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Exuberance and social contagion

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Abstract

Episodes of collective exuberance that recurrently hit the economy are, in this note, associated with sentiment propagation in a network of social relations. The pivotal role played by exuberant individuals will give place to a dynamic setting where limit cycles constitute the most plausible long-term outcome. Endogenous sentiment waves, with peaks and troughs of exuberance, are in this way identified in the context of a straightforward interaction scenario.

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1. Introduction

As thoroughly documented in Akerlof and Shiller (2009) and Shiller (2015), exuberance is a pervasive feature of economic activity and of the underlying human behavior. Prototypical and popular examples of markets where episodes of collective euphoria recurrently emerge include stock markets (Phillips *et al.,* 2011, 2015) and real estate (Huang, 2013; Kivedal, 2013). In a broader sense, extreme sentiments, which typically propagate fast across households and investors, are a source of observed aggregate business fluctuations and, therefore, they must be accounted for when assembling a comprehensive and rigorous theory of the macro economy.

An influential recent contribution in this direction has been offered by Angeletos and La'O (2013), who emphasize the role of changing sentiments in shaping business cycles. These authors suggest that imperfect communication in a local interaction market environment triggers a slow diffusion of information. As a result, sentiment shocks will hit the economy at a gradual pace and an inertia effect will preponderate, determining the kind of sluggishness that is documented, among others, in Sims (1998).

In the Angeletos-La'O model, sentiments are presented as an exogenous force capable of impacting the economy. Although a brief note is included suggesting that sentiment changes originate on rumor propagation, in the mentioned model this line of reasoning is not explored in detail; as in rumor spreading theory (see, e.g., Nekovee *et al.*, 2007 or Zhao *et al.*, 2012*a*), the population is separated in three categories (in the case, the uninformed, the exuberant and the informed) but no insights on the structure of the interaction are given.

In this note, sentiment contagion across a large population of individuals is explicitly modeled. A slight change in the designation of the categories of agents is introduced, namely one considers neutrality, exuberance and non-exuberance; these are attitudes each individual may have relatively to a given sentiment. A homogeneous network of degree 1 is assumed; in this network, at each date t, every agent will randomly contact with some other agent and, as a result of such contact, the individual attitude towards the sentiment might change.

Besides local direct interaction, sentiment switching is also determined, in the framework to propose, by an overall assessment that exuberant individuals make about population wide sentiment dynamics. This last assumption conducts to an equilibrium result characterized by the existence of a limit cycle. The limit cycle outcome suggests the presence of endogenous sentiment waves, with high and low exuberance periods alternating in time. The regular cycles originating in social contagion may constitute an important basis over which business fluctuations can be assessed; a similar claim is made by Beaudry *et al.* (2015), who associate observable irregular business cycles to a sequence of stochastic shocks that occur over an otherwise deterministic limit cycle foundation.

This note is organized as follows. Section 2 establishes a parallelism between the spreading of rumors, a widely explored subject in the scientific literature, and sentiment contagion. Section 3 characterizes the social contagion structure of analysis. In section 4, the main dynamic results are derived. Section 5 discusses the existence and stability of the limit cycle. Section 6 concludes by discussing how a limit cycle underlying the evolution of human sentiments might constitute a powerful piece in an integrated view on the generation and persistence of aggregate business fluctuations.

2. From Rumors to Sentiments

There is a voluminous amount of academic literature dealing with rumor propagation, in multiple contexts. Examples of popular applications include the dissemination of unconfirmed information in emergency scenarios (Huo *et al.*, 2013; Li and Ma, 2015), and the spread of gossip on on-line social media (*Zhao et al.*, 2012*b*; Jin *et al.*, 2013).

The prototype analytical model underlying most of the studies on the topic of rumor propagation corresponds to a simple dynamic system of equations originally addressed by Daley and Kendall (1965). This framework characterizes a process of social contagion, where the direct contact between members of a given population triggers a transition across states, specifically from the state of ignorance to the spreader state, and from this to the stifler category.

Because transitions occur through interaction between pairs of agents, the rumor typically spreads sluggishly over time, with an initial scenario of almost complete ignorance gradually giving place to a setting where everyone knows the rumor and where, eventually, everyone will end up by losing the initial compulsion to spread it.

The rumor propagation model might be interpreted as a useful framework to address many other social phenomena besides the dissemination of rumors in specific environments. In fact, it works as a metaphor for any social process of diffusion with origin in the direct contact between members of a population. Individuals share ideas, information, knowledge and sentiments, and all of these diffusion processes are addressable under the social epidemic setup usually applied to approach rumor spreading.

In the particular case of this paper, the mentioned framework is applied to sentiment spreading. Sentiments, as rumors, tend to spread among individuals as they contact with one another, as discussed in Zhao *et al.* (2014) and Gomes (2015). Low and high confidence levels or sentiments of pessimism and optimism, which are known to have a strong influence on the decisions of economic agents, might, then, be interpreted as subject to a similar kind of dynamics relatively to the one that underlies rumor propagation.

The framework proposed in the sections that follow receives direct inspiration from the rumor propagation setup, in the sense that it separates the population in three categories of agents, allowing agents to change category as they interact with one another in a degree 1 homogeneous network. Furthermore, probabilities of transition and the type of relations that trigger transitions are similar to those found in the prototype ignorant-spreader-stifler model.

However, despite the similarities, the adaptation of the analytical setting requires a few adjustments, which will become evident in the following section: first, the categories of agents receive new denominations - the neutral, the exuberant and the non-exuberant; second, while rumors have an irreversible nature (someone who knows the rumor will not recede to a state of ignorance), sentiments eventually retreat to the initial state; third, agents with a strong sentiment (the exuberant) will have a more active role in the propagation process than the agents in the other assumed categories (instead of a constant transition probability governing the decision of exuberant individuals to abandon this category, a probability contingent on the aggregate sentiment dynamics is taken).

The selected sentiment categories are supposed to conform with the evidence underlying the arguments in the first paragraph of the introduction. In most markets (financial and housing markets are good examples), periods of collective euphoria or apparent irrational exuberance are observed. Exuberance tends to set in at a relatively fast pace, as the reduced number of initial exuberant begins contaminating the agents who were unaware of any reason to change their based-on-fundamentals view of the market (to whom we call the 'neutral').

Exuberance will eventually fade out, as individuals continue to establish contact and realize that the factors underlying the generated sentiment bubble will not persist forever. Agents will then start a process of returning to neutrality, which contains two phases: first, they lose their exuberance (in the sense they do not force their point of view on others) but continue to behave in a way that feeds the bubble; as they contact with individuals that have returned to the neutrality state, the non-exuberant will also go back to such state. Meanwhile, another bubble may eventually emerge, turning this into a perpetual process that maintains markets in an out-of equilibrium position.

3. The Social Interaction Framework

Let x_t, y_t and z_t represent, respectively, the densities of neutral, exuberant and non-exuberant agents in a social network. At each date t, social interaction might move agents from one sentiment category to another according to a set of rules adapted from rumor propagation theory,

1) When a neutral individual meets an exuberant, the first will shift to the exuberance state with probability $\lambda \in (0,1]$;

2) When an exuberant meets another exuberant or a non-exuberant, the first turns into a non-exuberant with probability $\sigma \in (0,1]$;

3) When a non-exuberant meets an agent in the neutrality state, the non-exuberant becomes neutral with probability $\theta \in (0,1]$.

Exuberant individuals are assumed more attentive than the other agents; besides deliberating based on local contact, they will also make an assessment of the overall sentiment dynamics and decide not to change category when the number of agents sharing the sentiment is increasing relatively fast. Therefore, parameter σ in the above transition rules is replaced by function $\sigma_t = \frac{\bar{\sigma}}{2}[1 - \tanh(\kappa(\Delta y_t + \Delta z_t))]$, $\kappa > 0, \bar{\sigma} \in (0,1]$. Observe that if $\Delta y_t + \Delta z_t \to -\infty$ then $\sigma_t \to \bar{\sigma}$ and if $\Delta y_t + \Delta z_t \to +\infty$ then $\sigma_t \to 0$; when $\kappa \to \infty$, $\sigma_t = \bar{\sigma}$ for any $\Delta y_t + \Delta z_t < 0$ and $\sigma_t = 0$ for any $\Delta y_t + \Delta z_t > 0$.

Under the proposed transition rules and applying the law of mass action, sentiment dynamics are translated in a 2-dimensional system of difference equations,

$$\begin{cases} \Delta y_t = \lambda x_t y_t - \sigma_t y_t (y_t + z_t) \\ \Delta z_t = \sigma_t y_t (y_t + z_t) - \theta z_t x_t \end{cases} \tag{1}$$

with $x_t = 1 - y_t - z_t$.

4. Local Dynamics

Let $E = \{(y^*, z^*): \Delta y_t = 0, \Delta z_t = 0\}$ be the set of equilibrium points of system (1).

Proposition 1. *Equilibrium set E contains three points: the corner solutions e_1, $(y^*, z^*) = (0,0)$, and e_2, $(y^*, z^*) = (0,1)$, and the non-trivial equilibrium e_3, $(y^*, z^*) = \left(\frac{2\lambda\theta}{(2\lambda+\bar{\sigma})(\lambda+\theta)}, \frac{2\lambda^2}{(2\lambda+\bar{\sigma})(\lambda+\theta)}\right)$.*

Proof: Noticing that tanh(0)=0, in the long-term equilibrium $\sigma^* = \frac{\bar{\sigma}}{2}$. Applying, then, the equilibrium condition to system (1), one gets

$$\begin{cases} \lambda x^* y^* = \dfrac{\bar{\sigma}}{2} . y^* (y^* + z^*) \\ \dfrac{\bar{\sigma}}{2} . y^* (y^* + z^*) = \theta z^* x^* \end{cases}$$

The above system, solved with respect to the equilibrium values, has three solutions, namely those in the proposition ■

To address the local stability of each of the equilibria, one computes the Jacobian matrix associated to system (1),

$$J = \begin{bmatrix} 1+\lambda(1-2y^*-z^*)-\frac{\bar{\sigma}}{2}(2y^*+z^*)-\frac{\partial\sigma}{\partial y^*}y^*(y^*+z^*) & -\left(\lambda+\frac{\bar{\sigma}}{2}\right)y^*-\frac{\partial\sigma}{\partial z^*}y^*(y^*+z^*) \\ \frac{\bar{\sigma}}{2}(2y^*+z^*)+\frac{\partial\sigma}{\partial y^*}y^*(y^*+z^*)+\theta z^* & 1+\frac{\bar{\sigma}}{2}y^*+\frac{\partial\sigma}{\partial z^*}y^*(y^*+z^*)-\theta(1-y^*-2z^*) \end{bmatrix}, \tag{2}$$

with

$$\frac{\partial\sigma}{\partial y^*} = -\frac{\bar{\sigma}}{2}[\lambda(1-2y^*)-(\lambda-\theta)z^*]\,\kappa$$

$$\frac{\partial\sigma}{\partial z^*} = \frac{\bar{\sigma}}{2}[\theta(1-2z^*)+(\lambda-\theta)y^*]\kappa$$

Proposition 2. *Corner solutions* e_1 *and* e_2 *are not locally stable,* $\forall\lambda,\bar{\sigma},\theta \in (0,1]$*. The interior equilibrium,* e_3*, is locally stable under condition* $\kappa < \frac{1-\lambda}{2\bar{\sigma}}\left(\frac{2\lambda+\bar{\sigma}}{\lambda}\right)^2$.

Proof: For e_1, $J = \begin{bmatrix} 1+\lambda & 0 \\ 0 & 1-\theta \end{bmatrix}$, and for e_2, $J = \begin{bmatrix} 1-\frac{\bar{\sigma}}{2} & 0 \\ \frac{\bar{\sigma}}{2}+\theta & 1+\theta \end{bmatrix}$. In both cases, two non-negative real eigenvalues exist, one located inside and the other outside the unit circle.

For e_3, the Jacobian matrix is

$$J = \begin{bmatrix} 1+\lambda\chi-\alpha & -\theta\chi-\alpha \\ \alpha+\lambda(\beta-\chi) & 1-\theta(\beta-\chi)+\alpha \end{bmatrix},$$

with $\alpha \equiv \frac{\lambda\theta}{\lambda+\theta}$, $\beta \equiv \frac{\bar{\sigma}}{2\lambda+\bar{\sigma}}$, $\chi \equiv \frac{2\lambda^2\bar{\sigma}^2\theta}{(2\lambda+\bar{\sigma})^3(\lambda+\theta)}\kappa$.

Let $Tr(J)$ and $Det(J)$ represent, respectively, the trace and the determinant of matrix J. The conditions for stability are $1+Tr(J)+Det(J)>0$, $1-Tr(J)+Det(J)>0$, $1-Det(J)>0$. The first two conditions are satisfied, regardless of parameter values. The third condition requires $\theta\beta > (\lambda+\theta)(\alpha\beta+\chi)$. Solving the inequality with respect to κ, the inequation in the proposition is derived ■

The two exuberance-free steady-states are saddle-path equilibria; they will never be feasible long-term solutions unless $y_0 = 0$. Thus, if one assumes that at least one individual, no matter the dimension of the population, is exuberant, one can concentrate the analysis on e_3. The interior equilibrium is locally stable for a relatively low value of parameter κ; as progressively larger values of κ are assumed, stability will be eventually lost as the system undergoes a Neimark-Sacker bifurcation.

Fig. 1 displays, for e_3, the admissible region of local dynamics in the trace-determinant space. Given the imposed constraints on the values of parameters, the dynamics are circumscribed to a region that is delimited by inequalities $Tr(J)-1 < Det(J) < Tr(J)$ and $Tr(J) > 1$. The diagram makes it evident that the only possible bifurcation to occur in this case is a Neimark-Sacker bifurcation.

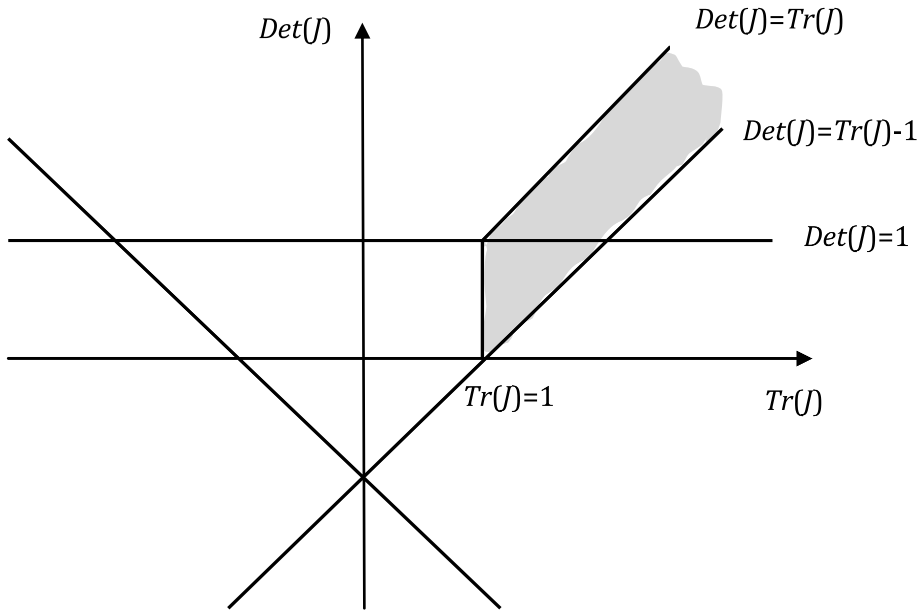

Fig. 1 – Trace-Determinant diagram.

5. The Limit Cycle

Numerical inspection of the dynamic behavior of y_t and z_t indicates that the Neimark-Sacker bifurcation gives place to a limit cycle for every value of κ larger than the respective bifurcation value.

To formally address the existence of a limit cycle, consider the following constraint over parameter values: $\lambda = \bar{\sigma} = \theta = 1$. Under this constraint, the bifurcation occurs at $\kappa = 0$, and therefore e_3 is unstable $\forall\kappa \in \mathbb{R}^+$. Fig. 2 represents an attractor for the relation between the two endogenous density variables, for $\kappa = 2.5$; the attractor takes the form of a closed invariant curve.

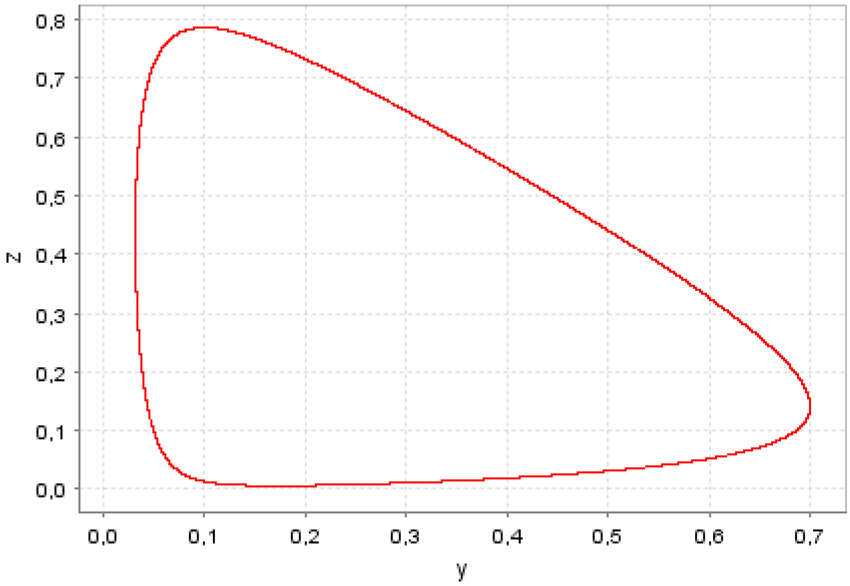

Fig.2 – Long-term attractor.

Both the graphical analysis and intuition suggest that the Neimark-Sacker bifurcation is supercritical. This signifies that independently of the initial state, and as long as this state does not coincide with any of the three unstable equilibrium points of the system, all orbits originating outside or inside the attractor will converge to it. The intuition is straightforward: if admissible values of both endogenous variables are confined to the interval [0,1] and the three equilibrium points are unstable, trajectories will not converge to any of them and, presumably, they follow the path towards the closed invariant curve. This is confirmed by the graphic: if the bifurcation were subcritical, the trajectories of the variables would be repelled from it and, in fact, the limit cycle would not be observable.

A proof of the existence of a limit cycle and of a supercritical bifurcation follows.

Proposition 3. *Let* $\lambda = \bar{\sigma} = \theta = 1$*. The dynamics of system (1) is characterized by the formation of a limit cycle. This limit cycle is locally attractive, i.e., the Neimark-Sacker bifurcation that generates it is supercritical.*

Proof: Marsden and McCracken (1976), enumerate the conditions for the existence of a closed invariant curve in a 2-dimensional difference equations system,

1) Non-hyperbolicity condition: $|\lambda(0)| = 1$;

2) Non-strong-resonance condition: ${\lambda_1}^k(0) \neq 1$ for k=1,2,3,4;

3) Transversality condition: $\left.\frac{d|\lambda(\kappa)|}{d\kappa}\right|_{\kappa=0} = d \neq 0$;

4) Genericity condition:

$$a = -Re\left[\frac{\left(1-2\lambda_1(0)\right)\left(\lambda_2(0)\right)^2}{1-\lambda_1(0)}c_{11}c_{20}\right] - \frac{1}{2}|c_{11}|^2 - |c_{02}|^2 + Re(\lambda_2(0)c_{21}) \neq 0,$$

with

$$c_{20} = \frac{1}{8}\left[\left(f_{yy} - f_{zz} + 2g_{yz}\right) + i\left(g_{yy} - g_{zz} - 2f_{yz}\right)\right]$$

$$c_{11} = \frac{1}{4}\left[\left(f_{yy} + f_{zz}\right) + i\left(g_{yy} + g_{zz}\right)\right]$$

$$c_{02} = \frac{1}{8}\left[\left(f_{yy} - f_{zz} - 2g_{yz}\right) + i\left(g_{yy} - g_{zz} + 2f_{yz}\right)\right]$$

$$c_{21} = \frac{1}{16}\left[\left(f_{yyy} + f_{yzz} + g_{yyz} + g_{zzz}\right) + i\left(g_{yyy} + g_{yzz} - f_{yyz} - f_{zzz}\right)\right]$$

If conditions 1 to 4 are satisfied, an invariant closed curve is formed at the bifurcation point. Furthermore, this curve is attracting (supercritical bifurcation) if $d > 0$ and $a < 0$.

In the above expressions, $\lambda_1(0)$, $\lambda_2(0)$ are the eigenvalues of the Jacobian matrix at bifurcation point $\kappa = 0$; in the current case they correspond to $\lambda_1(0), \lambda_2(0) = \frac{5}{6} \pm i\frac{\sqrt{11}}{6}$. The term $|\lambda(0)|$ is the modulus of the pair of complex conjugate eigenvalues at $\kappa = 0$. Functions f and g denote the r.h.s. of each of the two equations in system (1); the c terms involve partial derivatives of these functions evaluated at the bifurcation point.

The observance of the first two conditions is self-evident. Regarding the other two, computation indicates that

$$d = \frac{1}{27}$$

$$a = -\frac{11}{72}\sqrt{11} - \frac{173}{192}$$

Because $d > 0 \wedge a < 0$, one confirms the formation of closed invariant curves of an attracting nature ■

6. Conclusion

Sentiment shocks, understood as a source and a driver of business fluctuations, are gradually occupying their rightful place in mainstream macroeconomic theory. In order to strengthen the explanatory and predictive power of the theory, it is vital to identify and characterize the forces that generate and feed the episodes of collective euphoria. This note offers a tentative explanation on the causes of the intermittent resurgence of phases of strong exuberance. The explanation is based on social interaction and contagion, with the particularly attentive posture of exuberant individuals inducing the formation of limit cycles that support the persistence of sentiment waves that, in turn, will be reverberated into the economy.

The proposed interaction model should be interpreted as a baseline structure over which one can assess aggregate business decisions and market transactions. Evidently, business cycles are fed by technology shocks, policy actions, preference changes, stickiness on the adjustment of prices and wages, and many other factors recurrently identified in the literature. However, beyond all these features of the economy, there is an underlying natural tendency for businesses to reveal more vitality in some periods and less vitality on others, as a result of people's confidence or sentiments. As Keynes brilliantly put it, in his *General Theory*, people are commanded by their animal spirits and, therefore, psycho-sociological factors should not be disregarded when evaluating (macro)economic outcomes.

A possible avenue for future research that the offered analysis suggests consists in inquiring in what extent is it possible to separate purely economic driven fluctuations from those dictated by the dynamics of interaction and sentiment switching. It can be a meaningful empirical exercise to identify how countries with similar infrastructural conditions diverge in terms of the duration and intensity of their periods of recession and expansion possibly as a result of how societies are more or less prone to different kinds of social interaction.

Bringing social interaction considerations to the realm of stabilization policy might assist public authorities in understanding that the same policy recipes are not always adequate to confront the same economic issues, when these issues unfold in distinct social scenarios. How individuals interact and how exuberance emerges and vanishes are factors that matter to economic policy and that compromise the application of universal policy rules to different social and cultural environments in space and time.

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