UNEMPLOYMENT PERIOD QUANTITATIVE APPROACH THROUGH INFINITE SERVERS QUEUE SYSTEMS

***Manuel Alberto M. Ferreira*[1]*, José António Filipe*[1]*, Manuel Coelho*[2]**

[1]Instituto Universitário de Lisboa (ISCTE-IUL), BRU-IUL, ISTAR-IUL, Portugal

[2] Universidade de Lisboa (UL), SOCIUS/ISEG, SIRIUS/ISEG, Portugal

Abstract

This paper stands over the (Ferreira, Filipe and Coelho, 2014) work. There using results on the infinite servers queue systems with Poisson arrivals - M|G|∞ queues - busy period, it is presented an application of those queue systems in the unemployment periods time length parameters and distribution function study. It is now completed with an economic analysis aiming the evaluation of the assistance costs due. These queue systems are adequate to the study of many population processes, and this quality is brought in here. The results presented are mainly on unemployment periods length and their number in a certain time interval. Also, some questions regarding the practical applications of the outlined formulas are discussed.

Keywords: *Infinite servers queues, busy period, unemployment.*

JEL Classification: C18

AMS Classification: 60G99

1 RISING THE MODEL

In the queue systems used thoroughgoing this work

- The customers arrive according to a Poisson process at rate λ,
- Receive a service which time length is a positive random variable with distribution function $G(.)$ and mean α,
- Upon they arrive, each one finds immediately an available server[1],
- Each customer service is independent from the other customers' services and from the arrivals process,

[1]Or there is no distinction between the customer and its server, as it happens in the application to be considered in this work.

- The traffic intensity is $\rho = \lambda\alpha$.

That is: they are $M|G|\infty$ queues. It is easy to understand how these queues can be applied to the unemployment study. Then:

- λ is the rate at which occur the firings, supposed to happen according to a Poisson process
- The service time, paradoxically, is the time between the worker firing and the moment he/she finds a new job.

In any queue system, a busy period is a period that begins when a costumer arrives at the system finding it empty, ends when a costumer abandons the system letting it empty and in it there is always at least one customer present. So in a queuing system there is a sequence of idle and busy periods, during its operation time.

In the $M|G|\infty$ queue system, as in any queue system with Poisson arrivals, the idle periods have an exponentially distributed length with mean λ^{-1}.

But the busy period's distribution is much more complicated, being in general given by infinite sums which parcels are convolutions (Ferreira and Andrade, 2009). In spite of it, it is possible to present some results as it will be seen.

For what interests in this work

- A busy period is a period of unemployment
- An idle period is a period of full employment.

The results to be presented are on unemployment periods length and their number in a certain time interval.

A study in following also this approach, over a public health situation, can be seen in Ferreira (2014).

2 UNEMPLOYMENT PERIOD TIME LENGTH DISTRIBUTION

Designate D the random variable unemployment period length. According to the results known for the $M|G|\infty$ queue busy period length distribution, see (Ferreira and Andrade, 2009),

- $$E[D] = \frac{e^{\rho} - 1}{\lambda} \qquad (2.1)$$

  whichever is the worker unemployment time length distribution, see Takács (1962)

- As for $Var[D]$, it depends on the whole unemployment time length distribution probabilistic structure. But Sathe (1985) demonstrated that

$$\max \left[e^{2\rho}+e^{\rho}\rho^{2}\gamma_s^{2}-2\rho e^{\rho}-1;0\right] \le \lambda^{2}Var[D] \le \left[2e^{\rho}\left(\gamma_s^{2}+1\right)\left(e^{\rho}-1-\rho\right) \quad -\left(e^{\rho}-1\right)^{2}\right],$$

(2.2)

where γ_s the unemployment time length coefficient of variation

- If a worker unemployment time length distribution function is

$$G(t)=\frac{e^{-\rho}}{\left(1-e^{-\rho}\right)e^{-\lambda t}+e^{-\rho}}, t \ge 0, \quad (2.3)$$

the D distribution function is

$$D(t)=1-\left(1-e^{-\rho}\right)e^{-e^{-\rho}\lambda t}, t \ge 0 \qquad (2.4),$$

see Ferreira (1991)

- If the unemployment time length of a worker is such that

$$G(t)=1-\frac{1}{1-e^{-\rho}+e^{-\rho+\frac{\lambda}{1-e^{-\rho}}t}}, t \ge 0 \qquad (2.5)$$

the D distribution function is

$$D(t)=1-e^{-\left(e^{\rho}-1\right)^{-1}\lambda t}, t \ge 0 \qquad (2.6),$$

see (Ferreira, 1995)

- For α and ρ great enough (very intense unemployment conditions) since $G(.)$ is such that for α great enough $G(t) \cong 0, t \ge 0$,

$$D(t) \cong 1-e^{-\lambda e^{-\rho}t}, t \ge 0 \qquad (2.7),$$

see (Ramalhoto and Ferreira, 1994).

Note:

- As for this last result, begin noting that many probability distributions fulfill the condition $G(t) \cong 0, t \ge 0$ for α great enough. The exponential distribution is an example.

- As for the meaning of α and ρ great enough, computations presented in (Ramalhoto and Ferreira, 1994) show that for $\lambda = 1$, after $\rho = 10$ it is reasonable to admit (2.7) for many service time distributions.

Calling N_D the mean number of unemployed people in the unemployment period, if $G(.)$ is exponential

$$N_D = e^{\rho} \qquad (2.8).$$

For any other $G(.)$ probability distribution

$$N_D \cong \frac{e^{\rho(\gamma_s^2+1)}\left(\rho(\gamma_s^2+1)+1\right)+\rho(\gamma_s^2+1)-1}{2\rho(\gamma_s^2+1)} \qquad (2.9),$$

see (Ferreira and Filipe, 2010). Of course, multiplying (2.8) or (2.9), as appropriate, by the mean cost of each unemployment subsidy it is possible to estimate the assistance costs caused by the unemployment period.

Be $p_{1'0}(t)$ the probability that everybody is working at time *t*, being the time origin the unemployment period beginning. Being $h(t) = \frac{g(t)}{1-G(t)}$, where $g(t)$ is the probability density function associated to $G(.)$, the service time hazard rate function[2],

$$h(t) \geq \lambda \Rightarrow p_{1'0}(t) \text{ is non} - \text{decreasing} \qquad (2.10),$$

see Proposition 3.1 in (Ferreira and Andrade, 2009). And calling $\mu(1',t)$ the mean number of unemployed people at time *t*, being the time origin the unemployment period beginning instant

$$h(t) \leq \lambda \Rightarrow \mu(1',t) \text{ is non} - \text{decreasing} \qquad (2.11),$$

see Proposition 5.1 in (Ferreira and Andrade, 2009).

3 UNEMPLOYMENT PERIODS IN A TIME INTERVAL MEAN NUMBER

After the renewal processes theory, see Çinlar (1975), calling $R(t)$ the mean number of unemployment periods that begin in $[0,t]$, being $t = 0$ the beginning instant of an unemployment period, it is possible to obtain, see Ferreira (1995),

[2]That is: the rate at which unemployed people finds a new job.

$$R(t)=e^{-\lambda\int_0^t[1-G(v)]dv}+\lambda\int_0^t e^{-\lambda\int_0^u[1-G(v)]dv}du \qquad (3.1)$$

and, consequently,

$$e^{-\rho}(1+\lambda t)\le R(t)\le 1+\lambda t \qquad (3.2),$$

see Ferreira (2004).

Also,

A) $G(t)=\dfrac{e^{-\rho}}{(1-e^{-\rho})e^{-\lambda t}+e^{-\rho}}, t\ge 0$

$$R(t)=1+\lambda e^{-\rho}t \qquad (3.3)$$

B) $G(t)=1-\dfrac{1}{1-e^{-\rho}+e^{-\rho+\frac{\lambda}{1-e^{-\rho}}t}}, t\ge 0$

$$R(t)=e^{-\rho}+\left(1-e^{-\rho}\right)^2+\lambda e^{-\rho}t+e^{-\rho}\left(1-e^{-\rho}\right)e^{-\frac{\lambda}{1-e^{-\rho}}t} \quad (3.4)$$

C) $G(t)=\begin{cases}0, t<\alpha\\ 1, t\ge\alpha\end{cases}$

$$R(t)=\begin{cases}1, t<\alpha\\ 1+\lambda e^{-\rho}(t-\alpha), t\ge\alpha\end{cases} \qquad (3.5)$$

D) If the unemployment time length is exponentially distributed

$$e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)}+\lambda e^{-\rho}t\le R(t)\le e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)}+\lambda t \quad (3.6)$$

4 CONCLUSIONS

So that this model can be applied it is necessary that the firings occur according to a Poisson process at constant rate. It is a hypothesis that must be tested. Thus remain outside of this study, periods of unemployment caused by mass firings.

Among the results presented, (2.1), (2.2), (2.7) and (3.2) are remarkable for its simplicity and also for requiring only the knowledge of the firings rate λ, the mean unemployment time α, and the unemployment time variance.

The other results are little more complex and demand the goodness of fit test for the distributions indicated to the unemployment times.

References

[1] Çinlar, E.: *Introduction to Stochastic Processes*. Prentice Hall, Englewood Cliffs, NJ.1975.

[2] Ferreira, M. A. M.: Um sistema M|G|∞ com período de ocupação exponencial. In: *Actas das XV Jornadas Luso-Espanholas de Matemática* (C. Braumann, ed.), Vol. IV, Universidade de Évora, Évora, 1991, D63-D66.

[3] Ferreira, M. A. M.: *Comportamento transeunte e período de ocupação de sistemas de filas de espera sem espera*. PhD thesis. ISCTE-IUL, Lisboa,1995.

[4] Ferreira, M. A. M.: $M|G|\infty$ queue busy cycle renewal function for some particular service time distributions. In: *Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XII)* (M. Lukáèik, ed.), Virt, Slovakia, 2004, 42-47.

[5] Ferreira, M. A. M. and Andrade, M.: The Ties between the $M|G|\infty$ Queue System Transient Behaviour and the Busy Period. *International Journal of Academic Research*, **1**(2009), 84-92.

[6] Ferreira, M. A. M. and Filipe, J. A.: Economic Crisis: Using $M|G|\infty$ Queue Systems Busy Period to solve Logistics Problems in an Organization. *China-USA Business Review*, **9** (2010), 59-63.

[7] Ferreira, M. A. M., Filipe, J. A. and Coelho, M.: Unemployment Modelled Through M|G|∞ Systems. *International Journal of Latest Trends in Finance and Economic Sciences*, **4**(2014), 677-679.

[8] Ferreira, M. A. M.: The pandemic period length modelled through queue systems. In: *Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVII)* (R. Marian, P. Gezik, and L. Martin, eds.), Virt, Slovakia, 2014, 43-47.

[9] Ramalhoto, M. F. and Ferreira, M. A. M.: Some further properties of the busy period of a $M|G|\infty$ queue. *Central European Journal for Operations Research and Economics*, **4**(1996), 251-278.

[10] Sathe, Y. S.: Improved bounds for the variance of the busy period of the $M|G|\infty$ queue. *Advances in Applied Probability*, 17(1985), 913-914.

[11] Takács L.: *An introduction to queueing theory*. Oxford University Press. 1962.

Authors' addresses

Professor Catedrático Manuel Alberto M. Ferreira

Instituto Universitário de Lisboa (ISCTE-IUL), Escola de Tecnologias e Arquitectura (ISTA),

Departamento de Matemática, BRU-IUL, ISTAR-IUL

Av. das Forças Armadas, 1649-026 Lisboa, Portugal

Email: manuel.ferreira@iscte.pt

Professor Auxiliar com Agregação José António Filipe
Instituto Universitário de Lisboa (ISCTE-IUL), Escola de Tecnologias e Arquitectura (ISTA),
Departamento de Matemática, BRU-IUL, ISTAR-IUL
Av. das Forças Armadas, 1649-026 Lisboa, Portugal
Email: josé.filipe@iscte.pt

Professor Auxiliar com Agregação Manuel Coelho
Universidade de Lisboa (UL), Lisbon School of Economics and Management,
Departamento de Economia, SOCIUS/ISEG, SIRIUS/ISEG
Rua Miguel Lupi, 20, 1249-078, Lisboa, Portugal
Email: coelho@iseg.ulisboa.pt