

Unemployment Period Approach with Infinite Servers Queue Systems

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With results on the infinite servers queue systems with Poisson arrivals - $M|G|\infty$ queues - busy period, it is displayed an application of those queue systems in the unemployment periods time length parameters and distribution function study. These queue systems are adequate to the study of many population processes, and this quality is brought in here. The results presented are mainly on unemployment periods length and their number in a certain time interval. Also, some questions regarding the practical applications of the outlined formulas are briefly discussed.

Keywords: infinite servers queues, busy period, unemployment

The Model Rising

In the queue systems used thoroughgoing in this article, see for instance, Ferreira, Filipe, and Coelho (2016), the customers arrive, according to a Poisson process at rate λ and receive a service which time length is a positive random variable with distribution function $G(\cdot)$ and mean α . When a customer arrives it finds immediately an available server¹, each customer service is independent from the other customers' services and from the arrivals process and the traffic intensity is $\rho = \lambda\alpha$. That is: they are $M|G|\infty$ queues. It is easy to understand how these queues can be applied to the unemployment period study. Then λ is the rate at which occur the firings, supposed to happen according to a Poisson process and the service time, paradoxically, is the time between the worker firing and his/her discovery of a new job.

In any queue system, a busy period is a period that begins when a customer arrives at the system finding it empty, ends when a customer abandons the system letting it empty and there is always at least one customer present. So in a queuing system, there is a sequence of idle and busy periods, during its operation time. In the $M|G|\infty$ queue system, as in any queue system with Poisson arrivals, the idle periods have an exponentially distributed length with mean λ^{-1} . But the busy period's distribution is much more complicated, being in general given by infinite sums which parcels are convolutions (Ferreira & Andrade, 2009). Despite it, it is possible to present some results as it will be seen. For what interests in this work: a busy period is a period of unemployment and an idle period is a period of full employment.

The results to be presented are on unemployment periods length and their number in a certain time interval. A study in following also this approach, over a public health situation, can be seen in Ferreira (2014).

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¹ Or there is no distinction between the customer and its server, as it happens in the application to be considered in this work.

Unemployment Period Time Length Distribution

Designate D the random variable unemployment period length. According to the results known for the $M|G|\infty$ queue busy period length distribution, see Ferreira and Andrade (2009):

$$E[D] = \frac{e^\rho - 1}{\lambda} \tag{2.1}$$

whichever is the worker unemployment time length distribution, see Takács (1962).

As for $Var[D]$, it depends on the whole unemployment time length distribution probabilistic structure. But Sathe (1985) demonstrated that:

$$\max[e^{2\rho} + e^\rho \rho^2 \gamma_s^2 - 2\rho e^\rho - 1; 0] \leq \lambda^2 Var[D] \leq [2e^\rho (\gamma_s^2 + 1)(e^\rho - 1 - \rho) - (e^\rho - 1)^2] \tag{2.2}$$

where γ_s is the unemployment time length coefficient of variation.

For α and ρ great enough (very intense unemployment conditions) since $G(\cdot)$ is such that for α great enough $G(t) \cong 0, t \geq 0$:

$$D(t) \cong 1 - e^{-\lambda e^{-\rho} t}, t \geq 0 \tag{2.3}$$

see Ramalhoto and Ferreira (1996).

Calling N_D the mean number of unemployed people in the unemployment period, if $G(\cdot)$ is an exponential distribution:

$$N_D = e^\rho \tag{2.4}$$

For any other $G(\cdot)$, probability distribution:

$$N_D \cong \frac{e^{\rho(\gamma_s^2+1)} (\rho(\gamma_s^2+1)+1) + \rho(\gamma_s^2+1) - 1}{2\rho(\gamma_s^2+1)} \tag{2.5}$$

see Ferreira and Filipe (2010). Of course, multiplying Equation (2.4) or (2.5), as appropriate, by the mean cost of each unemployment subsidy, it is possible to estimate the assistance costs caused by the unemployment period.

Unemployment Periods in a Time Interval Mean Number

After the renewal processes theory (see Çınlar, 1975), calling $R(t)$ the mean number of unemployment periods that begin in $[0,t]$, being $t = 0$ the beginning instant of an unemployment period, it is possible to obtain (see Ferreira, 1995):

$$R(t) = e^{-\lambda \int_0^t [1-G(v)] dv} + \lambda \int_0^t e^{-\lambda \int_0^u [1-G(v)] dv} du \tag{3.1}$$

and consequently, the following simple upper and lower bounds for $R(t)$ (see Ferreira, 2004):

$$e^{-\rho} (1 + \lambda t) \leq R(t) \leq 1 + \lambda t \tag{3.2}$$

Conclusions

So that this model can be applied, it is necessary that the firings occur according to a Poisson process at a constant rate. It is a hypothesis that must be tested. Thus remain outside of this study, periods of unemployment caused by mass firings.

Among the results presented, Equations (2.1), (2.2), (2.3), (2.5), and (3.2) are remarkable for its simplicity and for requiring only the knowledge of the firings rate λ , the mean unemployment time α , and the unemployment time variance.

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