Understanding Firms Compensation Policy Using Fuzzy Sets

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***Abstract*—It has been noted in the literature that firms rarely follow a single theoretical model when designing their compensation policy. This study illustrates how a fuzzy cluster analysis can be helpful in understanding the way employees are rewarded according to firms' specificity and market conditions. For this purpose, we convert linked employer-employee data (LEED) into firm level data prior to fuzzy clustering. Then, we explore the particular distribution of firms on the emerged fuzzy partition to sort them by compensation policy and, eventually, to examine the potential factors behind a specific option.**

I. Introduction

This paper revisits and expands on the methodological aspects of a comprehensive study on compensation policies carried out in [21]. Compensation policy entails different dimensions of pay, notably pay level, pay grades, pay growth, pay flexibility, and pay penalty. Firms configure these dimensions according to their own specificity. We can therefore expect them to vary across firms, thus giving rise to a heterogeneous population. In this study, we explore the fuzzy sets theory to model the diversity of compensation policies, and further investigate the characteristics of firms that might affect those policies. Readers interested in furthering their understanding of aspects of economic theory on this subject may refer to [21].

The problem we are addressing can be framed as follows. The theoretical models for compensation policy are useful tools to segment firms according to the way they reward employees. Two broad and almost antagonistic models are identified as Internal Labor Market (ILM) and External Labor Market (ELM).

The ILM model follows the rationale of fostering a long-term relationship between employees and organizations while simultaneously protecting investments in firm-specific skills [9]. The wages are attached to jobs rather than to workers, grow with tenure, and more importantly, are not adjusted to the business cycles or other external factors. There is also a limited discretion on wages. The ELM model has the opposite characteristics and is highly responsive to market conditions.

However, employers generally use a variety of incentive devices to obtain more advantages from employees. This includes, among others, high pay to avoid shirking [16], reduction employee turnovers [15], rewarding high-performers through group or individual incentives [11] or differentiating the key group of employees [14]. Hence, we can expect firms to tailor the compensation policy so as to suit their specificities and, of course, to face the market conditions. Despite the expected diversity among firms, we nonetheless believe that some, say $c \geq 2$ models or, more appropriately, typologies of compensation policy prevail, and the way firms position themselves in the labor market is a matter of degree in each typology. This hybrid characteristic of compensation policies is stressed by Baker in [1]. Therefore, we use a fuzzy clustering approach to empirically examine firms' compensation policy in the Portuguese labor market. In a post-hoc analysis, we attempt to identify the factors explaining the emerged typologies.

II. Description of Data

The Portuguese LEED, called *Quadros de Pessoal* (*QP*), are our source data. The *QP* is a longitudinal data set compiled annually by the Ministry of Economy and Employment, and is mandatory for every firm with wage earners (see e.g. [4]). We limit our focus to large-sized firms, i.e. ≥ 250 workers. This option aims to prevent additional heterogeneity due to firms' size. The sample we are working with comprises $N = 669$ firms, and is related to the year of 2009. The longitudinal information, whenever necessary, is based on the period $2004 - 2009$.

Since the firms are at the heart of our analysis, we first convert the employee-wise data into firm level data, following a strategy similar to that of [12]. The resulting constructs are intended to reflect the mechanisms behind the design of a compensation policy, as detailed in [21]. We can divide the underlying variables into two major groups: internal and external [17]. The internal variables are, on substantive grounds, potentially important to profile the emerged compensation typologies. In turn, the external variables are used to *a posteriori* identify the factors behind each typology, and can be useful tools for predictive purposes. Below, we present each group separately and, where necessary, summarize the purpose of each variable.

A. Internal variables

There are $n_I = 15$ internal variables, which can be arranged in four distinct dimensions of pay, as follows.

Pay Levels (8):

- Hourly wage: position of the firm in the labor market;

- Firm / industry wage: position of the firm in the corresponding industry;
- Gini: inequality of wage distribution;
- Skewness: wages asymmetry;
- Entry wage: competition around skills;
- Education premium: value of general skills;
- Wage hierarchy: job hierarchy;
- Intra job dispersion: internal equity and incentive devices.

Pay Growth (3):

- Wage growth: (self-explanatory);
- Growth skewness: selective incentive devices;
- Tenure profile: firm-specific skills.

Pay Flexibility (3):

- Wage cushion: incentive device measuring the difference between the total wage and the wage that has been bargained;
- Wage adjustment: wage used as adjustment to the business cycle;
- Rent sharing: incentive device.

Pay Penalty (1):

- Gender wage gap: penalization of female workers.

B. External variables

The following $n_E = 13$ external variables are used to characterize fuzzy clusters in a post hoc analysis. Ten variables are numerical and three are categorical. For the latter ones we indicate, in parenthesis, the respective categories, thus making their meaning clearer. The numerical variables, all self-explanatory, are: Percentage of fixed term contracts; Percentage of part-time workers; Percentage of female workers; Percentage of young workers, Percentage of blue collar workers; Firm dimension (number of workers); Firm age; Firm growth (in terms of the number of workers); Sales per worker (as a proxy of productivity); and Coefficient of variation of firm's sales (as a measure of sales volatility). For the growth variable, which accounts for the evolution and the competitive dynamic, we trace firm data back to 2004.

The three categorical variables are: C_1 : Collective bargaining level (Single, Collective, Industry or Other agreement); C_2 : Share holding (National private capital, Public, Foreign, Mixed or Other); and C_3 : Industry affiliation (Less knowledge-intensive service (KIS), Medium-high technology industry (TI), Medium-low TI, Low TI, High technology KIS, Market KIS, Knowledge intensive (KI) financial service, Other KIS, High TI, Other less KIS, Primary sector, or Construction). The categories of the affiliation variable are in accordance with EUROSTAT [10].

In sum: we use internal variables to cluster firms by compensation policy and, subsequently, examine the potential factors behind a specific option, using external variables.

III. Data analysis

A. Fuzzy clustering

We opt for a fuzzy clustering based on a matrix factorization approach, since we are interested in profiling wage policies that are somewhat extreme. This is expected to allow us to position firms in a structure set out by extreme profiles [13]. The prior assumption behind this approach is that the data matrix can be decomposed into a product of two matrices.

Formally, suppose $\mathbf{X} = [x_{jk}] \in \mathbb{R}^{n \times N}$ is the data matrix, where $n \geq 2$ is the dimension of the feature space, and N is the sample size. In our case, $n = n_I = 15$ is the number of internal variables, and $N = 669$. We assume there are two matrices, $\mathbf{U}$ and $\mathbf{V}$, such that

$$\mathbf{X} = \mathbf{V}\mathbf{U}, \tag{1}$$

although, in practice, we find an approximate decomposition of $\mathbf{X}$. In this relation, $\mathbf{V} = [v_{ji}] \in \mathbb{R}^{n \times c}$ and $\mathbf{U} = [\mu_{ik}] \in [0,1]^{c \times N}$, $c \geq 2$, $\sum_{i=1}^{c} \mu_{ik} = 1, 1 \leq k \leq N$, and $0 < \sum_{k=1}^{N} \mu_{ik} < N$, $1 \leq i \leq c$. The factorization (1), together with the restrictions on μ_{ik}, maps $\mathbf{X}$ into a fuzzy c-partition. The columns of $\mathbf{V}$ are referred to as prototypes and μ_{ik} is the membership degree of kth data point, $\mathbf{x}_k$, in fuzzy cluster i. Furthermore, the product (1) has a geometrical interpretation; it configures a polytope, say $\mathbb{P}_c$, with c extreme points, spanned by c columns of the matrix $\mathbf{V}$, $\mathbf{v}_1$, $\mathbf{v}_2$, ..., $\mathbf{v}_c$. Therefore, the data points are, by assumption, drawn from $\mathbb{P}_c$ as convex combinations of the fuzzy c-partition prototypes. As usual, each data point $\mathbf{x}_k$ is represented in a fuzzy c-partition by the vector of membership degrees, $\boldsymbol{\mu}_k = (\mu_{1k}, \mu_{2k}, ..., \mu_{ck}) \in \mathcal{S}_c$, where $\mathcal{S}_c$ is the unit simplex

$$\mathcal{S}_c = \left\{ (a_1, a_2, ..., a_c) : 0 \leq a_i \leq 1 \wedge \sum_{i=1}^{c} a_i = 1 \right\},$$

and here, in particular,

$$\mathbf{x}_k = \sum_{i=1}^{c} \mu_{ik} \mathbf{v}_i.$$

A common procedure to estimate the matrices $\mathbf{U}$ and $\mathbf{V}$, given c, is to minimize the objective function

$$J_c = \frac{1}{2} \left\| \mathbf{X} - \mathbf{V}\mathbf{U} \right\|_F^2, \tag{2}$$

subject to the constraints on membership degrees μ_{ik} referred to above. Here, $\|\mathbf{A}\|_F$ is the Frobenius norm of the matrix $\mathbf{A}$. We note, however, that, given $\mathbf{V}$, the minimization of J_c in (2) reduces to N independent least squares problems. Therefore, a common solver can be used to estimate $\mathbf{U}$. This means the factorization of the data matrix $\mathbf{X}$ is ultimately dependent on the way we estimate the matrix of prototypes $\mathbf{V}$.

In this study, we use the archetypal analysis [7] approach to estimate $\mathbf{V}$. Accordingly, the prototypes, now archetypes, are themselves convex combinations of the data points,

$$\mathbf{v}_i = \sum_{k=1}^{N} \beta_{ki} \mathbf{x}_k, \ i = 1, 2, ..., c,$$

where $0 \leq \beta_{ki} \leq 1$ and $\sum_k \beta_{ki} = 1$. As a result, the estimation of the matrix $\mathbf{V}$ reduces here to the estimation of the coefficients β_{ki}. In practice, we use an alternate optimization between $\mathbf{U}$ and $\mathbf{V}$; the β_{ki} coefficients are estimated using the updating rule given by Ding, Li, and Jordan in [8]; the matrix $\mathbf{U}$ is estimated by means of MATLAB function *lsqlin()*, with 'interior-point algorithm' option [6].

We note that, in [21], $\mathbf{V}$ is estimated using the factorized fuzzy c-means algorithm [20], which is akin to that of [3]. It is a special case of the archetypal analysis, where the β weights are calculated from the membership degrees μ_{ik}, and not estimated explicitly.

B. Compensation typologies

Following [21], we use the Xie and Beni index [22] to assess the goodness-of-fit of the estimated fuzzy c-partitions. The best fit is achieved for $c = 3$ clusters, meaning that the compensation policies of the Portuguese labor market of large-sized firms can be modeled by a fuzzy 3-partition. This result, as well as the profile of the emerged fuzzy clusters displayed in Table I, is consistent with the earlier work. In this table, the archetypes are identified as A_1, A_2 and A_3; in the second column, we represent the sample average values, for comparison purposes.

TABLE I
CHARACTERIZATION OF FUZZY 3-PARTITION PROTOTYPES. THE SECOND COLUMN SHOWS THE MEAN VALUES OF THE SAMPLE AS A WHOLE.

Variable	Sample	A_1	A_2	A_3
Hourly wage	5.23	3.51	3.89	9.63
Firm / industry wage	0.03	−0.12	−0.08	0.38
Gini	0.24	0.19	0.23	0.27
Skewness	4.58	8.32	3.24	3.11
Entry wage	−0.13	−0.10	−0.12	−0.17
Education premium	0.34	0.28	0.40	0.29
Wage hierarchy	−0.64	−0.57	−0.66	−0.65
Intra job dispersion	0.25	0.22	0.23	0.33
Wage growth	0.04	0.04	0.04	0.06
Growth skewness	2.56	2.45	2.56	2.64
Tenure profile	0.19	0.17	0.20	0.19
Wage cushion	1.95	1.57	1.66	2.88
Wage adjustment	−0.34	−0.54	−0.32	−0.28
Rent sharing	0.08	0.09	0.06	0.12
Gender wage gap	0.21	0.24	0.19	0.21

We can label the compensation policies underlying the estimated archetypes as follows: A_1 – *Competitive*, A_2 – quasi-*Internal Labor Market* (q-*ILM*), and A_3 – *Incentive*. At first glance, A_1 and A_3 typologies represent two extreme types of compensation policy, while the q-*ILM* appears as an intermediate case. For example, A_1 has the lowest levels of wage (3.51); the incentive devices target a small proportion of employees who earn higher wages, while a large proportion of employees earn low wages (Skewness $= 8.32$); and wages are particularly procyclical as they fall in response to an increase in unemployment (Wage adjustment $= -0.54$). Firms clustered in A_3 pay the highest wage (9.63), and above the industry level (Firm/industry wage $= 0.38$). These firms use several incentive devices, notably wage cushion (2.88), differentiation of employees in same jobs (Intra-job dispersion $= 0.33$), reward high-performers through specific wage growth (Growth skewness $= 2.64$) and rent sharing schemes (0.12). The distinctive features of q-*ILM* (A_2) are the tenure wage profile, which suggests that individual wages grow with the length of contract (Tenure profile $= 0.20$) and the returns to education (Education premium $= 0.40$).

We note, however, that some characteristics occur across all typologies. Without going into detail, we stress the striking similarity of fuzzy clusters in the use of wages as an adjustment process. This evidence is in line with [5], since wages are highly responsive to macroeconomic conditions in Portugal.

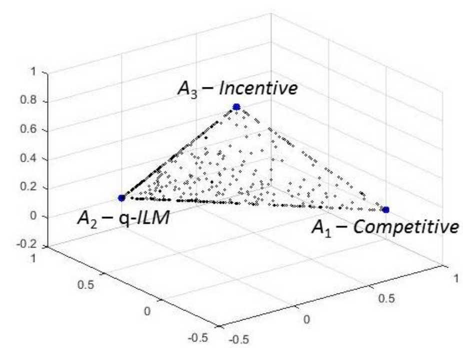

Fig. 1. An empirical distribution of firms on unit simplex $\mathcal{S}_3$.

In Fig. 1 we depict, in the unit simplex $\mathcal{S}_3$, the estimated membership degree vectors of every firm to gain a more accurate picture of their distribution on the polytope $\mathbb{P}_3$. If we further accept 0.90 as the threshold for the full membership in each fuzzy cluster, we realize (Table II) that only 35% of firms share the characteristics of a single typology. This means, most firms' compensation policy mixes characteristics of different typologies, as claimed in the literature [1]. However, a closer look at Table II evidences the concentration on the edges or, equivalently, in two fuzzy clusters, since 21% of firms have membership in $A_1 - A_2$, 23.6% in $A_2 - A_3$, and 6.7% in $A_1 - A_3$. In particular, more than 80% of firms are positioned on the path of edges $A_1 - A_2 - A_3$, including typologies.

TABLE II
DISTRIBUTION OF FIRMS ON THE FUZZY 3-PARTITION, AS REPRESENTED BY THE UNIT SIMPLEX $\mathcal{S}_3$ (MEMBERSHIP IN $A_i \wedge A_j$ MEANS NONZERO PARTIAL MEMBERSHIP IN FUZZY CLUSTERS A_i AND A_j, @0.90; OVERLAPPED CASES ARE ACCOUNTED FOR ONLY ONCE).

Membership in			Total (669)
A_1	A_2	A_3	
72	99	63	234
10.8%	14.8%	9.4%	35.0%
$A_1 \wedge A_2$	$A_2 \wedge A_3$	$A_1 \wedge A_3$	
146	158	45	349
21.8%	23.6%	6.7%	52.1%
$A_1 \wedge A_2 \wedge A_3$			86
			12.9%

C. Post-hoc analysis

A common procedure to study the potential factors behind a compensation policy would be to defuzzify the membership degrees, and subsequently analyze the pattern of external variables in each *crisp* typology. However, the distribution of firms on the path $A_1 - A_2 - A_3$ allows us to reasonably use the theorem below, which is established in [19]. It allows a mapping between multivariate data and a real interval, via fuzzy c-partition. The notation $A_i \to A_j$ indicates that we steadily move along the edge $A_i - A_j$, from the vertex A_i to the vertex A_j, $j > i$.

Theorem 1: Consider a fuzzy c-partition of the data matrix $\mathbf{X}$, and let $\mathcal{S}_c$ be the corresponding unit simplex. Let the vertices of $\mathcal{S}_c$ be enumerated as A_1, A_2, ..., A_c. Assume that each data point has membership in one or, at most, two consecutive fuzzy clusters, that is, positioned on the edge $A_i - A_{i+1}$, $1 \leq i \leq c-1$, of $\mathcal{S}_c$. Then, the utility function

$$\rho(k) = \sum_{i=1}^{c} \eta(i)\, \mu_{ik}$$

increases from $\eta(1)$ to $\eta(c)$ in the path of edges $A_1 \to A_2 \to \ldots \to A_c$, provided $\eta(i)$ is a strictly increasing sequence of real numbers. $\square$

In our example, if two firms k and k' are positioned on the edge $A_1 - A_2$, and $\rho(k) < \rho(k')$, then the firm k' is closer to q–*ILM* typology than the firm k; reciprocally, this firm is closer to the *Competitive* typology than the former. The utility function ρ is, therefore, a scalar indicator of firms' compensation policy, at least of those positioned on the path $A_1 - A_2 - A_3$. We explore this in an attempt to identify the potential factors that might be conditioning firms' individual option.

Even though the results of fuzzy clustering of firm data do not fully meet the conditions of the above theorem, we believe that it can be used as a good approximation. The major difficulty lies in the selection of the sequence $\eta(i)$, which can be subjective. Here, we consider a $0-1$ normalization of hourly wage in each typology (Table I, fist row), which leads to $\eta(1) = 0$, $\eta(2) = 0.06$, and $\eta(3) = 1$. We subsequently use the resulting utility function $\rho \in [0, 1]$ in a regression analysis, with the expectation of predicting the compensation policy by means of external variables. In sum, the variation of the utility function from 0 to 1 approximately translates the gradual movement of compensation policies from the *Competitive* typology to *Incentive* typology, passing through q-*ILM* (see Fig. 1); we therefore explore it for prediction purposes.

Due to the limited range of the utility function ρ, we opt for a Tobit regression model. This model can formally be written as a relationship between the observed outcome, here $\rho(k)$, and the latent variable of interest, $\rho^*(k)$, as follows [18]:

$$\rho(k) = \begin{cases} 0 & \text{if} \quad \rho^*(k) \leq 0 \\ \rho^*(k) & \text{if} \quad 0 < \rho^*(k) < 1 \\ 1 & \text{if} \quad \rho^*(k) \geq 1 \end{cases},$$

for each firm $k = 1, 2, \ldots, 669$. The n_E external variables, say $\mathbf{z} = (z_1, z_2, \ldots, z_{n_E})$, are linearly related to ρ^*, i.e.

$$\rho^*(k) = \alpha_0 + \alpha_1 z_{1k} + \alpha_2 z_{2k} + \ldots + \varepsilon_k,$$

where α_0 is the constant term, α_1, α_2, ..., are the Tobit regression coefficients, and ε_k is an independent normal random variable, with zero mean and constant variance σ^2. In this context, the α coefficient associated with a numerical variable has the same meaning as in ordinary least squares regression, whereas the coefficient of a categorical variable measures the impact of a given category when compared to the category that has been chosen as the reference. Some missing values in external variables are replaced by the corresponding mean value. The results of Tobit regression are displayed in Table III. The overall model fit, based on McKelvey & Zavoina's coefficient of determination, is $R^2 = 0.68$, which indicates a good predictive power.

TABLE III
TOBIT REGRESSION α COEFFICIENT ESTIMATES, AND THE ASSOCIATED STATISTICS, FOR THE EXTERNAL VARIABLES.

External Variable	α Coefficient	Std. Err.	p-value
% fixed term contracts	−0.1545	0.0406	0.000
% part-time workers	0.3752	0.0621	0.000
% female workers	−0.1571	0.0401	0.000
% young workers	−0.6205	0.0559	0.000
% blue collar workers	−0.4021	0.0363	0.000
Firm dimension	−0.0010	0.0005	0.045
Firm age	−0.0002	0.0002	0.281
Firm growth	0.0326	0.0317	0.304
Sales per worker	0.0783	0.0080	0.000
CV of firm sales	−0.0019	0.0292	0.947
C_1: Collective bargain. level			
Collective firms agreement	0.2521	0.0398	0.000
Single firm agreement	0.1836	0.0311	0.000
Others	0.1163	0.0254	0.000
C_2: Share holding			
Public	0.0938	0.0359	0.009
Foreign	0.0323	0.0214	0.131
Mixed	0.1116	0.0285	0.000
Others	−0.0667	0.0389	0.067
C_3: Industry affiliation			
Less KIS	−0.1093	0.0360	0.003
Medium-high TI	−0.0286	0.0407	0.485
Low TI	−0.0064	0.0362	0.859
High KIS	0.1919	0.0544	0.000
Market KIS	0.0815	0.0443	0.067
KI financial service	0.1859	0.0552	0.001
Other KIS	0.0140	0.0470	0.776
High TI	0.0139	0.0752	0.854
Other less KIS	−0.0133	0.0664	0.824
Primary sector	−0.1456	0.1031	0.156
Construction	0.0880	0.0403	0.029
Constant term α_0	−0.2206	0.1116	0.049

Dependent variable: $\rho(k)$; the reference categories of categorical variables are: C_1: Industry-level bargaining; C_2: National private capital; C_3: Medium-low TI. (KIS: knowledge-intensive service; TI: technology industry).
Obs. summary: 0 left-censored; 620 uncensored; 49 right-censored.

In general terms, when a numerical variable with positive α increases, the compensation policy moves in the direction *Competitive* $\to$ q-*ILM* $\to$ *Incentive*; the reverse holds true when α is negative. In the case of categorical variables, the coefficient of a given category weights its relative importance in moving in one or other direction. Looking at Table III, we realize that the workers with fixed-term contract prevail in firms closer to the *Competitive* typology, while part-time workers push firms to the *Incentive* typology. This only partially confirms the labor market segmentation theory, since we would expect both groups of workers to be present in firms sharing the characteristics of the former typology.

Female, blue-collar, and young workers tend to be associated with firms paying lower and flexible wages, that is, those closer to the *Competitive* typology. It is expected that high-skill workers (white collar workers) are employed in firms paying higher wages. Whereas no significant differentiation in wage typologies is found in manufacturing, the same does not hold in services, where significant industry differences are found, in accordance with knowledge intensity. Less knowledge intensive services (KIS) pushes them to *Competitive* typology, contrasting with more KIS (High technology KIS, market KIS, and KI Financial Services), which tends to push them in the opposite direction. As expected, the q-*ILM* and *Incentive* typologies are common in banking industry, while strategies to reduce labor costs are found in labor intensive services with low knowledge intensity.

Firms using decentralized bargaining systems (multiple or single firm arrangements) are closer to the *Incentive* typology, possibly because they allow more scope for rewarding skills. Larger firms are more prone to share characteristics of the *Competitive* typology; this finding contradicts some studies, since larger firms are likely to pay higher wages [2]. Most probably, this is due to the specificities of the Portuguese labor market, where a significant number of large firms follow generic low value added strategies, and use low-skilled workers. Finally, firms with higher sales per worker tend towards the *Incentive* typology, which signals an association between higher wages and labor productivity.

IV. Conclusion

We use a fuzzy cluster analysis to quantify the mixing characteristics of firms' compensation policy that has long been reported in the literature. We find three broad typologies in the Portuguese labor market, and realize that none fits most firms (65%) exactly. However, the heterogeneity within firms is seemingly not so extreme, since more than 80% of estimated compensation policies result from sharing, at most, two consecutive typologies. As a consequence, we extend the use of the fuzzy representation of firms, by taking advantage of a result that, under certain conditions, provides a scalar measure for multivariate data as mapped into a fuzzy c-partition. We are able to reasonably predict the expected compensation policy of a given firm by means of what we refer to as external variables. This can be a useful decision tool for policy makers. We note that some of our conclusions are in line with those reported in studies using different types of data analysis, which gives us confidence in the results achieved and in continuing this investigation under the framework of the fuzzy sets theory.

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