Generalization of the image method for a Minkowskian isotropic medium

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Abstract – **The Minkowskian isotropic medium (MIM) is a special kind of Tellegen medium that generalizes the perfect electromagnetic conductor (PEMC) concept. Moreover, this generalization provides a systematic way of addressing other well-known electromagnetic boundaries, such as the perfect electric conductor (PEC) and the perfect magnetic conductor (PMC). In this paper, we present a generalized image method for an MIM. We show that an MIM ground plane place in the vicinity of an electric dipole can be replace by electric and magnetic image dipoles.**

I. INTRODUCTION

The perfect electromagnetic conductor (PEMC) was defined by Lindell & Sihvola in [1] as an electromagnetic boundary which generalizes the perfect electric conductor (PEC) and the perfect magnetic conductor (PMC). In the literature, several studies analyzed the PEMC as a boundary [2]. Nevertheless, the PEMC may be as well regarded as special case of a Tellegen medium, that is, a nonbirefrigent nonreciprocal bi-isotropic medium. More recently, the PEMC was studied as a limiting case of a special Tellegen medium called the Minkowskian isotropic medium (MIM) [3]. The electromagnetic field in the interior of the PEMC can be unambiguously defined by considering this limit. The MIM concept was formulated in the Minkowskian spacetime by the 4D constitutive relation [3]

$$\mathbf{G} = \Gamma\ \mathbf{F} - M\ \mathbf{I}\mathbf{F} \tag{1}$$

where M is the admittance associated with the electromagnetic coupling and Γ is the principal part of the constitutive relation. The Maxwell and Faraday bivectors, **G** and **F**, are expressed using the geometric algebra (GA) by $\mathbf{G} = \mathbf{D} + c^{-1}\mathbf{I}\mathbf{H}$ and $\mathbf{F} = c^{-1}\mathbf{E} + \mathbf{I}\mathbf{B}$, respectively, where **I** is the unit quadrivector. The spacetime algebra (STA) establishes an easy way to distinguish observer-dependent and observer-independent entities. As a consequence, the 4D constitutive relation of a MIM is an invariant and thus, the MIM can be defined as a truly isotropic medium. In the usual Euclidean space, an MIM can be described as a Tellegen medium with a unit refractive index, i.e. $n^2 = \varepsilon\mu - \kappa^2 = 1$ where ε is the permittivity, μ is the permeability and κ is the Tellegen parameter. Both representations can be related according to $\varepsilon = \eta_0\,(M^2 + \Gamma^2)/\Gamma$, $\mu = 1/(\Gamma\,\eta_0)$ and $\kappa = M/\Gamma$.

As it is well known, radiation problems in a semi-infinite half-space terminated with either a PEC or PMC boundary can be solved with the help of image theory, such that the scattering of the primary electromagnetic fields can be expressed in terms of the fields radiated by some suitable "image source". Recently, the image method was been generalized for a PEMC boundary [4]-[5]. In this work, we further generalize the image method for radiation problems involving air-MIM interfaces. It is shown that the "image source" associated with a primary horizontal infinitesimal electric dipole radiating over a MIM consists of both infinitesimal electric and magnetic horizontal dipoles. The occurrence of a magnetic image dipole is a consequence of the electromagnetic coupling (bi-isotropy) of the MIM.

II. The special cases of a Minkowskian isotropic medium: the PEC, the PMC and the PEMC

The 4D constitutive relation of a PEMC is defined by $\mathbf{G} = -M\ \mathbf{IF}$, which corresponds to the limit of a MIM when $\Gamma = 0$. The corresponding 3D relations of a PEMC are the boundary conditions, $\mathbf{H} + M\ \mathbf{E} = 0$ and $\mathbf{D} - M\ \mathbf{B} = 0$. The PEC limit is obtained when: $M \to \infty$ or $\Gamma \to \infty$, such that the 4D constitutive relation given in (1) is reduced to $\mathbf{F} = 0$. On the other hand, the PMC is a particular case of a MIM when $M^2 + \Gamma^2 = 0$. In this case, the 4D constitutive relation (1) becomes $\mathbf{G} = 0$. Moreover, the simple isotropic medium (SIM) is obtained as a limit of a MIM, with $M = 0$, which implies a 4D constitutive relation $\mathbf{G} = \Gamma\ \mathbf{F}$. Using the parameterizations angles $0^\circ \le \mathrm{X} \le 90^\circ$ and $0^\circ \le \mathrm{Y} \le 90^\circ$, where $\varepsilon = \tan(\mathrm{X})$, $\mu = \tan(\mathrm{Y})$ and $\kappa^2 = \tan(\mathrm{X})\tan(\mathrm{Y}) - 1$ we can get a simple geometric picture of the special cases of a MIM (Fig. 1). The contours corresponding to constant κ and constant impedance $\eta = \sqrt{\varepsilon/\mu}$ are also represented in Fig. 1. The PEMC is obtained for any curve of η with the condition $\mathrm{X} = \mathrm{Y} = 90^\circ$. A SIM is defined by the condition $\mathrm{Y} = 90^\circ - \mathrm{X}$. The PEC and the PMC are defined by $\mathrm{X} = 90^\circ$ and $\mathrm{Y} = 90^\circ$, respectively. We should stress, that the PEC and the PMC are obtained for any curve of κ and not only for the PEMC case ($\kappa \to \infty$).

a)

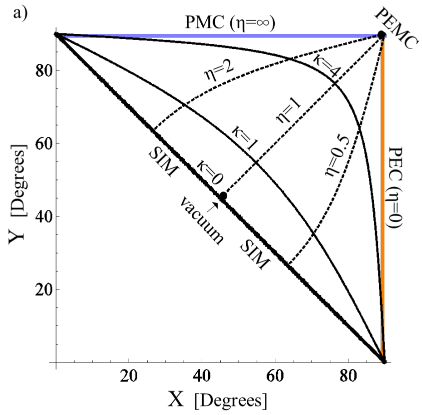

Fig. 1.Some particular cases of a MIM: the PEC, the PMC, the PEMC and the SIM.

III. The image theory for a Minkowskian isotropic medium

Let us consider a horizontal (x-oriented) infinitesimal electric dipole placed at a height h over a MIM semi-space, as depicted in Fig. 2). The primary source is described by the electric current density $\mathbf{J}_e(\mathbf{r},t) = -i\ \omega\ \mathbf{p}_e \delta(\mathbf{r} - \mathbf{r}_1)$, where $\delta(\mathbf{r} - \mathbf{r}_1)$ is the Dirac Delta function, $\mathbf{p}_e = p_e \hat{\mathbf{x}}$ is the electric dipole moment, and $\mathbf{r}_1 = (x_1, y_1, z_1)$ are the coordinates of the dipole. As is well-known, the primary radiation field $\mathbf{E}_i$ can be written in terms of the Green's function, $g_1 = e^{ik_0|\mathbf{r}-\mathbf{r}_1|}/(4\pi|\mathbf{r}-\mathbf{r}_1|)$, and of the vector potential $\mathbf{A} = \mu_0 g_1 \mathbf{p}_e$. The incident wave has the transverse electric components $\mathbf{E}_i = [E_{x,i} \quad E_{y,i}]^{\mathrm{T}}$. The reflected field can be determined by $\mathbf{E}_r = \overline{\mathbf{R}} \cdot \mathbf{E}_i$ where $\overline{\mathbf{R}}$ is the reflection matrix of the interface between the free space and the MIM given by

$$\overline{\mathbf{R}} = \frac{1}{\Delta}\begin{bmatrix} (ak_x k_y/k_z) + b & a(k_y^2 + k_z^2)/k_z \\ -a(k_x^2 + k_z^2)/k_z & -(ak_x k_y/k_z) + b \end{bmatrix}, \tag{2}$$

where $a = 2\kappa/k_0$, $b = \mu - \varepsilon$, $\Delta = \varepsilon + \mu + 2$, $k_z = (k_0^2 - k_x^2 - k_y^2)^{1/2}$, and (k_x, k_y) are the transverse wave numbers associated with the incident plane wave. We attempt to write the reflected wave as the field radiated by suitable image sources placed at the image point $\mathbf{r}_2 = (x_1, y_1, -z_1)$. Because of the MIM electromagnetic coupling, we expect that the image source is formed by the horizontal electric and magnetic infinitesimal dipole moments, $\mathbf{p}_{e,r} = p_{e,r}\hat{\mathbf{x}}$ and $\mathbf{p}_{m,r} = p_{m,r}\hat{\mathbf{x}}$, respectively. Thus, the corresponding magnetic and electric vector potentials, $\mathbf{A}'$ and $\mathbf{F}'$, are given by $\mathbf{A}' = \mu_0 g_2 \mathbf{p}_{e,r}$ and $\mathbf{F}' = \varepsilon_0 g_2 \mathbf{p}_{m,r}$, where the Green's function g_2 associated with the reflection image source is $g_2 = e^{ik_0|\mathbf{r}-\mathbf{r}_2|}/(4\pi|\mathbf{r}-\mathbf{r}_2|)$. The electric field radiated by the image sources, $\mathbf{E}_{im}$, is a superposition of the partial fields originated from electric and magnetic image dipoles

$$\mathbf{E}_{im} = [\nabla(\nabla \cdot \mathbf{p}_{e,r}) + k_0^2 (\bar{\mathbf{I}} \cdot \mathbf{p}_{e,r})] g_2 / \varepsilon_0 + i(\nabla \times \mathbf{p}_{m,r}) g_2 , \tag{3}$$

where $\bar{\mathbf{I}} = \hat{\mathbf{x}} \otimes \hat{\mathbf{x}} + \hat{\mathbf{y}} \otimes \hat{\mathbf{y}} + \hat{\mathbf{z}} \otimes \hat{\mathbf{z}}$ is the unity dyadic. By imposing that $\mathbf{E}_{im}$ is coincident with the field reflected by the MIM-air boundary $\mathbf{E}_r$, we can determine the electric and magnetic image dipole moment amplitudes, $p_{e,r}$ and $p_{m,r}$

$$p_{e,r} = \frac{1-(M\eta_0)^2 - (\Gamma\eta_0)^2}{(M\eta_0)^2 + (1+\Gamma\eta_0)^2}, p_{m,r} = -\frac{2M\eta_0^2}{(M\eta_0)^2 + (1+\Gamma\eta_0)^2} . \tag{4}$$

The PEC, PMC and PEMC cases be easily recovered using the proper conditions for the MIM admittances described in Section II.

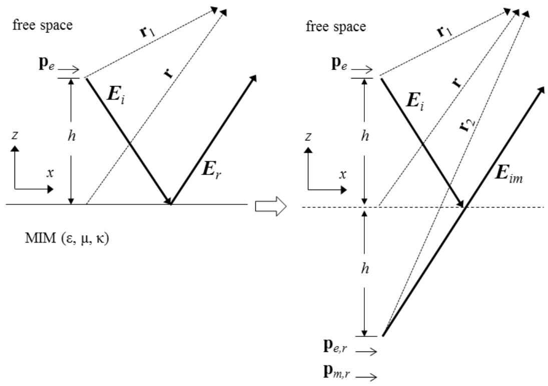

Fig. 2. Image method for a horizontal electric dipole placed over a MIM.

VI. Conclusion

The MIM concept comprises a vast class of electromagnetic boundaries as well as several classes of transparent media. In this wok, we show that radiation problems of electromagnetic sources placed over a planar vacuum-MIM interface can be addressed using the image method. Our theory generalizes the conventional image method associated with the well- know media: the PEC, the PMC and the PEMC.

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