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FISCAL MULTIPLIERS AND NON-SEPARABLE PREFERENCES IN A SMALL OPEN ECONOMY MODEL

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Abstract In Euro area countries fiscal policy plays a central role in the stabilization of business cycles mainly because monetary policy is defined for the whole area. A small open economy model is employed to assess the consequences of government expenditure shocks. The main novelty of this thesis is the inclusion of non-separable preferences between private consumption and government expenditure. Numerical simulation suggests that fiscal policy is more effective in a fixed exchange regime. In this regime, fiscal multipliers reach 1.6% on impact and 1% in the long-run.

Resumo Nos países da zona Euro a política orçamental tem um papel chave na estabilização dos ciclos económicos principalmente porque a política monetária é definida para o conjunto das economias. Um modelo de pequenas economias abertas é utilizado para avaliar as consequências de um choque nos gastos do governo. O contributo principal desta tese é a introdução de preferências não separáveis entre o consumo privado e os gastos do Estado. Uma simulação numérica sugere que a política orçamental é mais eficaz num regime de câmbios fixos. Neste regime, os multiplicadores orçamentais atingem 1.6% no impacto e 1% no longo prazo.

Keywords: fiscal policy, fiscal multiplier, non-separable preferences, small open economy, exchange rate regime, new Keynesian, DSGE model.
JEL Classification: E62, F41.

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Contents

1 Introduction

1.1 Motivation

The ability of governments to stimulate the economy has been considerably discussed in the literature. Recently, the consequences of the last worldwide crisis and the subsequent coordinated stimuli packages (e.g., American Recovery and Reinvestment Act of 2009 (ARRA) and European Economic Recovery Plan (EERP)) came under scrutiny. Articles gauging potential effects of government spending on the economy were written and theoretical models were adjusted to empirical findings.

The decision of using countercyclical fiscal policy is usually based on the concept of the fiscal multiplier, i.e., the potential increase in output following an expansion in government expenditures. Moreover, the response of private consumption to a government expenditure shock has played a central role in the discussion because private consumption is the largest component of aggregate demand and consequently the main driver of the multiplier. Furthermore, fiscal policy effectiveness is mostly linked to the complementarity or substitutability between private and public expenditures. If public-private consumption are complements, households experience higher utility if both are consumed together. Thus, private consumption is likely to increase with a public expenditure shock making fiscal policy effective. Otherwise, if substitutes, private consumption would be expected to decrease in response to a government stimulus.

In the aftermath of World War II, worldwide trade experienced a steep growth. Trade agreements, such as GATT, were signed aiming to promote the liberalisation of commerce and the abolition of trade barriers. During the 90's, the Maastricht Treaty created a single European market. This step led to more integration across economies and to an expansion of international relations inside EU. This development was a key contribution to the globalization process and impacted the transmission of fiscal policy.[1] As a consequence, each

[1] Milani (2012) discusses the impact of globalization in macroeconomic models.

time government aims to stimulate economy and purchases goods abroad, there is a leakage of potential benefits to foreign economic agents. In other words, a share of the domestically financed stimulus is very often allocated to imports which, in the end, stimulate other countries. This possibility might induce a free-riding behaviour in countries benefiting from foreign fiscal stimuli. These countries receive a share of others' fiscal stimulus via higher imports. In the end, this might lead countries to refrain from a larger stimulus. However, it should be highlighted that exports to foreign countries can also play a determinant role offsetting imports and counter-balance the negative cross-border spillovers. This is why the trade balance is commonly referred as an important instrument to gauge the effectiveness of fiscal policy. As Chinn (2013) and Chen et al. (2013) observe, most open economy models assume countries with a balanced current account. Though, an expansionary stimulus works differently depending whether the economy is balanced or unbalanced.

This investigation begins with a comprehensive literature review aiming to formulate a cutting-edge research question. First, an historical overview is made where earlier fiscal policy in open economy models are revisited. Afterwards is made an exhaustive empirical discussion of fiscal multipliers. Finally, recent theoretical literature is reviewed and shortcomings are identified. After having a clear picture of empirical and theoretical literature and being sure of some theoretical gaps that could be improved, a research question emerges. Are non-separable preferences able to properly reproduce empirically documented private-public consumption synchronism? To answer this question, Galí (2008, ch.7) model is augmented with two main novelties: (i) non-separable preferences over private consumption and government expenditure and (ii) private consumption and government expenditure with asymmetric propensities to import. The model is calibrated to the Portuguese economy. Special attention is given to the calculation of the private and public propensities to import. The main purpose of this thesis is to understand the effects of fiscal policy within a small open economy (SOE) and respective interaction with the currency union, similar to Portugal in the context of European Monetary Union (EMU).

The impulse response functions obtained in the numerical simulation for fixed exchange regime show private consumption and output responding more than one-for-one to an exogenous government expenditure shock. The transmission of the fiscal shock is made primarily through private consumption because it is bundled with government expenditure in a non-separable relation. In addition to the non-separability assumption, modelling private consumption and government expenditure as complements produces a strong reaction of private consumption. This reaction is strengthened by: (i) the degree of complementarity and (ii) the fixed exchange regime. Moreover, the model confirms a well-known result in the literature, fiscal policy is more effective in the fixed exchange regime. In this regime, fiscal multipliers reach 1.6% on impact and 1% in the long-run. In the end, a main policy implication is derived, fiscal stimuli should be directed to sectors with high degree of complementarity relative to private consumption and sectors with low propensity to import to avoid leakages of the stimulus to foreign countries.

1.2 Related Literature

As economies are becoming more interdependent, it is natural to use an open economy model to investigate the consequences of fiscal stimuli. The Mundell-Fleming model, dating from the 60's, is considered the first step-through to analyse the impact of fiscal policy in open economy context. About two decades later, in the 80's, the discussion about the effects of government purchases on economy is enhanced with several seminal contributions. Aschauer (1985), Karras (1994), Ni (1995) and Amano & Wirjanto (1998) published articles assessing the relation between private-public consumption but results were mixed and inconclusive. Barro (1981), Aiyagari et al. (1992), Baxter & King (1993) and Finn (1998) built simple neoclassical models to analyse the impact of temporary and persistent increases in government expenditure. According to Real Business Cycle (henceforth RBC) theory, when a government increases expenditure to stimulate the economy, output increases as well but there is a negative wealth effect where forward-looking households increase savings (decrease current consumption) to respond to future tax burdens. To counter-act the decrease in permanent income, households increase labour supply (to increase consumption) but this increment is insufficient to offset the wealth effect. RBC theory assumes that government budget is intertemporally optimized and debts are always paid. Thus, an increase in current spending will lead to an increase in taxes if not today, tomorrow. This mechanism, commonly referred as Ricardian equivalence, is an implication of most theoretical models assumptions.

Recently, already in the 21st century, these above-cited mainstream contributions were questioned in the light of empirical observations. Empirical literature on fiscal policy is focused at the size of fiscal multipliers, i.e., whether the reaction of GDP exceeds government stimulus. A first step to understand the transmission mechanism of fiscal shocks is to look at the feedback of private consumption to an increase in government expenditures. Blanchard & Perotti (2002), Fatás & Mihov (2001), Galí et al. (2007), Mountford & Uhlig (2009), Monacelli & Perotti (2010) find private consumption rising after a fiscal stimulus. In particular, Fatás & Mihov (2001) and Mountford & Uhlig (2009) document long-run fiscal multipliers around 1%. This conclusion supports the Keynesian rather than the neoclassical literature. Perotti (2005) using a panel of OECD countries estimated fiscal policy having a tepid impact on GDP, although, its impact has been rising since 1980's period. Barro & Redlick (2009), Hall (2009) and Ramey (2011b) estimate fiscal multipliers below or close to one which suggests that fiscal policy is ineffective in stimulating economy. Contrary to these findings, Beetsma et al. (2008) and Fisher & Peters (2010) find multipliers higher than one, in particular the authors estimate a peak multiplier of 1.6 and 1.5, respectively. Blanchard & Leigh (2013) and Coenen et al. (2013) assess the recent crisis episode and find that government consumption multipliers larger than one but the effects dissipate once the stimulus is removed. Only one conclusion emerges from this analysis, fiscal multipliers estimates depend on many aspects (e.g., methodology used to identify fiscal shocks and data which might vary in terms of periods or countries). As a result, fiscal multipliers do not converge to a single conclusion and leave the policy maker in trouble. Latest articles about fiscal multipliers focus on specific aspects that are worth analysing, such as (i) the degree of openness, (ii) cross-border fiscal spillovers, (iii) the current account balance, (iv) exchange rate regime and (v) business cycles and zero lower bound.

The globalization process that the world has been experiencing has a significant impact in the transmission of fiscal policy. Likewise, fiscal policy effectiveness is also related to the openness of economies and trade diversification. Open economies, depending on the degree of integration, are more vulnerable to foreign economic shocks. Aizenman & Jinjarak (2012) and Ilzetzki et al. (2013) investigated the impact of openness in fiscal multipliers and concluded that economies comparatively closed to trade (*"through trade barriers or larger internal markets"*[2]) have a fiscal multiplier exceeding one while those more open to trade have negative multipliers. Ilzetzki et al. (2013) estimate long-run multipliers in closed economies around 1 percentage point. Beetsma et al. (2008) find closed economies yielding a stronger answer to the stimulus, contrary to open economies where multipliers never exceed 1.

In open economy context, the Ricardian equivalence is not the only argument against expansionary fiscal policy. Unbalanced small open economies with high propensity to import are likely to leak a share of the government stimulus to foreign countries. Thus, an economy with trade surplus can free-ride and benefit from stimuli in other countries. Beetsma et al. (2006) unveiled that cross-border spillovers from a fiscal stimulus would be greater as economies are more integrated, i.e., when they have higher bilateral trade flows. Forni & Pisani (2010) underline that trade leakages tend to be higher immediately after the stimulus as long as the propensity to import expands more than exports multiplier does. Auerbach & Gorodnichenko (2013) found higher spillovers between two countries when both are in recession. Very often, international fiscal spillovers have as transmission channel the current account. Consequently, studying the role of the current account in the transmission of fiscal policy is crucial.

Economies are no longer closed, therefore current account imbalances have a substantial

[2] Ilzetzki et al. (2013, p.240)

impact in the transmission of macroeconomic policies. Countries with an unfavourable trade balance should be cautious with fiscal policy. Although government expenditure is more home-biased than private consumption, the current account response to a fiscal stimulus is still puzzling. It depends on two forces, the complementarity or substitutability between private-public consumption, but also on the characteristics of domestic and foreign goods (e.g., prices and/ or quality). For example, if the government chooses to purchase, in the respective country or abroad, a good which is a complement to private consumption, then private consumption is expected to rise. However, this increment might come from domestically or foreign produced goods depending on the characteristics of the goods in each country. It is not straightforward to conclude whether a fiscal stimulus will lead to an improvement or deterioration of the current account. This uncertainty about the response of the current account is reflected in the empirical studies subsequently presented. Abbas et al. (2011) conclude that a stimulus of 1 percentage point of GDP improves the current account, on impact, in 0.20 percentage points of GDP. However, M. O. Ravn et al. (2012) find fiscal stimulus producing a deterioration of trade balance. Kim & Roubini (2008) review the stylized fact of "twin deficits", i.e., a pro-cyclical relation between fiscal and current account deficits. The results obtained suggest "twin divergence", i.e., when government budget deteriorates, the current account improves. However, Monacelli & Perotti (2010) disagree with this view as their conclusion points towards the so-called "twin deficit". In the midway of these two perspectives, Corsetti & Müller (2008) find twin deficits more plausible in open economies comparatively to closed economies. The difficulty of measuring the impact of fiscal policy on the current account is clear in these findings.

The transmission of fiscal shocks may still depend on the exchange rate regime. Whether an economy has floating or pegged exchange rate matters a lot for fiscal policy efficacy. Until recently, most theoretical articles assume economies working under a flexible exchange rate regime and empirical research simply ignore the impact of different exchange rate regimes in fiscal multipliers. The second half of last century has some examples of important fixed exchange rate regimes. The Bretton Woods agreement (1944-1971) signed between 45 leading countries established a fixed (but adjustable) exchange rate regime of each currency relative to US dollar. There was also the possibility to convert dollars in gold at a fixed rate. The European Monetary System (EMS), 1979-1998, was based on two mechanisms, the European Currency Union (ECU) and the Exchange Rate Mechanism (ERM) which was itself divided into two instruments, the bilateral exchange rates and the foreign exchange band. In the meantime, European currencies floated relative to US dollar but European central banks manipulated their values. After 1998, the emergence of the EMU put an end to individual monetary and exchange rate policies across Europe creating a single monetary policy and a floating exchange rate relative to US dollar.

The EMU is many times considered a fixed exchange regime. An identical assumption is made in Nakamura & Steinsson (2014) who consider the US economy to be a fixed exchange regime and find fiscal multipliers of 1.5. S. H. Ravn & Spange (2012) examine the impact of fiscal policy in Denmark from 1983 to 2011, a period under currency peg. During this horizon, multipliers reach 1.3 (on impact and cumulative). This means that the stimulus cease once removed. Ilzetzki et al. (2013) found countries with flexible exchange rate having a neutral response to fiscal policy while countries with fixed exchange rates have comparatively higher multipliers. The findings of Corsetti et al. (2012b) and Born et al. (2013) point in the same direction, fiscal multipliers are larger when countries have an exchange rate peg or belong to a currency union.

Conventional macroeconomic theory advocates the use of fiscal policy when all monetary policy benefits are exhausted, that is, as an instrument of last resort. When economic environment start to deteriorate, the central bank should immediately decrease nominal interest rate to drive down real interest rate making economic agents update their expectations and consequently stimulate private consumption and investment. This should be put in place long before public spending is used as an expansionary measure. Although, the short-term interest rate is the most used monetary policy instrument it has some operational limits.

When the interest rate reaches zero and monetary policy is not being effective, the economy enters a liquidity trap. Substantial affordable liquidity is available but economic agents do not demand it. This situation is also referred as zero lower bound (ZLB) which is a slowdown time when the traditional monetary policy instruments are ineffective.

The Japanese experience over the last two decades motivated research about unconventional monetary measures and fiscal policy in liquidity traps. Cutting-edge research supports different effects of fiscal policy over the business cycle and when the nominal interest rate reaches the zero threshold. Auerbach & Gorodnichenko (2012) look at fiscal multipliers over business cycles and confirmed an impact of more than one-for-one during recessions in comparison to expansions where the multiplier is situated in the 0-0.5 interval. Similarly, Hall (2009) estimated a 1.7 government purchases multiplier conditional on the ZLB and Owyang et al. (2013) scan slack periods to document multipliers above unity in Canada but below one in the US. Corsetti et al. (2012b) evaluated the impact of financial crises on fiscal multipliers which were estimated to be 2.3 on impact and 2.9 cumulative. Finally, Bachmann & Sims (2012) analyse the impact of fiscal policy on economic agents confidence and found it rising after a public stimulus. In the end, it is concluded that fiscal policy is an appropriate countercyclical tool as output multipliers exceed one.

Over the years, fiscal multipliers have been extensively discussed in the literature. Reviewing the empirical results leads to the conclusion that literature has a broad range of views to support virtually any policy decision. Ramey (2011a) point that these multiple perspectives are considered an empirical puzzle as they do not converge to a single transmission mechanism.[3] The literature does not agree on the size of fiscal multipliers because researchers use different methodologies to identify and estimate the impact of fiscal shocks. Another heterogeneous issue is that most empirical articles provide estimates for fiscal multipliers but ignore whether they're impact, peak or cumulative values. Each methodology has drawbacks but two conclusions can be drawn from above: (i) generally private consumption rises after

[3] Perotti (2007) and Cogan et al. (2010) survey empirical literature on fiscal multipliers. Namely, the methodology, identification of fiscal shocks, data and alternative specifications of estimators.

a government stimulus (even though its response does not always yield a multiplier higher than one), (ii) fiscal multipliers are higher: in relatively closed economies, in fixed exchange regimes (currency unions), and when monetary policy is constrained by the zero bound.

Despite much debate on the complementarity or substitutability between private consumption and government expenditure, a pattern seems to emerge. Most of the times private consumption rises after expansionary fiscal policy. The increase in private consumption after the stimulus suggests a complementarity relation. However, fiscal multipliers are not necessarily higher than unity because the private consumption response might not be strong enough. As pointed out in Karras (1994) government spending has certain types of rival goods (e.g., free-lunches at school) while others are non-rival (e.g., national defence). Although, the author acknowledges that private consumption and public expenditure are, on average, complements. Monacelli & Perotti (2010) suggested to upgrade open economy models with a mechanism generating a positive response of private consumption whenever the government stimulates economy. One way to embed this suggestion is to include non-separable preferences over private consumption-leisure or private-public consumption. The latter case makes households' utility dependent on a bundle of private-public consumption, referred as effective consumption in the literature. Although both types of (private and public) consumption affect households' utility, households only choose the amount of private consumption. Moreover, private consumption and government expenditure might be complements or substitutes depending on the calibration of the parameters. This relation has a profound impact in the effectiveness of fiscal policy. If government stimuli are directed to goods which are complements to private consumption, then fiscal policy is likely to be effective as private consumption is crowded-in. On the other hand, if government expenditure substitutes private consumption, fiscal policy is not expected to succeed. A relevant research question would be whether non-separable preferences are able to properly reproduce empirically documented private-public consumption synchronism?

During the 90's emerged the New Open Economy Macroeconomics (NOEM) to overcome the shortcomings of Mundell-Fleming model. Obstfeld & Rogoff (1995), as precursor of NOEM, developed a perfect-foresight two-country general equilibrium model with imperfect competition and nominal rigidities in the goods and labour markets. Since then, two-country models evolved and gave origin to new Keynesian SOE models. Galí & Monacelli (2005) built a model where it is assumed a world economy populated by infinite small open economies. Afterwards, Galí & Monacelli (2008) developed this model to accommodate for fiscal policy and study its consequences in a SOE context. Along this article authors analyse the consequences of a fiscal stimulus in a currency union. The same is done in Ferrero (2009) but in a two-country framework. In the last years, theoretical models have been improved and complemented with relevant features such as: (i) rule-of-thumb consumers[4], (ii) habits in private consumption[5], (iii) the effect of zero lower bound[6] and (iv) non-separable preferences.

Following the suggestion of Monacelli & Perotti (2010), some authors incorporate non-separable preferences in theoretical models as a mechanism to generate a positive response of private consumption to government expenditure shocks. Non-separable preferences are included in theoretical models through two common ways, an Edgeworth mechanism or a constant elasticity of substitution (CES) index[7]. For example, private consumption and government expenditure are usually bundled in a new variable, the effective consumption which is represented as follows

$$\text{Edgeworth mechanism: } \widehat{C}_t \equiv (1-\vartheta)C_t + \vartheta G_t,$$

$$\text{CES: } \widehat{C}_t \equiv \left[(1-\vartheta)C_t^{1-\nu} + \vartheta G_t^{1-\nu}\right]^{\frac{1}{1-\nu}}.$$

Cortuk (2013), Bouakez & Rebei (2007), Linnemann & Schabert (2004) and Ercolani &

[4]Among relevant literature on rule-of-thumb consumers see Coenen & Straub (2005), Coenen et al. (2013) Galí et al. (2007) and Forni & Pisani (2010).

[5]Several recent articles introduce habits in private consumption such as Bouakez & Rebei (2007), Fève & Sahuc (2013), Coenen et al. (2013) and M. O. Ravn et al. (2012).

[6]For a representative perspective of recent theoretical literature about fiscal policy at ZLB see, among others, Christiano et al. (2011), Farhi & Werning (2012), Correia et al. (2013), Erceg & Lindé (2014), Fujiwara & Ueda (2013) and Ercolani & Azevedo (2013).

[7]Note that the CES bundle is identical to the Edgeworth mechanism if $\nu = 0$.

Azevedo (2013) use closed economy models with CES non-separable preferences to evaluate fiscal policy outcomes. The first three assume complementarity while the last considers substitutability. Cortuk (2013) instead of using a non-separable relation between private consumption and government purchases as the others, use government production. Ganelli & Tervala (2009) also employ a closed economy paradigm but introduce non-separability via Edgeworth complementarity. Bilbiie (2011) evaluate fiscal policy in a closed economy model with non-separable preferences over private consumption and leisure.

Ganelli (2003), Tervala (2005) and Iwata (2013) apply the Edgeworth mechanism in open economy models to explore the relation between private-public consumption. The first two use the Obstfeld-Rogoff (also called OR) framework and assume substitutability. Though, the last use an alternative two-country framework with complementarity assumption.

Finally, an analysis of the literature above leads to the conclusion that open economy models augmented with non-separable preferences are relatively scarce. Moreover, there is no new Keynesian SOE model measuring the impact of fiscal policy in a fixed exchange regime under non-separability. Furthermore, discussing the linkages of fiscal policy within each exchange rate regime is of interest. Particularly, in currency unions fiscal spillovers is a topic of special concern because stimuli can easily flood to neighbouring countries depending on the propensity to import. Afterwards, fiscal policy in fixed exchange regime is studied using a SOE model.

The remainder of the article is structured as follows: Section 2 describes the structure of a new Keynesian SOE model with non-separable preferences over private consumption and government expenditure. In Section 3, the model is calibrated and results from a numerical simulation are discussed. Finally, Section 4 concludes and points directions to further research.

2 The Model

This section elaborates Galí (2008) model[8] incorporating three main novelties: (i) government expenditure in the utility function, (ii) non-separable preferences over private consumption and government expenditure and (iii) private consumption and government expenditure with asymmetric propensities to import. The model assumes a world economy made of a continuum of infinitesimally SOE's distributed along the [0,1] interval. The SOE framework is also used in previous research such as Galí (2008), Galí & Monacelli (2008) and Benigno & De Paoli (2010). An assumption of the model is that preferences, market structure and technology except productivity shocks are equal across economies. As pointed out in Benigno & De Paoli (2010, p.1524), *"The [small open] economy is perfectly integrated with the rest of the world that is, there are no trade frictions (i.e., the law of one price holds) and capital markets are perfect (i.e., asset markets are complete).*" Market completeness also implies that marginal utilities are identical in every period and all countries.

2.1 Households

Each and every SOE has a representative household maximizing intertemporal utility. The utility function is made of two components, effective consumption ($\widehat{C}_t$) and hours of work (N_t),

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(\widehat{C}_t, N_t\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\widehat{C}_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{2.1}$$

where $\sigma^{-1} > 0$ is the Arrow-Pratt measure of relative risk aversion and the inverse of the intertemporal elasticity of substitution, φ is the inverse of the Frisch elasticity of labour supply and β is the subjective time discount factor. Effective consumption is a composite index of private consumption (C_t) and government expenditure (G_t). A non-separable relation between private and government consumption is assumed, i.e., the utilities of private consumption and government expenditure are no longer independent. An increase in gov-

[8]This section may be complemented reading Galí (2008, ch.7).

ernment expenditures leads to an increase (decrease) of private consumption if complements (substitutes). One of the first studies using non-separable preferences over private and public consumption was Amano & Wirjanto (1998). Such specification is also used in Bouakez & Rebei (2007), Coenen et al. (2013) and Cortuk (2013). Effective consumption is given by

$$\widehat{C}_t \equiv \begin{cases} \left[(1-\vartheta)C_t^{1-\nu} + \vartheta G_t^{1-\nu}\right]^{\frac{1}{1-\nu}}, & \text{for } \nu \neq 1, \\ C_t^{(1-\vartheta)} G_t^{\vartheta}, & \text{for } \nu = 1. \end{cases} \tag{2.2}$$

The parameter $\nu^{-1} > 0$ defines the intratemporal complementarity or substitutability between private consumption and public expenditure and is one of the key parameters for the magnitude of the fiscal multiplier, as well as σ. In the case public-private consumption are considered complements, an increase in one produces an increase in the other while if substitutes, a fiscal stimulus would crowd-out private consumption and make fiscal policy less effective or even ineffective. Following Amano & Wirjanto (1998): if $\sigma^{-1} > \nu^{-1}$, private consumption and government expenditure are Edgeworth-Pareto complements; if $\sigma^{-1} < \nu^{-1}$, then private-pubic consumptions are Edgeworth-Pareto substitutes; and finally if $\sigma^{-1} = \nu^{-1}$, then the goods are unrelated. This last equality also implies that preferences are additively separable in private and public consumption.

The log-linearisation of the system of equations (2.2) take the form

$$\widehat{c}_t \equiv \begin{cases} (1-\vartheta)\left(\dfrac{C}{\widehat{C}}\right)^{1-\nu} c_t + \vartheta \left(\dfrac{G}{\widehat{C}}\right)^{1-\nu} g_t, & \text{for } \nu \neq 1, \\ (1-\vartheta)c_t + \vartheta g_t, & \text{for } \nu = 1, \end{cases} \tag{2.3}$$

where C, G and $\widehat{C}$ are the steady state values of private, public and effective consumption, respectively.[9] Inside effective consumption index, $\widehat{C}_t$, there are two additional sub-indexes, C_t and G_t. The private consumption index is exactly equal to the one presented in Galí &

[9]Please check Appendix A.1 for a detailed linearisation of the expression above.

Monacelli (2005) and Galí (2008),

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{2.4}$$

As above, the complementarity or substitutability between types of consumption is critical to the size of the fiscal multiplier. The same happens in what respects to the relation between domestically and foreign produced goods. If these are complements, an increase of domestic private consumption will lead to an expansion of imports. Whereas if they are substitutes, the reverse happens.

One of the novelties in this work is to assume that government spending has a similar structure to private consumption. Now, contrary to Galí & Monacelli (2008) and Forni & Pisani (2010), there is no home bias in government expenditure, a fraction (χ) is purchased abroad.

$$G_t = \left[(1-\chi)^{\frac{1}{\eta}} \left(G_{H,t}\right)^{\frac{\eta-1}{\eta}} + \chi^{\frac{1}{\eta}} \left(G_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{2.5}$$

Private consumption and government expenditure are assumed to share a similar composition. Constant elasticity of substitution (CES) functions are used to aggregate a large sum of goods/varieties into just one.[10] $C_{H,t}$ and $G_{H,t}$ are indexes representing a basket of domestically produced goods which have as many j varieties as the numbers in the interval [0,1], i.e., a continuum of differentiated varieties.

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } G_{H,t} \equiv \left(\int_0^1 G_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}};$$

On the other hand, imports depend not only on j varieties produced abroad but also on i countries of origin. $C_{F,t}$ and $G_{F,t}$ are indexes measuring the quantity of imported goods from

[10] Constant elasticity of substitution aggregators are very often referred as Armington aggregators or Dixit-Stiglitz indexes, see Armington (1969) and Dixit & Stiglitz (1977), respectively.

each country. For instance, how much does Portugal import from each country?

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \text{ and } G_{F,t} \equiv \left(\int_0^1 (G_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}};$$

While, $C_{i,t}$ and $G_{i,t}$ measure the composition of imports in terms of varieties imported from each country i. Which is the structure of the Portuguese imports from Spain? From which sectors does Portugal import?

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } G_{i,t} \equiv \left(\int_0^1 G_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}};$$

Note that the goods purchased by the government and private sector are the same, and consequently there is just one domestic price ($P_{H,t}$) and one foreign price ($P_{F,t}$). Although, there might be different propensities to import, i,e., $\alpha \neq \chi$.

Take into consideration that $\vartheta \in [0,1]$ is the percentage of government spending in effective consumption and ν is the inverse of the intratemporal elasticity of substitution between private consumption and government spending. Parameters α and $\chi \in [0,1]$ measure the shares of imported goods of private consumption and government expenditure, respectively. $\varepsilon > 1$ is the Dixit-Stiglitz parameter for within-country consumption, $\eta > 0$ stands for the elasticity of substitution between domestically and foreign produced goods whereas γ accounts for the substitutability between goods produced in different foreign countries.

The households budget constraint is given by

$$P_t^C C_t + E_t \{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t, \tag{2.6}$$

where $P_t^C \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the CPI and C_t is the private consumption index, as in (2.4). D_t *"is the nominal payoff in period t+1 of the portfolio held at the end of period t (and which includes shares in firms)"*[11], W_t is the nominal wage paid to every hour

[11] Galí (2008, p.152)

worked by the household, T_t are lump-sum transfers which do not affect incentives to work, save or invest and $Q_{t,t+1}$ *"is the stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic household"*[11]. In this model, the small open economies have the property of perfect and complete financial markets, which implies the existence of a worldwide traded security. Total private consumption expenditures in domestically and foreign produced goods is given by $P_t^C C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$.[12]

The representative household of each SOE maximizes lifetime utility subject to a stream of budget constraints.[13] The optimality conditions are obtained maximizing households utility controlled by the budget constraint. The intratemporal optimality equation is given by

$$\frac{W_t}{P_t^C} = (1-\vartheta)^{-1} N_t^{\varphi} C_t^{\nu} \widehat{C}_t^{\sigma-\nu}. \tag{2.7}$$

Whereas, the intertemporal optimality equation (often called Euler equation) takes the form

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{\nu-\sigma} \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\} = Q_t, \tag{2.8}$$

where $Q_t = E_t \{Q_{t,t+1}\}$.

The above expressions are log-linearised with respect to their steady state values as follows

$$w_t - p_t^C = \varphi n_t + \nu c_t + (\sigma - \nu)\,\widehat{c}_t, \tag{2.9}$$

$$= \varphi n_t + \sigma c_t + (\nu - \sigma)\,(c_t - \widehat{c}_t)\,, \tag{2.10}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\nu}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right) + \frac{\nu-\sigma}{\nu}\left(E_t\{\widehat{c}_{t+1}\} - \widehat{c}_t\right), \tag{2.11}$$

$$= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - (\nu-\sigma)\,E_t\{(c_{t+1} - \widehat{c}_{t+1}) - (c_t - \widehat{c}_t)\} - \rho\right), \tag{2.12}$$

[12] P_t^C is identical to P_t in Galí (2008) but now there is a price index for private consumption and other for the government expenditure.

[13] This section follows the derivation of the budget constraint in the book Galí (2008) and Galí & Monacelli (2008). More details, please check Appendix A.2 or the respective references.

where lower case letters represent the logs of the respective variables, $E_t\left\{\pi^C_{t+1}\right\} = E_t\left\{p^C_{t+1}\right\} - p^C_t$ is CPI inflation[14] (with $p^C_t \equiv \log P^C_t$), $i_t \equiv -\log E_t\left\{Q_{t,t+1}\right\}$ is short-term nominal interest rate and $\rho \equiv -\log \beta$ is the steady state time discount rate.

2.2 Government

Taking into consideration that the government expenditure shares the same structure of private consumption, the government budget constraint is derived in a similar way to the households budget constraint. Through comparison to Appendix A.2, the government budget constraint is easily obtained[15]

$$P^G_t G_t + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq T_t + D_t, \tag{2.13}$$

where $P^G_t \equiv \left[(1-\chi)\left(P_{H,t}\right)^{1-\eta} + \chi\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the government price index, henceforth GPI and G_t is the government expenditure index as in (2.5). The remaining variables are identical to those in the households constraint. For simplicity it is assumed that government does not use bonds to finance expenditures, has a balanced budget in each and every period. Then equation (2.13) might be re-written as follows

$$P^G_t G_t \leq T_t. \tag{2.14}$$

Note that if $\alpha = \chi$, then $P^C_t = P^G_t = P_t$, that is if private and public sectors have the same propensity to import, there is just one price index, identical to the one in Galí (2008).

In this model, domestic and world government expenditure are exogenous, expressed in

[14]More details in Section 2.3 below.

[15]For a detailed derivation of the government budget constraint please check Appendix A.4.

log-deviation from the steady state and defined as an autoregressive process.[16]

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g, \qquad \text{where } \varepsilon_t^g \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\varepsilon_t^g}), \tag{2.15}$$

$$g_t^* = \rho_{g^*} g_{t-1}^* + \varepsilon_t^{g^*}, \qquad \text{where } \varepsilon_t^{g^*} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\varepsilon_t^{g^*}}), \tag{2.16}$$

where ρ_g and $\rho_{g^*} \in [0,1]$ are the autocorrelation parameters accounting for the persistence of the shock whereas ε_t^g and $\varepsilon_t^{g^*}$ represent the shocks to the domestic and world government expenditure, respectively. Note that if $\rho_g = 1$ the increase in government expenditures would be perpetual whereas if $\rho_g = 0$ the shock would last one period only.

2.3 Prices

The present section describes several hypotheses of the model which are regularly used below. It follows closely the definitions presented in Galí (2008). First of all, the bilateral terms of trade are expressed as $\mathcal{S}_{i,t} = \dfrac{P_{i,t}}{P_{H,t}}$ and refer to the ratio of country i price index (measured in domestic currency) relative to the home country price index. Likewise, the effective terms of trade are given by

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}. \tag{2.17}$$

The terms of trade in log-deviations from its steady state value is given by

$$s_t \equiv \log \mathcal{S}_t = \int_0^1 s_{i,t} di = p_{F,t} - p_{H,t}. \tag{2.18}$$

CPI is log-linearised assuming a symmetric steady state as follows

$$p_t^C \equiv (1-\alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t. \tag{2.19}$$

[16]Identical specification of fiscal shocks can be found in Linnemann & Schabert (2004), Coenen & Straub (2005), Forni & Pisani (2010) and Iwata (2013), for example.

and domestic CPI inflation is given by

$$\pi_t^C \equiv p_t^C - p_{t-1}^C = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} = \pi_{H,t} + \alpha\Delta s_t. \tag{2.20}$$

Similarly, the linearisation of GPI yields

$$p_t^G \equiv (1-\chi)p_{H,t} + \chi p_{F,t} = p_{H,t} + \chi s_t \tag{2.21}$$

and domestic GPI inflation is

$$\pi_t^G \equiv p_t^G - p_{t-1}^G = (1-\chi)\pi_{H,t} + \chi\pi_{F,t} = \pi_{H,t} + \chi\Delta s_t. \tag{2.22}$$

An important assumption of this model is that the law of one price holds for every variety of goods $j \in$ [0,1], implying the immediate translation of any foreign price variation into the domestic country through its imports. In other words, there is a complete exchange rate pass-trough to import prices in every time horizons or there are no trade frictions. Prices of each good j in the foreign country i measured in foreign currency, $P_{i,t}^i(j)$, are transformed into country i's prices in domestic prices through the bilateral nominal exchange rate, $\mathcal{E}_{i,t}$. This is equivalent to say that the following expression $P_{i,t}(j) = \mathcal{E}_{i,t}P_{i,t}^i(j)$ hold for each good $j \in$ [0,1]. Furthermore, if this identity hold for individual goods, it is also valid for the aggregate, i.e., $P_{i,t} = \mathcal{E}_{i,t}P_{i,t}^i$. Where $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ is the country i's price index for all varieties of goods measured in domestic currency and $P_{i,t}^i \equiv \left(\int_0^1 P_{i,t}^i(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ is the country i's aggregate price index in their own currency. Recall that $P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}}$. Plugging the law of one price for each economy i (instead of being for each good j) in the last equation and then log-linearising around the symmetric steady state, yields the following

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p_{i,t}^i\right) di = e_t + p_t^*,$$

where $e_t \equiv \int_0^1 e_{i,t} di$ is the effective nominal exchange rate in log-deviations from the steady state and $p^* \equiv \int_0^1 p^i_{i,t} di$ is the log of world price index. Any time there is a superscript with an asterisk, it denotes the integration over every country $i \in [0,1]$, that is world variables. Notice that, for the latter case, as it is a price index for the whole world, there is no difference between CPI (or GPI) inflation and domestic inflation[17]. The last result might be combined with the terms of trade, equation (2.18) and yields[18]

$$s_t = e_t + p^*_t - p_{H,t}. \tag{2.23}$$

Households bilateral real exchange rate is given by

$$\mathcal{Q}^C_{i,t} = \frac{P^{C,i}_t \mathcal{E}_{i,t}}{P^C_t} = \frac{P^C_{i,t}}{P^C_t}, \tag{2.24}$$

which, if linearised, can be expressed as

$$\begin{aligned} q^C_t \equiv \log \mathcal{Q}^C_{i,t} = \int_0^1 q^C_{i,t} di &= \int_0^1 \left(e_{i,t} + p^{C,i}_t - p^C_t \right) di, \\ &= e_t + p^{C,*}_t - p_{H,t} + p_{H,t} - p^C_t + p^*_t - p^*_t, \\ &= s_t + p_{H,t} - p^C_t + p^{C,*}_t - p^*_t, \\ &= (1-\alpha)\, s_t + \left(p^{C,*}_t - p^*_t \right). \end{aligned} \tag{2.25}$$

Looking closer at what is $\left(p^{C,*}_t - p^*_t \right)$, one may conclude that it cancel out because $\int_0^1 s^i_t di = 0$, for instance Galí (2008, p.161), thus

$$p^{C,*}_t = \int_0^1 p^{C,i}_t di = \int_0^1 (p^i_{i,t} + \alpha s^i_t) di = p^*_t. \tag{2.26}$$

[17]For a detailed demonstration of this result, check equation (2.26).

[18]In the simulation of the model is used a differenced version of equation (2.23) as follows $\Delta s_t = \Delta e_t + \pi^*_t - \pi_{H,t}$.

In words this means that the world consumer price index ($p_t^{C,*}$) is the integration of all economies consumer price indexes ($\int_0^1 p_t^{C,i} di$). In turn, each country consumer price index (p_t^C) is made of its respective domestic price index ($p_{H,t}$) and the propensity to import times the terms of trade (αs_t). Next, the log-linearisation of $\mathcal{Q}_{i,t}^C$ yields

$$q_t^C \equiv (1-\alpha)\, s_t. \tag{2.27}$$

Similarly, the government bilateral real exchange rate is

$$\mathcal{Q}_{i,t}^G = \frac{P_t^{G,i}\mathcal{E}_{i,t}}{P_t^G} = \frac{P_{i,t}^G}{P_t^G}. \tag{2.28}$$

which yields

$$q_t^G \equiv (1-\chi)\, s_t. \tag{2.29}$$

Again, note that if $\alpha = \chi$ all this analysis collapses to equations (2.30) and (2.31) where there is only one bilateral real exchange rate, $\mathcal{Q}_{i,t}$. The effective bilateral real exchange rate becomes the ratio of the two countries price indexes, both expressed in terms of domestic currency

$$\mathcal{Q}_{i,t} = \frac{P_t^i\mathcal{E}_{i,t}}{P_t} = \frac{P_{i,t}}{P_t}. \tag{2.30}$$

The log effective real exchange rate is given by

$$\begin{aligned} q_t \equiv \int_0^1 q_{i,t} di &= \int_0^1 \left(e_{i,t} + p_t^i - p_t\right) di, \\ &= e_t + p_t^* - p_{H,t} + p_{H,t} - p_t, \\ &= s_t + p_{H,t} - p_t, \end{aligned} \tag{2.31}$$

where $q_{i,t} = \log \mathcal{Q}_{i,t}$.

2.4 International Risk Sharing

The common approach in the SOE framework is to assume complete financial markets. The return on a cross-border security influences the intertemporal allocation of households' budget. The ratio current vs. future consumption depends on the expected return of the security. Country i's intertemporal optimization equation (in terms of domestic currency) is given by

$$\beta\left(\frac{C_{t+1}^{i}}{C_{t}^{i}}\right)^{-\nu}\left(\frac{\widehat{C}_{t+1}^{i}}{\widehat{C}_{t}^{i}}\right)^{\nu-\sigma}\left(\frac{\mathcal{E}_{i,t}}{\mathcal{E}_{i,t+1}}\right)\left(\frac{P_{t}^{C,i}}{P_{t+1}^{C,i}}\right)=Q_{t}. \tag{2.32}$$

Expressions (2.8) and (2.32) may be combined in order to derive the equilibrium condition for the internationally traded security. In equilibrium, it holds such that

$$C_{t}^{\nu}\widehat{C}_{t}^{\sigma-\nu}=\varpi_{i}\left(C_{t}^{i}\right)^{\nu}\left(\widehat{C}_{t}^{i}\right)^{\sigma-\nu}\mathcal{Q}_{i,t}^{C}, \tag{2.33}$$

where ϖ_i is a constant characterizing the relative composition of the balance sheet of each country. Log-linearising (2.33), integrating over [0,1] and having in mind that in aggregate $\varpi_i = \varpi = 1$, i.e., zero net foreign asset holdings, one can derive

$$\nu c_{t}+(\sigma-\nu)\,\widehat{c}_{t}=\nu c_{t}^{*}+(\sigma-\nu)\,\widehat{c}_{t}^{*}+q_{t}^{C}, \tag{2.34}$$

$$c_{t}=c_{t}^{*}+\frac{\sigma-\nu}{\nu}\left(\widehat{c}_{t}^{*}-\widehat{c}_{t}\right)+\frac{1-\alpha}{\nu}s_{t}, \tag{2.35}$$

where $c_t^* = \int_0^1 c_t^i di$ is the (log) index for world private consumption and $\widehat{c}_t^* = \int_0^1 \widehat{c}_t^i di$ is linearised expression for the effective world consumption. The complete markets hypothesis allows having an identity as (2.35) relating private and effective world consumption to the terms of trade (s_t) and the degree of openness (α).

2.5 Firms

2.5.1 Production Function

Every firm in this model uses labour (N_t) and technology (A_t) to produce a differentiated good $j \in [0,1]$. The production function of each firm (or good) does not include physical capital for simplicity[19] and is expressed as follows

$$Y_t(j) = A_t N_t(j), \tag{2.36}$$

where technology follows an AR(1) process, $a_t = \log A_t = \rho_a a_{t-1} + \varepsilon_t$. The linearisation of the aggregate production function is given by $y_t = a_t + n_t$. Solving the typical firms optimization problem[20] one obtains an expression for the real marginal cost, which is identical to every firm

$$mc_t = -\delta + w_t - p_{H,t} - a_t, \tag{2.37}$$

where $\delta \equiv \log(1-\tau)$ is a employment subsidy.

2.5.2 Price Setting

The model sets prices according to Calvo (1983), i.e., are adjusted in a staggered manner. In this framework, similar to Galí (2008), a fraction of firms are selected to re-optimize profits changing prices with regard to new contingencies. In other words, selected firms maximize the expected discounted profits. The probability of being selected to re-optimize is timely-independent of the last pricing decision. The price index resulting from this property is given by

$$\overline{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{mc_{t+k} + p_{H,t+k}\}, \tag{2.38}$$

[19]For a similar model with capital see Galí et al. (2007).
[20]More details about the optimization problem in Appendix A.9.

where $\theta \in [0,1]$. Every period a share $(1-\theta)$ of randomly selected firms are able to set new prices while the remaining share θ have to keep their prices fixed. Consider also that $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon - 1}\right)$ is the equilibrium markup in the flexible price state. This assumption leads to the inflation equation

$$\pi_{H,t} = \beta E_t \{\pi_{H,t+1}\} + \lambda \, \widehat{mc}_t, \tag{2.39}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ is a coefficient that relates the probability of resetting prices with the time discount rate.[21]

2.6 Equilibrium

2.6.1 The Demand Side

The demand side of this economy has two economic agents, the households and the government. Each consumes a certain quantity of each good variety j which need to be aggregated to obtain the demand for the whole economy. An assumption of this model is that goods market clear every period.

The demand of country i for good j is made of two parts, the domestic production which is domestically consumed but also the foreign production domestically consumed, i.e., the imports of each good j from each country i. The demand of good j in each country i is then defined as

$$\begin{aligned} Y_t(j) = (1-\vartheta)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right)^{-\gamma}\left(\frac{P_{F,t}^i}{P_t^{C,i}}\right)^{-\eta} C_t^i di\right] \\ +\vartheta\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[(1-\chi)\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta} G_t + \chi\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\right)^{-\gamma}\left(\frac{P_{F,t}^i}{P_t^{G,i}}\right)^{-\eta} G_t^i di\right]. \end{aligned} \tag{2.40}$$

This equation is similar to equation (24) in Galí (2008, p.160) but with two novelties, first government consumption is added to the demand for good j and second, each type of con-

[21] Please notice that when $\lambda = 0$, prices are fully flexible and $\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\}$.

sumption (private or public) has its own price, P_t^C and P_t^G.

Aggregate demand of each country i is the sum of every demand for each good variety j. Consequently, aggregate demand of each country i is the integration over [0,1] of the demand for each product type as $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Integrating expression (2.40) yields

$$
\begin{aligned}
Y_t = (1-\vartheta)&\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^{C,i}}\right)^{-\eta} C_t^i di\right] \\
+\vartheta&\left[(1-\chi)\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta} G_t + \chi \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^{G,i}}\right)^{-\eta} G_t^i di\right]. \qquad (2.41)
\end{aligned}
$$

Making some arithmetic operations, which are detailed in Appendix A.11, results in

$$
\begin{aligned}
Y_t = (1-\vartheta)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta}&\left[(1-\alpha)C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \left(\mathcal{Q}_{i,t}^C\right)^{\eta} C_t^i di\right] \\
+\vartheta\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}&\left[(1-\chi)G_t + \chi \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \left(\mathcal{Q}_{i,t}^G\right)^{\eta} G_t^i di\right]. \qquad (2.42)
\end{aligned}
$$

Equation (2.33) can be used to simplify (2.42), i.e.,

$$
\begin{aligned}
Y_t = (1-\vartheta)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t&\left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \left(\mathcal{Q}_{i,t}^C\right)^{\eta-\frac{1}{\nu}} \left(\frac{\widehat{C}_t}{\widehat{C}_t^i}\right)^{\frac{\sigma-\nu}{\nu}} di\right] \\
+\vartheta\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}&\left[(1-\chi)G_t + \chi \int_0^1 \left(\mathcal{S}_t^i \mathcal{S}_{i,t}\right)^{\gamma-\eta} \left(\mathcal{Q}_{i,t}^G\right)^{\eta} G_t^i di\right], \qquad (2.43)
\end{aligned}
$$

where $\mathcal{S}_t^i$ stands for effective terms of trade for country i and $\mathcal{S}_{i,t}$ is the bilateral terms of trade between the home economy and country i, identical to Galí (2008). To have a manageable solution for this model the last expression is first-order log-linearised around a symmetric steady state and integrated. Moreover, recalling that $\int_0^1 s_t^i di = 0$, one has

$$
\begin{aligned}
y_t = (1-\vartheta)&\left[-\eta\left(p_{H,t} - p_t^C\right) + c_t + \alpha\left[(\gamma-\eta)s_t + \left(\eta - \frac{1}{\nu}\right) q_t^C + \left(\frac{\sigma-\nu}{\nu}\right)\left(\widehat{c}_t - \widehat{c}_t^*\right)\right]\right] \\
+\vartheta&\left[-\eta\left(p_{H,t} - p_t^G\right) + (1-\chi)g_t + \chi\left[(\gamma-\eta)s_t + \eta q_t^G + g_t^*\right]\right], \qquad (2.44)
\end{aligned}
$$

where $\widehat{c}_t$ is the log-linearised effective consumption, $\widehat{c}_t^*$ is the world effective consumption and g_t^* is the world government expenditure. Substituting the following simplifications $\omega^C \equiv \gamma\nu + (\eta\nu - 1)(1-\alpha)$ and $\omega^G \equiv \gamma\nu + \eta\nu(1-\chi)$, the last expression becomes

$$y_t = (1-\vartheta)\left[c_t + \frac{\alpha}{\nu}\omega^C s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right] + \vartheta\left[(1-\chi)g_t + \frac{\chi}{\nu}\omega^G s_t + \chi g_t^*\right]. \quad (2.45)$$

Taking into consideration that $\int_0^1 s_t^i di = 0$, world output is given by

$$y_t^* = (1-\vartheta)c_t^* + \vartheta g_t^*, \quad (2.46)$$

where c_t^* is world private consumption. Isolating s_t, substituting $\Upsilon = (1-\vartheta)\frac{\alpha}{\nu}(\omega^C - 1) + \vartheta\frac{\chi}{\nu}\omega^G + \frac{(1-\vartheta)}{\nu}$ and also the world output, equation (2.45) turns into

$$y_t = y_t^* + (1-\vartheta)\left[(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t^* - \widehat{c}_t)\right] + \vartheta\left[(1-\chi)(g_t - g_t^*)\right] + \Upsilon s_t. \quad (2.47)$$

Combining the Euler equation (2.11) with equation (2.45) yields

$$\begin{aligned} y_t = {} & E_t\{y_{t+1}\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) - \left[\Lambda + \frac{\chi}{\nu}(1-\vartheta)\right]\vartheta\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta g_{t+1}^*\right\} \\ & + \Lambda\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta y_{t+1}^*\right\} - (1-\chi)\vartheta E_t\{\Delta g_{t+1}\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\left\{\Delta\widehat{c}_{t+1}^*\right\} \\ & - \left[\Upsilon + \Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\}, \end{aligned} \quad (2.48)$$

where $\Lambda \equiv \Upsilon - \frac{1-\vartheta}{\nu} = \left[(1-\vartheta)\frac{\alpha}{\nu}(\omega^C - 1) + \vartheta\frac{\chi}{\nu}\omega^G\right]$, and y_t is log-linearised aggregate demand.

Finally, net exports are expressed in deviations of the steady state output Y[22]

$$nx_t = \frac{NX_t}{Y} \approx \frac{1}{Y}\left[Y_t - \frac{P_t^C}{P_{H,t}}C_t - \frac{P_t^G}{P_{H,t}}G_t\right].$$

[22]For a complete derivation of the trade balance, check Appendix A.11.2

After applying a first order linear approximation to the last equation, one has

$$nx_t = y_t - (1-\vartheta)(c_t + \alpha s_t) - \vartheta (g_t + \chi s_t), \tag{2.49}$$

which, if combined with equation (2.45), yields

$$nx_t = (1-\vartheta)\left[\alpha\left(\frac{\omega^C}{\nu} - 1\right)s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right] + \vartheta\left[\chi\left(\frac{\omega^G}{\nu} - 1\right)s_t + \chi(g_t^* - g_t)\right]. \tag{2.50}$$

2.6.2 The Supply Side

A first step to obtain the natural level of output is to get the flexible prices marginal cost. In order to obtain it, one has to make some transformations in the optimal real marginal cost expression derived in Section 2.5

$$mc_t = -\delta + w_t - p_{H,t} - a_t, \tag{2.37}$$

$$mc_t = -\delta + \varphi y_t + \nu c_t^* + (\sigma - \nu)\widehat{c}_t^* + s_t - (1+\varphi)a_t, \tag{2.51}$$

which makes use of equations (2.9), (2.19), (2.35) and the linearised production function. Using (2.47) to replace s_t above produces

$$\begin{aligned} mc_t = {} & -\delta + \left(\frac{\Upsilon\varphi + 1}{\Upsilon}\right)y_t - \frac{1}{\Upsilon}y_t^* + \nu c_t^* + \left(\Lambda + \alpha\left(\frac{1-\vartheta}{\nu}\right)\right)\frac{1}{\Upsilon}(\sigma-\nu)\widehat{c}_t^* - (1+\varphi)a_t \\ & + (1-\alpha)\left(\frac{1-\vartheta}{\nu}\right)\frac{1}{\Upsilon}(\sigma-\nu)\widehat{c}_t - \vartheta(1-\chi)\frac{1}{\Upsilon}g_t + \vartheta(1-\chi)\frac{1}{\Upsilon}g_t^*. \end{aligned} \tag{2.52}$$

Under flexible prices the marginal cost is constant, $mc_t = -\mu$. The natural level of output is represented by

$$y_t^n = \Gamma_0 + \Gamma_{y^*}y_t^* + \Gamma_{c^*}c_t^* + \Gamma_{\widehat{c}^*}\widehat{c}_t^* + \Gamma_{\widehat{c}}\widehat{c}_t + \Gamma_g g_t + \Gamma_{g^*}g_t^* + \Gamma_a a_t, \tag{2.53}$$

$$\text{where } \Gamma_0 \equiv \left[\frac{\Upsilon(\delta-\mu)}{\Upsilon\varphi+1}\right],\ \Gamma_{y^*} \equiv \left[\frac{1}{\Upsilon\varphi+1}\right],\ \Gamma_{c^*} \equiv -\left[\frac{\Upsilon\nu}{\Upsilon\varphi+1}\right],$$
$$\Gamma_{\widehat{c}^*} \equiv -\left[\Lambda+\alpha\left(\frac{1-\vartheta}{\nu}\right)\right]\left(\frac{1}{\Upsilon\varphi+1}\right)(\sigma-\nu),\ \Gamma_{\widehat{c}} \equiv -\left(\frac{1-\vartheta}{\nu}\right)\left[\frac{1-\alpha}{\Upsilon\varphi+1}\right](\sigma-\nu),$$
$$\Gamma_g \equiv \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right],\ \Gamma_{g^*} \equiv -\left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right],\ \Gamma_a \equiv \left[\frac{\Upsilon(1+\varphi)}{\Upsilon\varphi+1}\right].$$

2.6.3 The Equilibrium Dynamics

Having in mind that the output gap is defined by

$$\tilde{y}_t \equiv y_t - y_t^n, \tag{2.54}$$

one can start by substituting (2.53) into (2.54), and finally using the resulting expression to substitute into (2.52) generates a relation between the output gap and the steady state (i.e., flexible prices) domestic real marginal cost

$$\widehat{mc}_t = \left(\varphi + \frac{1}{\Upsilon}\right)\tilde{y}_t. \tag{2.55}$$

If the last equation is used to substitute in (2.39) the new Keynesian Phillips curve (NKPC) is obtained as

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \tilde{y}_t, \tag{2.56}$$

where $\kappa_\alpha \equiv \lambda\left(\varphi + \frac{1}{\Upsilon}\right)$ is the slope coefficient.[23]

To obtain the dynamic IS equation for the open economy in terms of the output gap one has to start with the IS equation (2.48), then substitute the output gap (2.54) and finally

[23]Please notice that when $\kappa_\alpha = 0$ or $\lambda = 0$, prices are fully flexible and $\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\}$.

replace (2.53) which yields

$$\begin{aligned} \tilde{y}_t &= E_t\left\{\tilde{y}_{t+1}\right\} - \Upsilon\left(i_t - E_t\left\{\pi_{H,t+1}\right\} - r_t^n\right) \\ &- \left[\Upsilon + \Lambda + \left(\frac{1-\vartheta}{\nu}\right)\left(\frac{\alpha\Upsilon\varphi+1}{\Upsilon\varphi+1}\right)\right](\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}\right\}, \end{aligned} \tag{2.57}$$

where r_t^n is the natural (or Wicksellian) rate of interest of the domestic economy and has the following format

$$\begin{aligned} r_t^n \equiv \rho &+ \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \frac{1}{\Upsilon\varphi+1}\right]\frac{1}{\Upsilon}E_t\left\{\Delta y^*_{t+1}\right\} - \left[\frac{\nu}{\Upsilon\varphi+1}\right]E_t\left\{\Delta c^*_{t+1}\right\} \\ &- \left[\frac{\varphi}{\Upsilon\varphi+1}\right](1-\chi)\vartheta E_t\left\{\Delta g_{t+1}\right\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right]\left(\frac{\varphi}{\Upsilon\varphi+1}\right)(\sigma-\nu)\,E_t\left\{\Delta\widehat{c}^*_{t+1}\right\} \\ &- \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \chi + \frac{1-\chi}{\Upsilon\varphi+1}\right]\vartheta\frac{1}{\Upsilon}E_t\left\{\Delta g^*_{t+1}\right\} - \left[\frac{(1+\varphi)}{\Upsilon\varphi+1}\right](1-\rho_a)\,a_t. \end{aligned} \tag{2.58}$$

As already emphasized, the current model is based in Galí (2008, ch.7) open economy model. For $\vartheta = 0$, $\alpha = \chi$ and $\sigma = \nu$ the model is identical to Galí (2008, ch.7). Additionally, if $\alpha = 0$, the model becomes a closed economy as in Galí (2008, ch.3). In the end, it is possible to conclude that this model nests both Galí (2008) models.

3 Numerical Simulation

3.1 The Policy Experiment

The present section quantitatively analyses the results of a government expenditure shock in the model above elaborated. As referred earlier, fiscal policy is modelled as an exogenous government expenditure shock. Recall also that government expenditure is modelled through an autoregressive process as follows

$$g_t = \rho_g \; g_{t-1} + \varepsilon_t^g, \qquad \text{where } \varepsilon_t^g \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\varepsilon_t^g}). \tag{2.15}$$

In the simulation of the model, both fixed and flexible exchange rate regimes are scrutinized in order to derive relevant policy implications. Each regime gives a particular answer to fiscal shocks and consequently these distinct responses are analysed. Aiming to simulate an environment as the EMU, a fixed exchange regime is assumed across every SOE of the model. In such regime, there is just one currency and the nominal exchange rate does not change, i.e., $e_t = 0$. On the other hand, in a flexible exchange rate regime there is a different currency in every country, each central bank defines the short-term nominal interest rate according to a domestic inflation Taylor rule (DITR)[24], that is a rule stipulating the response of the nominal interest rate to changes in domestic inflation.

$$i_t = \rho_i i_{t-1} + \phi_\pi \pi_{H,t}, \tag{3.1}$$

where ρ_i is the autocorrelation parameter defining the inertia of interest rate adjustments and ϕ_π is the weight of domestic inflation.[25]

[24] More details in Taylor (1993).

[25] Monacelli (2004) and Galí & Monacelli (2013) develop an exchange rate regime rule accommodating both fixed and flexible exchange rate regimes depending on the the calibration of ϕ_e. According this rule, $i_t = \rho + \phi_\pi \pi_{H,t} + \dfrac{\phi_e}{1-\phi_e} e_t$, the central bank responds not only to the domestic inflation rate but also to fluctuations in the nominal exchange rate. Note that if $\phi_e = 0$ then the economy has a fully flexible exchange

3.2 Calibration

This subsection calibrates the SOE model aiming to study the effects of fiscal policy in the Portuguese economy. When no estimates are available for EMU, US is used as a proxy for EMU. The calibration used in the simulation of the model is based on existing literature and is, as much as possible, empirically plausible. First of all, recall that the the intratemporal elasticity of substitution between private consumption and government expenditure (ν^{-1}) and the households' intertemporal elasticity of substitution (σ^{-1}) are the parameters defining the effectiveness of fiscal stimuli. Therefore, an accurate estimation of these two parameters has a particular importance. Based in US data, the parameter ν is estimated in Bouakez & Rebei (2007, p.971) where the authors suggest to calibrate $\nu = 3$.[26] On the other hand, the parameter σ^{-1} is estimated in Havránek et al. (2013) who collect more than two thousand estimates for the intertemporal elasticities of substitution worldwide. In the end, the authors suggest the value of 0.5 as the most plausible intertemporal elasticity of substitution which is the same to calibrate $\sigma = 2$. Taking into consideration these two estimates and Amano & Wirjanto (1998) classification concerning substitutability and complementarity, private consumption and government expenditures are modelled as complements. Moreover, the calibration is in line with empirical results of Section 1 which concludes that private consumption and government expenditure are, on average, complements. There is a complementarity relation because private consumption gives a positive response to most of government stimuli (even though its response does not always lead to a multiplier higher than one).

Another important aspect concerns the Frisch elasticity of labour supply (φ^{-1}) which is fixed at 0.5 based in Chetty et al. (2011). Equally important are the private and public propensities to import, α and χ, respectively. These were calculated based in the Portuguese national accounts and Dias (2010, 2011) methodology.[27] The shares of steady state private

rate whereas $\phi_e \to 1$ then $e_t = 0$ which represent a fixed exchange rate regime.

[26] In Bouakez & Rebei (2007), the exponents of effective consumption have an inverse structure relative to equation (2.2). For more details check Appendix B.2.

[27] More details in Subsection 3.2.1 below.

consumption and government expenditures in effective consumption are calibrated according to the Portuguese economy, that is $\frac{C}{\widehat{C}} = 0.80$ and $\frac{G}{\widehat{C}} = 0.20$.[28] The choice of the monetary policy parameter follows Taylor (1993) original calibration, i.e. $\phi_\pi = 1.5$.[29] The remainder of the values come from Galí (2008, ch.7) open economy model.[30] Table 1 summarizes the parameters choice used in the simulation exercise.

Table 1: Baseline Calibration

Parameter	Description	Value
α	share of private imports	0.25
χ	share of public imports	0.10
ϑ	share of government expenditures in effective consumption	0.20
φ^{-1}	Frisch elasticity of labour supply	0.50
ν^{-1}	intratemporal elasticity of sub. btw private and public consumption	0.33
σ^{-1}	intertemporal elasticity of substitution of effective consumption	0.50
γ	substitutability between goods produced in different foreign countries	1.00
η	substitutability between domestic and foreign goods	1.00
ε	elasticity of substitution between varieties produced within countries	6.00
β	time discount factor	0.99
θ	share of firms unable to reset prices	0.75
ρ_g	autocorrelation of government expenditures	0.90
ϕ_π	weight of domestic inflation in Taylor rule	1.50
ρ_i	monetary policy smoothing parameter	0.90

[28]For additional details check the "Economic Bulletin 10/2014" of the Bank of Portugal, page 7. These values are widely used in the literature see, among others, Christiano et al. (2011), Erceg & Lindé (2014) and Bouakez & Rebei (2007).

[29]Duffy & Xiao (2011) showed that Taylor original calibration produces determinacy in the several model specifications used. Moreover, the authors demonstrate that introducing policy smoothing, i.e., make the interest rate depend on its past values, increase the determinacy of the models.

[30]Galí (2008) model is calibrated according to Cole & Obstfeld (1991), that is $\sigma = \eta = 1$ which became known in the literature as the Cole-Obstfeld case. Remember that when $\sigma = 1$ the utility function becomes logarithmic.

3.2.1 Propensities to Import

The propensities to import of the main components of GDP have been measured in several recent articles, see, among others Dias (2010, 2011) and Cardoso et al. (2013) measuring the import contents of each aggregate demand component for the Portuguese economy. Aiming to find a plausible estimate for the share of imports of private consumption and government expenditure and also verify whether it changed over the years, Dias (2010) methodology was applied to the most recent data for the Portuguese economy, the year 2008. The methodology uses Input-Output matrices released by Instituto Nacional de Estatística (INE)[31] to measure the (direct and indirect) propensities to import of each aggregate demand component. The direct effect accounts for the imports of inputs that each good or service use during the production process while the indirect effect measures imports within national production used as intermediate consumption in other production processes.[32] A summary of the results is presented below

Table 2: Percentage of Imports in each Final Demand Component in Portugal, 2008

	Direct	Indirect	Total
Households Consumption Expenditure	**0,128**	**0,138**	**0,266**
Government Consumption Expenditure	**0,015**	**0,084**	**0,099**
Gross Fixed Capital Expenditure	0,216	0,165	0,380
Exports	0,017	0,418	0,436
Total Final Demand	0,102	0,192	0,294

Source: Author's calculations based in INE data and Dias (2010, 2011) methodology.

Cardoso et al. (2013) using an identical but more aggregated methodology obtain identical results. Then, it may be assumed that the results above are corroborated. Nevertheless, in Corsetti et al. (2009) the government share of imports is calibrated to 6% highlighting the importance of government imports when analysing effectiveness of fiscal policy.

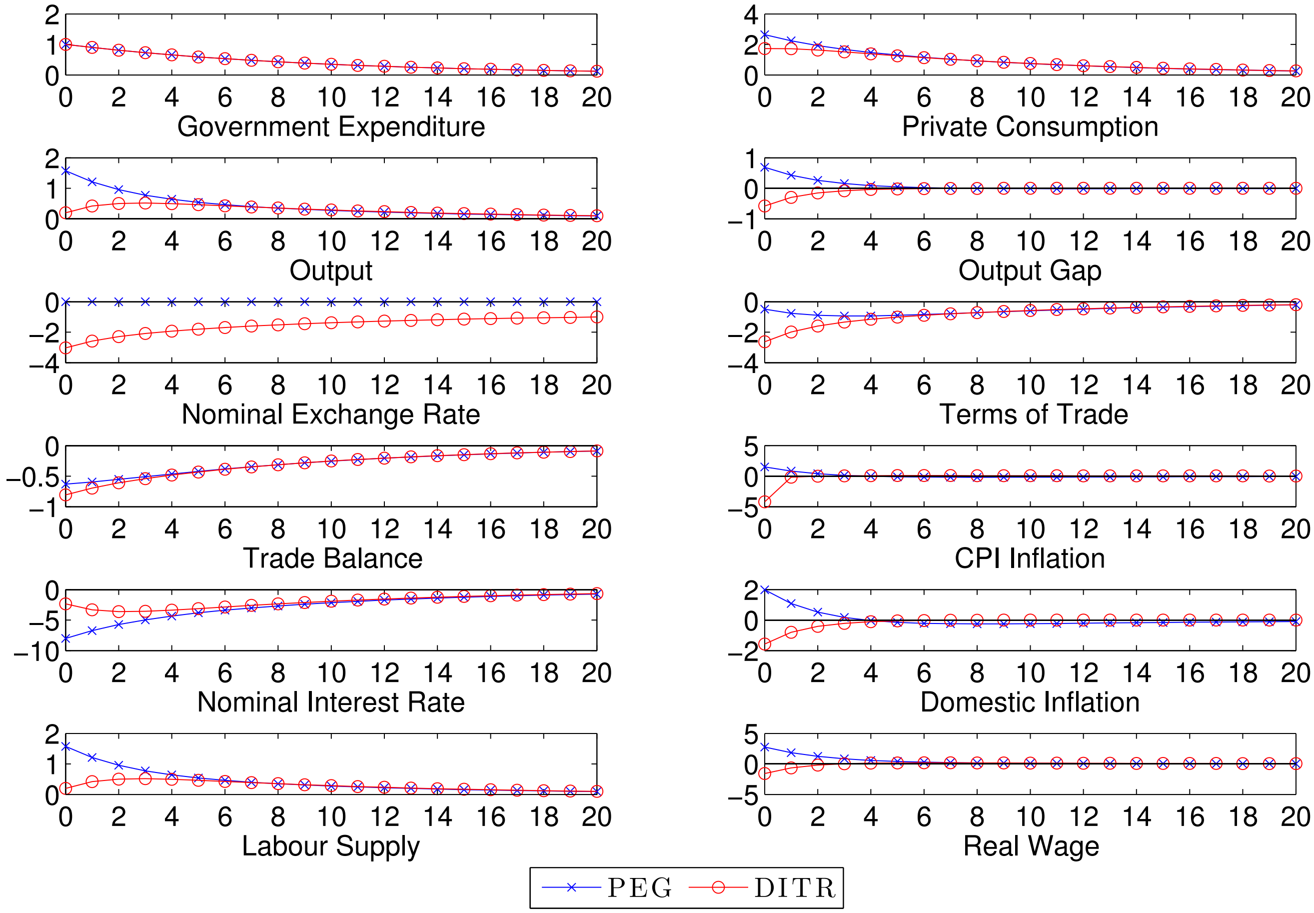
[31] More details in http://www.ine.pt.

[32] For more details, methodology and formulas, please check Dias (2010, 2011) available at request.

3.3 Discussion and Results

This subsection quantitatively analyses the dynamics of an expansionary shock in domestic government expenditure under the baseline calibration. Figure 1 depicts the response of economic variables to a 1 standard deviation increase in the steady state level of government expenditure. The vertical axis of impulse response functions measure the percentage deviations of the variables from the respective steady state values while the horizontal axis measures quarters. Domestic inflation, CPI inflation and nominal interest rate are represented as annualized quarterly rates. For the sake of comparison, the same government expenditure shock is analysed under pegged and flexible exchange rate. Figure 1 shows that both private consumption and output react positively to the simulation. With fixed exchange rate the reaction of output is more than one-for-one whereas in the flexible exchange regime output weakly responds to government expenditure. This leads to the conclusion that, in this model, fiscal policy is more effective under a fixed exchange rate regime. Following the stimulus, the nominal exchange rate in the flexible exchange regime depreciates. The shock also makes the trade balance and terms of trade deteriorate. Nominal interest rate decreases and labour supply increases in both exchange regimes. However, real wage only responds positively in the fixed exchange regime. The conclusion that fiscal policy is more effective in a fixed exchange regime is reinforced with an analysis of fiscal multipliers.

Figure 1: Impulse Response Functions for a Shock in Domestic Government Expenditure



Note: Recall that DITR stands for domestic inflation Taylor rule and represents a flexible exchange rate regime. PEG denotes a fixed exchange regime.

$$\text{Impact Multiplier} = \frac{\sum_{T=1}^{0} y_t}{\sum_{T=1}^{0} g_t} \qquad \text{Cumulative Multiplier} = \frac{\sum_{T=t}^{\infty} y_t}{\sum_{T=t}^{\infty} g_t} \tag{3.2}$$

Impact multipliers measure the immediate response of output in percentage of government stimulus.[33] In this model, the impact multiplier coincide with its peak. On the other hand, the cumulative multiplier also measures the response of output in percentage of government stimulus but it's the long-run average. Table 3 easily illustrates the above conclusion. The impact multiplier in fixed exchange regime is about 1.6 while in the long-run it falls to 1. Much lower are the multipliers in the flexible exchange regime where fiscal multipliers achieve 0.2 on impact and 0.7 cumulative.

Table 3: Fiscal Multipliers

	Fixed	Flexible
Impact	1.6	0.2
Cumulative	1.0	0.7

[33]Note that the specification of fiscal multipliers used in this model is different than the traditional one (where it is usually a ratio between variables measured in levels). In this case, the fiscal multiplier is a ratio of two variables measured in percentage deviations from respective steady state. If the initial values are not the same, the results might be biased. For example if Y=100 and G=20, a 1 percentage increase in each is not the same.

To simulate a government expenditure shock in EMU and have a better understanding of its consequences, a more comprehensive analysis of fixed exchange regime is made. The fixed exchange regime does not allow the currency to depreciate and consequently does not force private consumption to adjust to higher foreign prices (when converted to domestic currency). This makes the response of aggregate demand much stronger than it would be in the case of a flexible exchange rate. In flexible exchange regimes, the depreciation of the exchange rate establishes higher foreign prices when converted to domestic currency and mitigates a wave of imports. In the context of fixed exchange regime, represented in Figure 1, the trade balance is dampened with the increase in both private and public consumption. Once government also purchases abroad the stimulus weakens net exports. Take into account that the terms of trade, influenced by the exchange rate regime, also foster this unfavourable behaviour of the net exports.

Another important but expected finding concerns the labour supply which is found to increase and partly offset the negative wealth effect induced by the fiscal shock. The increase in labour supply is understandable in the light that households want to smooth their consumption and to do so need to work more. Remember that the fiscal stimulus is financed through a tax over households current income. Certainly, the impulse response function of labour supply is much pronounced due to the complementarity because households will want to consume public and private goods together. On the other hand, the short-term nominal interest decreases and real wage increases which will tend to further amplify the positive reaction of private consumption. Households intertemporally substitute future consumption for current consumption decreasing their savings due to its low return. In summary is possible to conclude that non-separability and complementarity between private consumption and government expenditure induce a positive response of private consumption to a government expenditure shock. Private consumption reaction is strengthened by: (i) the degree of complementarity $(\nu - \sigma)$, (ii) the fixed exchange regime, (iii) the increase in working hours and (iv) the decrease in the nominal interest rate.

Finally, notice that all world variables are invariant to any domestic shock because the home economy has an infinitesimally small size in comparison to the world economy. However, a shock of the world economy may affect the domestic economy through cross-border spillover effects.

One of the assumptions of this model is the Uncovered Interest Parity (UIP) defining the domestic economy interest rate as a function of world interest rate and expected change in the nominal exchange rate.[34] Naturally, in a fixed exchange regime, nominal interest rate should not vary. However, non-separable preferences ($\sigma \neq \nu$ and $\vartheta > 0$) break the UIP condition and thus nominal interest rate varies in response to a fiscal stimulus. From here a conclusion emerges, an exogenous government expenditure shock in a model with non-separable preferences has a profound impact in the households optimization problem such that the behaviour of nominal interest rate is distorted.[35] Appendix A.8 shows how UIP does not hold. In this model, the relation between domestic and foreign interest rates is represented by:

$$i_t = i_t^* + E_t \left\{ \Delta e_{t+1} \right\} - 2 \left(\sigma - \nu \right) \left(E_t \left\{ \Delta \widehat{c}_{t+1} \right\} - E_t \left\{ \Delta \widehat{c}_{t+1}^* \right\} \right). \tag{3.3}$$

From this equation a conclusion emerges, UIP does not hold as long as there is non-separability, i.e. $\sigma \neq \nu$. Next this conclusion is illustrated numerically.

[34]The UIP is represented as follows $i_t = i_t^* + E_t \left\{ \Delta e_{t+1} \right\}$, where i_t^* is the world interest rate.

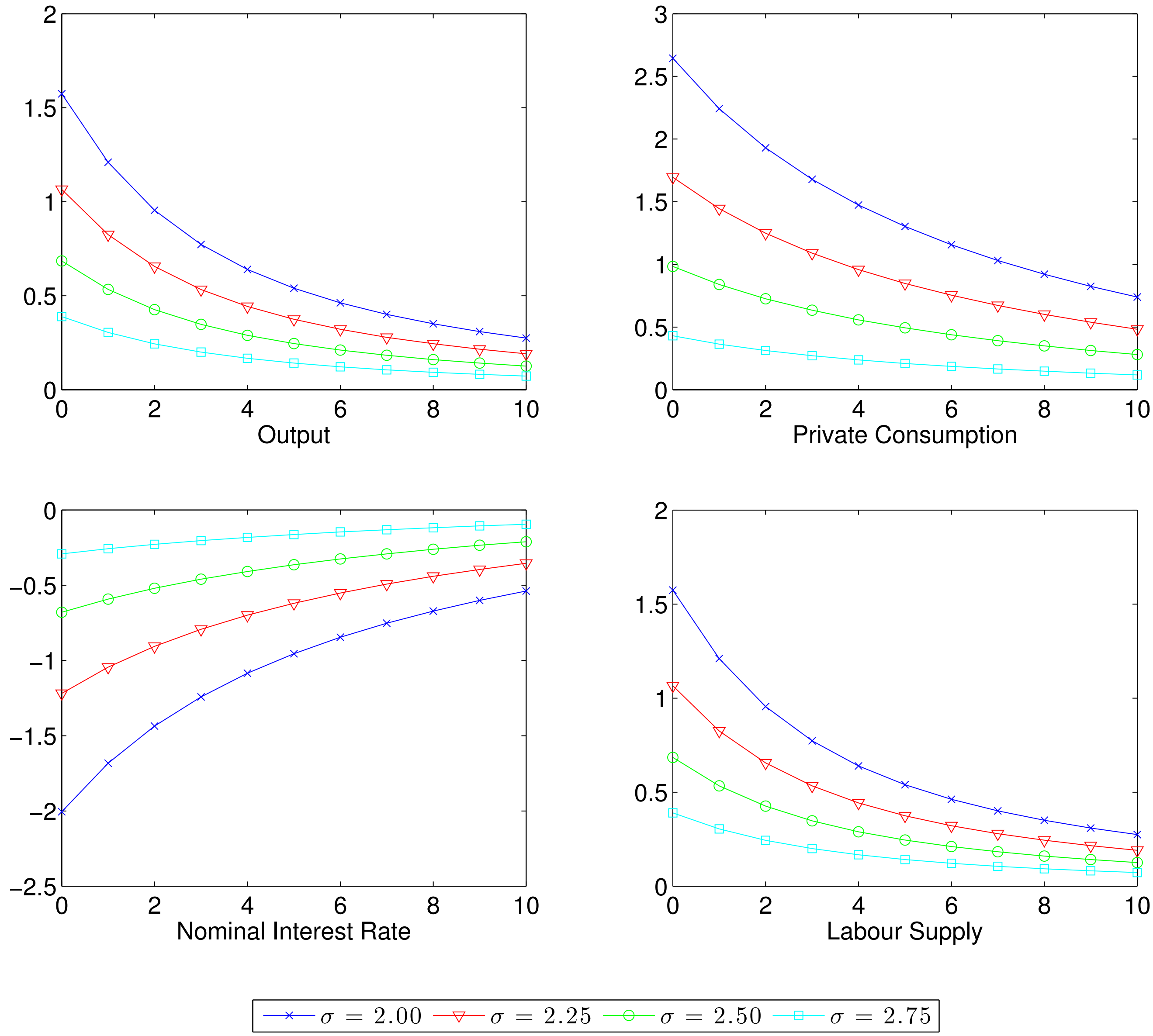
[35]Note that analysing fiscal policy in flexible exchange regime instead of fixed exchange regime solves the UIP puzzle. In flexible exchange regimes, nominal interest rate varies.

To investigate how fiscal multipliers depend on the degree of complementarity ($\nu - \sigma$), a sensitivity analysis is made to parameter σ. Figure 2 illustrates a sensitivity analysis to the inverse of the elasticity of intertemporal substitution (σ), one of the main drivers of the fiscal multipliers. Maintaining all other parameters in accordance to Table 1, parameter σ is increased (but is always below the calibrated value of ν to preserve complementarity assumption). As the inverse of the elasticity of intertemporal substitution of effective consumption converges to the inverse of the intratemporal elasticity of substitution between private consumption and government expenditure, the complementarity relation becomes weaker. Consequently, the response of output, private consumption and labour supply to fiscal stimulus are lower the closer the values of σ and ν. Fiscal multipliers also reflect this relationship between σ and ν. It is possible to conclude that the bigger the difference between ν and σ the higher will be the fiscal multiplier and consequently the effect of fiscal policy in the economy. But more important is finding that UIP puzzle is driven by the degree of complementarity. As $\sigma \to \nu$, private consumption and public spending become unrelated (neither complements nor substitutes) and then the UIP puzzle is solved.

Table 4: Fiscal Multipliers (with Fixed Exchange Rate Regime)

	$\sigma = 2.00$	$\sigma = 2.25$	$\sigma = 2.50$	$\sigma = 2.75$
Impact	1.6	1.1	0.7	0.4
Cumulative	1.0	0.7	0.5	0.3

Figure 2: Impulse Response Functions for a Shock in Domestic Government Expenditure



Note: The blue crossed line stands for the baseline calibration.

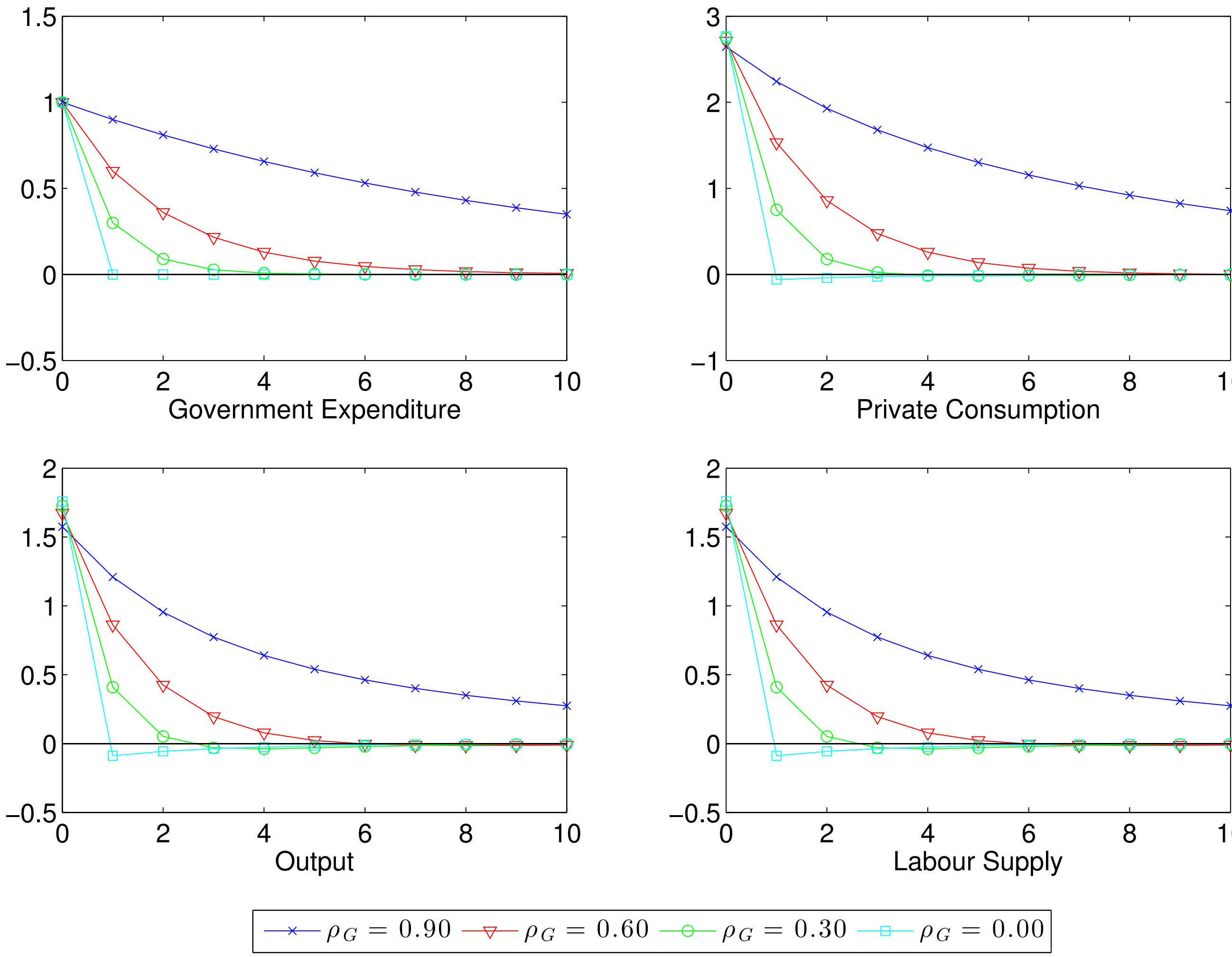
To investigate whether fiscal stimulus should be temporary or continue over time, an analysis is performed to the parameter ρ_G. Figure 3 makes a sensitivity analysis to the parameter ρ_G which defines the persistence of the government expenditure shock. All the parameters, except ρ_G, are listed in Table 1, the baseline calibration. From the figure is possible to observe that the impact multiplier is the same to every level of persistence. It is also possible to conclude that higher persistence leads to longer responses of each of the variables analysed (government expenditure, private consumption, output and labour supply). The fiscal spending shock impacts the economy via private consumption which also stimulates aggregate demand. The complementarity assumption makes households increase consumption as long as the stimulus lasts. Looking at the fiscal multipliers is at first sight surprising because cumulative multiplier increases as the persistence of the shock decreases. However, this unexpected result can be explained mathematically. There is an increase in cumulative fiscal multipliers while the persistence of government expenditure decreases because output decreases faster than government expenditure (even though it departs from higher values).[36] Then, it is possible to conclude that the degree of complementarity $(\nu - \sigma)$, the main driver of the multiplier, is not strong enough to make fiscal policy more effective with a higher level of government persistence.

Table 5: Fiscal Multipliers (with Fixed Exchange Rate Regime)

	$\rho_G = 0.9$	$\rho_G = 0.6$	$\rho_G = 0.3$	$\rho_G = 0.0$
Impact	1.6	1.6	1.6	1.6
Cumulative	1.0	1.3	1.4	1.5

[36]Recall that the fiscal multiplier is a ratio between output and government expenditure. For each ρ_G, the implicit autocorrelation parameter of output function is lower than the respective for government expenditure. Consider a ratio where the numerator is decreasing faster than denominator. Then, the lower the denominator, the higher the ratio will be.

Figure 3: Impulse Response Functions for a Shock in Domestic Government Expenditure



Note: The blue crossed line stands for the baseline calibration.

4 Conclusion

Are non-separable preferences able to properly reproduce empirically documented private-public consumption synchronism? Yes indeed, augmenting the Galí (2008, ch.7) open economy model with government expenditure under non-separable preferences with regard to private consumption produced fiscal multipliers in line with empirical results. A numerical simulation confirms a well-known result in the literature, fiscal policy is more effective in the fixed exchange regime as pointed out in Ilzetzki et al. (2013), Corsetti et al. (2012b) and Born et al. (2013). In this regime, fiscal multipliers reach 1.6% on impact and 1% in the long-run which are values close to the estimates of Nakamura & Steinsson (2014) and S. H. Ravn & Spange (2012). Glancing at the multipliers, fiscal policy seems an effective instrument to stimulate the economy, however the results of the numerical simulation should be carefully interpreted. Fève & Sahuc (2013) confirm that the degree of complementarity between public-private consumption governs the magnitude of the fiscal multiplier. A similar reasoning make Christiano et al. (2011) concluding that fiscal multipliers might be above or below unity depending on the specification of preferences.

Even though these findings can be subject to calibration criticisms, the choice of the main parameters is based in empirical estimates. A policy implication is derived, fiscal stimuli should be directed to sectors with high degree of complementarity. The sensitivity analysis shows that: (i) the higher the degree of complementarity, the bigger the fiscal multiplier and (ii) fiscal stimulus should be short-lived to maximize cumulative multipliers. According to Spilimbergo et al. (2009), the macroeconomic effects of fiscal policy are maximized if three conditions are observed. First of all, sectors with low propensity to import should be prioritized to avoid leakages of domestically financed stimulus to foreign countries. Second, the monetary conditions need to accommodate the fiscal stimulus, i.e., expansionary fiscal policy induce an increase in aggregate demand but not enough to create inflationary expectations and trigger a monetary policy reaction. Finally, the third aspect to take into account when deciding about expansionary fiscal policy is the financial health of the government. SOE have little scope for a substantial stimulus because to do so, countries need more savings than the respective endowment (government needs to finance expenditure abroad). Then, the ability to stimulate the economy is dampened by the prior financial balance and also by the need of keeping public deficit and public debt in a sustainable path.

In the end, it is possible to conclude that this was the first time non-separable preferences, fixed exchange regime and a government with propensity to import are combined in a new Keynesian SOE model. Additionally, other policy experiments may be performed and the model can still be enhanced with additional features. An alternative policy experiment would be to focus in international fiscal spillovers instead of fiscal multipliers. In the same perspective that domestic stimulus might leak to foreign countries, a stimulus in a foreign country might produce positive cross-border spillovers in the domestic economy. Regarding possible improvements, the private sector, in particular households, can accommodate some theoretical extensions such as: (i) rule-of-thumb consumers, a traditional Keynesian property as in Galí et al. (2007), (ii) habits in private consumption to increase persistence of business cycles and (iii) introducing wage rigidities. In the government side it would be interesting to look at the effect of fiscal policy with spending reversals, i.e. using a debt-stabilizing rule as in Corsetti et al. (2012a). Finally, Chudik & Straub (2010) argue that the SOE framework is not empirically valid because it assumes economies having similar size and a diversified trade structure. But having higher openness in certain economy does not mean that it has a diversified trade structure. These are some suggestions to refine the model and improve its robustness to empirical evidence. Finally, this thesis made a comprehensive analysis of fiscal policy in a new Keynesian SOE model. There are alternative frameworks (e.g. two-country models) to assess the transmission of fiscal policy in a currency union. However, further research is needed to have a better characterization of the transmission mechanism of fiscal policy and understand why UIP does not hold. This should be the starting point for future research about non-separable preferences in a fixed exchange regime.

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A Appendix 1

A.1 Linearisation of Effective Consumption

This section is based in Jensen (2012) and Bouakez & Rebei (2007). Starting with equation (2.2)

$$\widehat{C}_t = \left[(1-\vartheta)C_t^{1-\nu} + \vartheta G_t^{1-\nu}\right]^{\frac{1}{1-\nu}}, \tag{2.2}$$

$$\begin{aligned}\widehat{C}_t = \widehat{C} &+ \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1}{1-\nu}-1}(1-\vartheta)C^{-\nu}\left(C_t - C\right) \\ &+ \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1}{1-\nu}-1}\vartheta G^{-\nu}\left(G_t - G\right),\end{aligned}$$

$$\begin{aligned}\frac{\widehat{C}_t - \widehat{C}}{\widehat{C}} &= \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1}{1-\nu}-1}(1-\vartheta)C^{-\nu}\frac{C}{\widehat{C}}\frac{(C_t - C)}{C} \\ &+ \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1}{1-\nu}-1}\vartheta G^{-\nu}\frac{G}{\widehat{C}}\frac{(G_t - G)}{G},\end{aligned}$$

$$\widehat{c}_t = \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1-1+\nu}{1-\nu}}(1-\vartheta)C^{-\nu}\frac{C}{\widehat{C}}c_t + \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{1-1+\nu}{1-\nu}}\vartheta G^{-\nu}\frac{G}{\widehat{C}}g_t,$$

$$\widehat{c}_t = \frac{\left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{\nu}{1-\nu}}}{\widehat{C}}(1-\vartheta)C^{-\nu}Cc_t + \frac{\left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{\nu}{1-\nu}}}{\widehat{C}}\vartheta G^{-\nu}Gg_t,$$

Assuming that: $\widehat{C} = [(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}]^{\frac{1}{1-\nu}}$

$$\widehat{c}_t = \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{-(1-\nu)}{1-\nu}}(1-\vartheta)C^{1-\nu}c_t + \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{\frac{-(1-\nu)}{1-\nu}}\vartheta G^{1-\nu}g_t,$$

$$\widehat{c}_t = \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{-1}(1-\vartheta)C^{1-\nu}c_t + \left[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}\right]^{-1}\vartheta G^{1-\nu}g_t,$$

$$\widehat{c}_t = \frac{(1-\vartheta)C^{1-\nu}}{[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}]}c_t + \frac{\vartheta G^{1-\nu}}{[(1-\vartheta)C^{1-\nu} + \vartheta G^{1-\nu}]}g_t,$$

$$\widehat{c}_t = (1-\vartheta)\left(\frac{C}{\widehat{C}}\right)^{1-\nu}c_t + \vartheta\left(\frac{G}{\widehat{C}}\right)^{1-\nu}g_t. \tag{2.3}$$

A.2 Households Budget Constraint Derivation

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t. \quad \text{(A.1)}$$

Taking into consideration that the following two expressions are the demand functions for each variety of goods and the last denotes the optimal allocation of imports with respect to the country of origin. Replacing above yields the following

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}, \qquad C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}, \quad \text{(A.2)}$$

$$\int_0^1 P_{H,t}(j)\left(\frac{P_{H,t(j)}}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}dj + \int_0^1\int_0^1 P_{i,t}(j)\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon}\left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}djdi + E_t\left\{Q_{t,t+1}D_{t+1}\right\}$$
$$\leq D_t + W_tN_t + T_t,$$

$$\int_0^1 P_{H,t}(j)^{1-\varepsilon}P_{H,t}^{\varepsilon}C_{H,t}dj + \int_0^1\int_0^1 P_{i,t}(j)^{1-\varepsilon}P_{i,t}^{\varepsilon-\gamma}P_{F,t}^{\gamma}C_{F,t}djdi + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_{H,t}^{\varepsilon}C_{H,t}\int_0^1 P_{H,t}(j)^{1-\varepsilon}dj + P_{F,t}^{\gamma}C_{F,t}\int_0^1\int_0^1 P_{i,t}(j)^{1-\varepsilon}P_{i,t}^{\varepsilon-\gamma}djdi + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}},$$

$$P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}},$$

$$P_{H,t}^{\varepsilon}C_{H,t}P_{H,t}^{1-\varepsilon} + P_{F,t}^{\gamma}C_{F,t}\int_0^1 P_{i,t}^{1-\varepsilon}P_{i,t}^{\varepsilon-\gamma}di + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_{H,t}C_{H,t} + P_{F,t}^{\gamma}C_{F,t}\int_0^1 P_{i,t}^{1-\gamma}di + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}},$$

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P^C}\right)^{-\eta} C_t; \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P^C}\right)^{-\eta} C_t,$$

$$P_{H,t}(1-\alpha)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t + P_{F,t}\alpha\left(\frac{P_{F,t}}{P_t^C}\right)^{-\eta} C_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$\left(P_t^C\right)^{\eta} C_t\left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right] + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_t^C \equiv \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$\left(P_t^C\right)^{\eta} C_t\left(P_t^C\right)^{1-\eta} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t,$$

$$P_t^C C_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t + T_t. \tag{2.6}$$

A.3 Households Optimization Problem

In order to solve the households intra and intertemporal optimization problem one has to write down the Lagrangian for two consecutive periods and then derive the First Order Conditions (FOC) as follows

$$
\begin{aligned}
\mathcal{L} = \ldots + E_t\beta^t & \left(\frac{\left[(1-\vartheta)C_t^{(1-\nu)} + \vartheta G_t^{(1-\nu)}\right]^{\frac{1-\sigma}{1-\nu}}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} - \lambda_t \left(P_t^C C_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} - D_t - W_t N_t - T_t\right) \right) \\
+ E_{t+1}\beta^{t+1} & \left(\frac{\left[(1-\vartheta)C_{t+1}^{(1-\nu)} + \vartheta G_{t+1}^{(1-\nu)}\right]^{\frac{1-\sigma}{1-\nu}}}{1-\sigma} - \frac{N_{t+1}^{1+\varphi}}{1+\varphi} - \lambda_{t+1} \left(P_{t+1}^C C_{t+1} + E_{t+1}\left\{Q_{t+1,t+2}D_{t+2}\right\} - D_{t+1} - W_{t+1} N_{t+1} - T_{t+1}\right) \right) + \ldots
\end{aligned}
$$

First Order Conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= 0 \Leftrightarrow \beta^t \left((1-\vartheta)C_t^{-\nu}\widehat{C}_t^{\nu-\sigma} - \lambda_t P_t^C\right) = 0, \\
\frac{\partial \mathcal{L}}{\partial N_t} &= 0 \Leftrightarrow \beta^t \left(-N_t^{\varphi} + \lambda_t W_t\right) = 0, \\
\frac{\partial \mathcal{L}}{\partial D_{t+1}} &= 0 \Leftrightarrow \beta^t \left(-\lambda_t E_t\left\{Q_{t,t+1}\right\}\right) + \beta^{t+1}\left(\lambda_{t+1}\right) = 0.
\end{aligned}
$$

Intratemporal Optimization Equation

$$
\frac{W_t}{P_t^C} = (1-\vartheta)^{-1} N_t^{\varphi} C_t^{\nu} \widehat{C}_t^{\sigma-\nu}, \tag{2.7}
$$

$$
w_t - p_t^C = \varphi n_t + \nu c_t + (\sigma - \nu)\,\widehat{c}_t, \tag{2.9}
$$

$$
w_t - p_t^C = \varphi n_t + \sigma c_t + (\nu - \sigma)\left(c_t - \widehat{c}_t\right). \tag{2.10}
$$

Intertemporal Optimization Equation (Euler equation)

$$
\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t}\right)^{\nu-\sigma} \left(\frac{P_t^C}{P_{t+1}^C}\right) \right\} = E_t\left\{Q_{t,t+1}\right\}, \tag{2.8}
$$

$$
c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\nu}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right) + \frac{\nu-\sigma}{\nu}\left(E_t\left\{\widehat{c}_{t+1}\right\} - \widehat{c}_t\right), \tag{2.11}
$$

where $\pi^C_{t+1} = p^C_{t+1} - p^C_t$ is the CPI inflation, more details in equation (2.20) in t+1.

$$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi^C_{t+1}\right\} - (\nu - \sigma) E_t\left\{(c_{t+1} - \widehat{c}_{t+1}) - (c_t - \widehat{c}_t)\right\} - \rho\right). \qquad (2.12)$$

A.4 Government Budget Constraint

$$\int_0^1 P_{H,t}(j) G_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) G_{i,t}(j) dj di + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq T_t + D_t. \qquad (A.3)$$

Taking into consideration that the following two expressions are the demand functions for each variety of goods and the last denotes the optimal allocation of imports with respect to the country of origin. Replacing above yields the following

$$G_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} G_{H,t}, \qquad G_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} G_{i,t}, \qquad G_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} G_{F,t}, \qquad (A.4)$$

$$\int_0^1 P_{H,t}(j) \left(\frac{P_{H,t(j)}}{P_{H,t}}\right)^{-\varepsilon} G_{H,t} dj + \int_0^1 \int_0^1 P_{i,t}(j) \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} G_{F,t} dj di + E_t\left\{Q_{t,t+1} D_{t+1}\right\}$$
$$\leq T_t + D_t,$$

$$\int_0^1 P_{H,t}(j)^{1-\varepsilon} P^{\varepsilon}_{H,t} G_{H,t} dj + \int_0^1 \int_0^1 P_{i,t}(j)^{1-\varepsilon} P^{\varepsilon-\gamma}_{i,t} P^{\gamma}_{F,t} G_{F,t} dj di + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq T_t + D_t,$$

$$P^{\varepsilon}_{H,t} G_{H,t} \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj + P^{\gamma}_{F,t} G_{F,t} \int_0^1 \int_0^1 P_{i,t}(j)^{1-\varepsilon} P^{\varepsilon-\gamma}_{i,t} dj di + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq T_t + D_t,$$

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}},$$

$$P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}},$$

$$P^{\varepsilon}_{H,t} G_{H,t} P^{1-\varepsilon}_{H,t} + P^{\gamma}_{F,t} G_{F,t} \int_0^1 P^{1-\varepsilon}_{i,t} P^{\varepsilon-\gamma}_{i,t} di + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq T_t + D_t,$$

$$P_{H,t}G_{H,t} + P_{F,t}^{\gamma}G_{F,t}\int_0^1 P_{i,t}^{1-\gamma}di + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t,$$

$$P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma}di\right)^{\frac{1}{1-\gamma}},$$

$$P_{H,t}G_{H,t} + P_{F,t}G_{F,t} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t,$$

$$G_{H,t} = (1-\chi)\left(\frac{P_{H,t}}{P^G}\right)^{-\eta}G_t, \qquad G_{F,t} = \chi\left(\frac{P_{F,t}}{P^G}\right)^{-\eta}G_t,$$

$$P_{H,t}(1-\chi)\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}G_t + P_{F,t}\chi\left(\frac{P_{F,t}}{P_t^G}\right)^{-\eta}G_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t,$$

$$\left(P_t^G\right)^{\eta}G_t\left[(1-\chi)P_{H,t}^{1-\eta} + \chi P_{F,t}^{1-\eta}\right] + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t,$$

$$P_t^G \equiv \left[(1-\chi)P_{H,t}^{1-\eta} + \chi P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$\left(P_t^G\right)^{\eta}G_t\left(P_t^G\right)^{1-\eta} + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t,$$

$$P_t^G G_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq T_t + D_t. \tag{2.13}$$

A.5 Private Consumption Price Index (CPI)

Private consumption

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{2.4}$$

Minimizing total expenditure in private consumption relative to its composition

$$\mathcal{L} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + \lambda_t^C \left[C_t - \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right].$$

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{\lambda_t^C}\right)^{-\eta} C_t. \tag{A.5}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{\lambda_t^C}\right)^{-\eta} C_t. \tag{A.6}$$

Obtaining consumer price index (CPI)

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$C_t = \left[(1-\alpha)\left(\frac{\lambda_t^C}{P_{H,t}}\right)^{\eta-1} C_t^{\frac{\eta-1}{\eta}} + \alpha\left(\frac{\lambda_t^C}{P_{F,t}}\right)^{\eta-1} C_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$C_t = \left(\lambda_t^C\right)^{\eta}\left[(1-\alpha)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} C_t^{\frac{\eta-1}{\eta}} + \alpha\left(\frac{1}{P_{F,t}}\right)^{\eta-1} C_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$C_t = \left(\lambda_t^C\right)^{\eta} C_t \left[(1-\alpha)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} + \alpha\left(\frac{1}{P_{F,t}}\right)^{\eta-1}\right]^{\frac{\eta}{\eta-1}},$$

$$P_t^C \equiv \lambda_t^C = \left[(1-\alpha)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} + \alpha\left(\frac{1}{P_{F,t}}\right)^{\eta-1}\right]^{-\frac{1}{\eta-1}}.$$

The CPI equation is given by

$$P_t^C \equiv \lambda_t^C = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{A.7}$$

Applying a log-linearisation around a symmetric steady state yields

$$p_t^C \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t, \tag{2.19}$$

$$\pi_t^C \equiv (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} = \pi_{H,t} + \alpha\Delta s_t. \tag{2.20}$$

A.6 Government Expenditure Price Index (GPI)

Public Consumption

$$G_t = \left[(1-\chi)^{\frac{1}{\eta}}\left(G_{H,t}\right)^{\frac{\eta-1}{\eta}} + \chi^{\frac{1}{\eta}}\left(G_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{2.5}$$

Minimizing total expenditure in public consumption relative to its composition

$$\mathcal{L} = P_{H,t}G_{H,t} + P_{F,t}G_{F,t} + \lambda_t^G\left[G_t - \left[(1-\chi)^{\frac{1}{\eta}}\left(G_{H,t}\right)^{\frac{\eta-1}{\eta}} + \chi^{\frac{1}{\eta}}\left(G_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right].$$

$$G_{H,t} = (1-\chi)\left(\frac{P_{H,t}}{\lambda_t^G}\right)^{-\eta} G_t. \tag{A.8}$$

$$G_{F,t} = \chi\left(\frac{P_{F,t}}{\lambda_t^G}\right)^{-\eta} G_t. \tag{A.9}$$

Obtaining government price index (GPI)

$$G_t = \left[(1-\chi)^{\frac{1}{\eta}}\left(G_{H,t}\right)^{\frac{\eta-1}{\eta}} + \chi^{\frac{1}{\eta}}\left(G_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$G_t = \left[(1-\chi)\left(\frac{\lambda_t^G}{P_{H,t}}\right)^{\eta-1} G_t^{\frac{\eta-1}{\eta}} + \chi\left(\frac{\lambda_t^G}{P_{F,t}}\right)^{\eta-1} G_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$G_t = \left(\lambda_t^G\right)^\eta \left[(1-\chi)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} G_t^{\frac{\eta-1}{\eta}} + \chi\left(\frac{1}{P_{F,t}}\right)^{\eta-1} G_t^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$G_t = \left(\lambda_t^G\right)^\eta G_t \left[(1-\chi)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} + \chi\left(\frac{1}{P_{F,t}}\right)^{\eta-1}\right]^{\frac{\eta}{\eta-1}},$$

$$P_t^G \equiv \lambda_t^G = \left[(1-\chi)\left(\frac{1}{P_{H,t}}\right)^{\eta-1} + \chi\left(\frac{1}{P_{F,t}}\right)^{\eta-1}\right]^{-\frac{1}{\eta-1}}.$$

The GPI equation is given by

$$P_t^G \equiv \lambda_t^G = \left[(1-\chi)\left(P_{H,t}\right)^{1-\eta} + \chi\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{A.10}$$

Applying a log-linearisation around a symmetric steady state yields

$$p_t^G \equiv (1-\chi)p_{H,t} + \chi p_{F,t} = p_{H,t} + \chi s_t, \tag{2.21}$$

$$\pi_t^G \equiv (1-\chi)\pi_{H,t} + \chi\pi_{F,t} = \pi_{H,t} + \chi\Delta s_t. \tag{2.22}$$

A.7 International Risk Sharing

Intertemporal optimization equation for the SOE

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{\nu-\sigma} \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\} = E_t \left\{ Q_{t,t+1} \right\}. \tag{2.8}$$

Intertemporal optimization equation for each of the rest of world SOEs

$$\beta E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}^i}{\widehat{C}_t^i} \right)^{\nu-\sigma} \left(\frac{\mathcal{E}_{i,t}}{\mathcal{E}_{i,t+1}} \right) \left(\frac{P_t^{C,i}}{P_{t+1}^{C,i}} \right) \right\} = E_t \left\{ Q_{t,t+1} \right\}, \tag{2.32}$$

$$\beta E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}^i}{\widehat{C}_t^i} \right)^{\nu-\sigma} \left(\frac{\mathcal{Q}_{i,t}^C}{\mathcal{Q}_{i,t+1}^C} \right) \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\} = E_t \left\{ Q_{t,t+1} \right\}.$$

Equating both expressions, one has

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{\nu-\sigma} \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\} = \beta E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}^i}{\widehat{C}_t^i} \right)^{\nu-\sigma} \left(\frac{\mathcal{Q}_{i,t}^C}{\mathcal{Q}_{i,t+1}^C} \right) \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\},$$

$$E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{\nu-\sigma} \right\} = E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}^i}{\widehat{C}_t^i} \right)^{\nu-\sigma} \left(\frac{\mathcal{Q}_{i,t}^C}{\mathcal{Q}_{i,t+1}^C} \right) \right\},$$

$$C_t^{\nu} \widehat{C}_t^{\sigma-\nu} = \varpi_i \left(C_t^i \right)^{\nu} \left(\widehat{C}_t^i \right)^{\sigma-\nu} \mathcal{Q}_{i,t}^C, \tag{2.33}$$

where ϖ_i is *"a constant that will generally depend on initial conditions regarding relative net asset positions"*[37]. Log-linearising (2.33), integrating over i and assuming $\varpi_i = \varpi = 1$, i.e.,

[37] Galí (2008, p.157)

zero net foreign asset holdings

$$\nu c_t + (\sigma - \nu)\,\widehat{c}_t = \nu c_t^* + (\sigma - \nu)\,\widehat{c}_t^* + q_t^C, \tag{2.34}$$

$$\nu c_t + (\sigma - \nu)\,\widehat{c}_t = \nu c_t^* + (\sigma - \nu)\,\widehat{c}_t^* + (1 - \alpha)\,s_t,$$

$$\nu c_t = \nu c_t^* + (\sigma - \nu)\,(\widehat{c}_t^* - \widehat{c}_t) + (1 - \alpha)\,s_t,$$

$$c_t = c_t^* + \frac{\sigma - \nu}{\nu}\,(\widehat{c}_t^* - \widehat{c}_t) + \frac{1 - \alpha}{\nu} s_t, \tag{2.35}$$

where $c_t^* = \int_0^1 c_t^i di$ is the (log) index for world consumption.

A.8 Uncovered Interest Parity

The intertemporal optimality equation (often called Euler equation) takes the form

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{\nu - \sigma} \left(\frac{P_t^C}{P_{t+1}^C} \right) \right\} = Q_t, \tag{2.8}$$

where $Q_t = E_t \{Q_{t,t+1}\}$.

The above expressions is log-linearised with respect to their steady state values as follows

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\nu}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right) + \frac{\nu - \sigma}{\nu}\left(E_t\{\widehat{c}_{t+1}\} - \widehat{c}_t\right) \tag{2.11}$$

where lower case letters represent the logs of the respective variables, $E_t\left\{\pi_{t+1}^C\right\} = E_t\left\{p_{t+1}^C\right\} - p_t^C$ is CPI inflation[38] (with $p_t^C \equiv \log P_t^C$), $i_t \equiv -\log E_t\{Q_{t,t+1}\}$ is short-term nominal interest rate and $\rho \equiv -\log \beta$ is the steady state time discount rate.

Simplifying in order to use later

$$\nu \Delta E_t\{c_{t+1}\} = \left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right) + (\sigma - \nu)\left(E_t\{\Delta \widehat{c}_{t+1}\}\right). \tag{A.11}$$

[38] More details in Section 2.3 below.

Similarly for the world economy there are identical expressions.

$$\beta E_t \left\{ \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\nu} \left(\frac{\widehat{C}^*_{t+1}}{\widehat{C}^*_t} \right)^{\nu-\sigma} \left(\frac{P^{C,*}_t}{P^{C,*}_{t+1}} \right) \right\} = Q^*_t, \tag{A.12}$$

where $Q^*_t = E_t \left\{ Q^*_{t,t+1} \right\}$.

The above expressions is log-linearised with respect to their steady state values as follows

$$c^*_t = E_t \left\{ c^*_{t+1} \right\} - \frac{1}{\nu} \left(i^*_t - E_t \left\{ \pi^{C,*}_{t+1} \right\} - \rho \right) + \frac{\nu - \sigma}{\nu} \left(E_t \left\{ \widehat{c}^*_{t+1} \right\} - \widehat{c}^*_t \right) \tag{A.13}$$

Simplifying in order to use later

$$\nu \Delta E_t \left\{ c^*_{t+1} \right\} = \left(i^*_t - E_t \left\{ \pi^{C,*}_{t+1} \right\} - \rho \right) + (\sigma - \nu) \left(E_t \left\{ \Delta \widehat{c}^*_{t+1} \right\} \right). \tag{A.14}$$

Using international risk sharing expression derived in 2.4

$$\nu c_t + (\sigma - \nu) \widehat{c}_t = \nu c^*_t + (\sigma - \nu) \widehat{c}^*_t + q^C_t \tag{2.34}$$

Making the first differences in t+1 one has

$$\nu E_t \left\{ \Delta c_{t+1} \right\} + (\sigma - \nu) E_t \left\{ \Delta \widehat{c}_{t+1} \right\} = \nu E_t \left\{ \Delta c^*_{t+1} \right\} + (\sigma - \nu) E_t \left\{ \Delta \widehat{c}^*_{t+1} \right\} + E_t \left\{ \Delta q^C_{t+1} \right\} \tag{A.15}$$

Using equations (A.11) and (A.14) to substitute in (A.15) yields

$$\begin{aligned} &\left(i_t - E_t \left\{ \pi^C_{t+1} \right\} - \rho \right) + (\sigma - \nu) \left(E_t \left\{ \Delta \widehat{c}_{t+1} \right\} \right) + (\sigma - \nu) E_t \left\{ \Delta \widehat{c}_{t+1} \right\} = \left(i^*_t - E_t \left\{ \pi^{C,*}_{t+1} \right\} - \rho \right) \\ &+ (\sigma - \nu) \left(E_t \left\{ \Delta \widehat{c}^*_{t+1} \right\} \right) + (\sigma - \nu) E_t \left\{ \Delta \widehat{c}^*_{t+1} \right\} + E_t \left\{ \Delta q^C_{t+1} \right\} \end{aligned} \tag{A.16}$$

$$\left(i_t - E_t\left\{\pi^C_{t+1}\right\} - \rho\right) + 2\left(\sigma - \nu\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\}\right) = \left(i^*_t - E_t\left\{\pi^{C,*}_{t+1}\right\} - \rho\right) + 2\left(\sigma - \nu\right)\left(E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right)$$
$$+ E_t\left\{\Delta q^C_{t+1}\right\} \tag{A.17}$$

Using the first differences of (2.27) in t+1

$$E_t\left\{\Delta q^C_{t+1}\right\} \equiv (1-\alpha)\, E_t\left\{\Delta s_{t+1}\right\}. \tag{A.18}$$

Making the first differences of (2.23) in t+1

$$E_t\left\{\Delta s_{t+1}\right\} = E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\} - E_t\left\{\pi_{H,t+1}\right\}. \tag{A.19}$$

Substituting (A.19) into (A.18) and then substituting the resulting expression into (A.17) yields

$$\left(i_t - E_t\left\{\pi^C_{t+1}\right\} - \rho\right) + 2\left(\sigma - \nu\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\}\right) = \left(i^*_t - E_t\left\{\pi^{C,*}_{t+1}\right\} - \rho\right) + 2\left(\sigma - \nu\right)\left(E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right)$$
$$+ (1-\alpha)\left(E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\} - E_t\left\{\pi_{H,t+1}\right\}\right) \tag{A.20}$$

Simplifying one has

$$i_t - E_t\left\{\pi^C_{t+1}\right\} + 2\left(\sigma - \nu\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right) = i^*_t - E_t\left\{\pi^{C,*}_{t+1}\right\}$$
$$+ (1-\alpha)\left(E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\} - E_t\left\{\pi_{H,t+1}\right\}\right) \tag{A.21}$$

Using the result derived in (2.26), a conclusion emerges, $E_t\left\{\pi^*_{t+1}\right\} = E_t\left\{\pi^{C,*}_{t+1}\right\}$. Also having in mind equation (2.20) in t+1, $E_t\left\{\pi^C_{t+1}\right\} - E_t\left\{\pi_{H,t+1}\right\} = \alpha E_t\left\{\Delta s_{t+1}\right\}$. Replacing these results in the expression above yields

$$i_t + 2(\sigma - \nu)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right) = i^*_t + \alpha E_t\left\{\Delta s_{t+1}\right\} - E_t\left\{\pi^*_{t+1}\right\}$$
$$+ (1-\alpha)\left(E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\}\right) + \alpha E_t\left\{\pi_{H,t+1}\right\} \tag{A.22}$$

Substituting (A.19) again produces

$$i_t + 2(\sigma - \nu)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right) = i^*_t + \alpha\left(E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\} - E_t\left\{\pi_{H,t+1}\right\}\right)$$
$$- E_t\left\{\pi^*_{t+1}\right\} + (1-\alpha)\left(E_t\left\{\Delta e_{t+1}\right\} + E_t\left\{\pi^*_{t+1}\right\}\right) + \alpha E_t\left\{\pi_{H,t+1}\right\} \tag{A.23}$$

Simplifying yields the following

$$i_t = i^*_t + E_t\left\{\Delta e_{t+1}\right\} - 2(\sigma - \nu)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}^*_{t+1}\right\}\right). \tag{3.3}$$

The UIP does not hold in this model due to non-separability assumption.

A.9 Firms Optimization Problem

In this model, each firm minimize its production costs relative to the composition of production as follows

$$\mathcal{L} = \left(\frac{W_t}{P_{H,t}}\right) N_t + MC_t \left(Y_t - A_t N_t\right)$$

Moreover, each firm chooses the stock of labour which gives as first order condition

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Leftrightarrow \left(\frac{W_t}{P_{H,t}}\right) = MC_t A_t$$

if the identity above is linearised it yields

$$mc_t = -\delta + w_t - p_{H,t} - a_t \tag{2.37}$$

where $\delta \equiv \log(1 - \tau)$ is an employment subsidy as in Galí (2008, p.169).

A.10 Demand for Good j

$$Y_t(j) = (1-\vartheta)\left[C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di\right] + \vartheta\left[G_{H,t}(j) + \int_0^1 G^i_{H,t}(j)di\right] \tag{A.24}$$

Substituing $C_{H,t}(j)$, with the expression in (A.2), and $G_{H,t}(j)$, with the expression in (A.4).

$$Y_t(j) = (1-\vartheta)\left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \int_0^1 C^i_{H,t}(j)di\right] + \vartheta\left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} G_{H,t} + \int_0^1 G^i_{H,t}(j)di\right]. \tag{A.25}$$

Substituing $C_{H,t}$, with the expression in (A.2), and $G_{H,t}$, with the expression in (A.4).

$$\begin{aligned} Y_t(j) = (1-\vartheta)&\left[(1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P^C_t}\right)^{-\eta} C_t + \int_0^1 C^i_{H,t}(j)di\right] \\ +\vartheta&\left[(1-\chi)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P^G_t}\right)^{-\eta} G_t + \int_0^1 G^i_{H,t}(j)di\right]. \end{aligned} \tag{A.26}$$

Obtaining $\int_0^1 C^i_{H,t}(j)di$ and $\int_0^1 G^i_{H,t}(j)di$

Remember that

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t}; \qquad C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}; \tag{A.2}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P^C_t}\right)^{-\eta} C_t. \tag{A.6}$$

Substituting the last two into the first expression, one has

$$C_{i,t}(j) = \alpha\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon}\left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P^C_t}\right)^{-\eta} C_t. \tag{A.27}$$

Substituting i for h, yields

$$C_{H,t}(j) = \alpha\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P^C_t}\right)^{-\eta} C_t. \tag{A.28}$$

Remember that

$$G_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} G_{i,t}; \qquad G_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} G_{F,t}; \tag{A.4}$$

$$G_{F,t} = \chi\left(\frac{P_{F,t}}{P^G_t}\right)^{-\eta} G_t. \tag{A.9}$$

Substituting the last two into the first expression, one has

$$G_{i,t}(j) = \chi\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon}\left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P^G_t}\right)^{-\eta} G_t. \tag{A.31}$$

Substituting i for h, yields

$$G_{H,t}(j) = \chi\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\gamma}\left(\frac{P_{F,t}}{P^G_t}\right)^{-\eta} G_t. \tag{A.32}$$

Making the integral over the universe [0,1]

$$\int_0^1 C^i_{H,t}(j)di,$$
$$= \int_0^1 \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{C,i}_t}\right)^{-\eta} C^i_t di. \tag{A.29}$$

Changing i for F in the LOP, one has

$$P_{F,t} = \mathcal{E}_{i,t} P^i_{F,t}.$$

Improving the integral with this insight

$$\int_0^1 C^i_{H,t}(j)di,$$
$$= \int_0^1 \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{C,i}_t}\right)^{-\eta} C^i_t di. \tag{A.30}$$

Making the integral over the universe [0,1]

$$\int_0^1 G^i_{H,t}(j)di,$$
$$= \int_0^1 \chi \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{G,i}_t}\right)^{-\eta} G^i_t di. \tag{A.33}$$

Changing i for F in the LOP, one has

$$P_{F,t} = \mathcal{E}_{i,t} P^i_{F,t}.$$

Improving the integral with this insight

$$\int_0^1 G^i_{H,t}(j)di,$$
$$= \int_0^1 \chi \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{G,i}_t}\right)^{-\eta} G^i_t di. \tag{A.34}$$

Substituting these last two results into equation (A.26)

$$\begin{aligned} Y_t(j) = (1-\vartheta) &\left[(1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P^C_t}\right)^{-\eta} C_t + \int_0^1 \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{C,i}_t}\right)^{-\eta} C^i_t di\right] \\ +\vartheta &\left[(1-\chi)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P^G_t}\right)^{-\eta} G_t + \int_0^1 \chi \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{G,i}_t}\right)^{-\eta} G^i_t di\right]. \end{aligned} \tag{A.35}$$

Simplifying the last result

$$\begin{aligned} Y_t(j) = (1-\vartheta)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} &\left[(1-\alpha)\left(\frac{P_{H,t}}{P^C_t}\right)^{-\eta} C_t + \alpha\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{C,i}_t}\right)^{-\eta} C^i_t di\right] \\ +\vartheta\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} &\left[(1-\chi)\left(\frac{P_{H,t}}{P^G_t}\right)^{-\eta} G_t + \chi\int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^{G,i}_t}\right)^{-\eta} G^i_t di\right]. \end{aligned} \tag{2.40}$$

Now, the equation is similar to equation (24) in Galí (2008) but with two novelties, first government consumption is added to the demand for good j and second, each type of consumption (private or public) has its own price. Please notice P^C_t and P^G_t.

A.11 Aggregate Demand

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.36}$$

$$\begin{aligned} Y_t = (1-\vartheta) & \left[(1-\alpha) \left(\frac{P_{H,t}}{P_t^C} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^{C,i}} \right)^{-\eta} C_t^i di \right] \\ & + \vartheta \left[(1-\chi) \left(\frac{P_{H,t}}{P_t^G} \right)^{-\eta} G_t + \chi \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^{G,i}} \right)^{-\eta} G_t^i di \right]. \end{aligned} \tag{2.41}$$

Following the book Galí (2008), one has to apply five algebraic tricks in order to arrive at the final result.

1. Changing $\left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i} \right)^{-\gamma}$ for $\left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma}$,
2. Multiply and divide by $P_{H,t}^{-\eta}$,
3. Multiply and divide by $\left(P_t^C\right)^{-\eta}$ (in the part of private consumption),
4. Multiply and divide by $\left(P_t^G\right)^{-\eta}$ (in the part of government consumption),
5. Multiply and divide by $\mathcal{E}_{i,t}$.

After applying all these arithmetic operations, one has the following

$$\begin{aligned} Y_t = (1-\vartheta) & \left[(1-\alpha) \left(\frac{P_{H,t}}{P_t^C} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{P_t^C} \right)^{-\eta} \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma} \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{-\eta} \left(\frac{P_t^C}{\mathcal{E}_{i,t} P_t^{C,i}} \right)^{-\eta} C_t^i di \right] \\ & + \vartheta \left[(1-\chi) \left(\frac{P_{H,t}}{P_t^G} \right)^{-\eta} G_t + \chi \int_0^1 \left(\frac{P_{H,t}}{P_t^G} \right)^{-\eta} \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma} \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{-\eta} \left(\frac{P_t^G}{\mathcal{E}_{i,t} P_t^{G,i}} \right)^{-\eta} G_t^i di \right], \end{aligned} \tag{A.37}$$

$$Y_t = (1-\vartheta)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1 \left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^C\right)^{\eta} C_t^i di\right]$$
$$+\vartheta\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}\left[(1-\chi)G_t + \chi\int_0^1 \left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^G\right)^{\eta} G_t^i di\right]. \quad (2.42)$$

Employing equation (2.33) and assuming that $\varpi_i = \varpi = 1$ in equilibrium, one has the following

$$C_t^i = C_t\left(\frac{\widehat{C}_t}{\widehat{C}_t^i}\right)^{\frac{\sigma-\nu}{\nu}}\left(\mathcal{Q}_{i,t}^C\right)^{-\frac{1}{\nu}}. \quad (A.38)$$

Substituting the expression above, one has

$$Y_t = (1-\vartheta)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1 \left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^C\right)^{\eta-\frac{1}{\nu}}\left(\frac{\widehat{C}_t}{\widehat{C}_t^i}\right)^{\frac{\sigma-\nu}{\nu}} di\right]$$
$$+\vartheta\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}\left[(1-\chi)G_t + \chi\int_0^1 \left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^G\right)^{\eta} G_t^i di\right]. \quad (A.39)$$

In order to simplify $\left(\frac{\mathcal{E}_{i,t}P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta}$ one has to use the following three equations

1. LOP, as above $P_{F,t} = \mathcal{E}_{i,t}P_{F,t}^i$,
2. Terms of trade (ToT) between home economy and country i: $\mathcal{S}_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$,
3. Effective terms of trade (ToT) for country i: $\mathcal{S}_t^i = \frac{P_{F,t}}{P_{i,t}}$.

$$Y_t = (1-\vartheta)\left(\frac{P_{H,t}}{P_t^C}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha\int_0^1 \left(\mathcal{S}_t^i\mathcal{S}_{i,t}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^C\right)^{\eta-\frac{1}{\nu}}\left(\frac{\widehat{C}_t}{\widehat{C}_t^i}\right)^{\frac{\sigma-\nu}{\nu}} di\right]$$
$$+\vartheta\left(\frac{P_{H,t}}{P_t^G}\right)^{-\eta}\left[(1-\chi)G_t + \chi\int_0^1 \left(\mathcal{S}_t^i\mathcal{S}_{i,t}\right)^{\gamma-\eta}\left(\mathcal{Q}_{i,t}^G\right)^{\eta} G_t^i di\right]. \quad (2.43)$$

Log-linearising and integrating over [0,1] the last expression, and recalling that $\int_0^1 s_t^i di = 0$, one has

$$y_t = (1-\vartheta)\left[-\eta\left(p_{H,t}-p_t^C\right)+c_t+\alpha\left[(\gamma-\eta)\,s_t+\left(\eta-\frac{1}{\nu}\right)q_t^C+\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t-\widehat{c}_t^*)\right]\right]$$
$$+\vartheta\left[-\eta\left(p_{H,t}-p_t^G\right)+(1-\chi)g_t+\chi\left[(\gamma-\eta)\,s_t+\eta q_t^G+g_t^*\right]\right], \tag{A.40}$$

$$y_t = (1-\vartheta)\left[\eta\left(p_t^C-p_{H,t}\right)+c_t+\alpha\left[(\gamma-\eta)\,s_t+\left(\eta-\frac{1}{\nu}\right)q_t^C+\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t-\widehat{c}_t^*)\right]\right]$$
$$+\vartheta\left[\eta\left(p_t^G-p_{H,t}\right)+(1-\chi)g_t+\chi\left[(\gamma-\eta)\,s_t+\eta q_t^G+g_t^*\right]\right]. \tag{A.41}$$

Having in mind equations (2.19) and (2.21) and substituting yields

$$y_t = (1-\vartheta)\left[\eta\alpha s_t+c_t+\alpha\left[(\gamma-\eta)\,s_t+\left(\eta-\frac{1}{\nu}\right)q_t^C+\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t-\widehat{c}_t^*)\right]\right]$$
$$+\vartheta\left[\eta\chi s_t+(1-\chi)g_t+\chi\left[(\gamma-\eta)\,s_t+\eta q_t^G+g_t^*\right]\right]. \tag{A.42}$$

Using equations (2.27) and (2.29) to substitute. Also simplifying $\eta\alpha s_t$

$$y_t = (1-\vartheta)\left[c_t+\alpha\left[\gamma s_t+\left(\eta-\frac{1}{\nu}\right)(1-\alpha)\,s_t+\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t-\widehat{c}_t^*)\right]\right]$$
$$+\vartheta\left[(1-\chi)g_t+\chi\left[\gamma s_t+\eta\,(1-\chi)\,s_t+g_t^*\right]\right]. \tag{A.43}$$

Cutting all brackets one has

$$y_t = (1-\vartheta)\left[c_t+\alpha\gamma s_t+\alpha\left(\eta-\frac{1}{\nu}\right)(1-\alpha)\,s_t+\alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t-\widehat{c}_t^*)\right]$$
$$+\vartheta\left[(1-\chi)g_t+\chi\gamma s_t+\chi\eta\,(1-\chi)\,s_t+\chi g_t^*\right], \tag{A.44}$$

$$y_t = (1-\vartheta)\left[c_t + \frac{\alpha}{\nu}\left[\gamma\nu + (\eta\nu - 1)(1-\alpha)\right]s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right]$$
$$+ \vartheta\left[(1-\chi)g_t + \chi\left[\gamma + \eta(1-\chi)\right]s_t + \chi g_t^*\right]. \tag{A.45}$$

Substituting $\omega^C \equiv \gamma\nu + (\eta\nu - 1)(1-\alpha)$ and $\omega^G \equiv \gamma\nu + \eta\nu(1-\chi)$ one has[39]

$$y_t = (1-\vartheta)\left[c_t + \frac{\alpha}{\nu}\omega^C s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right] + \vartheta\left[(1-\chi)g_t + \frac{\chi}{\nu}\omega^G s_t + \chi g_t^*\right]. \tag{2.45}$$

World output is given by[40]

$$y_t^* = (1-\vartheta)c_t^* + \vartheta g_t^*, \tag{2.46}$$

because $\int_0^1 s_t^i di = 0$.

Substituting (2.35) into (2.45), one has

$$y_t = (1-\vartheta)\left[c_t^* + \frac{\sigma-\nu}{\nu}(\widehat{c}_t^* - \widehat{c}_t) + \frac{1-\alpha}{\nu}s_t + \frac{\alpha}{\nu}\omega^C s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right]$$
$$+ \vartheta\left[(1-\chi)g_t + \frac{\chi}{\nu}\omega^G s_t + \chi g_t^*\right]. \tag{A.46}$$

Isolating an expression similar to (2.46) to substitute

$$y_t = (1-\vartheta)c_t^* + \vartheta g_t^* + (1-\vartheta)\left[(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t^* - \widehat{c}_t) + \frac{1-\alpha}{\nu}s_t + \frac{\alpha}{\nu}\omega^C s_t\right]$$
$$+ \vartheta\left[(1-\chi)(g_t - g_t^*) + \frac{\chi}{\nu}\omega^G s_t\right]. \tag{A.47}$$

Substituting (2.46) and isolating s_t

[39] As in Galí (2008, equation (27) of ch.7)
[40] As in Galí (2008, equation (28) of ch.7)

$$y_t = y_t^* + (1-\vartheta)(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t^* - \widehat{c}_t) + \vartheta\left[(1-\chi)(g_t - g_t^*)\right]$$
$$+ \left[(1-\vartheta)\frac{\alpha}{\nu}(\omega^C - 1) + \vartheta\frac{\chi}{\nu}\omega^G + \frac{(1-\vartheta)}{\nu}\right] s_t. \quad \text{(A.48)}$$

Assuming that $\Upsilon = \left[(1-\vartheta)\frac{\alpha}{\nu}(\omega^C - 1) + \vartheta\frac{\chi}{\nu}\omega^G + \frac{(1-\vartheta)}{\nu}\right]$ and substituting in the last equation[41]

$$y_t = y_t^* + (1-\vartheta)(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t^* - \widehat{c}_t) + \vartheta(1-\chi)(g_t - g_t^*) + \Upsilon s_t. \quad \text{(2.47)}$$

Solving w.r.t. s_t to use below, one has

$$s_t = \frac{1}{\Upsilon}(y_t - y_t^*) - \left(\frac{1-\vartheta}{\Upsilon}\right)(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t^* - \widehat{c}_t) - \left(\frac{\vartheta}{\Upsilon}\right)(1-\chi)(g_t - g_t^*). \quad \text{(A.49)}$$

Making the first differences in t+1, one has

$$E_t\{\Delta s_{t+1}\} = -\left(\frac{1-\vartheta}{\Upsilon}\right)(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}^*\right\} - E_t\{\Delta\widehat{c}_{t+1}\}\right)$$
$$+ \frac{1}{\Upsilon}\left(E_t\{\Delta y_{t+1}\} - E_t\left\{\Delta y_{t+1}^*\right\}\right) - \left(\frac{\vartheta}{\Upsilon}\right)(1-\chi)\left(E_t\{\Delta g_{t+1}\} - E_t\left\{\Delta g_{t+1}^*\right\}\right). \quad \text{(A.50)}$$

A.11.1 Obtaining the IS Curve

Starting with (2.45) and isolating s_t

$$y_t = (1-\vartheta)\left[c_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right] + \vartheta\left[(1-\chi)g_t + \chi g_t^*\right] + \left[(1-\vartheta)\frac{\alpha}{\nu}\omega^C + \vartheta\frac{\chi}{\nu}\omega^G\right] s_t. \quad \text{(A.51)}$$

[41] As in Galí (2008, equation (29) of ch.7)

Solving the last equation w.r.t. c_t, one has

$$c_t = \left(\frac{1}{1-\vartheta}\right) y_t - \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*) - \left(\frac{\vartheta}{1-\vartheta}\right)\left[(1-\chi)g_t + \chi g_t^*\right] - \left(\frac{1}{1-\vartheta}\right)\left[(1-\vartheta)\frac{\alpha}{\nu}\omega^C + \vartheta\frac{\chi}{\nu}\omega^G\right] s_t. \tag{A.52}$$

Making the first-differences in t+1 yields

$$E_t\left\{\Delta c_{t+1}\right\} = \left(\frac{1}{1-\vartheta}\right) E_t\left\{\Delta y_{t+1}\right\} - \left(\frac{\vartheta}{1-\vartheta}\right)\left[(1-\chi)E_t\left\{\Delta g_{t+1}\right\} + \chi E_t\left\{\Delta g_{t+1}^*\right\}\right] - \alpha\left(\frac{\sigma-\nu}{\nu}\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}_{t+1}^*\right\}\right) - \left(\frac{1}{1-\vartheta}\right)\left[(1-\vartheta)\frac{\alpha}{\nu}\omega^C + \vartheta\frac{\chi}{\nu}\omega^G\right] E_t\left\{\Delta s_{t+1}\right\}. \tag{A.53}$$

Recalling the Euler equation from Subsection (2.1) and then substituting this last result, one has

$$E_t\left\{\Delta c_{t+1}\right\} - \left(\frac{\sigma-\nu}{\nu}\right) E_t\left\{\Delta\widehat{c}_{t+1}\right\} = \frac{1}{\nu}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right), \tag{2.12}$$

$$\left(\frac{1}{1-\vartheta}\right) E_t\left\{\Delta y_{t+1}\right\} - \alpha\left(\frac{\sigma-\nu}{\nu}\right)\left(E_t\left\{\Delta\widehat{c}_{t+1}\right\} - E_t\left\{\Delta\widehat{c}_{t+1}^*\right\}\right) - \left(\frac{\vartheta}{1-\vartheta}\right)\left[(1-\chi)E_t\left\{\Delta g_{t+1}\right\} + \chi E_t\left\{\Delta g_{t+1}^*\right\}\right] - \left(\frac{1}{1-\vartheta}\right)\left[(1-\vartheta)\frac{\alpha}{\nu}\omega^C + \vartheta\frac{\chi}{\nu}\omega^G\right] E_t\left\{\Delta s_{t+1}\right\} - \left(\frac{\sigma-\nu}{\nu}\right) E_t\left\{\Delta\widehat{c}_{t+1}\right\} = \frac{1}{\nu}\left(i_t - E_t\left\{\pi_{t+1}^C\right\} - \rho\right). \tag{A.54}$$

Multiply everything by $(1-\vartheta)$

$$E_t\left\{\Delta y_{t+1}\right\}-(1-\vartheta)\left(\frac{\sigma-\nu}{\nu}\right)\left((1+\alpha)E_t\left\{\Delta\widehat{c}_{t+1}\right\}-\alpha E_t\left\{\Delta\widehat{c}^{*}_{t+1}\right\}\right)$$
$$-\vartheta\left[(1-\chi)E_t\left\{\Delta g_{t+1}\right\}+\chi E_t\left\{\Delta g^{*}_{t+1}\right\}\right]-\left[(1-\vartheta)\frac{\alpha}{\nu}\omega^C+\vartheta\frac{\chi}{\nu}\omega^G\right]E_t\left\{\Delta s_{t+1}\right\}$$
$$=\left(\frac{1-\vartheta}{\nu}\right)\left(i_t-E_t\left\{\pi^C_{t+1}\right\}-\rho\right). \tag{A.55}$$

Using equation (2.20) in t+1, substituting above and then isolate Δs_{t+1}

$$E_t\left\{\Delta y_{t+1}\right\}-(1-\vartheta)\left(\frac{\sigma-\nu}{\nu}\right)\left((1+\alpha)E_t\left\{\Delta\widehat{c}_{t+1}\right\}-\alpha E_t\left\{\Delta\widehat{c}^{*}_{t+1}\right\}\right)$$
$$-\vartheta\left[(1-\chi)E_t\left\{\Delta g_{t+1}\right\}+\chi E_t\left\{\Delta g^{*}_{t+1}\right\}\right]-\left[(1-\vartheta)\frac{\alpha}{\nu}(\omega^C-1)+\vartheta\frac{\chi}{\nu}\omega^G\right]E_t\left\{\Delta s_{t+1}\right\}$$
$$=\left(\frac{1-\vartheta}{\nu}\right)\left(i_t-E_t\left\{\pi_{H,t+1}\right\}-\rho\right). \tag{A.56}$$

Substituting equation (A.50) and substituting $\Lambda=\Upsilon-\dfrac{1-\vartheta}{\nu}=\left[(1-\vartheta)\dfrac{\alpha}{\nu}(\omega^C-1)+\vartheta\dfrac{\chi}{\nu}\omega^G\right]$

$$E_t\left\{\Delta y_{t+1}\right\}-(1-\vartheta)\left(\frac{\sigma-\nu}{\nu}\right)\left((1+\alpha)E_t\left\{\Delta\widehat{c}_{t+1}\right\}-\alpha E_t\left\{\Delta\widehat{c}^{*}_{t+1}\right\}\right)$$
$$-\vartheta\left[(1-\chi)E_t\left\{\Delta g_{t+1}\right\}+\chi E_t\left\{\Delta g^{*}_{t+1}\right\}\right]-\frac{\Lambda}{\Upsilon}\left(E_t\left\{\Delta y_{t+1}\right\}-E_t\left\{\Delta y^{*}_{t+1}\right\}\right)$$
$$+(1-\vartheta)\frac{\Lambda}{\Upsilon}(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)\left(E_t\left\{\Delta\widehat{c}^{*}_{t+1}\right\}-E_t\left\{\Delta\widehat{c}_{t+1}\right\}\right)$$
$$+\vartheta\frac{\Lambda}{\Upsilon}(1-\chi)\left(E_t\left\{\Delta g_{t+1}\right\}-E_t\left\{\Delta g^{*}_{t+1}\right\}\right)=\left(\frac{1-\vartheta}{\nu}\right)\left(i_t-E_t\left\{\pi_{H,t+1}\right\}-\rho\right), \tag{A.57}$$

Simplifying the expression above yields

$$\left(1-\frac{\Lambda}{\Upsilon}\right)E_t\left\{\Delta y_{t+1}\right\}-\left[\frac{\Lambda}{\Upsilon}+\left(1-\frac{\Lambda}{\Upsilon}\right)\chi\right]\vartheta E_t\left\{\Delta g^{*}_{t+1}\right\}+\frac{\Lambda}{\Upsilon}E_t\left\{\Delta y^{*}_{t+1}\right\}$$
$$-\left(1-\frac{\Lambda}{\Upsilon}\right)(1-\chi)\vartheta E_t\left\{\Delta g_{t+1}\right\}+\left[\frac{\Lambda}{\Upsilon}+\left(1-\frac{\Lambda}{\Upsilon}\right)\alpha\right]\left(\frac{1-\vartheta}{\nu}\right)(\sigma-\nu)E_t\left\{\Delta\widehat{c}^{*}_{t+1}\right\}$$
$$-\left[\left(1+\frac{\Lambda}{\Upsilon}\right)+\left(1-\frac{\Lambda}{\Upsilon}\right)\alpha\right]\left(\frac{1-\vartheta}{\nu}\right)(\sigma-\nu)E_t\left\{\Delta\widehat{c}_{t+1}\right\}=\left(\frac{1-\vartheta}{\nu}\right)\left(i_t-E_t\left\{\pi_{H,t+1}\right\}-\rho\right). \tag{A.58}$$

Remember that $\Upsilon - \Lambda = \dfrac{1-\vartheta}{\nu}$. Multiplying all the terms in the equation above by $\left(\dfrac{\Upsilon}{\Upsilon - \Lambda}\right)$ and simplifying, yields[42]

$$
\begin{aligned}
& y_t = E_t\{y_{t+1}\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) - \left[\Lambda + \frac{\chi}{\nu}(1-\vartheta)\right]\vartheta\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta g^*_{t+1}\right\} \\
& + \Lambda\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta y^*_{t+1}\right\} - (1-\chi)\vartheta E_t\{\Delta g_{t+1}\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\left\{\Delta\widehat{c}^*_{t+1}\right\} \\
& - \left[\Upsilon + \Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\}. \qquad (2.48)
\end{aligned}
$$

Applying the three rules to check whether it converges to the one in the book, one may confirm that it does!

1. $\vartheta = 0$,
2. $\alpha = \chi$,
3. $\sigma = \nu$.

This implies the following $\Lambda = \dfrac{\alpha\left(\omega^C - 1\right)}{\sigma} = \dfrac{\alpha\Theta}{\sigma}$ and $\Upsilon = \dfrac{1+\alpha\left(\omega^C-1\right)}{\sigma} = \dfrac{1+\alpha\Theta}{\sigma} = \dfrac{1}{\sigma_\alpha}$ Equation 30 of Galí (2008, p.161)

$$
y_t = y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - \pi_{H,t+1} - \rho\right) + \alpha\Theta\Delta y^*_{t+1}
$$

A.11.2 The Trade Balance

Considering that the nominal value of the domestic production is given by the following equations (which is implied by equation (2.42))

$$
P_{H,t}Y_t \equiv P^C_t C_t + P^G_t G_t, \qquad (A.59)
$$

$$
Y_t \equiv \frac{P^C_t}{P_{H,t}}C_t + \frac{P^G_t}{P_{H,t}}G_t. \qquad (A.60)
$$

[42] As in Galí (2008, equation (30) of ch.7)

In words this means that:

Nominal value of domestically produced goods in domestic currency under Producer Currency Pricing

=

Nominal value of domestically consumed goods by households in domestic currency under Producer Currency Pricing

+

Nominal value of domestically consumed goods by the government in domestic currency under Producer Currency Pricing

+

Nominal value of net exports in domestic currency under Producer Currency Pricing.

Naturally the trade balance (or net exports) of the domestic economy is characterized through the following equation

$$NX_t \equiv Y_t - \frac{P_t^C}{P_{H,t}} C_t - \frac{P_t^G}{P_{H,t}} G_t. \tag{A.61}$$

If net exports are expressed in deviations of the steady state output Y it would come as

$$nx_t = \frac{NX_t}{Y} \approx \frac{1}{Y}\left[Y_t - \frac{P_t^C}{P_{H,t}} C_t - \frac{P_t^G}{P_{H,t}} G_t\right],$$

$$nx_t = y_t - (1-\vartheta)\left(c_t + \alpha s_t\right) - \vartheta\left(g_t + \chi s_t\right). \tag{2.49}$$

Remembering equation (2.45) and then combining it the (2.49) yields

$$y_t = (1-\vartheta)\left[c_t + \frac{\alpha}{\nu}\omega^C s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)\left(\widehat{c}_t - \widehat{c}_t^*\right)\right] + \vartheta\left[(1-\chi)g_t + \frac{\chi}{\nu}\omega^G s_t + \chi g_t^*\right], \tag{2.45}$$

Finally, the trade balance comes as follows[43]

$$nx_t = (1-\vartheta)\left[\alpha\left(\frac{\omega^C}{\nu}-1\right)s_t + \alpha\left(\frac{\sigma-\nu}{\nu}\right)(\widehat{c}_t - \widehat{c}_t^*)\right] + \vartheta\left[\chi\left(\frac{\omega^G}{\nu}-1\right)s_t + \chi\left(g_t^* - g_t\right)\right]. \tag{2.50}$$

A.11.3 From Firms Section

Starting with the linearised marginal cost derived above

$$mc_t = -\delta + w_t - p_{H,t} - a_t, \tag{2.37}$$

$$mc_t = -\delta + \left(w_t - p_t^C\right) + \left(p_t^C - p_{H,t}\right) - a_t. \tag{A.62}$$

Using equation (2.9) to substitute $\left(w_t - p_t^C\right)$ and equation (2.19) to substitute $\left(p_t^C - p_{H,t}\right)$

$$mc_t = -\delta + \varphi n_t + \nu c_t + (\sigma-\nu)\,\widehat{c}_t + \alpha s_t - a_t. \tag{A.63}$$

Using equation (2.35) to substitute c_t and also the linearised production function to substitute n_t one has[44]

$$mc_t = -\delta + \varphi y_t + \nu c_t^* + (\sigma-\nu)\,\widehat{c}_t^* + s_t - (1+\varphi)\,a_t. \tag{2.51}$$

Using equation (A.49) to substitute s_t

$$\begin{aligned} mc_t = {} & -\delta + \varphi y_t + \nu c_t^* + (\sigma-\nu)\,\widehat{c}_t^* - (1+\varphi)\,a_t + \frac{1}{\Upsilon}\left(y_t - y_t^*\right) \\ & - \left(\frac{1-\vartheta}{\Upsilon}\right)(1-\alpha)\left(\frac{\sigma-\nu}{\nu}\right)\left(\widehat{c}_t^* - \widehat{c}_t\right) - \left(\frac{\vartheta}{\Upsilon}\right)(1-\chi)\left(g_t - g_t^*\right). \end{aligned} \tag{A.64}$$

[43]As in Galí (2008, equation (31) of ch.7)
[44]As in Galí (2008, equation (34) of ch.7)

Simplifying one has[45]

$$mc_t = -\delta + \left(\frac{\Upsilon\varphi + 1}{\Upsilon}\right) y_t - \frac{1}{\Upsilon} y_t^* + \nu c_t^* + \left(\Lambda + \alpha\left(\frac{1-\vartheta}{\nu}\right)\right)\frac{1}{\Upsilon}\left(\sigma - \nu\right)\widehat{c}_t^* - \left(1+\varphi\right) a_t$$
$$+ \left(1-\alpha\right)\left(\frac{1-\vartheta}{\nu}\right)\frac{1}{\Upsilon}\left(\sigma-\nu\right)\widehat{c}_t - \vartheta(1-\chi)\frac{1}{\Upsilon} g_t + \vartheta(1-\chi)\frac{1}{\Upsilon} g_t^*. \tag{2.52}$$

To obtain y_t^n one has to substitute $mc_t = -\mu$ in equation (2.52) and solve w.r.t. y_t^n

$$\left(\frac{\Upsilon\varphi + 1}{\Upsilon}\right) y_t^n = \delta - \mu + \frac{1}{\Upsilon} y_t^* - \nu c_t^* - \left(\Lambda + \alpha\left(\frac{1-\vartheta}{\nu}\right)\right)\frac{1}{\Upsilon}\left(\sigma - \nu\right)\widehat{c}_t^* + \left(1+\varphi\right) a_t$$
$$- \left(1-\alpha\right)\left(\frac{1-\vartheta}{\nu}\right)\frac{1}{\Upsilon}\left(\sigma-\nu\right)\widehat{c}_t + \vartheta(1-\chi)\frac{1}{\Upsilon} g_t - \vartheta(1-\chi)\frac{1}{\Upsilon} g_t^*. \tag{A.65}$$

Isolating y_t^n

$$y_t^n = \left[\frac{\Upsilon(\delta-\mu)}{\Upsilon\varphi+1}\right] + \left[\frac{1}{\Upsilon\varphi+1}\right] y_t^* - \left[\frac{\Upsilon\nu}{\Upsilon\varphi+1}\right] c_t^* - \left[\Lambda + \alpha\left(\frac{1-\vartheta}{\nu}\right)\right]\left(\frac{1}{\Upsilon\varphi+1}\right)\left(\sigma-\nu\right)\widehat{c}_t^*$$
$$- \left(\frac{1-\vartheta}{\nu}\right)\left[\frac{1-\alpha}{\Upsilon\varphi+1}\right]\left(\sigma-\nu\right)\widehat{c}_t + \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right] g_t - \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right] g_t^* + \left[\frac{\Upsilon\left(1+\varphi\right)}{\Upsilon\varphi+1}\right] a_t. \tag{A.66}$$

Simplifying, yields the following[46]

$$y_t^n = \Gamma_0 + \Gamma_{y^*} y_t^* + \Gamma_{c^*} c_t^* + \Gamma_{\widehat{c}^*}\widehat{c}_t^* + \Gamma_{\widehat{c}}\widehat{c}_t + \Gamma_g g_t + \Gamma_{g^*} g_t^* + \Gamma_a a_t, \tag{2.53}$$

where $\Gamma_0 \equiv \left[\frac{\Upsilon\left(\delta-\mu\right)}{\Upsilon\varphi+1}\right]$, $\Gamma_{y^*} \equiv \left[\frac{1}{\Upsilon\varphi+1}\right]$, $\Gamma_{c^*} \equiv -\left[\frac{\Upsilon\nu}{\Upsilon\varphi+1}\right]$,
$\Gamma_{\widehat{c}^*} \equiv -\left[\Lambda + \alpha\left(\frac{1-\vartheta}{\nu}\right)\right]\left(\frac{1}{\Upsilon\varphi+1}\right)(\sigma-\nu)$, $\Gamma_{\widehat{c}} \equiv -\left(\frac{1-\vartheta}{\nu}\right)\left[\frac{1-\alpha}{\Upsilon\varphi+1}\right](\sigma-\nu)$,
$\Gamma_g \equiv \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right]$, $\Gamma_{g^*} \equiv -\left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right]$, $\Gamma_a \equiv \left[\frac{\Upsilon\left(1+\varphi\right)}{\Upsilon\varphi+1}\right]$.

The output gap is given by the following equation

$$\tilde{y}_t \equiv y_t - y_t^n. \tag{2.54}$$

[45]As in Galí (2008, equation (35) of ch.7)
[46]As in Galí (2008, equation (36) of ch.7)

Using equation (2.52) and substituting $y_t = \tilde{y}_t + y_t^n$, one has the following

$$mc_t = -\delta + \left[\frac{\Upsilon\varphi + 1}{\Upsilon}\right](\tilde{y}_t + y_t^n) - \frac{1}{\Upsilon}y_t^* + \nu c_t^* + \left[1 - (1-\alpha)\left(\frac{1-\vartheta}{\nu}\right)\frac{1}{\Upsilon}\right](\sigma - \nu)\,\widehat{c}_t^*$$
$$- (1+\varphi)\,a_t + (1-\alpha)\left(\frac{1-\vartheta}{\nu}\right)\frac{1}{\Upsilon}(\sigma-\nu)\,\widehat{c}_t - \vartheta(1-\chi)\frac{1}{\Upsilon}g_t + \vartheta(1-\chi)\frac{1}{\Upsilon}g_t^*. \qquad \text{(A.67)}$$

Making use of (2.53), substituting and simplifying yields the domestic real marginal cost

$$\widehat{mc}_t = \left(\varphi + \frac{1}{\Upsilon}\right)\tilde{y}_t. \qquad \text{(A.68)}$$

Substituting the expression above into (2.39)[47]

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \kappa_\alpha \tilde{y}_t, \qquad (2.56)$$

where $\kappa_\alpha \equiv \lambda\left(\varphi + \dfrac{1}{\Upsilon}\right)$.

A.11.4 Dynamic IS Equation

Starting with (2.48), substituting (2.54), one has

$$\tilde{y}_t + y_t^n = E_t\{\tilde{y}_{t+1}\} + E_t\left\{y_{t+1}^n\right\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \Lambda\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta y_{t+1}^*\right\}$$
$$- (1-\chi)\vartheta E_t\{\Delta g_{t+1}\} - \left[\Lambda + \frac{\chi}{\nu}(1-\vartheta)\right]\vartheta\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta g_{t+1}^*\right\}$$
$$+ \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}^*\right\} - \left[\Upsilon + \Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}\right\}. \qquad \text{(A.69)}$$

[47] As in Galí (2008, equation (37) of ch.7)

Substituting the first differences in t+1 of equation (2.53), yields

$$\begin{aligned}
\tilde{y}_t &= E_t\{\tilde{y}_{t+1}\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \Lambda\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta y^*_{t+1}\right\} + \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right]E_t\{\Delta g_{t+1}\} \\
&+ \left[\frac{\Upsilon(1+\varphi)}{\Upsilon\varphi+1}\right]E_t\{\Delta a_{t+1}\} - \left[\Lambda + \frac{\chi}{\nu}(1-\vartheta)\right]\vartheta\left(\frac{\nu}{1-\vartheta}\right)E_t\left\{\Delta g^*_{t+1}\right\} - (1-\chi)\vartheta E_t\{\Delta g_{t+1}\} \\
&- \left[\frac{\vartheta(1-\chi)}{\Upsilon\varphi+1}\right]E_t\left\{\Delta g^*_{t+1}\right\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\left\{\Delta\widehat{c}^*_{t+1}\right\} + \left[\frac{1}{\Upsilon\varphi+1}\right]E_t\left\{\Delta y^*_{t+1}\right\} \\
&- \left[\Upsilon - (1-\alpha)\left(\frac{1-\vartheta}{\nu}\right)\right]\left(\frac{1}{\Upsilon\varphi+1}\right)(\sigma-\nu)E_t\left\{\Delta\widehat{c}^*_{t+1}\right\} - \left[\frac{\Upsilon\nu}{\Upsilon\varphi+1}\right]E_t\left\{\Delta c^*_{t+1}\right\} \\
&- \left[\Upsilon + \Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\} - \left(\frac{1-\vartheta}{\nu}\right)\left[\frac{1-\alpha}{\Upsilon\varphi+1}\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\}.
\end{aligned} \tag{A.70}$$

Simplifying one has

$$\begin{aligned}
\tilde{y}_t &= E_t\{\tilde{y}_{t+1}\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - \rho\right) + \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \frac{1}{\Upsilon\varphi+1}\right]E_t\left\{\Delta y^*_{t+1}\right\} \\
&- \left[\frac{\Upsilon\varphi}{\Upsilon\varphi+1}\right](1-\chi)\vartheta E_t\{\Delta g_{t+1}\} - \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \chi + \frac{1-\chi}{\Upsilon\varphi+1}\right]\vartheta E_t\left\{\Delta g^*_{t+1}\right\} \\
&- \left[\Upsilon + \Lambda + \left(\frac{1-\vartheta}{\nu}\right)\left(\frac{\alpha\Upsilon\varphi+1}{\Upsilon\varphi+1}\right)\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\} - \left[\frac{\Upsilon\nu}{\Upsilon\varphi+1}\right]E_t\left\{\Delta c^*_{t+1}\right\} \\
&+ \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right]\left(\frac{\Upsilon\varphi}{\Upsilon\varphi+1}\right)(\sigma-\nu)E_t\left\{\Delta\widehat{c}^*_{t+1}\right\} + \left[\frac{\Upsilon(1+\varphi)}{\Upsilon\varphi+1}\right]E_t\{\Delta a_{t+1}\}.
\end{aligned} \tag{A.71}$$

The dynamic IS equation is given by[48]

$$\begin{aligned}
\tilde{y}_t &= E_t\{\tilde{y}_{t+1}\} - \Upsilon\left(i_t - E_t\{\pi_{H,t+1}\} - r^n_t\right) \\
&- \left[\Upsilon + \Lambda + \left(\frac{1-\vartheta}{\nu}\right)\left(\frac{\alpha\Upsilon\varphi+1}{\Upsilon\varphi+1}\right)\right](\sigma-\nu)E_t\{\Delta\widehat{c}_{t+1}\},
\end{aligned} \tag{2.57}$$

[48] As in Galí (2008, equation (38) of ch.7)

where r_t^n is the domestic economy Wicksellian interest rate which has the following format[49]

$$
\begin{aligned}
r_t^n \equiv\ & \rho + \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \frac{1}{\Upsilon\varphi+1}\right]\frac{1}{\Upsilon}E_t\left\{\Delta y_{t+1}^*\right\} - \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \chi + \frac{1-\chi}{\Upsilon\varphi+1}\right]\vartheta\frac{1}{\Upsilon}E_t\left\{\Delta g_{t+1}^*\right\} \\
& - \left[\frac{\varphi}{\Upsilon\varphi+1}\right](1-\chi)\vartheta E_t\left\{\Delta g_{t+1}\right\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right]\left(\frac{\varphi}{\Upsilon\varphi+1}\right)(\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}^*\right\} \\
& - \left[\frac{\nu}{\Upsilon\varphi+1}\right]E_t\left\{\Delta c_{t+1}^*\right\} - \left[\frac{(1+\varphi)}{\Upsilon\varphi+1}\right](1-\rho_a)\,a_t.
\end{aligned} \tag{2.58}
$$

[49] As in Galí (2008, equation (39) of ch.7)

B Appendix 2

B.1 Model Equations

This version of the model assumes: $\alpha \neq \chi$, $\vartheta > 0$ and $\sigma < \nu$. In other words this means that the Galí (2008) model is augmented with government expenditures in a non-separable way with regard to private consumption and have asymmetric propensities to import. Moreover, private consumption and public spending are modelled as complements because $\sigma < \nu$.

Dynamic IS Curve

$$\begin{aligned}\tilde{y}_t &= E_t\left\{\tilde{y}_{t+1}\right\} - \Upsilon\left(i_t - E_t\left\{\pi_{H,t+1}\right\} - r_t^n\right)\\ &- \left[\Upsilon + \Lambda + \left(\frac{1-\vartheta}{\nu}\right)\left(\frac{\alpha\Upsilon\varphi+1}{\Upsilon\varphi+1}\right)\right](\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}\right\}\end{aligned} \tag{2.57}$$

Linearised Effective Consumption

$$\widehat{c}_t \equiv \begin{cases}(1-\vartheta)\left(\frac{C}{\widehat{C}}\right)^{1-\nu} c_t + \vartheta\left(\frac{G}{\widehat{C}}\right)^{1-\nu} g_t & \text{for } \nu \neq 1\\ (1-\vartheta)c_t + \vartheta g_t & \text{for } \nu = 1,\end{cases} \tag{2.3}$$

"Euler equation" - Consumption combining equations (2.47)**,** (2.35) **and** (2.46)

$$\begin{aligned}c_t &= \left[\left(\frac{1}{1-\vartheta}\right) - \frac{1}{\Upsilon}\left(\frac{1-\alpha}{\nu}\right)\right] y_t^* + \frac{1}{\Upsilon}\left(\frac{1-\alpha}{\nu}\right) y_t - \frac{1}{\Upsilon}\left(\frac{1-\alpha}{\nu}\right)\vartheta\,(1-\chi)\,g_t\\ &+ \left[(1-\chi)\frac{1}{\Upsilon}\left(\frac{1-\alpha}{\nu}\right) - \left(\frac{1}{1-\vartheta}\right)\right]\vartheta g_t^* + \left[1 - \frac{1}{\Upsilon}\left(\frac{1-\alpha}{\nu}\right)(1-\vartheta)(1-\alpha)\right]\left(\frac{\sigma-\nu}{\nu}\right)\left(\widehat{c}_t^* - \widehat{c}_t\right)\end{aligned} \tag{B.1}$$

Natural Rate of Interest

$$\begin{aligned}r_t^n &\equiv \rho + \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \frac{1}{\Upsilon\varphi+1}\right]\frac{1}{\Upsilon}E_t\left\{\Delta y_{t+1}^*\right\} - \left[\Lambda\left(\frac{\nu}{1-\vartheta}\right) + \chi + \frac{1-\chi}{\Upsilon\varphi+1}\right]\vartheta\frac{1}{\Upsilon}E_t\left\{\Delta g_{t+1}^*\right\}\\ &- \left[\frac{\varphi}{\Upsilon\varphi+1}\right](1-\chi)\vartheta E_t\left\{\Delta g_{t+1}\right\} + \left[\Lambda + \frac{\alpha}{\nu}(1-\vartheta)\right]\left(\frac{\varphi}{\Upsilon\varphi+1}\right)(\sigma-\nu)\,E_t\left\{\Delta\widehat{c}_{t+1}^*\right\}\\ &- \left[\frac{\nu}{\Upsilon\varphi+1}\right]E_t\left\{\Delta c_{t+1}^*\right\} - \left[\frac{(1+\varphi)}{\Upsilon\varphi+1}\right](1-\rho_a)\,a_t\end{aligned} \tag{2.58}$$

New Keynesian Phillips Curve

$$\pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + \kappa_\alpha \tilde{y}_t \tag{2.56}$$

Natural Level of Output

$$y_t^n = \Gamma_0 + \Gamma_{y^*} y_t^* + \Gamma_{c^*} c_t^* + \Gamma_{\widehat{c}^*} \widehat{c}_t^* + \Gamma_{\widehat{c}} \widehat{c}_t + \Gamma_g g_t + \Gamma_{g^*} g_t^* + \Gamma_a a_t, \tag{2.53}$$

where $\Gamma_0 \equiv \left[\dfrac{\Upsilon (\delta - \mu)}{\Upsilon \varphi + 1} \right]$, $\Gamma_{y^*} \equiv \left[\dfrac{1}{\Upsilon \varphi + 1} \right]$, $\Gamma_{c^*} \equiv - \left[\dfrac{\Upsilon \nu}{\Upsilon \varphi + 1} \right]$,
$\Gamma_{\widehat{c}^*} \equiv - \left[\Lambda + \alpha \left(\dfrac{1 - \vartheta}{\nu} \right) \right] \left(\dfrac{1}{\Upsilon \varphi + 1} \right) (\sigma - \nu)$, $\Gamma_{\widehat{c}} \equiv - \left(\dfrac{1 - \vartheta}{\nu} \right) \left[\dfrac{1 - \alpha}{\Upsilon \varphi + 1} \right] (\sigma - \nu)$,
$\Gamma_g \equiv \left[\dfrac{\vartheta (1 - \chi)}{\Upsilon \varphi + 1} \right]$, $\Gamma_{g^*} \equiv - \left[\dfrac{\vartheta (1 - \chi)}{\Upsilon \varphi + 1} \right]$, $\Gamma_a \equiv \left[\dfrac{\Upsilon (1 + \varphi)}{\Upsilon \varphi + 1} \right]$.

Domestic Output

$$\tilde{y}_t \equiv y_t - y_t^n \tag{2.54}$$

Market Clearing

$$y_t = y_t^* + (1 - \vartheta)(1 - \alpha) \left(\frac{\sigma - \nu}{\nu} \right) (\widehat{c}_t^* - \widehat{c}_t) + \vartheta \left[(1 - \chi)(g_t - g_t^*) \right] + \Upsilon s_t \tag{2.47}$$

Technology Shock and Foreign Economy Shock (as in Galí (2008, p.174))

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a + correl * \varepsilon_t^{c^*} \\ c_t^* &= \rho_{c^*} c_{t-1}^* + \varepsilon_t^{c^*} \end{aligned}$$

Government Spending Shock and World Government Spending Shock

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g, \qquad \text{where } \varepsilon_t^g \sim \mathcal{N}(0, \sigma^2_{\varepsilon_t^g}) \tag{2.15}$$

$$g_t^* = \rho_{g^*} g_{t-1}^* + \varepsilon_t^{g^*}, \qquad \text{where } \varepsilon_t^{g^*} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_t^{g^*}}) \tag{2.16}$$

World Economy

$$\pi_t^* = \beta E_t \left\{ \pi_{t+1}^* \right\} + \kappa_\alpha y_t^* \tag{B.2}$$

World Output

$$y_t^* = (1-\vartheta) c_t^* + \vartheta g_t^* \tag{2.46}$$

World Effective Consumption

$$\widehat{c}_t^* = (1-\vartheta) \left(\frac{C}{\widehat{C}} \right)^{1-\nu} c_t^* + \vartheta \left(\frac{G}{\widehat{C}} \right)^{1-\nu} g_t^* \tag{B.3}$$

CPI Inflation

$$\pi_t^C = p_t^C - p_{t-1}^C \tag{2.20}$$

Domestic Price Level

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} \tag{B.4}$$

Consumption Price Index (CPI)

$$\pi_t^C = \pi_{H,t} + \alpha \Delta s_t \tag{2.19}$$

Terms of Trade

$$s_t = e_t + p_t^* - p_{H,t} \tag{2.23}$$

$$\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t} \tag{B.5}$$

Net Exports

$$nx_t = (1-\vartheta) \left[\alpha \left(\frac{\omega^C}{\nu} - 1 \right) s_t + \alpha \left(\frac{\sigma - \nu}{\nu} \right) \left(\widehat{c}_t - \widehat{c}_t^* \right) \right] + \vartheta \left[\chi \left(\frac{\omega^G}{\nu} - 1 \right) s_t + \chi \left(g_t^* - g_t \right) \right] \tag{2.50}$$

Labour Supply

$$n_t = y_t - a_t + \tag{B.6}$$

Real Wage

$$w_t - p_t^C = \varphi n_t + \sigma c_t + (\nu - \sigma)(c_t - \widehat{c}_t) \tag{2.10}$$

(Fixed) Exchange Rate

$$e_t = 0$$

Domestic Inflation Taylor Rule

$$i_t = \rho_i i_{t-1} + \phi_\pi \pi_{H,t} \tag{3.1}$$

B.2 Adjustment in Parameter ν

In Bouakez & Rebei (2007) effective consumption is represented as follows

$$\widehat{C}_t \equiv \left[(1-\vartheta) C_t^{\frac{\nu_B - 1}{\nu_B}} + \vartheta G_t^{\frac{\nu_B - 1}{\nu_B}}\right]^{\frac{\nu_B}{\nu_B - 1}}. \tag{B.7}$$

The authors estimated $\nu_B = 0.3320$. However, the effective consumption used in this model is slightly different from the above presented. Recalling equation (2.2) representing effective consumption

$$\widehat{C}_t \equiv \left[(1-\vartheta) C_t^{1-\nu} + \vartheta G_t^{1-\nu}\right]^{\frac{1}{1-\nu}}. \tag{2.2}$$

As the exponents differ, the parameter ν_B need to be translated to the notation used in this model which is done by equalling exponents, which if solved yields $\dfrac{1}{\nu_B} = \nu$. If $\nu_B = 0.3320$ is replaced, one has finds that $\nu \simeq 3$.

B.3 MATLAB Code

```
// This file aims at simulating the model of
//"Fiscal Policy and Non-separable Preferences in a Small Open Economy Model"
// Author: Nuno M. Castanheira - ISCTE-IUL
// Last revised 22.10.2014

var
ygap rn piH yn r y erate pH s pistar a ystar c nx n rw  gstar   cstar
chat      g       chatstar piC pC;

varexo
varepsilonCstar varepsilonA varepsilonG varepsilonGstar;

parameters
alpha  beta   varphi    vartheta   chi    nu  eta    gamma
theta      sigma  C_Chat  G_Chat
varepsilon  rho omegaC     omegaG     Upsilon    Lambda  lambda
kappaAL     mu  delta   Gamma0 GammaYstar GammaCstar GammaChatstar
GammaChat
GammaG GammaGstar GammaA    rhoG
rhoGstar    rhoA rhoCstar    rho_r      phi_pi;

//DEEP PARAMETERS
alpha =             0.25    ;
chi =               0.10    ;
beta =              0.99    ;
vartheta =          0.20    ;
varphi =            2.00    ;
sigma =             2.00    ;
nu =                3.00    ;
eta =               1.00    ;
gamma =             1.00    ;
theta =             0.75    ;
C_Chat =            0.80    ;
G_Chat =            0.20    ;
varepsilon =        6.00    ;
rho_r =             0.90    ;
phi_pi =            1.50    ;
```

```
//CALCULATIONS OF PARAMETERS
rho = - log(beta);
omegaC = gamma * nu + (eta * nu-1) * (1 - alpha) ;
omegaG =  gamma * nu + eta * nu * (1 - chi);
Upsilon = (1 - vartheta) *  alpha * (omegaC-1) / nu + vartheta * chi * omegaG /nu +
(1 - vartheta) / nu;
Lambda = (1 - vartheta) *  alpha * (omegaC-1) / nu + vartheta * chi * omegaG /nu;
lambda = (1 - beta * theta) * (1 - theta) / theta;
kappaAL = lambda * (varphi + (1 / Upsilon));
mu = log ((varepsilon)/(varepsilon - 1));
delta = mu + log(1 - alpha);
Gamma0 = (Upsilon * (delta - mu)) / (Upsilon * varphi + 1);
GammaYstar = (1/(Upsilon*varphi+1));
GammaCstar = - (Upsilon * nu) / (Upsilon * varphi + 1);
GammaChatstar = - (Lambda+ alpha * ((1-
vartheta)/nu))*(1/(Upsilon*varphi+1))*(sigma-nu);
GammaChat = - (((1 - vartheta)/nu)*((1-alpha)/(Upsilon*varphi+1)))* (sigma - nu);
GammaG = (vartheta * (1-chi)) / (Upsilon * varphi + 1);
GammaGstar = - (vartheta * (1-chi)) / (Upsilon * varphi + 1);
GammaA = (Upsilon * (1 + varphi)) / (Upsilon * varphi + 1);

//PERSISTANCE OF SHOCKS
rhoA        = 0.66;
rhoCstar    = 0.86;
rhoG        = 0.9;
rhoGstar    = 0.9;

model(linear);
//Dynamic IS Curve
ygap = ygap(+1) - Upsilon*(r - piH(+1) - rn) - (Upsilon + Lambda
+ ((1 - vartheta)/nu)*((alpha*Upsilon*varphi+1)/(Upsilon*varphi+1))) * (sigma-
nu)*(chat(+1)-chat);

//Linearisation of Aggregate Consumption
chat = (1-vartheta)*(C_Chat) ^(1-nu) *c + vartheta*(G_Chat) ^(1-nu) *g;
```

```
//Consumption equation - combining (29), (19) and (28)
c = ((1/(1-vartheta))-(1/Upsilon)*((1-alpha)/nu))*ystar + (1/Upsilon) * ((1-
alpha)/nu)*y
-(1/Upsilon)*((1-alpha)/nu)*vartheta*(1-chi)*g
+((1-chi)*(1/Upsilon)*((1-alpha)/nu)-(1/(1-vartheta)))*vartheta*gstar
+(1-(1/Upsilon)*((1-alpha)/nu)*(1-vartheta)*(1-alpha))*((sigma-nu)/nu)*(chatstar-
chat);

//Natural Rate of Interest - no rho because it is variation!
rn = (Lambda*nu/(1-vartheta)+1/(Upsilon*varphi +1))*(1/Upsilon)*(ystar(+1)-
ystar)
-(Lambda*nu/(1-vartheta)+chi+(1-chi)/(Upsilon*varphi+1))*vartheta*(1/Upsilon)*
(gstar(+1) - gstar)
-(varphi/(Upsilon*varphi+1))*(1-chi)*vartheta* (g(+1)-g)
+(Lambda+alpha/nu*(1-vartheta))*(varphi/(Upsilon*varphi+1))*(sigma-
nu)*(chatstar(+1) - chatstar)
-(nu/(Upsilon*varphi+1))*(cstar(+1)-cstar)-((1+varphi)/(Upsilon*varphi+1))* (1 -
rhoA) * a;

//New Keynesian Phillips Curve
piH = beta*piH(+1) + kappaAL * ygap;

//Natural level of output in log deviations - no Gamma0 because it is variations
yn = GammaYstar*ystar + GammaCstar * cstar + GammaChatstar * chatstar
+GammaChat*chat + GammaG * g + GammaGstar * gstar + GammaA * a;

//Domestic Output
y = ygap + yn;

//Market Clearing
y = ystar + (1-vartheta)*((1-alpha)*(sigma-nu)/nu)*(chatstar-chat)
+vartheta * ((1-chi)*(g-gstar)) + Upsilon*s;

//SHOCKS TO  DOMESTIC ECONOMY
//Technological Shock and Foreign Economy Shock
a = rhoA * a(-1) + varepsilonA;
g = rhoG*g(-1) + varepsilonG;
```

```
//WORLD ECONOMY
//pistar = beta * pistar(+1) + kappaAL * ystar;
pistar = 0;
cstar = rhoCstar * cstar(-1) + varepsilonCstar;

//World output (y*)
ystar = (1-vartheta) * cstar + vartheta * gstar;

//World Effective Consumption
chatstar = (1-vartheta)*(C_Chat)^(1-nu) * cstar + vartheta*(G_Chat)^(1-nu)
*gstar;

//Government Spending Shock and World Government Spending Shock
gstar = rhoGstar*gstar(-1) + varepsilonGstar;

//PRICES
//CPI inflation
piC = pC - pC(-1);

//Domestic Price Level
piH = pH - pH(-1);

//Government Price Index (CPI)
piC = piH + alpha * (s - s(-1));

//Variation in the Terms of Trade
s - s(-1) = erate - erate(-1) + pistar - piH;

//Trade Balance
nx = (1-vartheta)*alpha*((omegaC/nu-1)*s+((sigma-nu)/nu)*(chat-
chatstar))+vartheta*chi*((omegaG/nu-1)*s+(gstar-g));

//Employment
n = y - a;

//Real Wage
rw = varphi*n + nu*c + (sigma-nu)*chat;
```

```
//EXCHANGE RATE REGIME
//Fixed Exchange rate peg
erate = 0;

// Flexible Exchange rate regime - Taylor Rule
//r = rho_r * r(-1) + phi_pi * piH;

end;

//steady;
check;

//SIZE OF SHOCKS
shocks;
var varepsilonG = 1^2;
var varepsilonGstar = 1^2;
var varepsilonA = 0.0071^2;
var varepsilonCstar = 0.0078^2;
//var varepsilonA,varepsilonCstar = 0.3 * 0.0071 * 0.0078;
end;

stoch_simul (irf = 21, periods= 2100, nograph);

// END OF DYNARE CODE
```