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THE SENSITIVITY ANALYSIS OF VIEWS RELATED PARAMETERS IN THE BLACK-LITTERMAN MODEL

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Abstract

This dissertation attempts to carry out the sensitivity analysis in the Black-Litterman model which is a model trying to compensate for the lack of practical use of Modern Portfolio Theory.

We analyse whether the change in the views related parameters has an effect on the final portfolio weights by using samples of eight stocks. In particular, we focus on the view vector and the confidence level of the views and treat them as independent variables. In addition, we generate the results of tests showing that both of the variables will influence the portfolio weights reasonably. Moreover, we also adopt the indicator called Tracking Error Volatility to illustrate the change in the portfolio weights.

Besides, our findings reflect that the variation of the portfolio weights also depends on the small components contained by the two variables. According to our conclusions, the weights are especially sensitive to the change in the absolute view and the confidence level of the relative views. And the weight of the stock with a large market capitalization tends to differ a lot when the stock is mentioned in the view.

Keywords: Black-Litterman model, Asset Allocation, Sensitivity Analysis, Views of Investors

JEL Classification:

G11- Portfolio Choice; Investment Decisions

G23- Non-bank Financial Institutions; Financial Instruments; Institutional Investors

Resumo

Esta dissertação baseia-se na análise de sensibilidade no modelo *Black-Litterman*, que é um modelo que tenta compensar a falta de uso prático da *Modern Portfolio Theory*.

Por um lado, analisamos se a alteração nos parâmetros relacionados aos pontos de vista tem um efeito nos pesos finais do portfólio usando amostras de oito ações e em particular, concentramo-nos no vetor de visão e no nível de confiança das visualizações tratando-as como variáveis independentes.

Por outro lado, geramos os resultados de testes que mostram que ambas as variáveis irão influenciar os pesos do portfólio razoavelmente e a adotamos o indicador chamado Tracking Error Volatility para ilustrar a mudança nos pesos do portfólio. O estudo efetuado reflete assim que a variação dos pesos do portfólio também depende dos peguenos componentes contidos pelas duas variáveis.

Em suma, chegamos à conclusão que os pesos são especialmente sensíveis à mudança na visão absoluta e ao nível de confiança das visualizações relativas. E o peso da ação com uma grande capitalização de mercado tende a diferir muito quanto o stock é mencionado na visão.

Palavras-chave: modelo Black-Litterman, alocação de ativos, análise de sensibilidade, pontos de vista dos investidores

Classificação *JEL*:

G11- Portfolio Choice; Investment Decisions

G23- Non-bank Financial Institutions; Financial Instruments; Institutional Investors

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Of course, any omissions and errors present in this dissertation are the responsibility of the author.

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List of Abbreviations

BA	The Boeing Company
B-L model	Black-Litterman model
CAPM	Capital Asset Pricing Model
CVaR	Conditional Value at Risk
DOW	The Dow Chemical Company
Et al.	Et alia.
EW	Edwards Lifesciences Corporation
GS	The Goldman Sachs Group, Inc.
INTC	Intel Corporation
KO	The Coca-Cola Company
MPT	Modern Portfolio Theory
NEE	NextEra Energy, Inc.
OFAT	One-factor-at-a-time
SBUX	Starbucks Corporation
TEV	Tracking Error Volatility
VaR	Value at Risk

1. Introduction

Modern Portfolio Theory (MPT)[1], also known as mean-variance analysis, came into the public sight with the appearance of Harry Markowitz's theory of Portfolio Selection (Markowitz, 1952). The expected return and risk of the portfolio are connected through MPT providing a mathematic framework for specifying the optimal portfolio. However, although MPT is still treated as a vintage model, the theoretical importance is not enough to cover its weakness in the practical use. Due to the requirement for the expected returns of all assets and high sensitivity of the optimal asset weights to the return assumption used, the unreasonable nature of the results often occurs. In order to create a better option for institutional investors to implement asset allocation, the Black-Litterman Model was introduced in the Journal of Fixed Income by Fischer Black and Robert Litterman (1991) and developed in the following years.

The B-L model provides an intuitive solution to the two main problems involved in the MPT (Black and Litterman, 1992). One of the most innovative features is that the model allows investors to specify their particular views (Walters, 2014). Additionally, the investors can hold views either on the absolute returns on assets or on the relative returns on different assets. And it is not mandatory for them to come up with views on each asset. The investors are free to offer as many or as few views as they wish based on their forecasts (Black and Litterman, 1991).

The concept of confidence level is applied to the views in order to enhance the accuracy of the model as well. In spite of the ambiguous covariance matrix of the views and the uncertainty scalar which have been discussed for many times after the publication of the B-L model, the model tends to obtain optimal portfolios which begin from a set of neutral weights and tilt in the direction of the investor's views (Black and Litterman, 1992), while the authors hardly mentioned the precise nature of that phenomenon (He and Litterman, 1999). The model keeps being modified based on the original framework so that most investors can exploit it more properly and extract benefits. However, the B-L model is still incomplete and usually adopted subjectively according to investors' understanding.

Although all the parameters in the model have been well defined, some of them are quite confusing for investors to input into the model. In order to enhance the practicability of the

[1] Economist Harry Markowitz introduced Modern Portfolio Theory in an essay in 1952. And he was later awarded a Nobel Prize in economics because of this. The theory assumes that investors are risk averse and it works as a mathematical framework for generating a portfolio of assets whose expected return is maximized for a given level of risk, which is defined as variance. The key insight of MPT is that the risk and return of an asset should be assessed by how it contributes to the overall risk and return of a portfolio.

model, lots of research has been carried out regarding the appropriate value of uncertainty scalar and how to compute the covariance matrix of the views as the original papers provided little information about them, but much less content relates to exploring the sensitivity of the final portfolio weights to the change in the parameters.

The motivation for this dissertation comes from the lack of sensitivity analysis[2] in terms of the view vector and the confidence level of the views. Most papers focus on improving the model and making the theory more concrete, while it is also essential to better understand the parameters based on the existing findings and this dissertation is trying to cover the blank part.

The purpose of this dissertation is to discover the influence in the final weights of the assets in a portfolio caused by the change in the view vector and the confidence level of the views by applying the B-L Model, namely the sensitivity tests of the view vector and the confidence level of the views. This dissertation aims to offer a comprehensive analysis of these two special parameters which are the main additional elements compared to the traditional mean-variance analysis by choosing a proposed practical universe of investments, implementing the Black-Litterman model and adopting the direct measure of sensitivity and the assistant measure of indicator Tracking Error Volatility. According to the findings, the investors are able to have a general understanding of the importance of each component in these two parameters regarding the contribution to the final portfolio weights and they can allocate suitable resources to generate these two parameters in details. Nowadays people have a large quantity of access to various resources, while the costs of information are not negligible. Hence, it is of vital importance to gather target information rather than all the information in order to avoid extra costs.

The dissertation mainly contains three key points, respectively the sensitivity tests for the parameters, the conclusion of the tests and the suggestions to the investors. As the B-L model is a newborn investment tool and not familiar to most people, the concept of the model is identified briefly in the literature review (Chapter 2) as well as some past researches, current situation and future expectation. Chapter 3 mainly introduces how to implement the Black-Litterman model given specific assumptions. The methodology is developed in chapter 4 to present a concrete approach indicating how the tests work. Afterwards, chapter 5 includes the entire sample and the reason for the selection.

[2] Sensitivity analysis is the study of the variation of the uncertainty in the output of a mathematical model or system resulting from the variation of different sources of uncertainty in its inputs. It is also closely related to the uncertainty analysis, but they focus on various aspects.

At the end of this dissertation, referred to the last two chapters, all the results of the sensitivity tests, the main conclusions and the application in the real life are interpreted.

2. Literature Review

2.1. Model description

The Black-Litterman model was created to lead portfolio modeling to perform better in the practical investment situations (Litterman, 2003). The first original paper published in the Journal of Fixed Income in 1991 provides a brief intention of the model to the public (Black and Litterman, 1991), while it rarely shows the formulas for the given example which causes confusion to most readers. Then the second original paper was published in the Financial Analysts Journal in the following year by the same authors (Black and Litterman, 1992). This paper contains not only the rationale for the methodology, but also some information on the derivation, which extends the model to a higher level. The model became more concrete and understandable due to the enlargement.

However, this paper still does not list all the formulas but meanwhile it also encourages people to make a contribution to the model to complete the formulas afterwards. Researchers continue sharing their own perspectives to improve the formulas, calibrate the model, add extensions to the model or give examples to illustrate the model. Even though some of them are not consistent with each other, the active discussion is able to accelerate the pace of the model development.

Idzorek (2005) depicts the process of combining information in Figure 1 to show how the B-L model works basically.

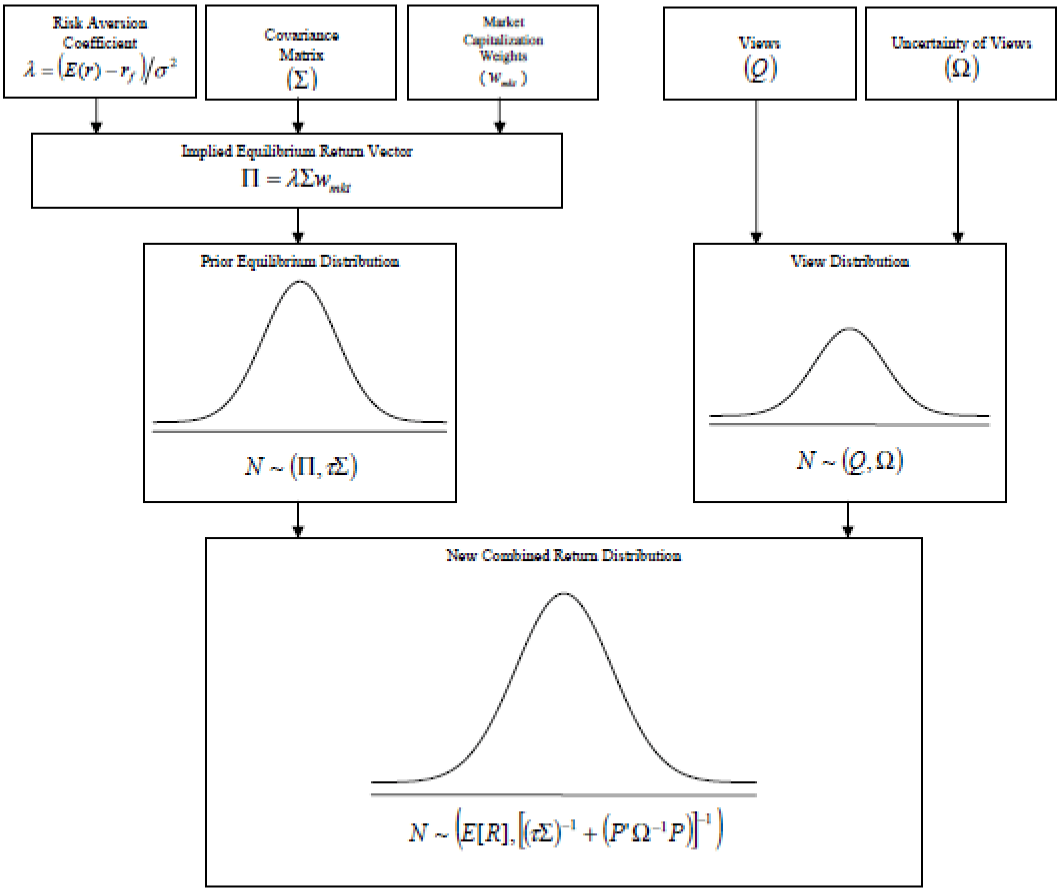

Figure 1: Deriving the New Combined Return Vector

Source: Idzorek, Thomas. (2005), A Step-By-Step guide to the Black-Litterman Model.

2.1.1. The Canonical Black-Litterman Reference Model

The Canonical Black-Litterman Reference Model aims to blend the investors' views with prior information that plays an essential role as a beginning point for estimation of asset returns views (Walters, 2014). The model follows General Equilibrium theory and starts with a neutral equilibrium portfolio. As the quadratic utility function is widely applied for the theory in practice, the equilibrium model is directly referred to the Capital Asset Pricing Model (CAPM) [3] which is familiar to most practitioners. According to the previous

[3] The capital asset pricing model (CAPM) is a model that describes the relationship between systematic risk and expected return for assets. And it is widely used for the pricing of risky securities, generating expected returns for assets given the risk of the assets and calculating costs of capital. The formula for computing the expected return of an asset is R = Rf + β(Rm - Rf), where β means Beta of the asset.

assumptions, the estimated mean excess return from the CAPM market portfolio becomes the prior base necessary to implement the Black-Litterman model.

The posterior distribution combines the prior portfolio and the conditional portfolio proportionally where the conditional portfolio is contributed by the views from the investors. The core formula for computing the posterior combined return vector (E[R]) is

$$E[R] = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right] \quad \textbf{(1)}$$

which can be alternated with another form

$$E[R] = \Pi + \left[\tau\Sigma P^T\left[P\tau\Sigma P^T + \Omega\right]^{-1}\right]\left[Q - P\Pi\right] \quad \textbf{(2)}$$

Both of them are known as the Black-Litterman 'master formula'. In the formula, the following notation is used:

$E[R]$	a N×1 vector of the posterior combined return where N represents the number of assets
Π	a N×1 vector of the implied equilibrium return
τ	a scalar indicating the uncertainty of the prior estimate of the mean returns
Σ	the covariance matrix of the excess returns for the assets
P	a K×N matrix of the asset weights within each view where K expresses the number of views
Ω	a diagonal K × K matrix of the covariance of the views and means the uncertainty in each view
Q	a K×1 vector of the returns for each view

After the computation of the return vector, the posterior variance of the estimated mean about the unknown mean M must be generated by applying the following formula

$$M = \tau\Sigma - \tau\Sigma P^T\left(P\tau\Sigma P^T + \Omega\right)^{-1}P\tau\Sigma \quad \textbf{(3)}$$

Later on, the covariance of returns about the estimated mean Σ_P is assumed as the sum of Σ and M when a condition is satisfied, which tells that the uncertainty in the estimates and the known covariance of returns about the unknown mean are independent. This process shows additional information will decrease the uncertainty of the model which accords with the reality. The final weights of the assets can be calculated either on the unconstrained efficient frontier or with constrains according to the needs of the investors.

2.1.2. The Alternative Reference Model

Besides the Canonical Reference Model, some authors also mentioned the Alternative Reference Model which is adopted especially by Satchell and Scrowcorft (2000). And some key assumptions in the canonical reference model are negated when the authors adopt this new concept. The parameter μ is still estimated while not treated as a random variable anymore (Walters, 2014). In other words, the value of τ becomes 1 in this case which is usually set as a small number close to 0 in the canonical reference model. Hence, a point estimate is applied in this model and τ does not play a role as a parameter. Now the parameter Ω works in a similar way to Σ and represents the covariance of the returns to the views. Since there is no confusion about posterior covariance in the Alternative Reference Model, the formula for the posterior combined return vector (E[R]) is rewritten as

$$E[R] = \Pi + \Sigma P^T \left[\left(P \Sigma P^T + \Omega \right) \right]^{-1} [Q - P\Pi] \quad \textbf{(4)}$$

As noticed, the term τ is removed from the original formula. In practice, both of τ and Ω are used to influence the weights of the prior and views from investors, while Ω remains in the formula. Due to the complicated feature of Ω compared to τ since Ω has separate elements for each view, the parameter Ω is kept on purpose and τ is gone. Based on the assumption, the new portfolio weights are equal to the equilibrium portfolio weights when there are no views specified by the investors. In this model, the formulas used to compute M and Σ_P are not required because of the disappearance of τ.

2.2. The development of the B-L model

During the last two decades, many authors have made a special contribution to the B-L model regarding various aspects. They mainly focus on the choices of the fundamental theorem or discussing about the value of Tau. In general, the development of model can be divided into three categories. The authors may decide to use Bayesian Theorem and Tau, or use non-Bayesian Theorem and Tau or use non-Bayesian Theorem but eliminate Tau.

2.2.1. Using Bayesian Theorem and Tau

The model including the Bayesian Theorem and Tau is also referred as the Canonical Reference Model, which is the initial type assumed by the original authors.

The original paper (Black and Litterman, 1991) indicates the Bayesian approach is applied to help investors solve the problem that the mapping between views and optimal portfolios can be very sensitive to small changes in the views when using mean-variance models. Besides, the confidence level of the views should also be pointed out in the Bayesian approach to realize the building of the posterior combined return distribution. The second paper (Black and Litterman, 1992) introduces the parameter τ which is accompanied by the covariance matrix of the excess returns for the assetsΣ. The parameter τ plays a role as a scalar and it is used to calibrate the covariance matrix. The value of τ is indicated to be close to zero in this paper because of the small uncertainty in the mean of the return assumed. However, τ is not well described and the approach to determining the exact value of τ is implicit, which creates confusion to investors in the practical implementation as well as resulting in substantial divergence in the following studies of different authors.

These two original papers offer the general framework of the model and the rationale for the methodology. On the one hand, the model remains incomplete reflected by the missing of formulas and ambiguity in the definition of some parameters although the given worked examples try to illustrate some concepts, but on the other hand, a large space is left for other researchers to improve the model when they hold intuitive opinions.

After a long break since the second original paper was published, detailed information about practical use of the Black-Litterman model was carried out (Bevan and Winkelmann, 1998). They show concrete procedures of how they incorporate the B-L model into their Asset Allocation process at Goldman Sachs with some calibrations of the model, which provides meaningful guidance to those who tends to apply the B-L model.

One year later, the paper written by one of the original authors was published (He and Litterman, 1999). The main purpose of the paper focuses on the supplement and summary of the mathematics of the B-L model. Another contribution made by them is to provide the details on the implementation of the model including several numeric examples, while the formulas are not complete.

Drobetz (2001) presents some research on the B-L model including the detailed description of the model and the intuition of interpreting the Degree of Confidence in a diagram to the public. The author also shows an example to illustrate the model and demonstrates that the B-L model actually help to alleviate some problems related to the use of traditional Markowitz (1952) approach.

After that, a group of researchers from Goldman Sachs provide an overview of how to implement the B-L model in the asset allocation process according to their own work experience (Litterman, et al, 2003). In this paper, they mainly pay attention to showing the practical use of the model rather than describing the model itself, which creates value for those who are familiar with the B-L model and have intention to build the model into their asset allocation process.

Blamont and Firoozy (2003) carried out the first discussion about setting the value of parameter τ under the canonical reference assumption at the same year. They explain the Canonical Reference Model in a clear way and mention that mixing is the reason for the decrease in variance of the posterior estimate. In their paper, $\tau\Sigma$ is interpreted as the standard error of estimate of the implied equilibrium return vector and τ is recommended to be one divided by the number of observations.

Later on, the direction of the study of the B-L model starts to shift to the views of investors, which is led by Beach and Orlo (2006). An innovative method to obtain the views is presented by them which is using GARCH model[4]. The views are quantitative rather than qualitative. However, the uncertainty of the views are not specified clearly, which sets constraints for the implementation of the B-L mode since the confidence level of the views is one of the special parameters in the model. In spite of the limitation, this approach is still considered as a useful tool in the case that the confidence level is neglected.

Moreover, one augmented model which integrates a factor model was introduced (Cheung, 2010). The author also comes up with a new concept which involves a joint estimation of the factor returns.

2.2.2. Using non-Bayesian Theorem and Tau

During the evolution of the B-L model, the type of the model changes due to the divergence of opinion among different authors. The model with these features is called the Hybrid Reference Model.

Satchell and Scowcroft (2000) published the first paper introducing this new type of model and it aims to demystify the B-L model. However, they come up with a new assumption instead of the Bayesian Theorem. They explain how to apply point estimates for the prior and the views in the paper. Besides, they recommend that the value of τ should be one, which is exactly contrary to how most authors deal with τ. Because of the limitation of their new model, the perspective is not supported by most people who adopt B-L model and mainly replaced by Meucci's model later. In spite of the weakness in some opinions, they try to help potential users to apply the B-L model by providing several examples and many mathematical expressions.

[4] The generalized autoregressive conditional heteroscedasticity (GARCH) process is an econometric term developed in 1982 by Robert F. The model mainly involves three steps. The first is to estimate a best-fitting autoregressive model. The second is to compute autocorrelations of the error term. The third is to test for significance. GARCH models are usually used in several areas including trading, investing, hedging and dealing.

One important contribution was made to the B-L model after that year by Qian and Gorman (2001). The authors make it possible to generate a conditional estimate of the covariance matrix of returns by expressing views on volatilities and correlations (Idzorek, 2005). They provide an intuitive way to demonstrate a conditional covariance other than Bayesian updating or Theil's mixed estimation (Theil, 1971 and 1978).

Herold (2003) shows the application of the B-L model to active portfolio management to the investors. He also uses point estimates and tracking error to check how much shrinkage to allow. In the paper, several measures are indicated to validate the reasonability of the views, which is one of the key contributions of the paper.

Afterwards, an essential paper called A STEP-BY-STEP GUIDE TO THE BLACK-LITTERMAN MODEL was published (Idzorek, 2005). The author gives an example with concrete process which makes it more convenient to apply the B-L model. He also points out the difference between the stated confidence level and the implied confidence level. Moreover, Idzorek explains his intuition about how to obtain $\boldsymbol{\Omega}$ based on the stated confidence level at the end of the paper. Although he leaves off this part of work with a partial example, other researchers can still take advantage of his findings.

Braga and Natale (2007) use the method called Tracking Error Volatility (TEV) to calibrate the uncertainty in the views. They illustrate the approach by carrying out an example with eighteen asset classes. In the paper, the sensitivities for the posterior estimates to the different views are tested. They also prove that the measure works efficiently to an active portfolio according to the comparison between an initial portfolio based on the DJ Stoxx implied equilibrium returns and a portfolio based on B-L returns. They treat TEV as a measure of distance from the prior portfolio and TEV is defined as

$$\mathbf{TEV} = \sqrt{\mathbf{w}_{\mathbf{actv}}^{\mathbf{T}} \boldsymbol{\Sigma} \mathbf{w}_{\mathbf{actv}}} \text{ where } \mathbf{w}_{\mathbf{actv}} = \mathbf{w}_{\mathbf{p}} - \mathbf{w}_{\mathbf{mkt}} \quad \mathbf{(5)}$$

In the same year, Martellini and Ziemann (2007) introduce a method to active management of a fund of hedge funds. Instead of the traditional way to run the reverse optimization, they choose VaR as the objective function and incorporate skewness and kurtosis into the CAPM model when they determine the neutral portfolio. They also invent a new approach to generating the confidence level of views. The authors decide to obtain rankings by using a factor model and covert the rankings into the confidence level of the views.

A group of researchers create a new method to measure the alignment of the views with the prior estimate (Bertsimas, Gupta and Paschalidis, 2012). They compare the view portfolio weights to the eigenvalues from the prior covariance matrix. Besides, they recommend replacing reverse optimization with different optimizations.

Michaud, Esch, and Michaud (2013) offer critical arguments against using Hybrid and Alternative Reference model. They neglect richer, state of modern econometrics and Bayesian statistics in the process.

2.2.3. Using non-Bayesian Theorem but eliminating Tau

The type of model is referred as the Alternative Reference Model. In practice, many authors put the Hybrid Reference Model and the Alternative Reference Model together into the same category because of the similarity of the features.

The first paper introduces the non-Bayesian variant of the model without τ was written by Fusai and Meucci (2003). After that, Meucci (2005) published a supplement to the previous paper and came up with a concept so called Beyond Black-Litterman model. In this model, Ω is used to control the degrees of freedom to shrinkage and τ is unnecessary.

Krishnan and Mains (2005) add an unpriced factor to the B-L model and name it Two-Factor Black-Litterman model. The factor is uncollected with the market and they provide an example to illustrate the influence of the new factor to the expected returns calculated from the B-L model.

Instead of assuming the normal distribution described initially to compute the neutral portfolio, Giacommeti, et al (2007) recommends to adopt Student T distribution. And they introduced variance VaR[5] and CVaR[6] as risk measures for the portfolio selection model.

Through the development of the B-L model, most authors apply the Canonical Reference Model in the early period. However, on the late stage, many researchers attempt to use the Alternative Reference Model because of the convenience and carry out their findings based on the Alternative Reference Model. Although some authors indicate that they apply the Alternative Reference Model in their papers, most results and techniques can also be applied to the Canonical Reference Model.

[5] Value at Risk (VaR) is a measure of the risk of investments. It estimates how much a set of investments might lose in a set time period given normal market conditions. VaR is often used by firms and regulators in the financial industry to forecast the amount of assets needed to cover possible losses.

[6] Conditional value at risk (CVaR) is a risk assessment technique which is often applied to reduce the probability that a portfolio will suffer large losses. It is performed by assessing the likelihood at a specific confidence level that a specific loss will exceed the value at risk. Mathematically speaking, CVaR is derived by taking a weighted average between the value at risk and losses higher than the value at risk.

2.3. The derivation of master formula

The Black-Litterman 'master formula', as one of the most important formulas in the model, is derived by using the standard Bayesian approach. In order to demystify the master formula, it is necessary to understand the concept of Bayesian inference, which states

$$\mathbf{p(A \mid B) = \frac{p(B|A)p(A)}{p(B)}} \qquad \textbf{(6)}$$

In this formula, the following notation is used:

p(A|B)	The posterior distribution in the B-L model.
p(B|A)	The conditional distribution in the B-L model.
p(A)	The prior distribution in the B-L model.
p(B)	The normalizing constant in the B-L model.

In the Black-Litterman model, p(A) is known as the prior distribution and it is defined as

$$\mathbf{p(A) = N（\Pi，\tau\Sigma），r_A \sim N（p(A)，\Sigma）(7)}$$

where r_A represents the total return on the market portfolio. The covariance matrix of the estimate τΣ is proportional to the covariance of the excess returns Σ by using the scalar τ and this assumption is given by Black and Litterman in order to simplify the process generating the missing covariance matrix.

The Canonical Reference model for the expected return in the Black-Litterman model is

$$\mathbf{r \sim N（\pi，\Sigma_r），\Sigma_r = \Sigma + \Sigma_\pi (8)}$$

where π is the estimate of the mean, Σ means the variance of the normally distributed expected returns and Σ_π is the variance of the unknown mean. Combined with formula (8), the formula (7) can be rewritten as

$$\mathbf{r_A \sim N（\Pi，(1+\tau)\Sigma）(9)}$$

According to the special feature of B-L model, the conditional distribution regarding the views is specified in view space as

$$\mathbf{p(B|A) = N（Q，\Omega）(10)}$$

However, the formula (10) has to be converted into asset space as

$$\mathbf{p(B|A) = N（P^{-1}Q，[P^T\Omega^{-1}P]^{-1}）(11)}$$

Finally, the new posterior distribution of the asset returns can be generated by blending the prior and conditional distribution. Based on the Bayes Theorem, the formula for the posterior is defined as

$$\mathrm{p(A\mid B)} = \mathrm{N}\left(\left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right], \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\right) \textbf{(12)}$$

The Black-Litterman model ‘master formula’ is able to be extracted from this expression and it plays a key role in the whole model.

2.4. The discussion of parameters

Many researchers have been discussing about the parameters in the Black-Litterman model since the model was initially introduced to the public. Given the ambiguity of several key parameters, the B-L model is usually implemented very differently and subjectively in some papers. In general, there are mainly four parameters which have been discussed and all of them are argued more than once, which are respectively Ω, τ, Σ_p and P. Moreover, there are at least two methods to generate the parameter for each one. Most of the methods are introduced given reasonable explanation, while some of them are still not convincing.

2.4.1. The parameter τ

Tau, as the most confusing parameter in the B-L model, is discussed by many authors during the development of the model. This parameter initially shows up in the second original paper introducing the B-L model (Black and Litterman, 1992). However, τ is only introduced by indicating that the meaning of $\tau\Sigma$ is the covariance matrix of the expected excess return rather than being explained exclusively. The authors recommend that τ should be close to zero given a reason that the uncertainty in the mean is much smaller than the variance in the return itself. But they neither define the parameter τ in a detailed way nor give a specific way to calculate the value of τ.

After that, several approaches to calibrating the value of τ are mentioned by different researchers. And in the Canonical Reference model, τ is usually defined as a scalar indicating the uncertainty of the prior estimate of the mean returns. One of the ways is to focus on falling back to basic statistics (Walters, 2014). According to the rule which indicates the variance of the mean estimate is proportional to the inverse of the number of samples when estimating the mean of a distribution, there are two important estimators of τ. One is the maximum likelihood estimator given by the expression as $= \frac{1}{T}$, and the other is the best quadratic unbiased estimator referred to $\tau = \frac{1}{T-k}$, where T represents the number of samples and k stands for the number of assets. The first definition is more widely accepted compared to the other one, and the method explained above is applied to the Canonical Reference Model. And the results are consistent with the values of τ on the range (0.025, 0.05) and used in some papers based on authors’ intuitions.

In terms of the Alternative Reference Model, Satchell and Scowcroft (2000) point out that τ is recommended to be one. Besides, they create a stochastic τ as well since they use point estimates and their model does not include any details about the precision of the estimate. In their model, τ is only used to control the amount of shrinkage of the views onto the prior.

Another form to deal with τ in the Alternative Reference Model is to neglect τ at all and eliminate this parameter from all the formulas, which is the same as that τ is on the order of one.

2.4.2. The parameter Ω

Differing from the parameter τ, the parameter **Ω** is clearly clarified as a diagonal K×K matrix of the covariance of the views, which means the uncertainty in the views and is inversely related to the confidence level of the views. However, the way to compute **Ω** is missing in Black and Litterman's papers even though the meaning of **Ω** is straight explained. In order to fill the gap, some authors make an effort to introduce several intuitive methods to calculate **Ω**, but there is little evidence showing which method can produce the best results.

The first way is to set **Ω** as a proportion to the variance of the asset returns, which is commonly used in the literature. He and Litterman (1999) use the following expression

$$\Omega = \mathbf{diag}[\mathbf{P}(\tau\Sigma)\mathbf{P}^{\mathbf{T}}] \quad (13)$$

The intuition behind the formula is equally weighting the views and the prior equilibrium weights. Meucci (2006) also uses the same way, but he expresses the formula in another form shown below

$$\Omega = \frac{\mathbf{1}}{\mathbf{c}}\mathbf{P}\Sigma\mathbf{P}^{\mathbf{T}} \quad (14)$$

where $\frac{1}{c}$ plays the same role as τ and c is greater than one.

The second way is applied when the views are generated by using a factor model. **Ω** can be derived by using the variance of the residuals from the model. Beach and Orlov (2006) introduce a way to generate the views by using the GARCH models. Given the assumption that residuals are independent and normally distributed, the variance of residuals can be calculated as a part of the regression based on the variance of the returns from the factor model.

The last method is to use an intuitive way described by Idzorek (2005). The author introduces a new approach at the end of his paper. He incorporates the confidence level of the views into his method which is an innovative idea and makes sense. The specified confidence levels are usually ignored when the users apply He and Litterman's method to compute **Ω**. However, the confidence level, as an important feature of the B-L model, should be exploited in some way. Idzorek mentions two formulas to link the confidence levels and the weights of the asset under different uncertainty in the view. The relations are shown below

$$\mathbf{Tilt_k} \approx (\mathbf{w_{100\%}} - \mathbf{w_{mkt}}) * \mathbf{C_k} \quad (15)$$

$$\mathbf{w_{k,\%}} \approx \mathbf{w_{mkt}} + \mathbf{Tilt_k} \quad (16)$$

And one new expression of the confidence level can be obtained after arranging these two formulas, which is

$$\mathbf{C_k} = (\mathbf{w_{k,\%}} - \mathbf{w_{mkt}}) / (\mathbf{w_{100\%}} - \mathbf{w_{mkt}}) \quad (17)$$

where

C_k	is the confidence level of the kth view
$w_{k,\%}$	is the weight of the asset under the kth view with a specified confidence level
$w_{100\%}$	is the weight of the asset under 100% certainty in the kth view
w_{mkt}	is the weight of the asset without views

Idzorek illustrates 7 steps to build a complete matrix Ω, in which the prior 6 steps are used to compute the value of ω_k that is the kth diagonal element of Ω and represents the uncertainty in the kth view. The author explains the concrete process to generate Ω, while the steps are quite complicated. Afterwards, Walters (2014) makes a contribution to simplify the set of formulas and reaches the conclusion

$$\Omega = \mathbf{P}\left[\left(\frac{1-\text{confidence}}{\text{confidence}}\right)\tau\Sigma\right]\mathbf{P^T} \quad (18)$$

Depending on the types of model chosen, the parameter τ can be eliminated if the users apply the Alternative Reference Model. Compared to the formula in the first vintage method, these two formulas seem similar. And the most obvious advantage of Idzoek's method is to make a full use of the confidence level, which enhance the reliability of the value of Ω.

2.4.3. The parameter Σ_p

Σ_p means the posterior covariance of returns. In most papers related to the B-L model, Σ_p is treated as the same as Σ, which shows most authors assume that the posterior covariance of returns is equal to the prior covariance of returns even though some special views are added to the model.

Another way to compute Σ_p is only mentioned by He and Litterman (1999), but it is more precise and includes one more parameter M meaning the variance of the posterior mean estimate about the unknown mean. In other words, M stands for the uncertainty in the posterior mean estimate. The following expression shows how to compute M

$$\mathbf{M} = [(\tau\Sigma)^{-1} + \mathbf{P}^{\mathbf{T}}\Omega^{-1}\mathbf{P}]^{-1}(19)$$

And the formula can be altered by using the Woodbury Matrix Identity, which is

$$\mathbf{M} = \tau\Sigma - \tau\Sigma\mathbf{P}^{\mathbf{T}}(\mathbf{P}\tau\Sigma\mathbf{P}^{\mathbf{T}} + \Omega)^{-1}\mathbf{P}\tau\Sigma(20)$$

Given the assumption that the uncertainty in the posterior mean estimate is independent of the known covariance of returns, the value of Σ_p can be derived as

$$\Sigma_{\mathbf{p}} = \Sigma + \mathbf{M}(21)$$

2.4.4. The parameter P

P is a peculiar parameter to the B-L model, which is a matrix identifying the assets involved in the views. In general, there are two ways to determine the elements in the matrix P.

One is introduced by Satchell and Scowcroft (2000). In their paper, they use an equal weighting scheme to express the weightings of assets related to the indirect relative view which is explained by an example in Chapter 5, and the corresponding value of the weighting is equal to one divided by the number of assets outperforming or underperforming. However, this means the view may have the same effect on two or more outperforming assets with different market capitalizations, which can cause extra error when the market capitalizations of assets differ a lot.

The other way is on the contrary to the previous one, which follows a market capitalization weighting scheme. The full explanation is that the weighting of the outperforming asset is proportional to the corresponding market capitalization divided by the total market capitalization of outperforming assets. In terms of the underperforming asset, the same logic is applied. This method is mainly used by He and Litterman (1999) and Idzorek (2005). During the practical implementation of the B-L model, the weights can be obtained based on the mixture of these two methods depending on the process used to estimate the view returns (Walters, 2014).

2.5. Future Development

In the early stage, the Black-Litterman model is commonly treated as a rocket science black box generating results in some mysterious way because of the unclear methodology (He and Litterman, 1999). Nowadays, the model gradually displays its real face to the public under the contribution made by many researchers. More and more investors start to add the B-L model to their Asset Allocation Process as a subsidiary investment tool.

However, there is still a long path for the Black-Litterman model to go compared to those vintage financial models. Although most people who have an understanding of the B-L model think highly of the pleasant features of the model, there is rare strong practical evidence showing that the model actually benefits the investors. Besides, the procedures of implementing the B-L model are not well standardized and researchers did not reach an agreement on the values of some key parameters, which substantially causes difficulty to the potential users in applying the model. Due to the reasons mentioned above, the B-L model is only known to a limited group of people and mainly used inside Goldman Sachs.

In general, the Black-Litterman model provides a creative intuition to capture the influence on the asset allocation. It is widely accepted that one of the most significant contributing factors is investors' views which can be either subjective or objective. So it makes sense to process the market equilibrium portfolio by adding the views. Nonetheless, lots of issues should be taken into consideration when incorporating the views into the model, which is not fully satisfied in the B-L model. That is what the B-L model should improve in the future.

Based on the existing research, it is of vital importance to reorganize all the steps and formulas related to the B-L model in order to offer a better understanding to the users. And the parameters should be defined clearly without confusion. Currently, the information is dropped in different papers and it is a hard decision for the investors to select the correct elements of the model. Although the framework of the B-L model is given, the content inside the model still needs to be arranged and polished. In terms of the formulas and parameters, some various opinions are ought to be merged or described with attached situations.

Regarding the views, the model should be able to guide the users to set their own views. In spite of the simple and fixed types of views, it is rather complicated to determine the views which is the main component influencing the results. Although the investors are free to set the views as they wish, more accurate views will enhance the performance of the model. As what Beach and Orlo (2006) have written in their paper, the views are possible to be generated in a quantitative way by inputting the information owned by the investors. Specifying the views in a quantitative way is likely to make the results more precise than using a qualitative method. Despite the increase in the difficulty, this is a good trend to develop the B-L model since the initial purpose of creating the model is to benefit the practical investment.

Moreover, the output of the model should be supported by more empirical outcomes. To check if both of them are consistent with each other is an effective approach to calibrate the model and demonstrate the availability of the B-L model, which requires a long time period as the long-term investment is likely to reflect the performance of the model in a rather regular way.

3. Implementation and Assumptions

In order to go through our sensitivity tests, one necessary condition is to decide all the details to implement the Black-Litterman model. As mentioned before, it is hard to find a series of steps recognized by most users and used to apply the B-L model due to the special characters of the model.

Because of the unavailability of a completely reliable method to use the B-L model, we have to extract the useful parts from papers related to the B-L model and make some reasonable assumptions to build a concrete method.

Before we illustrate all the steps, one of the most important assumptions we made is to apply the Canonical Reference Model. As explained previously, most calibrations of the B-L model in the recent 20 years can be adopted in the Canonical Reference Model even if some of them are initially used in the Alternative Reference Model. That means we accept to use the Bayesian Theorem and include τ in the B-L model.

The initial step is to determine the universe of the investments asset classes and calculate the daily returns of each asset.

The second step requires us to compute the historical covariance matrix for the assets. We need to calculate the covariance matrix from the highest frequency data available which is daily basis in our case and later on scale it up to annual basis. We decide to adopt the exponential weighting scheme to rely more on the recent data instead of the equally weighted scheme.

The next step is to figure out the market capitalizations of the assets and compute relative weights for each asset which are used to compute the implied equilibrium returns.

After that, we are supposed to use a reverse optimization method to obtain the CAPM equilibrium returns for the assets. The formula is shown below

$$\Pi = \lambda\Sigma w_{mkt} \quad (22)$$

where λ is the risk aversion coefficient and w_{mkt} means the market capitalization weight of the assets. It is necessary for us to determine the value of λ, which can be achieved by dividing the risk premium by the variance of the market excess returns (Grinold and Kahn, 1999). Typically, the risk aversion coefficient ranges from 2 through 4, so we assume that the investor's degree of risk aversion is 3 in our case.

The following step is to indicate three particular views on the US market based on the relevant events happened recently and the trend of the stock market. Then we have to transform them into inputs of the model. In terms of the parameter Ω, we decide to use the way in Idzorek's extension because of the relatively higher accuracy compared to He and Litterman's.

Then we can compute the posterior combined return $E[R]$ by using the 'master formula' after generating all the necessary inputs. In the formula, we choose the maximum likelihood estimator $\tau = \frac{1}{T}$ and use the market capitalization scheme to express P.

Finally, the optimal portfolio weights are calculated by adopting unconstrained mean variance optimization, which is shown by the formula

$$\mathbf{w_p} = \mathbf{E}[\mathbf{R}]\left(\lambda\Sigma_{\mathbf{p}}\right)^{-1} \quad \mathbf{(23)}$$

We make an assumption to include the uncertainty in the posterior mean estimate M when dealing with the covariance of posterior returns, which is likely to increase the precision of the model.

Through the whole process, we are allowed to compute all the elements in a standardized way and avoid generating ambiguous results.

4. Methodology

In order to implement the sensitivity tests regarding the view vector and the confidence level of the views individually, we have to separate these two parameters into two different categories of tests. After that, we need to apply the Controlling Variable Method which is also called One-factor-at-a-time (OFAT) to start our tests.

4.1. The view vector

In terms of the first category of sensitivity test, we set the view vector as the independent variable and determine the final weights of assets as the dependent variable. In order to provide a comprehensive analysis, we are ought to design tests for each type of views, respectively the absolute view, the direct relative view and the indirect relative view. The corresponding example is given to each one:

View 1	Asset A will have an absolute excess return of x
View 2	Asset B will outperform Asset C by y
View 3	Asset D and E will underperform Asset F and G by z

To make the results of our tests comparable, we plug different values into x, y and z individually or simultaneously and transform the views into the inputs which actually enter the model. The values of x, y and z are individually divided into 5 degrees from 0 to a given maximum value and we keep the difference same between every two consecutive degrees. Each group of inputs is coordinated as a vector (x, y, z). In total, there should be $5^3 = 125$ groups of inputs and group (0, 0, 0) represents a zero vector (0, 0, 0). Besides, in order to avoid the effect caused by the confidence level, we choose one group of confidence level of the views and keep it constant. We decide to select (100%, 100%, 100%) as the confidence level of the views, which stands for that we are exactly certain about our views. After generating the only changeable inputs relevant to views and other fixed inputs related to asset classes and views, we start to implement the Black-Litterman model. It is mandatory to repeat going through the B-L model until all the results are obtained. The comprehensive results contain the influence caused in the final weights due to the variety of types of views, the figures indicated in the views or the joint effects. After generating all the posterior portfolios by plugging different groups of inputs into the Black-Litterman model, we are able to measure the sensitivity directly by analyzing the relation between the changes in the view vector and the final portfolio weights. Besides, we can also use the indicator called Tracking Error Volatility (TEV) as an assistant tool to help us analyze the sensitivity resulting from change in the parameters.

4.2. The confidence level of the views

When it comes to the second category of sensitivity test, the independent variable becomes the confidence level of the views but the dependent variable remains the same. The value for each confidence level of the views will also be altered individually or simultaneously. Similarly, each confidence level of the views is divided into 4 degrees from 25% to 100%, which are respectively 25%, 50%, 75% and 100%. We coordinate each group as a point (a, b, c). So there are $4^3 = 64$ groups of inputs including the group (100%, 100%, 100%) which means all the views are exactly convincing. Now we select the view vector with the given maximum values decided before and keep it constant, which is on the contrary to the first type of sensitivity test.

Each group of confidence level should enter the model and produce a particular result. After collecting all the results, the direct measure regarding the change in the final portfolio weights caused by the change in the confidence level and the indicator TEV will be adopted as well.

The final results will be organized on a regular basis so that we are able to show the single or joint effect of each change.

5. Data Selection

In terms of the selection of sample data, first we look at the parameters required to implement the Black-Litterman model. In our case, before collecting the raw data, the most essential step is to determine the universe of the investments. Given the difficulty in finding and dealing with the market capitalization data as well as the return data for illiquid asset classes, which mainly refers to the complicated modeling since they are not traded transparently in liquid markets, we are going to exclude the real estate, private equity and commodities. We decide to choose eight stocks from totally different sectors. And there are two main advantages. One is that these stocks have low correction between each other. The other is that the diversified portfolio fits the behavior of investors well in reality. The following stocks are chosen – Intel Corporation (INTC), The Boeing Company (BA), The Coca-Cola Company (KO), The Dow Chemical Company (DOW), Edwards Lifesciences Corporation (EW), The Goldman Sachs Group (GS), Starbucks Corporation (SBUX) and NextEra Energy (NEE).

In order to compute the values of relevant parameters in our model, we collect a series of daily adjusted closing prices during last five years from February of 2012 to January of 2017. Due to the holidays, it is necessary to complete the stock prices on the missing weekdays by using interpolation method. Afterwards, we calculate the log return and generate the daily excess returns over the risk-free rate for each stock mentioned before by subtracting the daily risk-free rate on the corresponding day. The daily excess returns are prepared to build the covariance matrix of excess returns.

Besides, another important raw data is the market capitalization of each asset, which helps us to determine the Implied Excess Return Vector and plays an essential role in transforming the views into inputs. We collect the market capitalizations of stocks directly and divide the individual market capitalization by the total one to compute the corresponding weights.

In terms of the special information about the investor's views, we have to come up with three views to specify our forecast of the performance of stocks. Based on the financial market news, the following trends are predicted:

View 1	The stock DOW will have an absolute excess return of 1%
View 2	The stock KO will outperform the stock GS by 20 basis points
View 3	The stocks EW and SBUX will underperform the stocks BA and NEE by 0.28%

Furthermore, it is mandatory to express the views in a mathematic way as well. The numbers in the views are extracted to form the view vector Q. And the rest information builds the 3x8 matrix P in which only the stocks involved in the views are given weightings. The weightings in each row sum to 1 in the case of absolute view, while they sum to 0 in the case of relative view.

6. Results

As mentioned in the Methodology part, regarding the sensitivity test for the view vector, there are 125 groups of inputs. And 64 groups of inputs are prepared for the test about the confidence level of views. All the groups of inputs are supposed to enter the Black-Litterman model individually and the corresponding final portfolio weights are recorded. Besides, the changes in the asset weights are computed to show the pure effects cause by the difference in the independent variables regardless of the original asset weights.

6.1. The view vector

In order to provide a concrete analysis, it is necessary to divide the results into several sections depending on some given rules. The first rule to follow is picking out all the groups of inputs which contain the same x and put them together. The results are distributed to five sections since there are five different values for x. In terms of each section, the average asset weights, average change in the weights and average TEV are calculated and line charts are drawn for the results to show the change in a straightforward way. The same procedures are also applied to y and z.

Based on the processed results, the most obvious fact is that the final weight of stock INTC remains the same no matter how the view vector changes and the difference between the prior weight and the posterior weight is very close to zero, which is consistent with a key feature of the B-L model that indicates the weight of a particular asset will not be influenced if there are no views specified on that asset. The slight change results from the adding of the posterior variance of the estimated mean about the unknown mean M to the prior covariance matrix Σ.

Regarding the view 1, as shown in chart 1, the weight of stock DOW increases dramatically when x goes up. The highest weight reaches 21.89% when the excess return equals 1% and the lowest weight is -25.63% when the excess return is 0. A negative weight means the short selling of a stock is allowed. The weights of other stocks only differ slightly compared to the stock DOW due to the indirect influence caused by a higher absolute excess return of stock DOW indicated in the view 1. The results make sense because the investor will intend to allocate more capital to the stock DOW and less capital to other stocks when he holds the view specifying that the stock DOW is going to have a higher excess return.

According to the observation, the trend lines are linear for all the stocks which mean the change in the weight of any stock will be the same if the change in the excess return of stock DOW is equal. Based on the calculation, it shows the weight of stock DOW will go up by 11.88% when there is an increase of 0.25% in the excess return of stock DOW. We can conclude the weight of stock DOW is properly sensitive to the change in the proposed absolute excess return because the difference of the weights is observable and meanwhile it is under control with differing reasonably. Moreover, the trend lines are symmetric for the stock KO and GS as the view 2 assumes the stock KO will have a better or equivalent performance compared to the stock GS. The weight of stock KO is going to rise 3.52% when the excess return of stock DOW increases by 0.25%. In terms of stock GS, the situation is exactly opposite and the weight of stock GS will decrease by 3.52% given the same condition. The influence to the rest stocks is almost negligible.

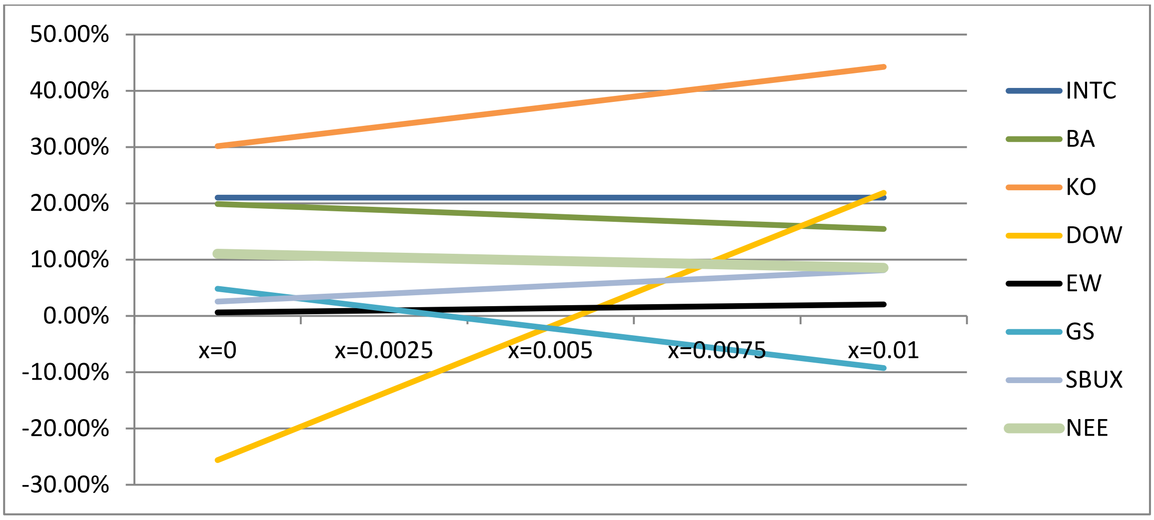

Chart 1: The average weights of stocks given specific values of x.

Looking at chart 2, the difference between the posterior weight and the prior equilibrium weight for each stock is displayed, which eliminates the effect of the equilibrium portfolio weights compared to chart 1. As mentioned before, most stocks have absolute changes less than 10% in the weights and converging to 0 when x increases. The weights of stocks DOW, KO and GS are quite sensitive to the change in the excess return of stock DOW and the weight of stock DOW has the highest sensitivity because of the straight impact from view 1.

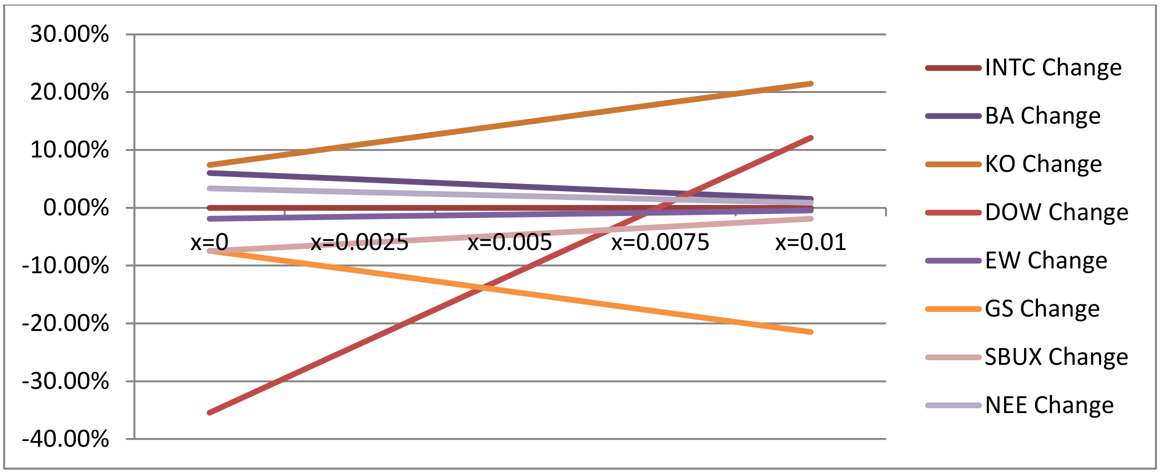

Chart 2: The difference between weights for each stock given specific values of x.

Besides, the indicator TEV which measures the distance from the prior weights also makes a contribution to our analysis. Although the linear trend line represents a stable sensitivity, the values of TEV are different when x changes. TEV goes down in the beginning and rises after a particular point while x is increasing. This indicates the sensitivity of portfolio weights to the change in the excess return of stock DOW is the lowest when x is around 0.75%. The conclusion is different from the direct measure because the indicator TEV also takes the covariance between stocks into consideration.

When it comes to the view 2, the chart 3 indicates that the stocks KO and GS become the key roles, which is consistent with the content expressed in the view 2. As anticipated, the weight of stock KO keeps increasing when stock KO has more and more favourable performance than stock GS. And there is a symmetric change for stock GS. The linear trend lines still describe all the stocks. The table shows the weight of stock KO will go up by 2.055% when y increases by 0.05%. The portfolio weights do not differ a lot except stocks KO and GS, which represents that the change in the view 2 has a weaker effect on the irrelevant stocks than the change in the view 1.

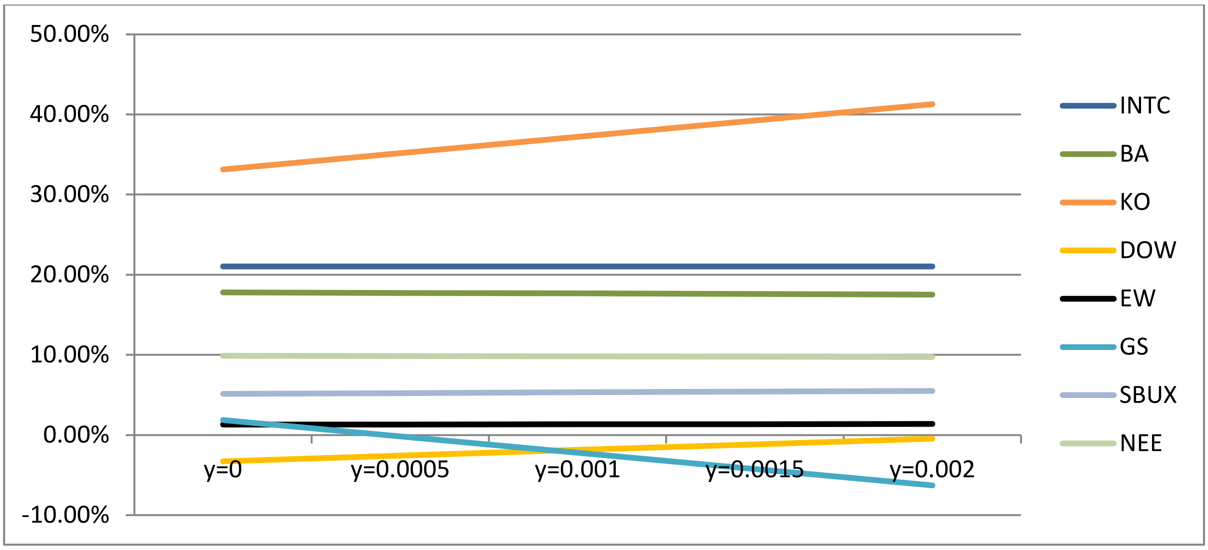

Chart 3: The average weights of stocks given specific values of y.

The chart 4 describing the trend of the difference between the prior and posterior weights tells a similar story. Most stocks have absolute changes less than 5% in the weights and the weights are slightly sensitive to the change in y.

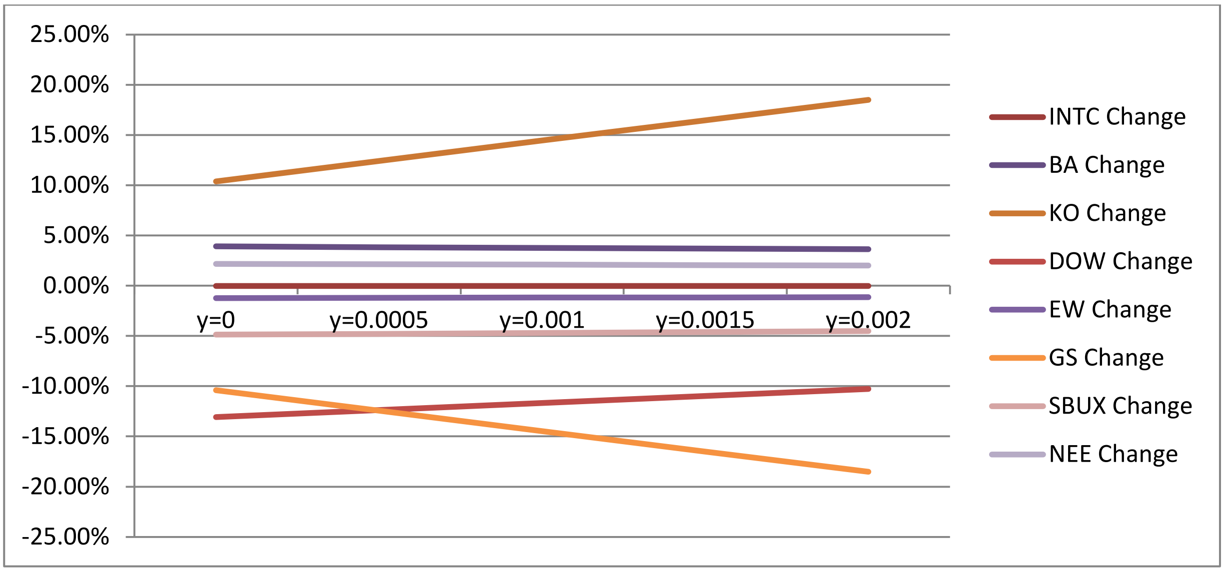

Chart 4: The difference between weights for each stock given specific values of y.

Based on TEV, the results demonstrate the sensitivity of the portfolio weights to the absolute difference between stocks KO and GS is rising up when y increases. In general, the sensitivity of the portfolio weights to the change in view 2 is considerably low compared to the change in view 1. In a more precise way, the portfolio weights are more sensitive to the change in the absolute view than the direct relative view.

In terms of the view 3, the chart 5 shows all the trend lines are still linear and there are four stocks whose weights differ obviously. Two of them have an upward trend and the remaining two are on the contrary. Among these four stocks, the slopes of the trend lines are different from each other. Besides, the trend lines for stocks BA and SBUX are steeper than the ones for stocks NEE and EW respectively. Knowing that the market capitalization of stock BA is greater than the one of stock NEE and the market capitalization of stock SBUX is greater than the one of stock EW, we can conclude the asset weight is more sensitive to the numeric change in an indirect relative view when the market capitalization of the asset is larger in the outperforming group or underperforming group. Regarding the outperforming group, there is a dump of 2.11% in the weight of stock BA when z rises 0.07%, while the weight of stock NEE only goes up by 1.47%. In the other side, the weight of stocks SBUX and EW drop 3.31% and 0.83% respectively given the same change in z.

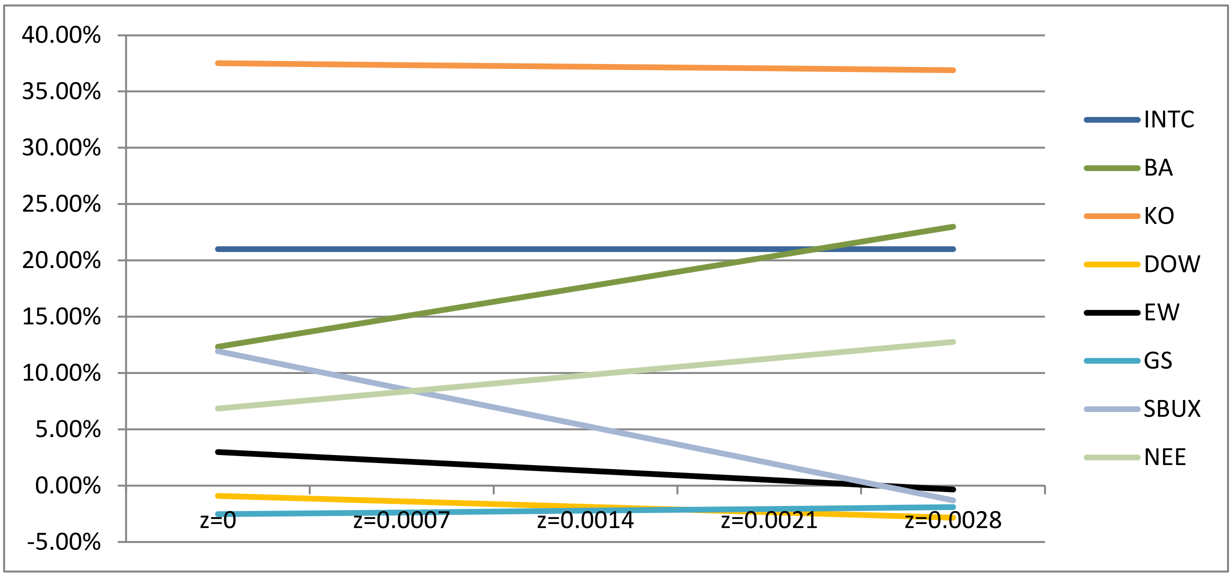

Chart 5: The average weights of stocks given specific values of z.

The chart 6 used to illustrate the changes between the prior and posterior weights shows an interesting fact that the trend lines for the four stocks specified in the view 3 intersect at one point when their posterior weights are equal to the prior weights.

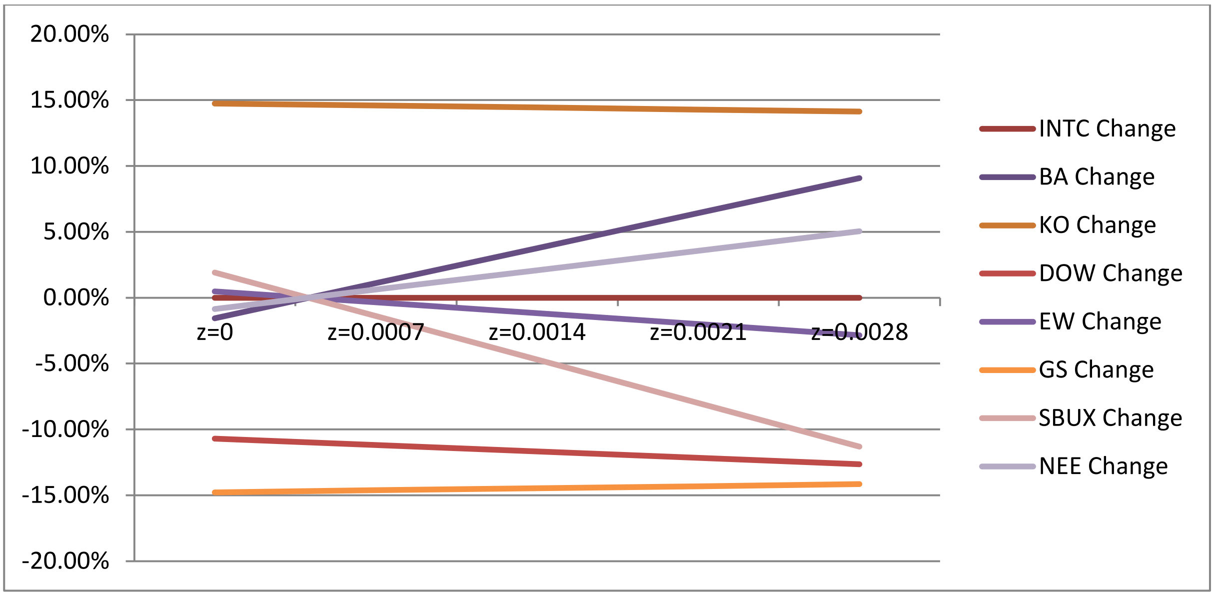

Chart 6: The difference between weights for each stock given specific values of z.

The trend and the value of TEV regarding the view 3 are similar to the previous one. TEV increases while z is rising, which represents the sensitivity of the portfolio weights to the change in the view 3 is becoming higher when z goes up according to the definition of TEV.

In order to analyse the effect caused by the change in the whole view vector, five special dependent groups are chosen, in which all the figures belong to the same scale. By comparing chart 7 to the previous charts, the trend line for each stock is similar to the one extracted from the chart which represents the view referring the stock. In other words, the chart is like a combination of the previous charts. As expected, the weight of stock DOW is the most sensitive one to the change in the view vector. It stands for that the portfolio weights are more sensitive to the change in an absolute view rather than the change in a relative view. Different from the results in the previous situations, now TEV fluctuates quite a lot due to the joint influence of the changes in three views.

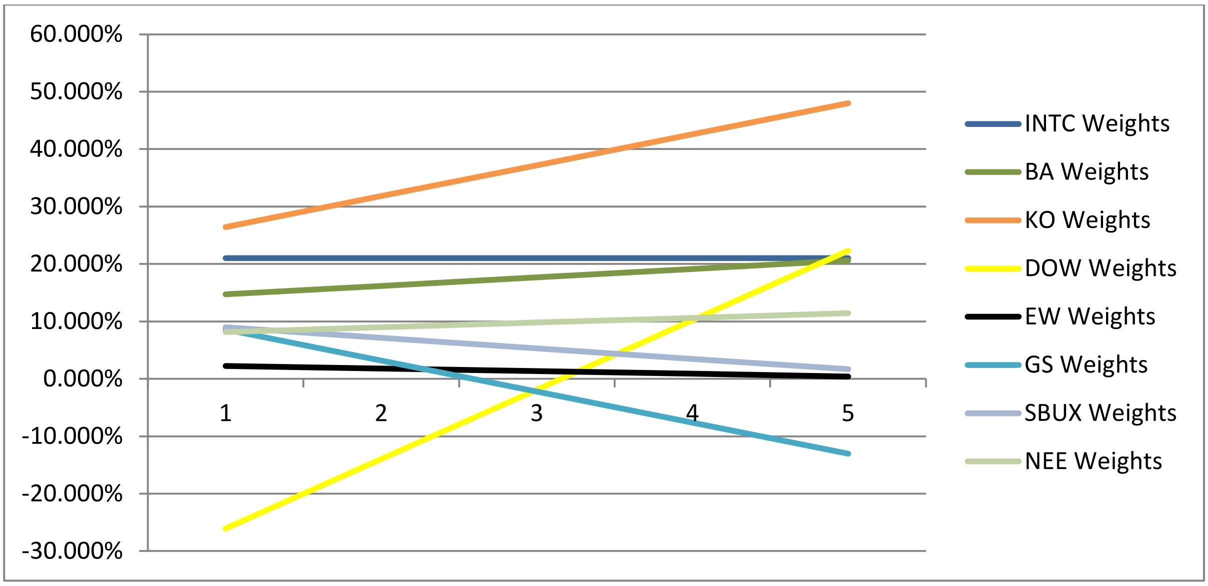

Chart 7: The portfolio weights given five special groups of inputs.

All in all, the sensitivity of the portfolio weights to the model parameter view vector depends on the type of the view. The asset weight is quite sensitive to the change in the related absolute view, which means the investor will adjust the weight of the particular asset in an obvious way when he holds different views on the absolute excess return of the asset by using the B-L model. Meanwhile, it also has an observable side effect on the other assets specified in the other views. However, unlike the standard optimization model which usually produces unreasonable nature of results like large short positions in many assets or large weights in the assets of markets with small capitalizations when there is a slight change in the expected return, the calibration of the portfolio weights makes sense resulting from the change in the absolute view. The indicator TEV interprets the sensitivity is changing in this case and reaches the lowest value when the proposed excess return of the asset is in the middle range.

The direct relative view and the indirect relative view have similar properties. The sensitivity of the relevant asset weights is moderate to the adjustment in the relative views. And there is little effect on the irrelevant assets. TEV shows the sensitivity is increasing when the proposed difference in the excess returns between the outperforming assets and the underperforming assets become larger.

The effect contributed by the change in the whole vector mainly keeps the individual characters of the views.

6.2. The confidence level

Among the 64 groups of inputs, like what have been done to the raw results in the previous case, the first step is to select all the groups which include the same confidence level a. The results produced by these groups are allocated to four sections. The average asset weights, average change in the weights and average TEV are calculated. The line charts are drawn for each section as well. The same procedures are also applied to b and c.

Regarding the confidence level of view 1, the line chart 8 shows the weight of stock DOW is the only one sensitive to the change in the confidence level a. Besides, the sensitivity is low because the weight of stock DOW only decreases by 2.54% when the confidence level held by the investor goes down by 25%. The trend line seems to be linear, which stands for constant sensitivity. However, the numeric results prove that the sensitivity is decreasing slightly given a lower confidence level. By looking at the chart, the distance tends to 0 when the confidence level is close to 0. This is consistent with the reality that the investor would like to choose the equilibrium market weight if he is uncertain about his view at all.

Regarding the indicator TEV, some are not available because of a negative value under the square root. So we exclude them and calculate the average of the rest. The values of TEV are basically constant around 1.2% no matter how the confidence level of view 1 differs, which means the portfolio weights keep the same sensitivity to the change in the confidence level of the absolute view.

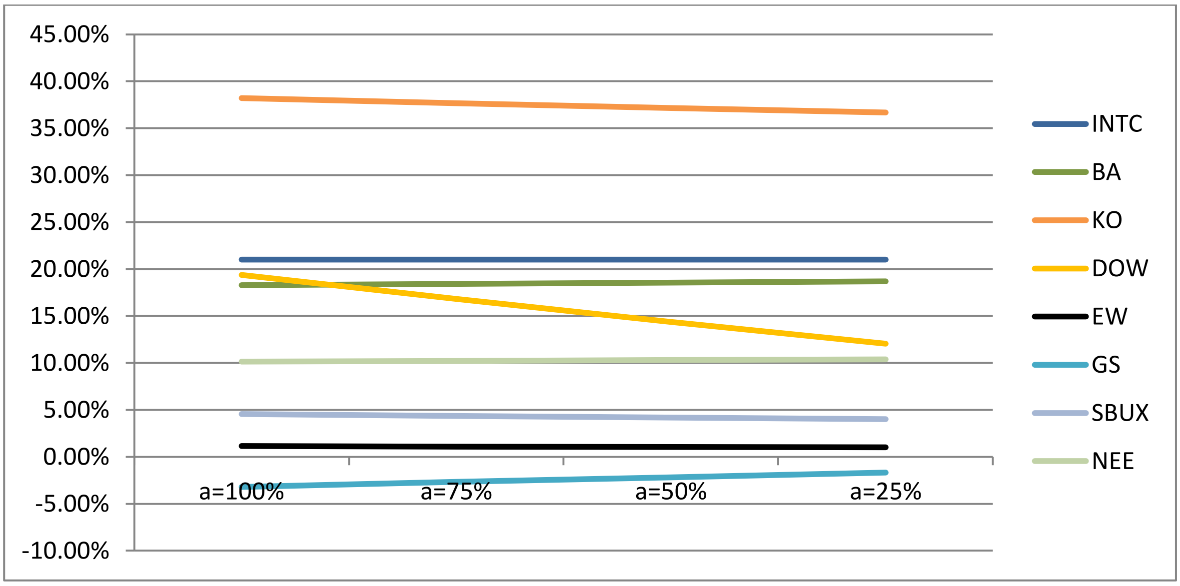

Chart 8: The average weights of stocks given specific values of a.

In terms of the confidence level of view 2, the weights of stocks KO and GS are quite sensitive to the adjustment of the confidence level b. Reasonably, the difference between the weights of two stocks specified in the view 2 becomes smaller when the investor is more uncertain about his view indicating stock KO will outperform stock GS. When the confidence level of view 2 drops from 100% to 75%, the weight of stock KO will decrease by 6.28% and there is a positive change in the weight of stock GS of 5.86%. Despite that the trend line looks to be linear in the chart, actually the change in the weight of stock KO caused by every decrease of 25% in the confidence level is different. The change tends to be smaller when the confidence level decreases, telling that the sensitivity of the weight of stock KO is going down. Based on the indicator TEV, it keeps falling when the confidence level becomes lower, which means the portfolio weights are less sensitive to the change in the confidence level of view 2 while b is decreasing.

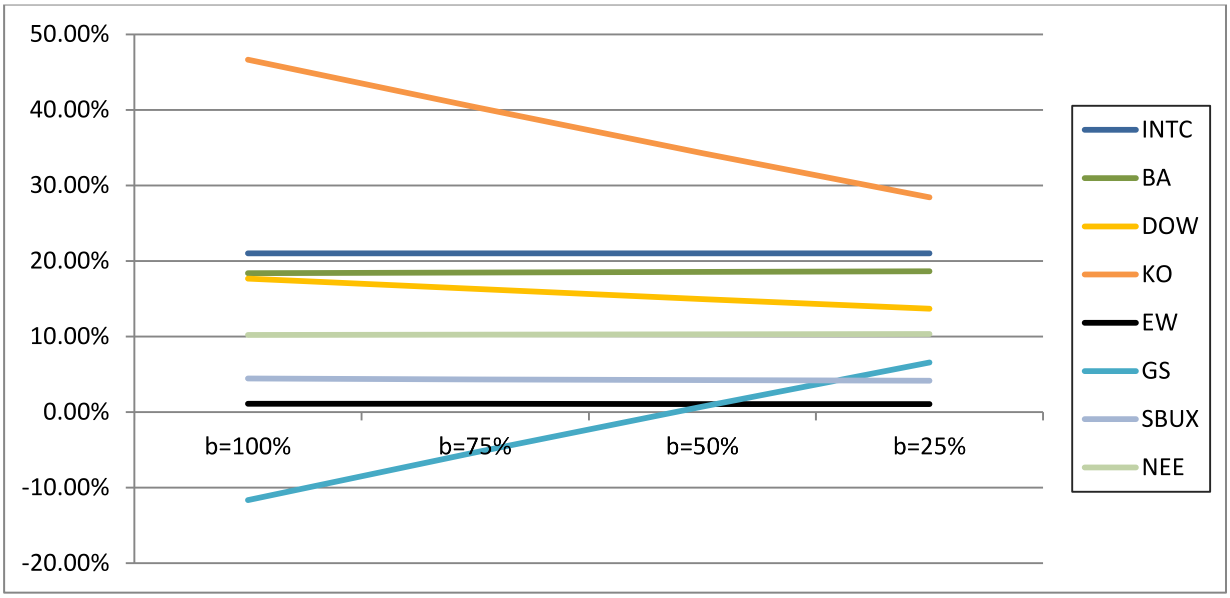

Chart 9: The average weights of stocks given specific values of b.

When it comes to the view 3, the chart shows the weights of the four stocks mentioned in the view 3 will change due to the variation of the confidence level. The slopes of the linear trend lines are quite small, which tells us the portfolio weights are not very sensitive to the change in the confidence level of view 3. When c dumps down from 100% to 75%, the weights of stocks BA and NEE will respectively decrease by 1.85% and 1.03%. On the contrary, the weights of stocks SBUX and EW will increase by 2.32% and 0.58%. The results also obey the rule summarized before that the stock with higher market capitalization has a larger change in the weight.

The sensitivity of the portfolio weights keeps the same feature as before telling that it decreases slightly when the investor lowers the confidence level. Moreover, there is a slight increase in TEV when the confidence level of view 3 decreases. According to this, we can conclude the sensitivity of the portfolio weights to the change in the confidence level of view 3 will become higher when the investor holds less certainty about the view. We should notice there is a conflict between the direct measure about the sensitivity and the indicator TEV due to their different definitions.

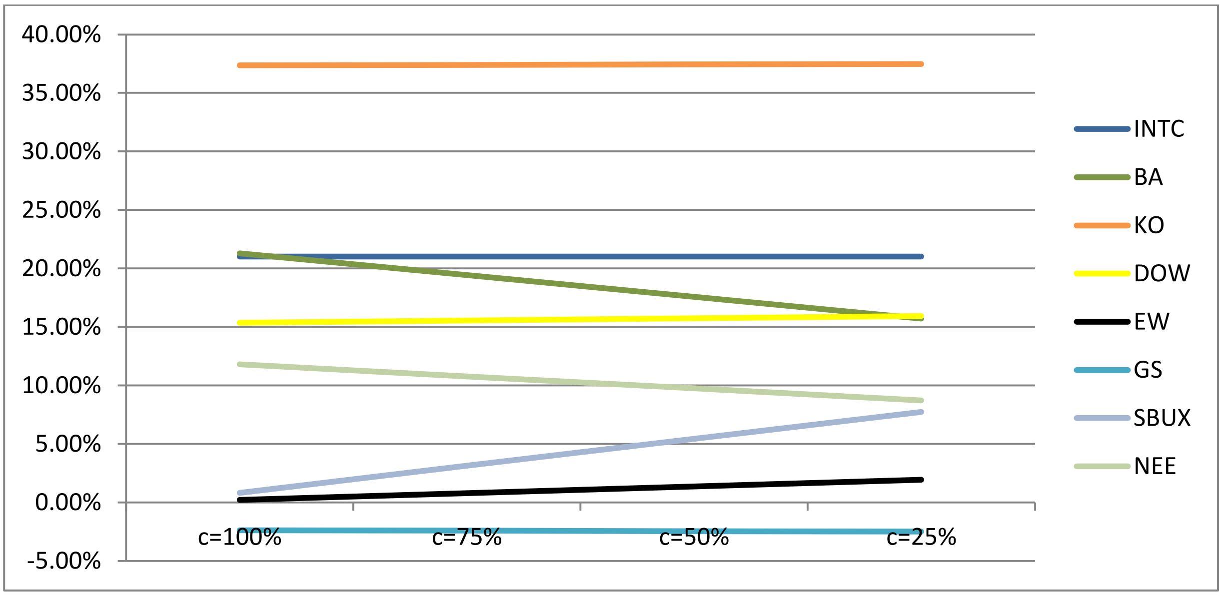

Chart 10: The average weights of stocks given specific values of c.

Besides, four special groups are chosen to analyse the joint effect caused by the same change in the three confidence levels simultaneously. In the chart, the change in the weight of the stock from the highest confidence level to the lowest confidence level is similar to the one in the previous chart which captures the particular view where the stock is specified. It shows the joint effect mainly consists of the effect caused by each individual view.

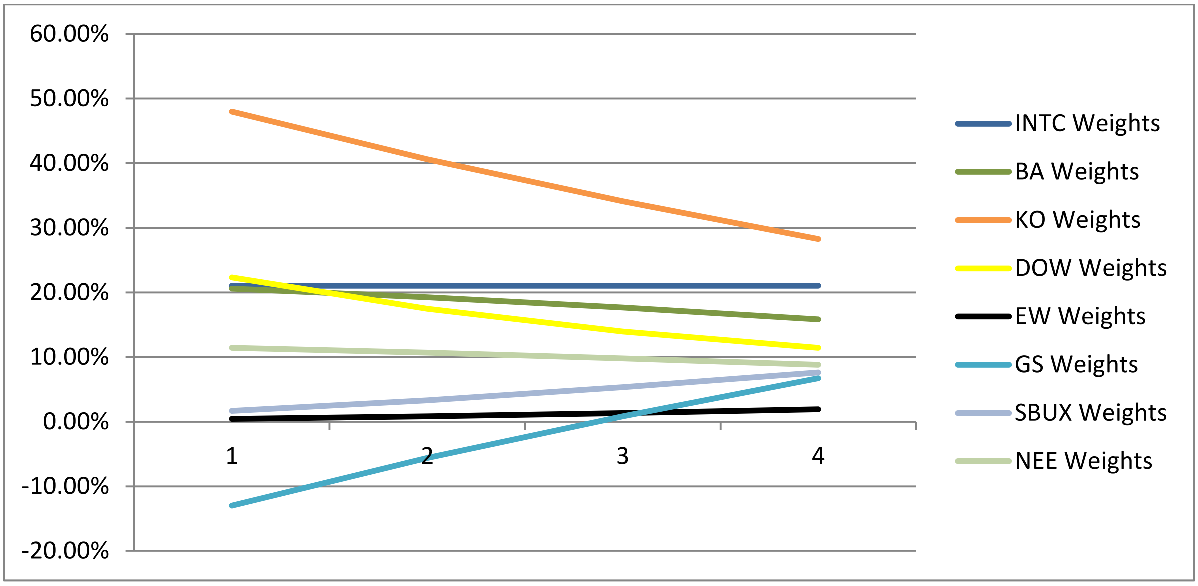

Chart 11: The portfolio weights given four special groups of inputs.

In general, the portfolio weights have the highest sensitivity to the change in the confidence level of the direct relative view. There is a dramatic difference in the weights of stocks mentioned in the view when the investor lowers the confidence level from 100% to 25%. And the sensitivity of the portfolio weights to the change in the confidence level of the absolute view is observable. Regarding the confidence level of the indirect relative view, the sensitivity of the weighs depends on the market capitalizations of assets. To be more concrete, the weight of the stock with higher market capitalization in the outperforming or underperforming group is more sensitive to the adjustment in the confidence level. Through the direct measure about the sensitivity, the results tell us the sensitivity of the portfolio weights to the change in the confidence level of each type of view drops while the confidence level is going down. All of the changes in the individual confidence level of the view have little influence on the weights of stocks not specified in the view.

Through the indicator TEV, we can notice the change in the confidence level of various types of views has different effects on the sensitivity of the portfolio weights. Regarding the confidence level of the absolute view, the sensitivity of the portfolio weights is almost constant. In terms of the confidence level of the direct relative view, there is a downward trend in the sensitivity of the portfolio weights when the investor is less confident about the view, while the trend is upward related to the confidence level of the indirect relative view.

7. Conclusion

The main purpose of the present dissertation is to demystify the sensitivity of portfolio weights to the change in the view vector and related confidence level in the Black-Litterman model. Many authors have discussed about some essential topics which are ambiguous in the Black-Litterman model. However, the sensitivity test is rarely mentioned in their research. And this dissertation aims to cover this blank part.

In order to implement the Black-Litterman model, we select eight stocks as the universe of investments subject to the availability of data about the excess return and market capitalization of asset classes.

Through the repetitive tests with various inputs, there are different expressions of the results based on two types of methods to measure the sensitivity of the portfolio weights. When we adopt the direct measure of sensitivity, our study indicates the sensitivity of the portfolio weights to the individual change in a component of the view vector or its confidence level depends on the type of the view.

Moreover, the weight of the asset mentioned in the absolute view is highly sensitive to the variation in the proposed excess return and it is less sensitive to the related confidence level. The situation regarding the direct relative view is exactly opposite to the previous one, which indicates a low sensitivity to the change in the figure specified in the view but a high sensitivity to the change in the confidence level of that view. In terms of the indirect relative view, the sensitivity to the individual change in both parameters is quite low. Nonetheless, among the assets mentioned in the indirect relative view, the asset with larger market capitalization the weight of asset with larger market capitalization is more sensitive to the change in both parameters.

In terms of the indicator TEV which focuses on measuring the sensitivity of the whole portfolio weights, the sensitivity will reach the lowest when the proposed excess return is set as a moderate value in the absolute view. However, the sensitivity keeps stable when its confidence level changes. Following an increase in the value placed in the direct relative view or the related confidence level, the sensitivity is rising. Although the sensitivity will be higher when the figure in the indirect relative view goes up, it will be lower when the investor is more certain about his view. Moreover, the whole portfolio weights are most sensitive to the change in the absolute view among three types of views and the confidence level of the direct relative view compared to the other two.

One of the main challenges in this dissertation is to standardize the procedures of implementing the Black-Litterman model. We make an effort to extract experience from the related literature and try to pick out those widely convincing parts.

Because of the limited access to the database, we are not able to select asset classes as the universe of investments which are supposed to capture the reality better. Nonetheless, we manage to generate reasonable results by using the chosen sample and we can infer our study is likely to be applied to other samples.

Based on our findings, we recommend the investor who tends to incorporate the Black-Litterman model into his asset allocation process to pay attention to the absolute view no matter how he is certain about the view. Besides, the confidence level of the relative view should also be decided as cautiously as possible.

Reference

Beach, Steven and Orlov, Alexei. 2006. ***An Application of the Black-Litterman Model with EGARCH-M-Derived views for International Portfolio Management.*** Working paper.

Bertsimas, Dimitris, Gupta Vishal and Paschalidis Loannis. 2012. Inverse Optimization: A New Perspective on the Black-Litterman Model. ***Operations Research***, Vol. 60, No. 6, November–December 2012, 1389–1403.

Bevan and Winkelmann. 1998. ***Using the Black-Litterman Global Asset Allocation Model: Three Years of Practical Experience.*** Goldman Sachs Fixed Income Research paper.

Black, Fischer and Litterman, Robert. 1991. Asset Allocation: Combining Investor Views withMarket Equilibrium. ***The Journal of Fixed Income***, 1.2:7-18.

Black, Fischer and Litterman, Robert. 1992. Global Portfolio Optimization. ***Financial Analysts Journal***, September/October, 28-43.

Braga, Maria Debora and Natale, Franceso Paolo. 2007. ***TEV Sensitivity to Views in Black-Litterman Model.*** 20th Australasian Finance & Banking Conference 2007 Paper.

Cheung, W. 2010. The Black-Litterman model explained. ***Journal of Asset Management***, October 2010, Volume 11, Issue 4, 229-243.

Christodoulakis, G.A. 2002. ***Bayesian Optimal Portfolio Selection: the Black-Litterman Approach.*** Unpublished paper. November. Available online at http://www.staff.city.ac.uk/~gchrist/Teaching/QAP/optimalportfoliobl.pdf.

Drobetz, Wolfgang. 2001. How to Avoid the Pitfalls in Portfolio Optimization? Putting the Black-Litterman Approach at Work. ***Financial Markets and Portfolio Management***, Volume 15, 2001 / Number 1, 59-75.

Firoozye Nick and Blamont, Daniel. 2003. ***Asset Allocation Model.*** Global Markets Research, Deutsche Bank, July 2003.

Fusai and Meucci, Attilio. 2003. Assessing Views. ***Risk Magazine***, 16, 3, S18-S21.

Giacometti, Rosella, Bertocchi, Marida, Rachev, Svetlozar T. and Fabozzi, Frank J. 2007. Stable distributions in the Black-Litterman approach to asset allocation. ***Quantitative Finance***, 7:4, 423 - 433

He and Litterman, Robert. 1999. ***The Intuition Behind Black-Litterman Model Portfolios***. Working paper available on SSRN.

Herold, Ulf. 2003. Portfolio Construction with Qualitative Forecasts. ***Journal of Portfolio Management***, Fall 2003, p61-72.

Idzorek, Thomas. 2005. ***A Step-By-Step guide to the Black-Litterman Model, Incorporating User-Specified Confidence Levels.*** Working paper.

Koch. 2004. ***The Black-Litterman Approach.*** Cominvest presentation, October 2004.

Krishnan, Hari and Mains, Norman. 2005. The Two-Factor Black-Litterman Model. ***Risk Magazine***, July 2005.

Litterman, et al. 2003. ***Modern Investment Management: An Equilibrium Approach by Bob Litterman and the Quantitative Research Group.*** Goldman Sachs Asset Management, Chapter 7.

Markowitz, H.M. 1952. Portfolio Selection. ***The Journal of Finance***, March, 77-91.

Martellini, Lionel and Ziemann, Volker. 2007. Extending Black-Litterman Analysis Beyond the Mean-Variance Framework. ***Journal of Portfolio Management***, Summer 2007, Vol 33, 4, 33-44.

Meucci, Attilio. 2005. ***Risk and Asset Allocation.*** Springer Finance.

Meucci, Attilio. 2006. ***Beyond Black-Litterman in Practice: A Five-Step Recipe to Input Views on non-Normal Markets.*** Working paper available on SSRN.

Michaud, R. O., D. N. Esch, and R. O. Michaud. 2013. Deconstructing Black-Litterman: How To Get The Portfolio You Already Knew You Wanted. ***Journal of Investment Management***, vol. 11, no. 1 (First Quarter): 6-20.

Qian, E. and Gorman, S. 2001. Conditional Distribution in Portfolio Theory. ***Financial Analysts Journal***, March/April, 44-51.

Satchell, S. and Scowcroft, A. 2000. A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Construction. ***Journal of Asset Management***, September, 138-150.

Sharpe, W.F. 1964. Capital Asset Prices: A Theory of Market Equilibrium. ***Journal of Finance***, September, 425-442.

Theil, H. 1971. ***Principles of Econometrics.*** New York: Wiley and Sons.

Theil, H. 1978. ***Introduction to Econometrics.*** New Jersey: Prentice-Hall, Inc.

Walters, Jay. 2014. ***The Black-Litterman Model in Detail.*** Working paper available on SSRN.

APPENDIX

APPENDIX I - CAPM equilibrium returns and the equilibrium portfolio weights

	Equilibrium Inputs							
Asset	**Intel Corporation (INTC)**	**The Boeing Company (BA)**	**The Coca-Cola Company (KO)**	**The Dow Chemical Company (DOW)**	**Edwards Lifesciences Corporation (EW)**	**The Goldman Sachs Group, Inc. (GS)**	**Starbucks Corporation (SBUX)**	**NextEra Energy, Inc. (NEE)**
Weights	21.04%	13.90%	22.76%	9.81%	2.52%	12.25%	10.02%	7.71%
Covariance Prior	0.004961018	0.002404093	0.00085337	0.003552147	0.001433744	0.001978606	0.000755229	0.000618456
	0.002404093	0.007346384	0.000883672	0.004425306	0.001057221	0.004024678	0.002449408	-0.000918605
	0.00085337	0.000883672	0.00183361	0.000869856	-0.000390216	-0.000102757	0.001337032	0.000424557
	0.003552147	0.004425306	0.000869856	0.007938187	-0.000367856	0.003574865	0.00227479	-0.000751548
	0.001433744	0.001057221	-0.000390216	-0.000367856	0.008341564	0.000249333	-0.002288503	0.001211485
	0.001978606	0.004024678	-0.000102757	0.003574865	0.000249333	0.0071006	0.002519805	-0.002284718
	0.000755229	0.002449408	0.001337032	0.00227479	-0.002288503	0.002519805	0.007330645	-0.002335178
	0.000618456	-0.000918605	0.000424557	-0.000751548	0.001211485	-0.002284718	-0.002335178	0.003683384
Prior Returns	0.697%	0.857%	0.285%	0.881%	0.128%	0.677%	0.550%	-0.052%

APPENDIX II - The Market Capitalizations of stocks and The View Vector P

Asset	Market Capitalization(Billion)
Intel Corporation (INTC)	166.09
The Boeing Company (BA)	109.71
The Coca-Cola Company (KO)	179.73
The Dow Chemical Company (DOW)	77.43
Edwards Lifesciences Corporation (EW)	19.86
The Goldman Sachs Group, Inc. (GS)	96.72
Starbucks Corporation (SBUX)	79.14
NextEra Energy, Inc. (NEE)	60.88
Total	789.56

P	INTC	BA	KO	DOW	EW	GS	SBUX	NEE
View 1	0	0	0	1	0	0	0	0
View 2	0	0	1	0	0	-1	0	0
View 3	0	0.64	0	0	-0.20	0	-0.80	0.36

APPENDIX III - Covariance Matrix

1-day Covariance Matrix (EWMA)								
INTC	1.90808E-05	9.24651E-06	3.28219E-06	1.36621E-05	5.5144E-06	7.61002E-06	2.90473E-06	2.37868E-06
BA	9.24651E-06	2.82553E-05	3.39874E-06	1.70204E-05	4.06623E-06	1.54795E-05	9.4208E-06	-3.5331E-06
KO	3.28219E-06	3.39874E-06	7.05235E-06	3.3456E-06	-1.50083E-06	-3.95219E-07	5.14243E-06	1.63291E-06
DOW	1.36621E-05	1.70204E-05	3.3456E-06	3.05315E-05	-1.41483E-06	1.37495E-05	8.74919E-06	-2.89057E-06
EW	5.5144E-06	4.06623E-06	-1.50083E-06	-1.41483E-06	3.20829E-05	9.58972E-07	-8.80193E-06	4.65956E-06
GS	7.61002E-06	1.54795E-05	-3.95219E-07	1.37495E-05	9.58972E-07	2.731E-05	9.69156E-06	-8.78738E-06
SBUX	2.90473E-06	9.4208E-06	5.14243E-06	8.74919E-06	-8.80193E-06	9.69156E-06	2.81948E-05	-8.98146E-06
NEE	2.37868E-06	-3.5331E-06	1.63291E-06	-2.89057E-06	4.65956E-06	-8.78738E-06	-8.98146E-06	1.41669E-05

Annualized Covariance Matrix (EWMA)								
INTC	0.004961018	0.002404093	0.00085337	0.003552147	0.001433744	0.001978606	0.000755229	0.000618456
BA	0.002404093	0.007346384	0.000883672	0.004425306	0.001057221	0.004024678	0.002449408	-0.000918605
KO	0.00085337	0.000883672	0.00183361	0.000869856	-0.000390216	-0.000102757	0.001337032	0.000424557
DOW	0.003552147	0.004425306	0.000869856	0.007938187	-0.000367856	0.003574865	0.00227479	-0.000751548
EW	0.001433744	0.001057221	-0.000390216	-0.000367856	0.008341564	0.000249333	-0.002288503	0.001211485
GS	0.001978606	0.004024678	-0.000102757	0.003574865	0.000249333	0.0071006	0.002519805	-0.002284718
SBUX	0.000755229	0.002449408	0.001337032	0.00227479	-0.002288503	0.002519805	0.007330645	-0.002335178
NEE	0.000618456	-0.0009186	0.000424557	-0.000751548	0.001211485	-0.002284718	-0.002335178	0.003683384

APPENDIX IV - The results of tests

	INTC	INTC Change	BA	BA Change	KO	KO Change	DOW	DOW Change	EW	EW Change	GS	GS Change	SBUX	SBUX Change	NEE	NEE Change	TEV
x=0	21.02%	-0.02%	19.90%	6.00%	30.16%	7.40%	-25.63%	-35.44%	0.64%	-1.88%	4.82%	-7.43%	2.54%	-7.48%	11.04%	3.33%	0.035
x=0.0025	21.02%	-0.02%	18.78%	4.88%	33.68%	10.92%	-13.75%	-23.56%	0.99%	-1.53%	1.30%	-10.95%	3.93%	-6.09%	10.42%	2.71%	0.027
x=0.005	21.02%	-0.02%	17.66%	3.77%	37.20%	14.44%	-1.87%	-11.68%	1.33%	-1.18%	-2.22%	-14.47%	5.32%	-4.70%	9.80%	2.09%	0.021
x=0.0075	21.02%	-0.02%	16.55%	2.65%	40.72%	17.96%	10.01%	0.20%	1.68%	-0.83%	-5.74%	-17.99%	6.71%	-3.32%	9.18%	1.47%	0.018
x=0.01	21.02%	-0.02%	15.43%	1.53%	44.24%	21.48%	21.89%	12.08%	2.03%	-0.48%	-9.26%	-21.51%	8.10%	-1.93%	8.56%	0.85%	0.021
	INTC	INTC Change	BA	BA Change	KO	KO Change	DOW	DOW Change	EW	EW Change	GS	GS Change	SBUX	SBUX Change	NEE	NEE Change	TEV
y=0	21.02%	-0.02%	17.80%	3.91%	33.14%	10.37%	-3.28%	-13.08%	1.29%	-1.22%	1.85%	-10.40%	5.14%	-4.88%	9.88%	2.17%	0.022
y=0.0005	21.02%	-0.02%	17.73%	3.84%	35.17%	12.41%	-2.57%	-12.38%	1.31%	-1.20%	-0.18%	-12.43%	5.23%	-4.79%	9.84%	2.13%	0.023
y=0.001	21.02%	-0.02%	17.66%	3.77%	37.20%	14.44%	-1.87%	-11.68%	1.33%	-1.18%	-2.22%	-14.47%	5.32%	-4.70%	9.80%	2.09%	0.024
y=0.0015	21.02%	-0.02%	17.59%	3.70%	39.23%	16.47%	-1.17%	-10.97%	1.36%	-1.16%	-4.25%	-16.50%	5.41%	-4.62%	9.76%	2.05%	0.025
y=0.002	21.02%	-0.02%	17.52%	3.63%	41.27%	18.50%	-0.46%	-10.27%	1.38%	-1.14%	-6.28%	-18.53%	5.49%	-4.53%	9.72%	2.01%	0.026
	INTC	INTC Change	BA	BA Change	KO	KO Change	DOW	DOW Change	EW	EW Change	GS	GS Change	SBUX	SBUX Change	NEE	NEE Change	TEV
z=0	21.02%	-0.02%	12.34%	-1.55%	37.51%	14.75%	-0.90%	-10.70%	2.99%	0.48%	-2.52%	-14.77%	11.93%	1.91%	6.85%	-0.86%	0.023
z=0.0007	21.02%	-0.02%	15.00%	1.11%	37.36%	14.59%	-1.38%	-11.19%	2.16%	-0.35%	-2.37%	-14.62%	8.63%	-1.40%	8.33%	0.61%	0.023
z=0.0014	21.02%	-0.02%	17.66%	3.77%	37.20%	14.44%	-1.87%	-11.68%	1.33%	-1.18%	-2.22%	-14.47%	5.32%	-4.70%	9.80%	2.09%	0.024
z=0.0021	21.02%	-0.02%	20.32%	6.43%	37.05%	14.29%	-2.36%	-12.16%	0.51%	-2.01%	-2.06%	-14.31%	2.01%	-8.01%	11.28%	3.57%	0.025
z=0.0028	21.02%	-0.02%	22.98%	9.09%	36.90%	14.13%	-2.84%	-12.65%	-0.32%	-2.84%	-1.91%	-14.16%	-1.29%	-11.32%	12.75%	5.04%	0.026

Group	x	y	z	INTC		BA		KO		DOW		EW		GS		SBUX		NEE		TEV
				Weights	Change	Weights	Change	Weights	Change	Weights	Change	Weights	Change	Weights	Change	Weights	Change	Weights	Change	
1	0	0	0	21.020%	-0.016%	14.717%	0.822%	26.405%	3.641%	-26.064%	-35.871%	2.254%	-0.262%	8.582%	-3.668%	8.980%	-1.043%	8.167%	0.456%	0.033
2	0.0025	0.0005	0.0007	21.020%	-0.016%	16.190%	2.295%	31.803%	9.040%	-13.967%	-23.774%	1.794%	-0.721%	3.183%	-9.067%	7.150%	-2.874%	8.984%	1.274%	0.025
3	0.005	0.001	0.0014	21.020%	-0.016%	17.663%	3.768%	37.202%	14.439%	-1.870%	-11.677%	1.335%	-1.180%	-2.216%	-14.465%	5.319%	-4.704%	9.801%	2.091%	0.020
4	0.0075	0.0015	0.0021	21.020%	-0.016%	19.135%	5.240%	42.601%	19.837%	10.227%	0.420%	0.875%	-1.640%	-7.614%	-19.864%	3.489%	-6.535%	10.619%	2.908%	0.028
5	0.01	0.002	0.0028	21.020%	-0.016%	20.608%	6.713%	47.999%	25.236%	22.324%	12.517%	0.416%	-2.099%	-13.013%	-25.263%	1.658%	-8.365%	11.436%	3.725%	0.025

	INTC	INTC Change	BA	BA Change	KO	KO Change	DOW	DOW Change	EW	EW Change	GS	GS Change	SBUX	SBUX Change	NEE	NEE Change	TEV
a=100%	21.02%	-0.02%	18.28%	4.39%	38.19%	15.43%	19.36%	9.55%	1.14%	-1.37%	-3.20%	-15.45%	4.55%	-5.47%	10.15%	2.43%	0.012
a=75%	21.02%	-0.02%	18.43%	4.53%	37.66%	14.90%	16.82%	7.01%	1.10%	-1.42%	-2.67%	-14.92%	4.37%	-5.66%	10.23%	2.52%	0.012
a=50%	21.02%	-0.02%	18.57%	4.67%	37.16%	14.39%	14.38%	4.57%	1.05%	-1.46%	-2.17%	-14.42%	4.19%	-5.83%	10.30%	2.59%	0.012
a=25%	21.02%	-0.02%	18.70%	4.81%	36.67%	13.91%	12.04%	2.24%	1.01%	-1.51%	-1.69%	-13.94%	4.02%	-6.00%	10.38%	2.67%	0.013
	INTC	**INTC Change**	**BA**	**BA Change**	**KO**	**KO Change**	**DOW**	**DOW Change**	**EW**	**EW Change**	**GS**	**GS Change**	**SBUX**	**SBUX Change**	**NEE**	**NEE Change**	**TEV**
b=100%	21.02%	-0.02%	18.36%	4.47%	46.63%	23.87%	17.68%	7.87%	1.12%	-1.40%	-11.65%	-23.90%	4.45%	-5.58%	10.19%	2.48%	0.017
b=75%	21.02%	-0.02%	18.45%	4.56%	40.35%	17.59%	16.29%	6.48%	1.09%	-1.43%	-5.36%	-17.61%	4.33%	-5.69%	10.24%	2.53%	0.013
b=50%	21.02%	-0.02%	18.54%	4.65%	34.28%	11.52%	14.96%	5.15%	1.06%	-1.45%	0.71%	-11.54%	4.23%	-5.80%	10.29%	2.58%	0.011
b=25%	21.02%	-0.02%	18.62%	4.73%	28.42%	5.65%	13.68%	3.87%	1.03%	-1.48%	6.57%	-5.68%	4.12%	-5.90%	10.34%	2.62%	0.008
	INTC	**INTC Change**	**BA**	**BA Change**	**KO**	**KO Change**	**DOW**	**DOW Change**	**EW**	**EW Change**	**GS**	**GS Change**	**SBUX**	**SBUX Change**	**NEE**	**NEE Change**	**TEV**
c=100%	21.02%	-0.02%	21.28%	7.39%	37.36%	14.60%	15.35%	5.55%	0.21%	-2.31%	-2.37%	-14.62%	0.82%	-9.20%	11.81%	4.10%	0.011
c=75%	21.02%	-0.02%	19.42%	5.52%	37.40%	14.64%	15.55%	5.75%	0.79%	-1.73%	-2.41%	-14.66%	3.14%	-6.89%	10.78%	3.07%	0.013
c=50%	21.02%	-0.02%	17.56%	3.67%	37.44%	14.68%	15.75%	5.94%	1.37%	-1.15%	-2.45%	-14.70%	5.44%	-4.58%	9.75%	2.04%	0.013
c=25%	21.02%	-0.02%	15.72%	1.82%	37.48%	14.72%	15.94%	6.14%	1.94%	-0.57%	-2.49%	-14.74%	7.73%	-2.29%	8.72%	1.01%	0.014

Group	a	b	c	INTC		BA		KO		DOW		EW		GS		SBUX		NEE		TEV
				Weights	**Change**	**Weights**	**Change**	**Weights**	**Change**	**Weights**	**Change**	**Weights**	**Change**	**Weights**	**Change**	**Weights**	**Change**	**Weights**	**Change**	
1	**100%**	**100%**	**100%**	21.02%	-0.02%	20.61%	6.71%	48.00%	25.24%	22.32%	12.52%	0.42%	-2.10%	-13.01%	-25.26%	1.66%	-8.37%	11.44%	3.73%	N/A
2	**75%**	**75%**	**75%**	21.02%	-0.02%	19.28%	5.38%	40.61%	17.84%	17.47%	7.66%	0.83%	-1.68%	-5.62%	-17.87%	3.31%	-6.71%	10.70%	2.99%	0.013
3	**50%**	**50%**	**50%**	21.02%	-0.02%	17.65%	3.76%	34.13%	11.37%	13.94%	4.13%	1.34%	-1.18%	0.85%	-11.40%	5.33%	-4.69%	9.80%	2.08%	0.012
4	**25%**	**25%**	**25%**	21.02%	-0.02%	15.83%	1.93%	28.26%	5.49%	11.44%	1.64%	1.91%	-0.61%	6.73%	-5.52%	7.60%	-2.43%	8.78%	1.07%	0.010