.005	0.005
ALL	**0.010**	**0.008**	**0.060**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 10%							
2007	0.018	0.060	0.791	0.201	0.039	0.009	0.018
2008	0.000	0.000	0.306	0.583	0.360	0.515	0.586
2009	0.000	0.001	0.317	0.486	0.486	0.487	0.564
2010	0.640	0.381	0.148	0.124	0.100	0.153	0.008
2011	0.002	0.008	0.746	0.795	0.771	0.493	0.637
2012	0.000	0.000	0.551	0.389	0.522	0.623	0.528
2013	0.175	0.355	0.610	0.772	0.012	0.025	0.024
2014	0.116	0.197	0.355	0.220	0.277	0.008	0.017
2015	0.708	0.370	0.145	0.238	0.377	0.352	0.487
2016	0.021	0.036	0.188	0.021	0.022	0.019	0.037
ALL	**0.460**	**0.031**	**0.008**	**0.000**	**0.000**	**0.000**	**0.000**

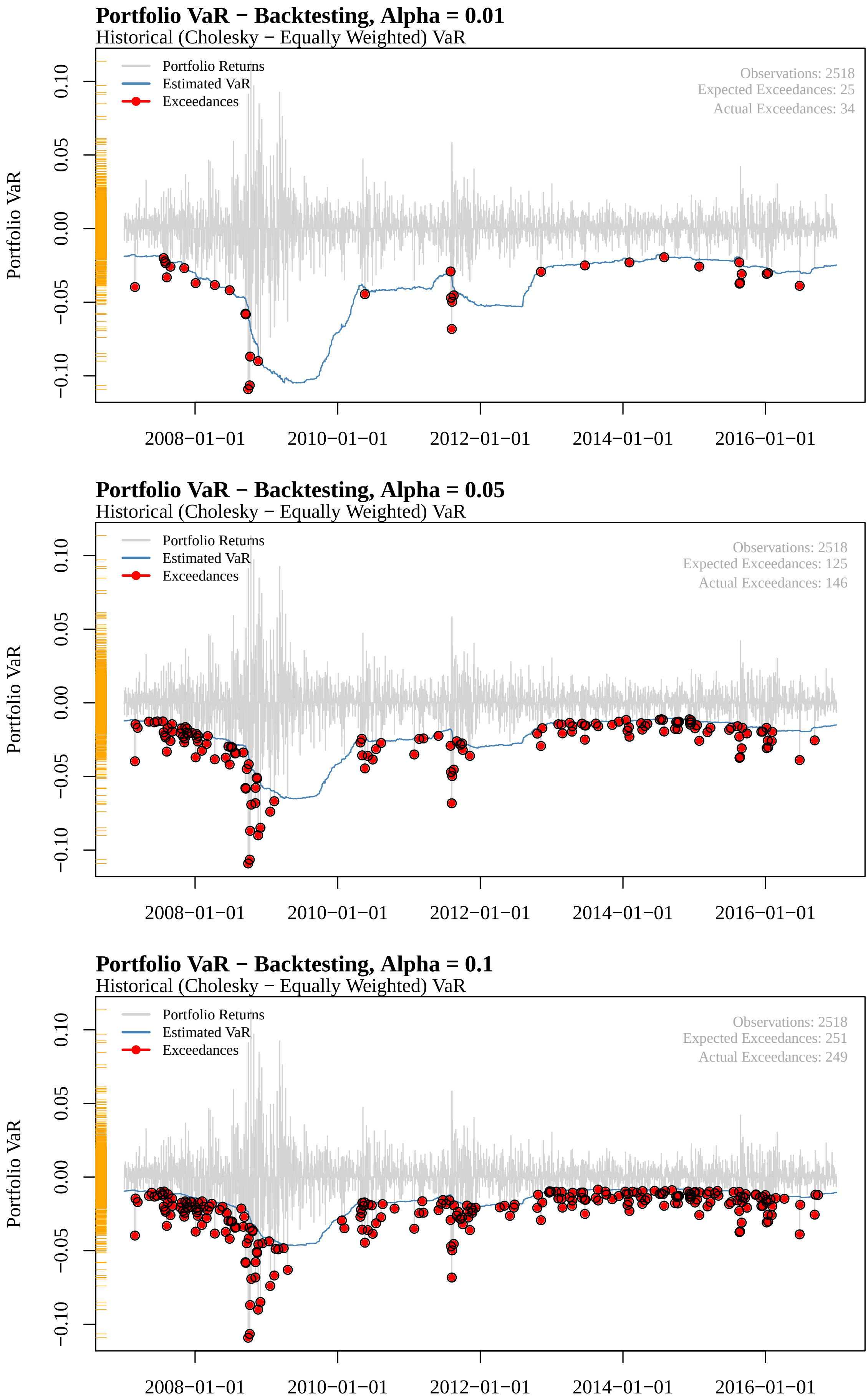


Figure A.10: Historical (Cholesky - Equally Weighted) VaR

Table A.10: Historical (Cholesky - Equally Weighted) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.019	0.053	0.646	0.809	0.263	0.109	0.049
2008	0.002	0.005	0.554	0.007	0.017	0.032	0.055
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2011	0.166	0.346	0.746	0.000	0.000	0.000	0.000
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2015	0.166	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.094**	**0.056**	**0.021**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.001	0.003	0.288	0.120	0.061	0.113	0.181
2008	0.000	0.000	0.286	0.250	0.205	0.172	0.065
2009	0.000	0.001	0.898	0.984	0.997	0.999	1.000
2010	0.155	0.280	0.599	0.267	0.404	0.524	0.349
2011	0.500	0.440	0.212	0.027	0.040	0.043	0.053
2012	0.001	0.004	0.846	0.963	0.990	0.997	0.000
2013	0.691	0.895	0.791	0.327	0.221	0.256	0.282
2014	0.003	0.003	0.047	0.003	0.009	0.000	0.000
2015	0.345	0.414	0.296	0.579	0.778	0.704	0.825
2016	0.155	0.181	0.125	0.094	0.063	0.039	0.025
ALL	**0.073**	**0.035**	**0.043**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 10%							
2007	0.162	0.306	0.532	0.210	0.120	0.138	0.206
2008	0.000	0.000	0.553	0.824	0.923	0.910	0.952
2009	0.000	0.000	0.647	0.810	0.261	0.108	0.167
2010	0.113	0.233	0.498	0.028	0.020	0.036	0.024
2011	0.435	0.683	0.686	0.852	0.471	0.268	0.385
2012	0.003	0.006	0.421	0.002	0.005	0.009	0.016
2013	0.493	0.269	0.099	0.180	0.025	0.041	0.029
2014	0.078	0.152	0.406	0.279	0.359	0.016	0.032
2015	0.078	0.042	0.055	0.107	0.213	0.341	0.416
2016	0.021	0.036	0.188	0.021	0.022	0.019	0.037
ALL	**0.852**	**0.032**	**0.006**	**0.000**	**0.000**	**0.000**	**0.000**

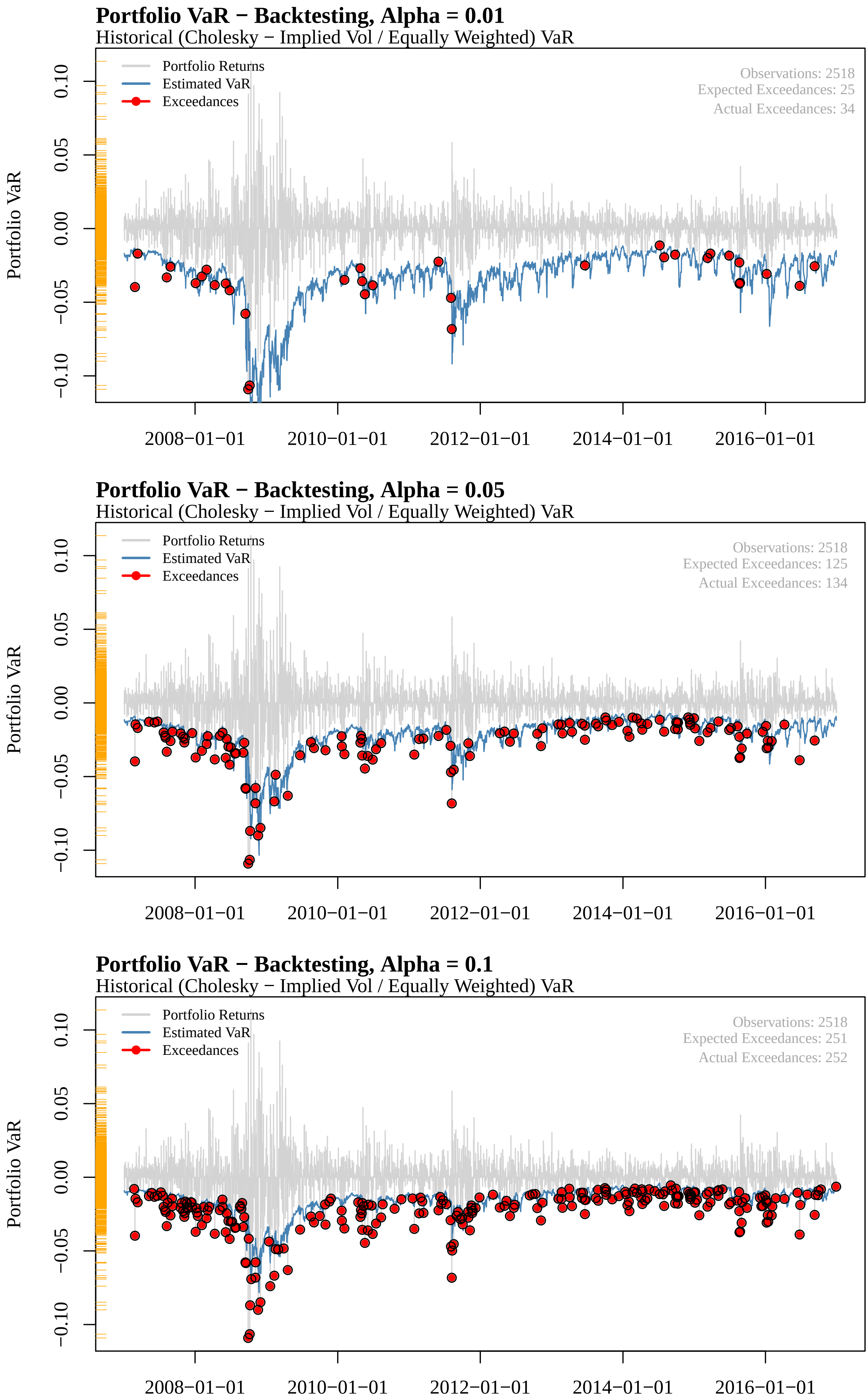


Figure A.11: Historical (Cholesky - Implied Vol / Equally Weighted) VaR

Table A.11: Historical (Cholesky - Implied Vol / Equally Weighted) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.384	0.642	0.796	0.935	0.977	0.991	0.997
2008	0.002	0.005	0.554	0.702	0.803	0.865	0.905
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.166	0.346	0.746	0.899	0.956	0.980	0.991
2011	0.768	0.923	0.847	0.000	0.000	0.000	0.000
2012	0.025	0.081	NaN	NaN	NaN	NaN	NaN
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.768	0.923	0.847	0.963	0.990	0.997	0.999
2015	0.061	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.094**	**0.056**	**0.021**	**0.005**	**0.011**	**0.021**	**0.034**
Alpha = 5%							
2007	0.220	0.136	0.245	0.502	0.554	0.593	0.727
2008	0.002	0.003	0.293	0.539	0.725	0.625	0.346
2009	0.078	0.174	0.647	0.810	0.888	0.376	0.487
2010	0.861	0.851	0.551	0.115	0.197	0.284	0.372
2011	0.637	0.540	0.464	0.054	0.094	0.135	0.181
2012	0.083	0.181	0.645	0.808	0.887	0.930	0.491
2013	0.691	0.895	0.791	0.616	0.372	0.399	0.417
2014	0.084	0.212	0.742	0.314	0.487	0.085	0.140
2015	0.691	0.423	0.141	0.320	0.382	0.425	0.458
2016	0.155	0.289	0.645	0.249	0.148	0.087	0.140
ALL	**0.463**	**0.688**	**0.654**	**0.023**	**0.044**	**0.004**	**0.004**
Alpha = 10%							
2007	0.316	0.357	0.334	0.222	0.166	0.275	0.396
2008	0.013	0.016	0.176	0.374	0.572	0.496	0.556
2009	0.021	0.027	0.310	0.594	0.788	0.900	0.956
2010	0.259	0.288	0.223	0.447	0.379	0.526	0.460
2011	0.238	0.496	0.920	0.789	0.677	0.724	0.810
2012	0.006	0.011	0.381	0.626	0.632	0.641	0.749
2013	0.493	0.269	0.099	0.067	0.010	0.018	0.028
2014	0.032	0.061	0.306	0.353	0.496	0.018	0.032
2015	0.563	0.180	0.055	0.159	0.297	0.447	0.401
2016	0.175	0.342	0.562	0.280	0.160	0.268	0.390
ALL	**0.989**	**0.349**	**0.134**	**0.006**	**0.002**	**0.000**	**0.001**

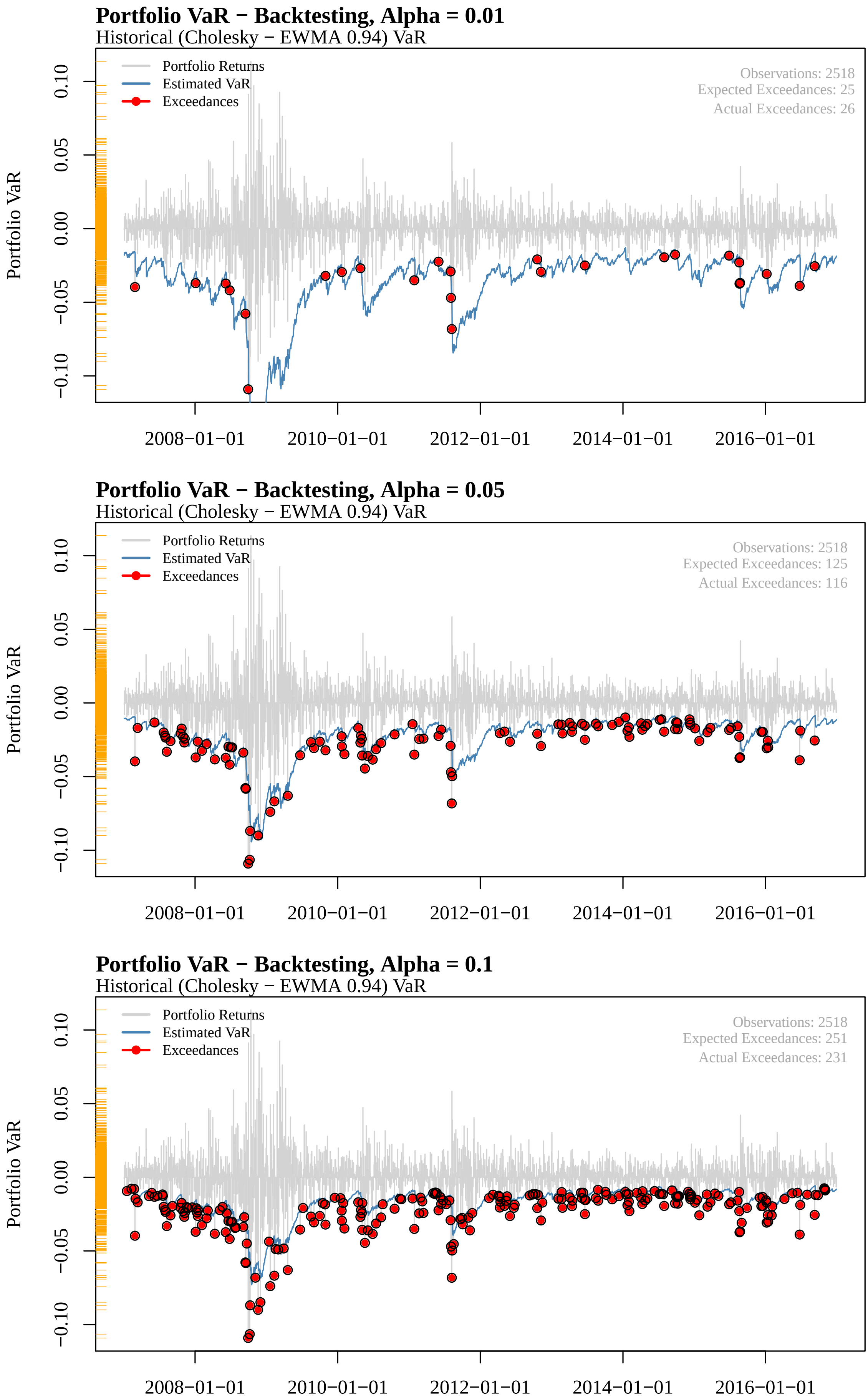


Figure A.12: Historical (Cholesky - EWMA 0.94) VaR

Table A.12: Historical (Cholesky - EWMA 0.94) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.276	0.550	0.949	0.996	1.000	1.000	1.000
2008	0.168	0.350	0.746	0.900	0.964	0.986	0.995
2009	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2010	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2011	0.166	0.346	0.746	0.000	0.000	0.000	0.000
2012	0.742	0.932	0.898	0.983	0.997	0.999	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2015	0.388	0.002	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.870**	**0.089**	**0.001**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.897	0.486	0.382	0.626	0.276	0.147	0.223
2008	0.232	0.143	0.247	0.354	0.349	0.134	0.141
2009	0.155	0.280	0.599	0.757	0.840	0.890	0.923
2010	0.691	0.895	0.791	0.327	0.512	0.669	0.787
2011	0.436	0.488	0.508	0.000	0.000	0.000	0.000
2012	0.014	0.043	0.744	0.899	0.956	0.980	0.990
2013	0.908	0.919	0.672	0.837	0.912	0.858	0.833
2014	0.141	0.326	0.781	0.254	0.231	0.027	0.050
2015	0.861	0.279	0.046	0.110	0.175	0.238	0.311
2016	0.034	0.032	0.020	0.004	0.011	0.002	0.004
ALL	**0.359**	**0.630**	**0.767**	**0.000**	**0.001**	**0.000**	**0.000**
Alpha = 10%							
2007	0.851	0.048	0.064	0.123	0.238	0.289	0.386
2008	0.337	0.133	0.120	0.206	0.261	0.303	0.418
2009	0.040	0.041	0.277	0.554	0.757	0.881	0.788
2010	0.259	0.499	0.724	0.885	0.633	0.528	0.185
2011	0.563	0.843	0.937	0.993	0.587	0.695	0.731
2012	0.276	0.475	0.600	0.423	0.112	0.179	0.253
2013	0.259	0.288	0.223	0.227	0.063	0.107	0.087
2014	0.238	0.244	0.202	0.079	0.137	0.001	0.002
2015	0.365	0.175	0.063	0.097	0.172	0.255	0.339
2016	0.175	0.163	0.137	0.019	0.016	0.034	0.063
ALL	**0.162**	**0.343**	**0.667**	**0.004**	**0.001**	**0.000**	**0.001**

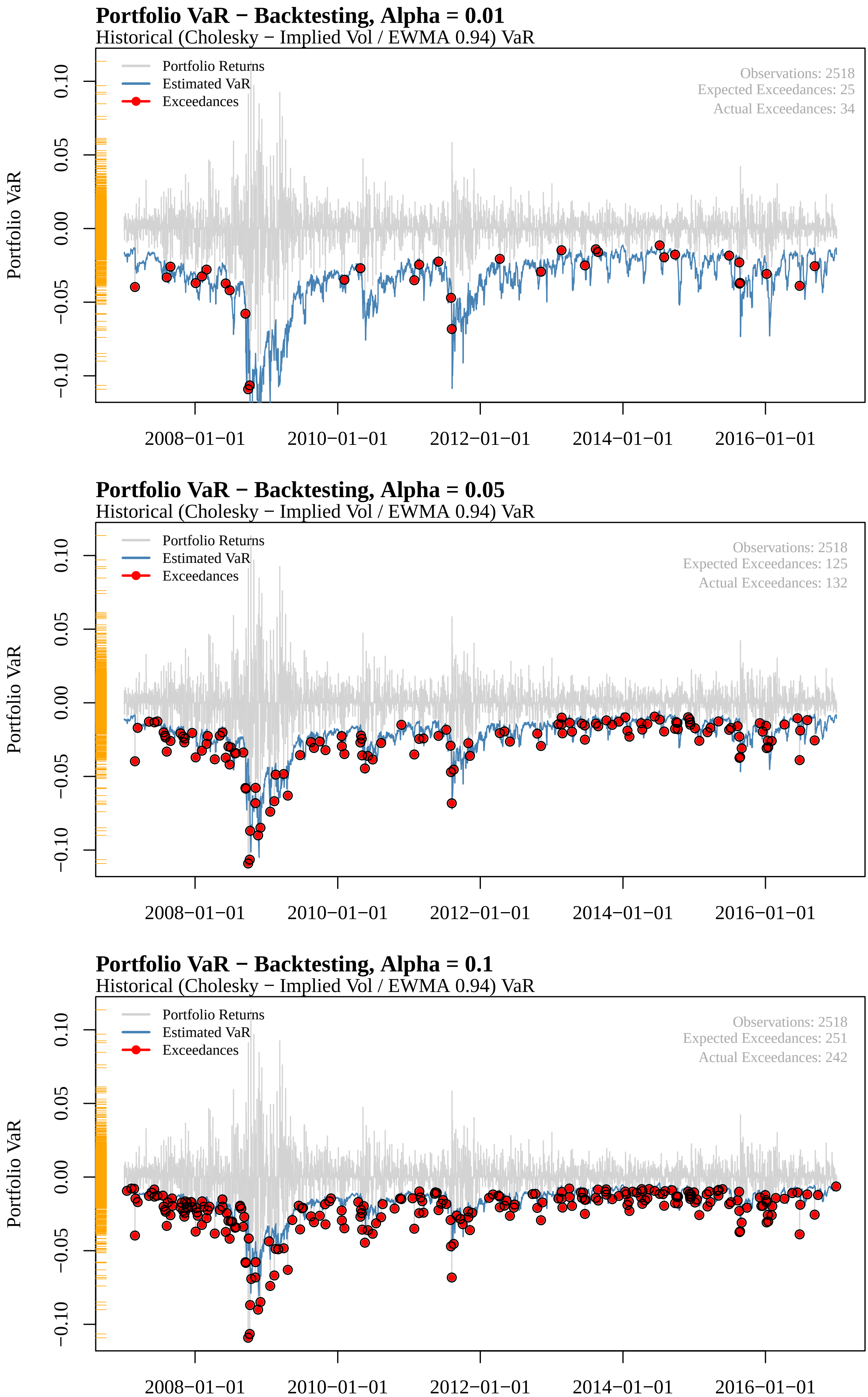


Figure A.13: Historical (Cholesky - Implied Vol / EWMA 0.94) VaR

Table A.13: Historical (Cholesky - Implied Vol / EWMA 0.94) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.763	0.921	0.847	0.963	0.990	0.997	0.999
2008	0.006	0.017	0.600	0.758	0.857	0.911	0.944
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2011	0.166	0.346	0.746	0.013	0.031	0.062	0.106
2012	0.742	0.932	0.898	0.983	0.997	0.999	1.000
2013	0.388	0.646	0.796	0.935	0.977	0.992	0.997
2014	0.768	0.923	0.847	0.963	0.990	0.997	0.999
2015	0.388	0.002	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.094**	**0.056**	**0.021**	**0.005**	**0.011**	**0.021**	**0.034**
Alpha = 5%							
2007	0.491	0.303	0.309	0.593	0.787	0.628	0.761
2008	0.007	0.018	0.400	0.555	0.309	0.386	0.250
2009	0.436	0.488	0.508	0.643	0.721	0.675	0.732
2010	0.861	0.851	0.551	0.115	0.197	0.284	0.372
2011	0.637	0.540	0.464	0.054	0.094	0.135	0.181
2012	0.014	0.043	0.744	0.899	0.956	0.980	0.990
2013	0.691	0.423	0.141	0.327	0.512	0.521	0.527
2014	0.226	0.140	0.246	0.090	0.103	0.006	0.013
2015	0.691	0.423	0.141	0.320	0.382	0.425	0.458
2016	0.637	0.665	0.394	0.484	0.537	0.556	0.640
ALL	**0.580**	**0.858**	**0.975**	**0.018**	**0.024**	**0.004**	**0.003**
Alpha = 10%							
2007	0.162	0.306	0.532	0.459	0.439	0.607	0.743
2008	0.013	0.030	0.391	0.646	0.808	0.840	0.849
2009	0.175	0.084	0.191	0.374	0.545	0.679	0.769
2010	0.113	0.273	0.781	0.767	0.893	0.901	0.913
2011	0.867	0.692	0.371	0.596	0.211	0.336	0.439
2012	0.024	0.077	0.913	0.464	0.671	0.622	0.756
2013	0.365	0.175	0.063	0.031	0.016	0.029	0.023
2014	0.169	0.208	0.242	0.504	0.713	0.047	0.086
2015	0.800	0.454	0.185	0.336	0.534	0.699	0.713
2016	0.069	0.132	0.356	0.428	0.422	0.588	0.726
ALL	**0.513**	**0.265**	**0.122**	**0.025**	**0.043**	**0.023**	**0.041**

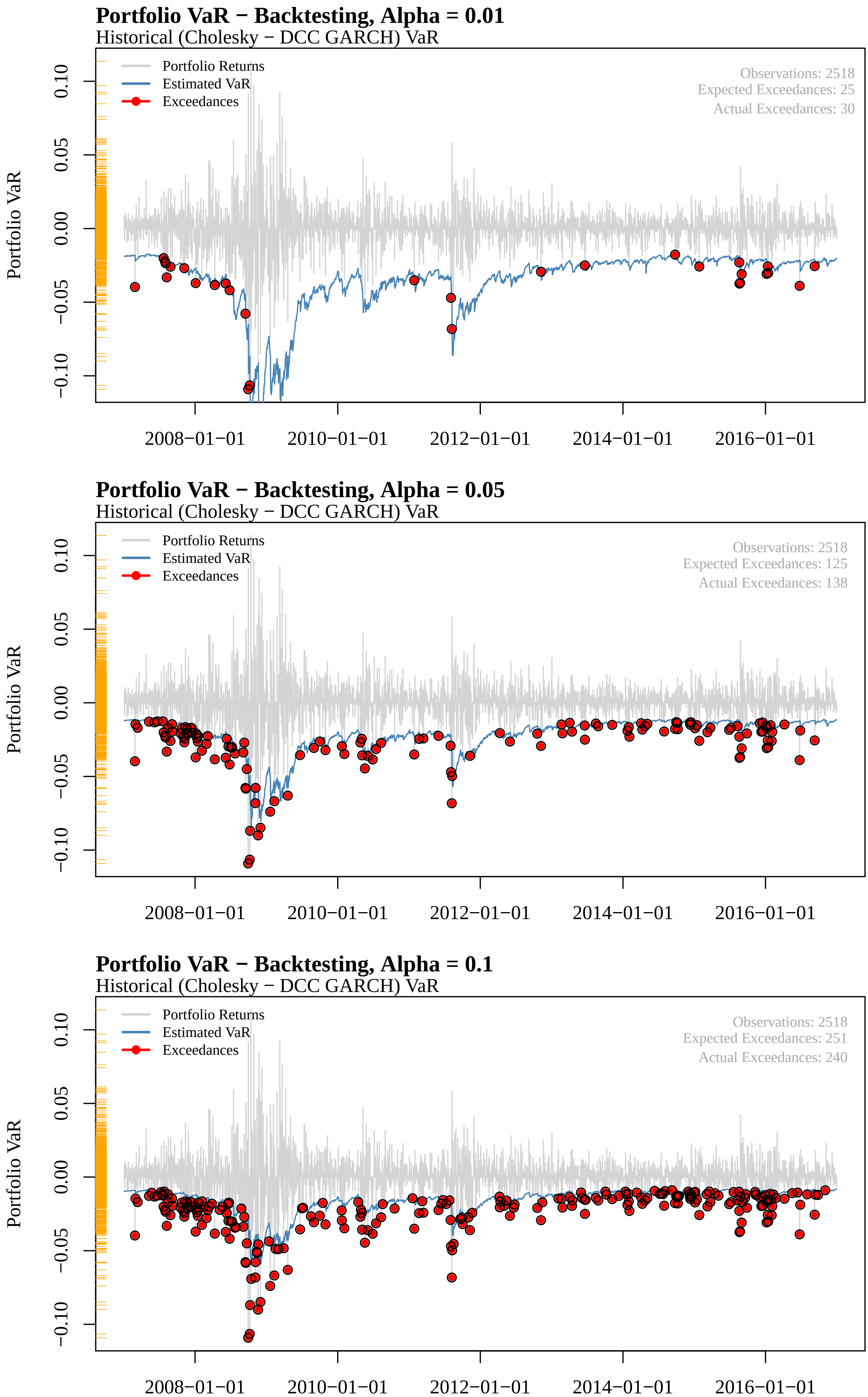


Figure A.14: Historical (Cholesky - DCC GARCH) VaR

Table A.14: Historical (Cholesky - DCC GARCH) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.019	0.053	0.646	0.809	0.263	0.109	0.049
2008	0.020	0.055	0.648	0.810	0.901	0.947	0.970
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2011	0.768	0.923	0.847	0.000	0.000	0.000	0.000
2012	0.278	0.553	0.949	0.996	1.000	1.000	1.000
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2015	0.166	0.003	0.000	0.000	0.000	0.000	0.000
2016	0.166	0.346	0.746	0.013	0.031	0.001	0.003
ALL	**0.349**	**0.097**	**0.005**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 5%							
2007	0.000	0.001	0.545	0.324	0.094	0.143	0.230
2008	0.000	0.000	0.475	0.661	0.484	0.398	0.475
2009	0.078	0.174	0.647	0.810	0.888	0.931	0.956
2010	0.436	0.488	0.508	0.487	0.596	0.673	0.648
2011	0.274	0.393	0.553	0.000	0.000	0.000	0.000
2012	0.004	0.016	0.795	0.934	0.977	0.991	0.997
2013	0.274	0.330	0.213	0.385	0.325	0.430	0.521
2014	0.500	0.791	0.907	0.058	0.081	0.001	0.002
2015	0.226	0.137	0.063	0.176	0.195	0.319	0.451
2016	0.861	0.240	0.039	0.014	0.005	0.001	0.002
ALL	**0.276**	**0.254**	**0.186**	**0.000**	**0.000**	**0.000**	**0.000**
Alpha = 10%							
2007	0.162	0.306	0.532	0.210	0.120	0.138	0.206
2008	0.001	0.002	0.549	0.802	0.905	0.881	0.946
2009	0.021	0.027	0.310	0.594	0.788	0.900	0.829
2010	0.021	0.071	0.907	0.456	0.663	0.810	0.901
2011	0.493	0.789	0.954	0.697	0.372	0.213	0.141
2012	0.001	0.004	0.437	0.547	0.613	0.662	0.700
2013	0.069	0.189	0.879	0.688	0.029	0.034	0.038
2014	0.169	0.327	0.549	0.185	0.194	0.002	0.005
2015	0.169	0.103	0.082	0.176	0.321	0.404	0.410
2016	0.365	0.640	0.787	0.150	0.273	0.248	0.351
ALL	**0.430**	**0.287**	**0.157**	**0.000**	**0.000**	**0.000**	**0.000**

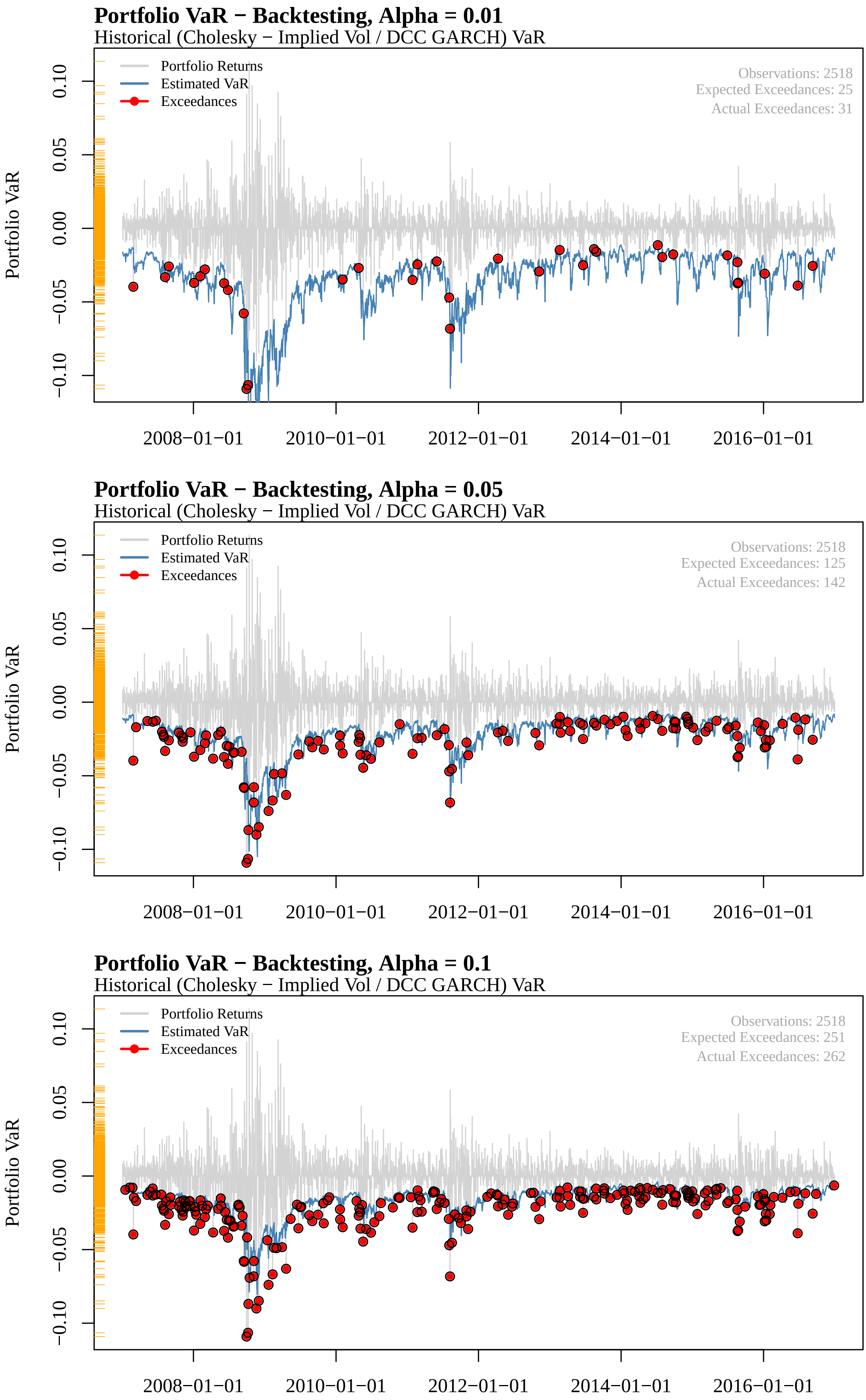


Figure A.15: Historical (Cholesky - Implied Vol / DCC GARCH) VaR

Table A.15: Historical (Cholesky - Implied Vol / DCC GARCH) VaR

Year	UC pvalue	CC pvalue	BCP L1 pvalue	BCP L2 pvalue	BCP L3 pvalue	BCP L4 pvalue	BCP L5 pvalue
Alpha = 1%							
2007	0.019	0.053	0.646	0.809	0.263	0.000	0.000
2008	0.002	0.005	0.554	0.702	0.803	0.865	0.905
2009	0.024	0.079	NaN	NaN	NaN	NaN	NaN
2010	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2011	0.388	0.646	0.796	0.001	0.002	0.005	0.011
2012	0.025	0.081	NaN	NaN	NaN	NaN	NaN
2013	0.273	0.546	0.949	0.996	1.000	1.000	1.000
2014	0.733	0.928	0.898	0.984	0.997	0.999	1.000
2015	0.388	0.002	0.000	0.000	0.000	0.000	0.000
2016	0.768	0.923	0.847	0.963	0.990	0.998	1.000
ALL	**0.261**	**0.089**	**0.008**	**0.001**	**0.002**	**0.000**	**0.000**
Alpha = 5%							
2007	0.013	0.033	0.457	0.758	0.740	0.418	0.562
2008	0.000	0.000	0.405	0.690	0.810	0.708	0.522
2009	0.436	0.488	0.508	0.643	0.721	0.675	0.732
2010	0.861	0.851	0.551	0.115	0.197	0.284	0.372
2011	0.637	0.540	0.464	0.054	0.094	0.135	0.181
2012	0.037	0.097	0.695	0.856	0.926	0.960	0.978
2013	0.637	0.705	0.432	0.560	0.621	0.676	0.717
2014	0.908	0.919	0.672	0.837	0.912	0.047	0.063
2015	0.226	0.333	0.356	0.653	0.588	0.555	0.697
2016	0.637	0.665	0.394	0.484	0.537	0.556	0.640
ALL	**0.149**	**0.272**	**0.456**	**0.248**	**0.257**	**0.004**	**0.005**
Alpha = 10%							
2007	0.422	0.486	0.397	0.184	0.110	0.187	0.277
2008	0.000	0.000	0.149	0.347	0.529	0.681	0.769
2009	0.069	0.056	0.246	0.504	0.706	0.840	0.830
2010	0.175	0.355	0.610	0.772	0.857	0.907	0.945
2011	0.326	0.599	0.802	0.677	0.381	0.421	0.550
2012	0.044	0.130	0.976	0.999	0.747	0.652	0.782
2013	0.365	0.175	0.063	0.031	0.016	0.029	0.023
2014	0.238	0.380	0.447	0.564	0.721	0.027	0.048
2015	0.169	0.103	0.082	0.218	0.381	0.465	0.295
2016	0.365	0.640	0.787	0.150	0.151	0.245	0.348
ALL	**0.501**	**0.391**	**0.221**	**0.006**	**0.005**	**0.001**	**0.002**

B Appendix: Portfolio Weights Simulation

Portfolio weights simulation results for the following models:

1. Historical (Cholesky - Equally Weighted) VaR
2. Historical (Cholesky - Implied Vol / Equally Weighted) VaR
3. Historical (Cholesky - EWMA 0.94) VaR
4. Historical (Cholesky - Implied Vol / EWMA 0.94) VaR
5. Historical (Cholesky - DCC GARCH) VaR
6. Historical (Cholesky - Implied Vol / DCC GARCH) VaR

Backtest period: **01-01-2007 / 31-12-2016 (10-years)**

Significance levels: **0.01, 0.05, 0.10**

Number of simulations: **500**

Portfolio weights simulation parameters: **Mean: 0.1, St.Dev: 0.05**

B.1 Historical (Cholesky Adjusted) VaR, Alpha = 1%

Table B.1: Portfolio Weights Simulation - Summary Statistics, Alpha 0.01

Results	Equally Weighted	IV/Equally Weighted	EWMA	IV/EWMA	DCC-GARCH	IV/DCC-GARCH
Actual Exceed.(Mean)	15.69	12.03	11.5	10.93	14.47	12.95
Actual Exceed.(St.Dev)	6.38	6.66	4.9	6.52	5.96	6.11
Range of Exceedances	0 - 38	1 - 34	1 - 27	1 - 33	1 - 35	1 - 34
Cum. % of Simulations < Expected	91.8%	95.4%	99.0%	96.6%	96.6%	97.2%

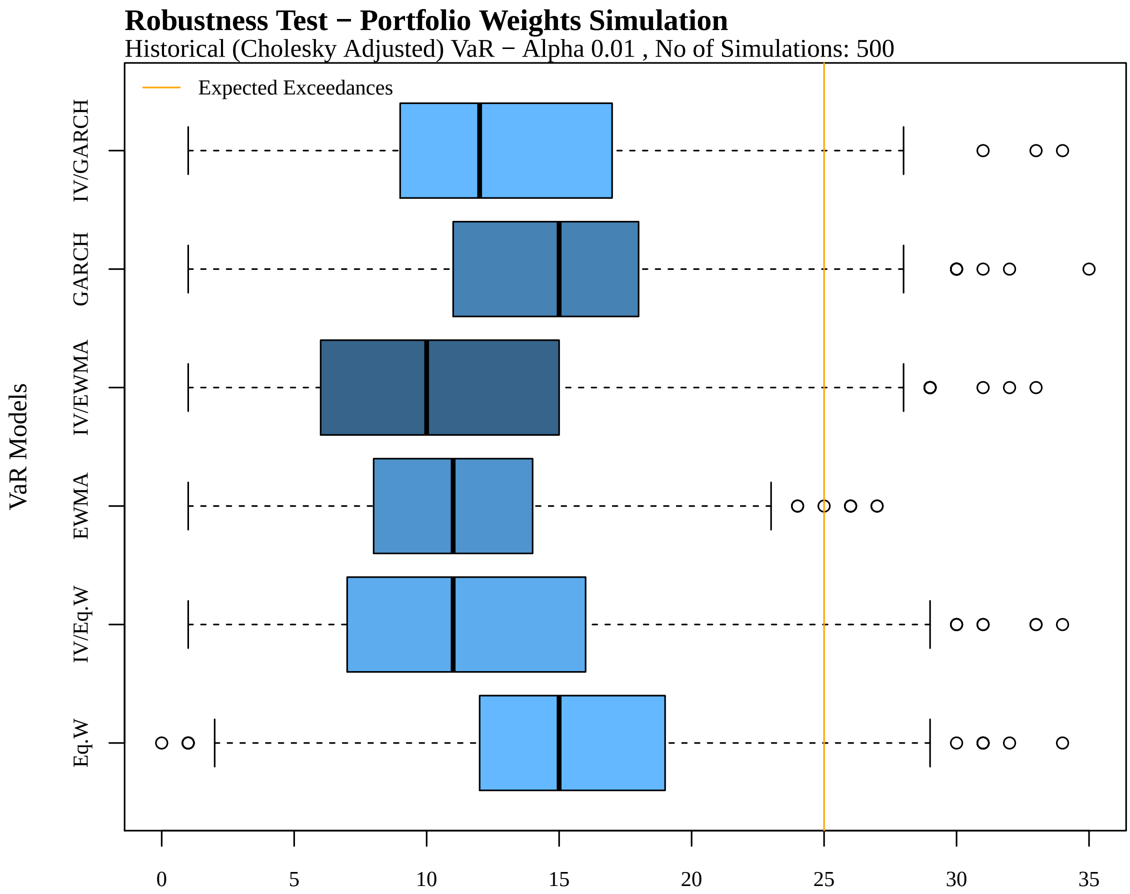


Figure B.1: VaR Models Robustness Test, Alpha = 0.01

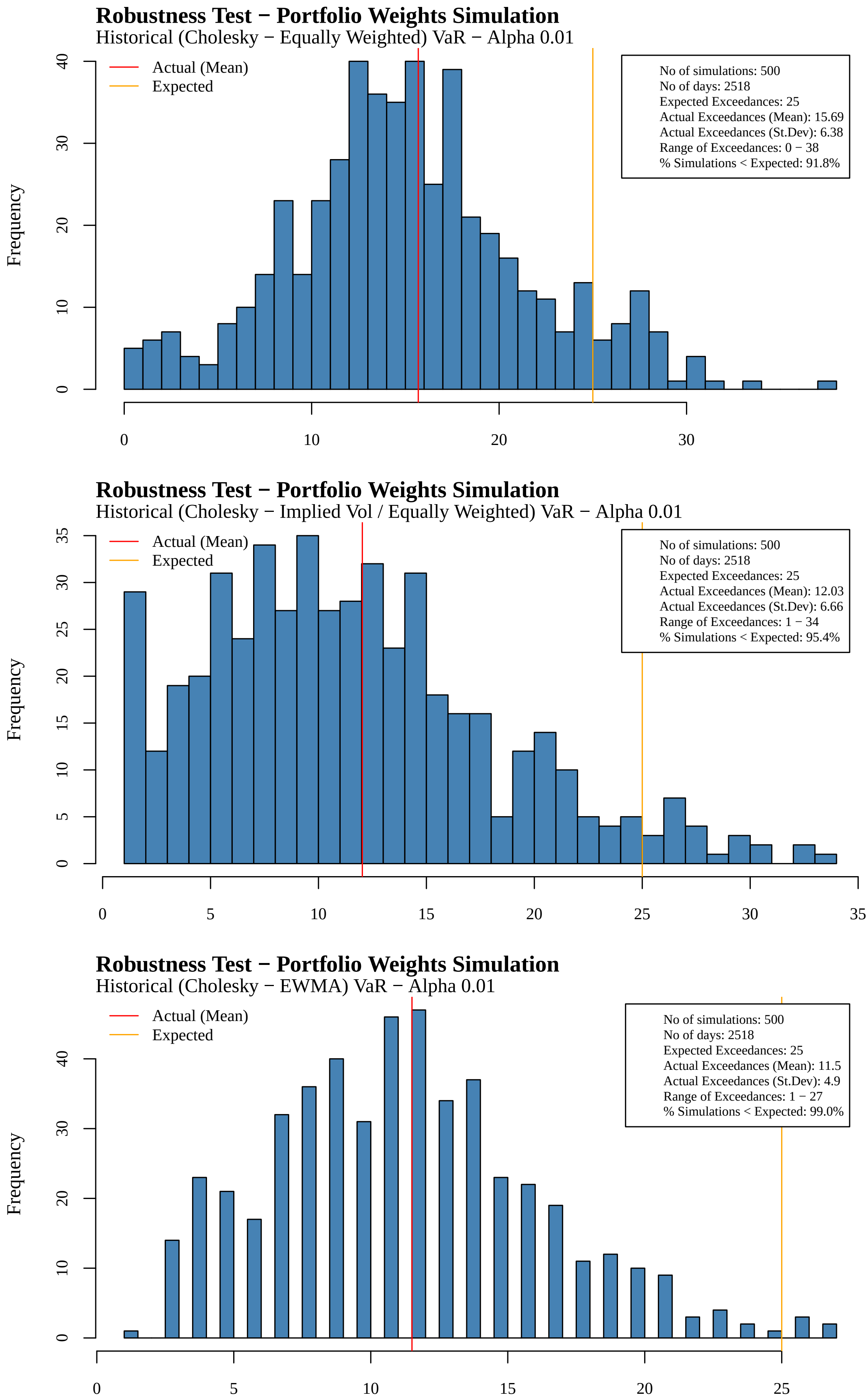


Figure B.2: Histogram of Exceedances, VaR 0.01 - Part 1

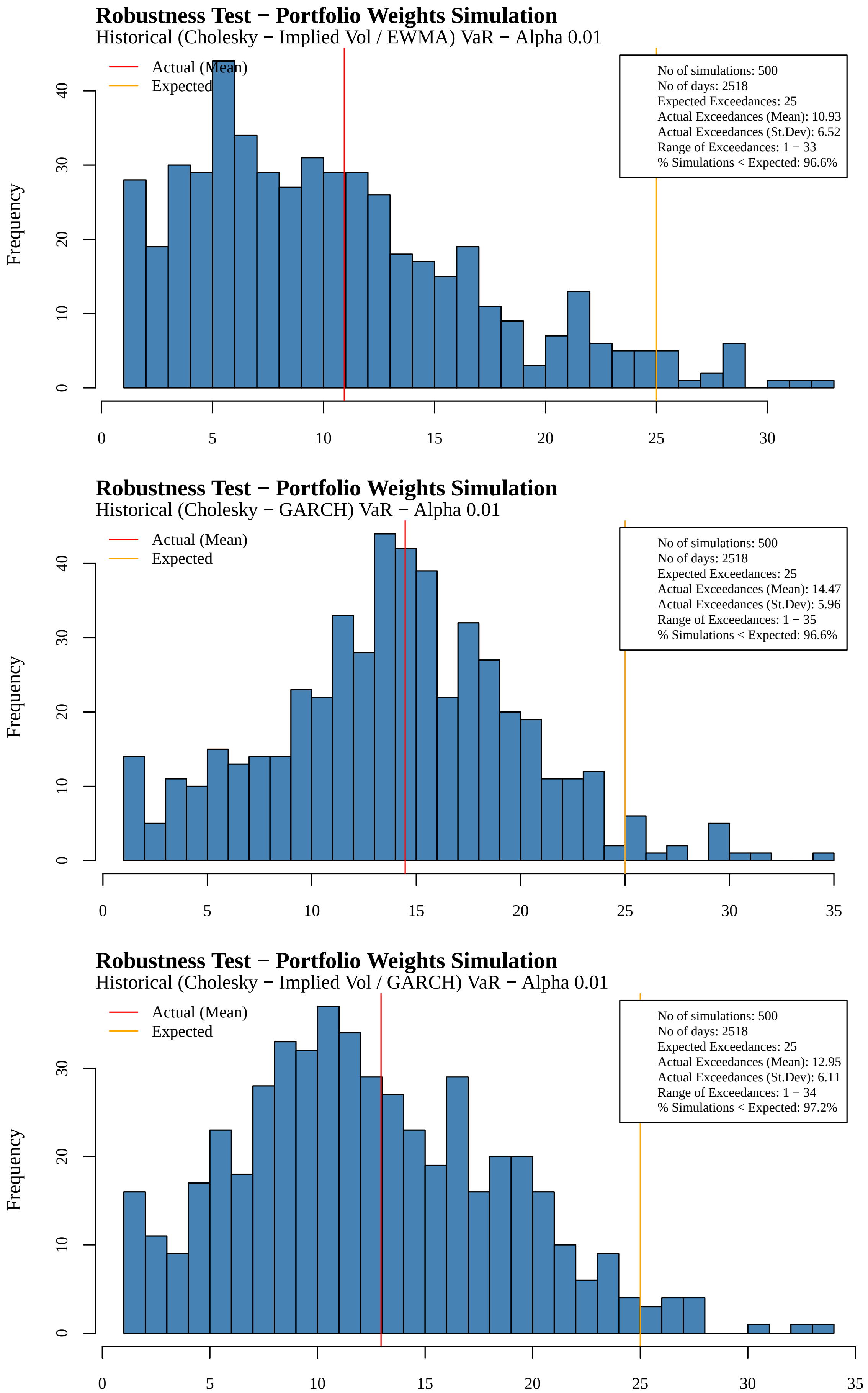


Figure B.3: Histogram of Exceedances, VaR 0.01 - Part 2

B.2 Historical (Cholesky Adjusted) VaR, Alpha = 5%

Table B.2: Portfolio Weights Simulation - Summary Statistics, Alpha 0.05

Results	Equally Weighted	IV/Equally Weighted	EWMA	IV/EWMA	DCC-GARCH	IV/DCC-GARCH
Actual Exceed.(Mean)	115.43	107.11	92.22	99.59	107.9	113.52
Actual Exceed.(St.Dev)	20.67	21.01	19.48	20.13	20.88	21.38
Range of Exceedances	43 - 163	38 - 170	34 - 142	34 - 158	34 - 168	39 - 176
Cum. % of Simulations < Expected	66.0%	81.0%	96.6%	90.2%	80.6%	70.0%

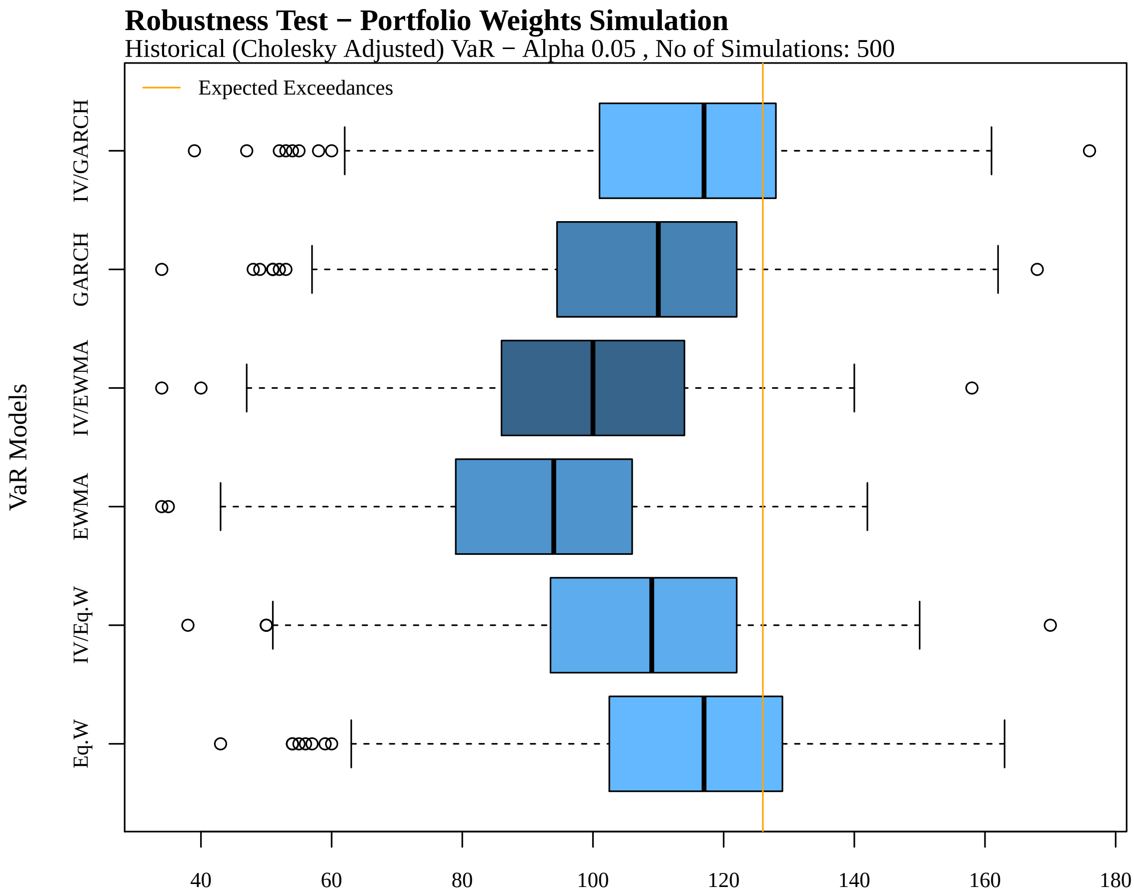


Figure B.4: VaR Models Robustness Test, Alpha = 0.05

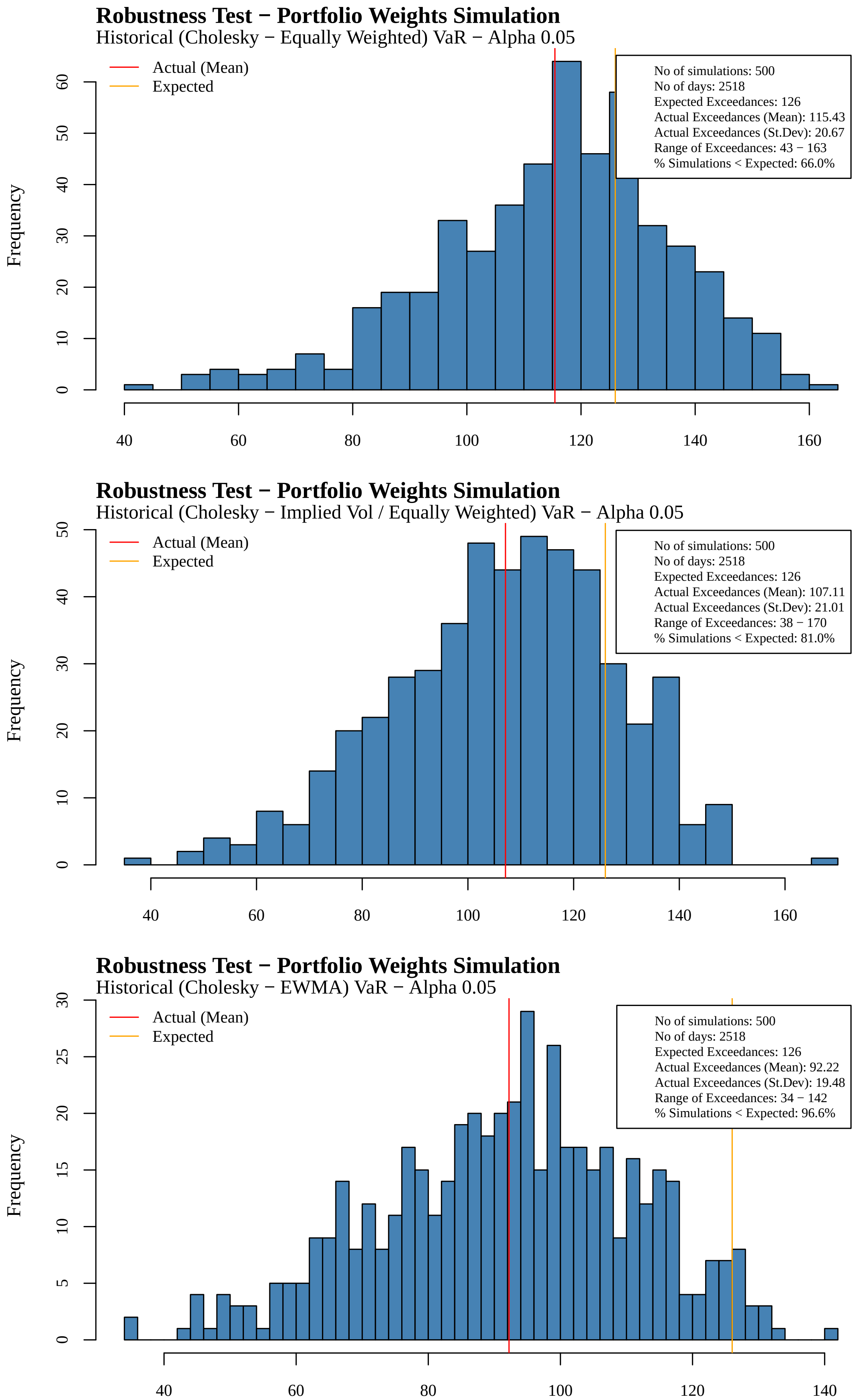
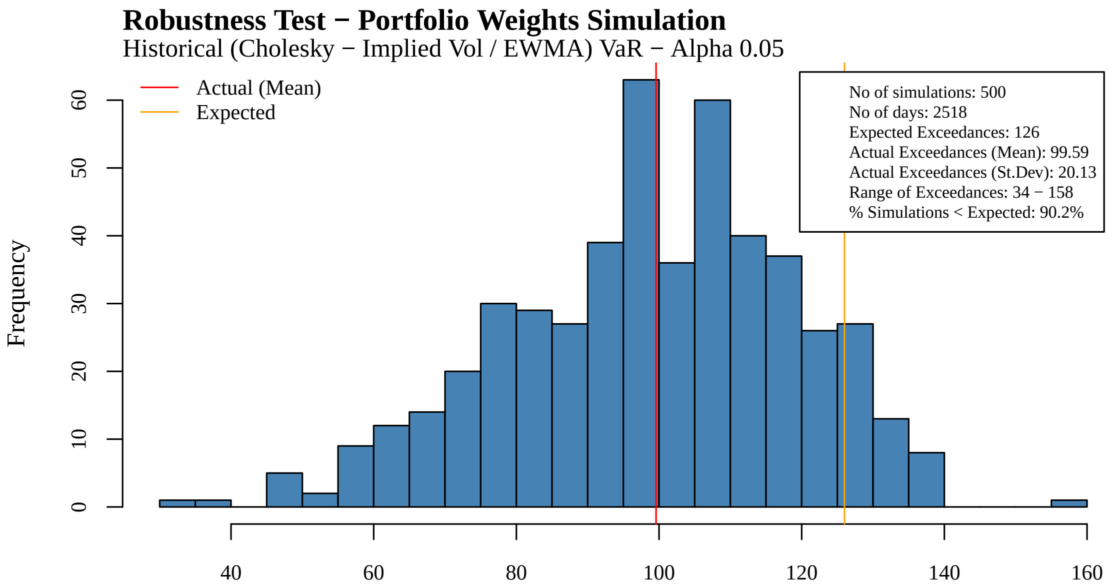
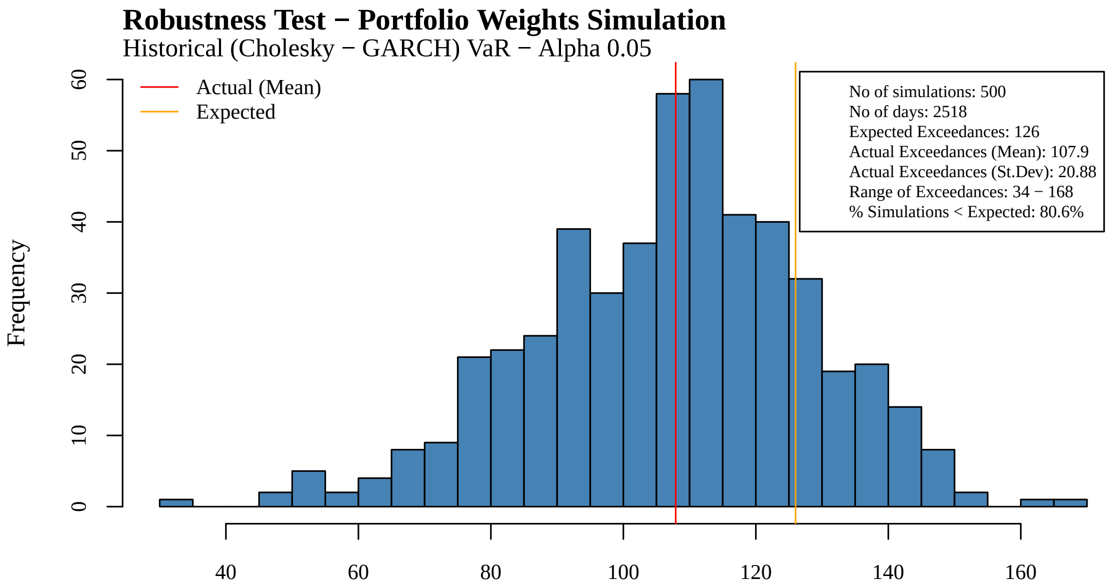


Figure B.5: Histogram of Exceedances, VaR 0.05 - Part 1





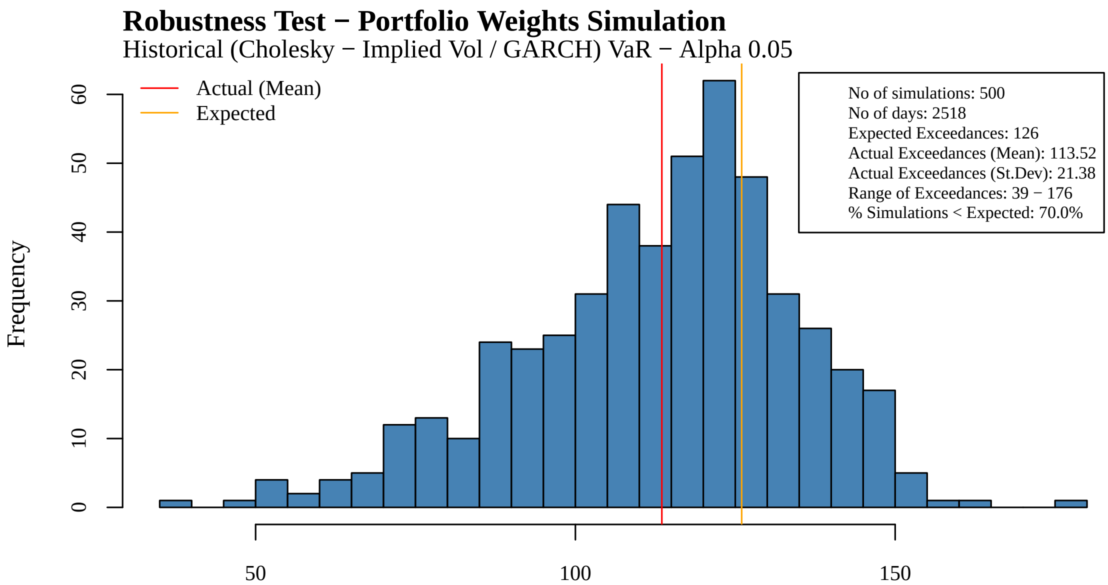


Figure B.6: Histogram of Exceedances, VaR 0.05 - Part 2

B.3 Historical (Cholesky Adjusted) VaR, Alpha = 10%

Table B.3: Portfolio Weights Simulation - Summary Statistics, Alpha 0.1

Results	Equally Weighted	IV/Equally Weighted	EWMA	IV/EWMA	DCC-GARCH	IV/DCC-GARCH
Actual Exceed.(Mean)	249.29	253.88	229.83	241.91	253.13	262.72
Actual Exceed.(St.Dev)	29.23	33.84	30.17	32.77	32.02	35.05
Range of Exceedances	137 - 355	135 - 379	117 - 332	127 - 351	128 - 364	144 - 386
Cum. % of Simulations < Expected	50.8%	43.2%	82.2%	62.8%	43.8%	32.2%

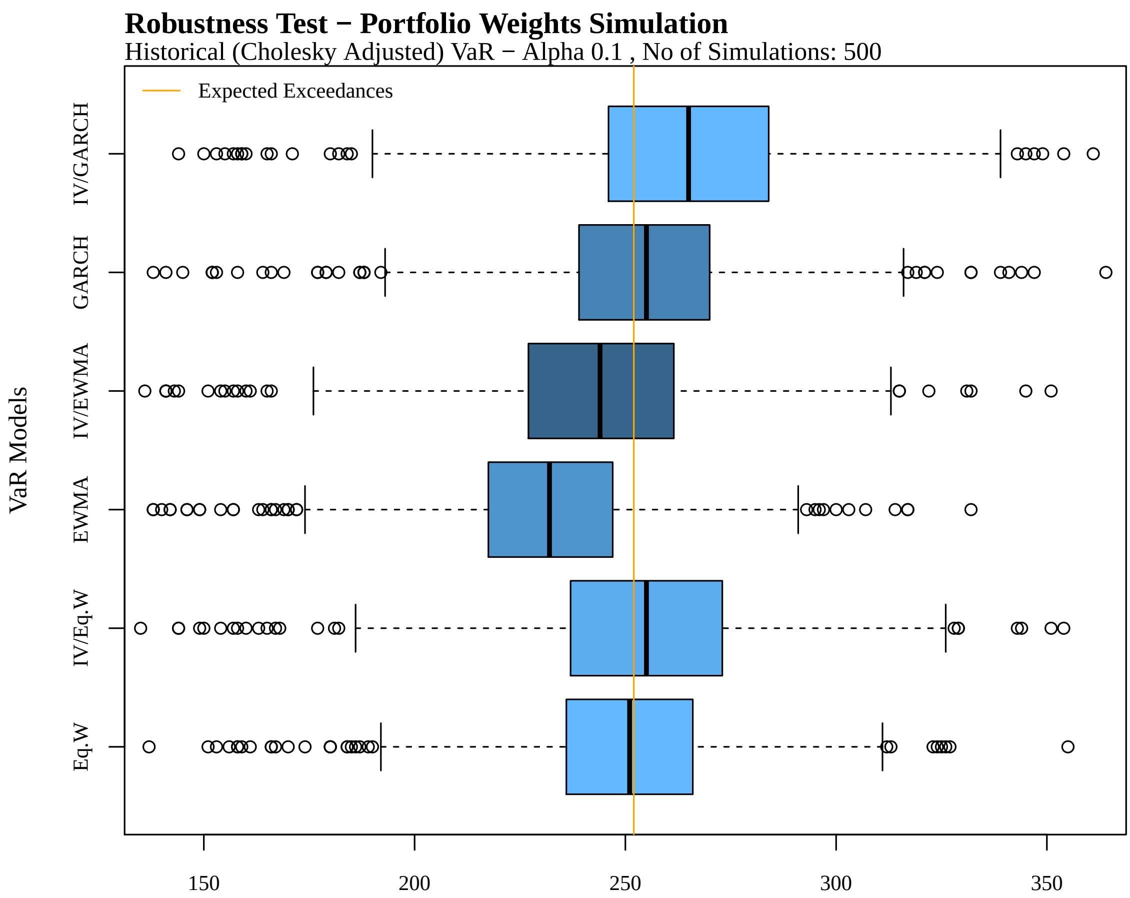
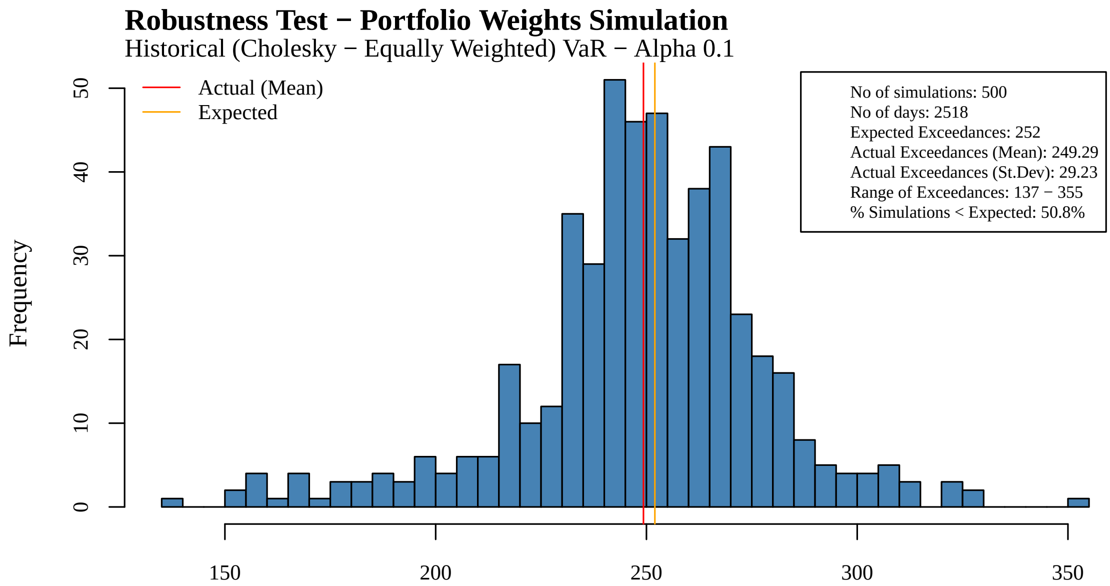
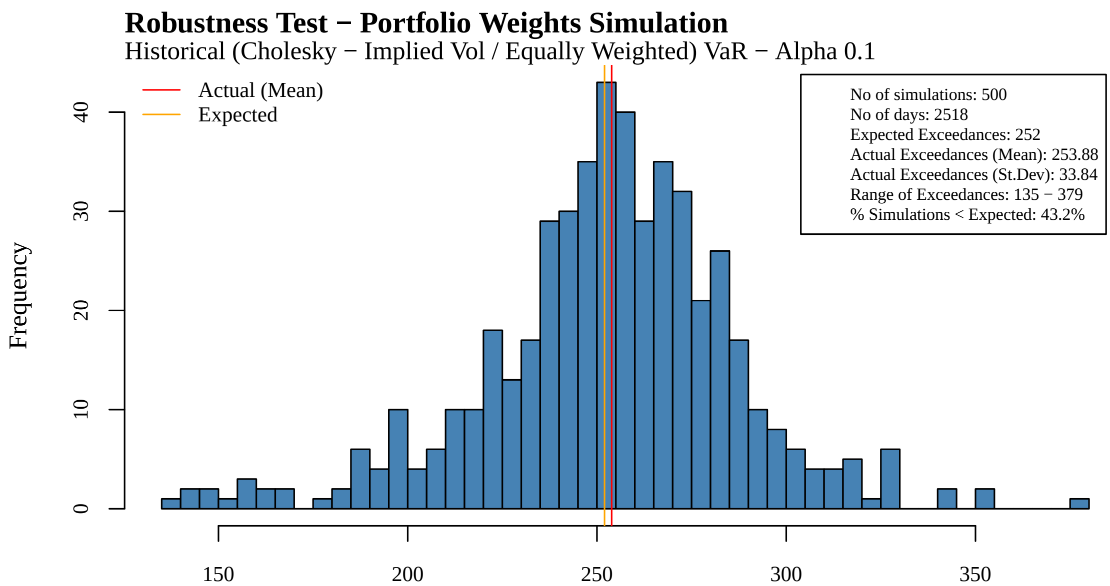


Figure B.7: VaR Models Robustness Test, Alpha = 0.10





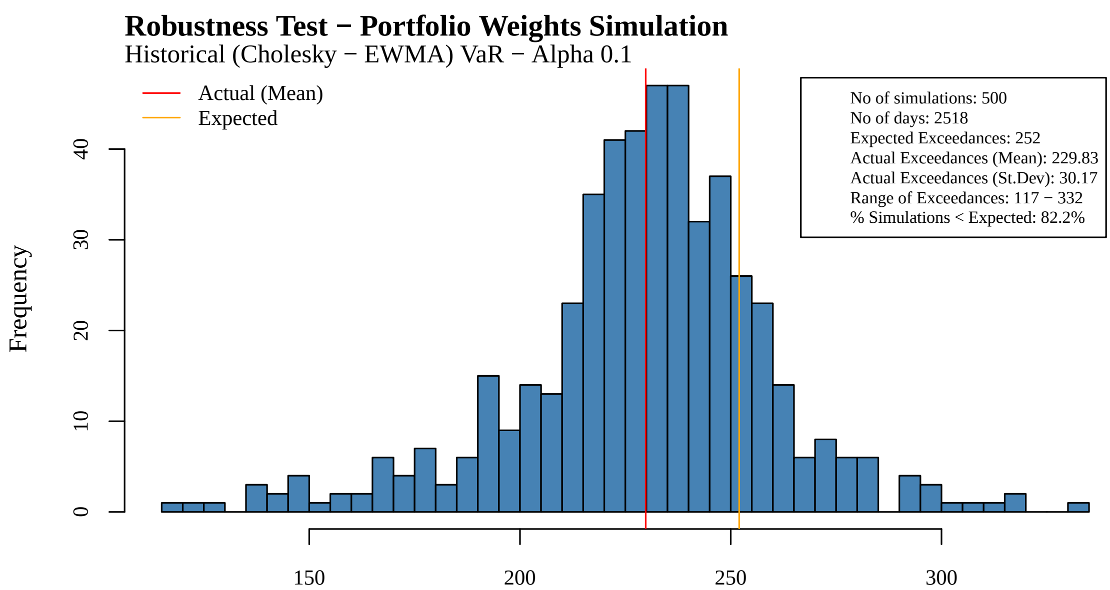


Figure B.8: Histogram of Exceedances, VaR 0.1 - Part 1

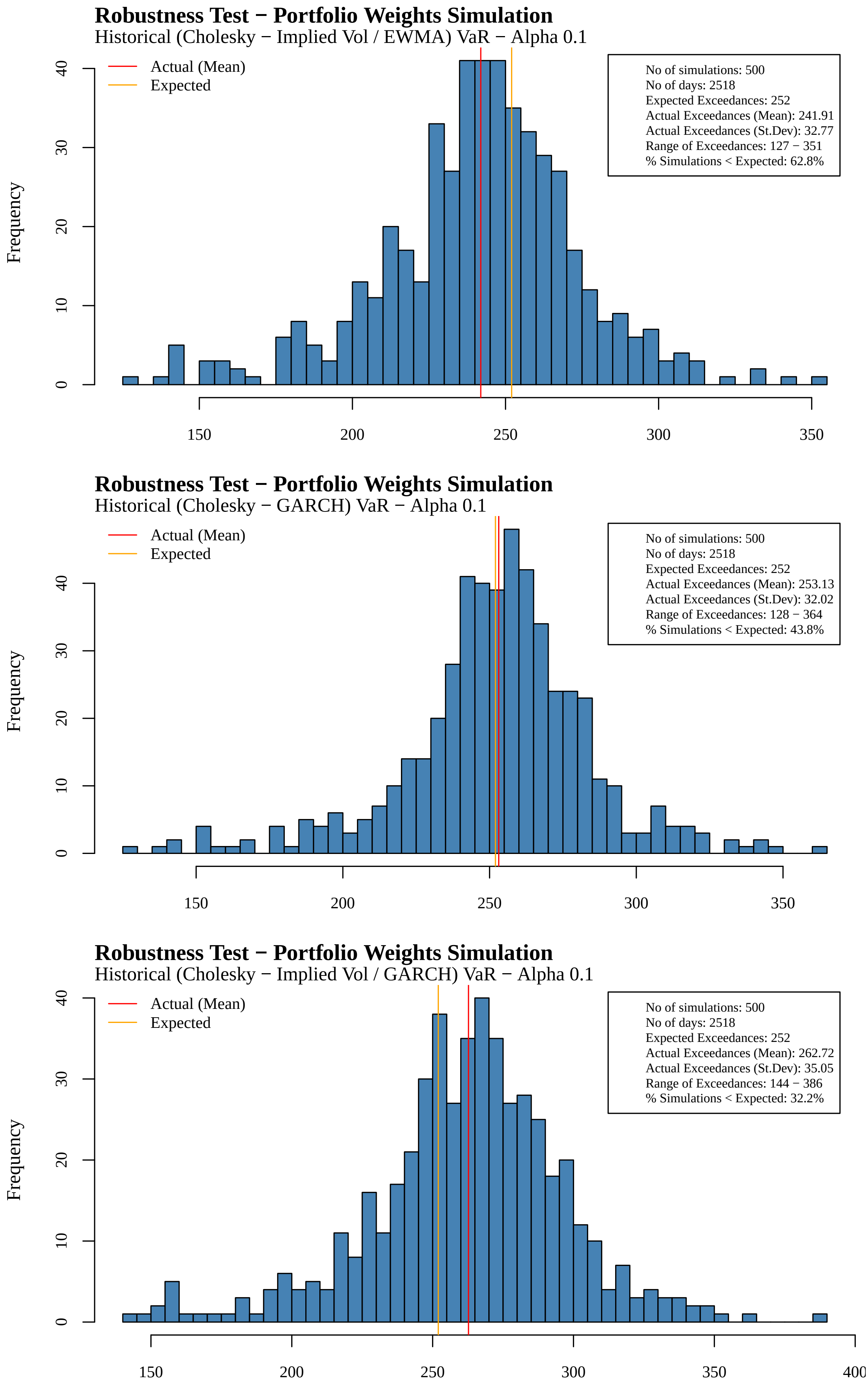


Figure B.9: Histogram of Exceedances, VaR 0.1 - Part 2

C Appendix: R Code

C.1 BCP Test

```
#Function for BCP test
#Arguments: TS of returns, TS of VaR, maximum lag for checking
    autocorrelation, vector of significance levels
bcp.test = function(tsret,tsvar,maxlag,sig){
  tsvar.sub = na.exclude(tsvar)
  l = length(tsret)
  obs = rownames(tsvar.sub)
  tsret.sub = tsret[obs,]
  tsexceeds1 = ifelse(tsret.sub >= -tsvar.sub[,1], 0, 1) - sig[1]
  ac1 = acf(tsexceeds1, lag.max = maxlag, plot = FALSE, type = "
     correlation")

  tsexceeds2 = ifelse(tsret.sub >= -tsvar.sub[,2], 0, 1) - sig[2]
  ac2 = acf(tsexceeds2, lag.max = maxlag, plot = FALSE, type = "
     correlation")

  tsexceeds3 = ifelse(tsret.sub >= -tsvar.sub[,3], 0, 1) - sig[3]
  ac3 = acf(tsexceeds3, lag.max = maxlag, plot = FALSE, type = "
     correlation")

  LB1 = vector(mode = 'numeric', length = maxlag)
  s = vector(mode = 'numeric', length = maxlag)
  p.value1 = vector(mode = 'numeric', length = maxlag)
  for (i in 1:maxlag){
    s[i] = ifelse(i==1,((ac1[[1]][i+1])^2)/(l-i),s[i-1]+((ac1[[1]][i
       +1])^2)/(l-i))
    LB1[i] = l*(l+2)*s[i]
```

```
    p.value1[i] = pchisq(LB1[i], i, lower.tail = F)
  }
  LB2 = vector(mode = 'numeric', length = maxlag)
  s = vector(mode = 'numeric', length = maxlag)
  p.value2 = vector(mode = 'numeric', length = maxlag)
  for (i in 1:maxlag){
    s[i] = ifelse(i==1,((ac2[[1]][i+1])^2)/(l-i),s[i-1]+((ac2[[1]][i
        +1])^2)/(l-i))
    LB2[i] = l*(l+2)*s[i]
    p.value2[i] = pchisq(LB2[i], i, lower.tail = F)
  }
  LB3 = vector(mode = 'numeric', length = maxlag)
  s = vector(mode = 'numeric', length = maxlag)
  p.value3 = vector(mode = 'numeric', length = maxlag)
  for (i in 1:maxlag){
    s[i] = ifelse(i==1,((ac3[[1]][i+1])^2)/(l-i),s[i-1]+((ac3[[1]][i
        +1])^2)/(l-i))
    LB3[i] = l*(l+2)*s[i]
    p.value3[i] = pchisq(LB3[i], i, lower.tail = F)
  }
  pvalue.mat = cbind(sig_1=p.value1,sig_2=p.value2,sig_3=p.value3)
  rownames(pvalue.mat) = paste("Lag",1:maxlag)
  colnames(pvalue.mat) = sig
  return(pvalue.mat)
}
```

C.2 Covariance Estimation - Equally Weighted

```
#Covariances, correlations in list of matrices, betas in dataframe,
    covariances of residuals and df for student t in list of matrices.
#Function that estimates covariances, correlations and betas based on
    last x days, equal weights, and place it in t+1
#Arguments: Dataset(xts), No. of days (window), Portfolio weights
volhist.mat = function(tsret,days,w){
  dims = dim(tsret)
  variance = list()
  variance[[1]] = index(tsret[(days+1):(dims[1]),])
  covmat = list()
```

```
  cormat = list()
  for (i in 1:((dims[1])-days)) {
    covmat[[i]] = cov(tsret[i:(i+days-1),])
    cormat[[i]] = cor(tsret[i:(i+days-1),])
  }
  variance[[2]] = covmat
  variance[[3]] = cormat
  variance[[4]] = 0 #Old list for Beta's (Systematic VaR)

  dflist = list()
    for (y in 1:((dims[1])-days)) {
      portret = vector(mode = 'numeric', length = days)
      dfvec = vector(mode = 'numeric', length = 3L)
      for (r in 1:days){
      portret[r] = rowSums(t(t(tsret[(y+r-1)][,-11]) * w))
      }
      mean1 = mean(portret)
      std1 = sqrt(var(portret))

      dfvec[1] = as.numeric(fitdistr((portret-mean1)/std1,"t")[["
          estimate"]][["df"]]) #Estimating df for standardized
          portfolio returns
      dfvec[2] = 3 #as.numeric(fitdistr((ret2-mean2)/std2,"t",start =
          list(m=0, s=1, df=ifelse(y==1,dfvec[1],(dfvec[1]+dflist[[y
          -1]][2])/2)))[["estimate"]][["df"]])  #Estimating df for
          standardized mapped portfolio returns
      dfvec[3] = 3 #as.numeric(fitdistr((ret3-mean3)/std3,"t")[["
          estimate"]][["df"]]) #Estimating df for standardized
          portfolio residuals
      dflist[[y]] = dfvec
    }
  variance[[5]] = 0 #Old list for covariance matrices of residuals
  variance[[6]] = dflist
  return(variance)
}
```

C.3 Covariance Estimation - EWMA

```
volewma.mat = function(tsret,lambda,days,vobject){
  dims = dim(tsret)
  vecnames = colnames(tsret)
  variance = list()
  variance[[1]] = index(tsret[1:(dims[1]),])
  covmat = list()
  covmat[[1]] = matrix(data = 0, nrow = dims[2], ncol = dims[2],
     dimnames = list(vecnames,vecnames))
  cormat = list()
  cormat[[1]] = matrix(data = 0, nrow = dims[2], ncol = dims[2],
     dimnames = list(vecnames,vecnames))
  for (i in 2:(dims[1])) {
    covmat[[i]] = matrix(data = NA, nrow = dims[2], ncol = dims[2],
       dimnames = list(vecnames,vecnames))
     for (x in 1:(dims[2])){
       for (y in 1:(dims[2])) {
         covmat[[i]][y,x] = (1-lambda) * tsret[(i-1),x] * tsret[(i-1),y
            ] + lambda * covmat[[(i-1)]][x,y]
       }
     }
    cormat[[i]] = cov2cor(covmat[[i]])
  }
  t1 = days+1
  t2 = length(variance[[1]])
  variance[[1]] = variance[[1]][t1:t2]
  variance[[2]] = covmat
  variance[[2]] = variance[[2]][t1:t2]
  variance[[3]] = cormat
  variance[[3]] = variance[[3]][t1:t2]
  variance[[4]] = 0 #Old list for Beta's (Systematic VaR)
  variance[[5]] = 0 #Old list for covariance matrices of residuals
  variance[[6]] = vobject[[6]]

  return(variance)
}
```

C.4 Covariance Estimation - DCC GARCH

```
volmgarch.mat = function(xtsret,start,refit,winsize,distrmod){
  #distrmod = ifelse(distrmod == 0, "norm", "sstd")
  spec = ugarchspec(variance.model = list(model = "sGARCH", garchOrder
      = c(1, 1),submodel = NULL, external.regressors = NULL, variance.
      targeting = FALSE), distribution.model = "norm", mean.model=list(
      armaOrder=c(1, 0), include.mean = T))
  mspec = multispec(replicate(dim(xtsret)[2], spec))
  dcc_spec = dccspec(mspec, distribution = c("mvnorm"))
  roll = dccroll(spec=dcc_spec, data=xtsret, n.ahead = 1, refit.every =
       refit, n.start = start, refit.window = "moving", window.size =
      winsize, keep.coef = TRUE)
  covars = rcov(roll)
  corels = rcor(roll)
  output = list()
  output[[1]] = as.Date(attributes(covars)$dimnames[3][[1]])
  output[[2]] = list()
  for (i in 1:(dim(covars)[3])){
    output[[2]][[i]] = covars[,,i]
  }
  output[[3]] = list()
  for (i in 1:(dim(corels)[3])){
    output[[3]][[i]] = corels[,,i]
  }

  output[[4]] = 0 #Old list of Beta's
  output[[5]] = roll
  return(output)
}
```

C.5 Covariance Estimation - Hybrid Implied Volatility

```
implied.cov = function(ivs,cormats,day,tsret){
  vec = NULL
  vec1 = c(1,seq(3,93,by=6))
  #Shifting IVs one day ahead, so IV from 2002-12-11 is placed in
     2002-12-12 since we use it as vol estimator for 2002-12-12.
  ivsub = ivs[1:(dim(ivs)[1]-1),vec1]
  ivsub[,1] = ivs[2:(dim(ivs)[1]),1]
  ivsub = ivsub[,c(1,1+keepcols)]
  ind = which(cormats[[1]] >="2002-12-11")
  ivsub$date = as.Date(ivsub$date)
  tmpdays = data.frame(date = cormats[[1]][ind])
  ivsub = left_join(tmpdays,ivsub,by="date")
  dims = dim(ivsub)

  variance = list()
  variance[[1]] = cormats[[1]][ind] #Dates
  covmat = list() #Implied Covariances
  for (i in 1:length(ind)) {
    #Covariance matrix = VRV, where V is diagonal matrix constructed of
         implied volatilities and R is correlation matrix constructed
        of EWMA, Eq.W or GARCH correlations
    covmat[[i]] = diag(ivsub[i,-1]*sqrt(day/252)) %*% cormats[[3]][(i+
        length(cormats[[3]])-length(ind))][[1]] %*% diag(ivsub[i,-1]*
        sqrt(day/252))
    dimnames(covmat[[i]]) = list(colnames(cormats[[3]][[1]]),colnames(
        cormats[[3]][[1]]))
  }
  variance[[2]] = covmat
  variance[[3]] = cormats[[3]][ind] #'Old' correlation matrix
  variance[[4]] = 0 #Old list for beta's
  variance[[5]] = 0 #Old list for residuals
  variance[[6]] = 0 #cormats[[6]][ind] just for v4.new.garch.new
  return(variance)
}
```

C.6 Parametric VaR

```
#Function that returns parametric normal and student-t VaR
#Arguments: XTS of returns, significance levels, object with
   covariances and betas, weights
param.var = function(xtsrets,sig,covlist,w){
  start.point = index(head(xtsrets,1))
  end.point = index(tail(xtsrets,1))
  p1 = match(start.point,covlist[[1]])
  p2 = match(end.point,covlist[[1]])
  covlist[[1]] = covlist[[1]][p1:p2]
  covlist[[2]] = covlist[[2]][p1:p2]
  covlist[[3]] = covlist[[3]][p1:p2]
  #covlist[[4]] = covlist[[4]][p1:p2,]
  #covlist[[5]] = covlist[[5]][p1:p2]
  covlist[[6]] = covlist[[6]][p1:p2]
  xtsrets.sub = xtsrets[covlist[[1]]]
  dims = dim(xtsrets.sub)
  stdev = matrix(data = NA, nrow = dims[1], ncol = 1)
  total.df = vector(mode = 'numeric', length = dims[1])
  syst.df = vector(mode = 'numeric', length = dims[1])
  spec.df = vector(mode = 'numeric', length = dims[1])
    for (i in 1:dims[1]){
      stdev[i,1] = sqrt(t(w) %*% covlist[[2]][[i]][-11,-11] %*% w) #
         Total

      total.df[i] = covlist[[6]][[i]][1]
    }

  var.xts.nor = xts(cbind(-qnorm(sig[1])*stdev,-qnorm(sig[2])*stdev,-
     qnorm(sig[3])*stdev), order.by = index(xtsrets.sub))
  var.ts.nor = timeSeries(var.xts.nor)

  var.xts.std = xts(cbind(-qt(sig[1],total.df)*sqrt((total.df-2)/total.
     df)*stdev,-qt(sig[2],total.df)*sqrt((total.df-2)/total.df)*stdev
     ,-qt(sig[3],total.df)*sqrt((total.df-2)/total.df)*stdev), order.
     by = index(xtsrets.sub))
  var.ts.std = timeSeries(var.xts.std)
```

```
  colnames(var.ts.nor) = c(paste(sig[1],"Total␣VaR"),
                           paste(sig[2],"Total␣VaR"),
                           paste(sig[3],"Total␣VaR"))

  colnames(var.ts.std) = c(paste(sig[1],"Total␣VaR"),
                               paste(sig[2],"Total␣VaR"),
                               paste(sig[3],"Total␣VaR"))

  output = list()
  output[[1]] = var.ts.nor
  output[[2]] = var.ts.std
  return(output)
}
```

C.7 Historical VaR - Non Adjusted

```
#Function that returns series of portfolio VaR (historical) at
   different significance levels
#Inputs: Time series (xts) of portfolio returns, rolling sample size,
   significance levels
hist.var = function(portrets, n, sig){
  dims = dim(portrets)
  dfhist = matrix(data = NA, nrow = dims[1], ncol = length(sig)+1)
  dfhist = data.frame(dfhist)
  colnames(dfhist) = c("date",sig)
  dfhist$date = index(portrets)

  for (i in (n+1):dims[1]){
    for (z in 1:3) {
      dfhist[i,z+1] = -quantile(portrets[(i-n):(i-1),],sig)[z]
    }
  }
  dfhist = xts(dfhist[-1], order.by=as.Date(dfhist[,1]))
  dfhist = timeSeries(dfhist["2007/2016"])
  return(dfhist)
}
```

C.8 Historical VaR - Cholesky Adjusted

```
#Arguments: xtsrets, n = size of rolling window, s = how many
   observations to skip due to 'the leading minor is not positive
   definite' error
hist.var.chol = function(xtsrets, n, sig, covlist, w, s){
  xtsrets.sub = xtsrets[covlist[[1]]] #returns with dates extracted
     from cov list (v1,v2,v4...)
  dims = dim(xtsrets.sub)
  dfhistchol = matrix(data = NA, nrow = dims[1], ncol = length(sig)+1)
  dfhistchol = data.frame(dfhistchol)
  colnames(dfhistchol) = c("date",sig)
  dfhistchol$date = index(xtsrets.sub)
  for (i in (n+1+s):dims[1]) {
    temprets = xtsrets.sub[(i-n):(i-1)] #creating temp subset of n
       returns
    cholrets = matrix(data = NA, nrow = n, ncol = (dims[2]-1))
    for (z in 1:n) {
      cholrets[z,] = temprets[z,1:10] %*% solve(chol(covlist[[2]][[(z+i
         -n-1)]][1:10,1:10])) %*% chol(covlist[[2]][[(i-1)
         ]][1:10,1:10])
    }
    portrets = rowSums(t(t(cholrets[,1:10] * w))) #1:10 because of
       excluding SPY

    for (y in 1:3) {
      dfhistchol[i,y+1] = -quantile(portrets,sig)[y]
    }
  }
  dfhistchol = xts(dfhistchol[-1], order.by=as.Date(dfhistchol[,1]))
  dfhistchol = timeSeries(dfhistchol)
  return(dfhistchol)
}
```

C.9 Portfolio Weights Simulation

```
#Generating portfolio weights 500 times with different seed
weights.mat = list()
for (w in 1:500){
  set.seed(w)
  weights = rnorm(9, mean = 0.1, sd = 0.05)
  weights[10] = 1 - sum(weights)
  weights.mat[[w]] = weights
}

#Cholesky decomposition 500 times with different portfolio weights
start.time = Sys.time()
cl <- makeCluster(15) #Use no of cores on Amazon EC2 instance
registerDoParallel(cl) #Set parallel backend
var.mat4 <- foreach(p=1:500, .packages=c('dplyr','timeSeries','xts','
   MASS','doParallel')) %dopar% {
  window(hist.var.chol(xts.returns1, 1000, significance, v4.new,
     weights.mat[[p]], 10),"2007-01-01","2016-12-31")
}
stopCluster(cl)
Sys.time() - start.time

#Generating portfolio returns 500 times with different weights
portrets.long = list()
for (p in 1:500){
  portr = rowSums(t(t(xts.returns1[,1:10]) * weights.mat[[p]]))
  portr = xts(portr, order.by = index(xts.returns1))
  names(portr) = "Portfolio"
  portr = timeSeries(portr)
  portrets.long[[p]] = window(portr,"2004-01-01","2016-12-31")
}
```