How to Statistically Model Coherent MPI in Optical Communications?

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Abstract: The Beta distribution is used to model coherent MPI in optical communications and its fitness to describe experimental results is evaluated. It fits quite well to symmetric scenarios, but has some troubles when skewness matters.
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1. Introduction

Multipath interference (MPI) is a common impairment encountered in optical communications and it is originated whenever a transmitted signal reaches the destination through two or more optical paths. When the difference between the propagation time of the different paths is much smaller than the laser coherence time the impairment is called as coherent MPI. In the presence of this impairment the signals propagating over the different paths are correlated, which leads to slow fluctuations in the received optical power, a phenomenon similar to multipath fading present in wireless communications [1]. The sources of coherent MPI are diverse ranging from light leakages in optical network nodes [2], to mode coupling in both bend-insensitive fibers [3] and few mode fibers.

A key question is how to statistically model those fluctuations. Contrary to wireless communications, where a large amount of statistical models is available to deal with fading, in optical communications the research in this subject is incipient and only Rice and the Beta distribution have been proposed in the literature [3], [4]. The main goal of this work is to contribute to model the coherent MPI in optical communications using as the starting point the experimental data given in [2, 3]. For that purpose we use as an input model the Beta distribution, and afterwards we estimate the parameters of this distribution from the experimental data using the moment matching technique. We use this distribution because it is able to model a wide variety of shapes and at the same time accommodate different types of skewnesses, which is an important factor in modeling the data from [2].

2. Coherent MPI statistical model

Consider an optical received signal which is impaired by coherent MPI due to the presence of N multiple propagation paths. Consider, also, that associated with the ith path there is the attenuation factor $\sqrt{\varepsilon_i}$ and the differential propagation delay τ_i, measured relatively to the signal, in such a way that the MPI level is defined as $\sqrt{\mathrm{MPI}} = \sum_{i=1}^{N}\sqrt{\varepsilon_i}$. In this situation the instantaneous received optical power is given by [4],

$$P \approx P_0 \,|\, 1 + \sum_{i=1}^{N} \sqrt{\varepsilon_i}\, \exp(-j\varphi_i) \,|^2 , \tag{1}$$

where P_0 denotes the average received optical power in the absence of MPI and $\varphi_i = 2\pi \nu_s \tau_i$ is the phase offset of the ith interfering field, due to multipath propagation, relative to the signal, with ν_s the signal frequency. The phase offset φ_i varies in a random manner following a uniform distribution between $[0, 2\pi]$, as a consequence of a number of uncontrollable disturbances like drifts in the laser source central frequency, variations in the differential delay coming from mechanical and thermal fluctuations, etc. In these conditions P is no longer constant and it is subject to slow fluctuations of random nature. We assume that these fluctuations can be modelled using a Beta distribution. In this case we can write from (1) that $P = P_0[1 + 4\sqrt{MPI}\,(\xi - \bar{\xi})]$, where $\xi \in [0,1]$ is a random variable with a Beta distribution, which permits to describe the probability density function (PDF) of P in the following form:

$$f_P(P;\alpha,\beta) = \frac{1}{4P_0\sqrt{MPI}}\, f_\xi\!\left(\frac{\dfrac{P}{P_0} - 1 + 4\sqrt{MPI}\,\bar{\xi}}{4\sqrt{MPI}} ; \alpha, \beta \right), \tag{2}$$

where $f_\xi(\xi;\alpha,\beta)$ is the PDF of ξ, and α and β are the distribution shaping parameters. In addition the expected value and variance of ξ are given by $\bar{\xi} = \alpha/(\alpha+\beta)$ and $\sigma_\xi^2 = \alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1]$, respectively.

3. Results and discussion

The Beta distribution can take a large variety of shapes depending on the values of the parameters α and β. For instance, when $\alpha = \beta$ the distribution is symmetric about the mean, while for $\alpha > 2$ and $1 < \beta < 2$ the distribution is negatively skewed, *i.e.* the tail on the left side of the density function is longer than the tail on the right side. Interestingly enough, the experimental results for the received intensity in the presence of coherent MPI in bend-insensitive fiber transmission, denoted here as Scenario 1, shows a symmetric distribution around its average value [3], while the experimental histograms for the Q function in the presence of coherent MPI in a WDM network node, denoted as Scenario 2, point out to a negatively skewed distribution [2]. Assuming that a Beta distribution applies to both the referred scenarios it would be useful to find the shape parameters that best fit the experimental data. To do so we use a moment matching technique, *i.e.* the sample mean $\bar{x}$ and the variance s^2 are computed from the set of the experimental samples $\{x_i\}_{i=1}^n$ and then these parameters are equated to $\bar{\xi}$ and σ_ξ^2, which permits to obtain the estimates $\hat{\alpha}$ and $\hat{\beta}$ for the shape parameters of the Beta function.

In Figs. 1(a) and 1(b) the PDF $f_P(P;\alpha,\beta)$ given by (2) is plotted as a function of the normalized optical power (P/P_0) for different values of the parameters α and β. These figures also include the experimental data from [2] and [3], respectively. Fig. 1(a), which corresponds to Scenario 1, was obtained assuming that $\alpha = \beta$ and a MPI level of –14 dB (the same value used in [3]). For this scenario we have obtained from the experimental data [3] $\bar{x} = 0.5$ and $s^2 = 0.032$ leading to $\hat{\alpha} = \hat{\beta} = 3.36$. The PDF for these parameters is represented in Fig. 1(a), from which we can confirm the symmetry about P/P_0, and observe that the Beta distribution fits quite well the sample data. This figure also shows that the density function widens as the value of the shape parameters decrease. The asymmetric case, corresponding to Scenario 2 is analyzed in Fig. 1(b), which is obtained considering a MPI level –30 dB (the same value used in [2]). For this scenario we have computed from the experimental data [2] $\bar{x} = 0.614$ and $s^2 = 0.051$, considering n = 100 samples, yielding $\hat{\alpha} = 2.24$ and $\hat{\beta} = 1.41$. The curve corresponding to these parameters is shown in Fig. 1(b), which also includes additional curves for other values of the shape parameters. As can be seen this curve does not fit well the sample data, especially at the lower tail, mainly because this data is highly asymmetric and we have not considered moments of order higher than two in the analysis.

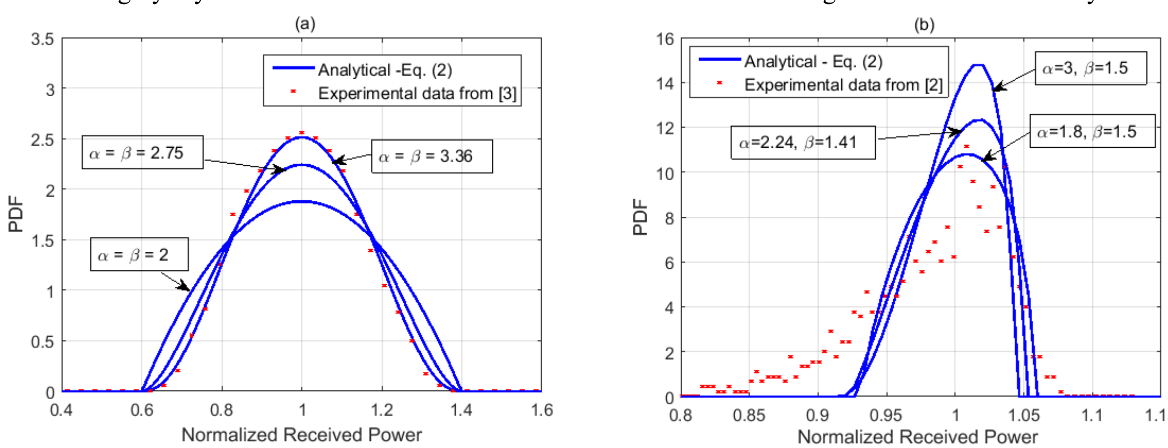

Fig. 1 PDF of the normalized received power: (a) Symmetric case (Scenario 1); (b) Asymmetric case (Scenario 2).

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5. References

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