Efficient Channel Estimation Using TCH Codes

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***Abstract*— In this paper, we consider the use of TCH codes to perform channel estimation in an OFDM system, using either data multiplexed pilots or superimposed pilots over the data. TCH codes possess several properties that allow us to use them efficiently in various applications which includes channel estimation, as we address in this paper. With this objective, several performance results were obtained through simulations which allowed the evaluation of the impact of different pilot power levels and modulations, as well as the comparison of TCH against other conventional pilots. In order to cope with the interference between pilots and data, an iterative receiver with interference suppression was employed for the superimposed pilots method.**

***Index Terms*— TCH codes, channel estimation, OFDM, data multiplexed pilots, implicit pilots.**

I. Introduction

In present-day communications, there is a need to mitigate the effects of multipath fading. This is possible by using Orthogonal Frequency Division Multiplexing (OFDM) since it has multicarrier modulation. The usable bandwidth in an OFDM system is divided into orthogonal sub-channels, allowing that a frequency selective channel to be converted into a non-frequency selective one [1][2][3]. This paper outlines a method to estimate the channel in an OFDM system by utilizing TCH codes with data multiplexed or implicit pilots.

TCH codes are error correcting codes first demonstrated in [4] and they are described as having a sturdy error correcting performance (oriented to transmit short and sensible information), as being able to use Fast Fourier Transform (FFT) to execute simple decoding, ideal rigid sizes and an excellent correlation performance. All of these features grant us an opportunity to utilize TCH codes in distinct applications based on digital transmission systems like error correction, spread spectrum systems or channel and phase estimation. The admirable correlation properties of these codes is what allows us to perform and study the channel estimation and to use them for the first time with this objective.

The main purpose of channel estimation is to compensate the effects of attenuation, fading and scattering suffered by the signal in the channel and the most prominent way of performing it consists in transmitting training sequences or pilots that characterizes the distortion that the channel causes, regarding attenuation and phase shift. The Least-Square (LS) estimation [5] and its improvement, the Minimum Mean-Square-Error (MMSE) estimation [6], are the two most used methods of channel estimation. The first is used when the distributions of channel and noise are unknown while the latter is utilized if the previous parameters are identified.

The pilot symbols used in channel estimation are regularly multiplexed with the data in both time and frequency domains [7] [8] [9]. However, this approach may originate an inefficient bandwidth use and to contradict this problem, a different method consisting in using implicit pilots was proposed in [10] and [11]. In this approach, the pilot symbols are superimposed over the data, increasing the pilots' density without sacrificing system capacity, though more power has to be spent on the pilot sequence.

The main objective of this paper, is to study the performance of channel estimation using data multiplexed and implicit pilots based on TCH codes. We analyze these methods by comparing results that illustrate the BER performance obtained through simulations. Following a similar approach to the one adopted in [12], an iterative receiver capable of performing joint detection and channel estimation is used in order to mitigate the mutual interference in the data and in the pilot symbols caused by the use of embedded pilots.

The paper is organized as follows. Section II portrays TCH codes and how they are built. Secondly, in Section III, the system characterization is presented, which contains information about the OFDM transmission frame structures and also the transmitter and the receiver we used. Section IV describes the channel estimation process and the results obtained are presented in Section V. Lastly, Section VI contains the appropriate conclusions taking into account the obtained results.

II. TCH Codes

TCH codes [4] [13] are a class of binary, non-systematic, non-linear and cyclic block codes, with length $n = 2^m$, where m represents any positive integer. These codes have successfully been utilized in various applications, including synchronization [14], coding [15][16] and Ultra-Wideband (UWB) systems.

A TCH block code can be identified as $TCH(n, k, t)$, where n represents the code length, k identifies the number of information bits in a code word and t is its error correcting capacity. They can be defined by Equations (1) and (2) that

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illustrate a set h generator base polynomials $P_i(x)$:

$$TCH(n,k) = \sum_{i=1}^{h} P_i(x), \tag{1}$$

$$P_i(x) \neq P_j(x^r) \mod n \quad i \neq j \forall t \in \mathbf{N}, \tag{2}$$

The error correcting capacity of the TCH codes depends on their minimum distance, represented by d_{min} in Equations (3) and (4), between those polynomials:

$$d_{min} \geqslant 2t + 1, \tag{3}$$

$$d_{min} \leqslant H_d[P_i(x), \{P_j(x^r)\} \mod n] \leqslant n - d_{min}, \tag{4}$$

Where H_d stands for Hamming distance. They are also balanced and, by using Basic TCH Polynomials (B-TCH Polynomials) of degree n, we can generate these codes:

$$P(x) = \sum_{i=0}^{(\frac{p-1}{2})-1} a_i x^{K_i}, \tag{5}$$

where the exponents K_i satisfy Equation 6:

$$a^{K_i} = 1 + a^{2i+1}, i = 0, 1, ..., (\frac{p-1}{2}) - 1, \tag{6}$$

p is a prime number with $p = n + 1 = 2^m + 1$. Prime numbers that obey this condition are called *Fermat* numbers:

$$F_i = 2^{2^i} + 1, \tag{7}$$

Only five numbers are known to obey the Fermat number rule, which means that we can only generate pure TCH polynomials, also designated as B-TCH Polynomials, for code lengths $n = 2, 4, 16, 256 \ and \ 65536$. Even though it is possible to build similar TCH codes that can be extended for other code lengths, this comes at the cost of losing the properties and also the ideal structure of the B-TCH polynomials. TCH codes originated by B-TCH polynomials have both good cross and auto-correlation, with the latter assuming only three-values: n, the value of the code polynomial, 0 and 4. This translates into a great advantage for higher sized TCH codes, such as TCH codes length $n \geqslant 256$. For higher n values, the sequences tend to get closer to a Dirac impulse, as depicted in Figure 1, showing an auto-correlation function of a B-TCH Polynomial of length 256, which are used in this paper, with offsets from -128 to 127, so that the peak is displayed in the center.

III. System Characterization

We describe a low-pass OFDM signal in (8) where X_k depicts the data symbols, N is the total number of sub-carriers and the symbol length is represented by T

$$\upsilon(t) = \sum_{k=0}^{N-1} X_k e^{\frac{2\pi kt}{T}}, 0 \leqslant t \leqslant T, \tag{8}$$

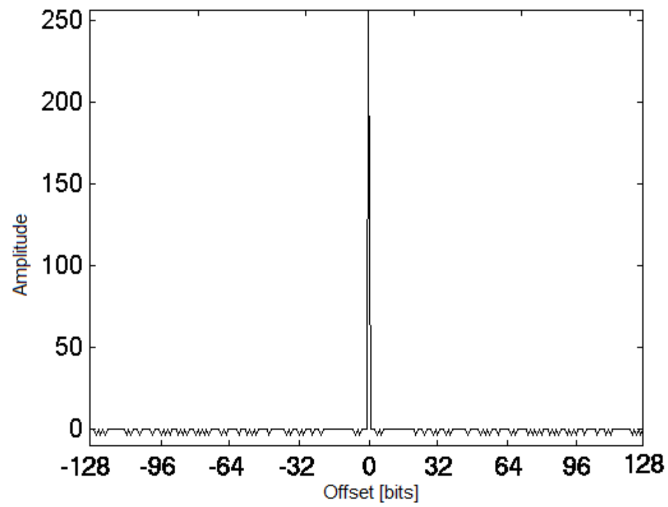

Fig. 1: Auto-correlation of a B-TCH Polynomial with $n = 256$.

There is a need to prevent intersymbol interference, so we insert a guard interval of length T_g immediately before the OFDM block. A cyclic prefix is transmitted during the guard interval so that the signals in the intervals $-T_g \leqslant t < 0$ and $T - T_g \leqslant t < T$ remain equal. The following equation represents an OFDM signal with a cyclic prefix

$$\upsilon(t) = \sum_{k=0}^{N-1} X_k e^{\frac{2\pi kt}{T}}, T_g \leqslant t \leqslant T, \tag{9}$$

A transmitter chain inspired in [12] is shown in Figure 2 and it combines QAM constellations with an OFDM transmission that can use data multiplexed or implicit pilots.

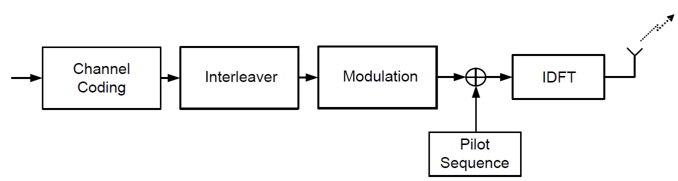

Fig. 2: Transmitter chain.

Figure 3 shows the frame structure that we consider for an OFDM system with N carriers using data multiplexed pilots, where only the first column of the pilot grid contains pilot symbols and the first column of the data grid is empty. Figure 4 illustrates a similar frame structure but this time implicit pilots are used. This means that all the positions in both grids are filled since the pilots are superimposed over the data. In both frame structures, the grids are built by utilizing an OFDM time block spacing in the time domain. We characterize the transmitted sequences as follows

$$\mathbf{X} = \mathbf{S} + \mathbf{C}, \tag{10}$$

where $\mathbf{S}$ describes an $N \times 1$ vector where the elements are complex valued modulated symbols drawn from an M-sized complex valued constellation and $\mathbf{C}$ is an $N \times 1$ vector that corresponds to $\mathbf{C} = DFT\{\mathbf{c}\}$ which is the DFT of a TCH codeword. In order to take advantage of the good auto-correlation properties of the the TCH codes, we utilize the DFT of these codes. The objective of using these auto-correlation properties is mainly for time synchronization purposes.

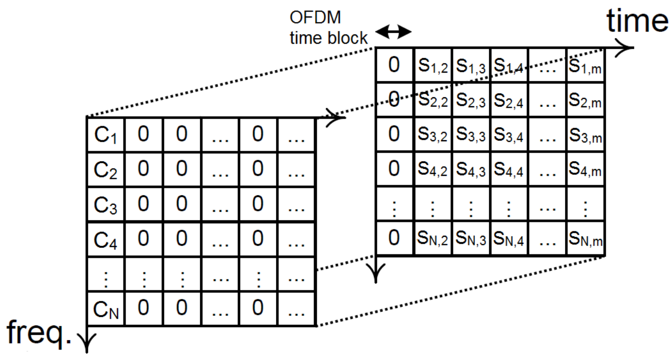

Fig. 3: Frame structure used for an OFDM transmission containing data multiplexed pilots where **C** represents a pilot symbol and **S** represents a data symbol.

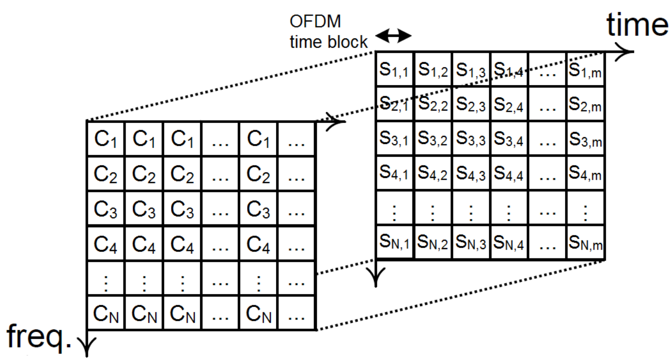

Fig. 4: Frame structure used for an OFDM transmission containing implicit pilots where **C** represents a pilot symbol and **S** represents a data symbol.

As we mentioned in Section I, transmitting superimposed pilots on data creates mutual interference. In order to reduce it, and also to attain reliable channel estimation and data detection, we propose a receiver based on a similar scheme [12], capable of performing these tasks via iterative processing. The structure of the referenced receiver is shown in Figure 5.

Firstly, the pilot symbols are removed from the sequence. Then, they enter the Channel Equalization block and after that, the sequences of equalized samples are demodulated into bit streams. These bit streams are processed so that an estimate of the transmitted signal, $\widehat{\mathbf{S}}$, can be reconstructed. In the following iteration, the reconstructed sequence can be utilized so that the channel estimates are enhanced.

IV. Channel Estimation

Many techniques regarding channel estimation on OFDM systems [17] can be used, each of them with its characteristics and many differences between them, like whether they use time or frequency domain samples, the complexity of the technique in question, performance and *a priori* information utilized. The latter can be made of sub-carrier's correlation in the frequency or time domains and the quantity of *a priori* information affects the estimation quality, so the more information of this type there is, the better the estimation.

If the overall channel impulse response is shorter than the N_G-sized cyclic prefix, we can describe the frequency domain received sequence as follows

$$\mathbf{R} = \mathbf{H}(\mathbf{S} + \mathbf{C}) + \mathbf{N}, \tag{11}$$

where $\mathbf{H}$ is an $N \times N$ diagonal matrix that stands for the channel frequency response and $\mathbf{N}$ represents an $N \times 1$ vector of noise samples in the frequency domain. Both $\mathbf{S}$ and $\mathbf{C}$ have been described in equation (10). This model directly matches the channel estimation based on implicit pilots method but, by establishing $\mathbf{S} = 0$, the model can likewise represent the data multiplexed pilots with a block of pilot symbols.

The receiver can employ an iterative approach based on [12] and therefore it is possible to obtain the frequency channel response. Each of the following steps is executed for each iteration q:

1) Data symbol estimates are removed from pilots. The resulting sequence becomes
$$\widetilde{\mathbf{R}}^{(q)} = \mathbf{R} - \widehat{\mathbf{S}}^{(q-1)}\widehat{\mathbf{H}}^{(q-1)}, \tag{12}$$
where $\widehat{\mathbf{S}}^{(q-1)}$ and $\widehat{\mathbf{H}}^{(q-1)}$ are the symbol and channel estimates from the previous iteration. When $q = 1$ we simply use $\widetilde{\mathbf{R}}^{(1)} = \mathbf{R}$. The described step is only applied when using superimposed pilots.
2) The channel frequency response estimates is calculated using
$$\widetilde{\mathbf{H}}^{(q)} = |\mathbf{\Lambda}|^{-2}\mathbf{\Lambda}^H\widetilde{\mathbf{R}}^{(q-1)}, \tag{13}$$
where $\mathbf{\Lambda} = diag(\mathbf{C})$, where diag$(\cdot)$ represents a diagonal matrix whose elements are contained in the vector used as argument. $|\mathbf{\Lambda}|$ denotes the element-wise absolute value operation and $(\cdot)^H$ depicts the conjugate transpose of a matrix/vector. After the first iteration, the estimates of data symbols can also be used as pilots for channel estimation refinement. In this case we use $\mathbf{\Lambda} = diag(\widehat{\mathbf{S}}^{(q-1)})$ for data multiplexed pilots and $\mathbf{\Lambda} = diag(\widehat{\mathbf{S}}^{(q-1)} + \mathbf{C})$ for implicit pilots.
3) We can augment the channel estimates by assuring that the corresponding impulse response has a duration N_G. This is accomplished by utilizing
$$\widehat{\mathbf{H}}^{(q)} = diag(\mathbf{F}\mathbf{T}\mathbf{F}^H\widetilde{\mathbf{h}}^{(q)}), \tag{14}$$
where
$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{N_G} \\ \mathbf{0}_{(N-N_G)\times N_G} \end{bmatrix}, \tag{15}$$
$\mathbf{0}_{N_{CF}\times(N-N_{CF})}$ represents a size $(N-N_G)N_G$ matrix full of zeros, while $\mathbf{I}_{N_G}$ depicts an $N_G \times N_G$ identity matrix. The $N \times N$ scaled discrete Fourier transform

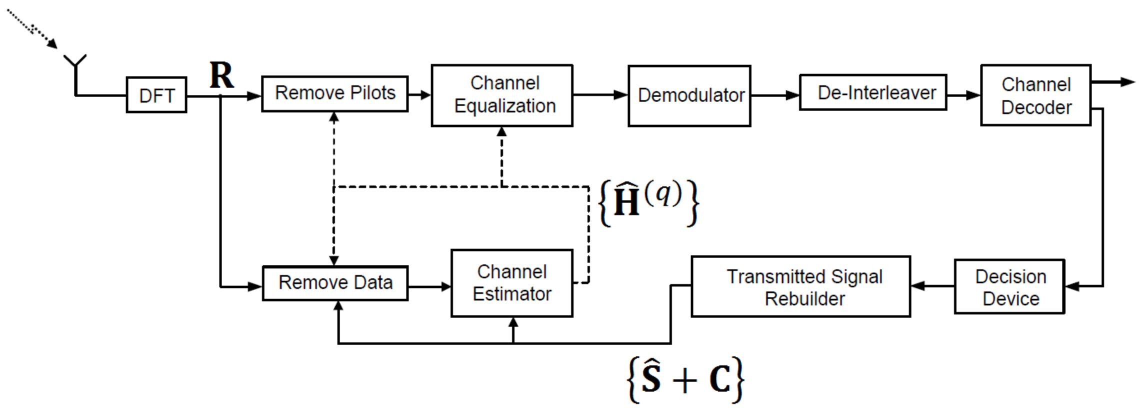

Fig. 5: Structure of the iterative receiver.

(DFT) matrix is represented by $\mathbf{F}$, such that $\mathbf{I}_N = \mathbf{F}^H\mathbf{F}$, and $\widetilde{\mathbf{h}}^{(q)}$ illustrates the $N \times 1$ vector that contains the diagonal of $\widetilde{\mathbf{H}}^{(q)}$.

V. Results

The BER performance results of all of the graphs present in this section were attained by performing Monte Carlo simulations with an 8 equal power tap Rayleigh fading channel, using 256 OFDM carriers. For Figure 6, we used QPSK modulation and we varied the pilot power values, which are relatively measured to the channel data, from 0 to -12 with jumps of -3 dB. We performed the channel estimation by utilizing data multiplexed pilots and, along with them, were sent blocks with TCH words of length 256. The channel encoders were rate $1/2$ turbo codes based on two identical convolutional codes with two constituent codes characterized by $G(D) = [1+D^2+D^3)/(1+D+D^3)]$[18]. 18 turbo decoding iterations were applied at the receiver and this receiver was conventional, meaning there was only one receiver iteration. A perfect estimation curve is shown for comparison purposes.

By analyzing these results, we verify that higher valued pilot powers, from 0 to -6 dB, translate into better results since the curves representing high values of pilot power are almost adjacent to the curve that portrays perfect estimation. For higher $E_s/N0$ values, the performance is slightly inferior, illustrated by the -9 and -12 dB curves that get further away from the perfect estimation curve.

In Figure 7, we maintained the same simulation conditions used in Figure 6 with the exception of two parameters: superimposed pilots with TCH codewords were used to perform the estimation, instead of data multiplexed pilots, and in the receiver we applied 3 turbo decoding iterations for each of the 6 receiver iterations in the iterative scheme.

Once again, the performance is better when the pilot power value is higher. The curves depicting -9 and -12 dB have a larger discrepancy from the perfect estimation curve than the same curves observed in Figure 6 because of interference between the data symbols and pilots, inherent to a scheme that uses implicit pilots. These results indicate that, to obtain

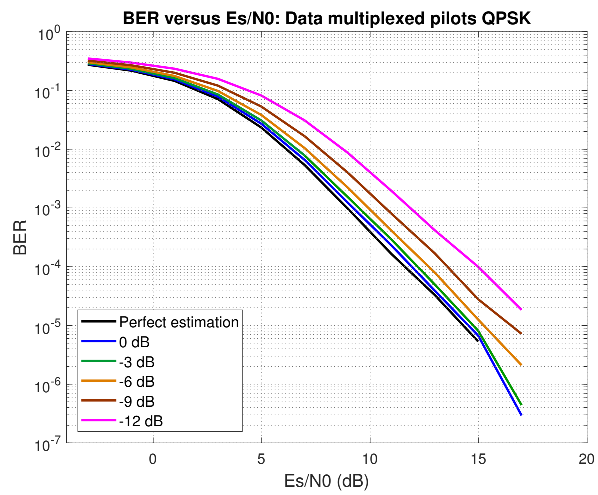

Fig. 6: BER performance of channel estimation utilizing QPSK modulation and data multiplexed pilots based on TCH codes for different pilot powers.

a good performance and to compensate the interference when using implicit pilots, it is necessary to use higher pilot power levels.

The results displayed in Figure 8 were obtained by using the same simulation parameters used to build Figure 7 but this time, the pilot power is fixed at 0 dB and the number of iterations in the receiver is gradually increased.

When the number of iterations used in the receiver is higher, the performance is better, which is visible by comparing the simulated curves with the perfect estimation one. Still, the difference in performance is small after we stop using a conventional receiver and it is almost indistinguishable for the highest simulated values of iterations, 4 and 8, meaning that the performance is not affected greatly by increasing the number of receiver iterations after a certain value is reached.

The results presented in Figure 9 were obtained by using either data multiplexed pilots or implicit pilots, while once again considering QPSK modulation and also by changing

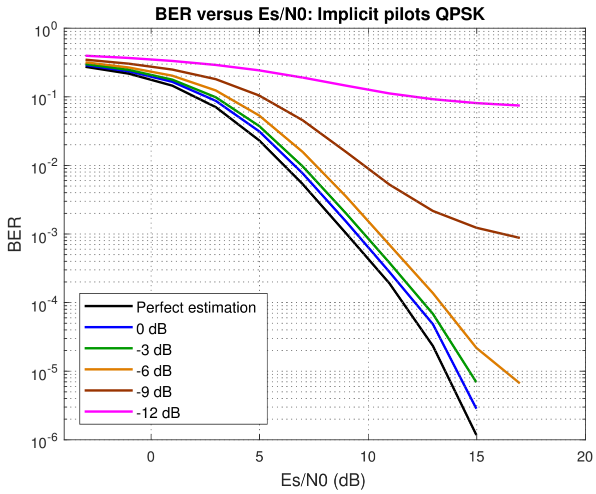

Fig. 7: BER performance of channel estimation utilizing QPSK modulation and superimposed pilots on TCH codes for different pilot powers.

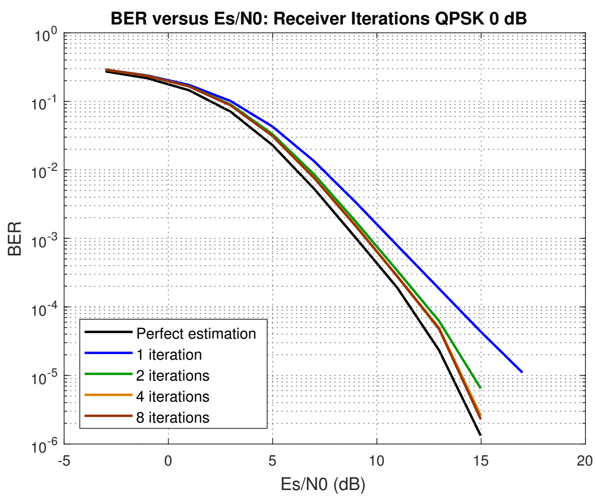

Fig. 8: BER performance of channel estimation utilizing QPSK modulation and superimposed pilots on TCH codes for different receiver iteration values.

the data sent, meaning it was either based on TCH codewords or conventional data. The channel coding and receiver structures used in Figures 6 and 7 were used to simulate the curves that represent data multiplexed pilots and implicit pilots in Figure 9, respectively. Based on the results demonstrated in Figure 6 and Figure 7, we considered single pilot power value, which is -4 dB. A curve regarding perfect estimation is also shown for reference.

All of the simulated cases possess an almost identical and very good performance, which is justified by observing the proximity between all of the curves and also because no BER floor is visible. The difference between the performance of data multiplexed pilots and implicit pilots is very small and even though data multiplexed pilots have a better performance, we can avoid spectral degradation by

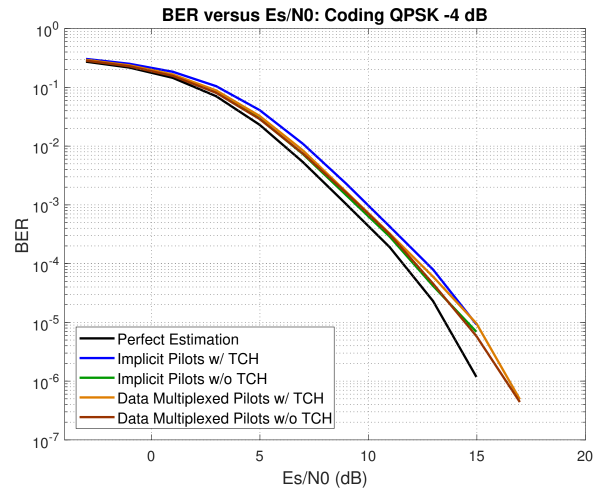

Fig. 9: BER performance of channel estimation utilizing QPSK modulation while considering different pilot approaches and based on TCH code words or conventional pilots.

using implicit pilots. Conventional pilots, labeled in Figure 9 as ”w/o TCH”, have a slightly better performance than the ones based on TCH code words but the difference is really small and TCH codewords have the benefit of possessing good synchronization properties.

Finally, Figure 10 was built using the same simulation conditions of Figure 9 with the exception of the modulation, which this time is 64-QAM instead of QPSK, and the pilot power, which is now a fixed value of -1.75 dB.

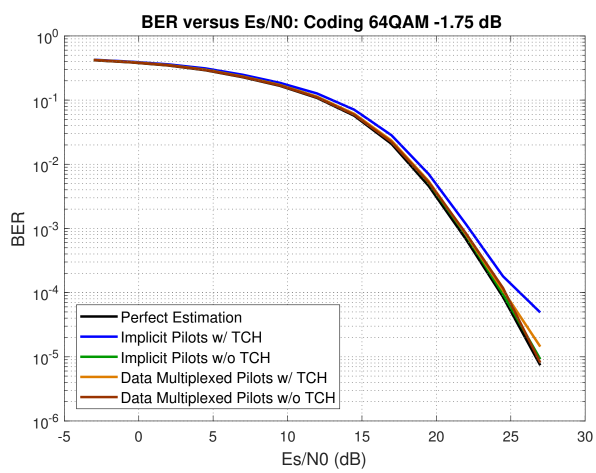

Fig. 10: BER performance of channel estimation utilizing 64-QAM modulation while considering different pilot approaches and based on TCH code words or conventional pilots.

The results show that with 64-QAM modulation we need higher BER values than the ones obtained from the QPSK simulations, as expected. Amplitude modulation methods are more susceptible to noise and that is the reason why we used a higher pilot power value than the one used in Figure 9.

But in this simulation, all the curves are closer to the perfect estimation curve, with the "Implicit Pilots w/TCH" showing a small detour starting around 25 dB.

VI. CONCLUSIONS

After studying and analyzing the use of TCH codes for channel estimation using two different pilot approaches, data multiplexed pilots and implicit pilots, we conclude that both approaches are reliable with each of them having its own advantages which are referenced in Section V. Using TCH codes for channel estimation is justified based on the fact that not only they have very similar performance levels when compared with conventional pilots but also that TCH codes have great synchronization properties, meaning that it is possible to simultaneously use them in the system for synchronization purposes, making TCH codes a better choice for channel estimation. Regarding the modulations used, QPSK has a better performance than 64-QAM but both present really good performances, showing that TCH codes can be successfully used with both modulations, as we expected.

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