ISCTE Business School
Instituto Universitário de Lisboa

MANAGERIAL FLEXIBILITY AND COMPETITIVE INTERACTION IN INVESTMENT DECISIONS: A DISCRETE-TIME AGENCY THEORETIC PERSPECTIVE

JOÃO CARLOS DA ROCHA E CUNHA MONTEIRO

Tese submetida como requisito parcial para obtenção do grau de
Doutor em Gestão
Especialidade Métodos Quantitativos

Orientador:
Prof. Doutor José António Candeias Bonito Filipe, Professor Auxiliar
ISCTE-IUL – Departamento de Métodos Quantitativos

Janeiro de 2013

MANAGERIAL FLEXIBILITY AND COMPETITIVE INTERACTION IN INVESTMENT DECISIONS: A DISCRETE-TIME AGENCY THEORETIC PERSPECTIVE

JOÃO CARLOS DA ROCHA E CUNHA MONTEIRO

Tese submetida como requisito parcial para obtenção do grau de
Doutor em Gestão
Especialidade Métodos Quantitativos

Júri:
Doutor Rui Manuel Campilho Pereira de Menezes
Doutora Maria Rosa Vidigal Tavares da Cruz Quartín Borges
Doutor Manuel Francisco Pacheco Coelho
Doutora Maria Isabel Craveiro Pedro
Doutor Manuel Alberto Martins Ferreira
Doutor José António Candeias Bonito Filipe

Janeiro de 2013

ISCTE Business School
Instituto Universitário de Lisboa

MANAGERIAL FLEXIBILITY AND COMPETITIVE INTERACTION IN INVESTMENT DECISIONS:
A DISCRETE-TIME AGENCY THEORETIC PERSPECTIVE

.

João Carlos da Rocha e Cunha Monteiro

À minha família

Agradecimentos

A minha primeira palavra de agradecimento vai para o meu orientador, Prof. Doutor José António Candeias Bonito Filipe, pela dedicação, compreensão, encorajamento e disponibilidade demonstrada no acompanhamento deste trabalho. O seu apoio foi verdadeiramente fundamental para a sua conclusão, pelo que por isso lhe ficarei eternamente grato.

Gostaria também de agradecer a todos os docentes deste programa doutoral pelos conhecimentos transmitidos, pela disponibilidade demonstrada e pela forma, cordata e amigável, como sempre me receberam e como sempre esclareceram qualquer dúvida apresentada. Não posso no entanto deixar passar a oportunidade para estender o agradecimento a todos os docentes que tive desde o início da minha formação.

Para os meus colegas de ventura deixo também palavras de gratidão. Foram uma parte importante daquele primeiro ano já tão distante, pelo que o companheirismo e entreajuda que todos demonstraram merecem também uma referência nesta página.

Tenho ainda que referir todos os meus colegas de trabalho e alunos, pela compreensão demonstrada pelas ausências e por algum eventual menor acompanhamento de um ou outro assunto.

Por fim, palavras de gratidão para a minha família e amigos. Por tudo o que eles sabem que foram nestes anos de trabalho.

Sumário

Este trabalho pretende incorporar numa perspectiva unificada três diferentes temáticas das finanças empresariais: a flexibilidade operacional presente em projectos de investimento através dos denominados modelos de opções reais; a existência de custos de agência em consequência de estratégias de financiamento que recorrem a capitais próprios e a capitais alheios; a concorrência entre empresas, em mercados não puramente concorrenciais, analisada através da teoria de jogos. Pretende-se com este trabalho trazer uma perspectiva integradora de análise que constitua uma mais valia para a literatura e que permite trazer novos prismas de abordagem a estas questões. Com efeito, até ao presente, a ligação entre estes diferentes aspectos, que afectam a tomada de decisões empresariais em mercados concorrenciais, ainda não se encontra estabelecida de uma forma unificada. Como tal, julgamos que a ligação aqui estabelecida pode contribuir para uma melhoria na compreensão dessa mesma tomada de decisões.

O trabalho desenvolvido inicia-se com uma revisão teórica dos principais conceitos e desenvolvimentos mais recentes em opções reais, teoria da agência e teoria dos jogos. A seguir, desenvolve-se um modelo a tempo discreto que unifica essas mesmas teorias. Tal modelo desenvolve-se a partir da análise de Mauer e Ott (2000) e da de Smit e Trigeorgis (2004). Posteriormente, procede-se a uma simulação e apresenta-se as conclusões da análise realizada. Os resultados encontrados demonstram que, na simulação realizada a existência de concorrência em mercados não puramente competitivos produz impactos significativos na tomada de decisões empresariais.

Palavras-chave: Opções Reais, Teoria da Agência, Teoria dos Jogos e Estrutura de Capitais.

JEL Classification System: C70; G31; G32.

Abstract

This work intends to incorporate into a unified perspective three different fields of corporate finance: the managerial flexibility present in investment projects through the so-called real options models; the existence of agency costs as a result of financing strategies that rely in a mix between equity and debt; the existence of competition between firms, in non-purely competitive markets, through game theory. The aim of this work is to produce an integrated perspective of analysis that constitutes a value added to the literature, bringing new angles of approach to these issues. Indeed, to date, the link between these different aspects that affect managerial decisions in competitive markets is not yet established in a unified point of view. As such, we believe that the connection established in the present research may contribute to an improved understanding of that decision-making process.

The work comprises two fundamental components. First, a theoretical review of concepts and latest developments in each of the different themes which are later combined. After such review, a discrete-time model that makes the connection between these theories is developed. Such model departs from the analysis of Mauer and Ott (2000) and Smit and Trigeorgis (2004). Afterwards, a numerical simulation is performed and the findings from such analysis are described. The results, from the simulation performed show that the existence of competition in non-purely competitive markets does produce a significant impact in managerial decisions.

Keywords: Real Options, Agency Theory, Game Theory and Capital Structure.

JEL Classification System: C70; G31; G32.

Resumo Executivo

A recente mudança no ambiente económico global produziu uma transformação significativa nas condições que afectam as decisões empresariais. De facto, a globalização conduziu a uma maior flexibilidade de gestão e a uma transformação na concorrência empresarial. Adicionalmente, a crise financeira provocou uma falta de liquidez nos mercados. Estas circunstâncias deram origem a uma mudança na forma como as oportunidades de investimento devem ser avaliadas por empresas globais.

A flexibilidade operacional pode traduzir-se na possibilidade de expansão para outros mercados para além dos inicialmente considerados. Tal expansão é hoje mais fácil de conseguir. Esta possibilidade é analisada na literatura através das opções reais.

Em mercados globais a competição é assegurada por empresas globais, que são escassas. Portanto, a concorrência à escala global é feita por um número limitado de empresas. Este tipo de competição é analisado na literatura através da teoria dos jogos.

A falta de liquidez nos mercados financeiros torna mais difícil a obtenção de financiamento e vem exacerbar os conflitos de interesses entre os diferentes detentores de capital de uma empresa. Tais conflitos são estudados na literatura através de teoria da agência.

A presente pesquisa pretende integrar estas teorias numa perspectiva unificada. Tal perspectiva deve ser capaz de analisar oportunidades de investimento no ambiente económico que as empresas globais encontram actualmente.

Mauer e Ott (2000) desenvolveram um modelo de opções reais com a existência de conflitos de agência entre capital próprio e capital alheio. Smit e Trigeorgis (2004) desenvolveram um modelo de opções reais com a existência de concorrência entre empresas. O modelo construído neste trabalho combina estas duas perspectivas. É um modelo a tempo discreto que analisa opções reais na presença de conflitos de agência entre capitais próprios e alheios em concorrência. Tal análise é realizada considerando a existência de duas empresas no mercado que partilham uma opção de crescimento. As empresas são financiadas por capitais próprios e alheios e o exercício da opção é financiado por uma emissão de capital próprio adicional. Sob esta configuração, equilíbrios de mercado alternativos são considerados, nomeadamente, Cournot-Nash e Stackelberg. Duas políticas alternativas são definidas: uma política óptima, que

maximiza o valor da empresa, e uma política secundária, que maximiza o valor dos capitais próprios da empresa.

Os resultados obtidos com a simulação numérica realizada demonstram que os custos de agência existem na presença de concorrência e levam a uma situação de menor investimento por parte das empresas. Tal ocorre devido à transferência de riqueza dos capitais próprios para os alheios. A despesa de capital necessária para exercer a opção de crescimento, sendo apenas financiada por capitais próprios, beneficia ambos os capitais iniciais, nomeadamente os capitais alheios. Com esta situação de redução de investimento, o valor da empresa é afectado. O valor obtido para a empresa pela política secundária é menor do que o valor obtido para a empresa com a execução da política óptima. Essa diferença no valor da empresa é o custo de agência do financiamento em capitais alheios.

Executive Summary

The recent change in the global economic environment produced a significant transformation in the conditions that affect managerial decisions. In fact, the increase in globalization led to higher managerial flexibility and a transformation in competition. Additionally, the financial crisis caused a lack of liquidity in financial markets. These circumstances originated a shift in the way investment opportunities should be analysed in global industries.

Managerial flexibility can translate itself into expansion to other markets besides the initial ones. Such expansion is today easier to achieve than it was before. This possibility is analysed in the existing literature through real options analysis.

In global markets competition is assured by global firms. However, the number of these firms is scarce. Therefore, global competition is made by a limited number of firms. This type of competition is analysed in the existing literature through game theory.

The lack of liquidity in financial markets makes financing harder to obtain and exacerbates the conflicts of interests between the different stakeholders of a firm. Such conflicts are studied in the existing literature through agency theory.

Departing from these theories, the present research integrates them in a unified perspective. Such perspective should be capable of analysing investment opportunities in the present economic environment global firms face.

Mauer and Ott (2000) developed a real options analysis under agency conflicts between equity and debt. Smit and Trigeorgis (2004) developed a real options analysis under competition. The framework constructed combines these models. A discrete-time real options analysis under agency conflicts between equity and debt in the presence of competition is developed. The setting under which such analysis is performed considers two firms in a market that share a growth option to expand its scale of operations for a fixed investment outlay. The firms are financed by both equity and debt and the exercise of the expansion option is financed by an additional equity issue. Under this setting, alternative market equilibriums are considered, namely, Cournot-Nash and Stackelberg. Two alternative managerial policies are defined: a first best policy, which maximizes the value of the firm; and a second best policy, which maximizes the value of the equity of the firm.

The results obtained with the numerical simulation performed demonstrate that agency costs exist in the presence of competition and lead to an underinvestment situation. The underinvestment occurs because of the wealth transfer effect from equity to debt. The additional capital expenditure necessary to exercise the growth option, being solely equity financed, benefits the existing stakeholders, namely debt. With this underinvestment situation, firm value is affected. The value for the firm given by the second best policy is lower than that obtained with the execution of the first best policy. This difference in firm value is the agency cost of debt financing.

Index

Index of Tables

Index of Figures

List of Abbreviations

CAPM:	Capital Asset Pricing Model
DTA:	Decision Tree Analysis
FCF	Free Cash-Flow
GBM:	Geometric Brownian Motion
IRR:	Internal Rate of Return
MVI:	Marginal Volatility of Investment
NPV:	Net Present Value
R&D:	Research and Development
ROA:	Real Options Analysis
WACC:	Weighted Average Cost of Capital

1. INTRODUCTION

The economic environment in which firms operate is in constant transformation. The growing globalization of the market economy affects managerial decisions and changes the paradigms under which such decisions are based on. In fact, globalization makes competition ever greater in a wide variety of economic sectors because of the easier access to other markets beside the internal ones. It also tends to make investment opportunities more flexible due to a broader applicability of technology to other purposes besides the original ones.

Today, in many economic industries the focus of competition is set at a global scale. Globalization of the market economy makes competition transferable to a world level. However, the possibility to compete at a global scale is only accessible to a limited number of firms. Therefore, such general increase in competition also causes a difference in the type of competition global firms have to face. In fact, such competition is being performed by a limited number of firms in each particular industry. We are witnessing an increase in competition by global firms that compete among themselves in different markets and in different products. In this setting, models that take into account the impact of one firm's decisions in the other firm's behaviour are the ones that better adjust to this economic environment. Therefore, game theoretic models of competition gain a renewed relevance.

At present, managerial flexibility is getting more and more present in investment opportunities. In fact, an investment opportunity, directed to a particular market, can more easily be replicated and developed to a broader one. In addition, technological breakthroughs can more easily be transferred to other industries. With globalization, access to external markets and the expansion of the initial concept to other realities is more easily performed. These two combined aspects lead to an increase in operational flexibility and highlight its present relevance. Models that incorporate such managerial flexibility are models that are best suited for today's economical environment. Therefore, real options models gain a renewed relevance.

Additionally, at present times we are facing tremendous constraints in financial markets. The recent financial crisis affected immensely the way in which financial markets operate and their capability to provide the necessary funding to firms. This increased difficulty results mainly from a lack of liquidity in these markets. Among others, three consequences from this situation are worth being mentioned for the purpose of the present research. Firstly, the financing of investments is now a harder task than it was before. Secondly, the problems between the different stakeholders of the firm tend to be worse than before. Thirdly, it is much more difficult for firms to rollover their initial debt issues. Models that take into account these different, yet complementary, aspects reflect better the actual economic environment. However, we shall concentrate the analysis in the problems that arise between the different stakeholders of the firm. Therefore, agency theoretic models gain a renewed relevance.

Despite the fact that the above mentioned effects are not all reflected in all industries, they are widespread in different magnitudes to different industries. However, for some particular industries they are all present. In fact, global firms which operate in markets where entry barriers do exist face all the above mentioned effects. They face a fearsome competition but only from a limited number of rivals. They generally possess high operational flexibility since they can easily proceed to other markets, hence they are global. They also possess technology that can easily be adopted by other industries, thus enlarging such operational flexibility. And finally, they also face financial constraints because of the lack of liquidity present in financial markets, which causes agency conflicts between their stakeholders.

As a consequence of all these combined effects, a reflection about the new conditions that affect managerial decisions is necessary, namely, decisions concerning investment opportunities. With this new economic environment investment decisions are particularly affected. Higher economic uncertainty increases the risk associated with expected future cash-flows. Lack of liquidity in financial markets increases the cost of equity and debt financing.

The present dissertation aims to implement a model that integrates these new problems into a unified perspective. The focus of the present research is the analysis of the impact of the financing structure in investment decisions that present managerial flexibility in a competitive market. However, such model must depart from previous work developed in the different fields of research that are being integrated. It departs from Smit and Trigeorgis (2004) and Mauer and Ott (2000).

It is well documented (Smit and Trigeorgis, 2004)[1] that market structures portray an influence on the firm's investment decisions. In a static approach to competition, it was shown that different market equilibriums, namely Cournot-Nash and Stackelberg, result in differences in the investment decisions of firms, and therefore, in firm value. In a duopoly setting, with both firms sharing a growth option and possessing an abandonment option, alternative competitive responses are analyzed. Departing from the monopolistic market structure as benchmark, the analysis derives the expressions for firm value under Cournot-Nash and Stackelberg equilibriums. Therefore, it is examined how such equilibrium competitive responses influence investment decisions and firm value through the differences in firm value compared to the monopolistic market structure.

It is also accepted that the exercise of growth options can, under certain financial structures, lead to an underinvestment problem, due to the existence of agency conflicts between equityholders and debtholders of the firm (Mauer and Ott, 2000 and Childs et al. 2005). In a typical underinvestment situation, equityholders decide to invest later[2] in a project (with similar risk characteristics to the existing portfolio of investment projects) when compared to the optimal investment timing because the increase in the asset base will increase the value of the debtholders' claims at the expense of equityholders. Rather than investing when it is optimal for the firm, equityholders tend to wait until the market evolves favourably and invest at a higher price of the underlying asset / project when the increase in value of the debtholders' claims is not accomplished at the expense of equityholders. Since the debtholders claim is fixed, they cannot expect to gain more than seeing their claim become riskless. This can occur either by a reduction on the volatility of the underlying asset or by an increase of the asset basis of the firm. If the increase is due to an additional investment performed by equityholders, the debtholders will benefit from it without having incurred in any additional cost. On the equityholders perspective, whatever return their additional investment yields, it is

[1] The important reference to be made concerning the present dissertation is the one-stage scenario, although a two-stage investment project was also considered. Under this alternative setting, it was shown that, no matter whether competition exists for the first-stage or second-stage investment, when a firm can obtain proprietary growth options or pre-empt competitive entry in the second-stage, the firm has great economic incentive to invest early in the first-stage even when investment returns are uncertain.

[2] This perspective represents the common real options perspective on the underinvestment problem. Traditionally underinvestment was seen as investing less than would be optimal in order to avoid the wealth transfer effects from equityholders to debtholders (Myers 1977). It is also possible to interpret underinvestment as delaying investment. However, as Mauer and Ott (2000) and Childs et al. (2005) refer, waiting for higher prices decreases the probability of the investment taking place, therefore leading to underinvestment.

going to be shared with the debtholders. They support all the costs and have to share the benefits. If equityholders wait to invest at a higher value of the underlying asset (project's present value), debtholders will have already benefited from this increase and whatever return equityholders get from the investment decision, it will no longer be shared with debtholders. This explains why, in the presence of pure expansion options, equityholders have an incentive to underinvest.

Departing from these theories, the present research addresses how different financial structures, for firms with shared growth options, can influence the firm's investment decisions in a context where competition is present. It is in accordance with the questions faced by firms under the new economic environment. The research addresses real options as a consequence of the operational flexibility we intend to incorporate. It also addresses a game theoretic approach to competition as a consequence of the analysis of competition by a reduced number of firms. Finally, it addresses agency conflicts as a result of the financial constraints to credit in the markets. It shall be studied if, under these conditions, capital budgeting decisions are affected. Namely, in a competitive market, with managerial flexibility present in the investment opportunities firms face, and with conflicts of interest between the different stakeholders of the firms, which are the managerial decisions that maximize firm value and which are the managerial decisions that firms actually take. Therefore, the work developed intends to incorporate the financial effect that can lead to the existence of underinvestment situations, through the appearance of agency conflicts, in the game theoretic approach that incorporates the effect of competition in management's flexibility.

The framework implemented consists on a two firm model in a context where both firms possess a shared growth option to expand their scale of operations and an abandonment option. The existence of these options is treated under a discrete-time approach. This methodology is not the most widely used in the literature, but it possesses the advantage of being more easily treatable and more suitable for a practical implementation[3]. Being the growth option shared by both firms, competition is therefore present. We intend to model it with the use of a game theoretic approach. The alternatives used to analyse the impact of competition are in accordance with Smit and Trigeorgis (2004). Therefore, Cournot-Nash and Stackelberg equilibrium will be

[3] This advantage is generally recognized in the existing literature, namely in Chevalier-Roignant et al. (2011).

analysed[4]. Additionally, the model developed integrates agency conflicts between stockholders and bondholders. The two firms considered are financed with both equity and debt. However, the financing of the growth option is made through an additional equity issue. This additional equity issue causes a wealth transfer from equityholders to bondholders, which is in accordance with Mauer and Ott (2000). The purpose is to analyse if this effect still holds in the presence of competition under a discrete-time model.

The wealth transfer effect is expected to delay the exercise of the growth option. The question to be answered in the present research is if such wealth transfer effect in the presence of competition still results in an underinvestment situation. Therefore, with this methodology, a unified perspective of these different factors is performed. It is therefore the opportunity to understand which decisions should be taken by firms that operate under such reality. We will analyse the results obtained from the model constructed with a numerical simulation.

In order to achieve such goal, the dissertation starts with a review of the most relevant theoretical concepts developed. The following chapters, chapter two to chapter five, describe the main aspects developed in the different bodies of literature that are being integrated. Chapter two presents a general perspective on the problem. Chapter three characterises real options analysis and describes the appearance and most relevant developments under such perspective. Afterwards, in chapter four, we perform an analysis of the conflicts of interest between the different stakeholders of the firm. Subsequently, the merging of these two different approaches is presented and described. In chapter five, we perform a characterization of game theory and present the most relevant developments of such theory. We end this initial approach with an analysis of the integration of game theory with real options analysis, describing real options games.

The remaining chapters of the dissertation present the research questions to be answered and the methodology from which we departed for the research, in chapter six, the description and implementation of the model with a description of the results obtained under a numerical simulation, in chapter seven, and, finally, the conclusions of the work follows, in chapter eight.

[4] The monopoly equilibrium will also be referred but it results from the lack of competition, and it is therefore less relevant for the analysis.

Having specified the basic pillars under which the present research is laid, its main objectives and the structure that will be adopted, it is now time to proceed with a general analysis of the problem according to the relevant literature in the field.

2. RESEARCH BACKGROUND

The introduction described the aim of the present research and sketched the problem to be answered. The theoretical review of the literature will be performed in the following chapters.

In this chapter, we present a general description of the fundamental building blocks of the different bodies of literature used in the present study. It intends to perform an introduction to the different subjects of analysis. It starts with the presentation of real options analysis (ROA), followed by a description of the conditions that link investment and financing decisions. The chapter ends with a description of game theory and of real options games.

2.1. Real options analysis (ROA)

The central paradigm for decision-making concerning investment projects has been the net present value (NPV) approach[5]. However, this approach neglects flexibility, or the capacity of a firm's management to choose among different future opportunities associated with the project.

Recognition of such managerial flexibility in investment opportunities originated the appearance of real options. In fact, the investment opportunity itself could be regarded as an option. It is a call option on the cash flows generated by the investment expenditure necessary (the exercise price) to trigger such cash flows. This was initially recognized by Myers (1977), and later reinforced by Kester (1984). Shortly after, Trigeorgis (1988) summarized such findings compiling the necessary adaptations in the inputs needed to value financial options to the ones needed to value real options.

Consideration of investment projects as real options allowed recognition of other options embedded in investment opportunities. The options to defer, expand, abandon, switch use or other managerial possibilities were added to the list. However, we will focus our attention on the options to defer (as a basic flexibility principle), to expand the scale of operations (growth option), and to abandon operations (as a limiting downside

[5] The relevance of the NPV criterion in current investment decision making will be referred in the next chapter.

situation). The recognition of these types of flexibility is as old as the recognition of real options itself. Myers (1977) made the first reference to the possibility management has to expand the scale of operations at a later phase of project development. Kensinger (1987) considered that, among other options, expansion of operations could be equivalent to a financial call option, and therefore it could be valued as one. When this recognition occurred, McDonald and Siegel (1986) had already deducted the option value of waiting to invest. Later, the abandonment option was identified and valued by Kemna (1988) and Myers and Majd (1990).

Such consideration of investment projects and their subsequent valuation as options enabled the partition of their value in three main groups. Firstly, its in-the-money value, which is nothing else than the value of the project if implemented immediately. This component of value is the NPV of the project, its value without consideration of any option associated with it. Secondly, one has to add the fundamental option present in such investments, the option to defer the investment related to the opportunity cost associated with immediate implementation. Finally, consideration of operational flexibility in management's decisions leads us to the consideration of other options that might be present in the investment opportunity that the firm faces.

The existence of such elements of value leads to implementation of ROA only whenever three different conditions are met. The first concerns the existence of uncertainty in the future. The second relates to the irreversibility of the investment decision, either totally or partially. The last condition considers the ability to delay the exercise of the option by the firm that holds it.

The joint existence of these three conditions is the cause of value for the investment project. It also implies that the use of ROA tends to delay investment when compared with traditional discounted cash flow analysis.

This last conclusion had already been formulated by Kester (1984). When he recognized investment projects as options he also defined the elements that make option value. Such elements are associated with the option's exercise time, the risk of the investment project, the level of risk-free interest rates and the existence (or not) of proprietary rights regarding the option's exercise. Therefore, he acknowledged that considering investment opportunities as options may lead to the deferral of the investment decisions. This recognition was also formulated by Trigeorgis (1988) and valued by McDonald and Siegel (1986). The optimal investment timing must be compared to that of a dividend distributing call option. This view was defined by Smit and Ankum (1993), and later by Dixit and Pindyck (1994). These initial works established a new setting for the analysis of investment opportunities.

2.2. Interactions between investment and financing decisions

Independence between investment and financing decisions can be traced back to Fisher's theorem of separation (Fisher, 1930). Afterwards, Williams (1938) contains the first exposition on the irrelevance of capital structure to firm value, without providing, however, a formal proof for the theorem of separation between investment and financing decisions.

Despite such contributions, the relationship between investment and financing decisions was still commonly reduced to the determination of the rate of return on investments discounted by a risk premium that was compared to the market rate of return. This rate represented a hurdle rate in the decision to accept or reject an investment opportunity. There was no theoretical support for the size of the risk premium and for the source and type of financing.

The celebrated paper from Modigliani and Miller (1958) proved the indifference between financing alternatives and the irrelevance of financing decisions to the market value of the firm. Consequently, it also demonstrated that the firm's investment decisions are independent of their financing policy. In fact, given the firm's investment policy and ignoring taxes and contracting costs, the firm's choice of financing policy does not affect the current market value of the firm. Despite the huge breakthrough in financial theory that such recognition enabled, it left unanswered the observed practice of corporate financing policies.

They assumed the inexistence of market imperfections, the riskiness of debt, symmetry of information and the inexistence of agency costs. Despite the restrictive nature of these assumptions, the results obtained still hold even if we relax most of them. However, if we consider that the financing policy of the firm plays a role in its market value, we must also realize that some of these assumptions do not hold, causing an impact in firm value.

The first assumption to be relaxed concerned the inexistence of taxes. Modigliani and Miller (1963) included corporate taxes in the analysis arriving to an all debt optimal capital structure due to the tax benefit of debt. Later, Miller (1977), Brennan and

Schwartz (1978) and de Angelo and Masulis (1980) developed this line of research analyzing the impact of personal as well as corporate taxes on market value.

A second field of research concerned the existence of bankruptcy costs. Baxter (1967) was the first to refer this possibility by specifying indirect bankruptcy costs as specific contracting costs which arise because the firm's investment policy and other resource allocation decisions are not fixed. Later, Stiglitz (1972), Kraus and Litzenberger (1973), Warner (1977) and Kim (1978) further developed this line of research, introducing direct bankruptcy costs and arguing that these costs can be a cause for the existence of an optimal capital structure.

These research developments showed that in the presence of taxation and potential bankruptcy costs the indifference proposition fails, so that the firm must choose an optimal financing method. However, the independence proposition still holds, so that the choice of financing method is independent of the investment decision. Therefore, if a relationship between investment and financing decisions is to be determined, other aspects must be considered.

The seminal work developed by Alchian and Demstez (1972) developed a new field of research related to the study of conflicting interests between different parties. They developed the concept of agency theory as the study of an agency relationship, in which one party (the principal) delegates in another (the agent), a certain task. The theory developed is concerned with the solution of two problems that can occur under such relationship. The first is the agency problem that arises when the interests of the principal and the agent conflict, being difficult or too expensive for the principal to verify the agent's work. The second is the risk sharing problem that arises when the principal and agent have different attitudes towards risk.

Later, in another seminal paper, Jensen and Meckling (1976) applied the concept to the finance field, using the agency framework to analyse the effects of conflicts of interests among stockholders, managers and bondholders on the investment and financing decisions of the firm. They argued that the capital structure problem involves the determination of the entire set of contracts among the different stakeholders of the firm, bearing in mind the existence of such conflicts.

This way, agency conflicts can arise not only between equity and bondholders (with equity as agent of debt), but also between managers and equityholders (managers as agents of equityholders).

The later sets of problems have been studied by Jensen (1986). They are commonly referred to as the agency problem of managerial discretion or as the agency problem of free cash flow (FCF). Equityholders want profitability, even at the expense of higher risks[6], while managers want growth and security[7]. The problems arise because growth is generally obtained sacrificing profitability (expensive mergers and acquisitions) while security requires the diversification of the firms' portfolio of businesses. The disciplinary role of debt has been shown to provide an efficient mechanism for controlling these conflicts and the market has been shown to perceive it (Jensen, 1985, 1986, Jensen and Smith, 1985 and Smith, 1986).

The first set of problems were initially developed by Myers (1977) and Galai and Masulis (1976), who argued that, in the presence of debt financing, if a conflict of interests between debtholders and stockholders emerges, the stockholders' decision will not necessarily be the one that debtholders would prefer. It was also argued that bondholders are aware of this, and will efficiently price debt so as to compensate them for any foreseeable loss of value to shareholders. It was shown that under these assumptions, the value-maximizing decision for shareholders will take both the project potential and the financing method into account. They show that the investment trigger prices, or the demand levels, at which the firm would invest, differ from those that would be used if the shareholders' goal is to maximise value the value of the firm in total, not just the shareholders' wealth. If this applies, firms may invest more/less and sooner/later than would be socially optimal.

This recognition led to the consideration of agency costs as the difference in firm value that result from the establishment by the firm of a second best policy instead of a first best policy. A second best policy is a policy that maximizes the value of the equity of the firm, while a first best policy is a policy that maximizes the value of the firm.

For investment decisions, these differences involve investing more or less than it would be optimal for the total value of the firm, or investing at different prices from the optimum. The investment biases can be classified as underinvestment problems,

[6] Ceteris paribus, higher risk raises the value of their claims. If we think of shares as call options on the assets of the firm, higher risks raise the expected value of the equity claims.

[7] Donaldson (1984) defines the objective function of managers as pursuing "corporate wealth" maximization. Corporate wealth represents the aggregate purchasing power available to management for strategic purposes comprehending cash, credit, and other corporate purchasing power. The existence of internal funding also releases managers from the market scrutiny and monitoring whenever funds are needed for financing investments.

initially formulated by Myers (1977) and overinvestment problems, initially presented by Jensen and Meckling (1976) and Galai and Masulis (1976).

In a typical underinvestment situation, as defined by Myers (1977), equityholders invest later than it would be optimal under a firm value maximizing policy. This happens because the increase in the asset base that results from the equity financed investment also reverts to the value of the debtholders' claims. Therefore, equityholders prefer to wait until the market evolves favourably and invest only when no increase in the debtholders' claims can be accomplished at their own expense. If the increase is due to an additional investment performed by equityholders, the debtholders will benefit from it without having incurred in any additional cost. On the equityholders perspective, the returns from their additional investment are to be shared with the debtholders. They support all the costs and have to share the benefits. If equityholders wait to invest only after debtholders have already benefited from the market favourable evolution, the returns from their additional investment will no longer be shared with debtholders. This explains why equityholders have an incentive to underinvest.

A typical overinvestment situation reflects the investment on a different risk class. In this case, equityholders chose to invest earlier (at a lower present value of the investment project[8]) in a riskier project (usually correlated with the current portfolio of projects in order to reduce diversification effects), thus increasing the overall risk of the firm. This increased risk diminishes the value of the debtholders claims. This situation can even lead to a substitution of the existing assets for other, riskier, ones. This is generally designated as the asset substitution problem. The logic underlying the wealth transfer is similar to the overinvestment situations but the wealth transfer effects are in this case much more severe.

The relationship between investment and financing decisions is now established. The conditions under which it can occur and the effects that debt might have in corporate investment decisions are specified. At this stage, we must consider such effect in corporate investment decisions whenever managerial flexibility is present.

[8] Similarly to the underinvestment situation, this description also assumes the real options perspective; traditionally overinvestment was seen as investing more than would be optimal in an investment project, or in some cases even investing in projects with negative NPV.

2.3. Interactions between investment and financing decisions under real options analysis

Literature on real options uses contingent-claims techniques to value and determine optimal exercise policies for the firm's capital budgeting options. However, this literature typically assumed all-equity financing and only recently began dealing with the impact that mixed financing structures might have in the analysis undertaken. Some earlier exceptions are Brennan and Schwartz (1984) that consider firm valuation in a setting in which bond covenants restrict financial policy and influence investment policy, Mello and Parsons (1992) that compare the operating decisions of a mine under all-equity financing to those when the mine is partially debt financed and maximizes levered equity value, Trigeorgis (1993) which illustrated interactions between financing and operating real options, and finally Mauer and Triantis (1994) that analyzed interactions between investment and financing decisions in a setting in which debt covenants constrain the firm's choice of policies to maximize firm value.

They analysed the interactions between a firm's dynamic investment decisions (option to invest), operating decisions (option to alter the scale of operations, open and shut down operations) and financing decisions (quantities of debt and/or equity). They further assumed several market imperfections. However, it was also assumed that managers chose the policies based on firm value maximization, thereby avoiding agency conflicts between debtholders and equityholders. Due to this fact, only firm value was modelled and there was no explicit valuation of either equity or debt. Debt financing was found to have a negligible impact on the firm's investment and operating policies. The incentive to invest earlier (due to the motivation of earning interest tax shields) that leveraged firms have, was offset by the loss of value on the option of waiting to invest. The net benefit, although present in some cases, was not large enough to significantly affect the investment policy. They concluded that the existence of debt financing had no impact in the determination of investment timing.

In practice, these conclusions allow managers to make their investment decisions independently of their decisions on capital structure whenever agency conflicts are not considered. Another significant conclusion of Mauer and Triantis (1994) concerns the fact that increased investment flexibility increases the value of the firm, allowing a greater debt capacity and, consequently, higher tax shields. Although Mauer and

Triantis (1994) relaxed several of the assumptions (risk free debt, transaction costs, bankruptcy costs), they continued to predict negligible interactions between financing and investment decisions.

In this sense, any possibility for interactions between financing and investment decisions may require the relaxation of one of the other assumptions that Mauer and Triantis (1994) did not relax, namely, the ‘no agency conflicts’ assumption[9]. Therefore, inclusion of agency conflicts in ROA followed.

Mauer and Ott (2000) argued that levered equityholders of a firm with assets in place and a growth option to expand the scale of operations have an underinvestment incentive. This perspective represents the common real options perspective on the underinvestment problem. Traditionally, underinvestment was seen as investing less than it would be optimal in order to avoid the wealth transfer effects from equityholders to debtholders (Myers 1977). However, it is also possible to interpret underinvestment as delaying investment rather than underinvesting. Mauer and Ott (2000) and Childs et al. (2005) point out that this delay in investment for expectation of higher prices decreases the probability of the investment takes place.

Mauer and Sarkar (2005) argued that equityholders have an incentive to overinvest in order to appropriate the benefits that derive from interest tax shields. Similarly to the underinvestment situation, this description also assumes the real options perspective. Traditionally, overinvestment was seen as investing more than it would be optimal in an investment project, or in some cases even investing in projects with negative NPV.

The theoretical arguments that support the relevance of the interactions between financing and investment decisions under ROA are solid. However, it is possible that managers do follow first best investment policies. It is also possible that debtholders assume equityholders will not follow opportunistic behaviours. One must consider if there is any evidence that consubstantiate the theoretical findings or if real practice diverts strongly from these theoretical developments. Empirical work in this area, covering the full scope of flexibility, is scarce. Real flexibility is not easy to capture empirically and the proxies some authors use are subject to debate[10].

However, a type of real flexibility that has already been subject to several empirical tests in its relation to financial structure is the growth option. It has been specifically

[9] The authors included in the debt contract a covenant that forced equityholders to follow a firm value maximization policy.
[10] Possibly the most common proxy used for defining firms that present this type of flexibility is a high market-to-book ratio.

tested the relationship between growth options and the level of leverage in the firm. The debate on the effect that growth options have on the leverage of the firm is extensively documented in the corporate finance literature ever since the seminal work of Myers (1977). The incentive of equity to underinvest when sharing the benefits of investing with debt led the author to suggest the low collateral value of the growth options. Jensen (1986) extended the theory on agency problems to FCF problems and argued that assets in place presented a high collateral value. According to both arguments, the debt capacity of real flexibility is expected to be lower than the debt capacity of the assets in place.

It is possible to express the diverse arguments concerning the interactions between financing and investment decisions in a real options framework in two basic hypotheses[11].

The first one is commonly referred to as the 'value hypothesis' and states that additional real flexibility will increase the value of the firm, thereby allowing greater debt capacity. The increase in firm value created by real flexibility lowers the default risk and the expected bankruptcy costs. Indirectly, it enhances the debt capacity of the firm, increasing the target debt to equity ratio and the associated debt tax-shield. Under this hypothesis, there is a positive relationship between real flexibility and financial leverage. The theoretical basis for this argument is portrayed in Mauer and Triantis (1994).

The second one is commonly referred to as the 'agency hypothesis', and states that additional flexibility will only exacerbate the agency conflicts, thereby reducing the debt capacity of the firm. Real flexibility exacerbates the opportunistic behaviour of equityholders, extending the range of actions these can take in order to expropriate debtholders of their wealth. Assuming this argument, debtholders increase the cost of financing thereby reducing the optimal debt ratio of the firm's capital structure. Under this hypothesis, the relationship between real flexibility and financial leverage is negative. It is in accordance, amongst others, with the theoretical works of Mauer and Ott (2000) and Mello and Parsons (1992).

Bradley, Jarell and Kim (1984) had already shown that low levels of leverage for firms were generally associated with the existence of growth options, confirming the

[11] Several authors empirically tested these two hypotheses in order to assess the impact of agency conflicts between debtholders and equityholders in a real options framework (see Mackay, 2003, Rajan and Zingales, 1995, Barclay Morellec and Smith, 2003, amongst others).

expectations of Myers (1977) and Jensen (1986) and using the high market-to-book ratio proxy for growth options. Long and Malitz (1985), Smith and Watts (1992) and Barclay, Smith and Watts (1995) presented evidence of a negative relationship between market-to-book value and financial leverage. This evidence was later corroborated by Rajan and Zingales (1995), further demonstrating that the relation is not only negative but significantly negative.

More recently, Barclay, Morellec and Smith (2003) extended the empirical literature on this effect, using a sample of 104.746 firm-year observations from the industrial corporate sector, covering the years from 1950 to 1999. The authors extended the previous theory and tested the hypothesis that the ratio of debt to assets in place should fall with an increase in growth options. The results they reached confirmed the hypothesis tested. Here, the evolution in theory becomes clear, beginning with the prediction of the low collateral value of growth options supported by early empirical studies. Later empirical tests showed a negative relation between the existence of growth options and financial leverage. Recent work has come to show that this relation is not only negative but increasing growth options significantly decrease the collateral value of assets in place.

Recently, the measures of real flexibility were refined and extended. In Mackay (2003), interactions between real flexibility and financial structure is examined in 17 manufacturing industries, using data from the Longitudinal Research Database and the Quarterly Financial Reports, comprising 2.028 firms over the period of 1977-1990.

The author formulates two basic hypotheses (once again, the value and the agency hypothesis) concerning the relationships between real flexibility and financial structure, later testing a series of competing hypothesis about several dimensions of real flexibility and financial structure. One of the measures tested concerns the maturity of the debt contracts. The author tests these competing hypotheses by regressing financial structure ratios on indicators of real flexibility and on other variables such as firm size.

The results suggest a significant relation between financial leverage and real flexibility thereby confirming the agency hypothesis. For firms that present a higher level of production flexibility, the degree of leverage and the maturity of debt tend to be shorter[12].

[12] However, in the specific case of investment flexibility in buildings, the results support the value hypothesis, indicating that restrictive covenants are able to control the agency conflicts enhancing the collateral value of the assets, and thereby increasing debt capacity.

The results on samples from small firms help to confirm the arguments that, due to their concentrated equity ownership and greater degree of real flexibility, small firms face severe agency conflicts between equityholders and debtholders. However, the results also show that small firms rely more on bank loans, suggesting that monitored lending helps to contain agency conflicts increasing the overall debt capacity of the firm.

Mackay (2003) tested the conflicting arguments in the literature. Although, overall, the work confirmed the agency hypothesis, it also showed the context under which this hypothesis gains more strength and the contexts where the value hypothesis has some validity. It also suggests that other variables should be taken into account in the models, in particular, firm size.

Agency conflicts exist in the 'real world'. Their importance is significant and, therefore, should be taken into account. As evidence seems to support, real flexibility exacerbates these conflicts, so their magnitude is larger when real options are involved. To ignore these conflicts translates into biased conclusions regarding the investment decisions, because in the end the existence of debt financing influences the investment decisions.

2.4. Game theoretic approach to competition

Game theory analyses strategic interactions among economic agents that produce outcomes according to the preferences of such agents. It is the study of strategic decision making. Therefore, it allows the study of competition among firms in a specific market.

The development of game theory is related to the initial work published by Von Neumann and Morgenstern (1944). This initial breakthrough was shortly after complemented with Nash's (1950a, 1951) equilibrium perspective under non cooperative competition and Nash's (1950b and 1953), Shapley (1953) and Gillies (1953) equilibrium for cooperative game theory. Extension of these notions of equilibrium to other types of interaction between players followed.

Being a method to analyse interactions between players, its application to economic analysis allowed for a better understanding of competitive interaction under market structures where imperfect competition is present. As a result, the scope of microeconomic theory is thus redefined under this new approach.

Application of game theoretic approaches to imperfectly competitive market structures is performed through consideration of some predefined strategy from the firm. Traditional perspectives, such as Cournot or Stackelberg gain a new insight under this approach. They set forward the strategy the firm adopts towards their competitors' actions attaining Nash equilibrium. Such equilibrium is reached in a price quantity relationship. Therefore, its application to competition between firms naturally developed.

Application of these principles to the investment strategies of firms is the following step. The next section shows the developments associated with the connection between those two different fields.

2.5. Competition under real options analysis

Incorporation of managerial flexibility and competitive interaction in a single and unified analysis of investment opportunities is a recent development in finance, economics and corporate strategy fields. Emergence of ROA and game theoretic approaches made such unified perspective a reality. Under this perspective when a firm considers an investment decision, it is engaged in a game not only against nature, but also against its rivals.

Trigeorgis (1988 and 1991) first mentioned a significant difference between real and financial options. The first ones can be proprietary or shared. When shared they have less economic value, since they can be exercised by anyone of their owners. Kester (1993) complemented this view with the reference to the impact of competition on the timing of investment. Trigeorgis (1991) analyzed the impact of competition on the optimal timing of project implementation using option methodology. Kester (1993) pointed out that in order to appropriate the full value of the option by corporations in the presence of competition they could exercise their options earlier than would otherwise do.

Such recognition boosted the research concentrated in the relationship between competition and investment in a real options perspective. Some studies took market structures as given, analysing the impact that different market structures have on the investment decisions. Others have focused on how competitive dynamics may lead to endogenous structural changes and influence investment behaviour.

However, the first association between game theory and ROA is made by Smets (1991), followed by Smit and Ankum (1993) in a discrete-time application, followed by Dixit and Pindyck (1994) in a continuous-time framework. Later, Grenadier (1996) applied a game theoretical approach to option exercise in the real estate market. Smit and Trigeorgis (1997) developed an integrated real options and game theoretical framework for strategic research and development (R&D) investments. The model developed illustrates the trade-off between the flexibility value and the strategic commitment value of R&D that interacts with market structure via altering the competitor's equilibrium quantity or changing the market structure altogether (e.g., from Cournot to Stackelberg or monopoly).

Since those pioneering works, research has evolved from a consideration of multi stage games, to a consideration of simultaneous or sequential games with single decisions, with different demand curves faced by competitors and different reaction functions by firms, in continuous or discrete time, with one or more than one stochastic variable. The relevant aspect is that the linkage between ROA and game theory is evolving rapidly allowing the analysis of competition in investment decisions containing operational and strategic flexibility.

Later, research studies integrating ROA and game theory, like Grenadier (2002) and Smit and Trigeorgis (2004), have focused on the effects of competitive structure and competitive dynamics on options value and hence on investment decisions. The model to be developed in the present research will do precisely that. It will depart from a certain competitive structure, assuming different competitive reactions. The impact that such different reactions produce in option value, and hence on investment decisions, will be initially analyzed.

2.6 Final notes

The general approach to the relevant literature in each field of study of the present research is thus concluded. This chapter provides a link between the different fields of research that will be integrated. The relevance of ROA becomes clear when dealing with managerial flexibility; the scenarios where interaction between investment and financing decisions do occur; the usefulness of game theoretic approaches to the analysis of competition between a limited number of firms; and finally, the integration of competition in ROA, through the appearance of real options games.

The following chapters will deepen the review performed in the present one. The next chapter focuses on investment decisions and the relevant methodologies to value them. The chapter after the next will describe the interactions between investment and financing decisions, exploiting the relevance of agency theory in such relationship. Finally, the last chapter of the literature review will focus on the effect of competition in the value of investment opportunities, merging game theory with real options theory.

3. INVESTMENT DECISIONS

In the present chapter of this dissertation we will perform a brief review of the most widely used techniques to assist management in the investment decisions to be taken. We start with a brief description of the relevance of investment to corporations, followed by the analysis of the main assumptions under which the major techniques to capital budgeting[13] rely on. This is followed by a detailed description of the most relevant traditional capital budgeting[14] procedures and then we finish with the most recent approaches to capital budgeting, which include managerial flexibility and financial structure.

3.1 Economic Relevance of Investment to Firms

An investment decision is essentially a choice about the timing of consumption. It is a decision of how much not to consume in the present in order that more can be consumed in the future. The optimal investment decision intends to maximize expected utility and be the source of value creation. For firms, value creation can be achieved essentially through investment.

Investment decisions are linked to new investment opportunities that the firm possesses. Such opportunities must be translated into investment projects that increase the value of corporations if, when undertaken, the present value of their future cash flows is higher than the capital expenditure necessary to implement it. Projects subject to analysis by management typically have associated an initial investment outlay, necessary for its implementation, and a stream of future cash flows. However, investment projects may have managerial flexibility, which translates into future options management may have on the cash flows associated with the project, and financial flexibility, which translates into different financing possibilities, that might affect the value of the investment and, therefore, the decision to be taken.

In order to perform the valuation of an investment project it is necessary to take into consideration all the parameters that influence its value. The future cash flows it

[13] The set of investment decisions rules is referred to as capital budgeting

[14] We shall refer to traditional capital budgeting as the initial techniques that were developed to assess the value of an investment project.

generates, its time of occurrence, the amount of the capital expenditure, the risk associated with the project, the future options the project might possess and its financing structure. An appropriate methodology to value an investment project has to consider all these aspects that are associated with it. The criterion used to select which investments should be undertaken needs such valuation.

3.2 Traditional Capital Budgeting

In this section we start with a brief review of the major theories that allowed the development of a methodology to value and select investment opportunities. It will be followed by a description of the most relevant capital budgeting techniques. The section ends with a presentation of the most significant weaknesses of these methodologies.

3.2.1 Theoretical Foundations

The valuation of investment opportunities was initially addressed by Fisher (1930). Fisher was the first to state the foundations of the necessary elements to perform capital budgeting. In their seminal work he introduced the concepts of NPV and Keynes's[15] equivalent, internal rate of return (IRR)[16]. The author, despite not including uncertainty in the analysis developed, stated, for the first time, the procedures to be taken by management in order to value investment opportunities.

He further demonstrated that the firm's investment decision is independent from the preferences of the owner, which delegates to management the capacity and the responsibility to select which investments should be made, and from the firm's financing decisions. These two propositions became known as Fisher's separation principle.

The first of such propositions demonstrated that the firms should only be concerned with the maximization of its objective function[17], which corresponds to the maximization of the expected utility of its current shareholders. To do this, it is not

---

[15] Keynes (1936) presents the marginal efficiency of capital, a concept that is precisely the one that results in the IRR.

[16] A description and presentation of these two concepts will be performed later.

[17] A firm's objective function results from the firm's characteristics. A firm is characterized by a production function that defines its ability to transform current resources into future consumption goods subject to an initial investment outlay.

necessary to know the utility function of their shareholders. However, in a perfect capital market (frictionless and perfectly competitive) under certainty, actions that maximize the price of the firm's shares maximize both the wealth and the utility of each current shareholder. This way, managers do not need to know the specific preferences of their shareholders. They only need to know the market discount rate and the cash flows of their investment projects to make optimal investment decisions. This framework represents the separation between the investment and operating decisions of firms from shareholders preferences.

The second of such propositions demonstrated that firms should separate their investment decisions from their financing decisions. The equilibrium rate necessary to equal demand and supply in the market for loanable funds is the one that allows total investments to be equal to total savings. What Fisher (1930) pointed out was that two firms with different financing (or demand for funds) structures but with the same investment possibilities, invest the same amount. Therefore, the financing structure does not affect the investment decisions of firms.

The independence between investment and financing decisions is also present in Williams (1938)[18]. In fact, it unveiled the same conclusions that are later present in Modigliani-Miller's first proposition, although it does not contain a formal proof of the theorem. Nevertheless, Williams (1938) concluded that the present value of a firm is the discounted value of its future net cash-flows and therefore it does not depend on the financing structure of such firm.

These two initial contributions had, however, some restrictive assumptions. The strongest of them all was related to the certainty that was present in the analysis of future cash-flows. Since investment choices involve present sacrifice for future benefit, and as the future is, by definition, uncertain, so are investments. Therefore, it was necessary to incorporate such uncertainty in the models.

State-preference theory[19], alongside with the notion of pure securities, allowed for the application of uncertainty in individual choices. Individual decision making under uncertainty is accomplished by the maximization of expected utility of end-of-period wealth. This way, firm decision making under uncertainty can be accomplished under those same principles.

[18] Williams work is more notoriously related to the deduction of the value of the firm through the discounted value of future cash-flows, namely dividends. For such deduction, the irrelevance of the financing structure in the value of the firm is present.
[19] Developed by Arrow (1964) and Debrew (1959).

Hirshleifer (1958; 1965; 1970) applied state-preference theory when he relaxed some of the assumptions taken by Fisher (1930), namely, the non-existence of a perfect capital market[20], the single period analysis[21] and the certainty associated with the future cash-flows.

It was shown that firms, when maximizing the price of current shares, are also maximizing current shareholders' expected utility when capital markets are perfectly competitive and frictionless as well as complete. The first condition (perfect competition and frictionless markets) ensures that firm actions will not be perceived to affect other firms market security prices, whereas the second (complete markets) ensures that the state-price spanned by the existing set of linearly independent market securities is unaffected by the firms' actions. Thus, firm actions affect shareholders' expected utility only by affecting their wealth through changes in the firms' current share price.

Under this setting, the acceptance of positive NPV investments increases the price of the firm's current stock and therefore the wealth and expected utility of all current shareholders in a perfect and complete capital market. However, if capital markets are not complete or not perfect, this is not necessarily true, because the firm's investment decisions may affect the price of other firms' shares or the feasible set of state-contingent payoffs.

With such findings, Hirshleifer (1958) demonstrated that, in the presence of uncertainty, the Fisher's separation principle still holds as long as capital markets are perfect and complete. Nevertheless, if capital markets are imperfect or incomplete Fisher separation principle no longer holds and the criteria used to take investment decisions no longer guarantees the maximization of all shareholders wealth.

The principles lay down by Fisher (1930) and supported by Hirshleifer (1958; 1965; 1970) in order to take optimal investment decisions were still in need of completion. It was still necessary to understand and validate[22] the independence between investment and financing decisions and it was also still necessary to determine a proper discount rate according to the risk class of the investment.

[20] This assumption results in a divergence between the lending and borrowing rate, the existence of an increasing marginal borrowing rate and capital rationing.

[21] Hirshleifer (1965) presented multi-period investment analysis, pointing out, with such analysis, the flaws and limitations of the internal rate of return for the first time and presenting the first restrictions to the use of the NPV criterion. Such multi-period analysis will not be subject of discussion here.

[22] The independence between investment and financing decisions was present in Fisher (1930) and Williams (1938), but a mathematical proof was still necessary. That is what is meant by validation of such hypothesis.

Modigliani and Miller (1958, 1963) wrote a seminal paper on cost of capital, corporate valuation and capital structure. They demonstrated that the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate appropriate to its risk class. Consequently, the method of financing is irrelevant to firm value. They also demonstrated how the cost of capital could be estimated and stated that the required return on equity capital increases with the debt to equity ratio because of a higher equity risk. Finally, it was also demonstrated that the appropriate rate to discount future cash-flows is the weighted average cost of capital. Such weighted average results from the proportions of equity and debt present in the financing structure of the firm.

Alongside with such developments concerning the cost of capital, estimation of the value of the firm and decision making under uncertainty, a quantification of uncertainty was still necessary. Markowitz (1952) established a framework where objects of choice are measurable. In fact, the mean-variance portfolio theory is statistical in nature and therefore provided such quantification of expected return and risk allowing for the establishment of decision rules concerning portfolio selection, maximizing expected utility of investors.

With such quantification, it was set the basis for an extension of market equilibrium in order to determine the market price for risk and the appropriate measure of risk for a single asset. The economic model was simultaneously developed by Sharpe (1963, 1964) and Treynor (1961) with subsequent developments by Mossin (1966), Lintner (1965, 1969) and Black (1972). The Capital Asset Pricing Model (CAPM) showed that the equilibrium rates of return on all risky assets are a function of their covariance with the market portfolio. Later, Ross (1976) determined that the return on any risky asset is seen to be a linear combination of various common factors that affect asset returns, generalizing the findings of the previous model in the model that became known as the Arbitrage Pricing Theory. These two important achievements provided a “price” for risk and allowed the establishment of an appropriate discount rate for future cash-flows. They enable us to price risky assets in equilibrium and, therefore, allowed us to estimate the cost of capital.

The cost of capital is seen to be a rate of return whose definition requires a project to improve the wealth position of the current shareholders of the firm. The original Modigliani-Miller (1958; 1963) work has been extended by using the CAPM so that a risk-adjusted cost of capital may be obtained for each project. When the expected cash-flows of the project are discounted back at the correct risk-adjusted rate, the result is the NPV of the project.

These were the basic concepts that allowed the widespread use of NPV as a tool to estimate investment value and to assist management into investment decision taking. In fact, Klammer (1972) and Schall, Sundem and Geijsbeek (1978) reported that a vast majority[23] of large firms used NPV as the criterion to analyse the implementation of large investments.

3.2.2 Capital Budgeting Techniques

Capital budgeting techniques should possess an essential property. They should unequivocally determine if a certain investment maximizes shareholders' wealth. Such property can be decomposed in the following separate criteria:

- All cash flows should be considered;
- The cash flows should be discounted at the opportunity cost of funds;
- The technique should select, from a set of mutually exclusive projects, the one that maximizes shareholders' wealth;
- Managers should be able to consider one project independently from all others.

Mutually exclusive projects are projects that cannot be jointly implemented. This means that, facing a set of mutually exclusive projects, management must select only one of them. The value-additivity principle implies that a valid criteria should evaluate projects independently, but the added value of separate projects must not change the decisions taken separately.

Next we will describe the most commonly used techniques in capital budgeting, namely the NPV and the IRR. We will also perform a comparison between these two different techniques.

3.2.2.1 Net Present Value (NPV)

NPV is the added value that a project can bring to a given firm. It represents the change in value that the firm will suffer from the implementation of an investment project. It is the difference between the future cash flows generated by the investment,

[23] Klammer (1972) reported that in 1959 only 19% of the firms used NPV, but by 1970, that number grew to 57%; Schall, Sundem and Geijsbeek (1978) reported that 86% of the large firms analysed used NPV.

discounted back to the present date at an appropriate rate considering its risk class, and the present value of the cash outlays necessary to implement it. In order to determine it, we need to define which are the cash-flows from a project, the capital invested in it and the opportunity cost of capital that is consistent with each of them.

The payoffs that a project generates are its FCF. It is nothing more than the after-tax operating earnings of a company, plus non-cash charges, subtracting investment in operating working capital, property, plant and equipment and other assets. It does not incorporate any financing-related cash flow such as interest expense or dividends. This definition of FCF highlights the fundamental elements of the concept.

FCF is the amount available to remunerate the financiers of the project. It is independent of the financing mixture that was used. Therefore, the stream of future cash flows that is considered under this methodology is the FCF that the project generates.

This methodology, therefore, consists solely on the estimation of those FCF, for the periods ahead, discounting them back to the present date at an appropriate rate. This appropriate rate is the cost of the capital invested in the project, the weighted average cost of capital (WACC).

$$NPV = \sum_{i=1}^{n} \frac{FCF_i}{(1+WACC)^i} - I_0 \tag{3.1}$$

This provides the present value of the future FCF that the project generates. The NPV of the project is obtained by deducting the amount of capital invested (I_0) to the present value of its future cash flows.

The decision rule under this methodology is one that reflects the added value that the project can bring to the firm. If the NPV is positive, the project in question will add value to the firm and should be implemented. If the NPV is negative, the project will not add value to the firm and should not be implemented. The NPV is the increase in shareholders' wealth generated by the project.

3.2.2.2 Internal Rate of Return (IRR)

The IRR on a project is the rate that equates the present value of the cash inflows to the present value of the cash outflows. It is the rate that makes the NPV of the project exactly zero. Hence, it is the rate of return on invested capital that the project is returning to the firm.

$$\sum_{i=1}^{n} \frac{FCF_i}{(1+IRR)^i} - I_0 = 0 \qquad (3.2)$$

The decision rule under the internal rate of return criterion is that firms must undertake projects that yield a return higher than the cost of capital. This means that the return generated from the investment taken is higher than the cost necessary to implement it and therefore compensates shareholders for the risk taken.

3.2.2.3 Comparison between NPV and IRR

Although similar, NPV and IRR can, in some situations, provide different answers to management concerning the projects to be implemented. It has to be referred that NPV is the only criterion that is necessarily consistent with maximization of shareholders' wealth. That can be inferred from the following aspects that make the superiority of NPV when compared to IRR.

The first is that NPV uses the appropriate discount rate, the one that reflects the market determined opportunity cost of capital, while IRR discounts future cash-flows at the IRR, therefore assuming that the time-value of money is the IRR. This assumption has become known as the reinvestment rate assumption. This implicit assumption causes IRR to violate a basic principle that capital budgeting techniques must possess, the one that refers that all cash-flows must be discounted at a rate that reflects the opportunity cost of the funds employed.

Secondly, it has to be noted that IRR also might violate the value-additivity principle, therefore preventing management to analyse projects independently.

Finally, the computation of IRR can, in some situations, provide multiple IRR for the same project whenever the sign of the cash-flows changes more than once. NPV is the criterion that performs the best, simply because it obeys all the necessary principles and it reflects the maximizations of shareholders' wealth.

3.2.2.4 Limitations of Traditional Capital Budgeting Methodologies

Traditional capital budgeting techniques are subject to some criticism. Such criticism relies in two fundamental reasons. The first is a growing dissatisfaction concerning these standard methodologies. The second is linked with the development of option valuation models, and the recognition of similarities that exist between financial options and some characteristics that are present in some investment opportunities.

The first of the above-mentioned factors is associated with the fact that current valuation techniques do not seem to capture all sources of value that are present in a given project or, capturing them, they might not value them properly .

NPV considers a project as a black box. It assumes a passive attitude from management since the decision to undertake the project is made. Management simply follows a certain operational strategy that was defined at the implementation of the project. It does not react to the future events that might affect the project.

The value of a project, with the incorporation of flexibility, will be incremented. Such increase in value that the project suffers will increase the likelihood of its implementation. Therefore, by not incorporating flexibility, NPV tends to undervalue the projects, which will lead to the rejection of projects that will indeed add value to the corporation.

A complementary procedure to NPV, decision tree analysis (DTA), developed in Magee (1964a; 1964b), recognises the different options management has in a certain investment at a later period of the project's life. It incorporates such options by building a tree that reflects the alternative actions management might adopt concerning those options. The cash flows relative to the subsequent periods are conditional upon those decisions. Therefore, in the valuation procedure, we have different cash flows relating to different options that were taken in the previous periods. Such differing and alternative cash flows are estimated until the final stage of the project's life. The determination of the different possible terminal values is the first stage of this valuation method, since they are conditional on management’s decisions. Management chooses the higher terminal values at the different branches of the decision tree. These different branches reflect the different options management possesses. The valuation methodology departs from these alternative terminal values and discounts them to the present date. The rate at which the alternative cash flows are discounted is the same despite their different risk. It is the same rate as the one that was used to value the project initially, with the use of the NPV approach. It is the weighted average cost of the capital that was invested in the project.

The value of the project is the discounted value of the FCF conditional on the options taken and weighted by the probabilities associated with those options, discounted back till the present date at the WACC, minus the investment expenditure made.

DTA overcomes the limitation associated with NPV, with the introduction of the flexibility present in the project. However, it is associated with two major difficulties. DTA defines subjective probabilities to the future events that can influence management's decisions, and it does not define an appropriate discount rate considering the proper risk class associated with each one of those future events.

One non-trivial limitation present in this methodology concerns the discount rate used to estimate the FCF present value. Applying the same rate to all of them is not the best procedure. Doing so, we are assuming that all these alternative cash flows possess the same risk. It is the risk that the initial project, without flexibility, possesses. Adjustment of this discount rate, taking into consideration their different risk, would have been the correct procedure to adopt.

Another limitation of this methodology is the probability implicit in undertaking all the options management possesses. DTA assumes an arbitrary probability of occurrence to all the different options it has incorporated in the project. It does not take into consideration the actual probability of exercise.

Under all these limitations that are associated with the traditional project valuation techniques, they only accomplish properly the task of project valuation when there is no managerial flexibility in the project.

The second factor mentioned above is linked with the development of option valuation models, and its application to value real assets. These models can capture the managerial flexibility present in a project and value it properly, overcoming some of the difficulties associated with the methodologies mentioned above. The development of ROA is directly related to this aspect of investment valuation.

3.3 Real Options Analysis (ROA)

This section starts by addressing the concepts and the main value drivers that are behind real options. It starts with the definition of options and an explanation of its main value drivers and its main valuation models[24].

Bearing in mind the similarities that exist between financial and real options, we present a framework for the problem of real options valuation and the methodologies to value these types of options. It is followed by a comparison between real options and options on financial securities with a thorough analysis of the main types of real options described in the literature. It is also performed a detailed review of the developments in real options models and an explanation of the main value drivers of real options are also performed.

Finally, we make a comparison between traditional capital budgeting and ROA. The similarities and the differences between these models are stated and explained, as well as the advantages and disadvantages of all of them.

3.3.1 Option pricing Framework

An option is the right, but not the obligation, to buy or sell a specific amount of a specific underlying asset at a specified price, termed the exercise price, and for a specified period of time, the maturity date of the option. A call option is an option to buy the underlying asset, whereas a put option is an option to sell the underlying asset.

Black and Scholes (1973) and Merton (1973) derived, for the first time, in a satisfactory manner, a closed form solution for the valuation of financial options and corporate liabilities. They derived a continuous time model that allowed the valuation of European[25] type financial options.

The model thus developed values options according to five different factors[26]. The first is, naturally, the price of the underlying asset. For a call option the higher the price of the underlying asset, the higher the value of the option contract. For a put option, the

[24] In this section we shall only present the initial valuation models, namely the Black-Scholes-Merton (1973) and the Cox-Ross-Rubinstein (1976).

[25] The term European applies to options that can only be exercised at the maturity date. American options, on the contrary, can be exercised at any date prior to the maturity date of the contract, making them harder to value.

[26] One other factor (a sixth) relates to the fact that the underlying asset returns any cash payout during the maturity of the option. We shall ignore it for the time being.

opposite is true, the higher the price of the underlying asset, the lower is the value of the option. The second aspect to influence option value is the exercise price. For a call option the higher the exercise price the lower the value of the option contract. For a put option the higher the exercise price, the higher the value of the option. The third factor is the length of time to maturity. The longer the time to maturity the higher the value of the options, because with more time to maturity there is a higher possibility that the value of the underlying asset departs further from the value of the exercise price. The others and less obvious factors are the instantaneous variance of the rate of return on the underling asset and the risk-free rate of return. The value of a call option increases with a higher instantaneous variance of the rate of return simply because it increases the possibility of higher payoffs, since the difference between the underlying asset's price and the exercise price can be higher, while the holder of an option is protected against the downside risk, because it can always not exercise his right to buy the underlying asset. The value of a put option increases with a higher volatility of the underlying assets' returns for precisely the same reasons. It is protected for the downside potential but can gain for decreases in the price of the underlying asset. The value of a call option increases as a function of the risk-free rate.

Black and Scholes (1973) have shown that it is possible to create a risk-free hedged position consisting of a long position in the underlying asset and a short position (where the investor writes a call) in the option. This insight allows them to argue that the rate of return on the underlying in the hedged position is nonstochastic. Therefore, the appropriate rate is the risk-free rate and as it increases so does the rate of return on the hedged position. The impact of the risk-free rate in the value of a put option is the opposite as on the value of a call option. As the risk-free rate increases the value of the put option decreases.

One crucial aspect of this model for option valuation is the fact that the expectations of the holders of the option are not taken into consideration regarding option value. One other aspect is that it does not also depend on the individuals' attitudes towards risk. Such attitudes are irrelevant to option value. All that is necessary is the consideration that individuals prefer more wealth to less so that arbitrage profits are eliminated. Finally, the only random variable is the underlying asset itself. The value of the option is therefore, a result of a set of directly observable variables and can be computed in a very straightforward manner.

Later, Cox, Ross and Rubinstein (1979) developed an alternative model to value financial options. They derived a discrete time model that allowed the valuation, not only of European type options, but also American type ones. They use a more intuitive approach based in the binomial distribution of the underlying. They achieved the same results as the previous model because the two alternative approaches to the option valuation problems are identical. In fact, Cox, Ross and Rubinstein (1979) explicitly referred the coherence between the continuous time and the discrete time approach, establishing the relationship that needs to prevail to assure such coherence. It is related to the translation between continuous time variable into discrete time ones, such as the annualized standard deviation of the returns into the up and down factor moves contained in the binomial option pricing formula. Thus, the binomial option pricing formula contains the Black-Scholes-Merton formula as a limiting case. An advantage of the binomial model to the continuous time one is its ability to value American put options, which is not possible to do under the Black-Scholes-Merton.

The establishment of a hedged position, that is present in both models, is essential to the option valuation problem. Such covered position is only possible by the recognition that it is possible to replicate an option with a different set of traded securities, thereby creating a so called synthetic option. The value of the option at the date of their valuation will be identical to the cost of building the replicating portfolio. Therefore, in order to apply these option valuation models, the main concern is to identify the existence of such a portfolio.

3.3.2 Definition and Characteristics of Real Options

An investment project can be seen as a portfolio of options. The analogy departs from the similarities that exist between some investment opportunities and options concerning their respective payoffs.

An option is the right, but not the obligation, to take an action at a predetermined cost, called the exercise price, for a predetermined period of time - the life of the option.

An option on a real asset – a real option - is nothing more than the possibility, for a determined price, to change the operating policies of an investment project, in accordance with the future scenarios or conditions that the environment surrounding the project suffers. Such possibilities will be exercised according to the conditions that management faces at the time those possibilities can be taken.

Accordingly, under this framework, it should be possible to value an investment project with the use of models that were developed to value financial options. When such valuation procedure is applied, it is termed ROA. In order to do so, it is necessary to correctly identify the options present in the investment decision and to obtain all the inputs needed to apply the financial options valuation models.

The analogy stated between real and financial options is, therefore also present in the factors that affect its value and that are incorporated in their valuation. The value of real options is also affected by the value of the underlying asset (the present value the project's future cash flows), the time to maturity of the option, the volatility of the underlying asset (uncertainty that affects the project's future cash flows), the strike price of the option (cost of the embedded option), the risk-free rate, and any dividends that might be distributed to the stakeholders of the firm.

Consideration of investment projects as options, regarding the flexibility that is present in those projects, makes it possible to make a partition of its value in three main groups.

Firstly, its in-the-money value, which is nothing more than the value of the project if implemented immediately. This component of value is the NPV of the project, its value without consideration of any option associated with it.

Secondly, one has to add the option to defer the investment (gathering more information on the project) related to the opportunity cost associated with immediate implementation. In an option framework, it is similar to immediate exercise of the option.

Finally, consideration of operating flexibility in management's decisions leads us to the consideration of other options that might be present in the investment opportunity that the firm in question faces. Such operational flexibility is linked to the capacity that management has to act throughout the life of the project. Through their actions, they can change the future cash flows associated with the initial investment, and, therefore, change the present value of the investment project today.

3.3.3 Valuation of Real Options

Considering the similarities that exist between an investment project and a financial option, the methodology developed to value an investment opportunity can be the one developed to value financial options.

The existence of a replicating portfolio is an essential feature of all the option valuation methods developed. Valuation of real options, due to the nature of the underlying asset, reveals some problems concerning the existence of such a replicating portfolio.

According to the body of literature that studies this approach, such a portfolio must be possible to build if, at least in theory, there is an asset, or a group of assets that is being subject of a transaction in the financial markets and is perfectly correlated with the value of the project. Such asset or group of assets should exist if financial markets are complete and efficient. This asset or group of assets is termed the "twin security", and it allows the valuation of real options using the same models that were developed to value financial options.

However, in the case that such twin security cannot be achieved, there is, according to this body of literature, a fundamental principle that is applied. The value of a non-traded project is the price that the project would have if it had been traded. This assumption is the most widely used in the methods constructed to value real options. It allows the consideration that the present value of the project can be considered as the value of the underlying asset in which the options are written.

Nevertheless, there are also other difficult parameters to obtain in order to apply the models firstly developed to value financial options. The most difficult one is the volatility of the underlying asset, being such an asset a real and non-traded security. If such a non-traded security does not have a perfectly correlated traded security, or group of securities, the methodology used to derive its volatility will have to be done through a simulation process. The uncertainties that affect the project will be incorporated, and through such simulation a value for the volatility of the investment project will be achieved.

The process that will be followed in the present dissertation to perform a project valuation using ROA, relies in these assumptions and simplifications, and can be decomposed in four different steps: calculation of the project's present value; incorporation of the uncertainty that surrounds the project; identification and inclusion of managerial flexibility; final and complete valuation of the project.

3.3.4 Types of real options

The main types of real options described in the literature relate to the major types of managerial flexibility we can find in investment projects. Next, we present a description of the most common ones.

3.3.4.1 Option to defer (delay)

The first type of real option considered, and inherent to most investment opportunities, is the option to defer[27] (or delay) an investment. It is simply the option that management possesses of delaying the investment expenditure necessary to implement an investment. It can add value to a firm if it waits to invest until the market develops sufficiently and some of the uncertainties associated with it disappear. It is particularly relevant when making an irreversible investment decision under uncertainty[28].

This type of option can be seen as a call option on the present value of expected cash inflows from the completed and operating project, with the exercise price being the investment expenditure necessary to implement it.

However, in some situations it can be a disadvantage to follow a wait-and-see strategy. It is the case when a project has a specified life[29] or when immediate investment pre-empts the entry of competitors.

Table 3.1: Deferral option inputs

Deferral Option	Call Option
Present Value of Expected Cash-flows	Stock Price
Present Value of Investment Outlays	Exercise Price
Length of Deferral Time	Time to Maturity
Time Value of Money	Risk-free Rate
Volatility of Project's Returns	Variance of Stock Returns

Source: Adapted from Smit and Trigeorgis (2004)

[27] This type of option was initially referred in Tourinho (1979), Titman (1985), McDonald and Siegel (1986), Paddock, Siegel and Smith (1988) and Ingersoll and Ross (1992).

[28] McDonald and Siegel (1986) examined the optimal timing of initiating a project, referring that deferment of an irreversible investment creates added value. Pindyck (1988) analyses the option to invest in irreversible capacity under product price uncertainty. Dixit (1989) considered the timing of a firm’s entry and exit decisions.

[29] Trigeorgis (1990, 1991) treated this effect analogous to a dividend payment. It is the case where a patent exists and therefore limits the life of the project.

3.3.4.2 Option to expand or contract

The growth or expansion option[30] is the possibility that management possesses to increase the scale of operations. It increases the value of the project if, when undertaken, the increase in the cash-flows generated by its exercise outperforms the investment expenditure necessary to implement it.

This type of option can be seen as a call option on a fraction of the value of the project (the percentage increase in the cash-flows generated by its expansion) with the exercise price being the investment expenditure necessary to expand the project.

The contraction option[31] is the opposite situation. It is the possibility that management possesses to contract (reduce) the scale of operations. It increases the value of the project because it allows, if market conditions turn out unfavourable, the firm to reduce part of its initial investment outlay.

This type of option can be seen as a put option on the part of the project that can be contracted (it can also be a percentage of the initial project value) with an exercise price equal to the part of the planned expenditures that can be avoided.

These options always tend to add value to an investment project. They increase the value of the project by the added flexibility they bring to management. They allow management to increase the cash-flows generated by the project if market conditions turn out to be better than it was initially expected, and allow management to cut the losses or adjust capacity if market conditions turn out worse than it was initially expected.

Table 3.2: Growth option inputs

Growth Option	Call Option
Fraction of Project Value	Fraction of Stock Price
Present Value of Extra Investment Outlays	Exercise Price
Length of Deferral Time	Time to Maturity
Time Value of Money	Risk-free Rate
Volatility of Project's Returns	Variance of Stock Returns

Source: Adapted from Smit and Trigeorgis (2004)

[30] This type of option was initially referred to by Myers (1977), Kester (1984; 1993), McDonald and Siegel (1985), Trigeorgis and Mason (1987), Trigeorgis (1988), Pindyck (1988) and Kemna (1988).
[31] This type of option was initially referred to by McDonald and Siegel (1985) and Trigeorgis and Mason (1987).

Table 3.3: Contraction option inputs

Contraction Option	Put Option
Fraction of Project Value	Fraction of Stock Price
Present Recovery Value	Exercise Price
Length of Deferral Time	Time to Maturity
Time Value of Money	Risk-free Rate
Volatility of Project's Returns	Variance of Stock Returns

Source: Adapted from Smit and Trigeorgis (2004)

3.3.4.3 Option to abandon or to switch use

The abandonment option[32] allows management to terminate operations for a salvage value. The option to switch use[33] allows manager to switch the use to its best alternative. This type of flexibility can be a significant source of value when market conditions have a high degree of demand uncertainty.

This abandonment option can be seen as a put option on total project value with an exercise price equal to a specified salvage value. The option to switch use can be seen as a put option on project value with an exercise price equal to the value of the project in its best alternative use.

Table 3.4: Abandonment option inputs

Abandon Option	Put Option
Present Value of Cash Inflows	Stock Price
Resale Value	Exercise Price
Length of Deferral Time	Time to Maturity
Time Value of Money	Risk-free Rate
Volatility of Project's Returns	Variance of Stock Returns

Source: Adapted from Smit and Trigeorgis (2004)

3.3.5 Differences between Real Options Analysis (ROA) and Traditional Capital Budgeting

ROA may overcome some of the limitations that are present in traditional capital budgeting, whenever managerial flexibility is present. It incorporates all the options available to management and discounts all the future cash flows associated with those options at the appropriate rate according to its risk class. The implementation of ROA is

[32] This type of option was initially referred to by Kemna (1988) and Myers and Majd (1990).
[33] This type of option was initially referred to by Margrabe (1978), Kensinger (1988), Kulatilaka (1988) and Aggarwal (1991).

derived from option pricing theory and takes into consideration two traditional capital budgeting methods, NPV and DTA. ROA starts with the first of the two methodologies referred, the estimation of the NPV of the project. Then it incorporates, the options that are present in the project, as in DTA. Finally, with the use of option pricing theory, it discounts the future cash flows at the appropriate rate considering its risk class. ROA leads, in presence of managerial flexibility, to a more accurate project valuation. ROA values managerial flexibility properly, improving the investment decisions that the management of a given firm has to undertake.

3.4 Final considerations on investment decisions

This chapter highlighted the fundamental aspects related to investments in real assets and to their valuation through relevant capital budgeting techniques. After an overview on the relevance of investments to corporations, we provided a description of the assumptions under which the valuation of investment should be performed. After that, we conducted a description of capital budgeting techniques, starting with the initial ones developed and ending with ROA. The conclusion to be drawn from such review is that, in the presence of flexibility, real options valuation techniques apply, since they are the only ones capable of capturing such operational flexibility, hence its relevance to the present research.

The next chapter performs a review of the conditions under which the financing of an investment can have influence in the valuation of such investment, namely the case where conflicts between the different stakeholders of the firm arise. It also performs a review of the relevant literature that analyses conflicts of interests between equityholders and debtholders in the presence of managerial flexibility.

4. AGENCY CONFLICTS BETWEEN EQUITYHOLDERS AND DEBTHOLDERS UNDER REAL OPTIONS ANALYSIS

In this chapter we intend to develop the fundamental research that integrates operational flexibility in a setting where agency costs are relevant in order to determine the decision to invest. We shall divide it in two different approaches. An initial, and necessarily brief, focus that presents the main empirical literature produced. This literature essentially examines the ‘value hypothesis’ and the ‘agency hypothesis’. Empirical evidence overwhelmingly supports the agency hypothesis, therefore providing a natural justification for the development of the theoretical models that followed. A subsequent, and larger, focus in the theoretical literature produced. This literature mainly developed multi-period dynamic models for the value of the assets incorporating different types of real options. The options to invest, risk-shift, shut-down, restart and switch use are the most common ones. These models incorporate a wide use of protective debt covenants and different debt maturities in their financing structure. Previous theoretical work and empirical evidence has shown it might mitigate agency conflicts between equity and debt.

The next section focuses on the empirical findings that relate operational with financial flexibility, namely with the debt capacity of the firm. Afterwards, we present theoretical models that were developed do analyse interactions between investment and financing decisions. Such presentation is structured according to the type of operational flexibility incorporated, distinguishing consideration and non consideration of agency conflicts. Both sections consider the option to invest in production capacity. However, a distinction is made between continuous and endogenously determined investment opportunities and discrete and exogenously determined ones.

4.1 Empirical literature

Bradley, Jarrel and Kim (1984) demonstrated that exists a low debt capacity in firms with high real flexibility. Later, Long and Malitz (1985), Smith and Watts (1992) and

Barclay, Smith and Watts (1995) also presented evidence of a negative relation between real flexibility and debt capacity. Rajan and Zingales (1995) reinforced such findings, demonstrating such relationship to be quite significant.

More recently, Barclay, Morellec and Smith (2006) extended the empirical analysis by testing the hypothesis that the ratio of debt to assets-in-place should fall with an increase in operation flexibility. The empirical findings demonstrated that such ratio fells with the existence of growth options.

The results achieved in Mackay (2003) suggest a significant (mostly negative) relation between real flexibility and the debt capacity of firms, thereby confirming the agency hypothesis. For firms that enjoy a higher level of production flexibility, the debt capacity and maturity of debt tend to be shorter supporting the agency hypothesis. However, in the specific case of investment flexibility, the results support the value hypothesis. According to Mackay (2003), these results imply that restrictive covenants, imposed by debtholders on equityholders, are able to control the agency conflicts in such cases, thereby enhancing the collateral value of the assets, and increasing debt capacity. The results for the small firms sample help to confirm the arguments that, due to their concentrated equity ownership and greater degree of real flexibility, small firms face severe agency conflicts between equityholders and debtholders. However, the results also show that small firms rely more on bank loans, suggesting that monitored lending helps to contain agency conflicts, increasing the overall debt capacity of the firm.

The empirical testing of the agency conflicts between equityholders and debtholders in the presence of operational flexibility confirms the existence of such conflicts. They are significant, they affect managerial decisions and firm value and therefore, they should be taken into account. As the evidence seems to support, real flexibility exacerbates agency conflicts and reduces the debt capacity of firms. To ignore the impact of agency conflicts on theoretical models, leads to biased conclusions regarding investment decisions.

4.2 Theoretical literature

The theoretical literature analysed intends to analyse the impact of agency conflicts between equityholders and debtholders in the exercise decisions of different real options firms face. In the models developed, the exercise of the real options is generally treated as irreversible, and the characteristics of the real options are exogenously determined. The exception is the option to invest. The literature incorporating the option to invest has treated investment in production capital from two different perspectives. One branch of the literature assumes that the possible rate of expansion of production capital is a continuous variable and is an endogenous decision of the firm. It is termed as full flexibility. The other branch of the literature assumes that the firm can only increase its production capital by discrete amounts, whose sizes are exogenously determined. It is termed as discrete flexibility. Other differences in the treatment of the option to invest concern the possibility to disinvest and the existence of assets-in-place, in which case the option to invest can be seen as a growth option. In the alternative case, the firm only holds an investment opportunity (the investment option itself).

4.2.1. Option to invest under full flexibility

This type of flexibility was the first one to be considered in the literature. Furthermore, initially interactions between investment decisions and financing decisions disregarded agency conflicts.

Brennan and Schwartz (1984) analyses the interactions between investment and financing decisions in a setting where the return on assets is uncertain, following geometric Brownian motion (GBM), and considering different protective covenants, in a dynamic debt model[34]. The protective covenants considered, prevented the firm from selling assets and set a minimum interest coverage ratio. They further considered that the investment decisions are set in order to maximize the value of equity. They concluded that, without tax advantage of debt financing, the impact of debt financing in terms of firm value was negligible. High levels of debt are found to reduce the value of the firm, because the higher proportion of the operational cash-flows that is required to service debt tends to inhibit further investment. The protective covenants considered, are in generally found to increase the value of the firm, but there are scenarios in which their existence reduces the value of the firm. This fact leads the authors to conclude that an optimal financing policy must take into consideration three different aspects. The

[34] There are essentially two different alternatives being modelled for debt maturity. One is static and it assumes that the debt principal and the maturity do not change as debt is rolled over, and new debt is issued at its market value. The alternative is to allow changes in the debt principal while keeping the same maturity.

first one is the choice of protective covenants, the second one is the choice of the initial capital structure and third one is the adjustment to the debt level considering the initial capital structure.

Dotan and Ravid (1985) use a one-period model where the price of the commodity sold is uncertain and follows GBM. The authors disregarded agency conflicts, and set the investment decisions aimed at maximizing the value of the firm. The existence of taxes and debt financing is found to influence the optimal investment decision of the firm, and an increase in taxes reduces investment. Increases in the tax rate increase the optimal debt level of the firm but reduces its investment level. Although the tax shields are increased, the accounting profits are reduced. Increases in debt financing are found to reduce the investment of the firm due to higher expectations of bankruptcy costs and loss of interest tax shields (an asymmetrical tax system is considered).

The difference between sequential and simultaneous decision-making is also analysed for both the investment and the financing decisions. Dotan and Ravid (1985), show that the simultaneous optimization of the investment and the corporate financing decisions yields higher firm and debt capacity values than when these decisions are sequential. This fact leads Dotan and Ravid (1985) to conclude that firm value maximization requires the simultaneous determination of the level of investment in production capital and the optimal level of debt.

The models that follow assume explicitly the existence of agency conflicts between equityholders and debtholders. Mao (2003) analyses the optimal investment level in production capital, assuming that the volatility of cash-flows is a linear function of the scale of the investment. It focuses on the marginal volatility of investment (MVI), which can be positive or negative, instead of the traditional focus on the volatility of the operational cash-flows of the assets-in-place or of the assets underlying the investment opportunity.

The incentives to overinvest or to underinvest are found to be a function of the MVI, but also of the cost of investment. Equityholders have an incentive to delay investment, in order not to share a significant portion of the benefits with debtholders. However, when investment increases the overall risk of the firm, equityholders can benefit at the expense of debtholders. In this situation, the risk-shifting incentives mitigate the debt overhang problem and the total agency cost of debt does not monotonically increases with leverage. The agency costs are measured as the difference in value between an unlevered firm and a firm with debt financing. Mao (2003) analyses four different scenarios, low and high growth firms[35] and positive and negative MVI. For high growth firms, there is a positive relation between the optimal leverage and MVI. For low growth firms, there is a negative relation between the optimal leverage and MVI.

Titman and Tsyplakov (2002) consider three different investment policies. The first policy considers the existence of an unlevered firm. The second policy considers maximization of firm value. The third policy considers maximization of equity value. The authors also distinguish between financing effects and agency effects. They further consider a static debt model and a dynamic debt model. Additionally, they analyse the possibility of distress, triggered by the breach of an exogenously determined interest coverage ratio. When firm value is below this exogenously defined distress trigger[36], the firm incurs a loss in value defined as costs of distress. Distress reflects a deterioration of the ability of the firm to meet its financial obligations, without having yet defaulted on any of them.

Titman and Tsyplakov (2002) find that the value of the firm with the dynamic debt model is 20% higher than with the static debt model. The static debt model generates significantly higher tax savings and financial distress costs, but lower agency costs than the dynamic debt model. The static debt model tends to overestimate the present value of taxes and deadweight costs (issuance, distress and bankruptcy costs), and underestimate the present value of agency costs, because of the impossibility to adjust the debt levels. In the dynamic debt model, the firm following a first-best policy retires debt as the price of the commodity falls, but the firm following a second best policy does not retire debt because of the protection awarded by limited liability.

The possibility to retire debt in the dynamic debt model and the unwillingness of equityholders following a second-best policy to do so, explain the higher agency costs of debt of the dynamic debt model relative to the static debt model.

The initial leverage for the firm with the dynamic debt model, for both investment policies, is lower than with the static model. However, this may be influenced by the fact that the choice of base-case parameters makes it easier to increase leverage than to reduce it[37]. The possibility of increasing debt later in time is analogous to the case of a

[35] The difference between low and high growth firms is reflected by the unit cost of investment. A low unit cost of investment is associated with high growth firms, a high unit cost of investment is associated with low growth firms.

[36] Although this ratio is defined as a function of the interest coverage, since the operational cash-flows are assumed to be a proportion of the firm value this interest coverage ratio can be expressed as a firm value ratio.

[37] The issuance costs of equity are higher than the issuance costs of debt.

firm with an option to invest, which may set its initial level of production capital lower, than in the case of a firm without the possibility of increasing its level of production capital later.

In both, static and dynamic debt models, the firm following a second-best investment policy presents lower debt capacity than that following a first-best investment policy. Furthermore, it also underinvests and distributes more cash to keep the interest coverage ratio high[38].

Titman, Tompaidis and Tsyplakov (2004) follow a similar procedure, focusing on the real estate lease market (office buildings) and analysing credit rationing and agency problems. Instead of investing in production capital, the firm analysed invests in the maintenance stock for its properties, which represents a proxy for quality. The higher the quality, the higher the lease rates the firm collects. Three different types of firms are considered: A restricted borrower, which follows the investment strategy followed by an unlevered firm. An unrestricted borrower with deep pockets, which follows a policy of equity maximization, and it defaults when equityholders are unwilling to provide extra funds (when the market value of equity is zero). An unrestricted borrower with empty pockets, which has limitations in access to external capital and can only issue equity when the value of the unlevered firm is higher than the debt principal.

The role of two different protective covenants is also analysed. One covenant requires the maintenance of a minimum quality level, and the other limits the distribution of dividends until repayment of the mortgage used to finance the investment.

For the case where protective covenants are absent, the unrestricted borrower has an incentive to underinvest relative to the restricted borrower, essentially because the former has a larger margin for adjusting the investment levels in the future. Higher investment flexibility is found to increase the agency costs of debt. However, the loss in firm value due to agency conflicts is still relatively small, and in most cases represents less than 1% of the unlevered firm value. Changes in short term rates do not

[38] In this model, there is a clear interaction between the distribution of cash-flow and the underinvestment incentives. They are two sides of the same coin, since the authors do not model the possibility to retain cash. The cash-flows generated are either invested or distributed and the coverage ratio is measured deducting investments. The question is which effect is more dominant. Is it the need to keep the coverage ratio high that induces a higher distribution of cash-flows, thereby reducing the ability of the firm to invest or are the incentives to underinvest, given the higher credit spreads, which allow the firm to distribute more cash, delaying distress but accelerating default given the lower value of the firm.

significantly influence agency costs, however, as long term interest rate increases agency costs are also expected to increase.

When protective covenants are considered, the covenant forcing the borrower to maintain the initial quality level of the property was found to substantially reduce credit spreads. However, both this covenant and the covenant limiting the distribution of cash induced a substantial overinvestment relative to the restricted borrower. In some situations, these covenants actually increase rather than decrease the agency costs of debt, which no longer result from an underinvestment problem but from a severe overinvestment problem.

Most authors have disregarded the possibility of disinvestment. Protective covenants prohibiting the sale of assets are easily enforceable, and for debtholders, the collateral value of the assets-in-place is usually assumed to be very important. One exception is Moyen (2007). It considers that the value of the firm is subject to random income shocks and five different scenarios are analysed. Three different investment policies are considered. The first one regards an unlevered firm. The second one regards a levered firm that maximizes the value of the firm. The third one regards a levered firm that maximizes the value of the equity of the firm. For each levered policy, two different types of debt maturity and debt coupon rates are considered. A short-term variable rate debt issue and a long-term fixed rate debt issue. The agency costs of debt are measured as the difference between the value of the firm following the first and second-best investment policies. Remarkably, the impact of the different types of maturity and interest rates on agency costs is found to be very small[39].

Morellec and Smith (2007) assume that the risk of the investment opportunities differs from the risk of assets-in-place. Additionally, a limit of positive NPV investment possibilities is defined, above which the returns of additional investment for the firm are negative. Morellec and Smith (2007) analyse the impact of risk management on the incentives of managers to overinvest, since managers are able to benefit from investments even below the zero NPV threshold. Although there is no explicit modelling of conflicts between equityholders and debtholders[40], the results show an

[39] As a percentage of the second-best firm the agency costs amount to 4.5% in the long term debt case and 5.2% in the short term variable debt case.

[40] There is only one investment policy and equityholders can only contract financing and hedging decisions. However, equityholders can chose from one of three different financing policies, all-equity, partial debt financing and partial debt financing with an efficient hedge.

increase in firm value when an efficient hedge is in place, due to the efficient control it enforces on the incentives of managers to overinvest and equityholders to underinvest.

4.2.2. Options to invest with discrete flexibility

This line of research also did not, initially, incorporate agency conflicts. Mauer and Triantis (1994) model an investment option in a firm with operational flexibility, and with the option to shutdown and resume operations. The price of the commodity this firm produces is uncertain, following GBM, and all the operational and investment policies aim at maximizing the value of the firm. It considers a dynamic multi-period model, where the value of the firm derives entirely from its future growth possibilities. The impact of debt financing in firm value is divided between purely debt financing effects (such as bankruptcy costs, tax shields and recapitalization costs) and operational effects. The operational effects are determined by the difference in value between a levered firm following an optimal levered operating policy, and a levered firm following the unlevered firm optimal operating policy.

Under these assumptions, Mauer and Triantis (1994) show that the corporate financing policy influences investment and operating decisions, and consequently the value of the operating firm. However, despite the operating value of the firm is sensitive to changes in the financing structure, the overall impact of the financial policy upon firm value is found to be very low and almost insignificant. Even though the levered firm generates additional income via interest tax shields, this positive effect is significantly reduced by recapitalization and adjustment costs, and more importantly, by overinvestment costs. The relatively low net impact of debt financing is explained by the trade-off between the tax shields positive effect, and the fact that this encourages earlier exercise of the real options causing overinvestment costs.

Mauer and Triantis (1994) analyse the impact of changes in several of the parameters of the model, such as operating adjustment costs (the cost to shut-down and resume operations) and recapitalization costs (cost of issuing debt). They show that firm value increases as operating adjustment and recapitalization costs decrease, which, in turn, increases the debt capacity of the firm, and the value of the interest tax shields. This occurs because, as these costs decrease, firm value increases and firm volatility decreases. So, to some extent, operational and financial flexibility may be seen as substitutes, because they have similar impact in firm value. Increased financial or operational flexibility also has a similar impact on the investment decisions. In both cases, additional flexibility (represented by lower operating adjustments and recapitalization costs) reduces the value of waiting to invest, by increasing the value of holding the underlying asset, and accelerates the investment decision.

Under all the simulations performed, they found that the impact of debt financing in terms of firm value is found to be relatively insignificant. This conclusion gives relevance to Modigliani-Miller theorem, since Mauer and Triantis (1994) have significantly relaxed their assumptions. One aspect that Mauer and Triantis (1994) leaves out concerns possible agency conflicts between equityholders and debtholders, since these authors assumed that protective covenants, determining that the policies followed would aim at maximizing the value of the firm, could be written and enforced.

Mauer and Ott (2000) follow a similar procedure to the one present in Mauer and Triantis (1994), but include the impact of agency conflicts. However, it analyses, in a classical ROA setting, agency conflicts concerning the exercise of a perpetual American option to invest, while Mauer and Triantis (1994) considered an option with a finite life. It is defined a first-best investment policy, which maximizes the value of the firm, and a second-best investment policy, which maximizes the value of the equity.

Mauer and Ott (2000) analyse a levered firm with assets in place holding an option to expand its existing production capacity (by an exogenously given coefficient). The investment in the expansion option is financed by an additional equity issue, which causes a conflict between equityholders and existing debtholders. Therefore, equityholders have an incentive to delay investment in a classical Myers' (1977) framework. The agency costs are analysed at the optimal debt level and decomposed between their operational and financial components. The estimated agency costs of debt for the base case parameters represent 2.2% of firm value. These agency costs are essentially financial costs - loss of interest tax shields and increased bankruptcy costs - because the operational component in the second-best policy is actually closer to the unlevered case[41].

Later, Mauer and Sarkar (2005) follow the procedure in Mauer and Ott (2000), but change the nature of the firm considered and the financing of the growth option. Mauer and Sarkar (2005) consider a firm that does not have any assets-in-place, and only holds

[41] The difference between the investment triggers in the unlevered and levered second-best investment policies is smaller than the difference between the investment triggers in the unlevered and levered first best investment policies.

an option to invest. The investment is partially financed by a commitment loan, which shifts the focus of the agency conflicts between equityholders and existing debtholders to equityholders and new debtholders. The conflicts analysed in Mauer and Sarkar (2005), relate to both Jensen and Meckling (1976) and Myers (1977), but are somewhat different from them. The conflicts differ from Jensen and Meckling (1976), because equityholders do not have the possibility of changing to higher risk projects. They are able to invest at price levels where the risk of default is higher, and thereby transfer the increase in the risk of default to debtholders while retaining the upside potential. In this sense, the incentives in Mauer and Sarkar (2005) represent the opposite problem to the one analysed in Myers (1977). Instead of delaying investment, in order not to suffer a transfer of wealth from equityholders to debtholders, equityholders decide to invest earlier, because they are able to capture some wealth from debtholders. Since the terms of a commitment loan are agreed upon at time zero, and do not change afterwards, equityholders are free and willing to invest at lower values of the underlying, because they can partially transfer part of the risk of default to debtholders.

The estimated agency costs of debt for the base case parameters represent 9.4% of firm value. Approximately half of these agency costs are operational costs, and the other half represents a loss in the net benefits of debt financing.

In both models, the agency conflicts significantly reduce the debt capacity of the firm. As the debt ratio increases so do the agency costs, and the debt capacity in the second-best investment policy is always lower than in the first-best investment policy.

Jou and Lee (2004) set up a similar model to Mauer and Sarkar (2005) but solve it by using dynamic programming (risk adjusted valuation method) instead of contingent claim analysis. This is the only paper that uses such a methodology when analysing interactions between investment and financing decisions in a ROA framework, but te analysis is performed at the moment the investment option is exercised, and not at time zero, as it is common. They also define two different investment policies. In a first-best policy the investment and financing decisions occur simultaneously, while in a second-best policy they occur sequentially. Firstly, the firm decides the optimal amount of debt financing and then it decides when it is optimal to invest. Jou and Lee (2004) observe evidence of Myers (1977) debt overhang problem. However, the debt overhang does not reduce the value of the firm, but it increases it. In the second-best policy, the debt capacity of the firm is higher.

Parrino and Weisbach (1999) considered the simple NPV rule for a firm with assets-in-place and where the investment is financed with debt. The investment decision is similar to a European investment option expiring at time zero. Therefore, the firm weights the decision to invest now with the decision of not investing at all. Parrino and Weisbach (1999) measures the distortions created by conflicts between equityholders and debtholders as the difference between the minimum rate of return required for the project to be accepted by equityholders, and the minimum rate of return required for the project to be accepted by equityholders and debtholders alike. It shows how levered equity maximizing firms have the incentive to turn down positive NPV projects with stable cash-flows, and to accept negative NPV projects with risky cash-flows. These agency problems are found to increase with leverage. When the correlation between the cash-flows of the investment opportunity and the cash-flows of the assets-in-place is high, overinvestment tends to occur, and when the correlation is low, underinvestment tends to occur.

Childs, Mauer and Ott (2005) developed a model with two stochastic processes, where there is uncertainty in the value of the assets-in-place and in the value of the assets underlying the investment opportunity. They allowed for the simultaneous possibility to expand or to risk-shift, by considering different characteristics of the assets underlying the investment possibility. They also considered the possibility to partially finance the investment with the sale of a proportion of the assets-in-place, creating a wide set of scenarios. These scenarios range from simple growth, when the assets underlying the investment possibility are equal to the assets-in-place, to the extreme case of asset substitution, when the assets underlying the investment possibility differ from the assets in- place and investment is entirely financed with the sale of all the assets-in-place.

The authors consider two different investment policies, a first-best policy, which maximizes the value of the firm and a second-best policy, which maximizes the value of the equity. The agency costs represent the difference between the values of the firm under these two alternative policies. Childs, Mauer and Ott (2005), find that, in the pure expansion case (no portion of the assets-in-place is exchanged when the option is exercised), there is an incentive to underinvest, while in the pure substitution case, there is an incentive to overinvest when the assets underlying the investment possibility are riskier than the assets-in-place. Similarly to all previous work analysing agency conflicts, with the exception of Jou and Lee (2004), Childs, Mauer and Ott (2005) predict that agency conflicts between equityholders and debtholders reduce the debt capacity of the firm. Additional real flexibility is found to exacerbate the opportunistic behaviour of agents, widening credit spreads and reducing the debt capacity of firms.

4.3 Final aspects concerning agency conflicts

The review performed in the present chapter highlighted the fundamental aspects of the linkage established between investment and financing decisions under ROA. The analysis of empirical research in the area allowed the validation of such relationship analysis. The theoretical models reviewed the set of possibilities for the present research. In fact, one of the models analysed will set the basis for the present research.

In the next chapter, our attention will now turn to the analysis of competition in investment decisions. Therefore, in the next chapter a review of such analysis will be performed.

5. COMPETITION UNDER REAL OPTIONS ANALYSIS

Real options games represent the integration of a game theoretic approach to competition under ROA. It departs from the observation that operational flexibility present in investment opportunities might be shared between firms and therefore, subject to competition. It is a recent field of research, but it is an emerging one due to its relevance in the analysis of competitive behaviour and capital budgeting decisions.

In the present chapter, we perform a review of the most relevant developments under this field of literature. An initial contextualization of game theory and its most significant advances shall be done. Afterwards, a description of the most relevant concepts in the analysis of a game will be set, and finally, a review of the most relevant developments in real options games concludes the chapter.

5.1 Game theory

Game theory is the approach by which interactions between interdependent players can be studied and understood. It is, therefore, one of the most useful tools whenever any firm’s actions are dependent on what its competitors will do. The main principle underlying game theory is that those involved in strategic decisions are affected not only by their own choices but also by the decisions of others.

In the present section, we will present the initial theoretical insights that allowed the appearance of game theory as well as the most recent developments that reinforced its relevance to social sciences in general and to economic theory in particular. Afterwards, there is a brief description of the main aspects that are necessary in order to characterize a game. We conclude this section with a description of the most relevant games followed by examples of the most relevant ones.

5.1.1 Theoretical foundations of game theory

The first studies of games in economic literature go back to Cournot (1838), Bertrand (1883) and Edgeworth (1925). These studies focused on the conditions behind the definition of the equilibrium conditions under an oligopoly market structure. In fact, they were directed to the determination of the price and quantities produced in an oligopoly market. However, these studies did not cause a great impact in economic literature at the time and, as a result, the games described there were not diffused in a large scale throughout economic literature.

Therefore, in reality, the starting point of the mathematical theory of games is generally attributed to the work published by Von Neumann and Morgenstern (1944)[42]. They introduced the extensive-form and normal-form representation of a game, defined the minmax solution and showed that such solution exists in all two players zero-sum games where the interests of the players were strictly opposed. Von Neumann and Morgenstern (1944) interpret the rational choices and social events through models of strategic games. Given a set of options, agents take the strategies they consider most advantageous as a function of probability calculus and utility maximization. The work introduced the notion that conflicts of interests could be analyzed mathematically and introduced the methodology to do so.

It was followed by Nash's (1950a and 1951) equilibrium perspective under non-cooperative competition[43] that extended the game-theoretic analysis to non-zero-sum games. Nash equilibrium corresponds to a set of mutually consistent strategies. Under such strategies no agent would be better off by changing their action. The action was defined by reaction to their competitor's actions. The competitors won't also be better off changing their actions, defined according to expectations on the competitor's reactions. A stable solution is thus achieved for the overall game, since no one possesses an incentive to change its action. This solution is a generalization of the equilibrium presented in Cournot and Bertrand and the starting point for most economic analysis under this perspective. Shortly after, Nash (1950b and 1953), Shapley (1953) e Gillies (1953) developed the equilibrium for cooperative game theory.

Some other developments in this initial period are also worth being mentioned. Schelling (1956) analysed agents' behaviour under bilateral negotiation and Aumann (1959) studied agents' behaviour under an infinite repetition of the same game. Later, Selten (1965) applied Nash equilibrium to dynamic, sequential games, and Harsanyi (1967) applied it to incomplete information games enabling its application to a wider scope of market structures. Later still, Selten (1975) and Kreps and Wilson (1982a,

---

[42] Their work, entitled "Theory of Games and Economic Behaviour" was the consequence of the first studies by Von Neumann concerning this subject in the mid twenties.

[43] For this type of games, it is worth mentioning the appearance of the "prisoner's dilemma" attributed to Tucker in an unpublished paper in 1950.

1982b) further analysed the impact of incomplete information in repeated games. The trembling hand perfect equilibrium and the sequential equilibrium concepts are the notions developed that enlarged the equilibrium perspectives. Other developments refined the initial Nash equilibrium concept under different perspectives, restricting even further the scope of analysis. Myerson (1978) presented the concept of proper equilibrium, developing the trembling hand equilibrium. Aumann (1974) presented the notion of correlated equilibrium as a more general perspective when compared to the Nash equilibrium concept. Maskin and Tirole (1988) presented the Markov perfect equilibrium as a set of strategies that form a subgame perfect equilibrium but under which each player move is determined by the other players last move, but not by other moves prior to the last[44].

The application of game theory to economic analysis[45] allowed a better understanding of competitive interaction under market structures where imperfect competition is present. The developments referred above made possible the establishment of sub optimal equilibrium under such market conditions. The scope of microeconomic theory is thus redefined under this new approach.

The application of game theoretical approaches to imperfectly competitive market structures is performed through consideration of some predefined strategy from the firm. Traditional perspectives, such as Cournot, Bertrand or Stackelberg gained a new insight under this approach. They set forward the strategy the firm adopts towards their competitors actions reaching Nash equilibrium. Such equilibrium is reached in a price quantity relationship. Therefore, game theory is the study of strategic decision making.

5.1.2 Game characterization

Having performed a brief review of some of the most relevant developments in game theory, it is now time to define the main aspects that are relevant to define the setting under which a game can be played. In this section, we describe the most relevant characteristics present in games.

---

[44] Hence the term Markov, due to the inexistence of “memory” in the players moves.

[45] Other initial relevant fields of application are political science, philosophy, and insurance, for example. Nowadays, its use has been spreading in many different social sciences, but also in other relevant fields of research.

5.1.2.1 Cooperation between players

The existence or lack of cooperation between players in a game completely changes the way a game is played. Therefore, it completely changes the conditions under which equilibrium is achieved.

A cooperative game is a game where players pursue their self-interest but have incentives to cooperate with other players. Naturally, that cooperation is beneficial to them. Players can communicate and negotiate, being able to celebrate binding agreements, pool their individual agreements and redistribute the total in a specified way. Under this type of games, players can coordinate their strategies and share the payoff that results from such coordination. A coalition is a subset of the set of players which is formed in order to coordinate strategies and to agree on how the total payoff is to be divided between those players. A coalition guarantees that each player gains at least as much as if they would gain if playing individually. The problems arise whenever stability is not present in the coalitions formed. The equilibrium concepts developed for this type of games try to define the conditions under which such stability can be found.

In a non-cooperative game[46] it is assumed that players cannot make a binding agreement. Communication can be made prior to the beginning of the game, but is not possible or it is imperfect after the game starts. Therefore, each non-cooperative outcome must be sustained by Nash equilibrium strategies. The principles under which such type of games reach the equilibrium are the most relevant developments depicted in the previous section of the present chapter. All the subsequent equilibrium perspectives that followed Nash equilibrium are essentially focused in non-cooperative games, since such type of games are the most relevant ones to economic theory[47].

5.1.2.2 Information Set

The information set is the information available at a given point in the game. It contains all the possible moves that could have taken place in the game, up to a particular time, according to what a particular player has observed. Therefore, it is the knowledge that a player has when taking a particular decision.

[46] Non-cooperative games are also called competitive games.
[47] At least, for the purpose of the present dissertation.

The definition of the information the players possess at any given point in time is a crucial element in the specification of a game. Such information influences decisively the strategies the players might choose.

A perfect information game is a game where the players, when taking an action, know everything that has happened in the game up to that point. Therefore, these games are the simplest ones. Under perfect information the players know all previous decisions of all the players in each decision node previous to the one they are at. A game of imperfect information is, by contrast, a game where the players do not know all the previous decisions the other players have taken.

A complete information game means that the complete structure of the game, including all the actions of the players and the possible outcomes, is common knowledge[48]. In real-life, most of the time, it may be unclear to each firm where its rival is at each point in time and so the assumption of complete information may be not realistic[49].

5.1.2.3 Order of play

The order of play concerns, essentially, the timing of the player's moves, but not necessarily so. Although by timing, it must be noted that we refer not to the temporal order of play but whether and when players know about other players' actions, when choosing their own actions. Therefore, the nature of the game, in what concerns the order of play, is intimately connected to the information set available. Sequential-move games are games in which the players choose their strategies one after the other. This type of games can relate to perfect information games. Simultaneous-move games are games in which the players choose their strategies at the same time. This type of games can relate to imperfect information games. However, a distinction must be made concerning the link between order of play and information set. The characterization of simultaneous-move games presented implies that all such games are games of imperfect information, but in fact, games where some moves are simultaneous and other are

[48] The distinction between incomplete and imperfect information is somewhat semantic. For instance, in research and development investment games, firms may have incomplete information about the quality or success of each other's research effort and imperfect information about how much their rivals have invested in research and development.

[49] It is quite common that a firm, before an investment decision, is uncertain about the strategic implications of its action, such as whether it will make its rival back down or reciprocate, whether its rival will take it as a serious threat or not.

sequential are games of imperfect information (despite the existence of sequential moves). This is so because at least in one move there was a lack of information for a particular player. Games of perfect information are games where no moves are simultaneous (and where no player ever forgets what has happened before).

5.1.2.4 Output

The output of the game represents the payoffs that accrue to the players. It is the increase in utility that players derive from playing the game. Such utility can be achieved totally at the expenses of the other participants or in spite of the other participants in the game.

Constant sum games are games where the total sum of the benefits from all the participants in the game is constant. What one player wins is equal to what the other player looses. The most relevant of such type of games is the zero-sum game. In this type of games, the total added value of the player’s benefits is equal to zero. If one player wins the other player looses. On the contrary, in variable sum games, the added value of the player’s benefits is not constant.

5.1.2.5 Representation

A game can be represented in a normal-form or in an extensive-form, although for some games, one of the two forms is more convenient for the analysis. In the normal-form representation, each player, simultaneously, chooses a strategy, and the combination of the strategies chosen by the players determines a payoff for each player. The extensive form representation of a game specifies the players in the game, when each player has the move, what each player can do at each of its opportunities to move, what each player knows at each of its opportunities to move, and the payoff received by each player for each combination of moves that could be chosen by players.

5.1.3 Game parameters

In this section we present the main parameters that allow the complete definition of a game.

The players are the individuals that take the actions in a game. The goal of each player, under the assumption of rationality[50], is the maximization of its utility. It takes the actions that are best suited to achieve such goal. Nature is not a player, but it takes random actions that have to be considered by all the players in the game. The games can be played by, at least, two players. This type of games is the most common one.

Actions, or moves, correspond to the choices a player can take. It is the best response to the other player's actions.

Strategies are the definition of the rules that tell each player which action to choose in a particular time of the game, given the information set that it possesses.

The payoff corresponds to the utility obtained by a player after all the players and nature has played their strategies and the game ended. It is also the expected utility for a particular player as a function of the strategies chosen by all the players in the game.

The result of the game represents the set of elements that is relevant after the game has been played. It allows the establishment of conclusions related to a particular game concerning the actions taken, the payoffs achieved and all other relevant variables after the game ends.

Therefore, the description of the game must include, at least, the players, the strategies and the payoffs. Apart from that, the players, the actions and the results of the game constitute the rules of the game. The objective is to use the rules of the game in order to determine the equilibrium. According to Luce and Raiffa (1957), the elements of a game are:

1. The game: A conflict of interests situation among a finite number of participants
2. Each player chooses a strategy among a set of strategies
3. The choices are made simultaneously, without observation of the other player's choices and without response to it[51].
4. The combination of the strategies chosen produces a result that is evaluated by each player according to its own preferences.

In game theory, the players (firms) are rational and possess a set of options available to them, the strategies. The expected result for a particular player, his payoff, is dependent on his moves and also on the moves of the other players in the game. The

[50] It is also assumed that such rationality is known to all the players in the game.
[51] This is true only for simultaneous games.

interaction between the players, which execute their own strategies, is the game. If the end result of all the player’s moves is stable, it is called the solution for the game.

5.1.4 Equilibrium of the game

The equilibrium of the game is a set of strategies that results from the adoption of the best individual strategy by each player. A set of strategies is the equilibrium of the game if each player maximizes its own payoff. The most relevant equilibrium concept for non-cooperative games is the Nash equilibrium.

Nash equilibrium occurs whenever each player’s strategies are a best response to the other players’ strategies. It is the most commonly used solution concept in game theory. In fact, if there is a set of strategies with the property that no player can benefit by changing its strategy while its opponent keeps its strategies unchanged, then that set of strategies and the corresponding payoffs constitute a Nash Equilibrium. This notion captures a steady state of the play of a strategic game in which each player holds the correct expectation about its rival's behaviour and acts rationally. The alternative, yet complimentary, notion of mixed strategy Nash equilibrium is designated to model a steady state game in which players' choices are not deterministic but regulated by probabilistic rules.

At this moment, it is also relevant to introduce the concept of subgame. A subgame is part of a game. According to Friedman (1991), it occurs from a certain stage of the initial game until its end and possesses all the qualitative characteristics of a game. A subgame always starts at a single decision node and contains all the subsequent nodes. Furthermore, if it contains part of an information set, it will contain all the nodes of such information set. Therefore, a set of strategies is a subgame perfect Nash equilibrium if the strategies of all players constitute Nash equilibrium for the entire game as well as for all the subgames of such game.

The subgame perfect Nash equilibrium is extremely relevant in perfect information sequential games. In fact, when players make their choices, supported in all previous choices that happened in the game, it is expected that those choices are the best responses for the subgame that is to be played. So, when it is necessary for a Nash equilibrium to be also an equilibrium for all the subgames present in the initial game, we have to assure that that particular set of strategies is the best response in all possible situations.

Finally, it is also worth being mentioned the concept of a Bayesian Nash equilibrium. It is the Nash equilibrium of the Bayesian version of the game[52], that is, the Nash equilibrium that we obtain once we take into consideration not only the strategic structure of the game but also the probability distributions over the firms' different (potential) characters or types.

Other, and more refined, notions of equilibrium in games do exist. However, they go beyond the scope of the present research and, therefore, there is no need to refer to them in this review.

5.1.5 Applications (Examples)

As a conclusion for this section we must mention some particularly important games that are relevant to the analysis to be performed in the present research, namely the prisoner's dilemma, Cournot and Stackelberg games.

The prisoner's dilemma:

This game constitutes one of the most well-known game theoretic problems. It departs from the assumption that two men are imprisoned accused of a crime. The law predicts different penalties according to the behaviour of the prisoners. If both prisoners (players) confess (cooperate) they will be imprisoned for 5 years. If one confesses (cooperate) and the other doesn't (doesn't cooperate), the one that confesses is set free and the one that doesn't is arrested for 10 years. If no one confesses (doesn't cooperate) they will be both imprisoned for 1 year. What is the best rational behaviour? Should the prisoners confess or remain silent?

The outcomes are best represented in normal form representation. The following table represents the situation of the game:

[52] A Bayesian version of a game consists of a finite set of potential types for each player, a finite set of perfect information games, each corresponding to one of the potential combinations of the players' different types and a probability distribution over a players' type (reflecting the beliefs of its opponents about its true type).

Table 5.1: The prisoner's dilemma

Strategies	Doesn´t Cooperate	Cooperate
Doesn´t Cooperate	-1 , -1	-10 , 0
Cooperate	0 , -10	- 5 , - 5

Source: Adapted from Filipe (2006)

Each player evaluates his two possible actions by comparing their personal payoffs in each column, since this shows which of their actions is preferable, just to themselves, for each possible action taken by their partner. Whenever one action for a player is superior to his other actions for each possible action by the opponent, we say that the first action strictly dominates the second one. It is clear that each player possesses a dominant strategy. In the prisoner's dilemma, then, confessing strictly dominates refusing for both players. Both players know this about each other, thus entirely eliminating any temptation to depart from the strictly dominated path. Thus both players will confess, and both will go to prison for 5 years.[53].

It is still worth being mentioned that the solution achieved for this particular game constitutes also a Nash equilibrium. In fact, both players choose not to cooperate because each player could not be better off by changing his decision. This is in accordance to the Nash equilibrium concept. However, it is not an efficient equilibrium, since both players could be better off if they could agree not to cooperate. Therefore, if one could enforce players to collude, the overall result of the game would be more efficient.

The prisoner's dilemma possesses many useful applications in practice. For the purpose of the present research it must be highlighted its usefulness to the definition of prices in oligopolistic markets or to the decision of entering a market. When faced with outputs similar to the ones presented in this example, firms must decide in accordance to the solution presented in this section.

Cournot[54]:

[53] If player II (column) confesses, then player I (line) gets a payoff of -5 by confessing and a payoff of -10 by refusing to cooperate. If player II refuses to cooperate, then player I gets a payoff of 0 by confessing and a payoff of -1 by refusing to cooperate. Therefore, player I is better cooperating regardless of what player II does. Player II, meanwhile, evaluates his actions by comparing his payoffs down each row, and comes to exactly the same conclusion that player I does.

[54] The notation and formulation presented is adapted from Filipe (2006).

This game represents the determination of the quantities to be produced by two[55] identical firms, with homogeneous products and with the objective of maximization of their profit, given by the difference between revenues and expenses. As such, the reward by participating in the game, for each firm, is given by:

$$\pi_i = RT_i - C_i \tag{5.1}$$

Where π_i represents the reward (profit), RT_i represents total revenues and C_i represents total costs for a particular firm and i = 1,2.

The function that represents the revenues for each firm is given by the price of sales times the quantity sold. The function is therefore, the following:

$$RT_1 = p(q) \times q_1 = [A - b(q_1 + q_2)]q_1 = Aq_1 - bq_1^2 - bq_1q_2 \tag{5.2}$$

And:

$$RT_2 = p(q) \times q_2 = [A - b(q_1 + q_2)]q_2 = Aq_2 - bq_2^2 - bq_1q_2 \tag{5.3}$$

Where $p(q)$ is the market price for that particular quantity, q is the total quantity produced and sold in the market, A and b are constants, q_1 and q_2 are the quantities produced and sold by firms 1 and 2. The function that represents the costs of the firms is given by the following expression:

$$C_i = cq_i \tag{5.4}$$

In which c represent a constant higher than zero. With this initial ser of functions, we have the following expressions:

$$\pi_1 = Aq_1 - bq_1^2 - bq_1q_2 - cq_1 \tag{5.5}$$

[55] It can also be applied to more than two firms.

And:

$$\pi_2 = Aq_2 - bq_2^2 - bq_1 q_2 - cq_2 \tag{5.6}$$

It is now necessary to maximize the function that represents the profit of each firm (being that the objective of each firm). That results in the subsequent expressions:

$$\frac{\partial \pi_1}{\partial q_1} = A - 2bq_1 - bq_2 - c = 0 \tag{5.7}$$

And:

$$\frac{\partial \pi_2}{\partial q_2} = A - bq_1 - 2bq_2 - c = 0 \tag{5.8}$$

These maximization conditions result in:

$$q_1 = \frac{A - bq_2^e - c}{2b} \tag{5.9}$$

And:

$$q_2 = \frac{A - bq_1^e - c}{2b} \tag{5.10}$$

Where q_i^e represents the quantity produced by each firm, which corresponds to its expected production. Therefore, we obtain the quantities that allow profit maximization for each firm given the expected quantities of the rival firm, the reaction functions for each firm. A firm chooses the quantities to produce that maximize its expected profit based on the expected production of the other firm. Therefore, the quantity to produce corresponds to the best response to the expected decision of the other firm. In order to achieve Nash equilibrium, it is necessary that the quantities that each firm chooses to produce correspond to the expected quantities by the other firms. No firm would be better off by changing their production. So, the following equalities should be achieved:

$$q_1^* = \frac{A-c}{3b} \tag{5.11}$$

And:

$$q_2^* = \frac{A-c}{3b} \tag{5.12}$$

These are the production quantities that correspond to Nash equilibrium.

Stackelberg[56]:

This classical model is also an oligopoly model. However, under this setting, firms choose their quantities sequentially. One firm, the leader firm, chooses the quantity to produce prior to the other firm, called the follower. The leader possesses an advantage and can, therefore, determine total production in the industry and limit the production of the follower to the quantity that maximizes its own profit.

The payoff function of the leader is given by the difference between revenues and expenses, as in the following expression:

$$\pi_1 = RT_1 - C_1 = p(q) \times q_1 - cq_1 = Aq_1 - bq_1^2 - bq_1q_2 - cq_1 \tag{5.13}$$

And:

$$\pi_2 = RT_2 - C_2 = p(q) \times q_2 - cq_2 = Aq_2 - bq_2^2 - bq_1q_2 - cq_2 \tag{5.14}$$

We can also establish that the prices and the production costs (for each firm) are given by:

[56] The notation and formulation presented is adapted from Filipe (2006).

$$p(q) = A - b(q_1 + q_2) \tag{5.15}$$

And:

$$C_i = cq_i \tag{5.16}$$

Therefore, the follower assumes the quantity produced by the leader and maximizes its profit by the maximization of its own profit function as in:

$$\frac{\partial \pi_2}{\partial q_2} = A - 2bq_1 - 2bq_2 - c = 0 \tag{5.17}$$

As a result, the reaction function of the follower is:

$$q_2 = \frac{A - bq_1 - c}{2b} \tag{5.18}$$

Knowing this reaction, the leader firm maximizes its own profit by:

$$\pi_1 = Aq_1 - bq_1^2 - bq_1 \frac{A - bq_1 - c}{2b} - cq_1 \tag{5.19}$$

With this framework the following equilibrium production quantities are achieved:

$$q_1^e = \frac{A - c}{2b} \tag{5.20}$$

And:

$$q_1^e = \frac{A - c}{4b} \tag{5.21}$$

Analysing the results, we have to conclude that the leader profits are higher than the ones obtained in the Cournot model, but the follower profits are lower than the ones obtained in the Cournot model. However, these results are in accordance with the anticipation, by the leader firm, of the reaction function of the follower. This is achieved through backward induction. The result obtained is also Nash equilibrium since no firm is better off by unilaterally changing its production quantities.

5.2 Real options games

Although corporate planners often recognize the practical importance of strategic considerations in investment decisions, it has until recently been unclear how strategic behaviour could be integrated with the contingent claim techniques employed in the real options literature. In fact, investment decisions are quite often very complicated and there is the intuition that, to improve the likelihood of deciding for a better outcome, firms must take into account, not only their own objectives, actions and possible strategies, but also the economic environments in which they operate. The combination of real option and game theories constitutes an attractive and promising framework to bring more sophisticated models to investment analysis. Next, we present a review of the most relevant contributions about this field of literature.

5.2.1 General approaches

The first reference found in the literature that relates competition and the analysis of investment opportunities with operational flexibility can be found in Kester (1984). He considered a finite maturity to real options because of the entrance of a competitor in the market that dilutes option value. Shortly after, Trigeorgis (1988) complemented this view referring that real options can be proprietary or shared. Later still, Trigeorgis (1991) modelled competition exogenously in order to identify major drivers to the problem. He studied the impact of competition on investment timing using standard ROA based on GBM. Competitive arrivals may reduce the value of a firm's own investment opportunity by taking away significant market share. Competition in this context can be modelled in one of two ways, depending on whether competitive entry is anticipated or random. Both suggest earlier investment. However, this framework is somewhat limited in that it does not explain what drives competitors' entry decisions.

After this initial breakthrough, the first approach in literature combining real options valuation with game theory concepts can be found in Smets (1991). It is a duopolistic model, with identical firms that leads to identical strategies. The model is developed in a continuous-time framework and has proven that, whenever a competitive advantage of a firm is not considered, an anticipation of the optimal investment timing exists due to a first-mover advantage.

Subsequently, Smit and Ankum (1993) also combined the real options approach of investment timing with basic principles of game theory and industrial organization. They examined a model where two firms share an investment option allowing them to enter the market for a (fixed) investment outlay. The evolution of the market and the decisions of the firms competing in such market are concurrently considered by use of a binomial lattice for the market value and a strategic-form game at each stage. If a firm enters first, as “leader", it grasps a higher market share in case the second firm subsequently enters. Unless one of the firms grasps this first-mover advantage, firms are assumed to be identical, receiving half of the market under simultaneous investment. In the symmetric case considered, the firms optimally choose to defer at the outset. At the next period, they invest simultaneously after an up-move but defer the investment after a down-move. Simultaneous immediate investment is not Pareto-optimal as both firms would be better of if they jointly deferred the investment. For the asymmetric case considered, one firm benefits not only from a payoff advantage but also from a strategic effect, since it preempts the rival firm in the up state and secures a leader position.

Shortly after, Dixit and Pindyck (1994) proposed a continuous-time symmetric-duopoly model, simplifying the original work by Smets (1991). An investment opportunity is available to two firms with each incurring a fixed investment cost upon exercise. In case of sequential investment, the leader earns monopoly rents after entering the market and the follower is faced with a single-agent optimal exercise problem. After the follower’s entry, the firms will form a duopoly, each earning the same amount. For low values of demand the value of the leader is lower than that of the follower and no firm entries the market. For intermediate levels of demand, there is an incentive to invest as leader, since the leader’s value exceeds that of the follower. This incentive dissipates the first mover advantage considered. Finally, for high levels of demand, both firms are operating in the market. Option premia are positive for both firms but the threat of preemption will cause the leader to invest earlier than in the monopoly case.

These initial formulations considered the decision whether or not to enter in a market. They further consider entry as a lumpy investment decision in a setting where firms possess information about the other firms that share the option to enter the market. However, under this setting, the initial breakthrough was achieved. Relaxation of the conditions assumed in this pioneering works was the logical next step.

So, Trigeorgis (1996) analyses the anticipation game, incorporating such effect in the trigger value necessary to exercise the option, thereby reducing it by the addition of a positive value to the dividend yield[57]. Grenadier (1996) used a game theoretical approach to value real options in real estate market. He develops a duopoly model involving completion delays that provides insights into the behaviours of property developers. Lambrecht and Perraudin (1994) analyse the impact of anticipation in the value and timing of investments in real assets. They conclude that the optimal investment is between the value defined by traditional capital budgeting and the value defined by ROA in a monopolistic context.

Smit and Trigeorgis (1997) used an integrated real options and game-theoretic framework for strategic R&D investments. They analyzed two-stage games where the growth option value of R&D depends on endogenous competitive reactions. In their model firms choose output levels endogenously and may have different (asymmetric) production costs as a result of R&D, investment timing differences or learning.

Huisman and Kort (1999) identify distinct equilibrium scenarios. The first one involves a preemptive investment sequence where the leader invests at a point where the leader and the follower are equal (rent equalization). A second equilibrium scenario involves tacit collusion with firms agreeing to invest at a later point in time. In addition, it is assumed that the value increment from leadership is larger than the value increment received by the follower upon investing. For highly volatile cash flows, firms are reluctant to exercise early, making tacit collusion likely to prevail.

Grenadier (1999) analyzes a setting where agents formulate option exercise strategies under imperfect information. The payoff received upon entry is not perfectly known to the firms, each receiving an independent private signal about the true

[57] Fudenberg and Tirole (1985) had already studied the effect of "preemption" in games of timing regarding the adoption of a new technology. They showed that the threat of preemption equalizes rents in a duopoly, although such result cannot be extended to the general oligopoly game.

underlying value. The firm may infer its rivals' private signals by observing their entry decisions. Grenadier discusses information cascades where firms ignore their private information and jump on the exercise bandwagon.

Weeds (2000) examined the impact of completion delays in the optimal investment strategies for R&D projects. The firm invests in research with the aim to acquire a patent giving it exclusive access to a new market. If the innovation is successful, the firm has the option to make an additional sunk investment to adopt the new technology. The entire R&D investment opportunity is a compound option where the value of the (first-stage) research option partly derives from the (second-stage) commercial investment option. The framework provides a rational explanation for the existence of sleeping patents, which are patents granted but kept in a stand-by or “sleep" mode. Policy-makers typically regard sleeping patents as anti-competitive devices employed by dominant firms to erect entry barriers. However, in this context sleeping patents may arise when options co-exist with completion uncertainty.

Lambrecht (2000) derives optimal investment strategies for two symmetric firms sharing the option to make a two-stage sequential investment, under incomplete information about the rival's profit. In the first stage, each firm is competing to acquire a patent enabling it to proceed to the second commercialization stage. Lambrecht (2000) derives conditions under which inventions are likely to be patented without being put to immediate commercial use. Sleeping patents are more likely to exist in an R&D portfolio when interest rates are low, volatility is high, and when the second-stage cost is high relative to the first-stage.

Boyer et al. (2001) extend previous pioneering contributions on strategic investment, most notably Gilbert and Harris (1984), Fudenberg and Tirole (1985), and Mills (1988). Decamps et al. (2002) investigate the impact of asymmetric costs on firms' investment strategies, and Thijssen (2002) discusses the problem of investment decisions under uncertainty and competition.

Weeds (2002) analyzed a patent race among duopolists and the effect of competitive pressure on firms' research activities. Two (symmetric) firms have the opportunity to launch a R&D project for a pre-determined cost. The first firm to succeed gains an exclusive patent, while the other firm is left with nothing. By comparing a competitive scenario with a monopolistic one, Weeds (2002) concludes that under competition one firm starts conducting research when the patent value reaches a certain threshold and the other firm initiates research later. Whereas in the monopolistic scenario, the larger and monopolistic firm starts conducting research later than the follower would do in the competitive setting. This later result challenges standard antitrust thinking in favour of joint research ventures.

Grenadier (2002) describes an oligopoly with symmetric firms that can increase capacity incrementally at any time at a cost per capacity unit. Given constant returns to scale, firms produce at full capacity and sell at the market-clearing price. Assuming symmetric investment strategies, the firms expand simultaneously and in the same proportion. Grenadier (2002) formulates the problem in a very tractable way and derives closed-form solutions for specialized cases. When the market approaches perfect competition, option values are getting less valuable. Grenadier (2002) assumed that output generation is costless with capacity being fully utilized (constant returns to scale).

Lambrecht and Perraudin (2003), assuming that firms have incomplete information about each other, formulate a full dynamic model of investment under uncertainty. In such model, the first-mover has an advantage and the approach used leads to a Bayesian Nash equilibrium in which each firm invests strategically.

Murto and Keppo (2002) develop a model combining real options and game theory where many firms compete for a single investment opportunity. The Nash equilibrium of the game is characterized under different assumptions regarding firms' information about each other's valuations for the project. Ziegler (2004) uses game theory to address several problems in finance, arguing that the payoff values of the economic agents can be obtained by using option pricing techniques. Inserting those payoffs into the strategic games between agents it is possible to analyze, more realistically, the value of strategic decisions. However, his methodology is unfortunately subject to a number of limitations such as the existence of path-dependent strategies[58] or strategies which are very difficult to parameterize[59].

Miltersen and Schwartz (2004) analyze patent-protected R&D investment projects when there is imperfect competition in the development and commercialization of the product. Competition in R&D not only may increase production and reduce prices, but

[58] A "path-dependent option" is an option whose payoff at exercise or expiry depends, in some non-trivial way, on the past history of the underlying asset price as well as its spot price at exercise or expiry. The "American option" is one of those cases.

[59] One important step in Ziegler's (2004) methodology consists in valuing the players' future uncertain payoffs using option pricing theory where all the players' possible actions enter the valuation formulas as parameters. However, for games where players have more than two choices at each node the use of these parameters introduces a high complexity in the derivation of the players' payoffs as well as in the computation of the game equilibrium.

it may also shorten the time of developing the product and increase the probability of a successful development. These benefits to society are offset by increased R&D investment costs in oligopolistic markets and lower aggregate value of the R&D investment projects.

Smit and Trigeorgis (2004, 2006) give an excellent literature review and illustrations about real options games. Shackleton et al. (2004) study the problem of entry decisions by competing firms in a two-player stochastic real options game.

Boyer et al. (2004b, 2007) consider, in continuous-time, a duopoly where symmetric firms add capacities by lumpy increments[60]. The only possible equilibrium at initial stages of the industry involves preemption[61]. When firms already hold substantial capacity, tacit collusion may be sustainable as industry equilibrium. Such equilibria are more likely in highly volatile or fast growing markets[62]. The possibility of collusion is more attractive to symmetric firms than to asymmetric ones.

Savva and Scholtes (2005) examine partnerships bilateral deals under uncertainty and downstream flexibility, focusing on the effect of options on the synergy set and analyzing cooperative and non-cooperative options. Paxson and Pinto (2005) derive a real options model for a duopoly market using two stochastic variables. They show that the degree of correlation between the two variables results in different value functions and investment thresholds, especially for the follower, and in the case where firms invest simultaneously in a non-preemption game.

Kort and Pawlina (2006) study the impact of investment cost asymmetry on the optimal real option exercise strategies and the value of firms in duopoly. They analyze the effects of firm heterogeneity (asymmetric investment costs) on optimal timing[63]. Three types of equilibria may arise depending on the magnitude of the first-mover advantage and the level of firm asymmetry. Cooperative equilibria involving simultaneous investments obtain for nearly homogenous firms with no real possibility to

---

[60] Boyer et al. (2004b) assume that reduced-form stage profits are the outcome of Bertrand price competition, whereas Boyer et al. (2007) consider Cournot quantity competition.

[61] When firms do not hold any existing capacity, tacit-collusion equilibria are ruled out as firms are not threatened with the loss of any existing rents. Due to preemption, the first industry-wide investment occurs earlier than what would be socially optimal (from the viewpoint of the industry participants). his distortion implies riskier entry and lower expected returns.

[62] In such a context, the conventional real options result that high volatility leads to investment postponement gets reinforced by the fact that higher volatility may result in a switch from the preemption equilibrium to a tacit-collusion equilibrium involving later investment and higher values.

[63] Dias and Teixeira (2003) and Joaquin and Butler (2000) discuss new market models with asymmetry in terms of production costs. Joaquin and Butler (2000) take an open-loop, pure-strategy approach. Dias and Teixeira (2003) use a closed-loop, mixed-strategy approach and prove that Joaquin and Butler's open-loop equilibrium obtains for a large cost differential.

gain a first mover advantage. Preemptive equilibria, characterized by the fear of preemption, emerges for mitigate asymmetry. Sequential (open-loop) equilibria in which the advantaged firm does not fear preemption and invests as a monopolist would do obtains for large asymmetry (this type of equilibrium does not occur in the symmetric case.) The relationship between firm values and the degree of asymmetry among firms is not clear-cut, since a higher degree of homogeneity may give rise to less efficient industry equilibria for both firms.

Thijssen et al. (2006) consider a duopolistic market and examine a first-mover vs. second-mover advantage (in the form of information spillovers via option exercise). Depending on specific parameters, the first or second-mover advantage may dominate, leading to preemption or a war of attrition. It is further shown that more competition does not necessarily lead to higher social welfare.

Novy-Marx (2007) extends the analysis for firms with different initial capacities. At any point in time the next firm to exercise its expansion option will always be the smallest firm. Simultaneous investment does not occur and firm heterogeneity results in a natural ordering of firm investments. Novy-Marx (2007) also challenges Grenadier's (2002) assertion that for an increasing number of firms, option values are continually eroded.

Bouis et al. (2009) extended the analysis in Dixit and Pindyck (1994) by considering a larger number of symmetric firms having the option to enter in a new market under complete information. To derive their main insights, they focus on a three-firm case and provide numerical analysis for larger oligopolies. Considering sequential as well as simultaneous investment decisions, Bouis et al. (2009) showed that the partitioning of the two equilibria hinges on a duopoly rent. They further showed that the leader in the three-firm case invests after the leader in duopoly but still before the monopolist. Therefore, increased competition (three firms rather than two) actually delays rather than hastens investment[64].

In Mason and Weeds (2009) a leader firm might be hurt by the follower's entry, in the case of a negative externality, or benefit from it, whenever in the presence of a positive externality. Therefore, the type of externalities impacts the investment schedule. The presence of negative externalities can hasten investment compared to the monopoly benchmark. With no first-mover advantage and no preemption, the leader

[64] For larger oligopolies, Bouis et al. (2009) argue that the number of expected future entrants is critical (especially whether this number is odd or even) and illustrate that the accordion effect sustains.

adopts the new technology at the cooperative trigger point. Otherwise, a preemptive sequential investment occurs where the follower adopts earlier than the cooperative solution.

Huisman and Kort (2009) allow firms to choose optimally their capacity, or production scale, at the time they enter the market. For low volatility, the follower chooses a higher capacity than the leader and for high volatility the leader chooses a higher capacity. Compared to the model without capacity choice, the monopolist and the follower invest later in more capacity for high volatility. Conversely, the leader will invest earlier in a higher capacity for higher volatility.

Novy-Marx (2009) derives Markov perfect equilibrium outcomes for duopolistic capacity competition. Two firms with differing initial capacity face isoelastic demand and an exogenous shock following a GBM. They have negligible operating costs. The author identifies and evaluates three distinct equilibria: Cournot, shared monopoly and preemptive preemption. He stresses that the notion of Stackelberg leaders where the larger firm profitably forecloses the market hinges on static market assumptions of zero-growth, no uncertainty, no depreciation and does not obtain in more general industry settings under uncertainty.

5.2.2 Discrete time games

Discrete-time games provide an intuitive introduction to some key insights by the use of numerical examples. This perspective is easier to implement and best suited for a practical application. In this section we shall review the most relevant discrete-time games under real options games.

The initial model developed under this setting was Smit and Ankum (1993). Despite the intuitive presentation of fundamental aspects relating competition with investment possibilities the framework was not continued until Smit and Trigeorgis (2001). They analyzed in discrete-time the trade-offs between managerial flexibility and commitment in a dynamic competitive setting under uncertainty. In fact, they extended the framework developed in Smit and Ankum (1993) by explaining the source of firm heterogeneity and quantifying the trade-off between commitment and flexibility.

Smit and Trigeorgis (2001) considered a scenario where two firms compete in two different stages of product development. Under this scenario, the early exercise of strategic investments can change later stages for the better. In fact, it can open new market opportunities or enhance the value of their investment options. Therefore, a firm can make a first-stage strategic investment possibly altering the later equilibrium strategic choices. Firms are initially assumed equal in the second competition stage but one firm may introduce some asymmetry by making this first-stage investment. Hence, the initial investment decision requires the firm to weigh the commitment cost against the expected future strategic benefits of commitment. For the different possible investment orderings considered, simultaneous, sequential or singular, it is defined a set of corresponding market outcomes, Cournot, Stackelberg or monopoly. These market outcomes are used to calculate the final payoffs. Following Fudenberg and Tirole (1984), the strategic effect of the committing first-stage investment depends on the type of competitive reaction and the nature of the commitment. The firm’s investment is either tough or soft. If firm (re)actions are strategic substitutes (as under Cournot quantity competition), the competing firm will engage less for an aggressive action by the rival firm. Conversely, firms' (re)actions can be strategic complements (as under differentiated Bertrand price competition). Smit and Trigeorgis (2001) construct and solve four numerical examples illustrating all possible combinations of competitive reaction and the investment type. Upfront investment is only optimal for the first firm to act in the two cases where the strategic effect is positive. For the cases with negative strategic effect, the first firm to act should not invest. It should benefit from increased uncertainty as its stage-two investment option becomes more valuable. But at the same time uncertainty erodes the value of committing as the upfront investment becomes riskier. Smit and Trigeorgis (2007, 2009) use this framework to assess R&D strategies and infrastructure investment decisions.

The original discrete-time analysis by Smit and Ankum (1993) considered that duopolists have perfect information regarding their rivals. That is to say, they know their production cost. This assumption was later relaxed by Zhu and Weyant (2003a, 2003b). They considered that two firms face a stochastic linear inverse demand and linear costs and that one firm has perfect information about the game, whereas the other firm knows its own cost but not its rival's, possessing only a believe about it with a certain degree of probability. If both firms invest simultaneously, the firm that possesses perfect information about the game optimally sets its own output in knowledge of both costs. The other firm forms expectations about its rival's quantity, selecting a Cournot quantity. For sequential investment decisions, the effect of incomplete information crucially depends on the order of the investment decisions. If the best-informed firm invests first, it may reveal its private cost, through its quantity choice, affecting the quantity decision of the rival and less-informed firm. On the other hand, if the less-informed party moves first, no new information is revealed to the best-informed firm. The analysis in Zhu and Weyant (2003a) focuses on single-stage decision making, while Zhu and Weyant (2003b) consider multiple time steps.

Finally, Murto el al. (2004) considered a different perspective concerning the possibilities to invest. In the previous analysis, each firm had a single investment opportunity. Murto et al. (2004) consider multiple interacting investment opportunities. Starting with a zero initial capacity, a firm can decide, at any time, to invest in order to increase capacity by a lump sum. Investment decisions are sequential with the first mover randomly chosen. Each investment subgame is described by the current firm capacities, the level of market demand, and present time. The optimal (Markov) expansion strategies involve investment-inducing demand thresholds and can be derived using dynamic programming. As these thresholds are increasing in a firm's installed capacity, the smallest firm is more likely to react to small demand shocks by expanding capacity. Symmetric firms can expand capacity by the same increment size and for the same investment cost, while in the asymmetric case analyzed, one of the firms considered invests in smaller lumps for a larger unit cash outlay. Despite its higher investment cost, the firm benefits from the situation since it can react quicker to changes in demand.

5.3 Final thoughts on real options games

In this chapter we introduced basic aspects of real option and game theory which are, implicit or explicitly, used in the real option models that consider the interactions between firms. We illustrated the effect of competition on the value of an investment decision and how the recognition of this fact by firms can change drastically the dynamics of their investment decisions. These models generally assume that the value of the investment is treated as a state variable that follows a known process. In the majority of the models, time is considered infinite and continuous. However, the advantage of discrete-time models is relevant for the purpose of the present research. The investment cost is sunk and fixed and the number of firms (players) holding the option to invest is usually two (duopoly). The focus of the analysis is the derivation of the firms' value functions and their respective investment trigger values, under the assumption that either firms are risk-neutral or that the stochastic evolution of the variables underlying the investment value is spanned by the current instantaneous returns from a portfolio of securities that can be traded continuously, without transaction costs, in a perfectly competitive capital market[65]. In addition, the investment game is usually played sequentially and treated as a "one-shot game" and the market for the project, underlying the investment decision, is considered to be complete and frictionless. In the games considered, in accordance to the legislation that defends competition, cooperation between firms is not allowed. In most games, however not in all of them, firms can only improve their profits by reducing the profits of opponents (zero-sum game) and are ex-ante symmetric and symmetric/asymmetric after the investment. Furthermore, the two most common games used in the real options literature are the "preemption game" and the "attrition game", both usually formulated as a zero-sum game. In the preemption game, it is assumed that there is a first-mover advantage that gives firms an incentive to be the first to invest and, in the attrition game, it is assumed that there is a second-mover advantage that gives firms an incentive to be the second in the investment.

The remaining chapters present the empirical analysis performed. We present the research questions, the models that served as a basis for the present research, and the deduction of the model developed under the framework of analysis adopted in the present research. Afterwards, we present the results of the model implementation and the corresponding conclusions.

[65] This is the generic assumption in ROA.

6. RESEARCH QUESTIONS AND METHODOLOGY

Having presented the most recent developments in three different branches of literature, relevant to the present dissertation, it is now time to define the methodological goals we aim to achieve with the current work and present the way to do so. Next, follows the presentation of the questions to be answered as well as the methodology from which we will depart to answer these questions.

6.1 Research questions

In the previous chapters of the present dissertation, it was initially demonstrated that interactions between agency theory and real options analysis provide a useful linkage between investment decisions and financing decisions. It was also established that under equity financing of the growth option an incentive to underinvest has been reported[66], as a consequence of the agency costs of debt. It was further demonstrated that interactions between game theory and ROA provide a useful linkage between investment decisions and competitive interaction. It was also demonstrated that different competitive responses under a duopoly market structure portray an influence in firm value[67]. Therefore, when facing both equity financing of the growth option and competition, does the incentive to underinvest still prevails? This is the essential question to be answered in the present research.

We depart from two identical firms financed by both equity and debt, which possess a shared growth option to expand its scale of operations. If we consider that such growth option will be financed solely by equity by both firms, does the effect predicted in Mauer and Ott (2000) still holds? In order to analyse that, conditions regarding the competitive reaction of the firms have to be assumed. First, we shall analyse the situation where both firms simultaneously decide concerning the exercise of the growth option. Under this setting, the firms will act according to Cournot-Nash equilibrium. Afterwards, we shall change this initial setting, considering sequential investment decision. Therefore, the firm that decides previously may act as leader, investing first, and the other as follower, investing later. Under this setting, the firms will act according

[66] Mauer and Ott (2000).
[67] Smit and Trigeorgis (2004).

to Stackelberg equilibrium. These two different alternatives regarding competitive responses by firms shall be included exogenously in line with Smit and Trigeorgis (2004). Firm value will be derived under these different settings.

In order to analyse these possibilities, consideration of different operating policies shall also be performed, a first best policy as the one that maximizes the value of the firm and a second best policy as the one that maximizes the value of the equity of the firm. The differences in the value of the firm thus obtained are the agency cost of debt and the loss in efficiency that the adoption of the second best policy generates.

The breakthrough that such analysis encompasses is the analysis of the effects of competition in the agency conflicts that were already studied and are already present in the literature.

6.2 Methodology

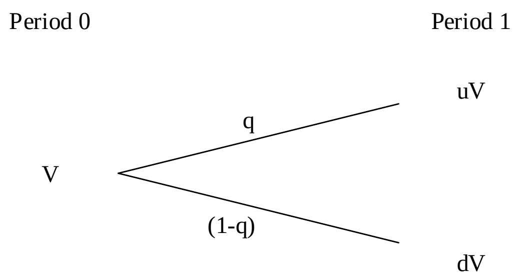
We now present and describe the setting under which the valuation model to be used, in order to analyse the impact that managerial flexibility and competitive interaction have in the valuation of investment opportunities, will be derived. It departs from the estimation of firm value under alternative behaviours for the firms in the duopolistic market structure. The first considers simultaneous reaction as in Cournot. The second considers sequential leader-follower reaction as in Stackelberg. We will present the reaction functions necessary to implement this different behavioural strategies, thus achieving different market equilibriums. Additionally, a monopolistic setting is also considered, in case one of the firms abandons the market. Afterwards, inclusion of managerial flexibility is done. The growth option and the abandonment option are included. Furthermore, the decision to be taken by the firms will also be conditioned by the financing structure adopted in the investment outlay necessary to exercise it. The firms are financed by both equity and debt at the outset, while the exercise of the investment is financed solely by equity. Under these financing structures two different strategies shall be considered. A first-best policy, that considers that managerial decisions are taken in order to maximize the value of the firm and a second-best policy, that considers that managerial decisions are taken in order to maximize the value of the equity of the firm. Computation of the differences in firm value that result from the adoption of these two different policies, allows for the determination of the agency cost of debt.

The methodology presented relies in two different models. The first approach is the one followed in Smit and Trigeorgis (2004) to value a strategic investment in R&D in a duopolistic market. They present a real options game that it will be applied to define a first approach to the problem under study. However, they disregarded the impact that differences in the financing structure of the firms can produce in the investment decision. In order to incorporate such impact, the model developed by Mauer and Ott (2000) shall also be presented. In this section we shall present the valuation techniques that are behind the general model developed in this research. However, we shall start with the description of a generic discrete-time growth option valuation, followed by the description of Smit and Trigeorgis (2004) and Mauer and Ott (2000).

6.2.1 Discrete-time Growth Option Valuation

The valuation of growth options in discrete-time models departs from the general multiplicative binomial option pricing approach developed by Cox, Ross and Rubinstein (1979). It is assumed that the underlying asset (in this setting, the cash flows generated by the initial investment outlay) follows a multiplicative binomial process over successive periods described by:

Figure 6.1: Evolution of firm value through time



Source: Author

Where the present value of the cash flows generated, *V*, may increase by the multiplicative factor, *u*, with probability *q*, to *uV* or decrease by the multiplicative factor *d*, with complementary probability *(1-q)*, to *dV*, at the end of the period. Therefore, at any point in time, the value of the underlying asset can be described by the following equations:

$$V_t = V_{t-1} \times u = uV \tag{6.1}$$

Or:

$$V_t = V_{t-1} \times d = dV \tag{6.2}$$

These expressions, alongside with the probabilities of an up move or of a down move, give the next expression for the value of the underlying:

$$V_t = uV \times q + dV \times (1 - q) \tag{6.3}$$

The multiplicative factors, *u* and *d*, represent the continuously compounded rate of return on the underlying asset, with:

$$d = \frac{1}{u} \tag{6.4}$$

And with *u* being equal to:

$$u = \frac{uV}{V} \tag{6.5}$$

As in the general binomial valuation process, the payoff to the call option at the expiration date is represented by the following expressions:

$$uC = Max(uV - I, 0) \tag{6.6}$$

$$dC = Max(dV - I, 0) \tag{6.7}$$

In the expressions above, I represents the additional investment outlay necessary to expand the scale of operations. With the terminal value for the call option on the cash flows of the project above derived, the value of such option at the present time follows:

$$C = \frac{[puC + (1-p)dC]}{(1+r)} \tag{6.8}$$

In the expression above r represents the risk-free rate. And with p being equal to:

$$p = \frac{[(1+r)-d]}{u-d} \tag{6.9}$$

If we extend this single period binomial model and subdivide the time to expiration of the growth option, T, into n equal subintervals, each of length $t = T/n$, the general binomial pricing formula can be represented as follows:

$$C = \frac{\sum_{j=0}^{n} \left\{ \frac{n!}{j!(n-j!)} \right\} p^{j} (1-p)^{n-j} Max\left(u^{j} d^{n-j} V - i, 0\right)}{(1+r)^{n}} \tag{6.10}$$

This expression adds the probability that firm value will take j upward jumps in n steps, each with risk neutral probability p.

In accordance with Cox, Ross and Rubinstein (1979), the parameters, u, d and p are chosen so that the mean and variance of the continuously compounded rate of return of the discrete binomial process are consistent in the limit with their continuous counterpart. The firm value will become log-normally distributed and the binomial distribution function will converge to the cumulative standard normal distribution function. The parameters must then be equal to:

$$u = e^{\left(\sigma\sqrt{t}\right)} \tag{6.11}$$

$$d = \frac{1}{u} \tag{6.12}$$

$$p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{t} \tag{6.13}$$

$$\mu = \ln r - \frac{1}{2}\sigma^2 \tag{6.14}$$

This is the standard procedure to value a call option under the binomial valuation model. The growth option can be valued like a call option. The adjustments necessary in order to apply such valuation model are the ones already described in a previous chapter. Essentially, one must consider the underlying asset as the additional cash flows generated by the additional investment expenditure. This additional investment expenditure is the exercise price of the option. The remaining parameters are the usual ones in the real options valuation framework.

6.2.2 Smit and Trigeorgis Real Options Games Valuation

In this section, a description of the model developed in Smit and Trigeorgis for the valuation of duopolistic firms with a shared growth option is performed. In the model developed it is necessary to introduce a monopolistic firm in order to compare the results obtained with the ones derived in a duopolist market structure. Therefore, we shall depart from firm valuation in a monopolistic setting and afterwards derive firm value in a duopolistic market.

6.2.2.1 Monopoly

A monopolistic structure implies that the firm is the only proprietary of the option in analysis, because the firm is the only one present in the market. In the framework developed, it shall be assumed a linear inverse demand function specified by:

$$P(Q, \theta_t) = \theta_t - Q \tag{6.15}$$

Where θ_t is the demand shift parameter[68], assumed to follow a lognormal diffusion process (or a multiplicative binomial process in discrete time), Q is the quantity produced by the firm and $P(Q)$ is the common market price as a function of total quantity Q. The initial investment represents the fixed costs, while the total variable costs depend on the size of production. The total variable production cost for the firm is given by:

$$C(Q) = c + \frac{1}{2} q Q^2 \tag{6.16}$$

Here, c and q are the linear and quadratic cost coefficients for the firm. The operating profit for the firm is given by:

$$\pi(Q, \theta_t) = PQ - C(Q) = \theta_t Q - c - \left(1 + \frac{1}{2} q\right) Q^2 \tag{6.17}$$

The gross project value (profit value), V, and the net present value, NPV, from the investment in the growth option, assuming perpetual annual operating cash flows (profits) and a constant risk-adjusted discount rate k, is obtained from:

$$V = \frac{\pi}{k} \tag{6.18}$$

$$NPV = V - I = \frac{\pi}{k} - I \tag{6.19}$$

With these equations it can be seen that the value maximizing quantity for a monopolistic firm is given by:

$$Q = \frac{\theta_t - c}{2 + q} \tag{6.20}$$

[68] Exogenous uncertainty in future market demand is assumed to be characterized by fluctuations in this parameter.

The firm can then set a monopolistic price equal to:

$$P = \frac{[\theta_t(1+q)+c]}{(2+q)} \tag{6.21}$$

This price enables the firm to enjoy monopoly profit value of:

$$V = \frac{(\theta_t - c)^2}{(4+2q)k} \tag{6.22}$$

These are the basic relationships that are necessary in order to determine the final outputs needed under a monopolistic market structure.

6.2.2.2 Duopoly

A duopolistic structure is characterized by the existence of two firms in the market. This means that decisions taken by one firm affect the market as well as their competitors. One firm's results depend not only of their own decisions but also from their rival's reactions.

Different forms of competition can be considered. Research primarily distinguishes between price and quantity competition. We will focus our attention in the later one. Two alternative approaches shall be analyzed.

The first one assumes a simultaneous reaction from both firms that originates an equal distribution of the profits among the two firms that share it. This approach results in the establishment of Cournot-Nash equilibrium.

The second one assumes a sequential reaction from the two forms present in the market that originates an unequal distribution of the profits among the two firms. The first firm to act is the leader taking a major stake of the market's profit. The second one is the follower keeping the remains left by the leader of the market. This approach results in the establishment of Stackelberg leader-follower equilibrium.

Cournot-Nash Equilibrium:

Specification of the traditional Cournot model is presented[69]. We are considering two firms producing a homogeneous product competing in the same market, with the objective of maximizing profits.

The assumptions necessary in order to reach this equilibrium are the same we applied previously, under a monopolistic structure. However another firm is introduced in the analysis.

We assume, as before, a linear inverse demand function. With two firms in the market, the demand function is specified by:

$$P(Q,\theta_t) = \theta_t - (Q_A + Q_B) \tag{6.23}$$

The notation is the same as before, but now θ_t is the demand shift parameter, assumed to follow a multiplicative binomial process, Q_A and Q_B are the quantities produced by firms A and B, respectively, and *P(Q)* is, as before, the common market price as a function of total quantity, now given by $(Q = Q_A + Q_B)$.

The total variable production cost for each firm is given by the same expression as before:

$$C(Q_i) = c_i + \frac{1}{2} q_i Q_i^2 \tag{6.24}$$

Here, c_i and q_i are, once again, the linear and quadratic cost coefficients for firm *i*. The second stage annual operating profits for each firm is now given by:

$$\pi_i(Q_i, Q_j, \theta_t) = PQ_i - C(Q_i) = [(\theta_t - c_i) - Q_j]Q_i - \left(1 + \frac{1}{2} q_i\right) Q_i^2 \tag{6.25}$$

[69] The analysis presented here follows the work developed by Smit and Trigeorgis (2004).

The gross project value (profit value), V_i, and the net present value, NPV_i, with the same assumptions as in monopoly are given by the same expressions:

$$V_i = \frac{\pi_i}{k} \tag{6.26}$$

$$NPV_i = V_i - I = \frac{\pi_i}{k} - I \tag{6.27}$$

Under quantity competition, the reaction function of each firm is downward sloping. Maximizing firm *i* own profit value over its quantity given that its competitor produces Q_j, each firm's reaction function is given by:

$$R_i(Q_i) = \frac{\theta_t - c_i - Q_j}{2 + q_i} \tag{6.28}$$

If both firms make a simultaneous production capacity investment, in the second stage a Cournot-Nash equilibrium outcome results. The equilibrium quantities, Q^*_A and Q^*_B are obtained by equating the reaction function of the two firms:

$$Q_i^* = \frac{(\theta_t - c_i)(2 + q_i) - (\theta_t - c_i)}{(2 + q_i)(2 + q_j) - 1} \quad \text{(6.29}$$

In the situation that shall be considered, where both firms invest simultaneously, a symmetric Cournot-Nash equilibrium may result if the firms are otherwise identical. Identity is verified if:

$$Q_A^* = Q_B^* = Q^* \tag{6.30}$$

$$c_A = c_B = c \tag{6.31}$$

$$q_A = q_B = q \tag{6.32}$$

This equality results, if $\theta_t > c$ in:

$$Q_i^* = \frac{(\theta_t - c)}{(3+q)} \tag{6.33}$$

$$V_i^* = \left(1 + \frac{1}{2}q\right)\frac{(\theta_t - c)^2}{(3+q)^2 k} \tag{6.34}$$

If $q = 0$, the symmetric Cournot equilibrium quantity simplifies to:

$$Q_i^* = \frac{1}{3}(\theta_t - c) \tag{6.35}$$

$$V_i^* = \frac{(\theta_t - c)^2}{9k} \tag{6.36}$$

The general model regarding simultaneous decisions is thus presented. An alternative solution, where firm's actions are sequential follows.

Stackelberg Equilibrium:

The general assumptions undertaken in the previous solution still hold. Differences that shall appear in the present approach depart from the inexistence of simultaneous reaction.

In case firm *i* invests first and firm *j* defers investment, the first becomes the leader and the later becomes the follower. The follower will set its quantity having first observed the leader's output according to its reaction function:

$$R_i(Q_i) = \frac{\theta_t - c_i - Q_j}{2 + q_i} \tag{6.37}$$

With this reaction function, the leader will then maximize the following function:

$$Q_i = \frac{(\theta_t - c_i)(2+q_i) - (\theta_t - c_j)}{(2+q_i)(2+q_j) - 2} \tag{6.38}$$

$$V_i = \frac{(\theta_t - 2c_i + c_j)^2}{8k} \tag{6.39}$$

Substituting the leader's optimal quantity from the above mentioned equation into the follower's reaction function referred to above gives the Stackelberg follower's quantity and profit value:

$$Q_j = \frac{\theta_t - 3c_j + 2c_i}{4} \tag{6.40}$$

$$V_j = \frac{(\theta_t - 3c_j + 2c_i)^2}{16k} \tag{6.41}$$

As expected, the follower's equilibrium quantity and profit value are lower than the ones of the leader.

6.2.3 Mauer and Ott agency-real-options model

In this subsection it is presented the model implemented by Mauer and Ott (2000). The interaction between investment and financing decisions is present due to a difference between the initial financing structure of the firms and the financing structure adopted to exercise the growth option. This difference in the financing structure originates two different operating policies that cause the agency costs of debt.

Let's start with their original basic assumptions. They considered one firm that produces a single commodity, infinitely divisible and produced continuously through time at a rate of one unit per year. The cost of producing it is *C*, which is constant through time. As the commodity is produced it is sold in a perfectly competitive market at a price P per unit. The evolution of the commodity price through time follows a geometric Brownian motion:

$$\frac{dP}{P} = \alpha dt + \sigma dz \tag{6.42}$$

Where the drift rate (α) and volatility (σ) are constants and dz is the increment of a standard Wiener process. The convenience yield of the commodity (δ) is assumed to be a constant proportion of its price, and the riskless security is assumed to yield a constant instantaneous interest rate of r per year. Operating profits are also taxed instantaneously at a constant rate τ .

They analyzed the value of an unlevered firm after exercise of the growth option. Firm value must satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 P^2 V_{PP}^U + (r-\delta)PV_P^U - rV^U + (P-C)(q)(1-\tau) = 0 \quad P > P_{Aq} \tag{6.43}$$

This unlevered firm value must satisfy three conditions: The abandonment option is worthless if the commodity price gets large; the firm has zero net salvage value at abandonment; and the abandonment price is optimally chosen. Inclusion of these restrictions in the equation leads to the following solution for the value of the unlevered firm:

$$V_q^U(P) = \left(\frac{P}{\delta} - \frac{C}{r}\right)(q)(1-\tau) - \left(\frac{P_{Aq}}{\delta} - \frac{C}{r}\right)\left(\frac{P}{P_{Aq}}\right)^{\beta_2}(q)(1-\tau) \tag{6.44}$$

Where the optimal abandonment price is given by:

$$P_{Aq} = \left(\frac{-\beta_2}{1-\beta_2}\right)\frac{\delta C}{r} \tag{6.45}$$

And with β_2 being equal to:

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0 \tag{6.46}$$

Next, the value of a firm with both equity and debt in its financing structure is calculated. The authors analyze the value of both equity and debt after the growth option is exercised. They assume that the firm's debt is permanent with a stated maturity and denote the promised coupon payment by R. Equity value must satisfy three boundary conditions. The first two consider that the firm has zero net salvage value at abandonment, and the abandonment price is optimally chosen. The other two are the value matching and optimality conditions that firm value must satisfy when the growth option is exercised.

With these restrictions the equity value is given by the following expression:

$$E_q(P) = \left[\frac{Pq}{\delta} - \frac{(Cq+R)}{r}\right](1-\tau) - \left[\frac{P_{Dq}q}{\delta} - \frac{(Cq+R)}{r}\right]\left(\frac{P}{P_{Dq}}\right)^{\beta_2}(1-\tau) \tag{6.47}$$

Where:

$$P_{Dq} = \left(\frac{-\beta_2}{1-\beta_2}\right)\left(\frac{\delta(C+R/q)}{r}\right) \tag{6.48}$$

The value of the debt of this same firm, after the exercise of the growth option is conditioned by the fact that at a high enough commodity price debtholders will almost surely receive the coupon value, and also by the fact that in bankruptcy they get firm value.

The value of the risky debt is therefore given by the following expression:

$$D_q(P) = \frac{R}{r} + \left[(1-b)V_q^U(P_{Dq}) - \frac{R}{r}\right]\left(\frac{P}{P_{Dq}}\right)^{\beta_2} \tag{6.49}$$

The total value of the levered firm after the growth option is exercised is the sum of these two different claims. It is the value of the unlevered firm plus the expected discounted value of tax shields on debt minus the expected discounted value of bankruptcy costs.

The first best exercising policy is chosen to maximize the value of the firm. The general solutions for the equity and debt values under this first best policy are given by:

$$E_F(P) = \left(\frac{P}{\delta} - \frac{(C+R)}{r}\right)(1-\tau) + K_1^F P^{\beta_1} + K_2^F P^{\beta_2} \qquad P_D < P < P_1^F \tag{6.50}$$

$$V_F^L(P) = \left(\frac{P}{\delta} - \frac{C}{r}\right)(1-\tau) + \frac{\tau R}{r} + K_3^F P^{\beta_1} + K_4^F P^{\beta_2} \qquad P_D < P < P_1^F \tag{6.51}$$

With β_1 being equal to:

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{6.52}$$

Where K_1^F, K_2^F, K_3^F, and K_4^F are constants to be determined and P_D is the endogenously determined commodity price at which the equityholders default on the debt before the growth option is exercised, and P_1^F is the commodity price at which the firm exercises the growth option. These general solutions must recognize the firm's limited liability whenever it defaults on debt payments, with this trigger price being determined in order to maximize equity value, as well as the fact that the trigger price of the growth option is chosen in order to maximize the value of the firm.

The second best exercising policy is chosen to maximize the equity of the firm. The general solutions for the equity and debt values under this second best policy are given by:

$$E_S(P) = \left(\frac{P}{\delta} - \frac{(C+R)}{r}\right)(1-\tau) + K_1^S P^{\beta_1} + K_2^S P^{\beta_2}, \qquad P_D < P < P_1^S \tag{6.53}$$

$$V_S^L(P) = \left(\frac{P}{\delta} - \frac{C}{r}\right)(1-\tau) + \frac{\tau R}{r} + K_3^S P^{\beta_1} + K_4^S P^{\beta_2} \qquad P_D < P < P_1^S \qquad (6.54)$$

Where K_1^F, K_2^F, K_3^F, and K_4^F are constants to be determined and P_1^S is the commodity price at which the firm exercises the growth option. These general solutions must take into account two differences regarding the previous set of assumptions. They must recognize that at abandonment the equity value is zero and the trigger price to exercise the growth option is chosen in order to maximize the value of the equity. This later difference is the fundamental one regarding the first and second best policy.

With this formulation the agency cost of debt can now be computed as:

$$AC(P) = V_F^L(P) - V_S^L(P), \qquad P_D < P < \max\left(P_1^F, P_1^S\right) \qquad (6.55)$$

It is the difference between the first and second best levered firm values.

6.3 Final remarks on methodology

The present chapter intended to describe the goals of the present research and the models upon which the research was based to achieve such goals. The goals are described in the first section of the present chapter. The models that "inspired" the present research are described in the second section of the present chapter.

This chapter sets the basis for the one that follows. In fact, the next chapter presents the model developed under the present research and conducts a numerical simulation that allows the questions placed to be answered.

7. IMPLEMENTATION AND RESULTS OBTAINED

The previous chapters presented the main concepts associated with agency conflicts and competition in a ROA framework. In this chapter a presentation of a discrete-time model that integrates these concepts shall be made. Our goal is to develop a firm valuation framework that takes into account a compound real option in the context of a duopolistic market structure. Such framework will also enable the valuation of the equity and debt value of the firm allowing, therefore, the measurement of the agency costs of debt under this context.

We shall start with the derivation of firm value in different market equilibriums. Afterwards, incorporation of managerial flexibility through a standard ROA model will be applied. Finally, valuation of equity and debt values follows, through the use of contingent claim analysis.

We depart from a market that is shared between two identical firms that face demand uncertainty. Under this setting, it is necessary to develop the analytical derivation of the reaction curves and the equilibrium outcomes for the Cournot, Stackelberg and monopoly equilibrium. Afterwards, derivation of firm values under these equilibrium market structures follows. It is additionally considered that both firms share a growth option and an abandonment option. The growth option will allow firms to expand its current scale of operations[70] by a certain multiplicative factor at a predetermined and fixed price (the exercise price of the option). The abandonment option permits the firms to leave the market by a salvage value. Valuation of managerial flexibility, and firm value under this setting, will be achieved through standard ROA valuation, departing from the equilibrium reached through game theoretic decisions for the exercise of the options.

It shall further be considered that both firms are financed by equity and debt and the exercise of the growth option shall only be equity financed. Therefore, we shall also present the contingent claim valuation in order to value the equity and the debt of each firm. However, because the growth option will be exercised by an additional equity issue, consideration of two different policies for the exercise of such option will be presented and valued. A first best policy, as the one that maximizes the value of the firm

[70] The increase in the scale of operations is exogenous to the model developed.

will be considered. A second best policy, as the one that maximizes the value of the equity of the firm will also be presented. The difference in firm value that results from the adoption of these two different policies is the agency cost of debt.

Finally, a different scenario concerning the financing of both firms will be presented. One firm will be only equity financed and the other firm will be financed by both equity and debt. Under this scenario, the resulting equilibrium reached under competition for the growth option will be analysed. Next, description of the model follows.

7.1 Discrete-time agency real options game valuation model

With two firms present in the market, we must start with the consideration that both firms face exogenous uncertainty in future market demand, which is in turn characterized by fluctuations in a demand parameter. It shall be assumed a linear inverse demand function of the form:

$$P(Q,\theta_t) = \theta_t - (Q_a + Q_b) \tag{7.1}$$

Where θ_t is the demand shift parameter, assumed to follow a multiplicative binomial process, *Qa* and *Qb* are the quantities produced by both firms present in the market and *P(Q)* is the common market price as a function of total quantity *(Qa* + *Qb)*.

The binomial process followed by the demand shift parameter follows the following path:

$$\theta_1 = \theta_0 \times u = u\theta \tag{7.2}$$

Or:

$$\theta_1 = \theta_0 \times d = d\theta \tag{7.3}$$

In the above expressions, the value of the demand parameter at time 0, θ_0, may increase by the multiplicative factor, *u*, to $u\theta$ or decrease by the multiplicative factor *d*, to $d\theta$, at time 1, considered the next time period. The multiplicative factors, *u*, *d*, are exogenous to the model but the relationship between them can be given by the following expression:

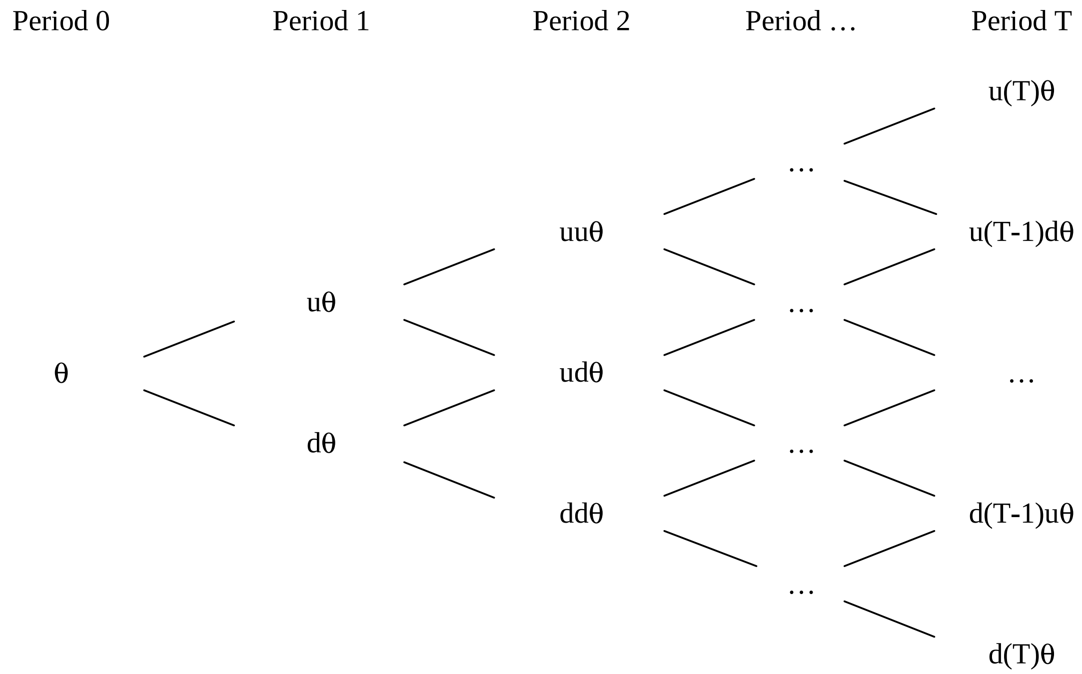
$$d = \frac{1}{u} \tag{7.4}$$

The probabilities associated with such movements are the actual probabilities. The multiplicative factor, *u*, has probability *q*, and the multiplicative factor *d*, has complementary probability *(1-q)*. Therefore, the value of the demand parameter at time 1, can be represented by:

$$\theta_1 = u\theta \times q + d\theta \times (1-q) \tag{7.5}$$

The extension of this framework to multiple periods is achieved through the repetition of the process described for one time period to the remaining periods considered. The general multiple period binomial process can be described by the following figure:

Figure 7.1: Evolution of demand parameter through time



Source: Author

Having derived the process for the evolution in time of the demand parameter it is now time to proceed to the analysis of firm value in the end node of the demand tree. The end node of the tree is the maturity date of the options considered and also the maturity date of the debt outstanding[71]. However, we shall start by analysing the value of the firms without including the decisions to be taken by the firms considering the exercise of the options and the payment of the debt. In order to do so, we must derive firm value under the two equilibrium alternatives to be considered, Cournot, Stackelberg, but also under the alternative monopolistic market structure.

The total variable production cost for a particular firm *i (i = A or B)* is given by:

$$C(Q_i) = c_i Q_i + \frac{1}{2} q_i Q_i^2 \tag{7.6}$$

Here, c_i and q_i are the linear and quadratic cost coefficients[72]. Therefore, the annual operating profit for each firm is given by[73]:

$$\pi_i(Q_i, Q_j, \theta_t) = PQ_i - C(Q_i) = [(\theta_t - c_i) - Q_j]Q_i - \left(1 + \frac{1}{2} q_i\right) Q_i^2 \tag{7.7}$$

The value of the firm, assuming perpetual annual operating cash-flows thereafter, corporate tax τ, and a constant risk-adjusted discount rate[74] κ, is given by:

$$V_i = \frac{\pi_i}{\kappa}(1 - \tau) \tag{7.8}$$

[71] Consideration of a different maturity for the debt outstanding will not produce any relevant change in the model derived. This assumption just simplifies the procedure.

[72] They can also be considered as the fixed and variable costs of the marginal cost function. This function can be expressed as $c_i + q_i Q_i$.

[73] This expression is the derivation of firm value at the exercise date of the operational managerial. The assumption present in the model is that debt matures at the same date. Therefore, firm’s profits do not possess any debt related costs.

[74] The assumption of this constant discount rate is in line with Smit and Trigeorgis (2004). In the financial structure assumed in this model, this assumption is also plausible since the maturity date of the options considered is also the maturity date of the original debt issue.

In order to obtain the reaction function of each firm under quantity competition it is necessary to maximize each firms profit function over its own given quantities. Each firm reaction function is thus:

$$R_i(Q_i) = \frac{\theta_t - c_i - Q_j}{2 + q_i} \tag{7.9}$$

If both firms equally share the market[75], they achieve Cournot-Nash equilibrium. In this scenario, the equilibrium quantities are obtained by equating both reaction functions. The end result is:

$$Q_i^* = \frac{(\theta_t - c_i)(2 + q_i) - (\theta_t - c_j)}{(2 + q_i)(2 + q_j) - 1} \tag{7.10}$$

Simplifying the above expression, by setting $q_i = q_j = 0$, we obtain the following expression for the quantities:

$$Q_i^* = \frac{\theta_t - 2c_i - c_j}{3} \tag{7.11}$$

These equilibrium quantities generate the following firm value:

$$V_i^* = \frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1 - \tau) \tag{7.12}$$

This is the general expressions for firm value under Cournot-Nash equilibrium. However, if one firm becomes the leader[76], and the other firm accommodates as a follower, we achieve Stackelberg equilibrium.

Under this equilibrium scenario, the follower sets its quantity, according to its reaction function to the leader's output. Because the leader is aware of such reaction

[75] This equality is obtained in a simultaneous decision game.

[76] This leadership is obtained in a sequential decision game. However, consideration of which firm decides first is exogenous to the model.

function, we can derive the quantity to be produced by the leader as defined in the previous expression (7.10) that we replicate:

$$Q_i = \frac{(\theta_t - c_i)(2+q_i) - (\theta_t - c_j)}{(2+q_i)(2+q_j) - 2} \tag{7.13}$$

Assuming, for the sake of simplicity that $q_i = q_j = 0$, we obtain the following expression for the quantity to be produced by the leader:

$$Q_i = \frac{\theta_t - 2c_i + c_j}{2} \tag{7.14}$$

And the firm value of the leader becomes:

$$V_i = \frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau) \tag{7.15}$$

Substituting the leader optimal quantity in the follower's reaction function we get the optimal follower quantity and the corresponding firm value as:

$$Q_j = \frac{\theta_t - 3c_j + 2c_i}{4} \tag{7.16}$$

$$V_j = \frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau) \tag{7.17}$$

Additionally, it must also be considered the possibility that only one firm remains in the market[77]. Under this scenario, the expression for the monopolist firm is given on the assumption that the rival firm produces nothing[78]:

---

[77] In the explanation of the model, it can not be ruled the out the possibility that one firm exercises the abandonment option and therefore, the firm remaining becomes a monopolist. Therefore, this possibility is also included.

[78] Once again with the assumption that $q_i = q_j = 0$.

$$Q_i = \frac{\theta_t - c_i}{2 + q_i} \qquad (7.18)$$

Being the only firm in the market, it can set a monopolistic price of:

$$P = \frac{\theta_t + c_i}{2} \qquad (7.19)$$

With such a price, the firm value becomes:

$$V_i = \frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau) \qquad (7.20)$$

Having derived firm value under the different equilibrium possibilities, it is now time to include managerial flexibility. That is represented by the possibility to expand the scale of operations, exercising the growth option, and by the possibility to abandon the market, exercising the abandonment option.

We shall start with the consideration of a growth option to expand its scale of operations. This type of option is typically assumed as an increment in firm value in exchange of the investment expenditure necessary to implement it. Therefore, it is like a call option on the increment in firm value with an exercise price equal to the investment expenditure necessary to implement it. The payoff at expiration date, for a call option with these characteristics can be represented by[79]:

$$G = Max(gV - I, 0) \qquad (7.21)$$

In the expressions above, I represents the additional investment outlay necessary to expand the scale of operations, G represents option value and gV represents the increment in the value of the firm. With the terminal value for the call option on the

[79] This expression departs from the assumption that the increase in capacity is absorbed by the demand. This is the general assumption under growth option valuation. Alternatively, we could have assumed that the increase in production would lead to alternative productive capacity and to a change in the equilibrium conditions. This alternative assumption would not significantly change the expressions obtained for the valuation of the growth option.

cash flows of the project above derived, the value of such option at a particular time is obtained by the general binomial valuation model of a call option:

$$G_{t-1} = \frac{\left[puG_t + (1-p)dG_t\right]}{(1+r)} \tag{7.22}$$

And with p, the risk neutral probability, being equal to:

$$p = \frac{\left[(1+r) - \left(\frac{\kappa}{1+\kappa}\right) - d\right]}{u-d} \tag{7.23}$$

In the above expression, $\kappa/(1+\kappa)$ represents the constant asset (dividend like) payout yield for a perpetual project (or firm)[80]. If we extend this single period binomial model and subdivide the time to expiration of the growth option, T, into n equal subintervals, each of length $t = T/n$, the general binomial pricing formula can be represented as follows:

$$G = \frac{\sum_{j=0}^{n}\left\{\frac{n!}{j!(n-j!)}\right\}p^j(1-p)^{n-j}Max\left(u^j d^{n-j} gV - I, 0\right)}{(1+r)^n} \tag{7.24}$$

This expression adds the probability that the firm will take j upward jumps in n steps, each with risk neutral probability p. These jumps are in accordance with the evolution of the demand parameter considered in the demand function.

It is now time to include the valuation methodology for the abandonment option. This type of option is typically assumed as a put option on the assets of the firm for the salvage value specified. The payoff at expiration date, for a put option with these characteristics can be represented by:

$$A = Max(X - V, 0) \tag{7.25}$$

[80] This assumption is also in line with Smit and Trigeorgis (2004).

In the expressions above, X represent the salvage value at which the firm can be abandoned, A represents option value and V represents the value of the firm.

With the terminal value for the put option on firm value above derived, the value of such option at a particular time is obtained by the general binomial valuation model for a put option:

$$P_{t-1} = \frac{\left[puP_t + (1-p)dP_t\right]}{(1+r)} \tag{7.26}$$

If we again extend this single period binomial model and subdivide the time to expiration of the growth option, T, into n equal subintervals, each of length $t = T/n$, the general binomial pricing formula can be represented as follows:

$$A = \frac{\sum_{j=0}^{n}\left\{\frac{n!}{j!(n-j!)}\right\}p^{j}(1-p)^{n-j}\,Max\left(X - u^{j}d^{\,n-j}V, 0\right)}{(1+r)^{n}} \tag{7.27}$$

This expression also adds the probability that the firm will take j upward jumps in n steps, each with risk neutral probability p. Once again, such probability is in accordance with the demand parameter considered initially.

With the two managerial possibilities present, it is now the time to develop the value of the firm after such flexibility is incorporated in the firm values derived. It results from the value of the firm obtained in accordance to the market equilibrium achieved, without consideration of flexibility, with the addition of these two managerial possibilities the firm possesses. At the maturity date of the options, the value of the firm with the addition of the growth option is given by:

$$V^{G} = V + Max\left[(g-1)V - I, 0\right] \tag{7.28}$$

This is equal to:

$$V^{G} = Max\left[gV - I, V\right] \tag{7.29}$$

At the maturity date of the options, the value of the firm with the addition of the abandonment option is given by:

$$V^{A} = V + Max(X - V, 0) \tag{7.30}$$

This is equal to:

$$V^{A} = Max(X, V) \tag{7.31}$$

Therefore, the value of the firm results from the initial value of the firm and these two managerial possibilities the firm possesses. At the maturity date of the options, it is given by:

$$V^{GA} = Max(V, gV - I, X) \tag{7.32}$$

By substituting firm value as in the different market equilibriums considered, we obtain the value of the firm under Cournot-Nash as:

$$V^{C} = Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau) - I, X\right) \tag{7.33}$$

Whereas the value of the leader firm under Stackelberg equilibrium is:

$$V_l^{S} = Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau) - I, X\right) \tag{7.34}$$

And the value for the follower as:

$$V_f^{S} = Max\left(\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau), g\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau) - I, X\right) \tag{7.35}$$

Under monopolistic equilibrium, the value of the firm becomes:

$$V^{M} = Max\left(\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau), g\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau) - I, X\right) \qquad (7.36)$$

The value of the firm with the managerial flexibility present at a date prior to the expiration date of the options considered follows a similar path to the one described for the value of the options when considered in isolation. Therefore, it can be computed as follows:

$$V^{GA} = \frac{\sum_{j=0}^{n}\left\{\frac{n!}{j!(n-j!)}\right\}p^{j}(1-p)^{n-j}Max\left(u^{j}d^{n-j}V, u^{j}d^{n-j}V + gu^{j}d^{n-j}V - I, A\right)}{(1+r)^{n}} \qquad (7.37)$$

After this computation, it is necessary to incorporate the agency conflicts that result from the additional equity issue necessary in order to exercise the growth option. Under the consideration that the firm is financed by both equity and debt, the total current market value of the firm, *V*, is the sum of the market value of the two securities. Therefore,

$$V = E + D \qquad (7.38)$$

Where *E* represents the market value of equity and *D* represents the market value of debt. Under this scenario, equity can be seen as a call option on the assets of the firm. The exercise value of such call option is the value of outstanding debt. The maturity of this option is the maturity of the debt. Under the present setting, such maturity date is the same as the maturity date of the options considered. Therefore, the general value for the equity of the firm at debt maturity can be represented as follows:

$$E_T = Max(V_T - F, 0) \qquad (7.39)$$

In this expression, *F* represents the face value of debt outstanding while E_T and V_T represent the equity and firm value at the maturity date of debt. Being a call option on the assets of the firm, the value of equity at any date before the maturity date of the debt contract, can be estimated as:

$$E = \frac{\sum_{j=0}^{n} \left\{ \frac{n!}{j!(n-j!)} \right\} p^j (1-p)^{n-j} Max\left(u^j d^{n-j} V - F, 0\right)}{(1+r)^n} \qquad (7.40)$$

Furthermore, the value of debt can be obtained by deducting to the value of the firm, the value of the equity. Alternatively, it can be computed as the difference between the value of the firm and the value of a call option on the assets of the firm with an exercise price equal to the face value of debt outstanding. Therefore, it can be given by:

$$D_T = V_T - Max(V_T - F, 0) \qquad (7.41)$$

The end result of this perspective is:

$$D_T = \min(V_T, F) \qquad (7.42)$$

The present value of this terminal value is obtained by the following expression:

$$D = \frac{\sum_{j=0}^{n} \left\{ \frac{n!}{j!(n-j!)} \right\} p^j (1-p)^{n-j} \min\left(u^j d^{n-j} V, F\right)}{(1+r)^n} \qquad (7.43)$$

This is the general model that will be used to obtain the market value of the firm as well as the market value of equity and debt. However, with the inclusion of managerial flexibility and consideration of debt financing, we get additional results for equity and debt value under the different market equilibriums considered.

In the presence of Cournot-Nash equilibrium without managerial flexibility, and applying (7.12) into (7.39), we get the expression for equity value as:

$$E^C = Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau) - F, 0 \right) \qquad (7.44)$$

And applying (7.12) into (7.42), we get the expression for debt value as:

$$D^{C} = \min\left(\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau), F\right) \tag{7.45}$$

In the presence of Stackelberg equilibrium without managerial flexibility, and applying (7.15) into (7.39) and (7.42), we get the expressions for equity and debt value for the leader as:

$$E_l^{S} = Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau) - F, 0\right) \tag{7.46}$$

$$D_l^{S} = \min\left(\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau), F\right) \tag{7.47}$$

Similarly, but applying (7.17) into (7.39) and (7.42) we obtain the equity and debt value for the follower as:

$$E_f^{S} = Max\left(\frac{(\theta_t - 3c_j + c_i)^2}{16\kappa}(1-\tau) - F, 0\right) \tag{7.48}$$

$$D_f^{S} = \min\left(\frac{(\theta_t - 2c_j + c_i)^2}{16\kappa}(1-\tau), F\right) \tag{7.49}$$

Finally, in the presence of monopoly equilibrium without managerial flexibility, and applying (7.20) into (7.39) and (7.42), we get the expressions for equity and debt value for the monopolist firm as:

$$E^{M} = Max\left(\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau) - F, 0\right) \tag{7.50}$$

$$D^M = \min\left(\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau), F\right) \tag{7.51}$$

When in the presence of managerial flexibility, the expressions for the value of the equity and for the value of the debt in Cournot-Nash equilibrium can be obtained by substituting (7.33) into (7.39) and (7.42):

$$E^C = Max\left[Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau) - I, X\right) - F, 0\right] \tag{7.52}$$

$$D^C = \min\left[Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{9\kappa}(1-\tau) - I, X\right), F\right] \tag{7.53}$$

Similarly, when in the presence of managerial flexibility, the equity and debt value of the leader firm can be obtained by substituting (7.34) into (7.39) and (7.42):

$$E_l^S = Max\left[Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau) - I, X\right) - F, 0\right] \tag{7.54}$$

$$D_l^S = \min\left[Max\left(\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau), g\frac{(\theta_t - 2c_i + c_j)^2}{8\kappa}(1-\tau) - I, X\right), F\right] \tag{7.55}$$

Whereas, the value of the follower can be derived by substituting (7.35) into (7.39) and (7.42):

$$E_l^S = Max\left[Max\left(\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau), g\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau) - I, X\right) - F, 0\right] \tag{7.56}$$

$$D_l^S = \min\left[Max\left(\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau), g\frac{(\theta_t - 3c_j + 2c_i)^2}{16\kappa}(1-\tau) - I, X\right), F\right] \tag{7.57}$$

Finally, the value for the monopolist firm is obtained by substituting (7.36) into (7.39) and (7.42):

$$E^M = Max\left[Max\left(\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau), g\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau) - I, X \right) - F, 0 \right] \tag{7.58}$$

$$D^M = \min\left[Max\left(\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau), g\frac{(\theta_t - c_i)^2}{4\kappa}(1-\tau) - I, X \right), F \right] \tag{7.59}$$

This set of expressions defines the model implemented in the present research. Additionally, the insight that determines the agency cost of debt is that two different policies shall be considered in order to exercise the growth option. The first-best policy considers that such option will be exercised in order to maximize the value of the firm. The second-best policy considers that such option will be exercised in order to maximize the value of the equity of the firm. The difference in firm value that results from these two different policies is the agency cost of debt.

For the first-best policy, the expressions are already derived, since they correspond to the maximization of firm value. Therefore, they are in accordance with the expressions derived for firm value. However, for the second-best policy, it must be noted that the expressions that were derived for the value of the equity of the firm do not correspond to the decision to be taken by firms that execute the second-best policy. The expression derived for the equity value corresponds to the value of equity that results from the maximization of firm value. It is not the maximization of equity value. Therefore, an expression for the maximization of equity value must be derived. The exercise of the option maximizes the equity value if firm value after exercise minus the value of debt outstanding and the value of the additional equity issue is higher than the value of the firm without exercise of the option minus the value of the debt outstanding. Under this premise, equity guarantees that the additional equity issue is not appropriated by the original debt. Therefore, the wealth transfer does not occur. Therefore, firm value under the second best policy can be derived from:

$$V_T = \begin{cases} gV - I & , gV - 2I - F \geq V - F \\ V & , gV - 2I - F < V - F \end{cases} \tag{7.60}$$

With this policy, derivation of firm value in the different market equilibriums reached is obtained by the straightforward procedure of substituting the expressions for firm value in (7.60). The equity and debt values are also obtained by the incorporation of the firm value obtained in the expressions previously derived for the equity and debt value. With the model fully described it is now the time to implement it. In the next section we shall perform a numerical simulation of the model constructed in order to analyse the results achieved with it.

7.2 Implementation and results

The implementation shall be made with a numerical analysis performed through a simulation of a set of parameters. We shall start by the definition of all the necessary parameters to perform such simulation, after that we shall proceed to the numerical analysis ending with the main conclusions to be withdrawn from the analysis made.

7.2.1. Parameters

The numerical analysis to be performed assumes a duopolistic market where both firms share a growth option to expand the scale of operations and an abandonment option. Both firms have equal operating costs and it is further assumed, initially, that both firms will have identical financing structures and both firms will finance the growth option in equal form (through an additional equity issue). The necessary parameters in order to implement the model are the following:

Table 7.1: Model parameters

θ_0	17,50		C_A	5,00
u	1,25		C_B	5,00
d	0,80		F	50,00
r_f	2,000%		X	0,00
κ	8,000%		g	3,00
τ	25,000%		I	50,00

Source: Author

This set of values is an adaptation of the set of values that were present in Smit and Trigeorgis (2004). Furthermore, some additional values were adjusted to the present market and legal conditions. With this set of values, we shall proceed to the valuation of the firm under the two different equilibrium perspectives, Cournot-Nash and Stackelberg.

7.2.2. Cournot-Nash equilibrium

Under the scenario without flexibility we get the following values for the value of the firm, its equity and its debt:

Table 7.2: Base case results under Cournot-Nash equilibrium

Va	118,49		*Vb*	118,49
Ea	84,67		*Eb*	84,67
Da	33,82		*Db*	33,82

Source: Author

With the inclusion of flexibility under the first-best policy, the values for the firm, its equity and its debt are naturally higher due to the existence, and exercise of the operational flexibility. They are presented below:

Table 7.3: First best policy results under Cournot-Nash equilibrium

Va	317,03		*Vb*	317,03
Ea	279,52		*Eb*	279,52
Da	37,51		*Db*	37,51

Source: Author

It must be referred that with the adoption of the growth option, the value of the debt increases significantly, 10.89%. The wealth transfer effect mentioned in the literature is also present in this formulation. The growth option is financed by an additional equity issue, but part of the benefits from such additional equity issue is transferred to the debtholders of the firm.

In order to prevent such transfer of value, management adopts a second-best strategy. Under this strategy, the adoption of the growth option is determined by the maximization of the equity value of the firm and not by the maximization of firm value. The results achieved under this strategy are presented below:

Table 7.4: Second best policy results under Cournot-Nash equilibrium

Va	305,91		*Vb*	305,91
Ea	272,08		*Eb*	272,08
Da	33,82		*Db*	33,82
Ea (additional)	18,49		*Eb (additional)*	18,49

Source: Author

It is clear from the above table that the value of the firms diminishes compared to the first-best policy firm value. The decrease in value is of 3.51% when compared to firm value with the adoption of the first-best policy. This decrease represents the agency cost of debt as a result of an underinvestment in the growth option, consequence of the change in the strategy adopted to exercise it. The value of the debt decreases to the initial debt value (in the scenario without flexibility), becoming this way clear that debt is not benefited from the additional equity issue. The value of the equity also diminishes, when compared to the one obtained under the first-best policy. However, the "savings" in the additional equity issue possess a present value of 18.49. This reduction in the equity issue necessary to exercise the growth option is an increase in the value to the equity (in a global perspective, including the initial equity and the additional equity issue) that largely compensates the loss originated from the reduction in the exercise of the growth option under the second best policy. The computation of this additional value is necessary in order to establish the difference between the two policies considered. In fact, by adopting the second-best policy, the firms exercise the growth option in a reduced number of branches of the demand tree. Therefore, the comparison between firm values in the two policies considered needs to include the additional investment in which the firms incur by adopting the first-best policy in comparison to the adoption of the alternative policy.

The results obtained under Cournot-Nash clearly show that under the premises assumed, the agency cost exists and is significant. It leads to a reduction in the value of the firm as a consequence of the underinvestment situation caused by the adoption of a second-best policy. These results, under this equilibrium perspective are identical to both firms, since they equally share the market.

7.2.3. Stackelberg equilibrium

In this scenario, without flexibility, we get the following values for the value of the firm, its equity and its debt:

Table 7.5: Base case results under Stackelberg equilibrium

Va	133,30		*Vb*	66,65
Ea	97,51		*Eb*	58,32
Da	35,79		*Db*	26,93

Source: Author

The firm value of the leader firm is naturally higher than that of the follower. Therefore, the value of the equity and the value of the debt are also higher. The follower possesses the same original debt outstanding as the leader, but with the smaller firm values, the market value of the debt is smaller, since the probability of default is higher for the follower than it is for the leader.

With the inclusion of flexibility under the first-best policy, the values for the firm, its equity and its debt are higher due to the exercise of the options that firms possess. They are presented below:

Table 7.6: First best policy results under Stackelberg equilibrium

Va	361,23		*Vb*	164,16
Ea	323,61		*Eb*	137,23
Da	37,62		*Db*	26,93

Source: Author

In this perspective, the value of the debt also increases but only for the leader. The same conclusions drawn under Cournot-Nash apply for the Stackelberg leader.

However, the magnitude of the wealth transfer is smaller than the one obtained previously. The leader's debt value increases 5.12%. The increase in debt value is smaller than before for one reason. Because the value of the leader firm is higher when compared to the Cournot firm, debt is less risky. This happens also in the initial valuation performed, without consideration of managerial flexibility. Therefore, the increase in debt value for the Stackelberg leader is smaller than that of the Cournot firm. Nonetheless, the value of debt for the Stackelberg leader is higher than the value of the debt for the Cournot firm (despite being almost identical). However, the main conclusion to be drawn is that for the leader, the wealth transfer effect also exists.

For the follower, the value of the debt remains identical to the no flexibility set[81]. Therefore, the wealth transfer effect does not occur under this parameter setting. In fact, the reduction in the quantity to be produced by the follower under this equilibrium perspective leads to a situation of deferred investment that avoids the wealth transfer effect from equity to debt. Therefore, the values obtained under the first- and second-best policies for the follower should be identical. That can be verified in the table below that presents the values obtained under the second-best policy:

Table 7.7: Second best policy results under Stackelberg equilibrium

Va	346,40		*Vb*	164,16
Ea	310,61		*Eb*	137,23
Da	35,79		*Db*	26,93
Ea (additional)	18,49		*Eb (additional)*	0,00

Source: Author

It can be seen that the values obtained for the follower, under this policy, are identical to the ones obtained with the first-best policy. Therefore, the agency cost is null for the follower, thus leading to an additional equity issue equal to zero. However, for the leader, the same situation as the one described under Cournot-Nash equilibrium exists. The value of the firm diminishes under this exercise policy in 4.10%, a reduction that is even higher than the one obtained in Cournot. The value of the debt is also

[81] This situation occurs under the premises considered in this framework. However, the extension of the binomial demand tree for more time periods should lead to an increase in the value of debt. In fact, the convergence of the binomial process to a continuous-time model, achieved through the expansion of the binomial model in more time periods, should make this situation clear.

reduced to the one obtained in the no flexibility scenario. Therefore, debt does not appropriate any part of the additional equity issue, which implies that the wealth transfer is avoided by management. The equity value also decreases, but if one adds the value of the additional equity issue, it is clear that the overall equity value is higher under this policy than it is under the previous one. Therefore, for the leader a similar situation to the one observed in Cournot obtains, whereas for the follower, the impact of the agency cots can be disregarded since they are inexistent.

7.2.4. General equilibrium

By general equilibrium it shall be analysed a combined situation between the equilibrium alternatives studied earlier. In each node of the tree four different alternative possibilities shall be considered. When both firms simultaneously invest a Cournot-Nash outcome is obtained, when one firm invests as a leader and the other subsequently invests as a follower a Stackelberg equilibrium occurs, and finally when both firms choose not to invest a Cournot-Nash equilibrium occurs again, but now without the investment in the growth option. Additionally, the possibility to abandon operations is also present. If one firm decides to abandon operations the remaining firm gains monopoly profits, and if both firms decide to abandon they will both be valued zero (the value of the abandonment possibility). The selection between these alternative possibilities is performed though a game theoretic perspective. Furthermore, all these four situations are also analysed under different policies. Three different sets of policies are combined. Initially, both firms adopt a first-best policy, then both firms adopt a second-best policy, and finally one of the firms adopts a first-best policy and the other adopts a second-best policy. In this latter scenario, it shall be considered that one firm is financed solely by equity (the firm that adopts the first-best policy) and the other firm is financed by both equity and debt. The exercise of the growth option is always made through an additional equity issue.

With all the different scenarios explained, we get the following values under a first best policy:

Table 7.8: First best policy results under general equilibrium

Va	317,03		*Vb*	317,03
Ea	279,52		*Eb*	279,52
Da	37,51		*Db*	37,51

Source: Author

Under this policy we get identical values for both firms. Since both firms are identical and follow identical policies, the overall outcome is the one previously obtained in Cournot equilibrium. Therefore, the values achieved are the ones already present in table 7.3 above.

If they both adopt the second best policy, the results achieved are presented below:

Table 7.9: Second best policy results under general equilibrium

Va	305,91		*Vb*	305,91
Ea	272,08		*Eb*	272,08
Da	33,82		*Db*	33,82
Ea (additional)	18,49		*Eb (additional)*	18,49

Source: Author

Once again, the results depict the results achieved under the second best policy in Cournot-Nash equilibrium. Therefore, the conclusions to be drawn replicate the conclusions that result from table 7.4 above.

The final scenario considered departs from the fact that one firm adopts a first-best strategy, the firm considered to be only equity financed, and the other adopts a second-best policy, the firm that is financed by both equity and debt. The results are shown in the table below:

Table 7.10: Mixed policies results under general equilibrium

Va	322,58		*Vb*	299,43
Ea	322,58		*Eb*	272,08
Da	0,00		*Db*	27,35
Ea (additional)	0,00		*Eb (additional)*	18,49

Source: Author

The results show that firm A, the one that is only equity financed, exercises the first-best policy, due to the inexistence of agency costs of debt, and that such policy results in a higher firm value than in the previous settings. The increase in firm value compared to the situation where both firms adopt the first-best policy is of 1.75%, and when compared to the setting where both firms adopt the second-best strategy is of 5.45%. This occurs because of the policy adopted by firm B. Being financed by both equity and debt, the firm adopts a second-best policy, thus avoiding the wealth transfer effect from the equity issue to the outstanding debt. This policy results in a reduction in firm value due to the underinvestment situation. The value of firm B suffers a reduction of 5.55% when compared to the value obtained under the first-best policy and of 2.12% when compared to the value obtained under the second-best policy. This reduction in firm value, compared to the first-best policy, is the reduction of higher magnitude observed. This reduction occurs for two different reasons. The first is similar to the reduction earlier observed that concerns the change in policy adopted, from first-best to second-best and the consequent underinvestment generated. The second relates to the maintenance of the first-best policy by firm A and the added value that it generates to firm A at the expenses of firm B. They no longer share the market equally and therefore firm B suffers a reduction in value to this fact. Therefore, under the simulation performed, the difference in the financing structure of the firms leads to a loss in firm value of the firm financed by both equity and debt and to an increase in firm value of the only equity financed firm.

The value of debt also suffers a reduction with the adoption of the second-best policy. This loss in value is entirely related to the loss in firm value and to the adoption of a strategic policy directed at not benefiting debt at the expenses of an additional equity issue. The equity value of firm B is equal to the equity value observed for the when both firms adopt a second-best policy (being this the only market value that does not suffer a reduction). The value of the equity under the second-best policy is higher than the one observed under the first-best policy if the additional equity issue is included. However, as it was already referred to previously, this additional equity issue (avoided under the second-best policy) must be included in the analysis.

Therefore, a change in the equilibrium outcome is produced if the firms posses a different capital structure at the outset. The firm that is only equity financed exercises the option in a wider range of tree branches. The firm that is equity and debt financed at the outset and finances the additional capital expenditure with an equity issue, exercises the growth option in a reduced range of tree branches. The equity financed firm gains value while the equity and debt financed firm suffers a loss in value. The underinvestment situation, as in Mauer and Ott (2000) is also present and is exacerbated under competition.

7.3. Final thoughts on results obtained

The analysis conducted in this chapter demonstrated the validity of the model developed and allowed the measurement of the effects of differences in the financing structure of the firms in the exercise policy of growth options and in firm value. It was demonstrated, under the simulation performed, that under Cournot-Nash and Stackelberg equilibrium the agency cost of debt, by the adoption of a second-best policy that maximizes the value of the equity of the firm, originates an underinvestment situation that causes a decrease in firm value. It was also demonstrated that under a general equilibrium perspective (that allows the establishment of different equilibrium outcomes) such incentive to underinvest exists, thus the reduction in firm value occurs for the equity and debt financed firm, while the only equity financed firm increases in value (since there is no incentive do adopt a second-best policy).

In the next and final chapter, a general conclusion of the research conducted is performed.

8. CONCLUSIONS

The economic environment that firms face is in constant transformation. At the present, such economic environment is characterized by higher operational (managerial) flexibility, competition and lack of liquidity in financial markets. This is the result of an increased globalization of the economy and of the crisis that affected financial markets. These events affect managerial decisions, particularly the ones related to capital budgeting. Therefore, a research conducted for the analysis of capital budgeting decisions under these new economic setting is extremely relevant in order to develop the current literature on the subject and to improve professional practice. This is the first conclusion to be withdrawn from the present dissertation. With the integration of three different theories into a unified perspective we aimed at an enhancement in the knowledge related to capital budgeting under competition and managerial flexibility, in the presence of agency conflicts between equity and debt. Such enhancement contributes to improved managerial decisions and therefore to added value in corporations.

In order to achieve it, we departed from a general approach to the problem through an analysis conducted on the fundamental literature on the subjects. A review of capital budgeting procedures, and the inclusion of operational flexibility in those procedures through the incorporation of option valuation models, was made right at the start. Later, a review on the relevance of agency conflicts between equity and debt in the analysis of investment opportunities and firm value was also performed, demonstrating the interaction between the investment and the financing decisions whenever such conflicts are present. Afterwards, a study on the impact of competition in investment decisions, trough the examination of real options games, was also performed. In fact, the model implemented departs from this different analysis. The aim is to analyse if these combined effects portray an influence in managerial decisions concerning investment opportunities and what are the costs associated with such influence. The methodology conducted to develop such model was also presented, describing the assumptions under which it was constructed. Later still, a numerical solution was implemented through a simulation set that enabled the extraction of results for analysis.

In this final chapter the conclusions to be withdrawn from the analysis are made. In order to clarify the main results achieved, these are separated between reflections about the model itself and considerations about the outcomes of the numerical simulation implemented. Finally, remarks regarding future possibilities of research will be presented.

The model developed departed from two previous works, one that integrated agency conflicts with ROA (Mauer and Ott, 2000), and another one that integrated ROA and competition (Smit and Trigeorgis, 2004). The first was in continuous-time and the second in discrete-time. Despite the widespread use of continuous-time models, we adopted the discrete-time perspective. This decision was based in the higher practical application of this type of models. In fact, the literature refers that one factor that might pose a barrier to the widespread use of real options models in corporation is the complexity continuous-time models possess. Therefore, the adoption of a discrete-time model can overcome such difficulty. On the other hand, models that analyse the interaction between investment and financing decisions tend to consider debt as "perpetual", in the sense that the rollover of the initial debt issue is considered feasible, and continuous-time models are best suited for such assumption. The model developed in the present dissertation does not consider so. In fact, due to the current situation of financial markets we opted to assume that the initial debt issue must be repaid at its maturity date. This assumption reinforced the possibility to develop a discrete-time model, since under these models it is necessary to establish a maturity for the options present. Therefore, the analysis developed gains in tractably, is best suited for adoption in real life practice and is more in accordance to actual conditions in financial markets.

The equilibrium conditions largely rely on the model developed by Smit and Trigeorgis (2004). In this particular aspect almost no particular innovative aspect is present in this dissertation. Nonetheless, the equilibrium defined is in line with Cournot and Stackelberg conditions and is, therefore, well established in the literature. However, in Smit and Trigeorgis (2004), such equilibrium conditions are not defined by the model itself but are exogenous to the model. In fact, the determination of Cournot or Stackelberg equilibrium results from an exogenous consideration of advantage of one firm over the other (or simply by reference to simultaneous investment decision or sequential investment decision) and does not result from endogenous conditions of each firm. In the model developed we follow a similar procedure but with differences in the financing structure of the firms, the equilibrium conditions change. Therefore, the equilibrium reached in the market is in part endogenous.

The simulation performed intended to define a set of conditions that enables the achievement of results from implementation of the model. That simulation allowed us to understand better the managerial decisions taken under the set of conditions defined. In fact, concerning a competitive market with a shared growth option in which firms face agency conflicts between equity and debt, the model attempted to illustrate the decisions that firms should take. The results, for the numerical simulations developed, are conclusive.

It became clear that under Cournot-Nash equilibrium an incentive to underinvest exists whenever the firm is financed by both equity and debt, and the growth option is financed solely by equity. A wealth transfer occurs between equity and debt. The solution to avoid such wealth transfer is to delay investment in the growth option, which generates a reduction in firm value. This reduction in firm value is the agency cost of debt.

It became also demonstrated that under Stackelberg equilibrium an incentive to underinvest exists in the leader firm. The reason for such underinvestment is the same as in the Cournot-Nash equilibrium. The reaction of the firms is identical and therefore, the effect is similar. However, and under the numerical simulation conducted, such effect is not present in the follower. Therefore, for the follower there is no agency cost and there is no difference between the first and second best exercise policy, which is an unexpected result.

Finally, it was also demonstrated that under the consideration of the different possibilities of equilibrium between firms, the effects mentioned above still prevail. In fact, when both firms are identical in operating and financing structure the Cournot-Nash solution is the one that prevails. The effects studied for this type of equilibrium are the ones that occur when the model allows for other types of equilibria. Whenever a difference exists between the financing structures of the firms, a change occurs in the equilibrium reached. Under some market demand evolution Cournot-Nash equilibrium occurs while in others, Stackelberg equilibrium occurs. The firm that is only equity financed adopts the only possible policy, a first best policy, while the firm that is equity and debt financed adopts a second best policy and suffers a reduction in firm value. The end result is that the agency costs still produce a negative effect in firm value in the presence of competition when firms possess managerial flexibility.

The path followed opens a different perspective of research in this field. In fact, the model constructed can easily be reformulated to incorporate other possibilities besides the ones considered. Namely, the possibility of consideration of other forms of financing is open wide. In fact, consideration of alternative financing structures for the firms or for the exercise of the growth option is a logical and natural step. We replicated the financing conditions present in Mauer and Ott (2000). A replication of the financing conditions present in Mauer and Sarkar (2005) is a useful step that will generate relevant conclusions for this field of research. It can also be analysed the use of instruments of debt that could mitigate the agency conflicts present, namely callable or convertible debt instruments.

Alternatively, other forms of equilibrium could also be analysed. In fact, the adoption of the Cournot and Stackelberg equilibrium conditions was justified with the adoption of the assumptions present in Smit and Trigeorgis (2004). Nonetheless, other possibilities do exist that could be studied. A Bertrand price competition is a logical development, also in line with the developments in Smit and Trigeorgis (2004).

Empirical testing of the present findings is another path that can be followed for the future. In fact, the testing of the present model could be made by its verification on oligopolistic sectors where innovation is present. Under ROA, empirical analysis is not yet very widespread. However, the validation of theoretical findings has a lot to gain with its empirical confirmation. The theoretical findings reached in this research are no exception.

The present research allowed the determination of the equilibrium conditions that might be present in competitive markets with shared growth options and abandonment options under agency conflicts between equity and debt. The comprehension of the factors that affect managerial decisions under these conditions is far from being reached, even because other aspects besides the ones here analysed interfere with those decisions. However, the breakthrough achieved in this research is one more step in the knowledge of those decisions.

Bibliography

Aggarwal, R. (1991), Justifying Investments in Flexible Manufacturing Technology, *Managerial Finance, 17*, 77-88.

Aguerrevere, F. (2003), Equilibrium Investment Strategies and Output Price Behaviour: A Real-options Approach, *Review of Financial Studies, 16*, 1239-1272.

Akerlof, G. (1970), The Market for Lemons: Quality Uncertainty and the Market Mechanism, *Quarterly Journal of Economics, 84*, 488-500.

Alchian, A., & Demsetz, H. (1972), Production, Information Costs, and Economic Organization, *American Economic Review, 5*, 777-795.

Amran, M., & Kulatilaka, N. (1999), *Real Options*, Boston, MA: Harvard Business School Press.

Arrow, K. (1964), Control in Large Organizations, *Management Science, 10*, 397-408.

Arrow, K, & Debreu, G. (1954), Existence of an Equilibrium for a Competitive Economy, *Econometrica, 22*, 265-290.

Aumann, R. (1959), Acceptable points in General Cooperative n-person Games, in R. D. Luce and A. W. Tucker (Eds.), *Contributions to the Theory of Games IV - Annals of Mathematics Study, Vol. 40*, Princeton, NJ: Princeton University Press.

Aumann, R. (1974), Subjectivity and Correlation in Randomized Strategies, *Journal of Mathematical Economics, 1*, 67-96.

Barclay, M., Morellec, E., & Smith, C. (2006), On the Debt Capacity of Growth Options, *The Journal of Business, 79*, 37-60.

Barclay, M., Smith, C., & Watts, R. (1995), The Determinants of Corporate Leverage and Dividend Policies. *Journal of Applied Corporate Finance, 7*, 4-19.

Barnea, A., Haugen, R., & Senbet, L. (1980), A Rationale for Debt Maturity Structure and Call Provisions in the Agency Theoretic Framework, *The Journal of Finance, 35*, 1223-1234.

Barnea, A., Haugen, R., & Senbet, L. (1985), *Agency Problems and Financial Contracting*, Englewood Cliffs, NJ: Prentice-Hall.

Basar, T., & Olsder, G. J. (1982), *Dynamic Non-Cooperative Game Theory*, New York, NY: New York Academic Press.

Baxter, N. D. (1967), Leverage, Risk of Ruin and the Cost of Capital, *The Journal of Finance, 22*, 395-403.

Bergin, J. (1992), A Model of Strategic Behaviour in Repeated Games, *Journal of Mathematical Economics, 21*, 113-153.

Bergin, J., & MacLeod, B. (1993), Continuous Time Repeated Games, *International Economic Review, 34*, 21-37.

Bertrand, J. (1883), Théorie Mathématique de la Richesse Sociale, *Journal des Savants, 67*, 499-508.

Black, F. (1972), Capital Market Equilibrium with Restricted Borrowing, *Journal of Business, 45*, 444–54.

Black, F., & Scholes, M. (1973), The Pricing of Options and Corporate Liabilities, *Journal of Political Economy, 81,* 637-659.

Bouis, R., Huisman, K., & Kort, P. (2005), Strategic Real Options: Three Firms, *Presented at the 9th Annual International Conference on Real Options,* Paris, France: Paris Dauphine University.

Bouis, R., Huisman, K. & Kort, P. (2009), Investment in Oligopoly under Uncertainty: The Accordion Effect, *International Journal of Industrial Organization, 27,* 320-331.

Bowman, E. H., & Moskowitz, G. T. (2001), Real Options Analysis and Strategic Decision Making, *Organization Science, 12,* 772-777.

Boyer, M., Gravel, E., & Lasserre, P. (2004a), Real Options and Strategic Competition: A Survey, *Working Paper, Economics and CIRANO*, University de Montreal.

Boyer, M., Lasserre, P., Mariotti, T. & Moreaux, M. (2004b), Preemption and Rent Dissipation under Price Competition, *International Journal of Industrial Organization*, *22*, 309-328.

Boyer, M., Lasserre, P., Mariotti, T. & Moreaux, M. (2001), Real Options, Preemption, and Dynamics of Industry Investments, *Department des Sciences Economiques Working Paper*, Université du Quebec à Montréal.

Boyer, M., Lasserre, P. & Moreaux, M. (2007), The Dynamics of Industry Investments, Working Paper, University of Montreal.

Brach, M. (2003), *Real Options in Practice,* Hoboken, NJ: John Wiley & Sons.

Bradley, M., Jarell, G., & Kim, E. (1984), On the Existence of an Optimal Capital Structure: Theory and Evidence, *The Journal of Finance, 39,* 857-878.

Brennan, M. J. & Schwartz, E.S. (1978), Corporate Income Taxes, Valuation and the Problem of Optimal Capital Structure, *Journal of Business, 51-1*, 103-114.

Brennan, M. J., & Schwartz, E. S. (1984), Optimal Financial Policy and Firm Valuation, *The Journal of Finance, 39,* 593-607.

Brennan, M. J., & Schwartz, E. S. (1985), Evaluating Natural Resources Investments, *Journal of Business, 58*, 135-157.

Brennan, M. J., Trigeorgis, L. (Eds.) (2000), *Project Flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options*, New York, NY: Oxford University Press.

Chevalier-Roignant, B., Flath, C., Huchzermeier, A, & Trigeorgis, L. (2011), Strategic Investment under Uncertainty: A Synthesis, *European Journal of Operational Research, 215*, 639-650.

Chevalier-Roignant, B., & Trigeorgis, L. (2011), *Competitive Strategy: Options and Games*, Cambridge, MA: The MIT Press.

Childs, P., Mauer, D., & Ott, S. (2005), Interactions of Corporate Financing and Investment Decisions: The Effect of Growth Options to Replace or Expand, *Journal of Financial Economics, 76*, 667-690.

Copeland, T., & Antikarov, V. (2001), *Real Options: A Practitioner's Guide*, New York, NY: Texere Publishing.

Copeland, T., & Weston, J. F. (1988), *Financial Theory and Corporate Policy*, Reading, MA: Addison Wesley.

Cournot, A. (1838), *Recherches sur les Principes Mathémathiques de la Théorie des Richesses*, Paris: M. Riviere.

Cox, J. C., Ingersoll, J. E. & Ross, S. A. (1985), A Theory of the Term Structure of Interest Rates, *Econometrica, 53*, 385-407.

Cox, J. C. & Ross, S. A. (1976), The Valuation of Options for Alternative Stochastic processes, *Journal of Financial Economics, 3*, 145-166.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979), Option Pricing: A Simplified Approach, *Journal of Financial Economics, 7*, 229-263

Dasgupta, P., & Maskin, E. (1986a), The Existence of Equilibrium in Discontinuous Economic Games, I: Theory, *Review of Economic Studies, 53*, 1-26.

Dasgupta, P., & Maskin, E. (1986b), The Existence of Equilibrium in Discontinuous Economic Games, II: Applications, *Review of Economic Studies, 53*, 27-41.

Dasgupta, P., & Stiglitz, J. (1980), *Industrial* Structure and the Nature of Innovative Activity, *Economic Journal, 90*, 266-293.

Dasgupta, P., & Stiglitz, J. (1981), Depletion under Technology Uncertainty, *Econometrica, 49*, 85-104.

De Angelo, H., & Masulis, R. (1980), Optimal Capital Structure under Corporate and Personal Taxation, *Journal of Financial Economics, 8*, 3-29.

Debrew, G. (1959), *Theory of Value*, New York: NY, John Wiley & Sons.

Décamps, J., Mariotti, T., & Villeneuve, S. (2002), Investment Timing under Incomplete Information, *Working Paper*, Université de Touluse.

Dias, M. & Teixeira, J. (2003), Continuous-time Option Games: Review of Models and Extensions, Part 1: Duopoly under Uncertainty, *Presented at the 7th Annual International Conference on Real Options*, Washington DC: USA, Georgetown University.

Dias, M., & Teixeira, J. (2010), Continuous-time Option Games: Review of Models and Extensions, *Multinational Finance Journal, 14*, 219-254.

Dixit, A. (1989), Entry and Exit Decisions under Uncertainty, *The Journal of Political Economy, 97*, 620-638.

Dixit, A., & Nalebuff, B. (1991), *Thinking Strategically: the Competitive Edge in Business, Politics, and Everyday Life*, New York, NY: Norton.

Dixit, A., & Pindyck, R. (1994), *Investment under Uncertainty*, Princeton, NY: Princeton University Press.

Donaldson, G. (1963), Financial Goals: Managers versus Stockholders, *Harvard Business Review, 41*, 116-129.

Dotan, A., & Ravid, S. A. (1985), On the Interaction of Real and Financial Decisions of the Firm under Uncertainty, *The Journal of Finance, 40*, 501-517.

Dutta, P., & Rustichini, A. (1995), Equilibria in Stochastic Games, *Journal of Economic Theory, 67*, 1-39.

Edgeworth, F. (1925), *Papers Relating to Political Economy, I-III*, London: Macmillan.

Ericsson, J. (2000), Asset Substitution, Debt Pricing, Optimal Leverage and Maturity, *Journal of Finance, 21*, 39-70.

Farzin, Y., Huisman, K., & Kort, P. (1998), Optimal Timing of Technology Adoption, *Journal of Economic Dynamics and Control, 22*, 779-799.

Fiani, R. (2004), *Teoria dos Jogos para Curso de Administração e Economia*, Rio de Janeiro, RJ: Elsevier North-Holland.

Figlewski, S., & Gao, B. (1999), The Adaptive Mesh Model: a New Approach to Efficient Option Pricing, *Journal of Financial Economics, 53*, 313-351.

Filipe, J. (2006), *O Drama dos Recursos Comuns: Um Caso de Aplicação da Teoria dos Jogos aos Comuns da Pesca*, Ph.D. Thesis, Departamento de Métodos Quantitativos, ISCTE.

Filipe, J. & Monteiro, J. (2008), Agency Conflicts and Competitive Interaction under Managerial Flexibility in Investment Decisions, *Proceedings of the 7th International Conference on Applied Mathematics (APLIMAT),* Bratislava: Slovak Republic, Slovak University of Technology in Bratislava, pp. 455-462.

Filipe, J. & Monteiro, J. (2011), A Framework for the Analysis of the Impact of Competition on Investment Decisions in the Presence of Managerial and Financial Flexibility, *Portuguese Journal of Quantitative Methods*, 1, 75-84.

Filipe, J. & Monteiro, J. (2012), Managerial Flexibility in Turbulent Times of Crisis, *International Journal of Latest Trends in Finance & Economic Sciences*, Forthcoming.

Fisher, I. (1930), *The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It*, New York, NY: Macmillan.

Friedman, J. (1991), *Game Theory with Applications to Economics*, London: Oxford University Press.

Fries, S., Miller, M., & Perraudin, W. (1997), Debt in Industry Equilibrium, *The Review of Financial Studies, 10*, 39–67.

Fudenberg, D. & Tirole, J. (1984), The Fat-cat Effect, the Puppy-dog Ploy, and the Lean and Hungry Look, *American Economic Review, 74*, 361-366.

Fudenberg, D., & Tirole, J. (1985), Preemption and Rent Equalization in the Adoption of New Technology, *Review of Economic Studies, 52*, 383-401.

Fudenberg, D., & Tirole, J. (1991), *Game Theory*, Cambridge, MA: The MIT Press.

Galai, D., & Masulis, R. (1976), The Option Pricing Model and the Risk Factor of Stock, *Journal of Financial Economics, 3*, 53-81.

Geske, R. (1979), The Valuation of Compound Options, *Journal of Financial Economics, 7*, 63-81.

Ghemawat, P., & Nalebuff, B. (1985), Exit, *RAND Journal of Economics, 16*, 184-194.

Gibbons, R. (1992), *A Primer in Game Theory*, London: FT - Prentice Hall.

Gilbert, R., & Harris, R. (1984), Competition with Lumpy Investment, *RAND Journal of Economics*, *15*, 197-212.

Gilbert, R., & Newbery, D. (1982), Preemption Patenting and the Persistence of Monopoly, *American Economic Review, 72*, 514-526.

Gillies, D. B. (1953), *Some Theorems on n-person Games*, Ph.D. thesis, Department of Mathematics, Princeton University.

Goetz, G. (2000), Strategic Timing of Adoption of New Technologies under Uncertainty: A Note, *International Journal of Industrial Organization, 18,* 369-379.

Grenadier, S. (1996), The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets, *Journal of Finance, 5*, 1653- 1679.

Grenadier, S. (1999), Information Revelation Through Option Exercise, *Review of Financial Studies, 12*, 95-129.

Grenadier, S. (2000), *Game Choices: The Interaction of Real Options and Game Theory*, London: Risk Books.

Grenadier, S. (2002), Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms, *Review of Financial Studies, 15*, 691- 721.

Grenadier, S., & Weiss A. (1997), Investment in Technological Innovations: An Option Pricing Approach, *Journal of Financial Economics, 44*, 397-416.

Hamada, R. (1969), Portfolio Analysis, Market Equilibrium and Corporation Finance, *The Journal of Finance, 24,* 13-31.

Hamada, R. (1971), Investment Decision with a General Equilibrium Mean-Variance Approach, *The Quarterly Journal of Economics, 85*, 667-683.

Hamada, R. (1972), The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks, *The Journal of Finance, 27*, 435-452.

Hamel, G., Doz, Y. L., & Prahalad, C. K. (1989), Collaborate with Your Competitors – and Win, *Harvard Business Review, 67,* 133-139.

Harsanyi, J. C. (1967), Games with Incomplete Information Played by Bayesian Players, I-III, *Management Science, 14,* 159-182, 320-334, 486-502.

Hart, S., & Mas-Collel, A. (1997), *Cooperation: Game Theoretic Approaches*, Berlin: Springer Verlag.

Heap, S., & Varoufakis, Y. (2004), *Game Theory: A Critical Text*, London: Routledge.

Hendricks, K. (1992), Reputations in the Adoption of a New Technology, *International Journal of Industrial Organization, 10,* 663-677.

Hendricks, K., & Wilson, C. (1985), Discrete versus Continuous Time in Games of Timing, *C. V. Starr Center for Applied Economics Working Paper*, New York University.

Hendricks, K. & Wilson, C. (1987), Equilibrium in Preemption Games with Complete Information, *C. V. Starr Center for Applied Economics Working Paper*, New York University.

Hendricks, K., Weiss, A., & Wilson, C. (1988), The War of Attrition in Continuous Time with Complete Information, *International Economic Review, 29,* 663-680.

Hicks, J. R. (1946), *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*, Oxford: Clarendon Press.

Hirshleifer, J. (1958), On the Theory of the Optimal Investment Decision, *The Journal of Political Economy, 66*, 329-52.

Hirshleifer, J. (1965), Investment Decisions under Uncertainty: Choice Theoretic Approaches, *Quarterly Journal of Economics, 79*, 509-536.

Hirshleifer, J. (1970), *Investment, Interest and Capital*, Englewood Cliffs, NJ: Prentice-Hall.

Howell, S., Stark, A., Newton, D., Paxson, D., Cavus, M., Pereira, J., & Patel, K. (2001), *Real Options: Evaluating Corporate Investment Opportunities in a Dynamic World*, London: FT - Prentice Hall.

Hsu, Y. W., & Lambrecht, B. (2007), Preemptive Patenting under Uncertainty and Asymmetric Information, *Annals of Operations Research*, *151*, 5-28.

Huisman, K. (2001), *Technology Investment: A Game Theoretical Options Approach*, Boston: Kluwer Academic Publishers.

Huisman, K. & Kort, P. (1999), Effects of Strategic Interactions on the Option Value of Waiting, Working paper, Tilburg University.

Huisman, K., & Kort, P. (2003), Strategic Investment in Technological Innovations, *European Journal of Operational Research*, *144*, 209-223.

Huisman, K., & Kort, P. (2004), Strategic Technology Adoption taking into account Future Technological Improvements: A Real Options Approach, *European Journal of Operational Research, 159*, 705-728.

Huisman, K., & Kort, P. (2009), Strategic Capacity Investment under Uncertainty, *Presented at the 13th Annual International Conference on Real Options,* Braga: Portugal, Universidade do Minho.

Hung, N., & Quyen, N. (1993), On the R&D Timing under Uncertainty, *Journal of Economic Dynamics and Control, 17,* 971-991.

Ingersoll, J., & Ross, S. (1992), Waiting to Invest: Investment and Uncertainty, *The Journal of Business, 65*, 1-29.

Jensen, M. (1985), When Unocal won over Pickens, Shareholder and Society Lost, *Financier*, *9*. 50-52.

Jensen, M. (1986), Agency Costs of Free Cash Flow, Corporate Finance and Takeovers, *American Economic Review, 76*, 323-329.

Jensen, M., & Meckling, W. (1976), Theory of the Firm: Managerial Behaviour, Agency Costs, and Ownership Structure, *Journal of Financial Economics, 3*, 305-360.

Jensen, M. & Smith, C. (1985), Stockholder, Manager and Creditor Interests: Applications of Agency Theory, in E. Altman and M. Subrahmanyam (Eds.), *Recent Advances in Corporate Finance*, Homewood, Ill.: Irwin, pp. 93-131.

Joaquin, D., & Butler, K. (2000), Competitive Investment Decisions - A Synthesis, in M. J. Brennan and L. Trigeorgis (Eds.), Project Flexibility, Agency and Competition: New Developments in the Theory and Applications of Real Options, New York, NY: Oxford University Press, pp. 324-339.

Jones, E. P., Mason, S. P., & Rosenfeld, E. (1984), Contingent Claims Analysis of Corporate Capital Structures: an Empirical Investigation, *The Journal of Finance, 39*, 611-625.

Jones, E. P., Mason, S. P., & Rosenfeld, E. (1985), *Contingent Claims Valuation of Corporate Liabilities: Theory and Empirical Tests*, Chicago: University of Chicago Press.

Jou, J-B., & Lee, T. (2004), The Agency Problem, Investment Decision and Optimal Financial Structure, *The European Journal of Finance, 10*, 489-509.

Kemna, A., (1988), Options in Real and Financial Markets, Ph.D. thesis, Erasmus University.

Kemna, A., (1993), Case Studies in Real Options, *Financial Management*, *22-3*, 259-270.

Kensinger, J. (1987), Adding the Value of Active Management into the Capital Budgeting Equation, *Midland Corporate Finance Journal, 5*, 31-42.

Kensinger, J. (1988), The Capital Investment Project as a Set of Exchange Options, *Managerial Finance, 14*, 16-27.

Kester, W. C. (1984), Today's Options for Tomorrow's Growth, *Harvard Business Review, 62*, 153-160.

Kester, W. C. (1993), Turning Growth Options into Real Assets, in R. Aggarwal (Ed.), Capital Budgeting under Uncertainty, Englewood Cliffs, NJ: Prentice-Hall, pp. 91-102.

Keynes, J. M. (1936), *The General Theory of Employment, Interest and Money*, Cambridge: Palgrave MacMillan Publishers.

Kim, E. H. (1978), A Mean-Variance Theory of Optimal Capital Structure and Corporate Debt Capacity, *The Journal of Finance, 33*, 45-63.

Klammer, T. (1972), Empirical Evidence on the Adoption of Sophisticated Capital Budgeting Techniques, *Journal of Business*, *45*, 387-397.

Kort, P., & Pawlina, G. (2006), Real Options in an Asymmetric Duopoly: Who Benefits from your Competitive Disadvantage?, *Journal of Economics & Management Strategy, 15*, 1-35.

Kraus, A., & Litzenberger, R. H. (1973), A State-Preference Model of Optimal Financial Leverage, *The Journal of Finance*, *28,* 911-922.

Kreps, D., & Wilson, R. (1982a), Sequential Equilibria, *Econometrica, 50*, 863-894.

Kreps, D., & Wilson, R. (1982b), Reputation and Imperfect Information, *Journal of Economic Theory, 27,* 253-279.

Kulatilaka, N. (1988), Valuing the Flexibility of Flexible Manufacturing Systems, *IEEE Transactions in Engineering Management, 35*, 250-257.

Kulatilaka, N., & Perotti, E. C. (1998), Strategic Growth Options, *Management Science, 44*, 1021-1031.

Kuhn, H. W. (1950), *Contributions to the Theory of Games, I*, Princeton, NY: Princeton University Press.

Kuhn, H. W. (1953), *Contributions to the Theory of Games, II,* Princeton, NY: Princeton University Press.

Lambrecht, B. (2000), Strategic Sequential Investments and Sleeping Patents, in M. J. Brennan & L. Trigeorgis (Eds.), *Project flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options,* New York, NY: Oxford University Press, pp. 297-323.

Lambrecht, B., & Perraudin, W. (1994), Option Games, Working Paper, University of Cambridge.

Lambrecht, B., & Perraudin, W. (2003), Real Options and Preemption under Incomplete Information, *Journal of Economic Dynamics and Control, 27*, 619–643.

Laraki, R., Solan, E., & Vieille, N. (2005), Continuous-Time Games of Timing, *Journal of Economic Theory, 120*, 206-238.

Leahy, J. (1993), Investment in Competition Equilibrium: The Optimality of Myopic Behaviour, *Quarterly Journal of Economics, 108,* 1105-1133.

Leland, H. E. (1994), Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *The Journal of Finance*, *49*, 1213-1252.

Leland, H. E. (1998), Agency Costs, Risk Management, and Capital Structure, *The Journal of Finance*, *53*, 1213-1243.

Lintner, J. (1965), The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, *7*, 13-37.

Lintner, J. (1969), The Aggregation of Investor's Diverse Judgements and Preferences in Purely Competitive Security Markets, *Journal of financial and Quantitative Analysis*, *4*, 347-400.

Long, M., & Malitz, I. (1985), The Investment-Financing Nexus: Some Empirical Evidence, *Midland Corporate Finance Journal*, *3*, 53-59.

Luehrman, T. (1998), Strategy as a Portfolio of Real Options, *Harvard Business Review*, *76*, 89-99.

MacKay, P. (2003), Real Flexibility and Financial Structure: An Empirical Analysis, *Review of Financial Studies*, *16*, 1131-1165.

Magee, J. F. (1964a), Decision Trees for Decision Making, *Harvard Business Review*, *42*, 126-138.

Magee, J. F. (1964b), How to Use Decision Tree in Capital Investment, *Harvard Business Review*, *42*, 95-106.

Mahoney, J. T. (2005), *The Economic Foundations of Strategy*, Thousand Oaks, CA: Sage Publications.

Majd, S., & Pindyck, R. (1987), Time to Build, Option Value, and Investment Decisions, *Journal of Financial Economics*, *18*, 7-27.

Mao, C. X. (2003), Interaction of the Debt Agency Problems and Optimal Capital Structure: Theory and Evidence, *Journal of Financial and Quantitative Analysis*, *38*, 399-423.

Margrabe, W. (1978), The Value of an Option to Exchange one Asset for another, *The Journal of Finance*, *33*, 177-186.

Markowitz, H. (1952), Portfolio Selection, *The Journal of Finance*, *7 (1)*, 77–91.

Maskin, E., & Tirole, J. (1988), A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixe Costs, *Econometrica*, *56-3*, 549-569.

Mason, R., & Weeds, H. (2001), Irreversible Investment with Strategic Interactions, *CEPR Discussion Paper*, London.

Mason, R., & Weeds, H. (2009), Investment, Uncertainty and Preemption, *International Journal of Industrial Organization, 28*, 278-287.

Mauer, D., & Ott, S. (1995), Investment under Uncertainty: the Case of Replacement Investment Decisions, *Journal of Financial and Quantitative Analysis, 30,* 581-605.

Mauer, D., & Ott, S. (2000), Agency Costs, Underinvestment, and Optimal Capital Structure, in M. J. Brennan and L. Trigeorgis (Eds.), *Project Flexibility, Agency, and Competition: New Developments in the Theory and Application of Real Options,* New York, NY: Oxford University Press, pp. 151-179.

Mauer, D., & Sarkar, S. (2005), Real Options, Agency Conflicts, and Financial Policy, *Journal of Banking and Finance, 29*, 1405-1428.

Mauer, D., & Triantis, A. J. (1994), Interaction of Corporate Financing and Investment Decisions: a Dynamic Framework, *The Journal of Finance, 49*, 1253-1277.

McDonald, R., & Siegel, D. (1985), Investment and the Valuation of Firms When There is an Option to Shut Down, *International Economic Review, 26*, 331-349.

McDonald, R., & Siegel, D. (1986), The Value of Waiting to Invest, *The Quarterly Journal of Economics, 101*, 707-728.

McGrath, R. G., & Nerkar, A. (2004), Real Options Reasoning and a New Look at the R&D Investment Strategies of Pharmaceutical Firms, *Strategic Management Journal*, *25*, 1-21.

Mella-Barral, P. (1999), The Dynamics of Default and Debt Reorganization, *Review of Financial Studies, 12*, 535-578.

Mella-Barral, P., & Perraudin, W. (1997), Strategic Debt Service, *The Journal of Finance, 52*, 531-557.

Mello, A. S., & Parsons, J. E. (1992), Measuring the Agency Cost of Debt, *The Journal of Finance, 47*, 1887-1904.

Merton, R. (1973), Theory of Rational Options Pricing, *Bell Journal of Economics, 4*, 141-183.

Merton, R. (1976), Option Pricing when Underlying Stock Returns are Discontinuous, *Journal of Financial Economics, 3,* 125-144.

Merton, R. (1998), Applications of Option Pricing Theory: Twenty Five Years Later, *American Economic Review, 88*, 323-349.

Miles, J., & Ezzell, J. (1980), The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification, *Journal of Financial and Quantitative Analysis*, *15*, 719-730.

Miller, M. (1977), Debt and Taxes, *The Journal of Finance*, *32*, 261-275.

Mills, D. (1988), Preemptive Investment Timing, *RAND Journal of Economics, 19*, 114-122.

Miltersen, K. & Schwartz, E. S. (2004), R&D Investments with Competitive Interactions, *Review of Finance, 8*, 355-401.

Modigliani, F., & Miller, M. (1958), The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review, 48*, 261-297.

Modigliani, F., & Miller, M. (1963), Corporate Income Taxes and the Cost of Capital: A Correction, *American Economic Review, 53*, 433-443.

Monteiro, J. (2004), *Re-evaluation of Mauer and Ott Agency-Real-Options Framework in a Complex Hybrid Structure Context*, Tese de Mestrado em Finanças, UCP.

Morellec, E., & Smith, C. (2007), Agency Conflicts and Risk Management, *Review of Finance, 11*, 1-23.

Mossin, J. (1966), Equilibrium in a Capital Asset Market, *Econometrica, 33*, 768-783.

Moyen, N. (2007), How Big Is the Debt Overhang Problem?, *Journal of Economic Dynamics & Control, 31*, 433-472.

Murphy, K. J. (1985), Corporate Performance and Managerial Remuneration: An Empirical Analysis, *Journal of Accounting and Finance, 7*, 11-42.

Murto, P. (2004), Exit in Duopoly under Uncertainty, *RAND Journal of Economics, 35*, 111-127.

Murto, P., & Keppo, J. (2002), A Game Model of Irreversible Investment under Uncertainty, *International Game Theory Review, 4*, 127-140.

Murto, P., Nasakkala, E., & Keppo, J. (2004), Timing of Investments in Oligopoly under Uncertainty: A Framework for Numerical Analysis, *European Journal of Operational Research, 157*, 486-500.

Myers, S. (1974), Interactions of Corporate Financing and Investment Decisions-Implications for Capital Budgeting, *The Journal of Finance, 29*, 1-25.

Myers, S. (1977), Determinants of Corporate Borrowing, *Journal of Financial Economics, 5*, 147-175.

Myers, S. (1987), Finance Theory and Financial Strategy, *Midland Corporate Finance Journal, 5*, 6-13.

Myers, S., & Majd, S. (1990), Abandonment Value and Project Life, in F. J. Fabozzi (Ed.), Advances in Futures and Options Research, vol. 4, Greenwich, CT: JAI Press, pp. 1-21.

Myers, S., & Majluf, S. (1984), Corporate Financing and Investment Decisions when Firms have Information that Investors do not Have, *Journal of Financial Economics, 13*, 187-221.

Myerson, R. B. (1978), Refinements of the Nash Equilibrium Concept, *International Journal of Game Theory, 7-2,* 73-80.

Nash, J. (1950a), The Bargaining Problem, *Econometrica, 18*, 155-162.

Nash, J. (1950b), Equilibrium Points in n-person Games, *Proceedings of the National Academy of Sciences of the United States of America, 36*, 48-49.

Nash, J. (1951), Non-Cooperative Games, *Annals of Mathematics, 54,* 286-295.

Nash, J. (1953), Two Person Cooperative Games, *Econometrica, 21*, 128-140.

Novy-Marx, R. (2007), An Equilibrium Model of Investment under Uncertainty, *Review of Financial Studies*, *20,* 1461-1502.

Novy-Marx, R. (2009), Preempting Preemptive Investment, Working Paper, University of Chicago.

Osborne, M. (2004), *An Introduction to Game Theory,* New York, NY: Oxford University Press.

Osborne, M., & Rubinstein, A. (1994), *A Course in Game Theory*, Cambridge, MA: MIT Press.

Paddock, J., Siegel, D., & Smith, J. (1988), Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases, *Quarterly Journal of Economics, 103,* 479-508.

Parrino, R., & Weisbach, M. (1999), Measuring Investment Distortions Arising from Stockholder-Bondholder Conflicts, *Journal of Financial Economics, 53,* 3-42.

Pawlina, G., & Kort, P. (2006), Real Options in an Asymmetric Duopoly: Who Benefits from your Competitive Disadvantage?, *Journal of Economics & Management Strategy, 15*, 1-35.

Paxson, D., & Pinto, H. (2005), Rivalry under Price and Quantity Uncertainty, *Review of Financial Economics, 14*, 209-224.

Pindyck, R. (1988), Irreversible Investment, Capacity Choice, and the Value of the Firm, *The American Economic Review, 78,* 969-985.

Pindyck, R. (1993), *Investments of Uncertain Cost,* Journal of Financial Economics, 34(1): 53-76.

Quigg, L. (1993), Empirical Testing of Real Option-Pricing Models, *Journal of Finance, 48*, 621-640.

Rajan, R., & Zingales, L. (1995), What do we know about Capital Structure? Some Evidence from International Data, *The Journal of Finance, 50*, 1421-1467.

Rasmusen, E. (2001), *Games & Information: An Introduction to Game Theory,* Oxford: Blackwell Publishing.

Reinganum, J. (1981a), On the Diffusion of New Technology: A Game-theoretic Approach, *The Review of Economic Studies, 48*, 395-405.

Reinganum, J. (1981b), Dynamic Games of Innovation, *Journal of Economic Theory, 25*, 21-41.

Reinganum, J. (1982), A Dynamic Game of R&D: Patent Protection and Competitive Behaviour, *Econometrica, 50,* 671-688.

Reinganum, J. (1983), A Note on the Strategic Adoption of a New Technology, *Journal of Optimization Theory and Applications*, *39*, 133-141.

Reiss, A. (1998), Investment in Innovations and Competition: An Option Pricing Approach, *Quarterly Review of Economics and Finance, 38*, 635-650.

Ross, S. (1973), The Economic Theory of Agency: The Principal's Problem, *American Economic Review*, *63*, 134-139.

Ross, S. (1976), The Arbitrage Theory of Capital Asset Pricing, *Journal of Economic Theory*, *13*, 341–60.

Ross, S. (1977), The Determination of Financial Structure: The Incentive-Signalling Approach, *The Bell Journal of Economics*, *8*, 23-40.

Rubinstein, M. (1973), A Mean-Variance Synthesis of Corporate Financial Theory, *The Journal of Finance, 28*, 167-181.

Rubinstein, M. (2003), Great Moments in Financial Economics: II. Modigliani-Miller Theorem, *Journal of Investment Management, 1*, 7-13.

Rumelt, R., Schendel, D., & Teece, D. (1994), Fundamental Issues in Strategy, in R. Rumelt, D. Schendel and D. Teece (Eds.), *Fundamental Issues in Strategy: A Research Agenda*, Boston, MA: Harvard Business School Press, pp 9-47.

Savva, N. & Scholtes, S. (2005), Real Options in Partnership Deals: The Perspective of Cooperative Game Theory, *Presented at the 9th Annual International Conference on Real Options*, Paris, France: Paris Dauphine University.

Schall, L., Sundem, G. & Geijsbeek, W. (1978), Survey and Analysis of Capital Budgeting References, *Journal of Finance*, *33*, 281-297.

Schelling, T. C. (1956), An Essay on Bargaining, *American Economic Review, 46*, 281-236.

Schwartz, E. S. (2004), Patents and R&D as Real Options, *Banca Monte dei Paschi di Siena Economic Notes*, 23-54.

Schwartz, E. S., & Moon, M. (2000), Evaluating Research and Development Investments, in M. J. Brennan & L. Trigeorgis (Eds.), *Project Flexibility, Agency and Competition: New Developments in the Theory and Application of Real Options*, New York, NY: Oxford University Press, pp. 85-105.

Schwartz, E. S., & Trigeorgis, L. (Eds.) (2001), *Real Options and Investment under Uncertainty*, Cambridge, MA: MIT Press.

Schwartz, E. S., & Zoraya, C. (2003), Investment under Uncertainty in Information Technology, Acquisition and Development Projects, *Management Science, 49*, 57-70.

Selten, R. (1965), Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragetragheit, *Zeitschrift fur die Gesamte Staatswissenschaft*, *121*, 321-324, 667-689.

Selten, R. (1975), Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games, International Journal of Game Theory, 4, 25–55.

Shackleton, M., Tsekrekos, A., & Wojakowski, R. (2004), Strategic Entry and Market Leadership in a Two-Player Real Options Game, *Journal of Banking & Finance, 28*, 179-201.

Shapley, L. (1953), Stochastic Games, *Proceedings of National Academy of Science*, *39*, 1095–1100.

Sharpe, W. (1963), A Simplified Model for Portfolio Analysis, *Management Science, 19*, 277-93.

Sharpe, W. (1964), Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *The Journal of Finance, 19,* 425-442.

Smets, F. (1991), Exporting versus Foreign Direct Investment, *Working Paper*, Yale University.

Smit, H. (2003), Infrastructure Investment as a Real Options Game: The Case of European Airport Expansion, *Financial Management, 32*, 5-35.

Smit, H., & Ankum, L. (1993), A Real Options and Game-Theoretic Approach to Corporate Investment under Competition, *Financial Management, 22*, 241-250.

Smit, H., & Trigeorgis, L. (1997), Flexibility and Competitive R&D Strategies, *Working Paper*, Columbia University.

Smit, H., & Trigeorgis, L. (2001), Flexibility and Commitment in Strategic Investment, in E. Schwartz & L. Trigeorgis (Eds.), *Real Options and Investment under*

Uncertainty, Classical Readings and Recent Contributions, Cambridge, MA: MIT Press, pp. 451-498.

Smit, H., & Trigeorgis, L. (2004), *Strategic Investment: Real Options and Games*, Princeton, NJ: Princeton University Press.

Smit, H., & Trigeorgis, L. (2006), Real Options and Games: Competition, Alliances and other Applications of Valuation and Strategy, *Review of Financial Economics, 15*, 95-112.

Smit, H., & Trigeorgis, L. (2007), Strategic Options and Games in Analysing Dynamic Technology Investments, *Long Range Planning, 40*, 84-114.

Smit, H., & Trigeorgis, L. (2009), Valuing Infrastructure Investment: an Option Games Approach, *California Management Review, 51*, 79-90.

Smith, C. (1976), Option Pricing Review, *Journal of Financial Economics, 3*, 3-51.

Smith, C. (1986), Investment Banking and the Capital Acquisition Process, *Journal of Financial Economics, 1-2*, 15-31.

Smith, C., & Watts, R. (1992), The Investment Opportunity Set, and Corporate Financing, Dividend, and Compensation Policies, *Journal of Financial Economics 32*, 262-292.

Stackelberg, H. von (1934), *Marktform und Gleichgewicht*, Vienna: Springer-Verlag.

Stenbacka, R., & Tombak, M. (1994), Strategic Timing of Adoption of New Technologies under Uncertainty, *International Journal of Industrial Organization, 12*, 387-411.

Stiglitz, J. (1969), A Re-Examination of the Modigliani-Miller Theorem, *The American Economic Review, 59*, 784-793.

Stiglitz, J. (1972), Some aspects of the pure theory of corporate finance: Bankruptcies and Takeovers, *The Bell Journal of Economics and Management Science, 3*, 458-482.

Stiglitz, J., & Weiss, A. (1981), Credit Rationing in Markets with Imperfect Information, *The American Economic Review, 71*, 393-410.

Subrahmanyam, M. (1990), The Early Exercise Feature of American Options, in S. Figlewski, W. Silber & M. Subrahmanyam (Eds.), *Financial Options - From Theory to Practice*, New York, NY: McGraw-Hill, pp. 135-183.

Thijssen, J. (2004), *Investment under Uncertainty, Coalition Spillovers and Market Evolution in a Game Theoretic Perspective*, Dordrecht: Kluwer Academic Publishers.

Thijssen, J., Huisman, K., & Kort, P. (2002), Symmetric Equilibrium Strategies in Game-Theoretic Real Option Models, *Center for Economic Research Discussion Paper*, Tilburg University.

Thijssen, J., Huisman, K., & Kort, P. (2006), The Effects of Information on Strategic Investment and Welfare, *Economic Theory, 28*, 399-424.

Tirole, J. (1988), *The Theory of Industrial Organization*. Cambridge, MA: MIT Press.

Titman, S. (1984), The Effect of Capital Structure on a Firm's Liquidation Decision, *Journal of Financial Economics*, *13*, 137-151.

Titman, S. (1985), Urban Land Prices under Uncertainty, *American Economic Review*, *75*, 505-514.

Titman, S., & Tsyplakov, S. (2002), A Dynamic Model of Optimal Capital Structure, *Working Paper*, University of Texas.

Titman, S., Tompaidis, S., & Tsyplakov, S. (2004), Market Imperfections, Investment Flexibility, and Default Spreads, *The Journal of Finance, 59*, 165-205.

Toft, K., & Prucyk, B. (1997), Options on Leveraged Equity: Theory and Empirical Tests, *The Journal of Finance, 52*, 1151-1180.

Tourinho, O. (1979), The Option Value of Reserves of Natural Resources, *Working Paper*, University of California at Berkeley.

Treynor, J. L. (1961), *Toward a Theory of Market Value of Risky Assets*, Unpublished manuscript.

Triantis, A., & Hodder, J. (1990), Valuing Flexibility as a Complex Option, *Journal of Finance, 45*, 549-565.

Trigeorgis, L. (1986), *Valuing Real Investment Opportunities: An Options Approach to Strategic Capital Budgeting*, Ph.D. Dissertation, Harvard Business School.

Trigeorgis, L. (1988), A Conceptual Options Framework for Capital Budgeting, in F. J. Fabozzi (Ed.), *Advances in Futures and Options Research, vol. 3*, Greenwich, CT: JAI Press, 145-167.

Trigeorgis, L. (1990), Valuing the Impact of Uncertain Competitive Arrivals on Deferrable Real Investment Opportunities, *Working Paper*, Boston University.

Trigeorgis, L. (1991), Anticipated Competitive Entry and Early Preemptive Investment in Deferrable Projects, *Journal of Economics and Business, 43*, 143-156.

Trigeorgis, L. (1993), Real Options and Interactions with Financial Flexibility, *Financial Management*, *22*, 202-224.

Trigeorgis, L. (1996), *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, Cambridge, MA: MIT Press.

Trigeorgis, L. & Mason, S. (1987), Valuing Managerial Flexibility, *Midland Corporate Finance Journal, 5*, 14-21.

Von Neuman, J., & Morgenstern, O. (1944), *Theory of Games and Economic Behaviour*, New York, NY: Wiley.

Warner, J. (1977), Bankruptcy Costs: Some Evidence, *The Journal of Finance, 32-2*, 337-347.

Weeds, H. (2000), Sleeping Patents and Compulsory Licensing: an Options Analysis, *Working Paper*, University of Warwick.

Weeds, H. (2002), Strategic Delay in a Real Options Model of R&D Competition, *Review of Economic Studies, 69*, 729-747.

Williams, J. B. (1938), *The Theory of Investment Value*, Cambridge, MA: Harvard University Press.

Williams, J. (1993), Equilibrium and Options on Real Assets, *Review of Financial Studies, 6*, 825-850.

Wilmott, P. (2000), *Paul Wilmott on Quantitative Finance*, London: John Wiley and Sons.

Zhu, K., & Weyant, J. (2003a), Strategic Exercise of Real Options: Investment Decisions in Technological Systems. *Decision Sciences, 34*, 643-675.

Zhu, K., & Weyant, J. (2003b), Strategic Exercise of Real Options: Investment Decisions in Technological Systems, *Journal of Systems Science and Systems Engineering*, *12*, 257-278.

Ziegler, A. (2004), *A Game Theory Analysis of Options: Corporate Finance and Financial Intermediation in Continuous Time,* Berlin: Springer-Verlag.

Annexes

BASE CASE SCENARIO UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	C_A	5,00	g	-	Va	118,49
u	1,25	C_B	5,00	I	-	Ea	84,67
d	0,80	F	50,00	X	-	Da	33,82
r_f	2,000%	K_e	8,000%			Vb	118,49
p	0,32	τ	25,000%			Eb	84,67
						Db	33,82

VALUE OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">118,49</td><td colspan="8">118,49</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">231,87</td><td colspan="4">231,87</td><td colspan="4">67,59</td><td colspan="4">67,59</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">440,32</td><td colspan="3">440,32</td><td colspan="2">138,70</td><td colspan="2">138,70</td><td colspan="3">35,47</td><td colspan="3">35,47</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">815,82</td><td colspan="2">815,82</td><td colspan="2">273,16</td><td colspan="2">273,16</td><td colspan="2">78,27</td><td colspan="2">78,27</td><td colspan="2">15,97</td><td colspan="2">15,97</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>1.482,44</td><td>1.482,44</td><td colspan="2">520,04</td><td colspan="2">520,04</td><td colspan="2">162,76</td><td colspan="2">162,76</td><td colspan="2">40,04</td><td colspan="2">40,04</td><td>4,90</td><td>4,90</td></tr>
</table>

BASE CASE SCENARIO UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	-	Va	118,49
u	1,25	CB	5,00	I	-	Ea	84,67
d	0,80	F	50,00	X	-	Da	33,82
rf	2,000%	Ke	8,000%			Vb	118,49
p	0,32	τ	25,000%			Eb	84,67
						Db	33,82

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="12">17,50</td></tr>
<tr><td colspan="6">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">84,67</td><td colspan="6">84,67</td></tr>
<tr><td colspan="6">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">21,88</td><td colspan="6">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">187,65</td><td colspan="2">187,65</td><td colspan="2">37,76</td><td colspan="4">37,76</td></tr>
<tr><td colspan="5">u</td><td>d</td><td>u</td><td colspan="5">d</td></tr>
<tr><td colspan="5">27,34</td><td colspan="2">17,50</td><td colspan="5">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="2">B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">392,27</td><td colspan="2">392,27</td><td>95,01</td><td>95,01</td><td colspan="2">11,40</td><td colspan="3">11,40</td></tr>
<tr><td colspan="4">u</td><td>d</td><td>u</td><td>d</td><td>u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="2">21,88</td><td colspan="2">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">766,80</td><td colspan="2">766,80</td><td>224,14</td><td>224,14</td><td>35,85</td><td>35,85</td><td colspan="2">0,00</td><td colspan="2">0,00</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td>u</td><td>d</td><td>u</td><td>d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="3">27,34</td><td colspan="2">17,50</td><td colspan="3">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td>B</td><td>A</td><td>B</td><td>A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>1.432,44</td><td>1.432,44</td><td colspan="2">470,04</td><td>470,04</td><td>112,76</td><td>112,76</td><td>0,00</td><td colspan="2">0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

BASE CASE SCENARIO UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	-	Va	118,49
u	1,25	CB	5,00	I	-	Ea	84,67
d	0,80	F	50,00	X	-	Da	33,82
rf	2,000%	Ke	8,000%			Vb	118,49
p	0,32	τ	25,000%			Eb	84,67
						Db	33,82

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">33,82</td><td colspan="8">33,82</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">44,22</td><td colspan="4">44,22</td><td colspan="4">29,83</td><td colspan="4">29,83</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="3">48,06</td><td colspan="2">43,69</td><td colspan="2">43,69</td><td colspan="3">24,07</td><td colspan="3">24,07</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">42,42</td><td colspan="2">42,42</td><td colspan="2">15,97</td><td colspan="2">15,97</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">40,04</td><td colspan="2">40,04</td><td>4,90</td><td>4,90</td></tr>
</table>

FIRST BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	317,03
u	1,25	C_B	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
r_f	2,000%	r_m	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">317,03</td><td colspan="8">317,03</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">648,49</td><td colspan="4">648,49</td><td colspan="4">167,34</td><td colspan="4">167,34</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.272,91</td><td colspan="3">1.272,91</td><td colspan="2">368,03</td><td colspan="2">368,03</td><td colspan="3">75,99</td><td colspan="3">75,99</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.398,43</td><td colspan="2">2.398,43</td><td colspan="2">770,45</td><td colspan="2">770,45</td><td colspan="2">185,79</td><td colspan="2">185,79</td><td colspan="2">25,54</td><td colspan="2">25,54</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.397,33</td><td>4.397,33</td><td colspan="2">1.510,13</td><td colspan="2">1.510,13</td><td colspan="2">438,28</td><td colspan="2">438,28</td><td colspan="2">70,13</td><td colspan="2">70,13</td><td>4,90</td><td>4,90</td></tr>
</table>

FIRST BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	317,03
u	1,25	CB	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
rf	2,000%	rm	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">279,52</td><td colspan="8">279,52</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">601,38</td><td colspan="4">601,38</td><td colspan="4">133,34</td><td colspan="4">133,34</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.224,85</td><td colspan="3">1.224,85</td><td colspan="2">319,97</td><td colspan="2">319,97</td><td colspan="3">47,72</td><td colspan="3">47,72</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.349,41</td><td colspan="2">2.349,41</td><td colspan="2">721,43</td><td colspan="2">721,43</td><td colspan="2">136,78</td><td colspan="2">136,78</td><td colspan="2">6,40</td><td colspan="2">6,40</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.347,33</td><td>4.347,33</td><td colspan="2">1.460,13</td><td colspan="2">1.460,13</td><td colspan="2">388,28</td><td colspan="2">388,28</td><td colspan="2">20,13</td><td colspan="2">20,13</td><td>0,00</td><td>0,00</td></tr>
</table>

FIRST BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	317,03
u	1,25	CB	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
rf	2,000%	rm	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">37,51</td><td colspan="8">37,51</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">47,12</td><td colspan="4">47,12</td><td colspan="4">34,00</td><td colspan="4">34,00</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="3">48,06</td><td colspan="2">48,06</td><td colspan="2">48,06</td><td colspan="3">28,26</td><td colspan="3">28,26</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">19,14</td><td colspan="2">19,14</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td>4,90</td><td>4,90</td></tr>
</table>

SECOND BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	305,91
u	1,25	C_B	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
r_f	2,000%	r_m	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE FIRMS

<table>
<tr><td colspan="12">17,50</td></tr>
<tr><td colspan="6">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">305,91</td><td colspan="6">305,91</td></tr>
<tr><td colspan="6">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">21,88</td><td colspan="6">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">639,75</td><td colspan="2">639,75</td><td colspan="2">154,75</td><td colspan="4">154,75</td></tr>
<tr><td colspan="5">u</td><td>d</td><td>u</td><td colspan="5">d</td></tr>
<tr><td colspan="5">27,34</td><td colspan="2">17,50</td><td colspan="5">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="2">B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.272,91</td><td colspan="2">1.272,91</td><td>354,82</td><td>354,82</td><td colspan="2">63,31</td><td colspan="3">63,31</td></tr>
<tr><td colspan="4">u</td><td>d</td><td>u</td><td>d</td><td>u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="2">21,88</td><td colspan="2">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.398,43</td><td colspan="2">2.398,43</td><td>770,45</td><td>770,45</td><td>165,87</td><td>165,87</td><td colspan="2">15,97</td><td colspan="2">15,97</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td>u</td><td>d</td><td>u</td><td>d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="3">27,34</td><td colspan="2">17,50</td><td colspan="3">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td>B</td><td>A</td><td>B</td><td>A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.397,33</td><td>4.397,33</td><td colspan="2">1.510,13</td><td>1.510,13</td><td>438,28</td><td>438,28</td><td>40,04</td><td colspan="2">40,04</td><td>4,90</td><td>4,90</td></tr>
</table>

SECOND BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">272,08</td><td colspan="8">272,08</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">595,53</td><td colspan="4">595,53</td><td colspan="4">124,92</td><td colspan="4">124,92</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.224,85</td><td colspan="3">1.224,85</td><td colspan="2">311,14</td><td colspan="2">311,14</td><td colspan="3">39,25</td><td colspan="3">39,25</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.349,41</td><td colspan="2">2.349,41</td><td colspan="2">721,43</td><td colspan="2">721,43</td><td colspan="2">123,44</td><td colspan="2">123,44</td><td colspan="2">0,00</td><td colspan="2">0,00</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.347,33</td><td>4.347,33</td><td colspan="2">1.460,13</td><td colspan="2">1.460,13</td><td colspan="2">388,28</td><td colspan="2">388,28</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

SECOND BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE ADITIONAL EQUITY ISSUE

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">18,49</td><td colspan="8">18,49</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">14,54</td><td colspan="4">14,54</td><td colspan="4">20,93</td><td colspan="4">20,93</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">0,00</td><td colspan="3">0,00</td><td colspan="2">21,94</td><td colspan="2">21,94</td><td colspan="3">21,06</td><td colspan="3">21,06</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">33,12</td><td colspan="2">33,12</td><td colspan="2">15,90</td><td colspan="2">15,90</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td></td><td></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2">50,00</td><td colspan="2">50,00</td><td>0,00</td><td>0,00</td></tr>
</table>

SECOND BEST POLICY UNDER COURNOT-NASH EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">33,82</td><td colspan="8">33,82</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">44,22</td><td colspan="4">44,22</td><td colspan="4">29,83</td><td colspan="4">29,83</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="3">48,06</td><td colspan="2">43,69</td><td colspan="2">43,69</td><td colspan="3">24,07</td><td colspan="3">24,07</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">42,42</td><td colspan="2">42,42</td><td colspan="2">15,97</td><td colspan="2">15,97</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">40,04</td><td colspan="2">40,04</td><td>4,90</td><td>4,90</td></tr>
</table>

BASE CASE SCENARIO UNDER STACKELBERG EQUILIBRIUM

θ	17,50	C_A	5,00	g	-	Va	133,30
u	1,25	C_B	5,00	I	-	Ea	97,51
d	0,80	F	50,00	X	-	Da	35,79
r_f	2,000%	r_m	8,000%			Vb	66,65
p	0,32	τ	25,000%			Eb	58,32
						Db	26,93

VALUE OF THE FIRMS

<table>
<tr><td colspan="12">17,50</td></tr>
<tr><td colspan="6">A - LEADER</td><td colspan="6">B - FOLLOWER</td></tr>
<tr><td colspan="6">133,30</td><td colspan="6">66,65</td></tr>
<tr><td colspan="6">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">21,88</td><td colspan="6">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">260,85</td><td colspan="2">130,43</td><td colspan="2">76,04</td><td colspan="4">38,02</td></tr>
<tr><td colspan="5">u</td><td>d</td><td>u</td><td colspan="5">d</td></tr>
<tr><td colspan="5">27,34</td><td colspan="2">17,50</td><td colspan="5">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="2">B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">495,36</td><td colspan="2">247,68</td><td>156,03</td><td>78,02</td><td colspan="2">39,90</td><td colspan="3">19,95</td></tr>
<tr><td colspan="4">u</td><td>d</td><td>u</td><td>d</td><td>u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="2">21,88</td><td colspan="2">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">917,79</td><td colspan="2">458,90</td><td>307,30</td><td>153,65</td><td>88,06</td><td>44,03</td><td colspan="2">17,97</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td>u</td><td>d</td><td>u</td><td>d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="3">27,34</td><td colspan="2">17,50</td><td colspan="3">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td>B</td><td>A</td><td>B</td><td>A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>1.667,75</td><td>833,87</td><td colspan="2">585,05</td><td>292,53</td><td>183,11</td><td>91,55</td><td>45,05</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

BASE CASE SCENARIO UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	-	Va	133,30
u	1,25	CB	5,00	I	-	Ea	97,51
d	0,80	F	50,00	X	-	Da	35,79
rf	2,000%	rm	8,000%			Vb	66,65
p	0,32	τ	25,000%			Eb	58,32
						Db	26,93

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A - LEADER</td><td colspan="8">B - FOLLOWER</td></tr>
<tr><td colspan="8">97,51</td><td colspan="8">58,32</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">215,18</td><td colspan="4">123,88</td><td colspan="4">43,93</td><td colspan="4">28,59</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">447,31</td><td colspan="3">247,68</td><td colspan="2">110,15</td><td colspan="2">68,13</td><td colspan="3">13,45</td><td colspan="3">10,46</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">868,77</td><td colspan="2">458,90</td><td colspan="2">258,28</td><td colspan="2">153,65</td><td colspan="2">42,32</td><td colspan="2">29,11</td><td colspan="2">0,00</td><td colspan="2">1,82</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>1.617,75</td><td>833,87</td><td colspan="2">535,05</td><td colspan="2">292,53</td><td colspan="2">133,11</td><td colspan="2">91,55</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td>0,00</td><td>2,75</td></tr>
</table>

BASE CASE SCENARIO UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	-	Va	133,30
u	1,25	CB	5,00	I	-	Ea	97,51
d	0,80	F	50,00	X	-	Da	35,79
rf	2,000%	rm	8,000%			Vb	66,65
p	0,32	τ	25,000%			Eb	58,32
						Db	26,93

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="12">17,50</td></tr>
<tr><td colspan="6">A - LEADER</td><td colspan="6">B - FOLLOWER</td></tr>
<tr><td colspan="6">35,79</td><td colspan="6">26,93</td></tr>
<tr><td colspan="6">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">21,88</td><td colspan="6">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">45,68</td><td colspan="2">39,13</td><td colspan="2">32,11</td><td colspan="4">21,88</td></tr>
<tr><td colspan="5">u</td><td>d</td><td>u</td><td colspan="5">d</td></tr>
<tr><td colspan="5">27,34</td><td colspan="2">17,50</td><td colspan="5">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="2">B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="2">48,06</td><td>45,88</td><td>36,00</td><td colspan="2">26,45</td><td colspan="3">15,75</td></tr>
<tr><td colspan="4">u</td><td>d</td><td>u</td><td>d</td><td>u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="2">21,88</td><td colspan="2">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td>49,02</td><td>49,02</td><td>45,74</td><td>30,82</td><td colspan="2">17,97</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td>u</td><td>d</td><td>u</td><td>d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="3">27,34</td><td colspan="2">17,50</td><td colspan="3">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td>B</td><td>A</td><td>B</td><td>A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>45,05</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

FIRST BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	361,23
u	1,25	C_B	5,00	I	50,00	Ea	323,61
d	0,80	F	50,00	X	0	Da	37,62
r_f	2,000%	r_m	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE FIRMS

<table>
<tr><td colspan="12">17,50</td></tr>
<tr><td colspan="6">A - LEADER</td><td colspan="6">B - FOLLOWER</td></tr>
<tr><td colspan="6">361,23</td><td colspan="6">164,16</td></tr>
<tr><td colspan="6">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">21,88</td><td colspan="6">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">735,44</td><td colspan="2">345,60</td><td colspan="2">192,33</td><td colspan="4">81,95</td></tr>
<tr><td colspan="5">u</td><td>d</td><td>u</td><td colspan="5">d</td></tr>
<tr><td colspan="5">27,34</td><td colspan="2">17,50</td><td colspan="5">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="2">B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.438,03</td><td colspan="2">694,99</td><td>420,04</td><td>188,16</td><td colspan="2">88,75</td><td colspan="3">33,40</td></tr>
<tr><td colspan="4">u</td><td>d</td><td>u</td><td>d</td><td>u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="2">21,88</td><td colspan="2">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.704,36</td><td colspan="2">1.327,67</td><td>872,89</td><td>411,93</td><td>215,15</td><td>86,34</td><td colspan="2">30,72</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td>u</td><td>d</td><td>u</td><td>d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="3">27,34</td><td colspan="2">17,50</td><td colspan="3">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td>B</td><td>A</td><td>B</td><td>A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.953,25</td><td>2.451,62</td><td colspan="2">1.705,15</td><td>827,58</td><td>499,32</td><td>224,66</td><td>85,14</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

FIRST BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	361,23
u	1,25	CB	5,00	I	50,00	Ea	323,61
d	0,80	F	50,00	X	0	Da	37,62
rf	2,000%	rm	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A - LEADER</td><td colspan="8">B - FOLLOWER</td></tr>
<tr><td colspan="8">323,61</td><td colspan="8">137,23</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">688,33</td><td colspan="4">306,48</td><td colspan="4">158,15</td><td colspan="4">60,07</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.389,98</td><td colspan="3">646,93</td><td colspan="2">371,98</td><td colspan="2">152,16</td><td colspan="3">60,22</td><td colspan="3">17,65</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.655,34</td><td colspan="2">1.278,65</td><td colspan="2">823,87</td><td colspan="2">362,91</td><td colspan="2">166,13</td><td colspan="2">55,53</td><td colspan="2">11,17</td><td colspan="2">0,00</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.903,25</td><td>2.401,62</td><td colspan="2">1.655,15</td><td colspan="2">777,58</td><td colspan="2">449,32</td><td colspan="2">174,66</td><td colspan="2">35,14</td><td colspan="2">0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

FIRST BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	361,23
u	1,25	CB	5,00	I	50,00	Ea	323,61
d	0,80	F	50,00	X	0	Da	37,62
rf	2,000%	rm	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A - LEADER</td><td colspan="8">B - FOLLOWER</td></tr>
<tr><td colspan="8">37,62</td><td colspan="8">26,93</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">47,12</td><td colspan="4">39,13</td><td colspan="4">34,18</td><td colspan="4">21,88</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="3">48,06</td><td colspan="2">48,06</td><td colspan="2">36,00</td><td colspan="3">28,53</td><td colspan="3">15,75</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">30,82</td><td colspan="2">19,55</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

SECOND BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	346,40
u	1,25	C_B	5,00	I	50,00	Ea	310,61
d	0,80	F	50,00	X	0	Da	35,79
r_f	2,000%	r_m	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A - LEADER</td><td colspan="8">B - FOLLOWER</td></tr>
<tr><td colspan="8">346,40</td><td colspan="8">164,16</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">723,79</td><td colspan="4">345,60</td><td colspan="4">175,55</td><td colspan="4">81,95</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.438,03</td><td colspan="3">694,99</td><td colspan="2">402,44</td><td colspan="2">188,16</td><td colspan="3">71,86</td><td colspan="3">33,40</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.704,36</td><td colspan="2">1.327,67</td><td colspan="2">872,89</td><td colspan="2">411,93</td><td colspan="2">188,59</td><td colspan="2">86,34</td><td colspan="2">17,97</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.953,25</td><td>2.451,62</td><td colspan="2">1.705,15</td><td colspan="2">827,58</td><td colspan="2">499,32</td><td colspan="2">224,66</td><td colspan="2">45,05</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

SECOND BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	346,40
u	1,25	CB	5,00	I	50,00	Ea	310,61
d	0,80	F	50,00	X	0	Da	35,79
rf	2,000%	rm	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A - LEADER</td><td colspan="8">B - FOLLOWER</td></tr>
<tr><td colspan="8">310,61</td><td colspan="8">137,23</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">678,11</td><td colspan="4">306,48</td><td colspan="4">143,44</td><td colspan="4">60,07</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">1.389,98</td><td colspan="3">646,93</td><td colspan="2">356,56</td><td colspan="2">152,16</td><td colspan="3">45,41</td><td colspan="3">17,65</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">2.655,34</td><td colspan="2">1.278,65</td><td colspan="2">823,87</td><td colspan="2">362,91</td><td colspan="2">142,85</td><td colspan="2">55,53</td><td colspan="2">0,00</td><td colspan="2">0,00</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>4.903,25</td><td>2.401,62</td><td colspan="2">1.655,15</td><td colspan="2">777,58</td><td colspan="2">449,32</td><td colspan="2">174,66</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

SECOND BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	346,40
u	1,25	CB	5,00	I	50,00	Ea	310,61
d	0,80	F	50,00	X	0	Da	35,79
rf	2,000%	rm	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE ADITIONAL EQUITY ISSUE

<table>
<tr><td colspan="16" align="center">17,50</td></tr>
<tr><td colspan="8" align="center">A - LEADER</td><td colspan="8" align="center">B - FOLLOWER</td></tr>
<tr><td colspan="8" align="center">18,49</td><td colspan="8" align="center">0,00</td></tr>
<tr><td colspan="8" align="center">u</td><td colspan="8" align="center">d</td></tr>
<tr><td colspan="8" align="center">21,88</td><td colspan="8" align="center">14,00</td></tr>
<tr><td colspan="4" align="center">A</td><td colspan="4" align="center">B</td><td colspan="4" align="center">A</td><td colspan="4" align="center">B</td></tr>
<tr><td colspan="4" align="center">14,54</td><td colspan="4" align="center">0,00</td><td colspan="4" align="center">20,93</td><td colspan="4" align="center">0,00</td></tr>
<tr><td colspan="6" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="6" align="center">d</td></tr>
<tr><td colspan="6" align="center">27,34</td><td colspan="4" align="center">17,50</td><td colspan="6" align="center">11,20</td></tr>
<tr><td colspan="3" align="center">A</td><td colspan="3" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="3" align="center">A</td><td colspan="3" align="center">B</td></tr>
<tr><td colspan="3" align="center">0,00</td><td colspan="3" align="center">0,00</td><td colspan="2" align="center">21,94</td><td colspan="2" align="center">0,00</td><td colspan="3" align="center">21,06</td><td colspan="3" align="center">0,00</td></tr>
<tr><td colspan="4" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="4" align="center">d</td></tr>
<tr><td colspan="4" align="center">34,18</td><td colspan="4" align="center">21,88</td><td colspan="4" align="center">14,00</td><td colspan="4" align="center">8,96</td></tr>
<tr><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td></tr>
<tr><td colspan="2" align="center">0,00</td><td colspan="2" align="center">0,00</td><td colspan="2" align="center">0,00</td><td colspan="2" align="center">0,00</td><td colspan="2" align="center">33,12</td><td colspan="2" align="center">0,00</td><td colspan="2" align="center">15,90</td><td colspan="2" align="center">0,00</td></tr>
<tr><td colspan="2" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="2" align="center">d</td><td colspan="2" align="center">u</td><td colspan="2" align="center">d</td></tr>
<tr><td colspan="2" align="center">42,72</td><td colspan="4" align="center">27,34</td><td colspan="4" align="center">17,50</td><td colspan="4" align="center">11,20</td><td colspan="2" align="center">7,17</td></tr>
<tr><td align="center">A</td><td align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td colspan="2" align="center">A</td><td colspan="2" align="center">B</td><td align="center">A</td><td align="center">B</td></tr>
<tr><td></td><td></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2" align="center">50,00</td><td colspan="2"></td><td></td><td></td></tr>
</table>

SECOND BEST POLICY UNDER STACKELBERG EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	346,40
u	1,25	CB	5,00	I	50,00	Ea	310,61
d	0,80	F	50,00	X	0	Da	35,79
rf	2,000%	rm	8,000%			Vb	164,16
p	0,32	τ	25,000%			Eb	137,23
						Db	26,93

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="16">17,50</td></tr>
<tr><td colspan="8">A</td><td colspan="8">B</td></tr>
<tr><td colspan="8">35,79</td><td colspan="8">26,93</td></tr>
<tr><td colspan="8">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">21,88</td><td colspan="8">14,00</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">45,68</td><td colspan="4">39,13</td><td colspan="4">32,11</td><td colspan="4">21,88</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">27,34</td><td colspan="4">17,50</td><td colspan="6">11,20</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">48,06</td><td colspan="3">48,06</td><td colspan="2">45,88</td><td colspan="2">36,00</td><td colspan="3">26,45</td><td colspan="3">15,75</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="4">8,96</td></tr>
<tr><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td></tr>
<tr><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">45,74</td><td colspan="2">30,82</td><td colspan="2">17,97</td><td colspan="2">8,99</td></tr>
<tr><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td></tr>
<tr><td colspan="2">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="2">7,17</td></tr>
<tr><td>A</td><td>B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td>A</td><td>B</td></tr>
<tr><td>50,00</td><td>50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">50,00</td><td colspan="2">45,05</td><td colspan="2">22,52</td><td>5,51</td><td>2,75</td></tr>
</table>

FIRST BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	317,03
u	1,25	C_B	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
r_f	2,000%	r_m	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE FIRMS

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">317,03</td><td colspan="10">317,03</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">648,49</td><td colspan="4">648,49</td><td colspan="4">167,34</td><td colspan="6">167,34</td></tr>
<tr><td colspan="8">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">1.272,91</td><td colspan="4">1.272,91</td><td colspan="2">368,03</td><td colspan="2">368,03</td><td colspan="4">75,99</td><td colspan="4">75,99</td></tr>
<tr><td colspan="6">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">2.398,43</td><td colspan="3">2.398,43</td><td colspan="2">770,45</td><td colspan="2">770,45</td><td colspan="2">185,79</td><td colspan="2">185,79</td><td colspan="3">25,54</td><td colspan="3">25,54</td></tr>
<tr><td colspan="4">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="2">d</td><td colspan="2">u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>4.397,33</td><td>4.953,25</td><td>2.451,62</td><td>1.482,44</td><td>1.510,13</td><td>1.705,15</td><td>827,58</td><td>520,04</td><td>438,28</td><td>499,32</td><td>224,66</td><td>162,76</td><td>70,13</td><td>85,14</td><td>22,52</td><td>40,04</td><td>0,00</td><td>5,51</td><td>2,75</td><td>4,90</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>4.397,33</td><td>2.451,62</td><td>4.953,25</td><td>1.482,44</td><td>1.510,13</td><td>827,58</td><td>1.705,15</td><td>520,04</td><td>438,28</td><td>224,66</td><td>499,32</td><td>162,76</td><td>70,13</td><td>22,52</td><td>85,14</td><td>40,04</td><td>0,00</td><td>2,75</td><td>5,51</td><td>4,90</td></tr>
</table>

FIRST BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	317,03
u	1,25	CB	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
rf	2,000%	rm	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE EQUITY OF THE FIRMS

17,50																			
A										**B**									
279,52										**279,52**									
u										**d**									
21,88										**14,00**									
A						B				A				B					
601,38						601,38				133,34				133,34					
u								**d**		**u**		**d**							
27,34								**17,50**				**11,20**							
A				B				A		B		A				B			
1.224,85				1.224,85				319,97		319,97		47,72				47,72			
u						**d**		**u**		**d**		**u**		**d**					
34,18						**21,88**				**14,00**				**8,96**					
A			B			A		B		A		B		A			B		
2.349,41			2.349,41			721,43		721,43		136,78		136,78		6,40			6,40		
u				**d**		**u**		**d**		**u**		**d**		**u**		**d**			
42,72				**27,34**				**17,50**				**11,20**				**7,17**			
A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D
4.347,33	4.903,25	2.401,62	1.432,44	**1.460,13**	1.655,15	777,58	470,04	**388,28**	449,32	174,66	112,76	**20,13**	35,14	0,00	0,00	0,00	0,00	0,00	**0,00**
B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D
4.347,33	2.401,62	4.903,25	1.432,44	**1.460,13**	777,58	1.655,15	470,04	**388,28**	174,66	449,32	112,76	**20,13**	0,00	35,14	0,00	0,00	0,00	0,00	**0,00**

FIRST BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	317,03
u	1,25	CB	5,00	I	50,00	Ea	279,52
d	0,80	F	50,00	X	0	Da	37,51
rf	2,000%	rm	8,000%			Vb	317,03
p	0,32	τ	25,000%			Eb	279,52
						Db	37,51

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">37,51</td><td colspan="10">37,51</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">47,12</td><td colspan="4">47,12</td><td colspan="4">34,00</td><td colspan="6">34,00</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">48,06</td><td colspan="4">48,06</td><td colspan="2">48,06</td><td colspan="2">48,06</td><td colspan="4">28,26</td><td colspan="4">28,26</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">49,02</td><td colspan="3">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="3">19,14</td><td colspan="3">19,14</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>22,52</td><td>40,04</td><td>0,00</td><td>5,51</td><td>2,75</td><td>4,90</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>22,52</td><td>50,00</td><td>40,04</td><td>0,00</td><td>2,75</td><td>5,51</td><td>4,90</td></tr>
</table>

SECOND BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	305,91
u	1,25	C_B	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
r_f	2,000%	r_m	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE FIRMS

<table>
<tr><td colspan="20">**17,50**</td></tr>
<tr><td colspan="10">**A**</td><td colspan="10">**B**</td></tr>
<tr><td colspan="10">**305,91**</td><td colspan="10">**305,91**</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">**21,88**</td><td colspan="10">**14,00**</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">639,75</td><td colspan="4">639,75</td><td colspan="4">154,75</td><td colspan="6">154,75</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">**27,34**</td><td colspan="4">**17,50**</td><td colspan="8">**11,20**</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">1.272,91</td><td colspan="4">1.272,91</td><td colspan="2">354,82</td><td colspan="2">354,82</td><td colspan="4">63,31</td><td colspan="4">63,31</td></tr>
<tr><td colspan="6">**u**</td><td colspan="4">**d u**</td><td colspan="4">**d u**</td><td colspan="6">**d**</td></tr>
<tr><td colspan="6">**34,18**</td><td colspan="4">**21,88**</td><td colspan="4">**14,00**</td><td colspan="6">**8,96**</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">2.398,43</td><td colspan="3">2.398,43</td><td colspan="2">770,45</td><td colspan="2">770,45</td><td colspan="2">165,87</td><td colspan="2">165,87</td><td colspan="3">15,97</td><td colspan="3">15,97</td></tr>
<tr><td colspan="4">**u**</td><td colspan="4">**d u**</td><td colspan="4">**d u**</td><td colspan="4">**d u**</td><td colspan="4">**d**</td></tr>
<tr><td colspan="4">**42,72**</td><td colspan="4">**27,34**</td><td colspan="4">**17,50**</td><td colspan="4">**11,20**</td><td colspan="4">**7,17**</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>**4.397,33**</td><td>4.953,25</td><td>2.451,62</td><td>1.482,44</td><td>**1.510,13**</td><td>1.705,15</td><td>827,58</td><td>520,04</td><td>**438,28**</td><td>499,32</td><td>224,66</td><td>162,76</td><td>70,13</td><td>85,14</td><td>22,52</td><td>**40,04**</td><td>0,00</td><td>5,51</td><td>2,75</td><td>**4,90**</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>**4.397,33**</td><td>2.451,62</td><td>4.953,25</td><td>1.482,44</td><td>**1.510,13**</td><td>827,58</td><td>1.705,15</td><td>520,04</td><td>**438,28**</td><td>224,66</td><td>499,32</td><td>162,76</td><td>70,13</td><td>22,52</td><td>85,14</td><td>**40,04**</td><td>0,00</td><td>2,75</td><td>5,51</td><td>**4,90**</td></tr>
</table>

SECOND BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">272,08</td><td colspan="10">272,08</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">595,53</td><td colspan="4">595,53</td><td colspan="4">124,92</td><td colspan="6">124,92</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">1.224,85</td><td colspan="4">1.224,85</td><td colspan="2">311,14</td><td colspan="2">311,14</td><td colspan="4">39,25</td><td colspan="4">39,25</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">2.349,41</td><td colspan="3">2.349,41</td><td colspan="2">721,43</td><td colspan="2">721,43</td><td colspan="2">123,44</td><td colspan="2">123,44</td><td colspan="3">0,00</td><td colspan="3">0,00</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>4.347,33</td><td>4.903,25</td><td>2.401,62</td><td>1.432,44</td><td>1.460,13</td><td>1.655,15</td><td>777,58</td><td>470,04</td><td>388,28</td><td>449,32</td><td>174,66</td><td>112,76</td><td>20,13</td><td>35,14</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>4.347,33</td><td>2.401,62</td><td>4.903,25</td><td>1.432,44</td><td>1.460,13</td><td>777,58</td><td>1.655,15</td><td>470,04</td><td>388,28</td><td>174,66</td><td>449,32</td><td>112,76</td><td>20,13</td><td>0,00</td><td>35,14</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

SECOND BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE ADITIONAL EQUITY ISSUE

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">18,49</td><td colspan="10">18,49</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">14,54</td><td colspan="4">14,54</td><td colspan="4">20,93</td><td colspan="6">20,93</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">0,00</td><td colspan="4">0,00</td><td colspan="2">21,94</td><td colspan="2">21,94</td><td colspan="4">21,06</td><td colspan="4">21,06</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">0,00</td><td colspan="3">0,00</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">33,12</td><td colspan="2">33,12</td><td colspan="3">15,90</td><td colspan="3">15,90</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>50,00</td><td>50,00</td><td>0,00</td><td>0,00</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>50,00</td><td>0,00</td><td>50,00</td><td>0,00</td><td></td><td></td><td></td><td></td></tr>
</table>

SECOND BEST POLICY UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	305,91
u	1,25	CB	5,00	I	50,00	Ea	272,08
d	0,80	F	50,00	X	0	Da	33,82
rf	2,000%	rm	8,000%			Vb	305,91
p	0,32	τ	25,000%			Eb	272,08
						Db	33,82

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="20">**17,50**</td></tr>
<tr><td colspan="10">**A**</td><td colspan="10">**B**</td></tr>
<tr><td colspan="10">**33,82**</td><td colspan="10">**33,82**</td></tr>
<tr><td colspan="10">**u**</td><td colspan="10">**d**</td></tr>
<tr><td colspan="10">**21,88**</td><td colspan="10">**14,00**</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">44,22</td><td colspan="4">44,22</td><td colspan="4">29,83</td><td colspan="6">29,83</td></tr>
<tr><td colspan="8">**u**</td><td colspan="4">**d** **u**</td><td colspan="8">**d**</td></tr>
<tr><td colspan="8">**27,34**</td><td colspan="4">**17,50**</td><td colspan="8">**11,20**</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">48,06</td><td colspan="4">48,06</td><td colspan="2">43,69</td><td colspan="2">43,69</td><td colspan="4">24,07</td><td colspan="4">24,07</td></tr>
<tr><td colspan="6">**u**</td><td colspan="4">**d** **u**</td><td colspan="4">**d** **u**</td><td colspan="6">**d**</td></tr>
<tr><td colspan="6">**34,18**</td><td colspan="4">**21,88**</td><td colspan="4">**14,00**</td><td colspan="6">**8,96**</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">49,02</td><td colspan="3">49,02</td><td colspan="2">49,02</td><td colspan="2">49,02</td><td colspan="2">42,42</td><td colspan="2">42,42</td><td colspan="3">15,97</td><td colspan="3">15,97</td></tr>
<tr><td colspan="4">**u**</td><td colspan="4">**d** **u**</td><td colspan="2">**d**</td><td colspan="2">**u**</td><td colspan="4">**d** **u**</td><td colspan="4">**d**</td></tr>
<tr><td colspan="4">**42,72**</td><td colspan="4">**27,34**</td><td colspan="4">**17,50**</td><td colspan="4">**11,20**</td><td colspan="4">**7,17**</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>22,52</td><td>40,04</td><td>0,00</td><td>5,51</td><td>2,75</td><td>4,90</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>22,52</td><td>50,00</td><td>40,04</td><td>0,00</td><td>2,75</td><td>5,51</td><td>4,90</td></tr>
</table>

MIXED POLICIES UNDER GENERAL EQUILIBRIUM

θ	17,50	C_A	5,00	g	3,00	Va	322,58
u	1,25	C_B	5,00	I	50,00	Ea	322,58
d	0,80	F	50,00	X	0	Da	0,00
r_f	2,000%	r_m	8,000%			Vb	299,43
p	0,32	τ	25,000%			Eb	272,08
						Db	27,35

VALUE OF THE FIRMS

17,50																			
A										**B**									
322,58										**299,43**									
u										d									
21,88										**14,00**									
A						B				A				B					
652,86						634,65				173,63				147,42					
u								d u				d							
27,34								**17,50**				**11,20**							
A				B				A		B		A				B			
1.272,91				1.272,91				374,62		347,14		82,31				55,94			
u						**d u**				**d u**				**d**					
34,18						**21,88**				**14,00**				**8,96**					
A			B			A		B		A		B		A			B		
2.398,43			2.398,43			770,45		770,45		195,74		154,26		30,31			10,40		
u				**d u**				**d u**				**d u**				**d**			
42,72				**27,34**				**17,50**				**11,20**				**7,17**			
A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D	A - I	A - I	A - D	A - D
4.397,33	4.953,25	2.451,62	1.482,44	**1.510,13**	1.705,15	827,58	520,04	**438,28**	499,32	224,66	162,76	70,13	**85,14**	22,52	40,04	0,00	5,51	2,75	**4,90**
B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D	B - I	B - D
4.397,33	2.451,62	4.953,25	1.482,44	**1.510,13**	827,58	1.705,15	520,04	**438,28**	224,66	499,32	162,76	70,13	**22,52**	85,14	40,04	0,00	2,75	5,51	**4,90**

MIXED POLICIES UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	322,58
u	1,25	CB	5,00	I	50,00	Ea	322,58
d	0,80	F	50,00	X	0	Da	0,00
rf	2,000%	rm	8,000%			Vb	299,43
p	0,32	τ	25,000%			Eb	272,08
						Db	27,35

VALUE OF THE EQUITY OF THE FIRMS

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">322,58</td><td colspan="10">272,08</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">652,86</td><td colspan="4">595,53</td><td colspan="4">173,63</td><td colspan="6">124,92</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">1.272,91</td><td colspan="4">1.224,85</td><td colspan="2">374,62</td><td colspan="2">311,14</td><td colspan="4">82,31</td><td colspan="4">39,25</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">2.398,43</td><td colspan="3">2.349,41</td><td colspan="2">770,45</td><td colspan="2">721,43</td><td colspan="2">195,74</td><td colspan="2">123,44</td><td colspan="3">30,31</td><td colspan="3">0,00</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>4.397,33</td><td>4.953,25</td><td>2.451,62</td><td>1.482,44</td><td>1.510,13</td><td>1.705,15</td><td>827,58</td><td>520,04</td><td>438,28</td><td>499,32</td><td>224,66</td><td>162,76</td><td>70,13</td><td>85,14</td><td>22,52</td><td>40,04</td><td>0,00</td><td>5,51</td><td>2,75</td><td>4,90</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>4.347,33</td><td>2.401,62</td><td>4.903,25</td><td>1.432,44</td><td>1.460,13</td><td>777,58</td><td>1.655,15</td><td>470,04</td><td>388,28</td><td>174,66</td><td>449,32</td><td>112,76</td><td>20,13</td><td>0,00</td><td>35,14</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td></tr>
</table>

MIXED POLICIES UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	322,58
u	1,25	CB	5,00	I	50,00	Ea	322,58
d	0,80	F	50,00	X	0	Da	0,00
rf	2,000%	rm	8,000%			Vb	299,43
p	0,32	τ	25,000%			Eb	272,08
						Db	27,35

VALUE OF THE ADITIONAL EQUITY ISSUE

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">0,00</td><td colspan="10">18,49</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">0,00</td><td colspan="4">14,54</td><td colspan="4">0,00</td><td colspan="6">20,93</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">0,00</td><td colspan="4">0,00</td><td colspan="2">0,00</td><td colspan="2">21,94</td><td colspan="4">0,00</td><td colspan="4">21,06</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">0,00</td><td colspan="3">0,00</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">0,00</td><td colspan="2">33,12</td><td colspan="3">0,00</td><td colspan="3">15,90</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td></td><td></td><td></td><td></td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>0,00</td><td>50,00</td><td>0,00</td><td>50,00</td><td></td><td></td><td></td><td></td></tr>
</table>

MIXED POLICIES UNDER GENERAL EQUILIBRIUM

θ	17,50	CA	5,00	g	3,00	Va	322,58
u	1,25	CB	5,00	I	50,00	Ea	322,58
d	0,80	F	50,00	X	0	Da	0,00
rf	2,000%	rm	8,000%			Vb	299,43
p	0,32	τ	25,000%			Eb	272,08
						Db	27,35

VALUE OF THE DEBT OF THE FIRMS

<table>
<tr><td colspan="20">17,50</td></tr>
<tr><td colspan="10">A</td><td colspan="10">B</td></tr>
<tr><td colspan="10">0,00</td><td colspan="10">27,35</td></tr>
<tr><td colspan="10">u</td><td colspan="10">d</td></tr>
<tr><td colspan="10">21,88</td><td colspan="10">14,00</td></tr>
<tr><td colspan="6">A</td><td colspan="4">B</td><td colspan="4">A</td><td colspan="6">B</td></tr>
<tr><td colspan="6">0,00</td><td colspan="4">39,13</td><td colspan="4">0,00</td><td colspan="6">22,50</td></tr>
<tr><td colspan="8">u</td><td colspan="4">d u</td><td colspan="8">d</td></tr>
<tr><td colspan="8">27,34</td><td colspan="4">17,50</td><td colspan="8">11,20</td></tr>
<tr><td colspan="4">A</td><td colspan="4">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="4">A</td><td colspan="4">B</td></tr>
<tr><td colspan="4">0,00</td><td colspan="4">48,06</td><td colspan="2">0,00</td><td colspan="2">36,00</td><td colspan="4">0,00</td><td colspan="4">16,69</td></tr>
<tr><td colspan="6">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="6">d</td></tr>
<tr><td colspan="6">34,18</td><td colspan="4">21,88</td><td colspan="4">14,00</td><td colspan="6">8,96</td></tr>
<tr><td colspan="3">A</td><td colspan="3">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="2">A</td><td colspan="2">B</td><td colspan="3">A</td><td colspan="3">B</td></tr>
<tr><td colspan="3">0,00</td><td colspan="3">49,02</td><td colspan="2">0,00</td><td colspan="2">49,02</td><td colspan="2">0,00</td><td colspan="2">30,82</td><td colspan="3">0,00</td><td colspan="3">10,40</td></tr>
<tr><td colspan="4">u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d u</td><td colspan="4">d</td></tr>
<tr><td colspan="4">42,72</td><td colspan="4">27,34</td><td colspan="4">17,50</td><td colspan="4">11,20</td><td colspan="4">7,17</td></tr>
<tr><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td><td>A - I</td><td>A - I</td><td>A - D</td><td>A - D</td></tr>
<tr><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td><td>0,00</td></tr>
<tr><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td><td>B - I</td><td>B - D</td></tr>
<tr><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>50,00</td><td>22,52</td><td>50,00</td><td>40,04</td><td>0,00</td><td>2,75</td><td>5,51</td><td>4,90</td></tr>
</table>