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A robust closed-form estimator for the GARCH(1,1) model

ABSTRACT

In this paper we extend the closed-form estimator for the GARCH(1,1) proposed by Kristensen and Linton (2006) to deal with additive outliers. It has the advantage that is *per se* more robust than the maximum likelihood estimator (ML) often used to estimate this model, it is easy to implement and does not require the use of any numerical optimization procedure. The robustification of the closed-form estimator is done by replacing the sample autocorrelations by a robust estimator of these correlations and by estimating the volatility using robust filters. The performance of our proposal in estimating the parameters and the volatility of the GARCH(1,1) model is compared with the proposals existing in the literature via intensive Monte Carlo experiments and the results of these experiments show that our proposal outperforms the ML and quasi-maximum likelihood (QMLE) estimators based procedures. Finally, we fit the robust closed-form estimator and the benchmarks to one series of financial returns and analyze their performances in estimating and forecasting the volatility and the Value-at-Risk.

JEL-Classifications: C22; C53; C58

Keywords: Additive Outliers; Autocorrelations; Robustness; Value-at-Risk; Volatility Forecasting

1. Introduction

Return series of financial assets typically exhibit high kurtosis, higher order dependence and volatility clustering. The generalized autoregressive conditional heteroscedastic model (GARCH) is the most popular model in parameterizing the higher order dependence and the evolution of volatility. Since its proposal in the literature by Bollerslev (1986), it has been extended in several directions. The first extension, proposed by Bollerslev (1987), allows the error of the GARCH model to follow a Student- t distribution in order to accommodate the high kurtosis of the data. However, it has been observed that the estimated residuals from this extended model still register excess kurtosis (see Baillie and Bollerslev, 1989; Teräsvirta, 1996). One possible reason for this occurrence is that some observations on returns are not fitted by a Gaussian GARCH model and not even by a t -distributed GARCH model. These observations may be influential (see Zhang, 2004, for a detailed definition of influential observation) since they can affect undesirably the estimation of parameters (see for example Fox, 1972; Van Dijk et al., 1999; Verhoeven and McAleer, 2000), the tests of conditional homoscedasticity (see Carnero et al., 2007; Grossi and Laurini, 2009), the out-of-sample volatility forecasts (see for instance Ledolter, 1989; Chen and Liu, 1993; Franses and Ghijssels, 1999; Grané and Veiga, 2010) and the risk measures (Grané

and Veiga, 2014). When this is the case, some authors denote them by outliers and distinguish between additive and innovational (or innovations) outliers. The first type is classified into two categories: additive level outliers (ALO), which exert an effect on the level of the series but not on the evolution of the underlying volatility, and additive volatility outliers (AVO), that also affect the conditional variance (see Hotta and Tsay, 2012; Sakata and White, 1998). Innovational outliers contrarily to additive outliers are outliers that may have a long-run effect on modeling the volatility. Less studies have focus on the effect of innovational outliers due mainly to the fact they are transmitted by the same dynamics of the series which makes their effect less relevant (see Carnero et al., 2007; Peña, 2001).

In the literature, there are two ways of dealing with outliers, either identify these observations and correct them before estimating the parameters and the volatility of the GARCH model or use robust methods. In this paper, we follow the second alternative and deal with additive outliers by robustifying the closed-form estimator for the GARCH(1,1) proposed by Kristensen and Linton (2006). This estimator has several advantages in comparison with the ML estimator often used to estimate the GARCH(1,1) model that are: it is easy to implement, it does not require the use of any numerical optimization procedure and initial starting values. The use of starting values might be a drawback since it can generate different outputs across popular packages (Brooks et al., 2001; McCullough and Renfro, 1999). Our proposal follows that of Kristensen and Linton (2006) and it is based on the autorregressive moving average (ARMA) representation of the squared GARCH model and on the use of the implied autocovariance and autocorrelation functions to obtain closed-form estimators of the parameters. The difference regarding the original estimator is that we replace the sample autocorrelation function by a robust estimator of this function. Very recently, Prono (2014) proposed a closed-form estimator that is based on second-order covariances and cross-order covariances that are also affected by the presence of outliers. Therefore, the robustification of Prono (2014)'s closed-form estimator would require robust measures of the cross-order covariances. We use the proposal of Teräsvirta and Zhao (2011) that is based on applying the Huber's and Ramsay's weights to the sample variance and autocovariances. In the time series literature, it is well known the importance of using robust estimators for measuring the time series dependence. The volatility is estimated by using robust filters proposed by Muler and Yohai (2008) and Carnero et al. (2012). Furthermore, the small sample properties of our proposal in the estimation of parameters and volatility are analyzed via intensive Monte Carlo experiments and compared to those of the existing alternatives in the literature. The results of these experiments show that our proposal is robust to the presence of additive outliers and outperforms the alternatives in terms of volatility estimation independently if the outliers are either isolated or patches, large or small. Finally, we illustrate our results empirically by fitting the robust closed-form estimator and the benchmarks to one series of financial returns in order to forecast the volatility and compute the Value-at-Risk (VaR) forecasts.

The remainder of the paper is organized as follows. In Section 2 we robustify the closed-form estimator by Kristensen and Linton (2006) and in Section 3 we propose to estimate its volatility by applying a robust filter. In Section 4, we perform intensive Monte Carlo experiments and show that in the presence of additive outliers the robust closed-form estimator provides more accurate estimates of the volatilities. Section 5 illustrates the performance of the new estimator in an empirical application, and finally, Section 6 summarizes our conclusions.

2. A robust closed-form estimator

In this Section, first we present the closed-form estimator of Kristensen and Linton (2006) that is based on the ARMA representation of the GARCH(1,1) model and on the use of the implied autocovariance and autocorrelation functions. Second, we propose to robustify this estimator by replacing the sample autocorrelation function by a robust estimator of this function. We use the robust estimator of the autocorrelations proposed by Teräsvirta and Zhao (2011) which is based on applying the Huber's and Ramsay's weights to the sample variance and autocovariances.

2.1. An ARMA representation of the GARCH(1,1) model

Let the GARCH(1,1) process be defined as:

$$y_t = \sigma_t \varepsilon_t \quad (1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (2)$$

where ε_t is an i.i.d process with mean zero and variance 1. Defining $x_t \equiv y_t^2$ we can write that

$$x_t = \alpha_0 + \phi x_{t-1} + \eta_t + \theta \eta_{t-1}, \quad (3)$$

where $\eta_t = x_t - \sigma_t^2$ is a martingale difference sequence, $\phi = \alpha_1 + \beta_1 > 0$ and $\theta = -\beta_1 < 0$. From equation (3), we observe that x_t is an ARMA(1,1) with parameters ϕ and θ , respectively. In order the ARMA process to be stationary it is imposed that $\phi < 1$.

The autocorrelation function of an ARMA(1,1) is given by:

$$\rho(1) = \frac{(1 + \phi\theta)(\phi + \theta)}{1 + \theta^2 + 2\phi\theta} \quad (4)$$

and

$$\rho(k) = \phi \rho(k-1), \quad k = 2, 3, \dots \quad (5)$$

(see Kristensen and Linton, 2006, for more details). Therefore, equation (4) can be expressed as a quadratic equation in θ as:

$$\theta^2 + b\theta + 1 = 0, \quad (6)$$

with $b \equiv \frac{\phi^2 + 1 - 2\rho(1)\phi}{\phi - \rho(1)}$. Given that $\phi < 1$, $\phi \neq \rho(1)$ and $\beta_1 > 0$, $b > 2$ is well defined and a solution to equation (6) is

$$\theta = \frac{-b + \sqrt{b^2 - 4}}{2}. \quad (7)$$

Finally,

$$\alpha_0 = \sigma^2(1 - \phi), \quad (8)$$

and $\sigma^2 \equiv E(y_t^2)$. Equations (4), (7) and (8) can be used to obtain closed-form estimators of α_0 , α_1 and β_1 . Kristensen and Linton (2006) estimated ϕ using the sample autocorrelations of order

1 and 2, that is, $\hat{\phi} = \hat{\rho}(2)/\hat{\rho}(1)$, where $\hat{\rho}(k) = \hat{\gamma}(k)/\hat{\gamma}(0)$ is the sample autocorrelation of order k and $\hat{\gamma}(k)$ is the corresponding sample autocovariance of order k with

$$\hat{\gamma}(k) = \frac{1}{T-k} \sum_{t=1}^{T-k} (x_{t+k} - \hat{\sigma}^2)(x_t - \hat{\sigma}^2) \quad (9)$$

and

$$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^T x_t. \quad (10)$$

Replacing $\hat{\phi}$ into equation (7), Kristensen and Linton (2006) obtain an estimator of θ , that is given by

$$\hat{\theta} = \frac{-\hat{b} + \sqrt{\hat{b}^2 - 4}}{2}, \quad \hat{b} \equiv \frac{\hat{\phi}^2 + 1 - 2\hat{\rho}(1)\hat{\phi}}{\hat{\phi} - \hat{\rho}(1)},$$

with $\hat{b} \geq 2$. Finally, this leads to the following estimators of the GARCH(1,1) model¹:

$$\hat{\alpha}_1 = \hat{\theta} + \hat{\phi}, \quad \hat{\beta}_1 = -\hat{\theta}, \quad \hat{\alpha}_0 = \hat{\sigma}^2(1 - \hat{\phi}). \quad (11)$$

2.2. The robustification

Our proposal is to replace the sample estimate of $\rho(\cdot)$ by a robust estimator that is the weighted autocorrelation function of x_t provided by Teräsvirta and Zhao (2011), that is,

$$\hat{\rho}_w(k) = \frac{\hat{\gamma}_w(k)}{\hat{\gamma}_w(0)}, \quad (12)$$

where

$$\hat{\gamma}_w(k) = \frac{\sum_{t=1}^{T-k} (x_{t+k} - \hat{\sigma}_w^2)(x_t - \hat{\sigma}_w^2)w_{t+k}w_t}{\sum_{t=1}^{T-k} w_{t+k}w_t}, \quad k = 0, 1, 2, \dots \quad (13)$$

and

$$\hat{\sigma}_w^2 = \frac{\sum_{t=1}^T x_t w_t}{\sum_{t=1}^T w_t}. \quad (14)$$

We use as weighting function that proposed by Ramsay (1977) that has the form

$$w_t = \exp\left(-a \frac{|x_t - \hat{\sigma}^2|}{\sigma_x}\right),$$

where $\sigma_x = \sqrt{\frac{1}{T-1} \sum_{t=1}^T (x_t - \hat{\sigma}^2)^2}$ and $a = 0.3$ (see Teräsvirta and Zhao, 2011; Carnero et al., 2014, for a similar value of a). Now the robust closed-form estimator for the GARCH(1,1) model are: $\hat{\alpha}_0^{KLr}$, $\hat{\alpha}_1^{KLr}$ and $\hat{\beta}_1^{KLr}$ with

¹See Kristensen and Linton (2006) for dealing with estimates of ϕ that are in the intervals $]-\infty; 0[$ and $]1; +\infty[$.

$$\widehat{\alpha}_0^{KLr} = \widehat{\sigma}_w^2(1 - \widehat{\phi}_w), \quad \widehat{\alpha}_1^{KLr} = \widehat{\theta}_w + \widehat{\phi}_w, \quad \widehat{\beta}_1^{KLr} = -\widehat{\theta}_w, \quad (15)$$

where $\widehat{\phi}_w = \widehat{\rho}_w(2)/\widehat{\rho}_w(1)$ and

$$\widehat{\theta}_w = \frac{-\widehat{b}_w + \sqrt{\widehat{b}_w^2 - 4}}{2}, \quad \widehat{b}_w \equiv \frac{\widehat{\phi}_w^2 + 1 - 2\widehat{\rho}_w(1)\widehat{\phi}_w}{\widehat{\phi}_w - \widehat{\rho}_w(1)}.$$

We have also tried other robust estimator by replacing the means in the sample autocorrelations and autocovariances by the corresponding medians, but we have decided not present them here due to the poorness of the results.² We analyze all the methods of estimation using the mean squared error (MSE) as in Kristensen and Linton (2006) and the biases in the estimation of the volatility as in Carnero et al. (2012).

3. Robust estimation of the volatility

There are in the literature some robust volatility estimators for the GARCH(1,1) model. The first is provided by Muler and Yohai (2008) and is given by:

$$\widehat{\sigma}_t^2 = \widehat{\alpha}_0 + \widehat{\alpha}_1 r_c \left(\frac{y_{t-1}^2}{\widehat{\sigma}_{t-1}^2} \right) \times \widehat{\sigma}_{t-1}^2 + \widehat{\beta}_1 \widehat{\sigma}_{t-1}^2, \quad (16)$$

where

$$r_c = \begin{cases} x, & |x| < c \\ c, & |x| \geq c \end{cases}$$

and $\widehat{\alpha}_0$, $\widehat{\alpha}_1$ and $\widehat{\beta}_1$ are estimated using the BM estimator (see Muler and Yohai, 2008, for details about this estimator).

Iqbal and Mukherjee (2010) propose a large class of M-estimators for estimating the parameters of an asymmetric GARCH model. We have not considered this class of estimators since it is based on an iterative algorithm that depends crucially on the starting values and according to Tinkl (2010) performs similarly to that proposed by Muler and Yohai (2008) in terms of asymptotic convergence.

The third is proposed by Carnero et al. (2012) and replaces the r_c in equation (16) by

$$r_{cpr} = \begin{cases} x, & |x| < c \\ 1, & |x| \geq c. \end{cases}$$

The parameters of the GARCH model are estimated, in this case, by maximizing the Student- t log-likelihood. We denote this estimator $QMLE - t$.

Our proposal is to estimate the volatility using:

$$\widehat{\sigma}_t^{2KLr} = \widehat{\alpha}_0^{KLr} + \widehat{\alpha}_1^{KLr} r_{cpr} \left(\frac{y_{t-1}^2}{\widehat{\sigma}_{t-1}^{2KLr}} \right) \times \widehat{\sigma}_{t-1}^{2KLr} + \widehat{\beta}_1^{KLr} \widehat{\sigma}_{t-1}^{2KLr}, \quad (17)$$

where the parameters are estimated with the robust closed-form estimator.

²Results are available from the authors upon request.

4. Monte Carlo experiments

In this section we present the results of two intensive Monte Carlo experiments. The aim of the first experiment is to evaluate the finite sample properties of the robust closed-form estimator in comparison to those of the benchmarks in estimating the parameters and the marginal variance. The second experiment has as aim to evaluate the performance of all estimators in estimating the volatility.

Regarding the first experiments, we have generated 1000 series of sizes $T = 500, 1000$ and 5000 with a GARCH(1,1) with parameters $\alpha_0 = 0.1$, $\alpha_1 = 0.1$ and $\beta_1 = 0.8$ (see Carnero et al., 2012, for similar parameter values). The simulated series are contaminated either with one isolated additive level outlier (ALO) or multiple isolated ALOs or patches of three outliers of sizes $\omega_{AO} = 0, 5, 10$ and 15 standard deviations of the original simulated series. In this experiment the outliers are placed randomly in the simulated series. Notice that the conditional mean equation of the GARCH(1,1) contaminated by ALOs is defined as:

$$y_t^* = \omega_{AO} I_T(t) + \sigma_t \varepsilon_t,$$

where ε_t is defined as before, ω_{AO} represents the magnitude (or size) of the additive level outlier and $I_T(t) = 1$ for $t \in T$ and 0 otherwise, representing the presence of the outlier at a set of times T . Equation (2) of the conditional variance remains the same, since this type of outliers only affect the level of the series.

Tables 1–3 reports the Monte Carlo means, standard deviations of the parameter and marginal variance ($mv = \alpha_0 / (1 - \alpha_1 - \beta_1)$) estimates, obtained with the closed-form estimators and the several alternatives based on QMLE, and the Monte Carlo MSE. Here, *QMLE* is equivalent to the Maximum Likelihood estimator since we assume that the error is Gaussian. In order to estimate the GARCH(1,1) by *QMLE* we use the Econometrics toolbox of Matlab R2013b. Regarding the QMLE based methods and the parameters α_1 and β_1 , the methods tend to overestimate these parameters and the *QMLE* – t provides the smallest MSEs. The closed-form estimators provide the largest biases and MSEs for the parameter estimates. However, if we increase the sample size above $T = 5000$, these biases almost disappear.³ On the other hand, if we focus on the estimates of the marginal variance the closed-form estimators beat always the QMLE based estimators for $T = 5000$. The exceptions are when we contaminate the series with multiple ALOs or patches of outliers of moderate and large sizes and the sample size is small. In these cases, the robust QMLE based estimators provide estimates of the marginal variance with smaller bias.

Carnero et al. (2012) provide evidence that the estimator that reports the smallest biases for the parameters is not necessarily the one that leads to good volatility estimates. The authors argue that the most relevant key for the accurate estimation of the volatility is the accurate estimation of the marginal variance. As in Carnero et al. (2012), the error in the estimation of σ_t^2 when there is an isolated outlier at time $t = \tau$ is given by:

$$\begin{aligned} \epsilon_t &= \hat{\sigma}_t^2 - \sigma_t^2 \\ &= (\hat{\alpha}_0 - \alpha_0) + (\hat{\alpha}_1 - \alpha_1) y_{t-1}^{*2} + (\hat{\beta}_1 - \beta_1) \sigma_{t-1}^2 + \hat{\beta}_1 (\hat{\sigma}_{t-1}^2 - \sigma_{t-1}^2) \\ &= \frac{(\hat{\alpha}_0 - \alpha_0)(1 - \hat{\beta}_1^{t-1})}{(1 - \hat{\beta}_1)} + (\hat{\alpha}_1 - \alpha_1) \sum_{i=0}^{t-2} \hat{\beta}_1^i y_{t-1-i}^2 + (\hat{\beta}_1 - \beta_1) \sum_{i=0}^{t-2} \hat{\beta}_1^i \sigma_{t-1-i}^2 \\ &\quad + \hat{\alpha}_1 \sum_{i=0}^{t-2} \hat{\beta}_1^i (y_{t-1-i}^{*2} - y_{t-1-i}^2) + \hat{\beta}_1^{t-1} (\hat{\sigma}_1^2 - \sigma_1^2). \end{aligned} \tag{18}$$

³Results are available from the authors upon request.

The expected error depends on the parameter biases, covariances, expectations of non-linear functions of the estimator and the initial estimate of σ_t^2 , $\hat{\sigma}_1^2$, that it is often set equal to the estimate of the marginal variance. Given that the presence of outliers affects the estimation of the autocorrelation function of the squared observations, the closed-form estimators of the parameters are affected and consequently the estimate of the marginal variance (see Carnero et al., 2007, for more details). In the simulations reported in Tables 1–3, we observe that, in general, closed-form estimators tend to overestimate α_0 and α_1 and underestimate β_1 , as the ML parameter estimators. Yet, for moderate and large sample sizes, the closed-form estimators estimate better the marginal variance. So, we expected that (a) the closed-form estimators and, in particular, the robust closed-form estimator would perform the best in estimating the volatility for these sample sizes and (b) the expected error in the estimation of σ_t^2 is positive at the moment that occurs the outlier. In fact, at period $\tau + 1$, y_τ^{*2} is large but σ_τ^2 is still not affected by the outlier and given the parameter biases, we expect that the error ϵ_τ is positive.

The second experiment provides evidence on this issue by generating 1000 series of size 1000 with parameter values similar to those used in the first Monte Carlo experiment. The series are contaminated at $t = 500$ and we consider isolated ALOs and patches of ALOs of size $\omega_{AO} = 0, 5, 10$ and 15 standard deviations of the original simulated series.

In Figure 1, we plot the Monte Carlo means of the volatility biases ($\hat{\sigma}_t - \sigma_t$). From the Figure we observe that all robust methods are better in estimating the volatility than the *QMLE*. The closed-form estimator of Kristensen and Linton (2006) presents also small volatility biases than the *QMLE*, and the *QMLE*– t performs better than the procedure proposed by Muler and Yohai (2008) (see Carnero et al., 2012, for similar results). Finally, the robust closed-form estimator performs the best in the presence of ALOs and patches of ALOs, except in one situation when the patch of ALOs is of size 15. In this case, the volatility estimated by the *QMLE* – t reacts less around the location of the patch of outliers. However, when the sample size increases till $T = 5000$ the robust closed-form estimator beats all the estimators in estimating the volatility.⁴

⁴Results are available from the authors upon request.

Table 1
Finite sample properties of the estimators

Monte Carlo means, standard deviations of the parameter and marginal variance (mv) estimates for the GARCH(1,1) model and mean square error (MSE). True parameters are: $\alpha_0 = 0.1$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$ and $mv = \alpha_0/(1 - \alpha_1 - \beta_1) = 1$. KL stands for the closed-form estimator proposed by Kristensen and Linton (2006), MY for the robust estimator proposed by Muler and Yohai (2008), $R_{TZ}KL$ for the robust closed-form estimator using the proposal of Teräsvirta and Zhao (2011) and $QMLE - t$ for the robust QMLE-t proposed by Carnero et al. (2012).

T		Estimation methods										
		$QMLE$	KL	MY	$R_{TZ}KL$	$QMLE - t$	MSE	MSE	MSE	MSE	MSE	
1 ALO size $5\sigma_y$	500	α_0	0.001	0.010	0.396	0.259	0.231	0.075	0.265	0.173	0.136	0.009
		α_1	0.104	0.056	0.045	0.213	0.132	0.079	0.041	0.163	0.112	0.014
		β_1	0.698	0.056	0.577	0.213	0.666	0.079	0.694	0.163	0.765	0.014
		mv	18.554	>1000	1.061	0.024	1.367	13.461	1.040	0.021	1.161	0.076
	1000	α_0	-0.000	0.010	0.263	0.112	0.155	0.025	0.122	0.050	0.119	0.004
		α_1	0.101	0.016	0.056	0.113	0.126	0.026	0.046	0.061	0.106	0.007
		β_1	0.762	0.016	0.687	0.113	0.746	0.026	0.833	0.061	0.785	0.007
		mv	8.585	>1000	1.031	0.010	1.373	0.757	1.018	0.009	1.122	0.050
	5000	α_0	0.000	0.010	0.141	0.022	0.114	0.001	0.085	0.005	0.108	0.001
		α_1	0.100	0.001	0.076	0.034	0.116	0.001	0.076	0.011	0.106	0.002
		β_1	0.795	0.001	0.783	0.034	0.792	0.001	0.840	0.011	0.793	0.002
		mv	0.000	1.004	1.006	0.002	1.260	0.089	1.001	0.002	1.078	0.015
1 ALO size $10\sigma_y$	500	α_0	0.002	0.010	0.716	0.672	0.230	0.075	0.579	0.520	0.142	0.011
		α_1	0.144	0.183	0.113	0.407	0.133	0.078	0.094	0.361	0.115	0.014
		β_1	0.513	0.183	0.300	0.407	0.667	0.078	0.402	0.361	0.766	0.014
		mv	120.189	>1000	1.226	0.078	1.361	12.670	1.166	0.052	1.269	0.149
	1000	α_0	-0.001	0.010	0.481	0.347	0.154	0.023	0.264	0.189	0.119	0.004
		α_1	0.124	0.095	0.031	0.244	0.126	0.026	0.025	0.161	0.108	0.005
		β_1	0.634	0.095	0.531	0.244	0.747	0.026	0.723	0.161	0.788	0.005
		mv	-52.966	>1000	1.113	0.024	1.383	0.767	1.082	0.017	1.177	0.063
	5000	α_0	0.000	0.010	0.165	0.034	0.114	0.001	0.082	0.006	0.107	0.001
		α_1	0.102	0.002	0.043	0.037	0.116	0.001	0.045	0.015	0.105	0.002
		β_1	0.785	0.002	0.795	0.037	0.792	0.001	0.874	0.015	0.796	0.002
		mv	-0.001	1.006	1.022	0.002	1.265	0.093	1.015	0.002	1.091	0.018
1 ALO size $15\sigma_y$	500	α_0	0.002	0.012	0.790	0.929	0.232	0.077	0.734	0.806	0.148	0.012
		α_1	0.211	0.259	0.197	0.423	0.133	0.080	0.185	0.434	0.118	0.015
		β_1	0.406	0.259	0.272	0.423	0.664	0.080	0.276	0.434	0.763	0.015
		mv	1260.833	>1000	1.490	0.281	1.354	13.124	1.363	0.166	1.335	0.231
	1000	α_0	-0.001	0.011	0.716	0.686	0.156	0.024	0.575	0.521	0.120	0.004
		α_1	0.167	0.203	0.100	0.397	0.126	0.026	0.075	0.344	0.109	0.005
		β_1	0.491	0.203	0.323	0.397	0.745	0.026	0.432	0.344	0.789	0.005
		mv	793.996	>1000	1.246	0.075	1.384	0.789	1.181	0.046	1.207	0.073
	5000	α_0	0.000	0.010	0.215	0.066	0.114	0.001	0.081	0.009	0.106	0.001
		α_1	0.108	0.007	0.017	0.053	0.116	0.001	0.018	0.021	0.105	0.002
		β_1	0.766	0.007	0.778	0.053	0.791	0.001	0.904	0.021	0.798	0.002
		mv	-0.003	1.010	1.049	0.004	1.264	0.093	1.035	0.003	1.100	0.019

Table 2
Finite sample properties of the estimators

Monte Carlo means, standard deviations of the parameter and marginal variance (mv) estimates for the GARCH(1,1) model and mean square error (MSE). True parameters are: $\alpha_0 = 0.1$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$ and $mv = \alpha_0/(1 - \alpha_1 - \beta_1) = 1$. KL stands for the closed-form estimator proposed by Kristensen and Linton (2006), MY for the robust estimator proposed by Muler and Yohai (2008), $R_{TZ}KL$ for the robust closed-form estimator using the proposal of Teräsvirta and Zhao (2011) and $QMLE - t$ for the robust QMLE-t proposed by Carnero et al. (2012).

T		Estimation methods										
		$QMLE$	KL		MY	$R_{TZ}KL$		$QMLE - t$				
			MSE	MSE	MSE	MSE	MSE	MSE	MSE	MSE	MSE	
3 ALOs size $5\sigma_y$	500	α_0	0.000	0.010	0.588	0.501	0.257	0.096	0.494	0.426	0.155	0.015
		α_1	0.100	0.130	0.074	0.323	0.135	0.096	0.062	0.292	0.120	0.016
		β_1	0.609	0.130	0.425	0.323	0.640	0.096	0.505	0.292	0.761	0.016
		mv	-1.486	>1000	1.193	0.063	1.425	3.128	1.161	0.050	1.405	0.337
	1000	α_0	0.000	0.010	0.365	0.216	0.159	0.027	0.198	0.127	0.125	0.005
		α_1	0.098	0.040	0.037	0.165	0.130	0.028	0.031	0.115	0.109	0.007
		β_1	0.716	0.040	0.626	0.165	0.740	0.028	0.779	0.115	0.785	0.007
		mv	0.002	1.025	1.097	0.020	1.399	0.325	1.079	0.016	1.225	0.081
	5000	α_0	0.000	0.010	0.146	0.026	0.114	0.001	0.084	0.006	0.108	0.001
		α_1	0.099	0.001	0.065	0.036	0.122	0.001	0.064	0.013	0.106	0.002
		β_1	0.791	0.001	0.791	0.036	0.789	0.001	0.853	0.013	0.796	0.002
		mv	0.001	1.001	1.019	0.002	1.314	0.126	1.014	0.002	1.099	0.018
3 ALOs size $10\sigma_y$	500	α_0	0.001	0.012	0.887	1.219	0.259	0.098	0.811	1.065	0.169	0.017
		α_1	0.143	0.260	0.243	0.448	0.135	0.099	0.242	0.455	0.132	0.015
		β_1	0.421	0.260	0.235	0.448	0.637	0.099	0.247	0.455	0.761	0.015
		mv	-3666.542	>1000	1.691	0.532	1.424	2.874	1.582	0.385	1.819	1.474
	1000	α_0	-0.001	0.011	0.765	0.801	0.158	0.024	0.673	0.673	0.134	0.006
		α_1	0.134	0.250	0.118	0.400	0.130	0.027	0.084	0.360	0.117	0.007
		β_1	0.430	0.250	0.314	0.400	0.741	0.027	0.391	0.360	0.782	0.007
		mv	1711.356	>1000	1.346	0.136	1.399	0.326	1.291	0.099	1.397	0.254
	5000	α_0	0.000	0.010	0.206	0.069	0.114	0.001	0.082	0.010	0.107	0.001
		α_1	0.106	0.007	0.022	0.057	0.123	0.001	0.022	0.022	0.106	0.001
		β_1	0.758	0.007	0.785	0.057	0.789	0.001	0.900	0.022	0.799	0.001
		mv	0.002	1.000	1.069	0.007	1.327	0.136	1.057	0.005	1.132	0.026
3 ALOs size $15\sigma_y$	500	α_0	0.002	0.014	0.873	1.629	0.263	0.102	0.788	1.472	0.188	0.024
		α_1	0.123	0.214	0.375	0.414	0.135	0.102	0.420	0.466	0.140	0.017
		β_1	0.451	0.214	0.271	0.414	0.633	0.102	0.230	0.466	0.755	0.017
		mv	6009.059	>1000	2.491	2.346	1.433	3.219	2.256	1.674	2.136	2.413
	1000	α_0	-0.003	0.013	0.914	1.284	0.159	0.026	0.848	1.131	0.140	0.007
		α_1	0.165	0.301	0.244	0.448	0.130	0.028	0.221	0.442	0.122	0.007
		β_1	0.351	0.301	0.234	0.448	0.740	0.028	0.260	0.442	0.780	0.007
		mv	-5318.376	>1000	1.746	0.585	1.399	0.326	1.627	0.418	1.506	0.345
	5000	α_0	0.001	0.010	0.366	0.240	0.114	0.001	0.147	0.085	0.107	0.001
		α_1	0.123	0.048	0.007	0.148	0.123	0.001	0.008	0.074	0.106	0.001
		β_1	0.678	0.048	0.673	0.148	0.789	0.001	0.859	0.074	0.800	0.001
		mv	0.002	1.017	1.149	0.024	1.328	0.136	1.125	0.018	1.151	0.030

Table 3
Finite sample properties of the estimators

Monte Carlo means, standard deviations of the parameter and marginal variance (mv) estimates for the GARCH(1,1) model and mean square error (MSE). True parameters are: $\alpha_0 = 0.1$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$ and $mv = \alpha_0/(1 - \alpha_1 - \beta_1) = 1$. KL stands for the closed-form estimator proposed by Kristensen and Linton (2006), MY for the robust estimator proposed by Muler and Yohai (2008), $R_{TZ}KL$ for the robust closed-form estimator using the proposal of Teräsvirta and Zhao (2011) and $QMLE - t$ for the robust QMLE-t proposed by Carnero et al. (2012).

T		Estimation methods										
		$QMLE$		KL		MY		$R_{TZ}KL$		$QMLE - t$		
			MSE		MSE		MSE		MSE		MSE	
1 patch of size $5\sigma_y$	500	α_0	0.001	0.010	0.563	0.240	0.356	0.137	0.620	0.415	0.160	0.017
		α_1	0.180	0.157	0.428	0.511	0.176	0.156	0.238	0.442	0.130	0.020
		β_1	0.484	0.157	0.098	0.511	0.515	0.156	0.223	0.442	0.748	0.020
		mv	-0.005	1.067	1.191	0.062	1.390	20.906	1.160	0.050	1.437	0.392
	1000	α_0	0.000	0.010	0.482	0.166	0.204	0.039	0.473	0.219	0.129	0.006
		α_1	0.143	0.047	0.377	0.411	0.143	0.045	0.261	0.342	0.115	0.008
		β_1	0.645	0.047	0.182	0.411	0.685	0.045	0.299	0.342	0.777	0.008
		mv	-0.001	1.015	1.096	0.020	1.415	12.883	1.078	0.016	1.242	0.104
	5000	α_0	0.000	0.010	0.330	0.068	0.121	0.001	0.277	0.039	0.108	0.001
		α_1	0.108	0.002	0.211	0.138	0.121	0.002	0.191	0.082	0.106	0.002
		β_1	0.776	0.002	0.465	0.138	0.782	0.002	0.535	0.082	0.794	0.002
		mv	-0.001	1.005	1.019	0.002	1.268	0.092	1.014	0.002	1.099	0.017
1 patch of size $10\sigma_y$	500	α_0	0.001	0.012	0.844	0.576	0.294	0.121	0.883	0.844	0.182	0.024
		α_1	0.257	0.338	0.496	0.635	0.140	0.127	0.307	0.526	0.144	0.020
		β_1	0.296	0.338	0.003	0.635	0.600	0.127	0.134	0.526	0.743	0.020
		mv	-1397.157	>1000	1.686	0.523	1.506	20.955	1.578	0.380	1.838	1.272
	1000	α_0	-0.001	0.011	0.660	0.325	0.169	0.029	0.731	0.568	0.137	0.007
		α_1	0.188	0.179	0.501	0.630	0.132	0.033	0.274	0.507	0.122	0.008
		β_1	0.465	0.179	0.007	0.630	0.730	0.033	0.160	0.507	0.776	0.008
		mv	0.001	1.047	1.344	0.135	1.499	12.989	1.289	0.099	1.415	0.256
	5000	α_0	0.000	0.010	0.509	0.172	0.118	0.001	0.485	0.167	0.107	0.001
		α_1	0.115	0.004	0.489	0.590	0.127	0.002	0.409	0.475	0.107	0.001
		β_1	0.752	0.004	0.034	0.590	0.783	0.002	0.132	0.475	0.798	0.001
		mv	-0.002	1.008	1.069	0.007	1.350	0.148	1.057	0.005	1.135	0.027
1 patch of size $15\sigma_y$	500	α_0	0.001	0.017	1.247	1.355	0.279	0.113	1.228	1.607	0.201	0.031
		α_1	0.373	0.414	0.497	0.640	0.135	0.115	0.363	0.560	0.153	0.022
		β_1	0.225	0.414	0.000	0.640	0.619	0.115	0.091	0.560	0.737	0.022
		mv	812.080	> 1000	2.479	2.301	1.545	25.414	2.247	1.648	2.185	2.613
	1000	α_0	-0.001	0.012	0.866	0.598	0.162	0.029	0.888	0.854	0.143	0.008
		α_1	0.251	0.302	0.502	0.639	0.130	0.030	0.322	0.525	0.126	0.008
		β_1	0.341	0.302	0.001	0.639	0.739	0.030	0.133	0.525	0.774	0.008
		mv	75.501	> 1000	1.741	0.578	1.521	14.888	1.624	0.414	1.533	0.404
	5000	α_0	0.000	0.010	0.569	0.222	0.115	0.001	0.560	0.285	0.108	0.001
		α_1	0.124	0.012	0.504	0.639	0.129	0.002	0.361	0.513	0.107	0.001
		β_1	0.723	0.012	0.001	0.639	0.785	0.002	0.139	0.513	0.799	0.001
		mv	-0.004	1.015	1.148	0.024	1.386	0.170	1.124	0.018	1.157	0.034

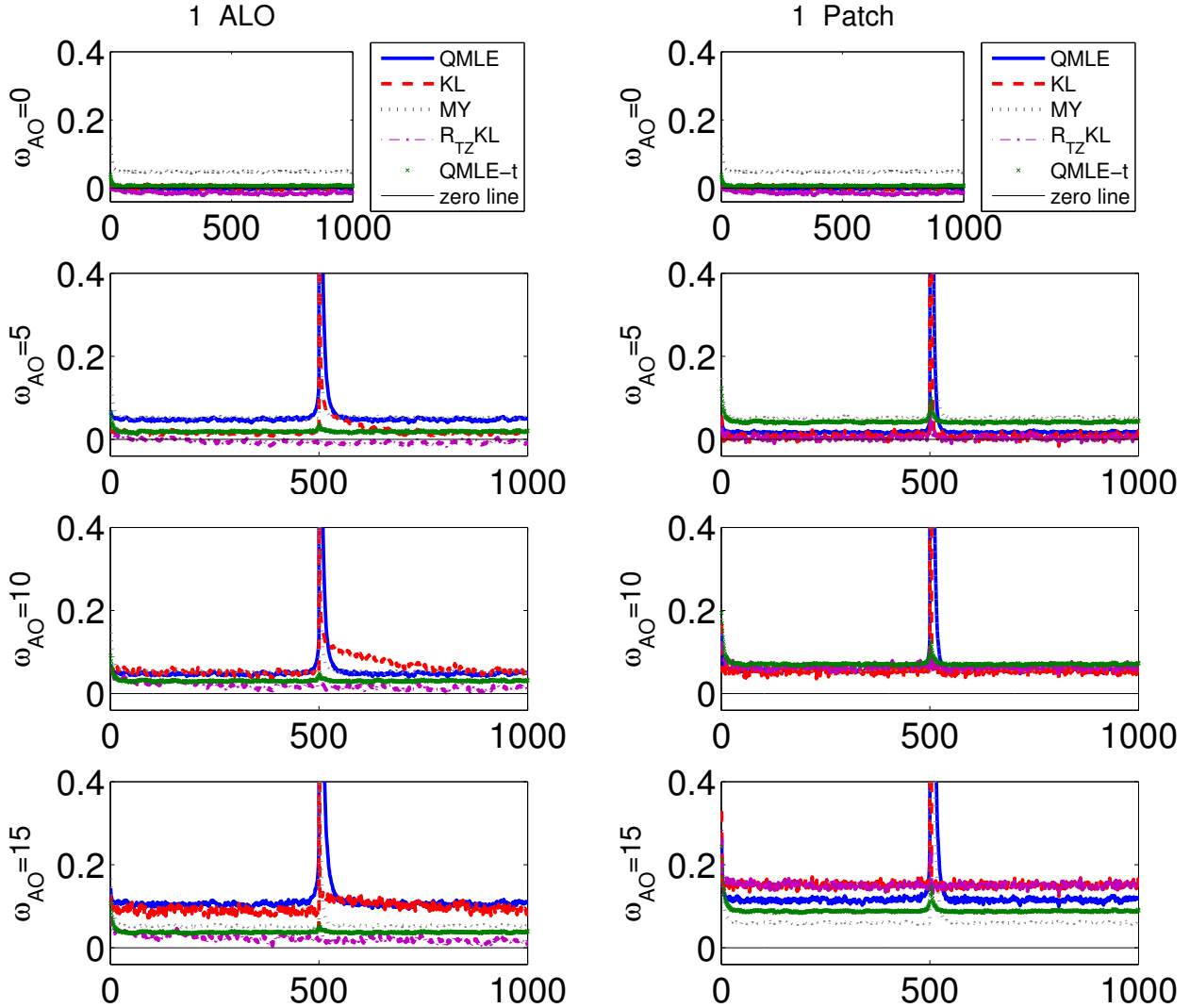


Figure 1. Monte Carlo means of the biases in the volatility estimated using a GARCH(1,1) model fitted to series of size $T=1000$.

Therefore, our findings reinforce the conclusions of Carnero et al. (2012) that the estimation methods that lead to accurate volatility estimates are those that estimate better the marginal variance.

5. Empirical application

In this section we analyze one daily financial time series of returns to illustrate the different volatility estimates of the volatility under the methods analyzed before via simulation. The series considered is the Nasdaq composite index. The data was collected from Yahoo Finance website (<http://finance.yahoo.com>) and spans the period of January 2, 1987–November 25, 2014.

Figure 2 depicts the return series, $y_t = (\log p_t - \log p_{t-1}) \cdot 100$, where p_t is the value at time t of the corresponding index and Table 4 reports some summary statistics and the results of the

Kiefer and Salmon (1983) test, which is a formal test of normality in the context of conditional heteroscedastic series.⁵ The test confirms the non Gaussianity of the two return series.

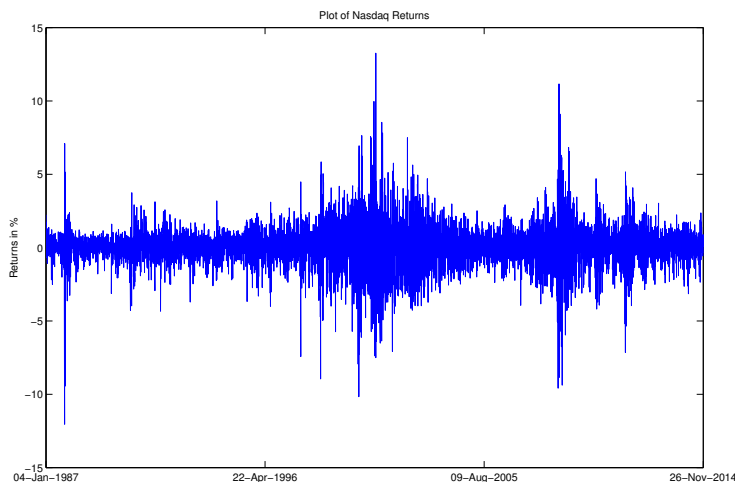


Figure 2. Returns in percentage of the Nasdaq composite index.

Table 4
Descriptive statistics

Stock index returns	Nasdaq
Mean	0.0370
Variance	2.1168
Skewness	-0.2323***
Kurtosis	10.5421***
KS_S	-20.2497
KS_K	640.7303

*** means that the skewness and kurtosis are significant at all relevant levels of significance.

Looking at Figure 2 and Table 4, we may appreciate several extreme observations that can be the cause of excess of kurtosis presented by the return series.

⁵The Kiefer and Salmon (1983) test is given by $KS_N = (KS_S)^2 + (KS_K)^2$, where $KS_S = \sqrt{\frac{T}{6}} \left[\frac{1}{T} \sum_{t=1}^T y_t^3 - \frac{3}{T} \sum_{t=1}^T y_t \right]$, $KS_K = \sqrt{\frac{T}{24}} \left[\frac{1}{T} \sum_{t=1}^T y_t^4 - \frac{6}{T} \sum_{t=1}^T y_t^2 + 3 \right]$ and y_t are the standardized returns. If the distribution of y_t is conditional $N(0, 1)$, then KS_S and KS_K are asymptotically $N(0, 1)$ and KS_N is asymptotically $\chi^2(2)$.

Table 5 reports the estimation of the parameters and the in-sample MSE and MAE of the volatility obtained with each of the estimation methods. The method proposed by Kristensen and Linton (2006) and its robustification provide quite similar parameters, although the second reports low estimated values for α_0 and α_1 and a high estimate for β_1 . Yet, the estimated marginal variance of y_t is slightly smaller for the robust closed-form estimator. In order to compare the models' goodness-of-fit, we calculate the in-sample MSE and MAE of the volatility assuming the squared returns as a proxy of the true nonobservable σ_t^2 . Regarding the in-sample MSE, the robustified closed-form estimator provides the smallest value of this measure followed by the closed-form estimator of Kristensen and Linton (2006). Yet, if we compare the in-sample MAEs, we observe that the closed-form estimator of Kristensen and Linton (2006) provides the smallest MAE followed closely by its robustified version. Therefore, we may conclude that the closed-form estimators provide the best fit to the data and the QMLE the worst.

Table 5
Estimation of the GARCH(1,1) parameters

Parameters	Estimation methods				
	<i>QMLE</i>	<i>KL</i>	<i>MY</i>	<i>R_{TZ}KL</i>	<i>QMLE - t</i>
α_0	0.0189	0.0021	0.0037	0.0018	0.0094
α_1	0.1105	0.0265	0.0676	0.0245	0.0900
β_1	0.8816	0.9725	0.9308	0.9746	0.9082
MSE	21.4321	15.5273	17.5932	14.9189	16.9960
MAE	3.9026	3.6680	3.8268	3.6875	3.7977

MSE and MAE are the in-sample mean squared error and the mean absolute error of the volatility, respectively. The nonobservable σ_t^2 is proxied by the squared returns.

On the other hand, when we consider the QMLE based estimators, the estimates of α_0 and α_1 are larger for the basic *QMLE* in comparison to the robust *QMLE* based estimators, while the estimate of β_1 is smaller.

Figure 3 depicts the estimated volatilities and the biases regarding the volatility obtained with the robust closed-form estimator. Looking at the Figure, we observe that the estimators *KL* and *R_{TZ}KL* are those that provide smaller estimates of the volatility, being the volatility of the second slightly smaller (see panel seven of Figure 3). The volatility estimated by the basic *QMLE* is the largest. The other robust *QMLE* based estimators provide smaller estimates of the volatility but larger than those of the closed-form estimators.

The implications of these results are important either for options pricing or risk management given that estimating volatility with robust estimators leads to smaller volatility estimates and consequently lower risk. In the case of options pricing lower volatility estimates indicate smaller expected fluctuations in underlying price levels and consequently lower option premiums for puts and calls.

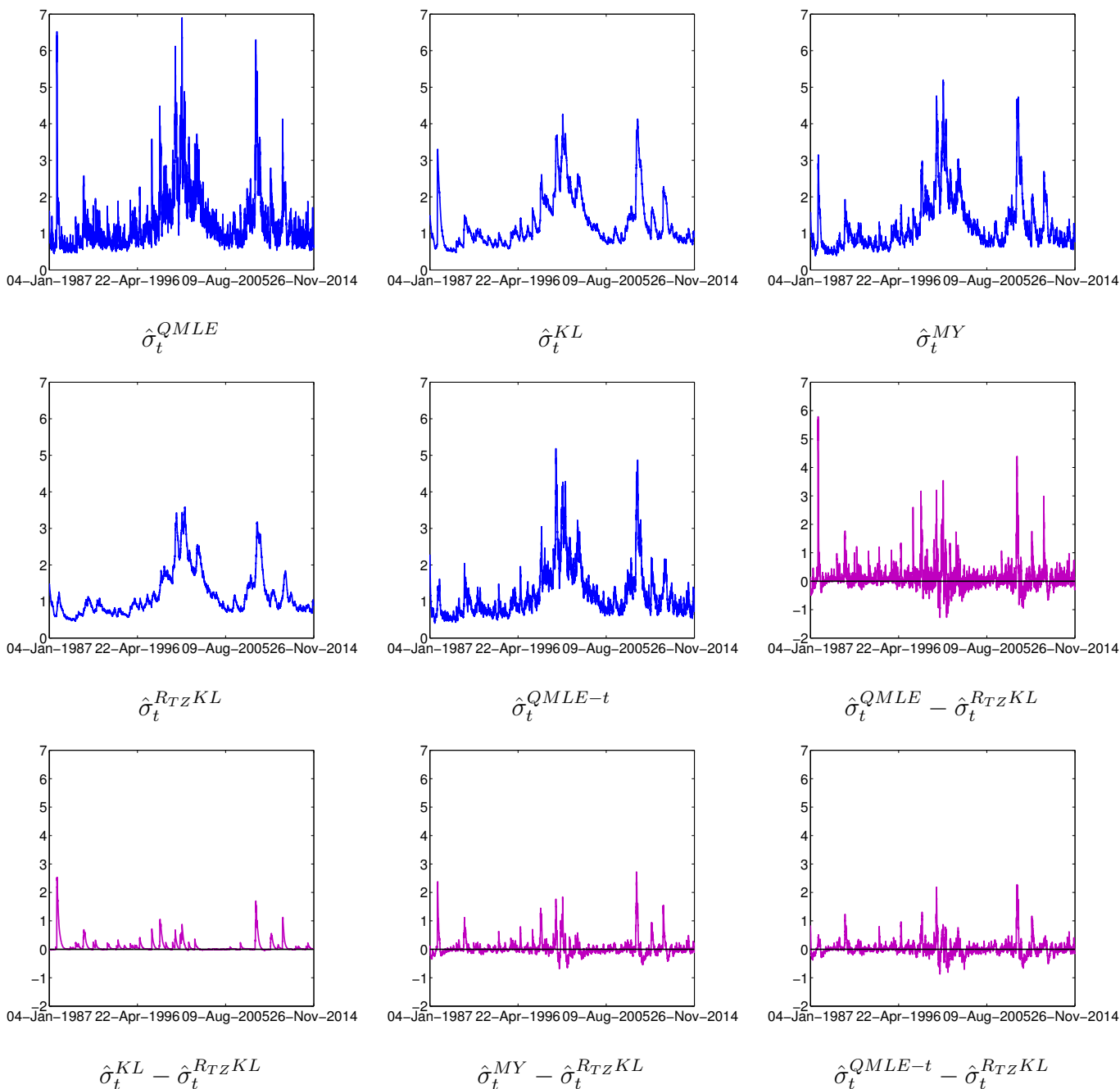


Figure 3. Estimated volatilities $\hat{\sigma}_t$ and biases.

5.1. Forecasting performance

In this subsection we perform an out-of-sample comparison of several GARCH(1,1) estimation proposals to calculate the one-day-ahead VaR of the daily Nasdaq returns. We split the sample in an in-sample period that ranges from January 2, 1987 to May 8, 2002 and an out-of sample period that spans the period May 9, 2002 to September 19, 2006. We obtain 1100 one-day-ahead

VaR forecasts as

$$VaR_{t+1|t}^\alpha = \widehat{\sigma}_{t+1|t} q_\alpha, \quad (19)$$

with $\alpha = 1\%$, q_α is the 1% quantile of the standard Normal distribution and $\widehat{\sigma}_{t+1|t}$ is the one-day-ahead volatility forecast. We use a recursive expanding window to calculate the VaRs.

In order to evaluate the performance of the methods in forecasting the VaR, Table 6 reports the failure rates for the 1100 VaR forecasts, the p-value of the conditional coverage test by Christoffersen (1998)⁶ and the p-value of the dynamic quantile test (DQtest) by Engle and Manganelli (2004)⁷. Since the calculation of the empirical failure rate defines a sequence of ones (VaR violation) and zeros (no VaR violation), we can test if the theoretical failure rate, f , is

⁶The first tests for the unconditional coverage (denoted LR_{uc}) and it is a standard likelihood ratio test (known also as Kupiec (1995)'s test) given by

$$LR_{uc} = -2\log [L(p; I_1, I_2, \dots, I_n) / L(\widehat{\pi}; I_1, I_2, \dots, I_n)] \overset{asy}{\sim} \chi^2(1),$$

where $\{I_t\}_{t=1}^n$ is the indicator sequence, p is the theoretical coverage, $\widehat{\pi} = n_1 / (n_0 + n_1)$ is the maximum likelihood estimate of the alternative failure rate π , n_0 is the number of zeros and n_1 is the number of ones in the sequence $\{I_t\}_{t=1}^n$.

The second tests for the independence part of the conditional coverage hypothesis (denoted LR_{ind}) and it is also a likelihood ratio test

$$LR_{ind} = -2\log [L(\widehat{\Pi}_2; I_1, I_2, \dots, I_n) / L(\widehat{\Pi}_1; I_1, I_2, \dots, I_n)] \overset{asy}{\sim} \chi^2(1),$$

where

$$\widehat{\Pi}_1 = \begin{pmatrix} n_{00}/(n_{00} + n_{01}) & n_{01}/(n_{00} + n_{01}) \\ n_{10}/(n_{10} + n_{11}) & n_{11}/(n_{10} + n_{11}) \end{pmatrix}, \quad \widehat{\Pi}_2 = \begin{pmatrix} 1 - \widehat{\pi}_2 & \widehat{\pi}_2 \\ 1 - \widehat{\pi}_2 & \widehat{\pi}_2 \end{pmatrix},$$

n_{ij} is the number of observations with value i followed by j and $\widehat{\pi}_2 = (n_{01} + n_{11}) / (n_{00} + n_{10} + n_{01} + n_{11})$. Finally, the third is a joint test of coverage and independence (denoted LR_{cc}) given by:

$$LR_{cc} = -2\log [L(p; I_1, I_2, \dots, I_n) / L(\widehat{\Pi}_1; I_1, I_2, \dots, I_n)] \overset{asy}{\sim} \chi^2(1).$$

⁷For computing Engle and Manganelli (2004)'s Dynamic Quantile test, $H_t(\alpha)$ is defined as $H_t(\alpha) = I_t(\alpha) - \alpha$ where $I(\alpha)$ is a vector composed by ones (VaR violations) and zeros (VaR no violations).

By the definition of VaR, we expect that the conditional expectation of $H_t(\alpha)$ given the past information must be zero. This assumption can be tested with the following linear regression model:

$$H_t(\alpha) = \beta_0 + \sum_{i=1}^P \beta_i H_{t-i}(\alpha) + \sum_{j=1}^K \gamma_j g_j(z_{t-j}) + \varepsilon_t, \quad (20)$$

where ε_t is an i.i.d process with zero mean and $g(\cdot)$ is a function of the past exceedances and of variable z_t .

Consider $H_0 : \beta_0 = \beta_1 = \dots = \beta_P = \gamma_1 = \dots = \gamma_K = 0$, and denote $\Psi = (\beta_0, \beta_1, \dots, \beta_P, \gamma_1, \dots, \gamma_K)^T$ the vector of the $P + K + 1$ parameters of the model. The statistics test are given by

$$DQ = \frac{\widehat{\Psi}^T X^T X \widehat{\Psi}}{\alpha(1 - \alpha)} \xrightarrow{\mathcal{L}} \chi^2(P + K + 1)$$

where X denotes the covariates matrix in equation (20). In our study, we select $P = 4$, $K = 4$ and $g(z_t) = VaR_t$ to account the influence of past exceedances up to four days (see Chen and Lu, 2012, for more details).

equal to 1%, i.e., $H_0 : f = 1\%$ vs. $H_1 : f \neq 1\%$. According to Christoffersen (1998), testing for conditional coverage is important in the presence of higher order dynamics and this author proposed a procedure that is composed of three tests.

Table 6
Failure rates and p -values for the null hypotheses $f = 1\%$

	Failure rates	LR_{uc}	LR_{ind}	LR_{cc}	DQtest
<i>QMLE</i>	0.019	0.007	0.366	0.017	0.006
<i>KL</i>	0.014	0.251	0.519	0.414	0.067
<i>MY</i>	0.019	0.007	0.366	0.017	0.002
<i>R_{TZ}KL</i>	0.015	0.092	0.465	0.183	0.072
<i>QMLE - t</i>	0.022	0.001	0.301	0.002	0.000

LR_{uc} , LR_{ind} , LR_{cc} stand for the LR test of unconditional coverage, the LR test of independence and the joint test of coverage and independence, respectively.

Looking at Table 6, we conclude that the VaR forecasts that are closer to the 1% nominal value are those obtained with the closed-form estimators. The VaR forecasts obtained with QMLE based methods tend to overreject and therefore the null hypotheses of the Christoffersen (1998) and Engle and Manganelli (2004) tests are rejected. The main conclusion is that the closed-form estimators seem to perform quite well in forecasting the volatility and the Value-at-Risk.

6. Conclusion

In the financial econometrics literature, it is well known that outliers affect the estimation of parameters and volatilities when using the traditional GARCH model and several robust alternatives have been actively investigated. All of them are based on QMLE methods and therefore are based on the use of numerical optimization procedures and starting values which might lead to different parameter and volatilities estimates.

In this paper we extend the closed-form estimator of the GARCH(1,1) proposed by Kristensen and Linton (2006) for dealing with additive level outliers by replacing the estimators of the sample autocorrelations by robust estimators of these autocorrelations. Moreover, we also use robust filters that exist in the literature to estimate the underlying volatility.

The Monte Carlo experiments together with the empirical application show that the closed-form estimators and in particular the robust closed-form estimator are more robust in terms of volatility estimation and Value-at-Risk forecasting than the basic ML estimator and some based robust QMLE estimators.

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