

# INTEREST RATE RISK IMMUNIZATION - THE IMPACT OF CREDIT RISK IN THE QUALITY OF IMMUNIZATION

## CASE STUDY: IMMUNIZATION WITH PORTUGUESE BONDS AND GERMAN BONDS

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**Abstract** – This paper contains an assessment of the interest rate risk present in Financial Institutions and the methods used for its immunization. The paper consists of two parts. The first part presents a theoretical review of the interest rate risk and how this risk can be immunized. Concepts such as Macaulay (1938) and Fisher & Weil (1971) duration and their limitations in the process of the approximation to the price of a considered bond will be highlighted. In the second part, the main indicators of the credit risk on bonds are analyzed. Based on market prices of Portugal's bonds and Germany's bonds, the quality of immunization is tested. The interest rate derivatives are then introduced as a method of hedging interest rate risk. Finally, an interview is conducted with the head of hedging the interest rate risk in one of the largest private banks in Portugal in order to identify the methods used to capture the interest rate risk and to understand how this risk is immunized. This research allows us to emphasize the importance of credit risk in an immunization strategy of interest rate risk. We conclude that interest rate hedging based on Fisher & Weil (1971) duration is not possible in a scenario of high volatility credit risk. Interest rate hedging based on interest rate swaps becomes more attractive to Financial Institutions.

**Keywords** - Duration, Interest Rate Risk, Immunization, Credit Risk.

### 1. Introduction

In late 2009, the first signs of a sovereign debt crisis in the Euro Zone appeared. In 2010, Greece and Ireland were the first countries to ask for support from the International Monetary Fund, followed by Portugal's request in 2011.

The European sovereign debt crisis resulted from a combination of complex factors and financial

globalization. A major reason was the ease of access to credit, from 1999 to 2007, which encouraged high-risk loans, and the global financial crisis, which began in 2007 and required a bailout of the financial sector.

Thus arose a crisis of confidence in financial markets, which led to the widening of bond spreads and credit default swaps between these countries and other members of the European Union, especially to Germany.

These financial indicators led to several downgrades in the banking sector and countries in the Euro Zone. Those downgrades were made by Standard & Poors and Moody's. Currently, Greece and Portugal are considered high risk investments (junk bonds). Given this, the yield to maturity of these countries reached values which were too high, making it unsustainable to get credit without the assistance of others.

With this high volatility in financial markets, it is essential to create an efficient hedge of various financial risks, including the interest rate risk and credit risk. Due to its importance, we will now define these two types of risk. The interest rate risk is the risk for a portfolio or business resulting from an adverse change in interest rates in the financial market. In turn, the credit risk is the risk of loss of principal or loss of a financial reward stemming from a borrower's failure to repay a loan or otherwise meet a contractual obligation.

Taking this into account, we can then consider interest rate risk immunization strategy as the strategy to ensure that, regardless of the evolution at the level of interest rates, the future value of an investment is at least equal to the value that would be obtained if interest rates did not change.

On this basis, several concepts have been developed over time. In interest rate risk immunization described by Bierwag (1987), Macaulay duration (1938) is a fundamental concept and represents the

price elasticity of a bond to changes in the interest rate. In the 1970s, Fisher & Weil (1971) criticize how the Macaulay duration was calculated and develop the Fisher-Weil duration.

## 2. Research Objectives

At a time when financial markets are increasingly volatile and that their impact on the results of Financial Institutions is increasing, it is important to study the perception of how Financial Institutions capture and hedge interest rate risk.

That said, it should be noted that this paper is focused precisely on interest rate risk and the methods by which this risk is mitigated by Financial Institutions. Various theories regarding this topic are to be presented and discussed, in order to support analysis to be done after this case study. In this paper, based on market prices of Portuguese bonds and German bonds, we will test the quality of interest rate risk immunization using the Fisher & Weil (1971) duration.

Portuguese bonds and German bonds were selected due to their stance at opposite sides regarding credit risk. Currently Portugal is seen as an investment with a high credit risk, while Germany is considered as the safest investment in Euro Zone.

It is intended to verify if the instability of the credit risk in the Euro Zone allows interest rate risk immunization using the concept of Fisher & Weil (1971) duration.

Later, based on interviews with those responsible for the interest rate risk at Financial Institutions, we will make conclusions about the way interest rate risk immunization is carried out in financial markets.

In the end, interest rate swaps will be introduced as a method used by Financial Institutions to hedge interest rate risk.

## 3. Literature review

In this section we will conduct a review of the financial literature on various topics regarding interest rate risk in order to analyze the quality of interest rate risk immunization using Fisher & Weil (1971) duration.

Initially we address issues such as interest rate risk and its implications for Financial Institutions. Later we define the concept of duration as an approximation for calculating the bond price after a change in interest rates in the financial market.

At the end of this section, we analyze the main models of interest rate risk immunization, with special emphasis at interest rate risk immunization using Fisher & Weil (1971) duration.

These concepts are critical to the performance of the case study and its conclusions. Based on Fisher & Weil (1971) duration we will test its applicability in interest rate risk immunization, using market prices of Portuguese bonds and German bonds.

### 3.1. Interest Rate Risk in Financial Institutions

One of the most important sources of risk in Financial Institutions is the interest rate risk which arises from the uncertainty regarding future interest rates. Fooladi & Gordon (2000) define the business sector and speculation as the main sources of the interest rate risk in Financial Institutions, stating that:

- Speculation is related to a bet made on the forecast of future interest rates. Thus, a forecast of lower interest rates in the future, investment must be made in bonds at fixed interest rates, in order to maximize the expectation of interest received. Given an opposite prediction of the movement in interest rates in the future, investment must be made in bonds at floating interest rates to track the rise in the interest rate. Regarding financing of Financial Institutions and forecasting a drop in interest rates in the future, funding should be conducted at a floating interest rate to minimize the expectation of interest paid. Given an opposite prediction of the movement in interest rates in the future, funding should be made at fixed interest rate in order to minimize the interest paid.

- By definition, the business sector of Financial Institutions, which is based on the trade-off between credit and deposits, the mismatch that may exist between the type of interest rate of assets<sup>1</sup> and liabilities<sup>2</sup> can cause high volatility in cash flows when the term structure of interest rates change.

Thus, the interest rate risk is the risk that results from an unfavorable change in interest rates in the financial market, resulting in a negative impact on the results of the Financial Institutions.

Pinheiro & Ferreira (2008) studied the speculation ability of Financial Institutions between 1980 and 2003 (having analyzed 371 Financial Institutions). According to the sign of the duration gap<sup>3</sup> a Financial Institutions have a forecast of interest rates in the future. Thus, a positive duration gap is a bet on rising interest rates in the future. In the opposite, a negative duration gap is a bet on falling interest rates in the future. They concluded that, in general, Financial Institutions failed to forecast interest rates in the future. This means that when Financial Institutions had a positive duration gap the interest rates on the

<sup>1</sup> A resource with economic value that a corporation owns or controls with the expectation that it will provide future benefits;

<sup>2</sup> A corporate legal debts or obligations that arise during the course of business operations;

<sup>3</sup> Asset duration less liability duration. Asset duration is the average duration of the portfolio's assets and liabilities duration is the average duration of the portfolio's liabilities.

financial market fell, and when duration gap was negative interest rates in the financial market rose. Both scenarios lead to losses in the financial statements of Financial Institutions.

Thus, Pinheiro & Ferreira (2008) suggest an active approach in interest rate risk immunization, reducing the volatility of cash flow and results of Financial Institutions.

Smith & Stulz (1985) argue that interest rate risk immunization is a way to create value for Financial Institutions. The main benefits identified by the authors are:

- The tax benefit in the Financial Institutions, because it allows the reduction of earnings volatility;
- Reducing the credit risk and therefore the probability of bankruptcy. This benefit is due to the reduction in the volatility of cash flows, and
- The reduction of agency costs, ie, reducing conflicts between management and stakeholders.

Froot et al. (1993) add that interest rate risk immunization allows funding at lower interest rates. This decrease in the cost of funding is associated with reduction of credit risk taken by creditors.

Pennings & Leuthold (2000) consider that the future contracts can develop a relationship of trust between the Financial Institutions.

Then, Pinheiro & Ferreira (2008) describe the main methods used to calculate the interest rate risk exposure of Financial Institutions:

- Funding gap is described as the allocation of assets and liabilities based on different maturities. This method is limited because book values are used and intermediate cash flow are neglected, as is the case of interest and repayment of capital; and
- According to the authors, the gap duration method involves calculating the duration of assets and liabilities. In section 4. we identify the limitations of the duration gap as an indicator of the interest rate risk.

Pinheiro & Ferreira (2008) refer to the increasingly important role of interest rate derivatives in the hedging strategy, with particular emphasis on the interest rate swap. The authors argue that this instrument allows for better adaptation to the needs of Financial Institutions and provide a better quality interest rate risk immunization.

Brewer III et al. (2001) argue that the flexibility of the interest rate swap allows Financial Institutions to adapt the portfolio to the forecast of interest rates in the future by exchanging a fixed interest rate for a

variable interest rate, and vice versa. It does not require initial investment from Financial Institutions.

In the case study presented in section 6. we will use the duration gap to test the quality of the interest rate risk immunization based on a portfolio consisting of Portuguese bonds and German bonds. In the same section we introduce interest rate derivatives as a hedging instrument used by Financial Institutions.

### 3.2. Bonds Duration

The duration is quite an old indicator in financial literature. Initially presented by Macaulay (1938), the duration is an indicator of the average time a bond needs to create its value.

The Macaulay duration is assumed based on two assumptions:

- Term structure of interest rates are constant for all maturities; and
- Changes in the term structure of interest rates are parallel.

Macaulay duration is then, on that basis, calculated as follows:

$$D_{MAC} = \frac{\sum_{t=1}^n \frac{t C_t}{(1+r)^t}}{P_0} \quad (1)$$

Where  $C_t$  is the cash flow received in  $t$ ,  $r$  is the discount factor of cash flows and  $P_0$  is the bond price.

This means, according to the formula shown, that bond duration is a weighted average of maturity of each of their cash flow. The weighting given to each of the maturities is equal to the proportion of the value of the bond that is equal to the cash flow that occurs at that maturity.

Later, Fisher & Weil (1971) expanded on the concept of duration which Macaulay had created. The Fisher-Weil duration requires only one assumption:

- Changes in the term structure of interest rates are parallel.

Unlike the concept of Macaulay duration, Fisher & Weil duration considers different interest rates for different maturities.

The Fisher & Weil formula duration is similar to Macaulay duration, except the cash flows of different periods are discounted using different interest rates. Thus, the Fisher & Weil duration is better suited to the financial market (see section 4.1. which includes analysis of the Term Structure of Interest Rates).

Fisher & Weil duration is calculated using the following formula:

$$D_{FW} = \frac{\sum_{t=1}^n \frac{t C_{(t)}}{(1+r_{tj})^t}}{P_0} \quad (2)$$

Where  $C_{(t)}$  is the cash flow received in  $t$ ,  $r_{tj}$  is the discount factor of the cash flow in period  $t$  and  $P_0$  is the bond price.

Macaulay duration is a particular case of Fisher & Weil duration when  $r_{0,1} = r_{0,2} = \dots = r_{0,T}$ , i.e. when the term structure of interest rates is constant for all maturities.

In the case study (section 6.) we use Fisher & Weil (1971) duration to test the quality of interest rate risk immunization using market prices of Portuguese bonds and German bonds.

Soon after the presentation of the concept by Macaulay, Hicks (1939) developed the interpretation of duration as a measure of price elasticity of the bond against movements in the term structure of interest rates.

Much later, with the work of Hopewell & Kaufman (1973), we arrive at the following expression to get the bond price when the term structure of interest rates changes:

$$\Delta P_0 = -D \frac{\Delta r}{(1+r)} \quad (3)$$

Where  $D$  is the duration,  $r$  is the interest rate and  $P_0$  is the price of the bond.

The analysis of this formula allows us to conclude that the duration of a bond indicates the percentage decrease in its price when the interest rate increases 100 basis points (1%). Thus, if interest rates increase (decrease) the value of the bond decreases (increases).

The interest rate risk decreases if the amount of exposure or duration of the portfolio is reduced.

Duration is a measure of the bond price sensitivity to movements in the interest rate. There are three drivers which influence the duration value and, consequently, the interest rate risk:

- The duration increases with maturity, but at a decreasing rate:

$$\frac{\partial D}{\partial n} > 0 \quad \text{and} \quad \frac{\partial^2 D}{\partial n^2} < 0$$

- The duration decreases with the increasing level of interest rates, because the discount factors

decrease more sharply for longer periods than for short periods:

$$\frac{\partial D}{\partial r} < 0$$

- The duration decreases with the increasing coupon rate of the bond, because the weight of cash flows increases over the nominal value.

The properties of the duration will allow us to understand the result obtained in the case study. The yield to maturity, maturity and coupon rate of the Portuguese bonds and German bonds are important indicators for the conclusions of the outcome of interest rate risk immunization.

### 3.3. Interest Rate Risk Immunization using Bonds Duration

An interest rate risk immunization strategy aims to ensure, in the present moment (time "0") that at the end of a given investment time horizon ("h" periods), and regardless of any developments that will occur concerning interest rates, the future value of the portfolio is at least equal to the value that would be obtained if interest rates did not change. This means that the total return rate of the portfolio is at least equal to that which would be obtained in a scenario of stable interest rates.

Because it is not necessary to have a vision of the future term structure of interest rates, interest rate risk immunization using duration is considered a passive strategy, which is very useful in a scenario of high volatility of interest rates.

Early versions of interest rate risk immunization using duration were created by Samuelson (1945) and Redington (1952). Later, interest rate risk immunization was demonstrated by Fisher & Weil (1971), using the following condition:

- A portfolio is immunized against any change in interest rates if its duration is equal to the investment time horizon.

Therefore, when investment time horizon is equal to Fisher & Weil duration, we have:

$$S_h^c \geq P_0^c \cdot (1+i_h)^h \quad \text{e} \quad TRR_h \geq i_h$$

Where  $P_0^c$  is the price of the obligation in the period  $c$ ,  $i_h$  is the interest rate at period  $h$  and  $P_0^c \cdot (1+i_h)^h = S_h^c$ .

Bierwag (1987a, Chapter 4) demonstrated the applicability of interest rate risk immunization using duration, by defining two concepts: price risk and reinvestment risk.

Price risk is characterized by the fact that any movement in the term structure of interest rates lead to a change in bond prices. The term structure of interest rates impacts bond prices as it is used in the discounted value of the future cash flows. If the bond is sold before maturity, an increase (decrease) in interest rates is unfavorable (favorable) for the investor as it implies a decrease (increase) in the bond price.

The reinvestment risk is characterized by the fact that any changes in the term structure of interest rates leads to the reinvestment of cash flows at different rates. Thus, an increase (decrease) in interest rates is favorable (unfavorable) to the investor because it creates investment opportunities at higher rates (lower).

Both price risk and reinvestment risk of the bond are the main drivers for the magnitude of interest rate risk.

When Fisher & Weil (1971) duration equals the investment time horizon, the two effects are of equal magnitude and opposite signs, and thus cancel each other out.

Bierwag & Roberts (1990) conducted a study about Canadian bonds between the period 1963 and 1986. They concluded that portfolios with higher duration are more sensitive to interest rate movements and the Macaulay duration explains 80% of the variation in the portfolio value. Later, using the same sample, Fooladi & Roberts (1992) studied interest rate risk immunization using duration. They assumed an investment time horizon of five years, with semiannual portfolio rebalancing. They concluded that interest rate risk immunization using duration was effective.

This information will enable a better understanding of the portfolio created in the case study to hedge interest rate risk.

### 3.4. Topics used on case study

The concepts presented in this section allow us to achieve the objectives set for the case study.

In this section, we describe the concept 'interest rate risk' and given the nature of the business sector, the Financial Institutions should take an active interest rate risk immunization strategy. We have introduced the concepts of duration and the way this can be used in an interest rate risk immunization strategy.

Based on the presentation of these concepts, we can now apply them, in particular the concept of Fisher & Weil (1971) duration, which is more suited to the reality of the financial market, to create a portfolio in order to hedge interest rate risk. This portfolio will be created using Portuguese bonds and German bonds.

The purpose of the case study is to verify if the Fisher & Weil (1971) duration is effective in implementing interest rate risk immunization, regardless of the issuer of the bonds and the credit risk volatility in financial markets.

## 4. Limitations of Bonds Duration

In section 3. we introduced the concept of duration as an indicator of interest rate risk. The duration has been the target of several complex studies and is difficult to implement in practice. Following from the above in this section, we can also add that Fisher & Weil (1971) duration is best known in financial literature and the most used in financial markets.

In this section we discuss the main limitations of duration as an approximation to the bond price, which is important because it will duly justify the results obtained in the case study.

### 4.1. Term Structure of Interest Rates

As mentioned, the concept of duration was introduced by Macaulay (1938) and was based on two assumptions:

- Term structure of interest rates are constant for all maturities, and
- Movements at the term structure of interest rate are parallel.

The first assumption Macaulay relied on, keeping term structures constant for all maturities, is not realistic because it is easy to see that interest rates change depending on the different maturities.

In the next graph we can observe the term structure of interest rates on 22 October 2012:



Figure 1 - Term Structure of Interest Rate

(Source: Bloomberg)

As we can see the term structure of interest rate currently assumes a crescent shape. This means that investors require a higher interest rate as the maturity

increases, i.e. the greater the period, the greater the interest rate required by the investor.

Based on financial market information (see graph above) we conclude that it is not possible to guarantee an adequate interest rate risk immunization using the Macaulay (1938) duration, because it assumes a fact that is not the present in the financial market today.

The Fisher & Weil (1971) duration is better suited to financial market conditions, since it does not assume a constant term structure of interest rate. For this reason the case study is performed using the Fisher & Weil (1971) duration.

**4.2. Parallel Movements in the Term Structure of Interest Rate**

The assumption made by Fisher & Weil (1971), which is that movements in the term structure of interest rates are parallel, is not characterized by what is happening in the financial markets. Movements in interest rates may take different magnitudes and opposite directions in different maturities. In the next graph we can see the change of the term structure of interest rates from the 15th to 19th October 2012:

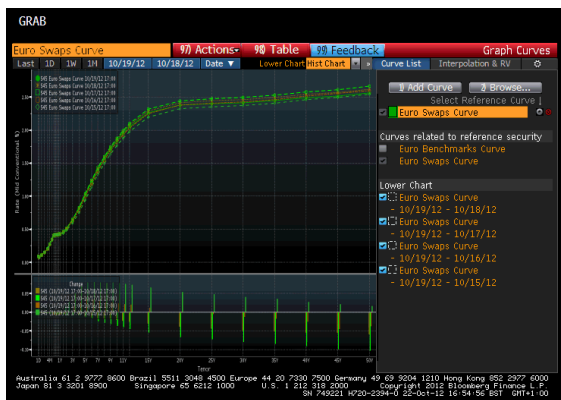


Figure 2 – Movements in the Term Structure of Interest Rate

(Source: Bloomberg)

As can be seen, the change of the term structure of interest rates was more significant in the long term compared to short term. In the long term there is volatility in the interest rate, while the short-term interest rates remained unchanged.

Based on financial market information (see graph above), we conclude that the movements of the term structure of interest rates are not parallel. The concept of parallel movements may, however, be useful for creating stress test scenarios. Based on variations of

the same magnitude in the short and long term of the term structure of interest rate (usually +/- 100 Basis Points) it is possible to get extreme scenarios of interest rates and calculate their impact on the results of Financial Institutions.

**4.3. Infinitesimal Movements in the Term Structure of Interest Rates**

Duration can be a good indicator of bond price sensitivity for infinitesimal movements in the term structure of interest rates. However, in financial literature there is no definition for what is considered to be an infinitesimal change of the term structure of interest rates. Sometimes we can see high volatility in the term structure of interest rates in the financial markets. Consider the following graph with the term structure of interest rate movements between the 26<sup>th</sup> to the 29<sup>th</sup> of September 2008 (Friday to Monday):



Figure 3 - Volatility in the Term Structure of Interest Rates

(Source: Bloomberg)

This term structure of the interest rate movements is related to the feeling of distrust between Financial Institutions that has developed in the financial markets after the collapse of Lehman Brothers in September 2008.

Duration is the first derivative of the relationship between bond price and interest rate. When there are big movements at the term structure of interest rates, using duration as an approximation of the bond price will not be efficient. Huge differences will arise between this approximation of the bond price and the bond price in financial markets.

Convexity is a measure of sensitivity of the bond duration to changes in interest rates. It is the second derivative of the relationship between the bond price and interest rates. It measure how the bond duration changes as interest rate changes.

In the following graph we can see the relationship between bond price and interest rate:

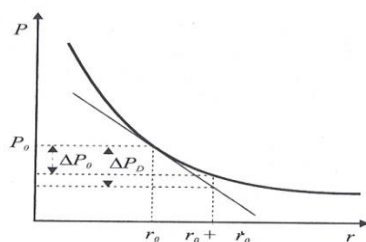


Figure 4 - Relationship between the Bond Price and the Interest Rate

(Source: Hull 2003)

When the interest rate varies from  $r_0$  to  $r_0^+$ , the variation in the bond price is  $\Delta P_0$ . When using duration to calculate the bond price, we calculate the change in bond price that would occur if the relationship between the interest rate and the bond price was linear. This means that we are moving along the tangent to the curve of the bond price, rather than along the curve itself.

For small movements in  $r_0$ , the tangent line is a good indicator of the bond price. However, for high movements in  $r_0$ , the error in the bond price is raised if the tangent line is used for calculating the bond price. The bond price error is the difference between  $\Delta P_D$  and  $\Delta P_0$ .

Using both duration and convexity the approximation to the bond price will be more accurate to the bond price in financial markets.

In the case study we chose to use only the Fisher & Weil (1971) duration. This option is related to the complexity that is obtained by using the convexity to hedge interest rate risk. In addition, the use of convexity would not bring additional conclusions to the case study.

#### 4.4. Impact of "Time" in the Bond Duration

The Macaulay and Fisher & Weil duration overlook the impact that "time" may have at calculating a bond price.

The effect of "time" shows that the bond price changes despite the fact that the term structure of interest rates remains unchanged. This happens because the present value of future cash flows is greater as we approach maturity.

The discount factor used to calculate the present value of future cash flows from the bond change according to the term structure of interest rates. Even if we maintain a fixed term structure of interest rates, the discount factor decreases as we approach the respective maturity of the bond.

Rakotondratsimba & Jarjir (2008) demonstrate the impact of "time" in bond price. They concluded that the approximation to the bond price, using duration can lead to significant errors and suggested adding a residual term that reflects the "time" of the bond.

In order to reduce this impact in the case study we choose to conduct quarterly rebalancing of the portfolio.

#### 4.5. Proportions of Bonds vs. Nominal Amount of Bonds

Academic examples about interest rate risk immunization are carried out using proportions of bonds regardless of nominal amount per bond.

In the financial market, we can find bonds with different nominal amounts. The most common are bonds where the nominal amount of each bond is a thousand currency units, which involves the purchase/sale of bonds in multiples of thousand units which thereby may affect the quality of interest rate risk immunization.

The following illustration is a bond issued by Portugal in August of 2012:

GRAB		PORTUGAL T-BILL PORTB 0 06/21/13 95.7550/97.0300 (4.47/3.09) BGN @16:15	
PORTB 0 06/21/13 Corp		Feedback	Page 1/11 Description: Bond
20 Bond Description		34 Notes	10 Buy 47 Sell 77 Settings
21 Issuer Description			
22 Bond Info		Identifiers	
1 Bond Info	Name: BILHETES DO TESOURO	BB Number	EJ2204687
2 Addtl. Info	Type: Sovereign	ISIN	PTPTJ1GE0016
3 Covenants	Security Information	BGGID	BBG0032NP122
4 Guarantors	Mkt of Issue: Euro-Zone	Bond Ratings	
5 Bond Ratings	Country: PT	Currency: EUR	Moody's: NA
6 Identifiers	Rank: Unsecured	Series	S&P: NA
7 Exchanges	Coupon: 0	Type: Zero Coupo	Fitch: NA
8 Line Parties	Can Free		DBRS: NA
9 Fees, Restrict	Day Cnt: ACT/360	Iss Price: 96.12700	Issuance & Trading
10 Schedules	Maturity: 06/21/2013		Amt Issued/Outstanding
11 Coupons			EUR: 1,000,000.00 (M) /
Quick Links	BULLETT		EUR: 1,000,000.00 (M)
12 ALLQ Pricing	Issue Spread		Min Piece/Increment
13 QRD Quote Reck	Calc Type: (7)INTEREST@MTY		1.00 / 1.00
14 Top Trade Hist	Announcement Date: 05/30/2012		Par Amount: 1.00
15 CACS Corp Action	Interest Accrual Date: 06/08/2012		Book Runner
16 CF Prospectus	1st Settle Date: 06/08/2012		Exchange: EURONEXT-LISBON
17 CN Sec News	1st Coupon Date		
18 HDG Holders	AHS YLD: 3.834%		
19 Send Bond			

Figure 5 - Treasury Bills

(Source: Bloomberg)

Portugal intends to get cash inflow of EUR 960 Million. For that issued 1 billion of treasury bills with a nominal amount of EUR 1. Thus, the purchase/sale of treasury bills has to be in multiples of a unit.

In the case study (section 6.) Portuguese bonds and German bonds were selected with a nominal amount of EUR 0.01.

#### 4.6. "Embedded Derivatives" in Bonds

The Macaulay and Fisher & Weil duration overlooks the impact of embedded derivatives on bond duration. However, the existence of embedded derivatives affects the quality of approximation to the bond price.

The evolution of financial engineering created bonds with embedded derivatives. The most used

derivatives on bonds are call options and put options, which allows the issuer/holder of the bond to collect repayment/prepayment before maturity.

Consider the following example that led to the financial crisis started in the United States in 2007. Mortgage-backed securities are bonds where the investor's return depends on the development of a mortgage credit portfolio. Based on a reduced duration of these bonds (neglecting the existence of an embedded derivative) investors thought that exposure to interest rate risk was low. However, when interest rates rose, many borrowers failed to pay their mortgage credit and investors in mortgage-backed securities recorded losses substantially higher than they expected.

Given the limitation of the duration on bonds with embedded derivatives, Bierwag (1997) developed the option-adjusted spread model, which consisted of calculating a spread that would be used to discount cash flows from a bond. A higher spread indicates a higher sensitivity and, therefore, higher price volatility with respect to interest rate movements.

In the case study (section 6.), bonds without embedded derivatives were selected in order to only conclude about the impact of credit risk in the quality of interest rate risk immunization.

#### 4.7. Transaction Costs inherent in Bonds

Transaction costs are present in every trade in financial markets. Therefore, while this point does not correspond to a limitation of the duration, is it nevertheless inserted in this section because transaction cost impacts the quality of interest rate risk immunization. The main transaction costs are:

- Fees charged on purchase/sale of bonds, and
- Custodian costs charged by an agent for holding company's assets.

Portfolio rebalancing should be done regularly in order to maintain equality between the duration portfolio and the investment time horizon. However, the frequency of portfolio rebalancing must take into account the transaction costs which influence the quality of interest rate risk immunization.

In conducting the case study (section 6.), we do not consider transaction costs as they do not influence the conclusions.

#### 4.8. Topics used on case study

This section assumes a particularly significant importance in the case study because it permits an understanding of the portfolio created for interest rate risk immunization.

In this chapter we may indeed see the main limitations of duration. In conducting the case study we can eliminate/reduce the impact of the following limitations:

- Term structure of interest rates, as we used the concept of Fisher & Weil (1971) duration;
- Impact of "time" in bond duration, which was reduced by performing a higher frequency of portfolio rebalancing (in the case study we perform quarterly portfolio rebalancing);
- "Proportions of bonds vs. Nominal amount of bonds", which was eliminated by selecting bonds with nominal amount of EUR 0.01, and
- "Embedded Derivatives", which were eliminated by selecting bonds without embedded derivatives.

In the conclusions of the case study we consider the limitations of the duration that were not eliminated/reduced:

- Parallel movements in the term structure of interest rate;
- Infinitesimal movements in the term structure of interest rates, and
- Transaction costs of bonds.

### 5. Credit Risk in the Euro Zone

In the case study we intend to demonstrate that the credit risk is the main limitation on the quality of interest rate risk immunization using Fisher & Weil duration. In this regard, Portuguese bonds and German bonds have been selected because they are on opposite sides of credit risk in Euro Zone. On one side we have Portugal, where credit risk reached high levels in recent years. On another side is Germany, where credit risk decreased considerably. However, in both cases we have seen credit risk volatility in recent years.

Credit risk is associated with uncertainty about the future cash flows of a bond. In an adverse economic scenario there is a possibility that an issuer of bonds cannot meet their obligations, including payment of interest and the reimbursement amount at the maturity of the bond.

In this section we will study the credit risk present in bonds issued by Portugal and Germany. Topics about credit risk will be discussed, such as credit spread practiced on credit derivatives, credit ratings assigned by major rating agencies and yield to maturity of bonds.



### 5.1. Credit Spread practiced in Credit Derivatives

A credit default swap is a contract where the buyer of the swap makes payments up until the maturity date of a contract. Payments are made to the seller of the swap. In return, the seller agrees to pay off a third party debt if this party defaults on the loan. A credit default swap is considered insurance against non-payment.

In the following figure we can see a credit default swap when there is no default of the third party:

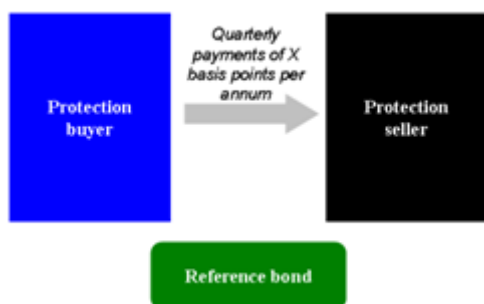


Figure 6 – Credit Default Swap (No Default of the third party)

(Source: Markit)

In the next figure we can see a credit default swap when there is a default of the third party:

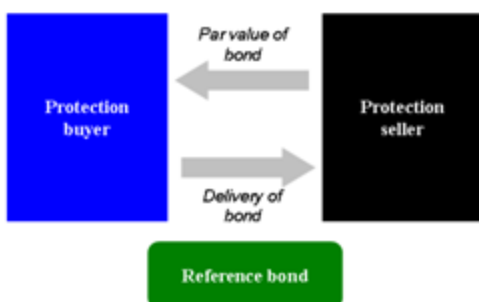


Figure 7 - Credit Default Swap (Default of the third party)

(Source: Markit)

Credit default swaps are used to protect the ability of a third party to fulfill their financial obligations. Thus, the higher the probability of default of a third party the greater the interest rate seen in the financial market to buy protection. The spreads found for credit default swaps have become an indicator to measure the credit risk of a third party.

Despite being traded over-the-counter<sup>4</sup>, we have seen an increasing standardization of credit default swaps which facilitates the comparison of credit risk among the reference entities.

<sup>4</sup> A security traded via dealer network as opposed to on a centralized exchange.

Based on spreads realized on credit default swaps between 2010 and 2012, we present the evolution of the credit curve for debt issued by Portugal and Germany:

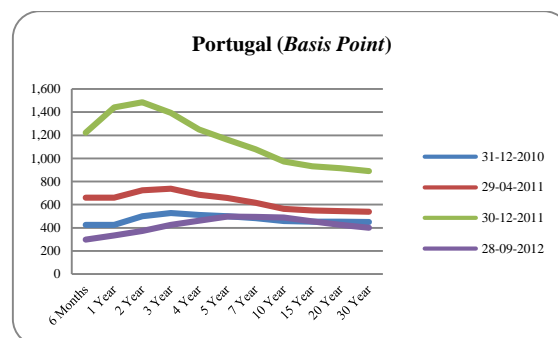


Figure 8 - Credit Curve of Portugal

(Source: Markit)

Currently we have seen tremendous volatility in spreads practiced on credit default swaps for Portugal. This means a higher bond price volatility and, consequently, the respective quality of interest rate risk immunization.

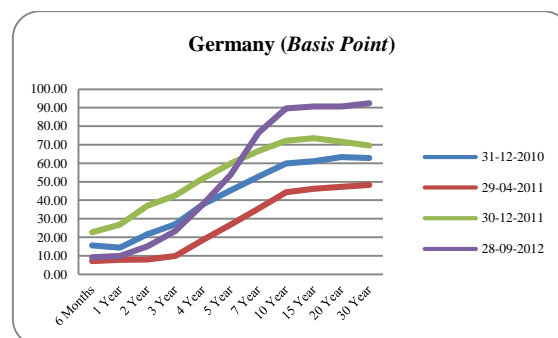


Figure 9 - Credit Curve of Germany

(Source: Markit)

Spreads seen on credit default swaps for Germany are the lowest in the Euro Zone. Despite the slight increase in late September 2012, the spreads do not exceed the barrier of 100 basis points.

Credit risk in bonds has a strong impact on its cash flows and its duration. Facing a high credit risk, there could be two plausible scenarios which result in partial/total default and bond restructuring. In a scenario of partial/total default the duration decreases considerably due to the declining cash flows received. On the other hand, in a scenario of bond restructuring, the duration increases due to the extension of the bond maturity.

Both Macaulay and Fisher & Weil duration overlook the credit risk in a bond and its impact on the approximation to the bond price. Fooladi et al. (1997a) claim that the use of Macaulay and Fisher & Weil duration should be limited and suggest introducing an adjustment of credit risk.

Given these considerations, the following questions arise: What characterizes a bond without credit risk? Can we identify a bond which will never have credit risk?

An asset is considered credit risk free whenever it is possible to predict its cash flows with a high degree of confidence. In these cases a default scenario is not a possibility.

Thus private companies cannot be considered free of credit risk, since even the largest companies have always default risk (although it may be reduced). Take as an example the collapse of Lehman Brothers and its impact on the financial sector.

Securities issued by governmental entities are the only bonds able to be considered without credit risk. This is due to the fact that governments can control some economic mechanisms, as is the case of monetary and tax policy, which helps to significantly increase the likelihood of fulfilling their responsibilities.

However, currently we can observe high credit risk in the sovereign debt of some countries. Portugal, which was once considered an investment without credit risk, now cannot finance its debt at sustainable interest rates. Germany, which today is considered the safest investment in the Euro Zone, was considered a high risk investment after the Second World War. The definition of an asset without credit risk should be framed in time.

Since the beginning of the single currency – the Euro, Portugal has no control of monetary policy (the power of issuing money). Although Portugal has tax autonomy to pursue the objective of fulfilling their financial responsibilities, as can be seen in the next graph, the credit risk in Portugal is very high when compared with the credit risk of Germany. Currently, the Portuguese credit derivatives are traded with a coupon rate that is very high and very volatile. On the other hand, German credit derivatives are traded with a low coupon rate, however it has some volatility. In this graph, we can see the difference between the credit curves of Portugal and Germany in 2012:

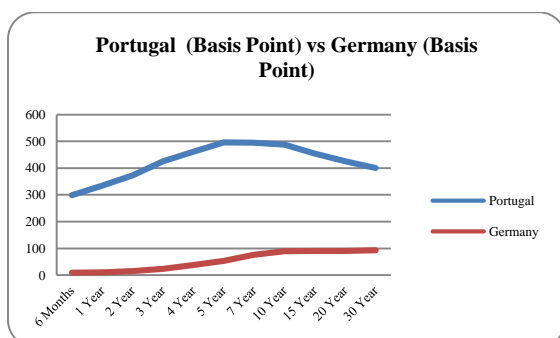


Figure 10 - Credit Curve of Portugal and Germany

(Source: Markit)

Thus the need to respond to the following questions arises: Is it possible to make an interest rate risk immunization using Portuguese bonds and German bonds? Or the volatility of the credit risk prejudice the quality of immunization?

In the case study we will test the quality of the interest rate risk immunization using Fisher & Weil duration and creating a portfolio with Portuguese bonds and German bonds.

## 5.2. Credit Ratings of the Issuers

In the work of Babbel et al. (1999) it is suggested that the formula of duration should include an adjustment factor that reflects the credit risk of the issuer. Each credit rating class is assigned an adjustment factor which is related to the volatility of the yield to maturity of its bonds.

The rating assigned by the specialized agencies is becoming increasingly important in financial markets. Despite strong criticism that the rating agencies have been influenced by the media and investors<sup>5</sup>, the truth is that the financial market continues to use their analysis as a base of credit risk of the issuers.

Through ratings, a probability of default of the issuer of the bond is assigned. An issuer with a high rating has a reduced probability of default and thus a low credit risk. In contrast, an issuer with a low rating has a high probability of default and consequently a higher credit risk.

In the following table we can see the evolution of the credit rating of Portugal and Germany assigned by Standard & Poor's and Moody's:

Entity	Date	S&P Rating	Fitch Rating
Portugal	21-01-2009	A+	AA
Portugal	24-03-2010	A+	AA-
Portugal	27-04-2010	A-	AA-
Portugal	23-12-2010	A-	A+
Portugal	24-03-2011	BBB	A+
Portugal	24-03-2011	BBB	A-
Portugal	29-03-2011	BBB-	A-
Portugal	01-04-2011	BBB-	BBB-
Portugal	24-11-2011	BBB-	BB+
Portugal	13-01-2012	BB	BB+

Figure 11 - Credit Rating of Portugal

(Source: Bloomberg)

<sup>5</sup> In Australia a Federal Court punished Standard & Poor's for assigning the maximum rating (AAA) for assets of dubious quality, known as toxic assets. Those assets led to the financial crisis started in 2008 in United States.

Since 2009 we have seen several downgrades for Portugal debt. Currently, Portuguese debt is considered a Junk Bond by major rating agencies.

Entity	Date	S&P Rating	Fitch Rating
Germany	17-08-1983	AAA	N/A
Germany	10-08-1994	AAA	AAA

Figure 12 - Credit Rating of Germany

(Source: Bloomberg)

Germany has the highest rating and is considered a safe investment in the Euro Zone.

### 5.3. Yield To Maturity of Bonds

Yield to maturity is an indicator of the credit risk to the extent that the higher the risk, the higher the return required by investors. Thus, issuers with less credit risk in the financial market issue bonds with a low yield to maturity. In contrast, issuers with greater credit risk issue bonds on the financial market with higher yield to maturity.

The evolution of the yield to maturity on the secondary market depends on the perception of credit risk of the issuer. Later on we will analyze the volatility of yield to maturity of Portugal and Germany and conclude on the impact that may exist in the quality of interest rate risk immunization.

In the next graph we can observe the yield to maturity of Portugal and Germany from 2004 to 2012:

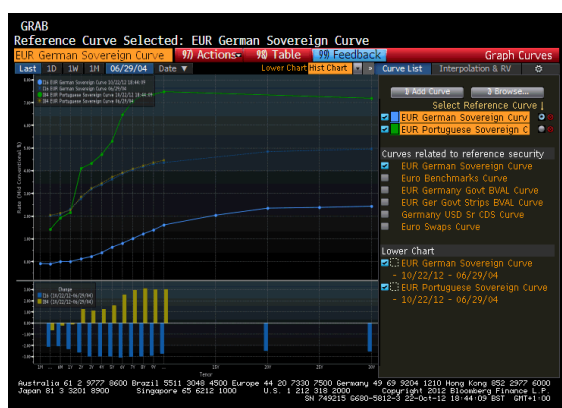


Figure 13 - Yield Curve of Portugal and Germany

(Source: Bloomberg)

In 2004 the yield to maturity of Portugal and Germany were similar in different maturities. However, currently the Portuguese yield to maturity increased significantly and in contrast, German yield to maturity decreased.

Fons (1990) conducted a study to measure the impact of credit risk in corporate bond duration and concluded that its duration is always shorter than the Macaulay duration. This is because the present value of the cash flow is always less than the actual value of cash flow.

### 5.4. Topics used on case study

After analyzing the credit spread on credit default swaps, credit ratings and yield to maturity we concluded that Portugal currently has a high credit risk. In contrast, Germany has the lowest credit risk in the Euro Zone.

In addition, we find that there is an inherent volatility of credit risk in both countries, although volatility in German bonds is smaller. The credit risk is reflected in the bond price and its duration.

It was due to this volatility that Portuguese bonds and German bonds were selected to test the interest rate risk immunization. With this, we want to test if the credit risk volatility provides a good quality of interest rate risk immunization.

In the next section, we present the case study and we draw the appropriate conclusions.

## 6. Case Study: Interest Rate Risk Immunization

Is interest rate risk immunization using Fisher & Weil (1971) duration effective in a scenario of extreme credit risk volatility in the Euro Zone?

With the case study we intend to give an answer to this question. However, before we move to the case study it is necessary to show why interest rate risk immunization is important.

Since the creation of the single currency – the Euro, the financial market has seen high interest rate volatility for different maturities. In the next graph we can see the evolution of Euribor from 1999 until late 2011:

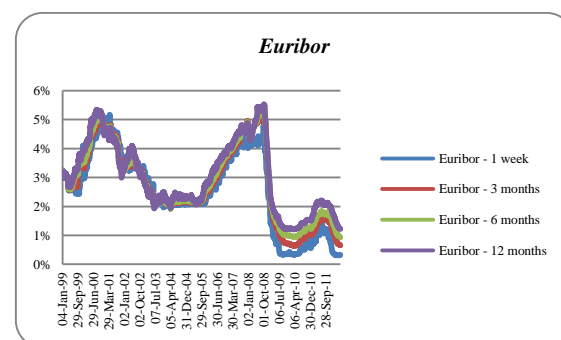


Figure 14 - Euribor

(Source: Bloomberg)

Euribor crossed the 5% twice in late 2000 and the third quarter of 2008. Since then, Euribor fell, reaching a historic low, and at the end of October 2012 the Euribor for 1 Week, 3 Months, 6 Months and 12 Months were 0.079%, 0.197%, 0.385% and 0.611%, respectively.

With the instability in financial markets in recent years, interest rate risk is identified as the main challenge for the Financial Institutions. Considering the Financial Institutions balance sheet, which have in their asset investments at variable interest rate (mortgage credits) and in their liability funding at a fixed interest rate (client deposits), an active interest rate risk immunization has become urgent.

In addition to the interest rate volatility observed in the Euro Zone in recent years, there is also high credit risk volatility. Since the financial crises started in 2008 the Euro Zone has seen successive downgrades from rating agencies, sought financial assistance from the International Monetary Fund and seen a partial default of Greece.

In section 1. we introduced the concept of duration as a measure of approximation to the bond price. The Fisher & Weil duration (see formula (2)), which is more realistic to the financial markets, will be used in the case study. Section 1 also presented the concept of interest rate risk immunization. To this end, we proceeded to define two concepts: "Price risk" and "reinvestment risk". When Fisher & Weil (1971) duration equals the investment time horizon, the two effects underlying these risks are equal and, being of opposite signs, cancel each other out. Thus, a total return rate of the portfolio equal to that which would be obtained in a scenario of stable interest rates is guaranteed.

The concepts of duration, interest rate risk and credit risk created the theoretical and contextual knowledge to proceed with the analysis proposed in this paper, which is the interest rate risk immunization based on the German bonds and Portuguese bonds. Later, interest rate derivatives as a hedging instrument will be introduced as the method used by Financial Institutions.

### **6.1. Interest Rate Risk Immunization using Fisher & Weil Duration**

In order to test the quality of interest rate risk immunization in an environment of high credit risk volatility in the Euro Zone Portuguese bonds and German bonds were selected. As we observed earlier, Portuguese and German debt represent two opposite sides of credit risk in the Euro Zone. Portuguese debt

is considered as a high risk investment and German debt as the safest investment in Euro Zone.

The period between 2005 to 2012 was selected to test the quality of interest rate risk immunization. The aim was to check the quality of interest rate risk immunization before and during the financial crisis begun in 2008, i.e., before and during the period of high credit risk volatility in the Euro Zone.

Thus, through Bloomberg, prices of Portuguese bonds and German bonds were obtained during the period from 2005 to 2012 (in figures 24 to 27 in annex we can see debt issued by Portugal and Germany).

A future liability was introduced to be a benchmark for the development of the portfolio created with Portuguese bonds and German bonds. It has started in March 2005 and will mature in December 2013, with a nominal value of EUR 100 Million.. The interest rates used in the present value of the future cash flows of the liability were taken from Bloomberg (in figure 23 in appendix we can see the present value of the future liability over time).

Based on Portuguese bonds and German bonds, a portfolio was created with identical duration as the future liability. Thus the immunization condition articulated by Fisher & Weil (1971) and demonstrated by Bierwag (1987) is guaranteed.

According to all Portuguese bonds and German bonds available, two Portuguese bonds and two German bonds were selected to ensure the same duration as the future liability (in figures 28 to 31 in appendix we can see a description of the selected bonds).

Based on duration and present value of the future liability in 2005 (duration of 8.8 years and a present value of EUR 72,991,615) and investing in Portuguese bonds and German bonds with the same duration, the objective was, in each period, to ensure the portfolio value was equal to or greater than the present value of the future liability.

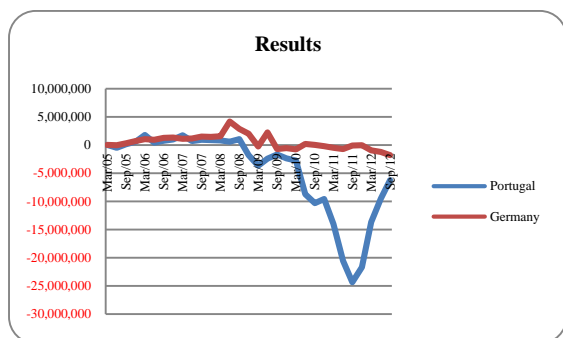
On a quarterly basis the portfolio was rebalanced in order to match the future liability duration. The portfolio was bought and sold based on the dirty price<sup>6</sup>. This guarantees the Fisher & Weil duration will be relatively equal to the future liability duration.

The coupons received from the Portuguese bonds and German bonds were included in the portfolio value and invested in the next portfolio rebalancing.

The calculations were carried out in Microsoft Excel and transaction costs were not considered. in order to simplify the calculations.

<sup>6</sup> A bond pricing quote referring to the price of a bond that includes the present value of all future cash flows, including interest accruing on the next coupon payment.

In figures 32 and 33 in appendix we can see the portfolio details, including nominal amounts, durations and results in each period. The next graph allows us to observe the result over the interest rate risk immunization period:



**Figure 15 - Results of Interest Rate Risk Immunization**

(Source: Author)

As we can see, the result of interest rate risk immunization based on Portuguese bonds and German bonds was negative. In late September 2012, the result of interest rate risk immunization based on Portuguese bonds was a loss of EUR - 6.2 million. The result of interest rate risk immunization based on German bonds was EUR - 1.7 million which, despite being less severe, is still negative.

This means that the portfolio value was lower than the present value of the future liability.

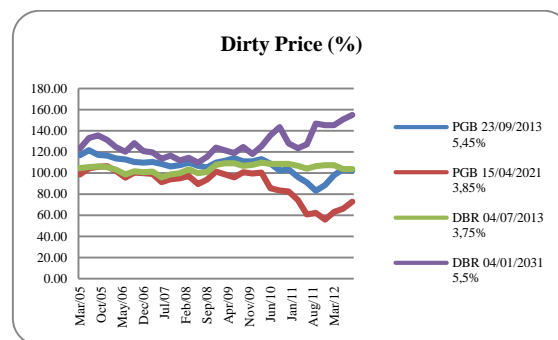
Responding to the question posed at the beginning of the section, the interest rate risk immunization based on Portuguese bonds and German bonds in a scenario of high credit risk volatility was not possible.

As we can see the results of interest rate risk immunization before the financial crisis, which started in 2008 in the United States, was close to zero. Regardless of bonds used in the portfolio (Portuguese bonds or German bonds) it was possible to obtain good quality interest rate risk immunization based on Fisher & Weil (1971) duration. However, since the beginning of the financial crisis (2008) there is some volatility in the results, with particular emphasis on the Portuguese bonds.

We conclude therefore that interest rate risk immunization based on Fisher & Weil (1971) duration is ineffective during a period of high credit risk volatility.

In the following graph we consider the reasons that made it impossible to hedge interest rate risk based on Portuguese bonds and German bonds. For this

purpose, the next graph shows the evolution of bond prices throughout the immunization period:



**Figure 16 - Evolution of Bond Price**

(Source: Bloomberg)

As can be seen, the bonds prices of Portugal and Germany have been very unstable in recent years. Despite some recovery in the last quarter, the price of Portuguese bonds has tended to decline. In the opposite direction, the price of German bonds has increased.

In November 2011, interest rate risk immunization based on Portuguese bonds recorded the worst result of the period (negative, standing at about EUR - 25 million). This period is related to the fall in bond prices registered in the financial market. As we can see, in November 2011 the Portuguese bonds decreased about 40% to 50%.

After this period the bond price rose again. However, the bond PGB 04/15/2021 - 3.85%, which will reach maturity in 2021, is fairly penalized due to the uncertainty being experienced in the financial market for a possible partial default of Portuguese debt in the long term and perhaps a drop of the single currency – the Euro.

The price of the bond PGB 23/09/2013 - 5.45% has a lower default risk because the financial market believe that the aid of International Monetary Fund will continue after 2013. Thus, the default risk is lower in the short term and its price will reach 100% as maturity approach.

For German bond prices, it is worth highlighting the fact that the bond DBR 04/07/2013 - 3.75% will reach maturity in 2013, and because of that, its price is close to par (100%).

The bond DBR 04/0/2031 - 5.5%, maturing in 2031, is being used as a refuge for investors and, because of that, demand has raising the bond price. Note that this increase in the price of German bonds coincides with

the decrease in the price of Portuguese bonds (PGB 04/15/2021 - 3.85%), which means that investors are exchanging Portuguese bonds for German bonds, further pressing the fall in the price of Portuguese bonds and rise in the price of German bonds.

The next graph shows the evolution of the yield to maturity of Portuguese bonds and German bonds between 2005 and 2012:

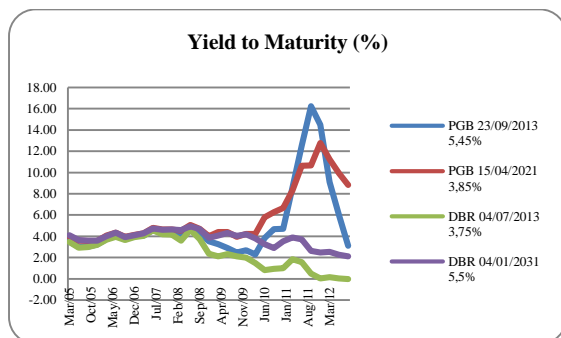


Figure 17 - Evolution of Yields to Maturity

(Source: Bloomberg)

As was expected, the yield to maturity of Portuguese bonds was very volatile and reached high levels in November 2011. This period coincides with the worst result recorded in interest rate risk immunization with Portuguese bonds.

Despite a lower volatility of yield to maturity in German bonds, it has decreased during the immunization period. In September 2012 the yield to maturity of bond DBR 07/04/2013 - 3.75% reached negative values, which means that investors were willing to pay to invest in German bonds.

In the next graph we can see the evolution in the future liability and Portuguese bonds duration during the years 2005 to 2012:

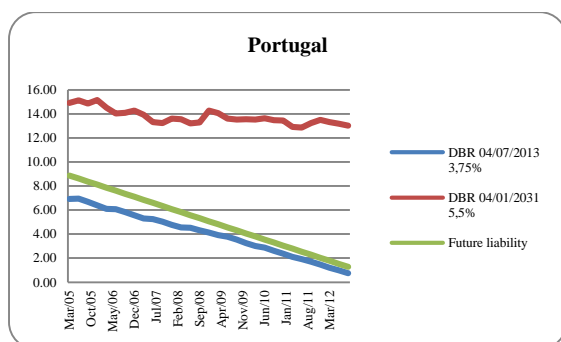


Figure 18 - Bond Duration of Portugal

(Source: Bloomberg)

In late 2007 the Portuguese bonds duration rose slightly. The bond PGB 04/15/2021 - 3.85%, which has a longer maturity, is more volatile because of the uncertainty present in Euro Zone.

Next, we see the same analysis but with German bonds during the years 2005 to 2012:

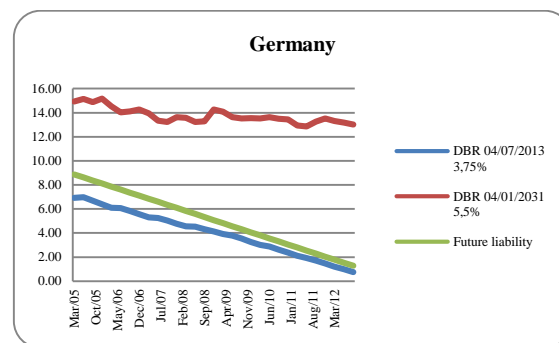


Figure 19 - Bond Duration of Germany

(Source: Bloomberg)

The bond DBR 04/0/2031 - 5.5% registered very unstable behavior throughout the immunization period. This volatility requires a portfolio rebalancing quite often to equalize liability duration with portfolio duration.

In both Portuguese bond and German bond duration, the longer the maturity of the bonds, the greater the duration volatility.

Given the above results, we conclude that interest rate risk immunization based on Fisher & Weil (1971) duration is not effective in periods of high credit risk volatility. Regardless of the bonds used in the interest rate risk immunization, i.e., bonds with high credit risk (e.g. Portugal) or low credit risk (e.g. Germany), interest rate risk immunization using Fisher & Weil (1971) duration is not effective.

The main factor in the failure of interest rate risk immunization is the bond price volatility, originated by changes in the credit risk of the issuer.

The question that arises after the completion of the case study is to know how Financial Institutions manage interest rate risk, considering the fact that interest rate risk immunization based on the Fisher & Weil (1971) duration is not possible in a scenario of high credit risk volatility.

Thus, an interview was conducted among those responsible for managing the interest rate risk in a Portuguese Financial Institution, in order to conclude how the interest rate risk is captured and managed. We intend, therefore, to present a method of interest rate risk immunization effective in an environment of high credit risk volatility.

## 6.2. Interest Rate Risk Immunization in Financial Institution

The interview was conducted with two traders responsible to manage the interest rate risk in a Portuguese Financial Institution (FI\_A), whose

identity is not indicated for reasons of confidentiality of the information provided. Based on this interview, we obtained information that allows us to understand the interest rate risk hedging strategy used in IF\_A.

Since the fall of Lehman Brothers in late 2008, we have seen an increase in the cost of funding for Financial Institutions. Currently, with the sovereign debt crisis in the Euro Zone, the funding cost for Financial Institutions is becoming very expensive.

With rising financing costs, Financial Institutions have focused their attention on the cheapest form of financing possible, i.e. client deposits. Competition among Financial Institutions for client deposits is so aggressive that the Bank of Portugal felt the need to intervene through a penalty in the ratio of consumption of capital for Financial Institutions that offer deposits with interest rates higher than the market rate plus 3%.

In the example of client deposits, Financial Institutions incur interest rate risk whenever interest rates decrease in the financial markets, therefore payments of interest incurred on deposits with fixed interest rates do not decrease. Thus, in a falling interest rates scenario, the financial institution is hampered because it has a fixed interest rate for deposits. In a rising interest rates scenario, the financial institution would benefit because interest payments would not rise.

In the first half of 2012, the IF\_A had about EUR 6.5 billion in client deposits with a fixed interest rate. Given the exponential increase in client deposits in recent years, it was necessary to adopt an active interest rate hedge strategy. The interview was thus focused on practical examples of hedging the interest rate risk in client deposits.

According to information provided, the hedge of client deposits is accomplished by interest rate derivatives, especially interest rate swaps. The choice of this instrument comes down to its flexibility and because it does not require initial investment (unlike the immunization method using duration, where an investment is required in creating a portfolio).

Another benefit of using interest rate swaps, according to the interviewees, is that this type of instrument is not under credit risk, which makes its present value less volatile. As we have seen in the case study, the bond price volatility was the main cause of the ineffectiveness of interest rate risk immunization.

The credit risk in the interest rate derivatives is mitigated by cash collateral deposited in the Financial Institutions. This method has gained increasing importance in the financial markets and is currently essential to the transaction of derivatives between Financial Institutions.

The next figure describes the interest rate hedging in client deposits:

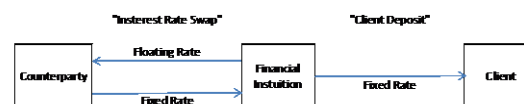


Figure 20 – Interest Rate Hedging

(Source: Author)

According to interviewees, the interest rate hedge of client deposits is done through an interest rate swap with a counterparty, where IF\_A receives a fixed interest rate and pays a floating interest rate. The purpose of this operation is to eliminate the risk of fixed interest rates, getting exposure to a floating interest rate.

This means that an increase in the interest rate leads to a gain in client deposits and a loss on the interest rate swap. If the interest rate decreases, there is a loss on client deposits and a gain on interest rate swap. Through interest rate hedge these two effects are eliminated so that there is no impact on the results of the financial institution, regardless the change in the interest rate.

In order to capture the exposure to interest rate risk, the financial institution department is responsible for managing the interest rate risk, taking a report from an application support with the name of Kondor +. This report allows IF\_A to follow the evolution of the interest rate risk and check basis point value of client deposits over different periods.

The concept of basis point value is similar to the concept of Fisher & Weil (1971) duration. However, there is a difference regarding the change in the term structure of interest rates assumed in the calculation. While duration assumes a 1% change in the yield curve, the basis point value assume a change of 0.01%. This means that the basis point value correspond to 1% of its duration.

Thus, the duration limitations identified in section 4. also are applied in the calculation of basis point value, as indicated by the interviewees. Like duration, the maturity and coupon rate are also the main drivers

of basis point value and consequently, the interest rate risk.

According to interviewees, the interest rate risk is based on basis point value. Whenever it is necessary interest rate swaps are negotiated/liquidated to reduce the basis point value gap<sup>7</sup> and consequently reduce the interest rate risk.

The difference in these two methods of hedging is the instrument used to make the interest rate risk immunization. In the case study we perform an interest rate risk immunization based on a portfolio, while in IF\_A interest rate risk immunization is based on an interest rate swap.

In the case study, the bond price was very volatile due to the credit risk that affects the quality of interest rate risk immunization. In IF\_A, an interest rate swap is used to hedge interest rate risk, and as mentioned by the interviewees, has no credit risk and subsequently its value is less volatile.

As demonstrated in the case study, the credit risk volatility influences the quality of interest rate risk immunization. As interest rate derivatives have no credit risk, Financial Institutions started to use it as a hedging instrument.

The results obtained with the interview allow us to support the conclusion obtained in the case study, which is interest rate risk immunization using Fisher & Weil (1971) duration is ineffective in an environment of high credit risk volatility.

On this basis we can see that interest rate risk immunization is more effective when using interest rate swaps. Having described the process in IF\_A interest rate hedge, we present a definition of interest rate swaps in the next section.

### 6.3. Interest Rate Risk Immunization using Interest Rate Swaps

As was mentioned in the interview conducted with Financial Institutions, interest rate derivatives are important in the process of interest rate hedge. Because of that it is important to explain this instrument more thoroughly.

Financial derivatives are an instrument whose value is linked to or derived from other assets. Their uses are varied from risk management, arbitrage and speculation, depending on the objectives of investors.

Interest rate derivatives are one of the innovations of great importance in the field of financial engineering. Growth has been exponential and the current amount of outstanding contracts is USD 402,611 billion, and

the interest rate swap is the interest rate derivatives most traded instrument in financial markets.

The following graph shows interest rate swap growth in the last decade:

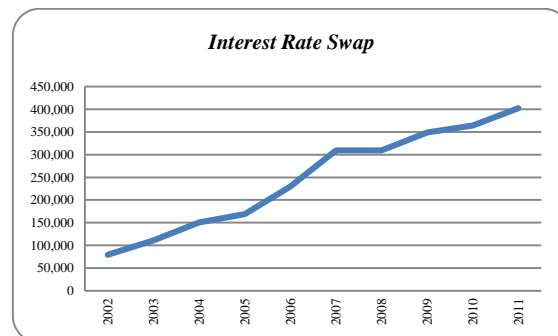


Figure 21 - Growth of Interest Rate Swap (Nominal)

(Source: Bloomberg)

An interest rate swap is defined as a contract in which two parties agree to exchange for a predetermined period of time, two streams of interest payments, each of whose flows are calculated based on different interest rate indices but contain the same reference value, referred to as the 'nominal'.

In the illustration below we can see the cash flows present in interest rate swap:

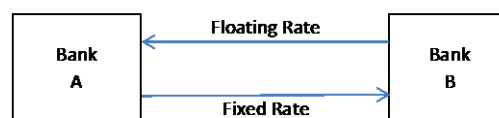


Figure 22 – Interest Rate Swap

(Source: Author)

The use of interest rate swaps gives several advantages such as they:

- Do not require an initial investment;
- Are flexible because they are adapted to Financial Institutions needs in terms of maturity, coupons rate and nominal amount, and
- Contain no credit risk because there is cash collateral between parties.

Interest rate swaps are used as hedging instruments because they allow for the transfer between a fixed rate and floating interest rate.

### 6.4. Topics from the case study

Upon completion of the case study and after conducting interviews among those responsible for managing the interest rate risk of a Financial

<sup>7</sup> Basis point value gap is the net of client deposits basis point value and interest rate swaps basis point value.



Institution, we are able to point out the main findings of the two methods of interest rate risk hedging.

Both methods used different indicators of the interest rate risk but both have the same interpretation. In the case study we used the concept of Fisher & Weil (1971) duration which showed us the percentage decrease of the bond price when the interest rate increases by 1%. In IF\_A the concept of basis point value is used which shows us the percentage increase of the client deposits value when the interest rate increases 0.01%.

The difference between the two methods lies in the instrument used for interest rate risk immunization. In the case study Portuguese bonds and German bonds were used. This method requires a large initial investment (purchase of bonds) and there is higher credit risk volatility, which impacts the quality of interest rate risk immunization.

In IF\_A interest rate swaps are used as a hedging instrument. This instrument does not require initial investment and its value is less volatile due to the lack of credit risk, allowing for an effective interest rate risk immunization. Additionally, the interest rate swaps allow for more flexibility for the needs of the Financial Institutions.

Given the above, we conclude that the main difference between the case study presented in this section and the method carried out by the Financial Institutions is basically the hedging instrument used to mitigate the interest rate risk.

## 7. Conclusion

The main objective of this paper to determine whether the instability observed in Euro Zone has an impact on interest rate risk immunization strategy.

We start the paper with a review of the financial literature on the subject of interest rate risk, which was defined as an adverse change in interest rates in the financial market, resulting in a negative impact on the results of the Financial Institutions.

Later, the concept of duration was introduced, initiated by Macaulay (1938), as being the average time a bond needs to generate its value. Macaulay (1938) makes two assumptions in the duration calculation:

- Term structure of interest rates are constant for all maturities, and
- Movements in the term structure of interest rate are parallel.

The first assumption made by Macaulay is not suited to the reality of the financial market. In figure 1

(section 4.), a crescent term structure of interest rates was shown.

Later, Fisher & Weil (1971) developed on the Macaulay duration, which took only one assumption:

- Movements in the term structure of interest rate are parallel.

Thus, the Fisher & Weil duration is more realistic in the financial market because the term structure of interest rates is not assumed constant. It is based on the Fisher & Weil duration that was created a portfolio in the case study to hedge a future liability.

The Fisher & Weil (1971) study shows that a portfolio is immunized against a change in interest rates as long as its duration is equal to the investment time horizon. In the case study analysis a portfolio was created in order to match this condition defined by Fisher & Weil (1971).

Bierwag (1987a, Chapter 4) explains interest rate risk immunization through two concepts: Price risk and reinvestment risk. The price risk is characterized by the fact that any changes in term structure of interest rates leads to a change in bond prices and the reinvestment risk is characterized by the fact that any changes in term structure of interest rates leads to reinvestment of cash flows at different rates. When the Fisher & Weil (1971) duration equals the investment time horizon, the two effects are of equal magnitude and opposite signs, thus cancelling each other out. This in turn guarantees a total return rate of the portfolio equal to that obtainable in a scenario of stable interest rates.

In the case study, we tested the applicability of interest rate risk immunization based on Fisher & Weil (1971) duration in an environment of high credit risk volatility. To this end, we selected Portuguese bonds and German bonds due to their contrasting credit risk profiles. While Portugal is considered a high risk investment, Germany is the safest investment in the Euro Zone.

Based on the case study we conclude that the interest rate risk immunization based on Fisher & Weil (1971) duration is not possible in an environment of high credit risk volatility. We found that until mid 2008 it was possible to implement interest rate risk immunization. However, with the financial crisis (started at the end of 2008) interest rate risk immunization using Fisher & Weil (1971) duration was ineffective.

Conducting an interview with those responsible for the interest rate risk management in a Portuguese Financial Institution we found that interest rate derivatives are used as a hedging instrument, in particular interest rate swaps. The choice of this instrument is due to the fact that it does not require an

initial investment and allows for more flexibility to the Financial Institution's needs. In addition, due to the fact that there is no credit risk in interest rate swaps, interest rate risk immunization becomes more effective. Credit risk is mitigated in this instrument because there is cash collateral involved between parties.

## 8. Future research directions

After conducting research on interest rate risk immunization, several interesting topics were not developed because it was not the purpose of this paper, are left open through some ideas:

- In the case study we found that the yield to maturity of German bonds reached negative values in September 2012. This means that investors were willing to recognize losses on investments in German bonds. It would be interesting to analyze these yields to maturity on German bonds based on the present economic environment in the Euro Zone. What are the impacts of these yields to maturity in the remaining members of the Euro Zone and the investors themselves?

- Credit ratings, assigned by rating agencies, are indicators of credit risk for particular entities. Given this, and because the rating agencies do not explain the fact that Lehman Brothers, an investment bank which was assigned the highest rating by the several specialized agencies, collapsed in September 2008, it would be interesting to analyze to what extent the ratings assigned by specialized agencies are trustworthy indicators of credit risk and if these agencies are independent and impartial in their credit risk evaluations.

## Acknowledgements

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## Appendix

**Figure 23 - Present Value of Future Liability**

(Source: Bloomberg)

Date	Duration	Euro Swap Curve	Liability	
			Future	Present Value
31-03-2005	8,88	3,61%	100.000.000	72.991.615
30-06-2005	8,63	3,08%	100.000.000	76.980.615
30-09-2005	8,37	3,15%	100.000.000	77.109.737
30-12-2005	8,12	3,36%	100.000.000	76.476.938
31-03-2006	7,87	3,90%	100.000.000	74.032.524
30-06-2006	7,61	4,18%	100.000.000	73.225.398
29-09-2006	7,36	3,89%	100.000.000	75.520.253
29-12-2006	7,11	4,17%	100.000.000	74.809.929
30-03-2007	6,86	4,25%	100.000.000	75.153.641
29-06-2007	6,60	4,79%	100.000.000	73.424.007
28-09-2007	6,35	4,60%	100.000.000	75.157.756
31-12-2007	6,09	4,58%	100.000.000	76.140.339
31-03-2008	5,84	4,20%	100.000.000	78.662.587
30-06-2008	5,58	5,11%	100.000.000	75.720.612
30-09-2008	5,33	4,71%	100.000.000	78.272.097
31-12-2008	5,07	3,24%	100.000.000	85.070.521
31-03-2009	4,82	2,61%	100.000.000	88.331.653
30-06-2009	4,57	2,77%	100.000.000	88.265.474
30-09-2009	4,31	2,53%	100.000.000	89.781.431
31-12-2009	4,06	2,56%	100.000.000	90.261.503
31-03-2010	3,81	2,02%	100.000.000	92.652.457
30-06-2010	3,56	1,73%	100.000.000	94.090.632
30-09-2010	3,30	1,72%	100.000.000	94.520.636
31-12-2010	3,04	1,88%	100.000.000	94.473.801
31-03-2011	2,79	2,58%	100.000.000	93.127.687
30-06-2011	2,54	2,30%	100.000.000	94.390.326
30-09-2011	2,29	1,54%	100.000.000	96.566.985
30-12-2011	2,03	1,31%	100.000.000	97.382.475
30-03-2012	1,78	1,06%	100.000.000	98.144.036
29-06-2012	1,53	0,83%	100.000.000	98.740.630
28-09-2012	1,28	0,41%	100.000.000	99.479.677

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Figure 24 – Treasury Bonds issued by Portugal

(Source: Bloomberg)

R	Ticker	ISIN	Amnt Issued(MM)	First Settle Date	Maturity	Mkt Type	Coupon	Coupon Freq	Coupon Type	Curr
1	PGB	PTCON4E0005	9.98	01/01/1940	PERPETUAL	PERPETUAL	4.000	S/A	FIXED	EUR
2	PGB	PTCON1E0008	2.49	12/01/1941	PERPETUAL	PERPETUAL	3.500	Q	FIXED	EUR
3	PGB	PTCON0E0007	15.42	02/01/1942	PERPETUAL	PERPETUAL	2.800	Q	FIXED	EUR

Figure 25 - Treasury Bills issued by Portugal

(Source: Bloomberg)

R	Ticker	ISIN	Amnt Issued(MM)	First Settle Date	Maturity	Mkt Type	Coupon	Coupon Freq	Coupon Type	Curr
1	PORTB	PPRTE1E0017	2131.71	01/20/2012	12/21/2012	BULLET	ZERO	NA	ZERO COUPON	EUR
2	PORTB	PPRTE2E0015	1166.76	02/21/2012	02/22/2012	BULLET	ZERO	NA	ZERO COUPON	EUR
3	PORTB	PPRTE3E0014	3572.13	03/22/2012	03/23/2012	BULLET	ZERO	NA	ZERO COUPON	EUR

Figure 26 - Treasury Bonds issued by Germany

(Source: Bloomberg)

R	Ticker	ISIN	Amnt Issued(MM)	First Settle Date	Maturity	Mkt Type	Coupon	Coupon Freq	Coupon Type	Curr
1	DER	DE0001134468	3750.00	06/20/1980	06/20/2016	BULLET	6.000	A	FIXED	EUR
2	DER	DE0001134492	750.00	09/20/1986	09/20/2016	BULLET	5.625	A	FIXED	EUR
3	DER	DE0001134922	10250.00	01/04/1994	01/04/2024	BULLET	6.250	A	FIXED	EUR

Figure 27 - Treasury Bills issued by Germany

(Source: Bloomberg)

R	Ticker	ISIN	Amnt Issued(MM)	First Settle Date	Maturity	Mkt Type	Coupon	Coupon Freq	Coupon Type	Curr
1	BUBLL	DE0001115962	2000.00	11/02/2011	10/31/2012	BULLET	ZERO	NA	ZERO COUPON	EUR
2	BUBLL	DE0001116082	3000.00	01/25/2012	01/23/2013	BULLET	ZERO	NA	ZERO COUPON	EUR
3	BUBLL	DE0001116020	3000.00	02/23/2012	02/23/2013	BULLET	ZERO	NA	ZERO COUPON	EUR

Figure 28 – Portuguese Bond (ISIN: PTOTEYOE0007)

(Source: Bloomberg)

Page	Bond Info	Issuer Information	Identifiers
1	Name	OBRIGACOES DO TESOURO	BB Number
2	Type	Sovereign	ISIN
3	Covenants		PTOTEYOE0007
4	Guarantors		BBG00006C0D58
5	Security Information		
6	Mkt of Issue	Euro-Zone	Bond Ratings
7	Country	PT	Moody's
8	Rank	Sr Unsecured	Series
9	Inv Parties	Coupon 3.85	Type
10	Fees, Restrict	Con Freq Annual	Fixed
11	Schedules	Day Cnt ACT/ACT	Iss Price
12	Coupons	Maturity 04/15/2021	Reoffer
13	ALLQ Pricing	Issue Spread 12.50bp vs FRTR 4 1/2	19
14	ORD Quote Reac	Calc Type (335)PORTUGAL-DOM	SETL
15	TDH Trade Hist	Announcement Date 02/16/2005	Min Piece/Increment
16	CACS Corp Actn	Interest Accrual Date 02/23/2005	Par Amount
17	CF Prospectus	1st Settle Date 02/23/2005	Book Runner
18	CN Sec News	1st Coupon Date 04/15/2006	Exchange
19	HDS Holders	LONG 1ST CPN. ADD'L €1.108BEN (16VR) ISS'D 4/18/05. ADD'L €900MM ISS'D 5/16/05.	Multiple

Figure 29 – Portuguese Bond (ISIN: PTOTEGOE0009)

(Source: Bloomberg)

Page	Bond Info	Issuer Information	Identifiers
1	Name	OBRIGACOES DO TESOURO	BB Number
2	Type	Sovereign	ISIN
3	Covenants		PTOTEGOE0009
4	Guarantors		BBG00005ZYH6
5	Security Information		
6	Mkt of Issue	Euro-Zone	Bond Ratings
7	Country	PT	Moody's
8	Rank	Unsecured	Series
9	Inv Parties	Coupon 5.45	Type
10	Fees, Restrict	Con Freq Annual	Fixed
11	Schedules	Day Cnt ACT/ACT	Iss Price
12	Coupons	Maturity 09/23/2013	100.0100
13	ALLQ Pricing	Issue Spread	Amnt Issued/Outstanding
14	ORD Quote Reac	Calc Type (335)PORTUGAL-DOM	SETL
15	TDH Trade Hist	Announcement Date 05/19/1998	EUR 9,737,917.26 (M) / EUR 5,980,917.26 (M)
16	CACS Corp Actn	1st Coupon Date 09/23/1998	0.01 / 0.01
17	CF Prospectus	Exchange Notice Date 10/02/2012	Par Amount
18	CN Sec News	Exchange Expiration Date 10/03/2012	Book Runner
19	HDS Holders	AMT OUT REFLECTS POST REVENUE FUNDS. ADD'L €288848T ISS'D 3/3/99. €253141M 3/31/99. ADD'L TOWN ISS'D 08/00 FOR THE REPO MKT. ADD'L €700MM ISS'D 7/22/02.	Exchange

Figure 30 – German Bond (ISIN: DE0001135176)

(Source: Bloomberg)

GRAB			
DEUTSCHLAND REP DBR5 ½ 01/04/31 150.2000/150.4200 (2.14/2.12) BGN @14:58			
DBR 5 ½ 01/04/31 Corp 93 Feedback Page 1/11 Description: Bond			
Bond Description		Issuer Description	
<b>Pages</b>			
1) Bond Info			
2) Addtl Info		Identifiers	
3) Covenants		BB Number EC3022802	
4) Guarantors		ISIN DE0001135176	
5) Bond Ratings		BBGID BBG00003FDB5	
Security Information			
Mkt of Issue Euro-Zone		Bond Ratings	
Country DE		Moody's Aaa	
Currency EUR		S&P NR	
Rank Unsecured		Series 00	
Fitch AAA		DBRS AAA	
Coupon 5.5		Type Fixed	
Cpn Freq Annual		Iss Price 99.64000	
Day Cnt ACT/ACT		Maturity 01/04/2031	
Maturity 01/04/2031		Issuance & Trading	
BULLET		Amt Issued/Outstanding	
Issue Spread		EUR 17,000,000.00 (M) /	
Calc Type (60) GERMAN BONDS		EUR 17,000,000.00 (M)	
Announcement Date 10/17/2000		Min Piece/Increment	
Interest Accrual Date 10/27/2000		0.01 / 0.01	
1st Settle Date 10/27/2000		Par Amount 0.01	
1st Coupon Date 01/04/2002		Book Runner	
LONG 1ST CPN, €2,032.75 BLN RETAINED FOR MKT INTERVENTION.ADD'L €5BLN ISS'D		Exchange Multiple	
1/01@102.71%, ADD'L €1BLN ISS'D 2/01.ALSO EMTS.ADD'L €5BLN ISS'D 01/02.ALSO EUROMTS.			

Figure 31 – German Bond (ISIN: DE0001135234)

(Source: Bloomberg)

GRAB			
DEUTSCHLAND REP DBR3 ¾ 07/04/13 102.4800/102.4900 (-0.01/-0.02) BGN @14:58			
DBR 3 ¾ 07/04/13 Corp 93 Feedback Page 1/11 Description: Bond			
Bond Description		Issuer Description	
<b>Pages</b>			
1) Bond Info			
2) Addtl Info		Identifiers	
3) Covenants		BB Number ED0323820	
4) Guarantors		ISIN DE0001135234	
5) Bond Ratings		BBGID BBG00003YXS2	
Security Information			
Mkt of Issue Euro-Zone		Bond Ratings	
Country DE		Moody's Aaa	
Currency EUR		S&P NR	
Rank Unsecured		Series 03	
Fitch AAA		Composite AAA	
Coupon 3.75		Type Fixed	
Cpn Freq Annual		Iss Price 98.55000	
Day Cnt ACT/ACT		Maturity 07/04/2013	
Maturity 07/04/2013		Issuance & Trading	
BULLET		Amt Issued/Outstanding	
Issue Spread		EUR 22,000,000.00 (M) /	
Calc Type (60) GERMAN BONDS		EUR 22,000,000.00 (M)	
Announcement Date 06/24/2003		Min Piece/Increment	
Interest Accrual Date 07/04/2003		0.01 / 0.01	
1st Settle Date 07/04/2003		Par Amount 0.01	
1st Coupon Date 07/04/2004		Book Runner	
€3.115 BLN RETAINED FOR MKT INTERVENTION. ADD'L €7BLN ISS'D 8/03 @96.48% & €7BLN/9/03		Exchange Multiple	
@95.8%. LISTED ALSO IN MILAN.			

Figure 32 – Interest Rate Risk Immunization using Portuguese Bonds (Values in Euros)

Date	Dirty Price (%)		Fisher & Weil Duration (Years)			Bond Portfolio (%)		Investment Bond Portfolio	Investment Future Liability	Immunization Result	Nominal Amount		Coupon Received	
	PGB 23/09/2013 5,45%	PGB 15/04/2021 3,85%	PGB 23/09/2013 5,45%	PGB 15/04/2021 3,85%	Future Liability	PGB 23/09/2013 5,45%	PGB 15/04/2021 3,85%				PGB 23/09/2013 5,45%	PGB 15/04/2021 3,85%	PGB 23/09/2013 5,45%	PGB 15/04/2021 3,85%
Mar-05	116,87	98,72	6,79	11,95	8,88	0,59	0,41	72.991.615	-72.991.615	0	37.150.859	29.955.422	0	0
Jun-05	121,82	104,28	6,61	11,85	8,63	0,61	0,39	76.493.048	-76.980.615	-487.567	38.613.103	28.247.205	0	0
Set-05	117,15	105,88	6,63	11,57	8,37	0,65	0,35	77.248.370	-77.109.737	138.633	42.684.320	25.730.237	2.104.414	0
Dez-05	116,41	106,49	6,34	11,25	8,12	0,64	0,36	77.088.864	-76.476.938	611.927	42.222.967	26.234.544	0	0
Mar-06	113,84	101,85	6,05	10,82	7,87	0,62	0,38	75.795.797	-74.032.524	1.763.272	41.224.535	28.342.126	0	1.010.030
Jun-06	112,97	95,47	5,78	10,98	7,61	0,65	0,35	73.628.658	-73.225.398	403.260	42.191.803	27.197.654	0	0
Set-06	110,55	100,54	5,82	10,82	7,36	0,69	0,31	76.286.318	-75.520.253	766.066	47.737.365	23.387.082	2.299.453	0
Dez-06	109,87	99,75	5,55	10,49	7,11	0,68	0,32	75.779.314	-74.809.929	969.384	47.213.640	23.963.934	0	0
Mar-07	110,47	99,25	5,31	10,22	6,86	0,69	0,31	76.864.661	-75.153.641	1.711.020	47.676.289	24.378.562	0	922.611
Jun-07	108,76	91,40	5,03	10,24	6,60	0,70	0,30	74.133.773	-73.424.007	709.766	47.586.417	24.485.504	0	0
Set-07	106,11	93,96	5,07	10,09	6,35	0,75	0,25	76.096.149	-75.157.756	938.393	53.427.146	20.649.620	2.593.460	0
Dez-07	107,44	95,09	4,82	9,85	6,09	0,75	0,25	77.035.626	-76.140.339	895.287	53.614.952	20.437.421	0	0
Mar-08	109,83	97,07	4,60	9,69	5,84	0,76	0,24	79.508.321	-78.662.587	845.734	54.813.164	19.892.308	0	786.841
Jun-08	106,69	89,52	4,40	9,85	5,58	0,78	0,22	76.285.716	-75.720.612	565.105	55.978.845	18.503.056	0	0
Set-08	105,24	93,77	4,41	9,75	5,33	0,83	0,17	79.311.055	-78.272.097	1.038.959	62.412.680	14.536.244	3.050.847	0
Dez-08	109,82	101,35	4,20	9,72	5,07	0,84	0,16	83.271.301	-85.070.521	-1.799.220	63.846.340	12.982.796	0	0
Mar-09	111,77	98,51	3,98	9,47	4,82	0,85	0,15	84.651.900	-88.331.653	-3.679.753	64.116.944	13.183.161	0	499.838
Jun-09	114,21	95,94	3,74	9,62	4,57	0,86	0,14	85.873.435	-88.265.474	-2.392.039	64.584.446	12.626.623	0	0
Set-09	111,22	100,85	3,68	9,42	4,31	0,89	0,11	88.085.914	-89.781.431	-1.695.516	70.452.131	9.645.657	3.519.852	0
Dez-09	111,17	99,51	3,38	8,97	4,06	0,88	0,12	87.920.027	-90.261.503	-2.341.476	69.489.196	10.721.423	0	0
Mar-10	113,29	100,57	3,14	8,74	3,81	0,88	0,12	89.918.445	-92.652.457	-2.734.013	69.898.936	10.670.304	0	412.775
Jun-10	109,06	85,52	2,87	8,64	3,56	0,88	0,12	85.357.237	-94.090.632	-8.733.395	68.967.193	11.858.489	0	0
Set-10	102,26	83,56	2,74	8,28	3,30	0,90	0,10	84.190.402	-94.520.636	-10.330.234	74.010.504	10.184.929	3.758.712	0
Dez-10	103,36	82,42	2,50	8,00	3,04	0,90	0,10	84.891.675	-94.473.801	-9.582.126	74.001.796	10.195.849	0	0
Mar-11	96,13	74,21	2,20	7,45	2,79	0,89	0,11	79.098.286	-93.127.687	-14.029.401	72.964.449	12.068.592	0	392.540
Jun-11	91,17	60,79	1,91	7,33	2,54	0,88	0,12	73.859.645	-94.390.326	-20.530.681	71.569.968	14.160.016	0	0
Set-11	83,16	62,16	1,76	7,12	2,29	0,90	0,10	72.219.723	-96.566.985	-24.347.262	78.321.038	11.403.457	3.900.563	0
Dez-11	88,53	55,78	1,54	6,62	2,03	0,90	0,10	75.701.254	-97.382.475	-21.681.221	77.201.628	13.180.281	0	0
Mar-12	98,02	63,07	1,35	6,61	1,78	0,92	0,08	84.496.631	-98.144.036	-13.647.405	79.144.074	10.965.950	0	507.441
Jun-12	103,57	66,25	1,13	6,93	1,53	0,93	0,07	89.234.107	-98.740.630	-9.506.523	80.250.094	9.237.008	0	0
Set-12	102,35	72,82	0,95	6,80	1,28	0,94	0,06	93.237.225	-99.479.677	-6.242.452	86.033.863	7.113.609	4.373.630	0

Figure 33 - Interest Rate Risk Immunization using German Bonds (Values in Euros)

Date	Dirty Price (%)		Fisher & Weil Duration (Years)			Bond Portfolio (%)		Investment Bond Portfolio	Investment Future Liability	Immunization Result	Nominal Amount		Coupon Received	
	DBR 04/07/2013 3,75%	DBR 04/01/2031 5,5%	DBR 04/07/2013 3,75%	DBR 04/01/2031 5,5%	Future Liability	DBR 04/07/2013 3,75%	DBR 04/01/2031 5,5%				DBR 04/07/2013 3,75%	DBR 04/01/2031 5,5%	DBR 04/07/2013 3,75%	DBR 04/01/2031 5,5%
Mar-05	104,74	123,35	6,92	14,92	8,88	0,75	0,25	72.991.615	-72.991.615	0	52.609.389	14.501.836	0	0
Jun-05	105,75	133,36	6,97	15,14	8,63	0,80	0,20	76.948.653	-76.980.615	-31.962	57.998.329	11.707.831	1.972.852	0
Set-05	106,15	135,62	6,69	14,88	8,37	0,79	0,21	77.445.589	-77.109.737	335.852	57.970.779	11.729.395	0	0
Dez-05	105,49	131,37	6,40	15,17	8,12	0,80	0,20	77.207.398	-76.476.938	730.460	58.839.824	11.522.621	0	645.117
Mar-06	103,20	124,43	6,11	14,53	7,87	0,79	0,21	75.057.469	-74.032.524	1.024.945	57.559.195	12.584.692	0	0
Jun-06	98,76	120,20	6,07	14,04	7,61	0,81	0,19	74.130.982	-73.225.398	905.584	60.521.346	11.946.640	2.158.470	0
Set-06	101,54	128,41	5,83	14,10	7,36	0,81	0,19	76.792.965	-75.520.253	1.272.712	61.627.651	11.071.855	0	0
Dez-06	100,87	120,91	5,56	14,28	7,11	0,82	0,18	76.157.452	-74.809.929	1.347.523	62.096.470	11.184.396	0	608.952
Mar-07	101,29	119,49	5,31	13,94	6,86	0,82	0,18	76.260.793	-75.153.641	1.107.152	61.806.484	11.430.217	0	0
Jun-07	95,90	113,47	5,24	13,33	6,60	0,83	0,17	74.561.654	-73.424.007	1.137.647	64.650.307	11.069.262	2.317.743	0
Set-07	98,64	116,44	5,03	13,24	6,35	0,84	0,16	76.662.476	-75.157.756	1.504.720	65.221.117	10.585.683	0	0
Dez-07	99,89	111,99	4,78	13,62	6,09	0,85	0,15	77.587.797	-76.140.339	1.447.458	66.171.275	10.258.047	0	582.213
Mar-08	103,48	114,30	4,57	13,57	5,84	0,86	0,14	80.197.762	-78.662.587	1.535.176	66.599.297	9.870.554	0	0
Jun-08	99,93	109,96	4,54	13,22	5,58	0,88	0,12	79.901.543	-75.720.612	4.180.932	70.349.461	8.733.896	2.497.474	0
Set-08	100,99	115,25	4,33	13,29	5,33	0,89	0,11	81.109.537	-78.272.097	2.837.440	71.372.800	7.837.194	0	0
Dez-08	107,80	124,00	4,13	14,28	5,07	0,91	0,09	87.092.135	-85.070.521	2.021.614	73.287.996	6.519.797	0	431.046
Mar-09	109,36	121,46	3,91	14,08	4,82	0,91	0,09	88.068.896	-88.331.653	-262.757	73.305.732	6.503.827	0	0
Jun-09	109,15	118,89	3,79	13,63	4,57	0,92	0,08	90.493.708	-88.265.474	2.228.234	76.341.089	6.029.349	2.748.965	0
Set-09	106,83	124,62	3,54	13,53	4,31	0,92	0,08	89.066.369	-89.781.431	-715.061	76.915.688	5.536.771	0	0
Dez-09	107,73	118,16	3,25	13,56	4,06	0,92	0,08	89.710.404	-90.261.503	-551.099	76.742.366	5.952.515	0	304.522
Mar-10	109,99	125,24	3,01	13,53	3,81	0,92	0,08	91.860.788	-92.652.457	-791.669	77.182.305	5.566.159	0	0
Jun-10	108,63	135,79	2,88	13,64	3,56	0,94	0,06	94.298.245	-94.090.632	207.613	81.354.493	4.359.885	2.894.336	0
Set-10	108,52	143,49	2,62	13,50	3,30	0,94	0,06	94.537.871	-94.520.636	17.235	81.674.657	4.117.761	0	0
Dez-10	108,66	127,98	2,37	13,45	3,04	0,94	0,06	94.246.561	-94.473.801	-227.240	81.453.422	4.482.563	0	226.477
Mar-11	106,96	123,41	2,11	12,93	2,79	0,94	0,06	92.656.776	-93.127.687	-470.911	81.145.400	4.749.543	0	0
Jun-11	104,24	127,06	1,93	12,87	2,54	0,94	0,06	93.663.450	-94.390.326	-726.876	84.829.845	4.121.695	3.042.953	0
Set-11	106,60	146,89	1,70	13,25	2,29	0,95	0,05	96.483.903	-96.566.985	-83.082	85.916.431	3.333.149	0	0
Dez-11	107,45	145,28	1,46	13,52	2,03	0,95	0,05	97.341.208	-97.382.475	-41.267	86.286.948	3.185.303	0	183.323
Mar-12	107,29	145,40	1,21	13,32	1,78	0,95	0,05	97.209.624	-98.144.036	-934.412	86.334.967	3.149.870	0	0
Jun-12	103,70	150,89	1,00	13,19	1,53	0,96	0,04	97.515.445	-98.740.630	-1.225.185	89.969.063	2.798.081	3.237.561	0
Set-12	103,77	155,06	0,75	13,03	1,28	0,96	0,04	97.699.461	-99.479.677	-1.780.216	90.124.863	2.693.812	0	0