Stock Exchange Competition in a Simple Model of Capital Market Equilibrium*

Sofia B. Ramos† Ernst-Ludwig von Thadden‡

Final version December 2007

Abstract

This paper uses a simple model of mean-variance capital markets equilibrium with proportional transactions costs to analyze the competition of stock markets for investors. We assume that equity trading is costly and endogenize transactions costs as variables strategically influenced by stock exchanges. Among other things, the model predicts that increasing financial market correlation leads to a decrease of transaction costs, an increase in cross-border trading activity, and to a decrease in the home bias of international equity flows. These predictions are consistent with the recent evolution of international stock markets.

JEL classification: G11, G15, G29.

Keywords: Stock Exchange Competition, Capital Markets Equilibrium, Transactions Costs, Home Bias, Cross-border Equity Flows.

---

*We thank Michael Brennan, Vihang Errunza, Thierry Foucault, Hong Liu, René Stulz and an anonymous referee for helpful comments. Ramos acknowledges financial support from Fundacao para a Ciencia e Tecnologia. Von Thadden acknowledges support from the Swiss National Science Foundation through the NCCR "FinRisk" and from Deutsche Forschungsgemeinschaft through SFB 504.

†ISCTE-Business School and CEMAF, e-mail: sofia.ramos@iscte.pt.

‡Universität Mannheim and CEPR, e-mail: vthadden@pool.uni-mannheim.de

1 Introduction

The objective of this paper is to analyze the competition between stock exchanges in the framework of asset pricing theory. We do this by considering a simple mean-variance capital market equilibrium model with transactions costs and by endogenizing the transactions costs as variables strategically influenced by stock exchanges. This perspective integrates insights from the asset pricing and the industrial organization literature and thus brings together two approaches to the study of stock exchanges that have evolved largely independently up to now.

We use this framework to investigate the determinants of transactions costs and trading volume for competing stock exchanges. Starting in the mid-1980s with the London Stock Exchange, European stock exchanges began a process of liberalization, which led to more profit-oriented organizations and strategies across Europe, and ultimately to serious competition between European stock markets. With the advent of cross-listings of European firms on the NYSE and Nasdaq and the continuing debate of the optimal trading structure of the American exchanges, this competition went global in the 1990s.[1]

Up to now, the literature has analyzed competition between stock exchanges as competition for the listing of firms (prominent examples of this literature are Chemmanur and Fulghieri (2006), Foucault and Parlour (2004), and Huddart, Hughes and Brunnermeier (1999)). This type of competition has indeed become more important since the 1990s, in particular for large firms (see Pagano, Roell, and Zechner (2002)). However, at least as important is stock exchange competition for investors. Transactions fees, market liquidity, disclosure rules, and the characteristics of the firms listed on the exchange are all important determinants of the attractiveness of a stock exchange to investors. Stock exchanges are therefore in principle subject to two-sided competition in the sense of Rochet and Tirole (2004): the more attractive it is for firms to list on the exchange, the more attractive it is for investors to trade on this exchange, and vice versa.

As a first step towards a fuller analysis of such two-sided competition, the present paper analyzes stock exchange competition for investors, taking the listing decisions of firms as given. Interestingly, Table I shows that, at least until recently, the vast majority of listed firms around the globe has

[1] See, for example, McKinsey-JPMorgan (2002)

listed on a local stock exchange. In fact, in 2002 the only major European stock exchange with large foreign turnover was London. Table I also shows that while the share of foreign firms listed on some stock exchange can reach 35 % of total listings in numbers (in Switzerland with 140 foreign listings out of 398), the total value of foreign share trading on all exchanges (except London) is negligible.

Table I
Value of Shares Traded and Number of Companies Listed for Selected Stock Exchanges

Note: Data for 2002, main and parallel markets. Remaining percentages are investment funds.
Source: World Federation of Exchanges.

	Total Value	of Trading	Number of	Listed Comp.
	Domestic	Foreign	Domestic	Foreign
Euronext	98%	1%	1114	N.A.
Frankfurt	92%	8%	715	219
Hong Kong	100%	0%	968	10
Milan	91%	9%	288	7
London	47%	53%	1890	382
Nasdaq	96%	3%	3268	381
NYSE	91%	7%	1894	472
Madrid	99%	1%	2986	29
Tokyo	99%	0%	2119	34
Zurich	97%	2%	258	140

In line with the figures in Table I, we therefore model stock exchanges as trading platforms for local assets and analyze their competition for investors who wish to diversify their portfolios across those assets. While our abstract model is a priori more general, it is particularly interesting to use it to investigate the determinants of international stock exchange competition for two reasons. First, global cross-border portfolio investment has increased substantially since the late 1980s.[2] As documented by Tesar and Werner

[2] The best documentation and analysis of the strong increase in international portfolio flows is by Portes and Rey (2005), but their sample ends in 1996. Less detailed evidence is regularly provided in the IMF's *Capital Markets Reports* and in the various national balance of payments statistics.

(1998) and the IMF's Coordinated Portfolio Investment Surveys (see Table A1 in the appendix), the percentage equity holdings of investors in domestic equity (the home bias) decreased on average from well above 90 percent in the late 1980s to below 70 percent in 2002. Second, almost all countries have witnessed a process of concentration of stock exchanges towards one national exchange, and sometimes (as in the case of Euronext) even a supranational exchange. Therefore, the trading of foreign equity has typically been channeled through the respective national exchanges, who have been forced to compete vigorously for this order flow.

In our model, stock exchanges charge fees and commissions to profit from this trading. High fees benefit stock exchanges directly, but hurt them indirectly because they distort investors' portfolio choices away from the assets traded on that exchange. The optimal fee size balances these two effects, just as in standard oligopoly theory. What is new in our work is that we explicitly derive the demand for stock exchange services, namely the intermediation of market transactions, in a fully-fledged capital markets model with proportional transactions costs.[3] Because of the relative simplicity of the model, we obtain an explicit solution, which nevertheless is non-trivial and has some surprising features.

One such feature is that in equilibrium those investors who in a world without transactions costs would reduce their exposure to an asset, can be net buyers of this asset. This result is surprising and, as we show, cannot happen in a model with only one single risky asset. In the context of international portfolio allocation, this means that the home bias of local investors may be reinforced by trading under transactions costs. However, as mentioned above, the recent experience in international capital markets has been a systematic erosion of the home bias. Our analysis allows to identify the conditions on the variance-covariance structure of asset returns and endowments that are consistent with this development.

Under these conditions, we find that in the unique subgame-perfect Nash equilibrium of the competition game between stock exchanges, stronger market integration, as measured by an increase in the correlation of local asset payoffs, leads to a decrease in transactions costs. This effect mainly stems from the decreasing demand for international diversification by investors, which erodes foreign exchanges' market power, and is consistent with the

[3] Shy and Tarkka (2001) also endogenize brokerage fees, but in a model with exogenous trading demand.

recent trend in Europe away from cross-country allocation strategies and towards industry-based allocation strategies.[4] Similarly, we predict that stock exchanges of markets that are less well integrated with the rest of the world should have higher transactions costs, which is consistent with the international comparative data provided by Domowitz, Glenn and Madhavan (2001).

We further find that equilibrium fees depend negatively on exogenous trading costs (trading costs that cannot be directly controlled by the stock exchange). Hence, exchanges have an interest in setting rules and using technology that lower these costs. This is, of course, exactly what has happened in the last 10 - 20 years in stock exchanges around the world: exchanges have adopted automated trading mechanisms, improved clearing and settlement procedures, and implemented trading rules that reduce trading costs. This has made these exchanges more attractive and allowed them to reduce trading fees less than they would have otherwise been forced to.[5]

Our work also yields predictions on international equity flows. While an increase in international payoff correlations has a direct negative impact on trading volume because of reduced hedging demand from investors, it also has an indirect positive effect through reduced endogenous stock market fees. The model predicts that transactions costs adjust to accommodate the decreased hedging demand of investors, which in turn stimulates cross-border portfolio investment. As we show, this indirect effect dominates the direct effect, leading to an overall increase in trading volumes and to an erosion of the home bias, without, however, fully eliminating it. This has exactly been the trend in Europe since the 1990s: increasing stock market correlation together with increasing cross-border trading volumes has led to a reduction in the home bias. While this may also be due to factors outside our model, such as increased spillovers in real activity or increased stock market participation, our model is consistent with this observation.

Two branches of the literature are related to this paper: general equilibrium models of asset pricing under transaction costs and models of exchange competition. As mentioned above, the latter literature mainly studies competition for the listing of firms or the trading of securities between different exchanges and thus has a different focus than our work. Relevant issues in that context are economies of scale in trading (Demsetz (1968)), liquidity

[4] See Galati and Tsatsaronis (2001) and Adjaoute and Danthine (2004).

[5] Domowitz (2001) estimates that, all other things equal, average trading costs are lower by 33-46 basis points in markets that are largely automated.

effects (Pagano, 1989), transportation costs (Gehrig (1998)), economies of scope (Pirrong (1999), and network externalities (Di Noia (2001).

Steil (2002) and Domowitz and Steil (2002) have argued that improvements in trading technology in the 1990s, most notably the advent of electronic trading, have facilitated and increased competition between stock exchanges, thus increasing entry and reducing transactions costs. Our work is consistent with this argument, but goes beyond it in at least two respects. First, as stock market transaction costs have a first-order effect on stock market trading, it is important to analyze this impact explicitly. We show that the mechanics of this impact are non-trivial, characterize them, and use the results to evaluate the incentives of stock exchanges to adjust transactions costs. An example of a prediction of our theory that demonstrates the need for a careful analysis is that decreasing transactions costs can lead to decreasing or increasing stock prices.[6] Second, we consider other determinants of market activity and stock exchange competition beyond intermediation costs, such as market correlation, volatility, or the distribution of asset holdings. These factors are at least as important as cost factors and are particularly relevant for general equilibrium considerations. It is also worth noting that increased entry has not necessarily been a dominant factor in international stock market activity, as the opposite, namely stock exchange consolidation, has played an important role in several key markets (such as the creation of Euronext).

In the asset pricing literature, a key contribution has been Vayanos' (1998) mean-variance general equilibrium model of several risky assets with proportional transactions costs in continuous time. In his model, agents are born without assets, buy them when they are young, and sell them off gradually when getting older in order to finance consumption. Hence, at any point in time, there are two groups of agents ready to trade, one with no assets at all and the other with a full portfolio. This structure is not well suited to study questions such as ours, where investors with differently composed non-trivial portfolios want to rebalance their portfolios. By abstracting from life-cycle and similar questions, our simpler one-period model is more flexible and may also be useful as a work-horse for other applications.

Models without life-cycle considerations up to now have mostly focused on settings with one risky asset and one riskless asset. Building on the continuous-time analyses of portfolio choice of Constantinides (1979), Con-

[6] See also Vayanos (1998) for this type of "unconventional" finding.

stantinides (1986) has analyzed equilibrium under proportional transactions costs and finds that while the impact of transactions costs on trading behavior (characterized by a "no-trade region" in endowment space) can be substantial, the impact on asset returns is small, due to adjustments in dynamic trading strategies. Basak and Cuoco (1998) have analyzed equilibrium in a market in which one group of investors is excluded from trading the (one) risky asset, which can be viewed as the limiting case of infinite transactions costs. Recently, progress has been made in the study of portfolio problems with several risky assets (see, in particular, Liu (2004)), but this work does not analyze market equilibrium. Similarly, Dybvig (2005) analyzes a one-period mean-variance portfolio rebalancing problem with proportional transactions costs and obtains instructive explicit solutions. But like Liu (2004), he does not consider market equilibrium. The few papers that study markets with several assets under transactions costs typically assume "variable proportional costs", i.e. transactions costs that are effectively quadratic in the quantity traded.[7] This has the advantage that transactions costs are a second-order effect for small trades and thus that Constantinides' no-trade region disappears. If one is interested in the impact of transactions costs on trading activity, the full first-order effect is, however, important, because transactions costs affect each traded unit equally.

The rest of the paper is organized as follows: Section 2 sets out the model. Section 3 describes the equilibrium in the asset market and Section 4 derives the optimal behavior of stock exchanges. Section 5 delivers the main comparative statics results and discusses empirical evidence. Section 6 concludes. The appendix contains some details of a longer proof and its generalization to the case of more than two risky assets and investors. The appendix also contains a table with data on the international home bias.

2 The Model

The model considers two countries, $i = 1, 2$, with the same currency (i.e. we ignore exchange rate risk). For each country there is one risky asset and one riskless asset. We interpret the risky asset as a representative asset of the economy, similar to a stock market index. We consider two risky assets for mathematical convenience and sketch the analysis for the n-asset case in the

[7] See, e.g., Brennan and Subrahmanyam (1996) or Fernando (2003).

appendix. In this 2-country approach, we interpret one of the two countries as the "reference" country and the other as the rest of the world.

The riskless asset has a gross return normalized to 1 in both countries and can be traded without frictions.[8] Shares of risky assets (stocks) are perfectly divisible and in positive supply ($s_i > 0, i = 1, 2$). There is one round of asset trading and pricing. Let p_i be the price of risky asset i and $\widetilde{F}_i$ its payoff at the end of the period. The (2×1) vector of payoffs $\widetilde{\mathsf{F}}$ is normally distributed with mean μ and variance

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

As usual, we denote by $|\Sigma| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2$ the determinant of Σ, and by $\rho = \sigma_{12}/(\sigma_1\sigma_2)$ the correlation coefficient.

There is a continuum of mass 1 of investors located in the two countries who are identical except for their initial endowments. Investors in country j ("type j investors") hold the amount e_i^j of asset i per capita. The total mass of type-j investors is ω^j ($\omega^1 + \omega^2 = 1$), hence total asset supply is $s_i = \sum_j \omega^j e_i^j$, $i = 1, 2$. We denote the difference of endowments for asset i between investor type 1 and 2 by

$$\Delta_i \equiv e_i^1 - e_i^2. \tag{1}$$

Thus investor type 1 owns more of asset i per capita than investor type 2 iff $\Delta_i > 0$ (but the total amount of asset i in the hands of type 1-investors, $\omega^1 e_i^1$, may be smaller than the total amount in the hand of type 2-investors).

As discussed in the introduction, in practice domestic investors hold more local assets than foreign investors. Table A1 in the appendix provides data illustrating this "home bias". We take this asymmetry of international asset

[8] If the riskless asset represents bonds, this is a simplification. See Biais, Declerck, Dow, Portes, and von Thadden (2006) for a survey of bond market frictions. It is conceptually cleaner to interpret the riskless asset as monetary savings.

holdings as given[9] and assume

$$\Delta_1 > 0, \Delta_2 < 0. \tag{2}$$

Each investor has initial wealth W_0 and exponential utility with coefficient of absolute risk aversion $\theta > 0$. Investors maximize expected utility from final wealth $\widetilde{W^j}$. They can trade assets incurring a proportional transaction cost $\mathsf{T} = (T_1, T_2)'$ in the two assets, and borrowing and short selling is allowed. Denoting the amount of asset i bought by investor j by $t_i^j \in \mathsf{R}$, final wealth is

$$\widetilde{W^j} = W_0 + \mathsf{e}^{j\prime}\widetilde{\mathsf{F}} + \mathsf{t}^{j\prime}\left(\widetilde{\mathsf{F}} - \mathsf{p}\right) - \sum_{i=1}^{2}\left|t_i^j\right| T_i \tag{3}$$

By the normality assumption, preferences are in fact mean-variance, and investor j solves the following problem:

$$\max_{t^j} \Phi(\mathsf{t}^j; \mathsf{p}, \mathsf{T}) \equiv E\widetilde{W^j} - \frac{1}{2}\theta var\widetilde{W^j} \tag{4}$$

For each asset, there is one exchange (which we interpret as the national stock exchange) on which the asset can be traded.

We assume that stock exchanges set trading fees to maximize the revenues from trading that accrue to themselves and their members. Until recently, most stock exchanges were mutuals controlled by the dealers and banks trading on them, whose trading revenue critically depended on the exchange's policies.[10] With the demutualization of many stock exchanges since the 1990s, exchanges have adopted explicit profit maximizing objectives. Table II provides international evidence on stock exchange transactions costs, which include some measures of illiquidity and market impact costs. Aiyagari and Gertler (1991) provide a broader documentation of stock transactions

[9] See Lewis (1999) for an excellent survey of the literature on the home bias. Our analysis can easily be generalized to groups of investors who are not differentiated by their country of residence but by other criteria. For example, we have also analyzed the case of two types of investors, $j = A, B$, where investor A is large in both markets: $\Delta_1 = e_1^A - e_1^B > 0$ and $\Delta_2 > 0$. This specification is less convincing descriptively in the context of international stock market competition, but would be appropriate for the study of specialized national stock exchanges or the competiton between specialist market makers.

[10] See Foucault and Parlour (2004), Domowitz, Glen and Madhavan (2001), and McKinsey-JPMorgan (2002) for a more comprehensive discussion.

costs until the late 1980s, when the recent wave of international portfolio flows took off. To put these figures in perspective, it is useful to remember that in 1998 the revenue from transactions services at the NYSE alone was $165 million.[11]

Table II
One-Way Transactions Costs for Trading Stock in Selected Countries

Data for 2001, 1st quarter, in basis points. Direct costs: Commissions, trading, clearing, and other transactions fees. Indirect costs: Price impact of trade and also one-half of the bid- ask spread according to Domowitz, Glen and Madhavan (2000, 2001).U.K.: Average of buy and sell orders. Source: Elkins/McSherry.

	Direct	Indirect	Total
Belgium	19.9	7.2	27.1
Canada	17.2	29.5	46.7
Finland	22.6	22.6	45.2
France	22.0	13.8	35.8
Germany	21.6	8.9	30.5
Italy	23.2	17.8	41.0
Japan	14.7	4.7	19.4
Netherlands	21.3	3.1	24.4
Spain	23.9	15.3	39.2
Sweden	21.8	11.1	32.9
Switzerland	26.0	12.6	38.6
UK	42.0	8.3	50.3
NYSE	14.3	14.9	29.2
US OTC	2.4	34.2	36.6

In our model, transactions costs therefore consist of two components, $T_i = f_i + d_i$, where f_i are fees and d_i are other transactions costs, such as

[11] This represented 23% of the NYSE's total revenue. The remaining $552 million came from listing fees and sale of data (which we ignore in this analysis), and from clearing and settlement services, which are strongly correlated with transactions services. The corresponding data for Nasdaq were: revenues from transactions services amounted to $127 million, out of a total of $705 million (Source: Foucault and Parlour (2004)).

taxes, communication costs, information costs and other access costs.[12] As we do not model the stock exchanges' market micro structure, we interpret the stock exchange broadly as a group of actors intermediating the trade of a given set of stocks and interpret the f_i as comprising market maker and brokerage fees as well as direct stock exchange fees. While the other costs d_i are exogenous, exchanges (broadly interpreted) determine their fees f_i in order to maximize profits. These fees represent a transfer from investors to stock exchanges.

Therefore, total returns to stock exchange i are $f_i \sum_j \omega^j \left|t_i^j\right| = 2f_i\omega^1 \left|t_i^1\right|$, where $\omega^j \left|t_i^j\right|$ is total trade in asset i by investor class j. This assumes that both sellers and buyers pay the cost T_i.[13] The profit of exchange i then is

$$\pi_i = 2\omega^1 \left|t_i^1\right| (f_i - c_i) - K_i \tag{5}$$

where c_i is a constant unit cost of intermediating trade and K_i a fixed cost. For simplicity, we set $K_i = 0$. Exchange i chooses f_i such as to maximize π_i.

Stock exchange competition is modeled as a normal-form game. The two players are the two exchanges, their strategies are $f_i \in [0, \infty)$, and their payoff functions are the profit functions π_i as defined in (5). In making their decisions, stock exchanges rationally anticipate investor behavior t^j. The overall game has two stages and complete information, since the stock exchanges are assumed to know investors' preferences and endowments. To solve the game we use backwards induction. In the second stage, investors make investment decisions, taking transactions costs as given. In the first round exchanges simultaneously choose f_i, and we look for a Nash equilibrium in these choices.

3 Equilibrium in the asset market

In this section, we study equilibrium in the asset market, taking the decisions of stock exchanges as given. This analysis will then be folded back into the

[12] As Hau (2001) has documented, these costs typically depend on the investor's location. It is not difficult to include differential trading costs ($d_i^{\text{home}} < d_i^{\text{abroad}}$) in our analysis. This generalization would yield smaller international capital flows and a reduced erosion of the home bias. If we interpret Bekaert, Harvey, and Lundblad's (2005) measure of asset trade liberalization as a proxy for reduced access costs for foreigners, this theoretical result is consistent with their empirical finding.

[13] An alternative would be to consider a fee on trading volume, which would give half of the figure above.

study of the equilibria of the competition between stock exchanges in the next section. To simplify some formulas, we restrict attention to the empirically relevant case of positive correlation, $\rho \geq 0$.

We begin by deriving individual net asset demand (details are in the appendix). Investor j's objective function (4) written out is

$$\Phi(\mathsf{t}^j; \mathsf{p}, \mathsf{T}) = W_0 + \mathsf{e}^{j\prime}\boldsymbol{\mu} + \mathsf{t}^{j\prime}(\boldsymbol{\mu} - \mathsf{p}) - \sum \left|t_i^j\right| T_i - \frac{1}{2}\theta(\mathsf{e}^j + \mathsf{t}^j)'\Sigma(\mathsf{e}^j + \mathsf{t}^j)$$

The objective function is not differentiable at $t^j = 0$. Yet, despite the lack of differentiability, the solution to the portfolio problem is straightforward (see Constantinides (1979) for a similar argument). Because Φ is strictly concave and differentiable on each of the four orthants of the (t_1, t_2) - plane, interior solutions of the problem when restricted to an orthant are unique and given by the first-order condition

$$\theta\Sigma(\mathsf{e}^j + \mathsf{t}^j) = \boldsymbol{\mu} - \mathsf{p} - \begin{pmatrix} sign(t_1^j)T_1 \\ sign(t_2^j)T_2 \end{pmatrix}. \tag{6}$$

Comparing the possible interior solutions in the four orthants then shows that at most one such solution exists. Therefore, the first-order condition is also sufficient. If there is no interior solution, there can be a solution on the axes. In particular if transactions costs of one risky asset are sufficiently large relative to those of the other and the latter are sufficiently small, then there is a solution on the axis, where only the demand for one asset is non-zero. Otherwise net asset demand is zero. Hence, there exists a "no-trading region" (Constantinides, 1979): net asset demand is zero if initial endowments are too close to the optimal allocation without transactions costs.

Putting the individual net demands by the two investor groups together shows that there can be four types of asset market equilibria. First, there is the possibility that one investor buys both assets and the other sells both. Second, there is the possibility that one investor buys asset 1 and the other buys asset 2. Third, there is the possibility that investors trade only one asset. And fourth, there is the no-trade equilibrium.

Given the simple structure of our model, it is possible to calculate the equilibria explicitly, which we do in the appendix. The result is given in the following proposition. It is useful to label the four full-trading equilibria by their directions of trade: (δ_1, δ_2), where δ_i is $+1$ if investor type 1 buys asset i and is -1 if he sells.

Proposition 1 *Equilibria in the asset market exist under the following conditions:*

- *A full-trading equilibrium of type* (δ_1, δ_2) *exists if and only if*

$$\sigma_2^2 T_1 - \delta_1 \delta_2 \sigma_{12} T_2 + \delta_1 \frac{\theta}{2} |\Sigma| \Delta_1 < 0 \quad (7)$$

$$\sigma_1^2 T_2 - \delta_1 \delta_2 \sigma_{12} T_1 + \delta_2 \frac{\theta}{2} |\Sigma| \Delta_2 < 0 \quad (8)$$

Equilibrium prices and trades in asset $i = 1, 2$ $(l \neq i)$ *are*

$$p_i^* = \mu_i - \theta \left(\sigma_i^2 s_i + \sigma_{12} s_l \right) - \delta_i T_i \left(\omega^1 - \omega^2 \right), \quad (9)$$

$$t_i^1 = -\omega^2 \Delta_i - \frac{2\omega^2}{\theta |\Sigma|} \left(\delta_i \sigma_l^2 T_i - \delta_l \sigma_{12} T_l \right). \quad (10)$$

- *For* $i = 1, 2$, *an equilibrium with* $t_i^1 \neq 0$ *and* $t_l^1 = 0$, $l \neq i$, *exists if and only if*

$$T_l \geq \frac{1}{\sigma_i^2} \left| \frac{1}{2} \Delta_l |\Sigma| \theta - \sigma_{12} T_i \right| \quad (11)$$

$$\text{and } T_i < \frac{\theta}{2} \left| \Delta_i \sigma_i^2 + \Delta_l \sigma_{12} \right|. \quad (12)$$

The equilibrium price and trade in asset i *is*

$$p_i^* = \mu_i - \theta \left(\sigma_i^2 s_i + \sigma_{12} s_l \right) - \delta_i T_i \left(\omega^1 - \omega^2 \right), \quad (13)$$

$$t_i^1 = -\frac{\omega^2}{\theta \sigma_i^2} \left[\theta (\Delta_i \sigma_i^2 + \Delta_l \sigma_{12}) + 2 \delta_i T_i \right]. \quad (14)$$

- *If none of the previous conditions hold, there is no trade.*

The proposition shows that aggregate endowments or relative investor size (ω^j) play no role for the existence of equilibrium. The key parameters entering the existence conditions are the differences in per capita endowments between investor 1 and 2, $\Delta_i = e_i^1 - e_i^2$. For example, an equilibrium in which investor 1 buys both assets (the case $\delta_1 = \delta_2 = 1$) exists if the Δ_i are sufficiently small compared to T_i (conditions (7) and (8)).

Proposition 1 can easily be understood in two benchmark cases. The first is the case of no transactions costs, the standard CAPM. In this case, equilibrium exists, is of the type $(\delta_1, \delta_2) = (-1, 1)$, and agent 1's trade is

$$t_i^1 = -\omega^2 \Delta_i, \forall i. \tag{15}$$

Hence, trade in an asset is simply negatively proportional to the investor's relative position in the asset, Δ_i (see (1)). The investors with large endowments ($\Delta_i > 0$) sell the asset while the investors with small endowments ($\Delta_i < 0$) buy. After one round of trading without transactions costs the home bias disappears, and both types of investors hold $\omega^1 e_i^1 + \omega^2 e_i^2$ of each asset.

Another simple benchmark is that of no correlation between the two assets ($\sigma_{12} = 0$), which is equivalent to the case of a single risky asset. In this case, conditions (7) and (8) reduce to $T_i + \delta_1 \frac{\theta}{2} \sigma_i^2 \Delta_i < 0$, $i = 1, 2$, showing that in equilibrium an investor buys an asset if and only if he holds sufficiently little of it, compared to that asset's transactions costs, sells if he owns sufficiently much, and otherwise there is no trade.

The less obvious part of Proposition 1 is the interaction between return correlations, transactions costs, and endowment differentials in the determination of equilibrium. As to be expected, the trading volume of each risky asset goes down if its transactions costs increase. But as in Vayanos (1998), prices can increase or decrease with transactions costs. Because of correlation, for the trading decision of any one asset also the trading costs of the other asset are relevant. Their effect depends on whether the investors in equilibrium trade in the same direction for both assets or not.[14] If one investor class buys/sells both assets in equilibrium, trade in one asset increases the higher are the transaction costs of the other asset (the assets behave like substitutes). Inversely, if each investor class buys one asset and sells the other ($\delta_1 \delta_2 = -1$), investors trade less of an asset the higher are the transaction costs of the other asset (the assets behave like complements). Whether $\delta_1 \delta_2$ is positive or negative is, of course, endogenous, and we characterize its sign below.

The equilibrium price when only one asset is traded has the same structure as that for full trading, but is simpler as there is no hedging demand.

[14] If in equilibrium $t_1^1 > 0, t_2^1 > 0$, an increase in T_1 decreases t_1^1 and increases t_2^1. If $t_1^1 > 0, t_2^1 < 0$, an increase in T_1 decreases t_1^1 and decreases $|t_2^1|$.

We now discuss what type of equilibria can exist as a function of transcations costs T, endowment differentials Δ, and asset risk Σ. For this discussion, it is useful to introduce the parameters

$$\overline{T}_i = \frac{\theta}{2} \left| \Delta_i \sigma_i^2 + \Delta_l \sigma_{12} \right|, l \neq i. \tag{16}$$

Below we will see that Proposition 1 implies that an equilibrium in which asset i is traded exists only if $T_i < \overline{T}_i$.

Our basic assumption that $\Delta_1 > 0$ and $\Delta_2 < 0$ (investor 1 owns more of asset 1, investor 2 more of asset 2), immediately implies, after an inspection of (7) and (8), that $\delta_1 = 1$ and $\delta_2 = -1$ (investor 1 buys asset 1 and sells asset 2) is impossible in equilibrium. Furthermore, it is clear that an equilibrium in which investor 1 sells asset 1 and buys asset 2 will exist if transactions costs are not too high (this is the "natural" direction of trade that erodes the home bias and occurs without transactions costs).

Yet, the "uni-directional" equilibria $\delta_1 \delta_2 = 1$ are also possible. Investor 1 will buy both assets if and only if the two conditions in (7) and (8) for $\delta_1 = \delta_2 = 1$ are compatible. There exist transactions costs T for which this is true if and only if $\sigma_1^2 \Delta_1 + \sigma_{12} \Delta_2 < 0$. Similarly, one can see that investor 2 will buy both assets for some values of T if and only if $\sigma_{12} \Delta_1 + \sigma_2^2 \Delta_2 > 0$. In the remaining case, $\Delta_1 \sigma_1^2 / \sigma_{12} > -\Delta_2 > \Delta_1 \sigma_{12} / \sigma_2^2$, only the "natural" equilibrium $\delta_1 = -1, \delta_2 = 1$ is compatible with $\Delta_1 > 0, \Delta_2 < 0$.

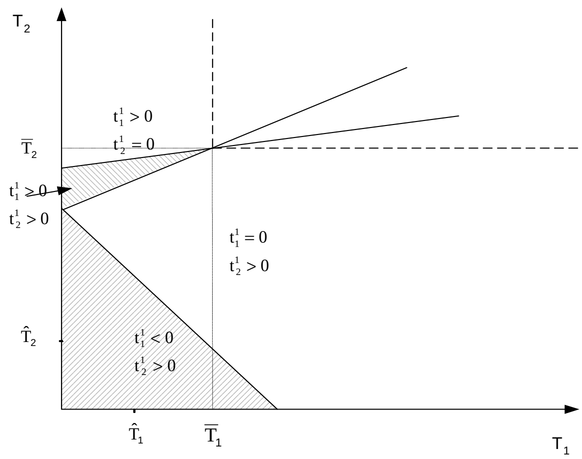

Figure 1: The trading region if $0 < \Delta_1 \sigma_1^2 / \sigma_{12} < -\Delta_2$

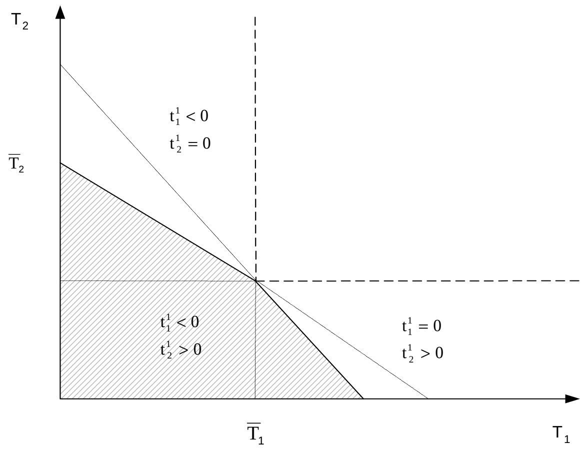

Figure 2: The trading region if $\Delta_1\sigma_1^2/\sigma_{12} > -\Delta_2 > \Delta_1\sigma_{12}/\sigma_2^2 > 0$

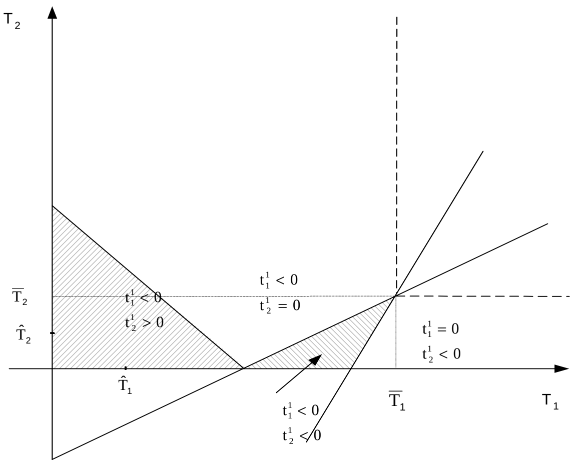

Figure 3: The trading region if $0 < -\Delta_2 < \Delta_1\sigma_{12}/\sigma_2^2$

Figures 1 - 3 present the result of the above analysis graphically in T-space. If transactions costs are not too unequal and sufficiently small (the hatched surface that contains the origin in the three figures), then, as in the case with no transactions costs, investor 1 will sell asset 1 in equilibrium and investor 2 sell asset 2. If T_1 is relatively small and T_2 relatively large, this behavior is impossible, and Figure 1 shows that investor 1 may rather buy both assets (the upper triangle). Yet, this happens only if Δ_1 is sufficiently small compared to $|\Delta_2|$, i.e., if investor 1 does not own much more of asset 1 than investor 2 (in other words, if the home bias for asset 1 is not too strong).

In this "unidirectional" equilibrium the relatively larger (smaller) investors buy (sell) the asset, thus reinforcing the home bias. This happens if the correlation between assets is sufficiently high and the foreign asset is expensive to trade. In this case, domestic investors prefer to re-balance their portfolio by taking on even more of the domestic asset.

Conversely, if T_1 is relatively large and T_2 relatively small, investor 2 may buy both assets (the right-hand triangle in Figure 3). For this, it is necessary that Δ_1 and T_1 be large and $|\Delta_2|$ and T_2 small. Now the direction of trade for asset 2 is different from the case without transactions costs. The trading areas of such "distorted" decisions are marked by relatively big discrepancies between transaction costs and strongly differing endowments in one asset and by relatively small differences in endowments in the other asset.

An interesting feature of Figures 1 and 3 is the non-convexity of the full-trading region in T - space. The non-convexities reflect the "distortion" of trading motives through the presence of transactions costs described above.

The relative size of endowments also determines the single-asset-trading equilibria. An inspection of the conditions in Proposition 1 then yields the following result.

Proposition 2 *The asset market equilibrium exists and is unique.*

Proof. The above discussion has shown that the four types of full-trading equilibria are mutually exclusive. Furthermore, it is straightforward to see that the conditions for single-trading equilibria imply different parameter values from the full-trading conditions. Figures 1 - 3 provide the graphical illustration. ■

4 Competition between Exchanges

We now use the results of the last section to analyze the interaction between the stock exchanges in the determination of transactions costs.

A potential problem for the analysis is the non-convexity of the "full-trading region" in T - space (see Figures 1 and 3).[15] In fact, as the analysis of Section 3 has shown, the set $\mathcal{T} = \{(T_1, T_2) \in \mathsf{R}^2_+$; there is trade in both assets in equilibrium$\}$ is only convex for

$$\Delta_1\sigma_1^2/\sigma_{12} > -\Delta_2 > \Delta_1\sigma_{12}/\sigma_2^2. \tag{17}$$

Yet, empirically, this constellation is the only relevant one. As shown in Section 3, the unique asset market equilibrium in this case takes the form $\delta_1 = -\delta_2 = -1$, which means that both investor types sell the asset of which they hold more and buy the one of which they hold less. Hence, in equilibrium trade erodes the home bias. Furthermore, comparing the equilibrium trade with transactions costs, (10), to the one without, (15), shows that in the equilibrium in constellation (17), trade does not eliminate the home bias. This is consistent with available data. Table A1 in the appendix presents the most recent data we are aware of from the Coordinated Portfolio Investment Survey (CPIS) by the IMF, which show that the home bias has decreased between 1997 and 2002, and markedly so in many countries, with the exception of some East Asian countries (for obvious easons).[16]

Hence, we proceed under the assumption (17). Then total trading volume, given in Proposition 1, is

$$v_i\left(T_i, T_l\right) = \omega^1\omega^2\left[\left|\Delta_i\right| - \frac{2}{\theta\left|\Sigma\right|}\left(\sigma_l^2 T_i + \sigma_{12}T_l\right)\right], l \neq i \tag{18}$$

By Proposition 1, the full-trading region $\mathcal{T}$ is the set of all $(T_1, T_2) \in \mathsf{R}^2_+$ that satisfy conditions (7)-(8):

$$\sigma_1^2 T_2 \quad < \quad -\frac{\theta\left|\Sigma\right|\Delta_2}{2} - \sigma_{12}T_1 \tag{19}$$

$$\text{and } T_2\sigma_{12} \quad < \quad \frac{\theta\left|\Sigma\right|\Delta_1}{2} - \sigma_2^2 T_1 \tag{20}$$

On the boundary of the full-trading region we have $v_i = 0$ and $v_j > 0$, if exactly one of the inequalities in (19)-(20) binds. The boundary separates the

[15] In the transactions literature following Constantinides (1979), the terms "trading region" or "no-trading region" usually refer to endowment space. For our purposes we cast the analysis in transactions costs space.

[16] Note that condition (17) essentially means that the correlation coefficients are small. Of course, the low correlations between domestic markets in reality are precisely what give rise to the home bias puzzle.

full-trading region from the two single-asset-trading regions. By Proposition 1, the region where asset i is traded alone is given by

$$T_l \geq \frac{1}{\sigma_i^2}\left|\frac{1}{2}\theta\Delta_l\left|\Sigma\right| - \sigma_{12}T_i\right| \tag{21a}$$

$$\text{and } T_i < \overline{T}_i, l \neq i \tag{21b}$$

where $(\overline{T}_1, \overline{T}_2)$ is the intersection of the lines in (19)-(20) and given by (16).

Call the single-asset trading regions $\mathcal{S}_i = \{(T_1, T_2) \in \mathsf{R}_+^2$; in equilibrium there is trade only in asset $i\}$. By Proposition 1, trading volume in asset i if the other asset is not traded is

$$v_i\left(T_i, T_l\right) = \frac{\omega^1\omega^2}{\sigma_i^2}\left(\sigma_i^2\left|\Delta_i\right| - \sigma_{12}\left|\Delta_l\right| - 2\frac{T_i}{\theta}\right), l \neq i, \forall i. \tag{22}$$

Each exchange maximizes profits given the behavior of the other exchange by setting its fees f_i. Exchange i's formal problem is

$$\begin{aligned} \max_{f_i \geq 0} & \quad v_i(d_i + f_i, T_l)(f_i - c_i) \\ \text{where } v_i \text{ is given by (18) if } (T_1, T_2) & \in \mathcal{T}, \\ \text{by (22) if } (T_1, T_2) & \in \mathcal{S}_i, \\ \text{and } v_i & = 0 \text{ otherwise} \end{aligned}$$

When choosing its level of fees, each exchange must solve the standard oligopoly trade-off: increasing the fee increases the revenue per transaction, but decreases the total volume of transactions. In the model, two elements are outside the exchanges' control: their cost structure (given by the marginal cost level c_i) and the exogenous component of trading costs (given by d_i). It is clear that if this total cost level,

$$k_i = c_i + d_i$$

is too high, there cannot be an equilibrium with trade. However, if this level is not too high, there will be trade. The next proposition provides a precise characterization of the exchanges' optimal policies.

Proposition 3 *Assume that* $\Delta_1\sigma_1^2/\sigma_{12} > -\Delta_2 > \Delta_1\sigma_{12}/\sigma_2^2$ *and that* $k_i < \overline{T}_i$, $i = 1, 2$ *(as defined in (16)). The competition between the exchanges has* *a unique Nash equilibrium, in which exchange i, $i = 1, 2$, sets*

$$
\begin{aligned}
f_i^* \;&=\; \frac{1}{4\sigma_1^2\sigma_2^2 - \sigma_{12}^2}\left[\theta\left|\Sigma\right|\left(\sigma_i^2\left|\Delta_i\right| - \frac{1}{2}\sigma_{12}\left|\Delta_l\right|\right) - \sigma_{12}\sigma_i^2(c_l + d_l)\right. \\
&\quad \left. -(2\sigma_1^2\sigma_2^2 - \sigma_{12}^2)d_i + 2\sigma_1^2\sigma_2^2 c_i\right],
\end{aligned}
\tag{23}
$$

($l \neq i$) and makes strictly positive profits.

Proof. We first solve a simplified problem for each exchange, by assuming that v_i is given by (18) for all $\mathsf{T} \in \mathsf{R}_+^2$ (i.e., we solve the problem as if the exchanges were always pricing in the full-trading region). To simplify notation, let $y_i = f_i - c_i$. In this problem, exchange i optimally chooses, for y_l given,

$$
y_i^* = \frac{\theta\left|\Sigma\right|}{4\sigma_l^2}\left|\Delta_i\right| - \frac{\sigma_{12}}{2\sigma_l^2}(y_l + k_l) - \frac{k_i}{2}
$$

Solving for (y_1, y_2) and substituting back for f_i, c_i, and d_i yields (23). We now show that $\mathsf{T}^* = \mathsf{f}^* + \mathsf{d} \in \mathcal{T}$. A straightforward calculation shows that T^* satisfies (19)-(20) if and only if

$$
k_2 \;<\; -\frac{\sigma_{12}\sigma_2^2}{2\sigma_1^2\sigma_2^2 - \sigma_{12}^2}k_1 - \frac{\theta\left|\Sigma\right|}{2}\frac{\sigma_{12}\Delta_1 + 2\sigma_2^2\Delta_2}{2\sigma_1^2\sigma_2^2 - \sigma_{12}^2} \tag{24a}
$$

$$
k_2 \;<\; -\frac{2\sigma_1^2\sigma_2^2 - \sigma_{12}^2}{\sigma_{12}\sigma_1^2}k_1 - \frac{\theta\left|\Sigma\right|}{2}\frac{\sigma_{12}\Delta_2 + 2\sigma_1^2\Delta_1}{\sigma_{12}\sigma_1^2} \tag{24b}
$$

The straight lines defined by (24a) and (24b) in $k_1 - k_2$ - space intersect at $(\overline{T}_1, \overline{T}_2)$. Therefore, the condition on k in the Proposition implies that k satisfies (24a) and (24b). Furthermore, it is straightforward to verify that the inequalities $f_i > c_i$ hold if and only if (24a) and (24b) hold. Hence, the f^* defined by (23) yields $\mathsf{T}^* \in \mathcal{T}$ (i.e. leads to trade in the full-trading region) and positive profits for the exchanges.

The check whether exchanges have incentives to deviate from T^* into a single-asset-trading region is trivial, because each exchange can only price itself out of the market (see Figure 2). Thus T^* is a Nash equilibrium. It is unique, because there is no other equilibrium in the full-trading region, as shown above, and there can be no equilibrium in a single-trading region, because then the excluded stock exchange can price itself into the market and make a profit. ■

In this equilibrium both exchanges are active and make positive profits. By inspection, equilibrium fees are driven by operational costs c_i, the difference of endowments Δ_i, exogenous costs d_i, and the variance-covariance structure of asset returns. If (differently from our assumption) operational and exogenous costs are high for one exchange and low for the other, there is typically an equilibrium in which only one exchange operates. As this is not relevant empirically, we shall ignore this case.

5 Determinants of Transactions Costs and Trading Volume

We now analyze the determinants of stock market transactions costs and trading volume. We first use the results of Section 4 to investigate how the asset structure and the trading environment influence the determination of transactions costs. This amounts to the comparative statics analysis of (23). For the remainder of the paper we assume that the conditions of Proposition 3 are satisfied, i.e. that

$$k_i < \frac{\theta}{2}\left(|\Delta_i|\,\sigma_i^2 - |\Delta_l|\,\sigma_{12}\right) \text{ and } |\Delta_i|\,\sigma_i^2 > |\Delta_l|\,\sigma_{12}, l \neq i. \tag{25}$$

5.1 Transactions costs

The main exogenous variables driving behavior in our model are the assets' volatilities σ_i, the correlation between the asset returns ρ, the endowment differentials Δ_i, investor risk aversion θ, and the cost variables c_i and d_i. We summarize their impact on equilibrium fees in the following proposition.

Proposition 4 *Equilibrium fees f_i^* for asset i*

- *decrease with market integration as measured by the correlation ρ and with the investors' exogenous trading costs d_i,*
- *increase with the home asset's variance σ_i, the endowment differential $|\Delta_i|$, the exchange's marginal cost c_i, and with investor risk aversion θ.*

Proof. Writing (23) in terms of ρ instead of σ_{12} yields

$$f_i^* = \frac{1-\rho^2}{4-\rho^2}\theta\left(\sigma_i^2\,|\Delta_i| - \frac{1}{2}\rho\sigma_1\sigma_2\,|\Delta_l|\right) - \frac{\rho\sigma_i}{(4-\rho^2)\sigma_l}(c_l+d_l) - \frac{2-\rho^2}{4-\rho^2}d_i + \frac{2}{4-\rho^2}c_i$$

The results for $|\Delta_i|$, θ, c_i, and d_i are obvious. The results for ρ and σ_i are obtained by differentiation, using (25). ■

The intuition behind Proposition 4 is simply a combination of changing demand for diversification by investors with price competition by exchanges. In particular, an inspection of equilibrium trading volumes (18) shows that they are decreasing in the correlation of asset returns, because diversification demand decreases. This in turn implies that the price competition between the exchanges intensifies, leading to reduced fees.

The result also has business implications, as it indicates that stock exchanges can set higher fees by listing less correlated assets. This way they can achieve market power by, e.g., specializing in regions, or as Nasdaq, in industries.

In a similar vein, our model implies a positive relationship between transactions costs and volatility. This relationship is well-known empirically and often interpreted as transactions costs causing volatility through reduced market liquidity. Our model suggests the reverse causality: the higher the variance of the asset, the more investors want to trade it for risk diversification purposes and therefore the smaller is the relative impact of transactions costs. Conversely, when the stocks become less risky, fees are pressured to decrease. Note, however, that in a dynamic model, higher volatility may come from higher exogenous trading costs (d_i), which would be consistent with the traditional view.

A further important determinant of transactions costs are operational costs of stock exchanges. It has been widely remarked that technological intermediation costs for exchanges have decreased dramatically during the 1990s. Proposition 4 shows that these reduced costs c_i are partially passed through to customers in our model in the form of lower fees f_i. Note that the impact of costs on fees is higher when stock market correlation is stronger ($\partial^2 f_i^* / \partial c_i \partial \rho > 0$).

High values of $|\Delta_i|$ in our model can be interpreted as a large home bias: local investors hold much of the local stock. Proposition 4 shows that a stronger home bias implies higher transactions costs. According to our theory, a strong home bias leads to larger diversification demand, thus increasing trade and allowing exchanges to charge higher fees. Alternatively one could argue that a strong home bias shields the local stock market from global pressure and allows stock exchanges to charge higher fees. In contrast to our theory, this theory would imply reduced trading for stronger home-biased markets.

Finally, Proposition 4 shows that the investors' exogenous trading costs d_i have a negative impact on fees. The intuition is simple: As trading depends only on total transactions costs, an increase in exogenous transactions costs must at least partially be compensated by reduced endogenous transactions costs if exchanges maximize profits.

5.2 Trading volume

Changes in the trading environment, such as the international variance-covariance structure of asset returns, have a direct and indirect impact on international stock market trading. The direct impact is the one given by Proposition 1, holding transactions costs fixed. The indirect impact comes through the endogenous changes in transactions costs identified in Proposition 3. Inserting the equilibrium value of $T_i^* = d_i + f_i^*$ into (18) yields the total effect and shows that equilibrium trading volume is given by

$$\begin{aligned} v_i(T_i^*, T_l^*) &= \frac{\omega^1 \omega^2}{\theta(1-\rho^2)(4-\rho^2)\sigma_1^2\sigma_2^2} \big(2\theta\sigma_1^2\sigma_2^2(1-\rho^2)\,|\Delta_i| \\ &\quad +\theta\sigma_i\sigma_l^3\rho(1-\rho^2)\,|\Delta_l| - 2\sigma_l^2(2-\rho^2)(c_i+d_i) - 2\sigma_1\sigma_2\rho(c_l+d_l)\big) \end{aligned} \quad (26)$$

From (26) the impact of the exogenous parameters in our model on equilibrium trading volume can be calculated as follows.

Proposition 5 *Equilibrium trading volume $v_i(T_i^*, T_l^*)$ in asset i*

- *decreases with the exchange's marginal cost c_i, and with the investors' exogenous trading costs d_i*
- *increases with correlation ρ, volatility σ_i, the size of the home bias $|\Delta_i|$, and risk aversion θ.*

Among the six variables of interest, the strength of the home bias ($|\Delta_i|$), risk aversion (θ), return correlation (ρ), and volatility (σ_i) influence both the trading volume directly (see (18)) and indirectly via endogenous transactions costs. For all of these variables, the indirect effect counteracts the direct effect. Interestingly, the indirect effect of reduced trading fees dominates the direct effect of correlation on turnover, leading to a positive predicted overall effect.

Furthermore, it is interesting to note that the two cost parameters c_i and d_i influence fees and turnover differently. Reduced exogenous trading costs (d_i) increase trading fees and reduced intermediation costs (c_i) decrease them, in both cases because stock exchanges pass cost savings on to the investors only imperfectly. On the other hand, (26) shows that reductions in both types of costs increase trading volume. The reason for this asymmetry is the interaction between the pricing decisions by both stock exchanges in equilibrium.

5.3 Interpretation

The 1980s and 90s have seen dramatic changes in competition between stock exchanges across the world. Because our model is static, it is not a very good yardstick for such time-series evidence. Yet, the static model can be compared to this evidence by "rolling over" the one-period equilibrium, because our CARA-Gaussian framework eliminates wealth effects from trading, and equilibrium prices in (9) only depend on aggregate endowments, not their distribution.

Assuming therefore that investors trade in periods $t = 1, 2, ...$, asset prices at date t are p_i^t, and payoffs $\widetilde{F}_i^{t+1}$ are i.i.d. and occur one period later, with correlation ρ^{t+1} as in Section 2, asset returns in $t+1$ are given by $(\widetilde{F}_i^{t+1} + p_i^{t+1})/p_i^t$. As viewed from time t, $\widetilde{F}_i^{t+1}$ is random and p_i^{t+1} deterministic. Hence, the correlation of returns is

$$corr(\frac{\widetilde{F}_1^{t+1} + p_1^{t+1}}{p_1^t}, \frac{\widetilde{F}_2^{t+1} + p_2^{t+1}}{p_2^t}) = corr(\frac{\widetilde{F}_1^{t+1}}{p_1^t}, \frac{\widetilde{F}_2^{t+1}}{p_2^t}) = \rho^{t+1}$$

Using this simple extension of our model, we can compare it to the empirical evolution of stock market correlations, transactions costs, and home market biases in the last 20 years. As Table III shows by means of two examples, the correlation of returns on all major stock exchanges has increased steadily from the 1970s to the 2000s. There has been some debate about whether this trend is due to increased "fundamental" integration of the underlying economies or whether it rather reflects a financial phenomenon.[17] Be this as it may, economic integration has certainly been important in Europe in the 1980s and 1990s and in much of the global economy since

[17]See Adam, Jappelli, Menichini, Padula, and Pagano (2002), Goetzmann, Li and Rouwenhorst (2005), or Canova, Ciccarelli and Ortega (2007).

the 1990s, which clearly mirrors the increased correlation of financial returns documented in Table III. The ultimate answer to the question whether "real" developments drive financial ones or vice versa is of little importance to our argument, we simply take the increased correlation between stock markets as a building block of our theory.

Table III
Average Stock Market Correlations 1973 - 2003

The table shows average correlations calculated from DataStream market indexes (our calculations). The data provided by DataStream is weekly returns calculated in US dollars, from January 1, 1973 to November 25, 2003 (1613 observations). The subsamples are 1973-1982 (522 observations), 1983-1992 (522 observations) and 1993-2003 (569 observations). Countries are Australia (Aus), Canada (Ca), France (Fr), Germany (Ger), Italy (It), Japan, (Jap), the Netherlands (NL), Switzerland (CH), United Kingdom (U.K), United States (U.S).

	U.S., U.K., Ger, Fr	U.S., U.K., Ger, Fr, Aus,Ca, It, Jap, NL, CH
1973-1982	0.32	0.33
1983-1992	0.46	0.44
1993-2003	0.71	0.57

With regard to trading costs, it is well known that costs have decreased world-wide since the liberalization of stock exchanges beginning in the 1980s. An important catalyzing event has been the so-called Big Bang, a set of reforms that liberalized the London stock market in 1986, with a major impact on commissions and spreads. Pagano and Roell (1990) report a fall of one third on large transaction costs after the Big Bang. These reforms were soon followed by other continental exchanges. In 1989, Paris liberalized members' commissions, reduced stamp duty and implemented an automated traded system. Progressively, similar reforms were adopted all over the world, leading to further drastic declines in transactions costs since the mid-90s.[18] In

[18] See, in particular, Bekaert, Harvey and Lumsdaine (2002).

the course of such reforms, technological intermediation costs have decreased substantially during the 1990s, most notably through the advent of electronic trading platforms. Domowitz (2001) reports that, all other things equal, total trading costs have decreased by 33 to 46 basis points in markets that are widely automated. Automazation was largely completed by the beginning of the century: according to Jain (2005) in 2002, 101 stock exchanges out of an international sample of 120 had electronic trading systems.

As we have argued, higher exogenous transactions costs depress (endogenous) fees, which does not only hurt investors but also the stock exchanges. This explains why exchanges in the last decade have assumed a more active role in organizing and imposing rules on trade. In fact, many of the reforms of stock exchanges have been fostered by exchanges themselves, and part of their efforts have been directed at reducing these "undesirable competitive" costs. There are several examples, such as the lobbying for the decrease of taxes, imposing stricter transparency standards for firms (e.g. Deutsche Börse and Stockholm's Borssen). Partnerships with financial data providers aim at decreasing information costs, mergers of clearing houses and settlement systems, as the one of Clearnet and LCH, intend to improve costs associated with cross border trading.

Increased stock market activity since the 1980s has been accompanied by a strong increase in cross-border trading and also lead to a continuous erosion of the home bias.[19] While as late as in the late 1980s most domestic markets were hardly exposed to international portfolio investment, this has changed in the 1990s and this change has accelerated since the late 1990s. The slow erosion of the home bias in the 1990s has been described, in particular, by Tesar and Werner (1998) and Lewis (1999). Table A1 in the appendix documents the evolution of the home bias between 1997 and 2002 for 23 countries surveyed in the Coordinated Portfolio Investment Survey by the IMF. If one excludes the countries hit by the East Asian crisis (Indonesia, Malaysia, South Korea, Thailand) for obvious reasons, equity holdings by investors in their home country in 1997 were on average 83.0 percent and declined by almost 15 percentage points to 68.8 until 2002. While 68.8 percent still represent a sizeable home bias, the decline from the estimated values above 90 percent in the late 1980s is truly remarkable.

In summary, the evolution of stock exchanges around the world from the

[19] On cross-border portfolio flows, see in particular Portes and Rey (2005) and Lane and Milesi-Ferretti (2004).

late 1980s to the early 2000s presents the following qualitative picture: a decrease in intermediation costs, fees, and the home bias, and an increase in international portfolio flows and return correlations. These co-movements are consistent with the predictions of our model in the dynamic interpretation given at the beginning of this subsection.

6 Conclusion

This paper has analyzed the competition between stock exchanges under the premise that stock exchange fees constitute revenues for the exchanges and transactions costs for the market participants. Using a standard mean-variance capital market equilibrium model with transactions costs, we have thus endogenized transactions costs as variables strategically influenced by stock exchanges.

Applied studies typically take transactions costs as exogenous and try to identify their impact on trading. In their empirical study, Domowitz and Steil (2002), for example, estimate that a decrease of 10 percent in trading costs yields an 8 percent increase in trading volume and a 1.5 percent decrease in the cost of capital to blue-chip listed companies. These estimates show how important the effects are that we identify in this paper, but also how important it is to understand their driving forces. The present paper contributes to this task by identifying fundamentals that drive transactions costs and trading volumes simultaneously, and by shedding light on their interaction.

In principle, the framework that we have adopted in this paper allows us to study the impact of stock exchange competition on stock market activity quite generally. International portfolio investment is a particularly natural application of our model, as the vast majority of listed firms still is listed on a domestic stock exchange. Hence, the portfolio of firms listed on a domestic stock exchange corresponds largely to the national index, and as a consequence, international portfolio diversification occurs largely via the choice of different national exchanges. More generally, if stocks are not cross-listed, stocks listed on different stock exchanges are only imperfect substitutes for the investors' portfolio problem, and the competition for investors between stock exchanges takes on an element of strategic complementarity. This type of competition also exists in contexts other than that of international portfolio flows, for example between exchanges specializing in different types of stock, such as the NYSE and Nasdaq. In this case, the endowment differentials that drive trade in our model cannot be interpreted as international portfolio biases, but rather correspond to other, more conventional portfolio imbalances.

The present paper has restricted attention to stock exchange competition for investors. This complements the existing literature that has focused on competition for firms, but still is incomplete. In fact, if stocks can be traded on several exchanges or platforms, stock exchange competition in principle takes the form of two-sided price competition, for the listing of firms on the one side and for the trading activity of investors on the other. This is an example of the more general phenomenon of two-sided markets discussed by Rochet and Tirole (2006), in which the success on one side of the market depends on that of the other and vice versa. Linking our work with the existing work on stock exchange competition for firms in the perspective of two-sided competition is a promising next step on the research agenda.

7 Appendix

1. Proof of Proposition 1:

To simplify the description of investor j's problem let, for given prices p,

$$A_i^j = \mu_i - p_i - \theta\left(\sigma_i^2 e_i^j + \sigma_{12} e_l^j\right), l \neq i$$

Because the problem is concave in each quadrant, it suffices to check the directional derivatives on the axes to verify that the problem is globally concave. It therefore has a unique solution, with asset demand either in the interior of an orthant or on the axes or equal to zero:

Lemma A1: *The investor's portfolio problem has a unique solution, given as follows.*

R1 *If* $\sigma_1^2\sigma_2^2(A_1^j - T_1) > \sigma_1^2\sigma_{12}(A_2^j - T_2) > \sigma_{12}^2(A_1^j - T_1)$, *investor* j *buys both assets, and*

$$t_i^j = \frac{\sigma_l^2}{\theta\left|\Sigma\right|}\left(\mu_i - p_i - T_i\right) - \frac{\sigma_{12}}{\theta\left|\Sigma\right|}\left(\mu_l - p_l - T_l\right) - e_i^j > 0, \quad i = 1, 2, l \neq i.$$

R2 *If* $\sigma_{12}^2(A_2^j + T_2) > \sigma_2^2\sigma_{12}(A_1^j + T_1) > \sigma_1^2\sigma_2^2(A_2^j + T_2)$, *investor* j *sells both assets, and*

$$t_i^j = \frac{\sigma_l^2}{\theta\left|\Sigma\right|}\left(\mu_i - p_i + T_i\right) - \frac{\sigma_{12}}{\theta\left|\Sigma\right|}\left(\mu_l - p_l + T_l\right) - e_i^j < 0, \quad i = 1, 2, l \neq i.$$

R3 *If* $\sigma_i^2\sigma_{12}(A_l^j + T_l) < \min(\sigma_1^2\sigma_2^2(A_i^j - T_i), \sigma_{12}^2(A_i^j - T_i))$, $i = 1, 2, l \neq i$, *investor* j *sells asset* l *and buys asset* i, *and*

$$\begin{aligned} t_i^j &= \frac{\sigma_l^2}{\theta\left|\Sigma\right|}\left(\mu_i - p_i - T_i\right) - \frac{\sigma_{12}}{\theta\left|\Sigma\right|}\left(\mu_l - p_l + T_l\right) - e_i^j > 0 \\ t_l^j &= \frac{\sigma_i^2}{\theta\left|\Sigma\right|}\left(\mu_l - p_l + T_l\right) - \frac{\sigma_{12}}{\theta\left|\Sigma\right|}\left(\mu_i - p_i - T_i\right) - e_l^j < 0. \end{aligned}$$

R4 *If* $\left|A_i^j\right| > T_i$ *and* $T_l \geq sign(A_l^j)\left(A_l^j - \frac{\sigma_{12}}{\sigma_i^2}\left(A_i^j - sign(A_i^j)T_i\right)\right), l \neq i$, *investor* j *only trades asset* i*:*

$$t_i^j = \frac{1}{\theta\sigma_i^2}(A_i^j - sign(A_i^j)T_i) \neq 0, t_l^j = 0.$$

R5 *If none of the above conditions hold, investor j does not trade:* $t^j = 0$.

The Lemma is tedious to derive but quite standard. Equilibrium is now obtained by market clearing as follows.

The market clearing condition for a full trading equilibrium of type (δ_1, δ_2) is:

$$s_i = \frac{\sigma_1^2\sigma_2^2}{\theta\sigma_i^2\left|\Sigma\right|}\left(\mu_i - p_i\right) - \frac{\sigma_{12}}{\theta\left|\Sigma\right|}\left(\mu_l - p_l\right) + \sum_{j=1}^{2}\omega^j\left(-\frac{\sigma_1^2\sigma_2^2}{\theta\sigma_i^2\left|\Sigma\right|}T_i\delta_i + \frac{\sigma_{12}}{\theta\left|\Sigma\right|}T_l\delta_l\right) \tag{27}$$

where the rightmost term is the component of equilibrium price that varies with the different trading decisions. Solving (27) simultaneously for both assets and using Lemma A1 for the possible buy-sell combination regimes yields (9). Equilibrium trades (10) are then obtained by combining (9) with the different expressions of R1 - R3 in Lemma A1.

Full trade is possible if and only if

$$-2\sigma_l^2T_i + 2\sigma_{12}T_l > \theta\left|\Sigma\right|\Delta_i \vee \theta\left|\Sigma\right|\Delta_i > 2\sigma_l^2T_i - 2\sigma_{12}T_l, \text{ for } \delta_i = \delta_l$$

or

$$-2\sigma_l^2T_i - 2\sigma_{12}T_l > \theta\left|\Sigma\right|\Delta_i \vee \theta\left|\Sigma\right|\Delta_i > 2\sigma_l^2T_i + 2\sigma_{12}T_l, \text{ for } \delta_i = -\delta_l.$$

This yields conditions (7) and (8) and completes the proof of the full-trading regime in Proposition (1).

The characterization of the partial-trading regimes (one asset is traded in equilibrium, the other not) is similar, using condition R4 of Lemma A1.

2. The case $n > 2$ (sketch):

The investor's objective function is as in Section 3. We clearly can restrict attention to equilibria in which all investors trade all assets (full-trading equilibria). Such equilibria can be associated with vectors $\delta^j \in \{-1, 1\}^n$, $j = 1, ..., n$, where $\delta_i^j = sign(t_i^j)$. The matrix $\delta = (\delta^j)_{j=1,\ldots,n}$ describes the directions of all equilibrium trades. Let

$$\boldsymbol{\tau}^j = \begin{pmatrix} \delta_1^j T_1 \\ \\ \delta_n^j T_n \end{pmatrix}$$

denote the signed vector of transactions costs. By local strict concavity, investor j's trade is given by the n-dimensional version the first-order condition (6),

$$\theta\Sigma(\mathsf{e}^j + \mathsf{t}^j) = \boldsymbol{\mu} - \mathsf{p} - \boldsymbol{\tau}^j$$

Market clearing then implies

$$\Sigma^{-1}(\boldsymbol{\mu} - \mathsf{p}) - \Sigma^{-1}(\sum_j \omega^j \boldsymbol{\tau}^j) = \theta \mathsf{s}$$

which yields

$$\mathsf{p} = \boldsymbol{\mu} - \sum_j \omega^j \boldsymbol{\tau}^j - \theta\Sigma\mathsf{s} \tag{28}$$

$$\mathsf{t}^j = \mathsf{s} - \mathsf{e}^j + \frac{1}{\theta}\Sigma^{-1}(\sum_k \omega^k \boldsymbol{\tau}^k - \boldsymbol{\tau}^j) \tag{29}$$

(28) and (29) generalize (9) and (10) in the main text and fully characterize an equilibrium of type δ. Differently from the case $n = 2$, the number of potential equilibrium configurations becomes large for $n > 2$. While for $n = 2$ there are 4 potential configurations (investor 1 sells both assets, buys both assets, sells asset 1 and buys asset 2, or buys asset 1 and sells asset 2), it can be shown that for $n = 3$ there are already 216 potential equilibrium configurations (not all of the 8^3 different matrices $\delta \in \{-1, 1\}^{3 \cdot 3}$ can occur because it is impossible to have 3 buyers or 3 sellers of one asset). However, as in the 2-asset case, it is straightforward to calculate specific equilibria of interest. For example, the equilibrium in which each of the three investors sells the asset of which he has most and buys the two assets of which he has least, exists and is characterized by 9 inequalities in $T_1 - T_2 - T_3 -$ space.

Table A1

	1997	2002
Australia	88.01	81.36
Austria	71.49	45.56
Belgium	64.92	46.56
Canada	80.36	73.57
Chile	99.41	90.83
Denmark	78.16	63.38
Finland	94.07	69.43
France	83.68	77.53
Indonesia	99.9	99.62
Italy	78	59.7
Japan	92.56	89.61
Malaysia	98.28	98.5
Netherlands	70.48	43.19
New Zealand	82.17	65.81
Norway	84.76	48.11
Portugal	85.78	78.86
Singapore	84.3	68.43
South Korea	97.62	99.06
Spain	90.91	89.1
Sweden	80.08	58.09
Thailand	98.79	99.77
United Kingdom	78.43	69.94
United States	90.08	88.64

Figure 4: Home Bias 1997 - 2002

The table shows the home bias in selected countries (equity holdings of investors in home country in percent)
Source: IMF Coordinated Portfolio Investment Survey (CPIS)

References

[1] Adam, K., Jappelli, T., Menichini, A., Padula, M., Pagano, M., 2002. Methodologies to measure the evolution of capital market integration in the European Union, EU Commission (http://europa.eu.int/comm/internal_market/en/update /economicreform/020128_cap_mark_int_en.pdf).

[2] Adjaouté, K., Danthine, J.P., 2004. Portfolio diversification: alive and well in Euroland. Applied Financial Economics 14, 1225-1231.

[3] Aiyagari, R., Gertler, M., 1991. Asset returns with transactions costs and uninsured individual risk: a stage III exercise. Journal of Monetary Economics 27, 309-331.

[4] Basak, S., Cuoco, D., 1998. An equilibrium model with restricted stock market participation. Review of Financial Studies 11, 309-41.

[5] Bekaert, G., Harvey, C., Lumsdaine, R., 2002. Dating the integration of world equity markets. Journal of Financial Economics 65, 203-247.

[6] Bekaert, G., Harvey, C., Lundblad, C., 2005. Does financial liberalization spur growth?. Journal of Financial Economics 77, 3-56.

[7] Biais, B., Declerck, F., Dow, J., Portes, R., von Thadden, E.L., 2006. Corporate bond markets in Europe: transparency, liquidity, and efficiency. Unpublished report, CEPR, London.

[8] Brennan, M., Subrahmanyam, A., 1996. Market microstructure and asset pricing: on the compensation for illiquidity in stock returns. Journal of Financial Economics 41, 441-464.

[9] Canova, F., Ciccarelli, M., Ortega, E., 2007. Similarities and convergence in G7-cycles. Journal of Monetary Economics 54, 850-878.

[10] Chemmanur, T., Fulghieri, P., 2006. Competition and cooperation among exchanges: a theory of cross-listings and endogenous listing standards. Journal of Financial Economics 82, 455-489.

[11] Constantinides, G., 1979. Multiperiod consumption and investment behavior with convex transaction costs. Management Science11, 1127-1137.

[12] Constantinides, G., 1986. Capital market equilibrium with transaction costs. Journal of Political Economy 94, 842-862.

[13] Demsetz, H., 1968. The cost of transacting. Quarterly Journal of Economics 82, 33-53.

[14] Di Noia, C., 2001. Competition and integration among stock exchanges in Europe: network effects, implicit mergers and remote access. European Financial Management 7, 39-72.

[15] Domowitz, I., 2001. Liquidity, transaction costs and reintermediation in electronic markets. eBusiness Research Center Working Paper.

[16] Domowitz, I., Glen, J., Madhavan, A., 2001. Liquidity, volatility and equity trading costs across countries and over time. International Finance 4, 221-255.

[17] Domowitz, I., Steil, B., 2002. Securities Trading. In: Steil, B., Victor, D., Nelson, R. (Eds), Technological Innovation and Economic Performance. Princeton University Press; 2002. p.314-326.

[18] Dybvig, P., 2005. Mean-Variance Portfolio Rebalancing with Transactions Costs. Manuscript Washington University, St. Louis.

[19] Fernando, C., 2003. Commonality in liquidity: transmission of liquidity shocks across investors and securities. Journal of Financial Intermediation 12, 233- 254.

[20] Foucault, T., Parlour, C., 2004. Competition for Listings. RAND Journal of Economics 35, 329-355.

[21] Galati, G., Tsatsaronis, K., 2001. The impact of the euro on Europe's financial markets. Bank of International Settlements Working Paper N^{o}100.

[22] Gehrig, T., 1998. Competing markets. European Economic Review 42, 277-310.

[23] Goetzmann, W., Li., L., Rouwenhorst, G., 2005. Long-Term global market correlations. Journal of Business 78, 1-38.

[24] Hau, H., 2001. Location matters: an examination of trading profits. Journal of Finance 56, 1959-1983.

[25] Huddart, S., Hughes, J., Brunnermeier, M., 1999. Disclosure requirements and stock exchange listing choice in an international context. Journal of Accounting and Economics 26, 237-269.

[26] Jain, P., 2005, Financial market design and the equity premium: electronic versus floor trading. Journal of Finance 60, 2955-2986.

[27] McKinsey/JP Morgan, 2002. The future of equity trading in Europe, London.

[28] Lane, P., Milesi-Ferretti, G., 2004, Financial globalization and exchange rates. Working Paper 05/03, International Monetary Fund.

[29] Lewis, K., 1999. Trying to explain home bias in equities and consumption. Journal of Economic Literature 37,571-608.

[30] Liu, H., 2004. Optimal consumption and investment with transactions costs and multiple risky assets. Journal of Finance 59, 289-338.

[31] Pagano, M., 1989. Endogenous market thinness and stock price volatility. Review of Economic Studies 56, 269-287.

[32] Pagano, M., Röell, A., 1990, Trading systems in European stock exchanges: current performance and policy options. Economic Policy 5, 63-115.

[33] Pagano, M., Röell, A., Zechner, J., 2002., The geography of equity listing: why do companies list abroad?, Journal of Finance 57, 2651-2694.

[34] Pirrong, C., 1999, The organization of financial exchanges markets: theory and evidence. Journal of Financial Markets 2, 329-357.

[35] Portes, R., Rey, H., 2005. The determinants of cross-border equity flows, Journal of International Economics 65, 269-296.

[36] Rochet, J.C., Tirole, J., 2006, Two-Sided Markets: A Progress Report. The RAND Journal of Economics 37, 645-667.

[37] Steil, B., 2002, Changes in the ownership and governance of securities exchanges: causes and consequences. Brookings-Wharton Papers on Financial Services, 61-91.

[38] Shy, O., Tarkka, J., 2001. Stock exchange alliances, access fees and competition. Bank of Finland Discussion Papers 22.

[39] Tesar, L., Werner, I., 1998. The internationalization of securities markets since the 1987 crash. Brooking-Wharton Papers on Financial Services, 281-372.

[40] Vayanos, D., 1998. Transactions costs and asset prices: a dynamic general equilibrium model. Review of Financial Studies 11, 1-58.